Lecture 22

Writing Cache-friendly Code

CPSC 275
Introduction to Computer Systems

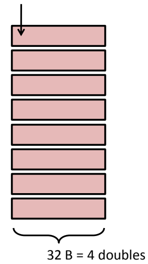
A Higher Level Example

```
int sum_array_rows(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (i = 0; i < 16; i++)
        for (j = 0; j < 16; j++)
            sum += a[i][j];
    return sum;
}
```

```
int sum_array_cols(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (j = 0; j < 16; j++)
        for (i = 0; i < 16; i++)
            sum += a[i][j];
    return sum;
}
```

Ignore the variables sum, i, j

assume: cold (empty) cache,
a[0][0] goes here



Writing Cache Friendly Code

- Make the common case go fast
  - Focus on the inner loops of the core functions
- Minimize the misses in the inner loops
  - Repeated references to variables are good (temporal locality)
  - Stride-1 reference patterns are good (spatial locality)

Matrix Multiplication Example

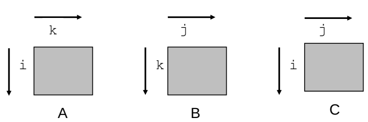
- Description:
  - Multiply N x N matrices
  - $O(N^3)$ total operations
  - N reads per source element
  - N values summed per destination
    - but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Variable `sum` held in register

Miss Rate Analysis for Matrix Multiply

- Assume:
  - Line size = 32 bytes (big enough for 4 doubles)
  - Matrix dimension (N) is very large
  - Cache is not even big enough to hold multiple rows
- Analysis Method:
  - Look at access pattern of inner loop



Layout of C Arrays in Memory

- C arrays allocated in *row-major* order
  - each row in contiguous memory locations
- Stepping through columns in one row:

```
for (i = 0; i < N; i++)
    sum += a[0][i];
```

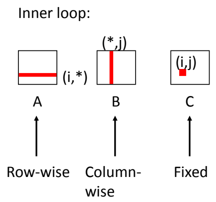
  - accesses successive elements
  - exploits spatial locality
- Stepping through rows in one column:

```
for (i = 0; i < n; i++)
    sum += a[i][0];
```

  - accesses distant elements
  - no spatial locality!

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

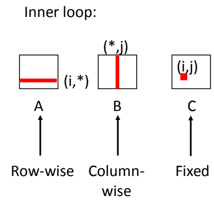


Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
```

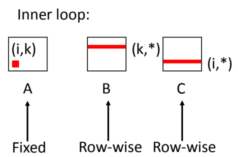


Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

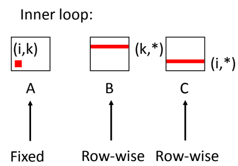


Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

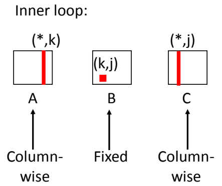


Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

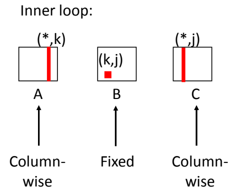


Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Matrix Multiplication (kji)

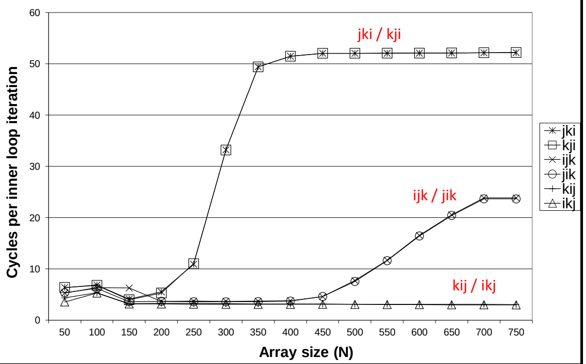
```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Core i7 Matrix Multiply Performance

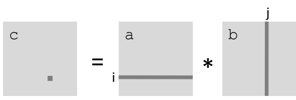


Optimizations for Memory Hierarchy

- Write code that has locality
  - Spatial: access data contiguously
  - Temporal: make sure access to the same data is not too far apart in time
- How to achieve?
  - Proper choice of algorithm
  - Loop transformations

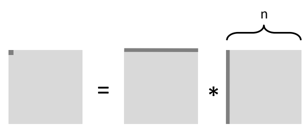
Example: Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n + j] += a[i*n + k]*b[k*n + j];
}
```

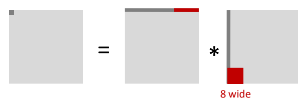


Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache block = 64 bytes = 8 doubles
  - Cache size C << n (much smaller than n)
- First iteration:
  - n/8 + n = 9n/8 misses (omitting matrix c)

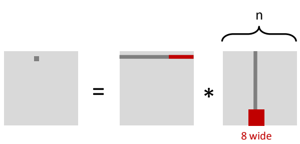


  - Afterwards in cache: (schematic)



Cache Miss Analysis

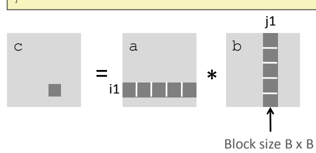
- Assume:
  - Matrix elements are doubles
  - Cache block = 64 bytes = 8 doubles
  - Cache size C << n (much smaller than n)
- Other iterations:
  - Again: n/8 + n = 9n/8 misses (omitting matrix c)



- Total misses:
  - $9n/8 * n^2 = (9/8) * n^3$

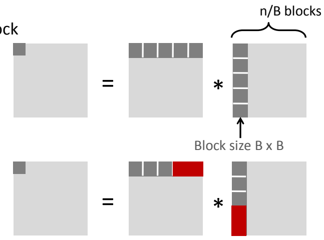
Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i1++)
                    for (j1 = j; j1 < j+B; j1++)
                        for (k1 = k; k1 < k+B; k1++)
                            c[i1*n + j1] += a[i1*n + k1]*b[k1*n + j1];
}
```



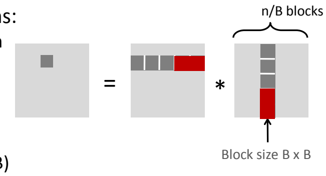
Cache Miss Analysis

- Assume:
  - Cache block = 64 bytes = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks ■ fit into cache: $3B^2 < C$
- First (block) iteration:
  - $B^2/8$ misses for each block
  - $2n/B * B^2/8 = nB/4$ (omitting matrix c)
  - Afterwards in cache (schematic)



Cache Miss Analysis

- Assume:
  - Cache block = 64 bytes = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks ■ fit into cache: $3B^2 < C$
- Other (block) iterations:
  - Same as first iteration
  - $2n/B * B^2/8 = nB/4$
- Total misses:
  - $nB/4 * (n/B)^2 = n^3/(4B)$



Summary

- No blocking: $(9/8) * n^3$
- Blocking: $1/(4B) * n^3$
- If B = 8 difference is 4 * 8 * 9 / 8 = 36x
- If B = 16 difference is 4 * 16 * 9 / 8 = 72x
- Suggests largest possible block size B, but limit $3B^2 < C$!
- Reason for dramatic difference:
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array element used O(n) times!
  - But program has to be written properly

Cache-Friendly Code

- Programmer can optimize for cache performance
  - How data structures are organized
  - How data are accessed
    - Nested loop structure
    - Blocking is a general technique
- All systems favor "cache-friendly code"
  - Getting absolute optimum performance is very platform specific
    - Cache sizes, line sizes, associativities, etc.
  - Can get most of the advantage with generic code
    - Keep working set reasonably small (temporal locality)
    - Use small strides (spatial locality)
    - Focus on inner loop code

Practice Problems

- Read CSaPP Sec. 6.4 and try 6.18 and 6.19