

ASSIGNMENT 1

A Collatz-Type Conjecture on the Set of Rational Numbers

Due 1:15 PM, Sep 14

[L. COLLATZ, 1937]. For any sequence of natural numbers, x_0, x_1, \dots , where

$$x_{i+1} = \begin{cases} x_i/2 & \text{if } x_i \text{ is even.} \\ 3x_i + 1 & \text{if } x_i \text{ is odd.} \end{cases}$$

will eventually reach 1.

We may extend this conjecture to the set of rational numbers. Let θ be the following function on $\mathbf{N} \times \mathbf{N}$:

$$\theta(p, q) = \begin{cases} (p - q, 3q) & \text{if } p \geq q \\ (2p, q - p) & \text{if } p < q \end{cases}$$

We make the following conjecture:

For every (p, q) , if we apply θ repeatedly, we eventually end up with $(0, m)$ for some m .

To show Collatz Conjecture experimentally:

1. (20 points) Write a C program (`collatz_i.c`) which will verify Collatz Conjecture for all integers $1 < x < N$ for some large N .
2. (30 points) Write a C program (`collatz_r.c`) which will verify Collatz Conjecture for all rational numbers of the form p/q where $1 < p, q < N$ for some large N .

Handin: Your code must be fully documented and uploaded at the course website.