Electromagnetics & Anisotropy

Half-Waveplates, Quarter-Waveplates, and Uniaxial Media

Kevin Lindstrom
kevin.lindstrom@uconn.edu

Department of Electrical and Computer Engineering
University of Connecticut

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Figure 1: A picture of a half-waveplate (HWP) from Thorlabs [1]. HWPs rotate the polarization of LP light by 90° and quarter-waveplates (QWP) convert LP light to CP and visa versa.

[1] Thorlabs, `https://www.thorlabs.com/newgrouppage9.cfm?objectgroup_id=711`.

The Displacement Field

The electric displacement field $\mathbf{D}$ (with units $\mathrm{C} \cdot \mathrm{m}^{-2}$) is defined as

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon \mathbf{E} \tag{1}$$

where $\mathbf{P}$ is the polarization of the material, and the red part of the equation is true given an isotropic dielectric [2].

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The perpendicular and parallel boundary conditions (BC) for the displacement field on an interface are

$$(\mathbf{D}_2 - \mathbf{D}_1) \cdot \hat{\mathbf{n}}_{12} = \sigma_f \quad \text{and} \quad (\mathbf{D}_2 - \mathbf{D}_1) \times \hat{\mathbf{n}}_{12} = (\mathbf{P}_2 - \mathbf{P}_1) \times \hat{\mathbf{n}}_{12} \tag{2}$$

where σ_f is the free (unbounded) surface charge density, and $\hat{\mathbf{n}}_{12}$ is the normal vector of the interface from medium 1 to medium 2.

[2] D. J. Griffiths, *Introduction to Electrodynamics* (Pearson, 2013).

The Displacement Field & Anisotropy

What is Anisotropy?

Materials are *Anisotropic* if their properties have **directional dependence**—permittivity could take two different values depending on if the $\mathbf{E}$ field is in the $\hat{\mathrm{x}}$ or $\hat{\mathrm{y}}$ direction [3].

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Materials are *Anisotropic* if their properties have **directional dependence**— permittivity could take two different values depending on if the $\mathbf{E}$ field is in the $\hat{\mathbf{x}}$ or $\hat{\mathbf{y}}$ direction [3].

For anisotropic media, we use the relative permittivity tensor (rank 2), which must be a Hermitian matrix [4, 5].

$$\bar{\epsilon}_r = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \quad \text{where} \quad \epsilon_{ij} = \epsilon_{ji}^* \tag{3}$$

Equation (1) becomes

$$\mathbf{D} = \epsilon_0 \bar{\epsilon}_r \mathbf{E} \tag{4}$$

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Uniaxial Media

Calcite, mica, and quartz are examples of *uniaxial media*, where the permittivity tensor takes the form

$$\bar{\epsilon}_r = \begin{bmatrix} \epsilon_r^e & 0 & 0 \\ 0 & \epsilon_r^o & 0 \\ 0 & 0 & \epsilon_r^o \end{bmatrix} \tag{5}$$

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Here, the x-axis is the *extraordinary* axis, and y/z-axes are the *ordinary* axes [5].

Wave Propagation

- The permittivity has two values
- **Waves propagate at different speeds depending on their polarization**

[5] F. Ran, "Waves in Anisotropic Media," `https://courses.cit.cornell.edu/ece303/Lectures/lecture17.pdf` (2007)

Half-Waveplate Diagram

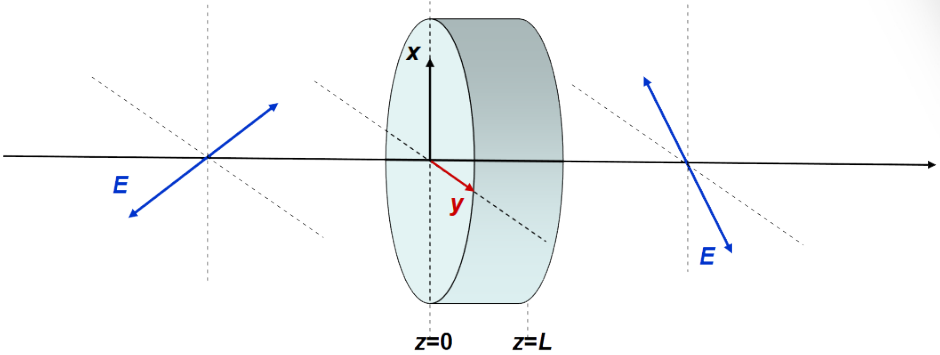

Figure 2: A half-waveplate (HWP) in the xy-plane with length L with an incident plane wave at $z = 0$ [5]. HWPs rotate the polarization of an incident plane wave $90°$. We choose $z < 0$ to be region 1, $0 < z < L$ to be region 2, and $z > L$ to be region 3.

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The Wave Equation

Using Faraday's Law, Ampere's Law, and Equation (4) we obtain the wave equation [3, 5]

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) = -j\omega\mu_0 \nabla \times \mathbf{H}(\mathbf{r}) = \omega^2 \mu_0 \mathbf{D}(\mathbf{r}) \tag{6}$$

which becomes

$$\nabla^2 \mathbf{E}(\mathbf{r}) = -\omega^2 \mu_0 \mathbf{D}(\mathbf{r}) = -\omega^2 \mu_0 \epsilon_0 \bar{\epsilon}_r \mathbf{E}(\mathbf{r}) \tag{7}$$

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For normal incidence, a wave polarized in $\hat{\mathbf{x}}$ stays in $\hat{\mathbf{x}}$ in the dielectric

$$\mathbf{E}_1^i(z) = \hat{\mathbf{x}} E_0 e^{-j\beta_0 z} \quad \Rightarrow \quad \mathbf{E}_2^t(z) = \hat{\mathbf{x}} T_{12}^e E_0 e^{-j\beta^e z} \tag{8}$$

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and $\hat{\mathbf{y}}$ stays in $\hat{\mathbf{y}}$

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The Half-Waveplate

How do they work?

For just $\hat{x}$ or $\hat{y}$ incident polarization the polarization inside the dielectric does not change. How does a HWP cause the polarization to rotate?

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where (for a nonmagnetic material)

$$\beta^e = \omega\sqrt{\mu_0 \epsilon_r^e \epsilon_0} \quad \text{and} \quad \beta^o = \omega\sqrt{\mu_0 \epsilon_r^o \epsilon_0} \quad (11)$$

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The field at $z = L$ becomes

$$\mathbf{E}_2^t(z)\Big|_{z=L} = \frac{E_0}{\sqrt{2}} \left(\hat{\mathbf{x}} T_{12}^e e^{-j\beta^e L} + \hat{\mathbf{y}} T_{12}^o e^{-j\beta^o L}\right) \quad (12)$$

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The Half-Waveplate Cont.

For 90° polarization change we want to flip $\hat{\mathrm{x}}$ or $\hat{\mathrm{y}}$, so we choose Equation (13)[1] such that Equation (12) becomes Equation (14) [3, 5].

$$L(\beta^e - \beta^o) = (2m+1)\pi \quad \text{where} \quad \forall m \in \mathbb{Z} \tag{13}$$

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We have obtained the desired polarization change, with limitations...

- ϵ_r^e and ϵ_r^o are ω dependent
- The device must be a precise thickness L

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The Quarter-Waveplate

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Hence, if $T_{12}^e \approx T_{12}^o$

LP $\rightarrow$ CP & CP $\rightarrow$ LP

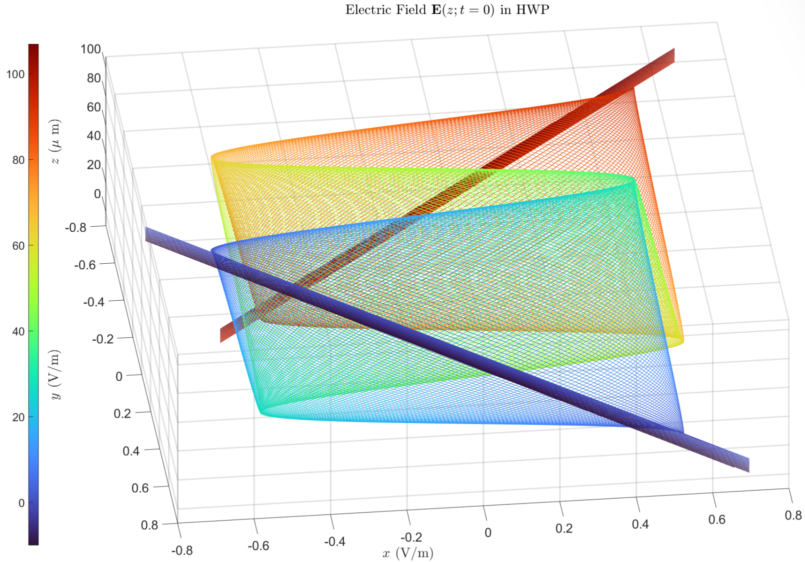

Figure 3: Forward propagating Electric Field before, inside, and after a quartz HWP, $L = 96.933$μm and $m = 1$. Color bar helps visualize propagation in z. Reflections not considered, discontinuities of $\vec{\mathscr{E}}(z; t = 0)$ would vanish they were.

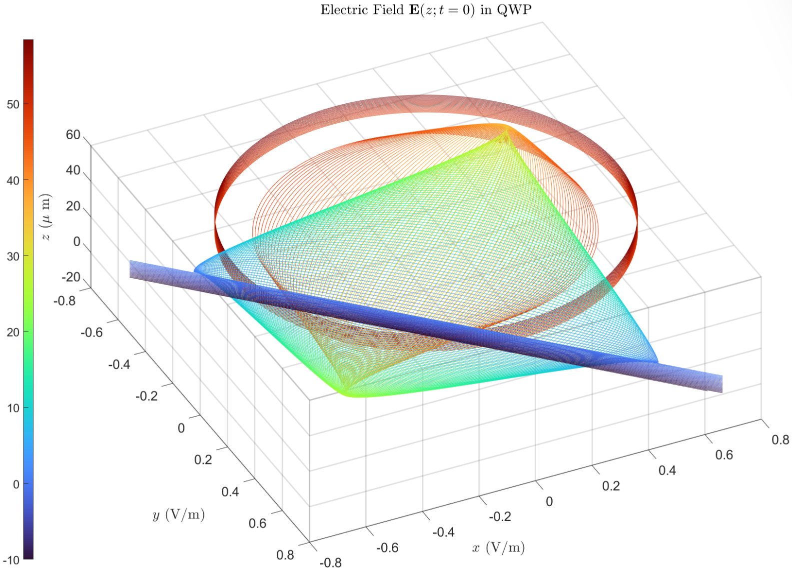

Figure 4: Forward propagating Electric Field before, inside, and after a quartz QWP, $L = 48.4667$µm and $m = 1$. Color bar helps visualize propagation in z. Reflections not considered, discontinuities of $\vec{\mathscr{E}}(z; t = 0)$ would vanish they were.

Total Field after the Second Interface ($z > L$)

Half-wave plate:

$$\mathbf{E}_3(z) = \xi(z) \sum_{n=0}^{\infty} e^{-j(2n+1)\beta^o L} \begin{bmatrix} -T_{12}^e(\Gamma_{23}^e)^{2n} T_{23}^e \\ T_{12}^o(\Gamma_{23}^o)^{2n} T_{23}^o \\ 0 \end{bmatrix} \tag{17}$$

Quarter-wave plate:

$$\mathbf{E}_3(z) = \xi(z) \sum_{n=0}^{\infty} e^{-j(2n+1)\beta^o L} \begin{bmatrix} -j(-1)^n(-1)^m T_{12}^e(\Gamma_{23}^e)^{2n} T_{23}^e \\ T_{12}^o(\Gamma_{23}^o)^{2n} T_{23}^o \\ 0 \end{bmatrix} \tag{18}$$

where

$$\xi(z) \equiv \frac{E_0}{\sqrt{2}} e^{-j\beta_0(z-L)} \quad \text{and} \quad \forall m \in \mathbb{Z}$$

Figures of Merit

Figures of Merit

We define $R^{\%}$ as the ratio of the power of first internally reflected and then transmitted wave $\mathbf{E}_3^{trrt}$ to the power of the only forward propagating wave $\mathbf{E}_3^{tt}$

$$R^{\%} = \frac{|\mathbf{E}_3^{trrt}|^2}{|\mathbf{E}_3^{tt}|^2} \cdot 100 = \frac{\left[T_{12}^{e}(\Gamma_{23}^{e})^2 T_{23}^{e}\right]^2 + \left[T_{12}^{o}(\Gamma_{23}^{o})^2 T_{23}^{o}\right]^2}{\left[T_{12}^{e} T_{23}^{e}\right]^2 + \left[T_{12}^{o} T_{23}^{o}\right]^2} \cdot 100 \qquad (19)$$

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We define the difference in the extraordinary and ordinary transmission coefficients to be ΔT

$$\Delta T = |T_{12}^e T_{23}^e - T_{12}^o T_{23}^o| \qquad (20)$$

where ideally $\Delta T \approx 0$.

Designed Quartz Waveplates

For a quartz waveplate where $\epsilon_r^e = 2.4130$ and $\epsilon_r^o = 2.3847$ at $\lambda = 590$ nm and non-magenetic $\mu = \mu_0$ [6]

Plate Type	L (μm)	m	ΔT	$R^{\%}$
HWP	96.933	1	0.001	0.215%
HWP	226.178	3	0.001	0.215%
QWP	48.467	1	0.001	0.215%
QWP	113.089	3	0.001	0.215%

Table 1: Designed quantities and figures of merit.

[6] G. Ghosh, “Dispersion-equation coefficients for the refractive index and birefringence of calcite and quartz crystals,” Opt. Commun. **163**, 95-102 (1999).

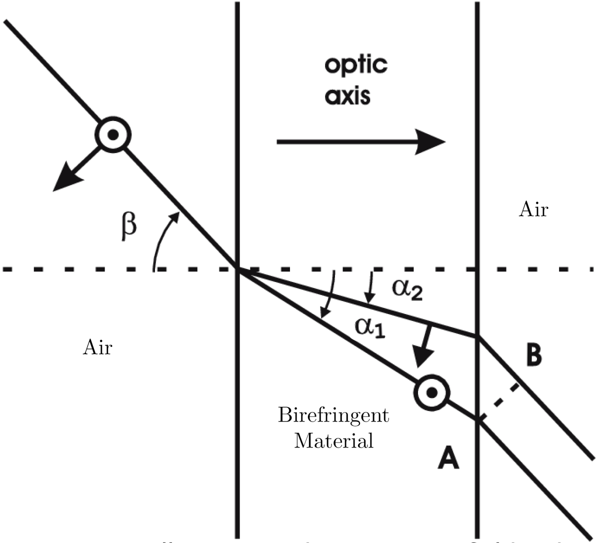

Figure 5: We obtain two spatially separated propagating fields when we consider the case of oblique incidence on a **birefringent material** [7].

[7] A. Lutich, M. Danailov, s. Volchek, V. Yakovtseva, V. Sokol, and S. Gaponenko, "Birefringence of Nanoporous Alumina: Dependence on Structure Parameters," Appl. Phys. B **84**, 327-331 (2006).

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Disclosures: The authors declare no conflicts of interest.

Data Availability: LaTeX and MATLAB code for this project and its reports can be found on the author’s github: https://github.com/kevinlindstrom/ECE5201_TermProject.

Questions?