CS70 In Simpler Terms - Note 1

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1 Sets and their Properties

This section contains definitions that you are expected to be familiar with throughout the course.

1. Cardinality: $|A|$ number of items in a set (*e.x.* $A = a, b, c$ *Then the cardinality of* A*, denoted* $|A|$*, is* 3.)
2. Subset: $\subseteq$ (*e.x.* $A \subseteq B$ *reads* A *is a subset of* B.)
3. Proper Subset: $\subset$ (*e.x.* $A \subseteq B$ *reads* A *is a proper subset of* B *so* A *includes all subsets of* B*, but not* B *itself.*)
4. Disjoint Sets: 2 sets are said to be disjoint if $A \cap B = \varnothing$
5. Relative Complement of A in $B = B - A = B \backslash A$, which means everything that is in B but not in A.
6. Cartesian Product: cross product of sets (*e.x.* $A \times B = \{(a, b) \mid a \in A, b \in B\}$.)
7. Power Set: $\mathscr{P}(S)$: The set of all subsets of S.(*e.x. let* $S = \{1, 2\}$*, then* $\mathscr{P}(S) = \{\{\},\ \{1\},\ \{2\},\ \{1, 2\}\}$). The cardinality of $\mathscr{P}(S)$ is 2^k, where k is the cardinality of S.
8. $\forall$ = for *all* a.k.a. the Universal Quantifier
9. $\exists$ = there *exists* some a.k.a. Existential Quantifier

2 Propositions

Propositions are statements/expressions that are either true or false. A *Tautology* is a proposition that always evaluates to true. A *Contradiction* is a proposition that is always false.
I will assume that everyone is familiar with and/not/or logic so I will not go in depth here.

1. Conjunction: $\wedge$ = and
2. Disjunction: $\wedge$ = or
3. Negation: $\neg$ = not

4. Implication: P $\Rightarrow$ Q
5. Contrapostive of the above: $\neg Q \Rightarrow \neg P \equiv P \Rightarrow Q \equiv \neg P \vee Q$
6. Converse: $Q \Rightarrow P$: not equivalent to $P \Rightarrow Q$

Oftentimes, making a truth table will simplify propositional logic a lot. Below is a simple truth table that you might find useful to keep in mind.

p	q	$p \Rightarrow q \equiv \neg p \wedge q$	$p \iff q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

I have also listed some common examples of statements that are true. They supplement the concept that the Universal Quantifier($\forall$) distributes over conjunction($\wedge$), and the Existential Quantifier($\exists$) distributes over disjunction($\vee$).

1. $\exists y \forall x P(x, y) \equiv \forall x \exists y P(x, y)$
2. $\forall x \forall y (P(x, y) \wedge Q(x, y)) \equiv \forall x (\forall y P(x, y)) \wedge (\forall y Q(x, y))$
3. $\exists x \exists y (P(x, y) \vee Q(x, y)) \equiv \exists x (\exists y P(x, y)) \vee (\exists y Q(x, y))$
4. $\exists x \forall y (P(x, y) \vee Q(x, y)) \not\equiv \exists x (\forall y P(x, y)) \vee (\forall Q(x, y))$

3 Proofs

For this course, there are just a few simple proofs that you need to know.

- Direct Proof: Assume P $\Rightarrow$ Q
- Proof by Contraposition: Assume $\neg$Q $\Rightarrow$ $\neg$P
- Proof by Contradiction: Assume $\neg$P...R...$\neg$R. Since $\neg$P $\Rightarrow$ ($\neg$R $\wedge$ R) is a contradiction, so P.
- Proof by Mathematical Induction: Arguably the most common proof you will be tested on. Just follow the steps, and you will be fine.

  1. Base case: Just like how in recursive functions you need a base case to STOP on, you need a base case to START from. *e.x. For* $n = 0$, *the statement is true.* ✓
  2. Inductive Hypothesis: Assume what you are trying to prove is true for all n up until some arbitrary k. (*e.x. Assume for* $0 < n \leqslant k$, *the statement is true.*). Sometimes you have to make your hypothesis stronger by making more assumptions than is given. This is known as strong induction, and you utilize this when you realize your original weak induction does not work.
  3. Inductive Step: Show that for $n = k + 1$, the statement is true. This usually takes some case-work to prove, but just do a few practice problems and it will get pretty easy.