CS70 In Simpler Terms - Note 7

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1 Uniform Distribution

$Unif\{1, 2, ...n\}$, deals with the set $= \{1, 2, ...n\}$

- Rolling Dice problems - $n = 6$
- $Pr(X = x) = \frac{1}{n} = \frac{1}{6}$ in the case of rolling a dice
- $E(X) = \sum_{x=1}^{n} x\frac{1}{n} = \frac{1}{n}\frac{(n+1)n}{2} = \frac{n+1}{2} = \frac{7}{2}$ for dice
- $E(X^2) = \frac{1}{n}(1^2 + 2^2 + ...n^2) = \frac{1}{n}\frac{n(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6}$
- $Var(X) = E(X^2) - E(X)^2 = \frac{n^2-1}{12}$

2 Bernoulli Distribution

$$Pr(X = x) = \begin{cases} 1 - p, & x = 0 \\ p, & x = 1 \\ 0, & otherwise \end{cases}$$

- $P(Success) = p, P(Fail) = 1 - p$
- $P(X = x) = (1 - p)^{1-x}p^x$ $1 - x$ *failures until success on xth trial*
- $E(X) = 0 * (1 - p) + 1 * P(X = 1) = p$
- $E(X^2) = 0^2 * (1 - p) + 1^2 * p = E(X) = p$
- $Var(X) = p - p^2 = p(1 - p)$

3 Indicator r.v. Revisited

Indicator r.v. essentially follow the Bernoulli Distribution

- Let X_i = indicator r.v. and $X_i = 1$ if trial i is a success, and 0 otherwise
- $X = X_1 + X_2 + ...X_n$
- $E(X) = E(X_1 + ...X_n) = E(X_1) + ...E(X_n)$
- $E(X_i) = Pr(trialiisasuccess) = p \Rightarrow E(X) = np$
- $Var(X) = Var(X_1) + Var(X_2) + ...Var(X_n) = np(1 - p)$

4 Binomial Distribution

$Bin(n, p)$

- Flipping coin problems
- $Pr(X = i) = \binom{n}{i} p^i (1-p)^{n-i}$ for $i = 0, 1, ...n$
- if $X \sim Bin(n, p)$, $\sum_{i=0}^{n} Pr(X = i) = 1$
- Practical Example: Error Erasure Code
  - $n+k$ packets, and each packet has probability p of getting lost ($(1-p)$ of successfully transfering)
  - $X \sim Bin(n+k, 1-p)$
  - Probability of receiving at least n packets $= Pr(X \geqslant n) = \sum_{i=n}^{n+k} Pr(X = i) = \sum_{i=n}^{n+k} \binom{n+k}{i} (1-p)^i p^{n+k-i}$