Electromagnetics & Anisotropy

Half-Waveplates, Quarter-Waveplates, and Spatial Light Modulators

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April 27, 2023

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The Displacement Field

The electric displacement field $\mathbf{D}$ (with units $\mathrm{C} \cdot \mathrm{m}^{-2}$) is defined as

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon \mathbf{E} \tag{1}$$

where $\mathbf{P}$ is the polarization of the material, and the red part of the equation is true given an isotropic dielectric [1].

The perpendicular and parallel boundary conditions (BC) for the displacement field on an interface are

$$(\mathbf{D}_2 - \mathbf{D}_1) \cdot \hat{\mathbf{n}}_{12} = \sigma_f \quad \text{and} \quad (\mathbf{D}_2 - \mathbf{D}_1) \times \hat{\mathbf{n}}_{12} = (\mathbf{P}_2 - \mathbf{P}_1) \times \hat{\mathbf{n}}_{12} \tag{2}$$

where σ_f is the free (unbounded) surface charge density, and $\hat{\mathbf{n}}_{12}$ is the normal vector of the interface from medium 1 to medium 2.

The Displacement Field & Anisotropy

What is Anisotropy?

Materials are *Anisotropic* if their properties have **directional dependence**. Specifically for us: a material's permittivity could take two different values depending on if the field is in the $\hat{\mathrm{x}}$ or $\hat{\mathrm{y}}$ directions [2].

For anisotropic media, we use the relative permittivity tensor (rank 2), which must be a Hermitian matrix [3, 4].

$$\bar{\epsilon}_r = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \quad \text{where} \quad \epsilon_{ij} = \epsilon_{ji}^* \tag{3}$$

Equation 1 becomes

$$\mathbf{D} = \epsilon_0 \bar{\epsilon}_r \mathbf{E} \tag{4}$$

Uniaxial Media

Calcite, mica, and quartz are examples of *uniaxial media*, where the permittivity tensor takes the form

$$\bar{\epsilon}_r = \begin{bmatrix} \epsilon^e & 0 & 0 \\ 0 & \epsilon^o & 0 \\ 0 & 0 & \epsilon^o \end{bmatrix} \tag{5}$$

The x-axis is the *extraordinary* axis, and y/z-axes are the *ordinary* axes [4].

Wave Propagation

We see that the permittivity in uniaxial media depends on the direction of the fields. Furthermore, we know the propagation constant depends on the permittivity, and we will see that **waves propagate at different speeds depending on their polarization**.

Half-Wave Plates

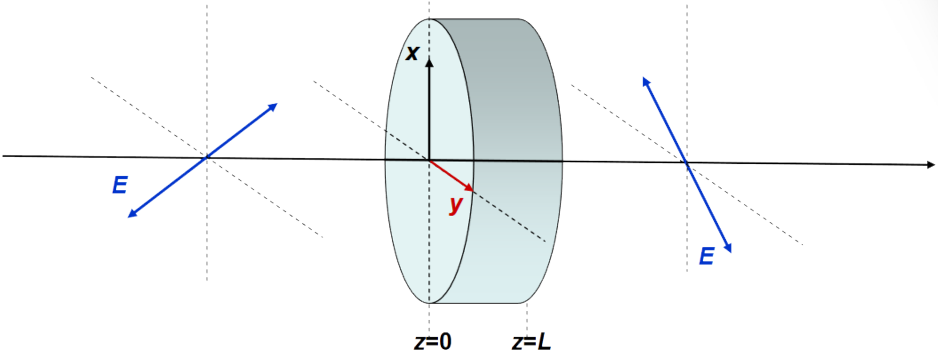

Figure 1: A half-wave plate (HWP) in the xy-plane with length L with an incident plane wave at $z = 0$ [4]. HWPs rotate the polarization of an incident plane wave $90°$. We choose $z < 0$ to be region 1, $0 < z < L$ to be region 2, and $z > L$ to be region 3.

The Wave Equation

Using Faraday's Law, Ampere's Law, and Equation 4 we obtain the wave equation [4, 2]

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) = -j\omega\mu_0 \nabla \times \mathbf{H}(\mathbf{r}) = \omega^2 \mu_0 \mathbf{D}(\mathbf{r}) \tag{6}$$

which becomes

$$\nabla^2 \mathbf{E}(\mathbf{r}) = -\omega^2 \mu_0 \mathbf{D}(\mathbf{r}) = -\omega^2 \mu_0 \epsilon_0 \bar{\epsilon}_r \mathbf{E}(\mathbf{r}) \tag{7}$$

For normal incidence, a wave polarized in $\hat{\mathbf{x}}$ stays in $\hat{\mathbf{x}}$ in the dielectric

$$\mathbf{E}_1(\mathbf{r}) = \hat{\mathbf{x}} E_0 e^{-j\beta z} \quad \Rightarrow \quad \mathbf{E}_2(\mathbf{r}) = \hat{\mathbf{x}} T_{12}^e E_0 e^{-j\beta^e z} \tag{8}$$

and $\hat{\mathbf{y}}$ stays in $\hat{\mathbf{y}}$

$$\mathbf{E}_1(\mathbf{r}) = \hat{\mathbf{y}} E_0 e^{-j\beta z} \quad \Rightarrow \quad \mathbf{E}_2(\mathbf{r}) = \hat{\mathbf{y}} T_{12}^o E_0 e^{-j\beta^o z} \tag{9}$$

The Half-Wave Plate

How do they work?
If for $\hat{\mathrm{x}}$ and $\hat{\mathrm{y}}$ incident polarization, the polarization inside the dielectric does not change, how does a HWP cause the polarization to change?

Consider an incident wave LP in $\hat{\mathrm{x}}$ and $\hat{\mathrm{y}}$ [4]

$$\mathbf{E}_1(\mathbf{r}) = \left(\frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{2}}\right) E_0 e^{-j\beta z} \Rightarrow \mathbf{E}_2(\mathbf{r}) = \frac{E_0}{\sqrt{2}}\left(\hat{\mathbf{x}} T_{12}^{e} e^{-j\beta^{e} z} + \hat{\mathbf{y}} T_{12}^{o} e^{-j\beta^{o} z}\right) \quad (10)$$

where

$$\beta^{e} = \omega\sqrt{\mu_0 \epsilon^{e}} \quad \text{and} \quad \beta^{o} = \omega\sqrt{\mu_0 \epsilon^{o}} \quad (11)$$

The field at $z = L$ becomes

$$\mathbf{E}_2(\mathbf{r})\big|_{z=L} = \frac{E_0}{\sqrt{2}}\left(\hat{\mathbf{x}} T^{e} e^{-j\beta^{e} L} + \hat{\mathbf{y}} T^{o} e^{-j\beta^{o} L}\right) \quad (12)$$

The Half-Wave Plate Cont.

For 90° polarization change we want to flip $\hat{\mathrm{x}}$ or $\hat{\mathrm{y}}$, so we choose

$$L(\beta^o - \beta^e) = (2m+1)\pi \quad \text{where} \quad m \in \mathbb{Z} \tag{13}$$

such that equation 12 becomes [4]

$$\mathbf{E}_2(\mathbf{r})\big|_{z=L} = \frac{E_0}{\sqrt{2}} \left(\hat{\mathbf{x}} T^e e^{-j\beta^e L} + \hat{\mathbf{y}} T^o e^{-j\beta^o L} \right) \tag{14}$$

Citations

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Funding: This research is currently unfunded.
Acknowledgments: The author thanks T. Saule for his helpful contributions.
Disclosures: The authors declare no conflicts of interest.

Data Availability: Code for this project can be accessed on the author's github account. All of the current relevant data is shown in this report. Once a viable focus is achieved, the data will also be shown in the author's github as well.

Simulation Repository & Algorithm Code

Questions?