1) Problem 2.9-2(a) (which means Fig. 2.1-10 (a)) (Plot only the Amplitude Spectrum)

2) Problem 2.9-2 (d)(which means Fig. 2.1-10 (d)) (Plot only the Amplitude spectrum)

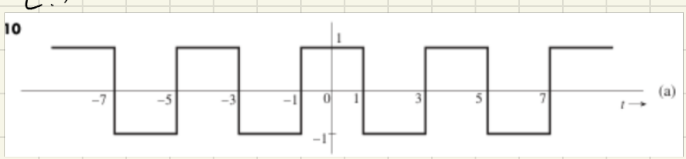
3) Problem 2.9.3 Write down the exponential Fourier series for the signal g(t) given in the problem. What is the fundamental period of this signal? (Ignore other parts of the problem, such as (a), (b) and (c))

4) Problem 3.1-1 (a)

5) Problem 3.1-2 (a): Do this problem only for Fig. 3.1-2(a)

2.9-2 For each of the periodic signals in Fig. P2.1-10, find exponential Fourier series and sketch the corresponding spectra.

1) 2.9-2 (a)



$$D_n = \frac{1}{T_0}\int_T g(t)\, e^{-jn\omega_0 t}\, dt$$

$$T_0 = 4 \qquad \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$g(t) = \begin{cases} 1 & -1 < t < 1 \\ -1 & 1 < t < 3 \end{cases}$$

$$D_n = \frac{1}{4}\left(\int_{-1}^{1} 1\, e^{-j\frac{\pi}{2}nt}\, dt + \int_{1}^{3}(-1)\, e^{-j\frac{\pi}{2}nt}\, dt\right)$$

$$\frac{1}{4}\left(\left(-\frac{1}{j\frac{\pi}{2}n}\right) e^{-j\frac{\pi}{2}nt}\Big|_{-1}^{1} + \left(\frac{1}{j\frac{\pi}{2}n}\right) e^{-j\frac{\pi}{2}nt}\Big|_{1}^{3}\right)$$

$$\frac{1}{4}\left(\frac{2}{j\frac{\pi}{2}n}\right)\left(-e^{-j\frac{\pi}{2}nt}\Big|_{-1}^{1} + e^{-j\frac{\pi}{2}nt}\Big|_{1}^{3}\right)$$

$$\frac{1}{j\pi n}\left(-e^{-j\frac{\pi}{2}n} + e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n3} - e^{-j\frac{\pi}{2}n}\right)$$

$$D_n = \frac{1}{j\pi n}\left(-2e^{-j\frac{\pi}{2}n} + e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n3}\right)$$

$$\frac{1}{j\pi n} e^{-j\frac{\pi}{2}n}\left(-2 + e^{j\pi n} + e^{-j\pi n}\right)$$

$$D_0 = \frac{1}{4}\left(\int_{-1}^{1} 1\, e^{0}\, dt + \int_{1}^{3} -1\, e^{0}\, dt\right)$$

$$\frac{1}{4}\left(t\Big|_{-1}^{1} + (-t)\Big|_{1}^{3}\right)$$

$$1/4\,(1-(-1) + -3-(-1))$$

$$1/4\,(0)$$

$$D_0 = 0$$

Calculation

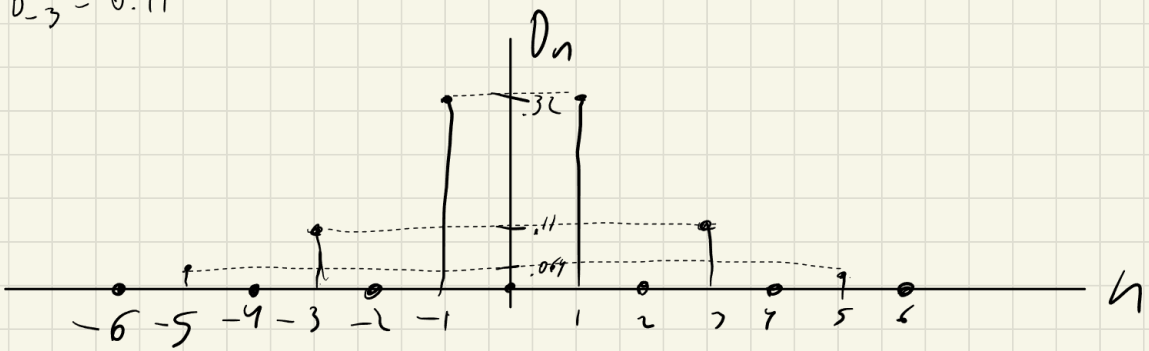
$D_1 = 0.32 + 0j$ $\quad D_4 = 0$

$D_2 = 0$ $\quad D_5 = 0.064$

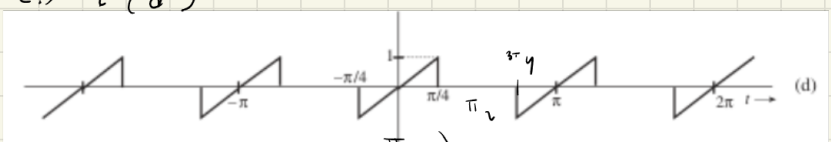
$|D_3| = 0.11$ $\quad D_6 = 0$

$D_{-1} = 0.32 \qquad D_{-5} = 0.064$

$D_{-3} = 0.11$



2) 2.8-2 (d)



$\pi/2$

slope $= \frac{2}{\pi/2} = \frac{4}{\pi}$

$T_0 = \pi \qquad \omega_0 = \frac{2\pi}{\pi} = 2$

$g(t) = \frac{4}{\pi}t \; ; \; -\pi/4 < t < \pi/4$

$D_n = \frac{1}{T_0}\int_{T_0} g(t)\, e^{-j\omega_0 n t}\, dt$

$\int u\,dv = uv - \int v\,du$

$= \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{4}{\pi} t\, e^{-j2nt}\, dt$

$= \frac{4}{\pi^2}\int_{-\pi/4}^{\pi/4} t\, e^{-j2nt}\, dt$ ($u = t$, $dv = e^{-j2nt}dt$; $v = \frac{1}{-j2n}e^{-j2nt}$, $du = 1\,dt$)

$= \frac{4}{\pi^2}\left(t\left(\frac{1}{j2n}\right)e^{-j2nt} + \int \frac{1}{j2n} e^{-j2nt}\, dt\right)$

$= \frac{4}{\pi^2}\left(t\left(\frac{1}{j2n}\right)e^{-j2nt} - \frac{1}{(j2n)^2}e^{-j2nt}\right)\Big|_{-\pi/4}^{\pi/4} = \frac{4}{\pi^2}\frac{1}{j2n}\left(t\, e^{-j2nt} - \frac{1}{j2n}e^{-j2nt}\right)_{-\pi/4}^{\pi/4}$

$= \frac{2}{\pi^2 jn}\left(\left(\frac{\pi}{4}e^{-jn\pi/2} - \frac{1}{j2n}e^{-jn\pi/2}\right) - \left(-\frac{\pi}{4}e^{jn\pi/2} - \frac{1}{j2n}e^{jn\pi/2}\right)\right)$

$\frac{2}{\pi^2 jn}\left(\frac{\pi}{4}e^{-jn\pi/2} - \frac{1}{j2n}e^{-jn\pi/2} + \frac{\pi}{4}e^{jn\pi/2} + \frac{1}{j2n}e^{jn\pi/2}\right)$

$D_n = \frac{2}{\pi^2 jn}\left(\frac{\pi}{2}\cos(n\pi/2) + \frac{1}{n}\sin(n\pi/2)\right)$

$D_0 = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{4}{\pi} t\, e^{-j2\cdot 0 t}\, dt = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{4}{\pi} t\, dt = \frac{4}{\pi^2}\left(\frac{t^2}{2}\right)\Big|_{-\pi/4}^{\pi/4} = \frac{2}{\pi^2}t^2\Big|_{-\pi/4}^{\pi/4}$

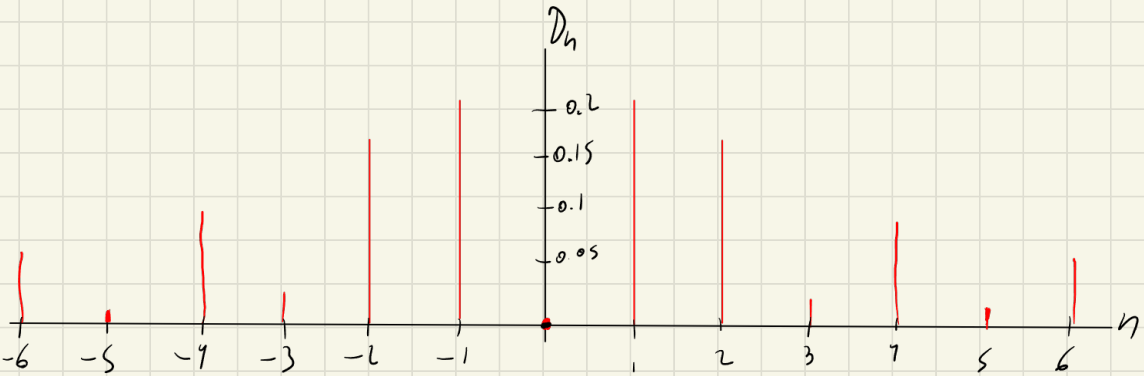
$$\frac{2}{\pi^2}\left(\left(\pi/4\right)^2-\left(-\pi/4\right)^2\right)=\frac{2}{\pi^2}\left(\frac{\pi^2}{16}-\frac{\pi^2}{16}\right)=0$$

$$D_0=0 \qquad D_n=\frac{2}{\pi^2 jn}\left(\frac{\pi}{2}\cos(n\pi/2)+\frac{1}{n}\sin(n\pi/2)\right)$$

★ Calculator ★

$|D_{-1}|=|D_1|=0.203$ $|D_{-2}|=|D_2|=0.16$ $|D_{-3}|=|D_3|=0.023$

$|D_{-4}|=|D_4|=0.080$ $|D_{-5}|=|D_5|=0.008$ $|D_{-6}|=|D_6|=0.05$



3)

2.9-3 A periodic signal $g(t)$ is expressed by the following Fourier series:

$$g(t)=\sin 2t+\cos\left(5t-\frac{2\pi}{3}\right)+2\cos\left(8t+\frac{\pi}{3}\right)$$

(a) Sketch the amplitude and phase spectra for the trigonometric series.

(b) By inspection of spectra in part **(a)**, sketch the exponential Fourier series spectra.

(c) By inspection of spectra in part **(b)**, write the exponential Fourier series for $g(t)$.

$g(t)=\sin 2t+\cos(5t-2\pi/3)+2\cos(8t+\pi/3)$

$$g(t)=\frac{e^{j2t}-e^{-j2t}}{2j}+\frac{e^{j(5t-2\pi/3)}-e^{-j(5t-2\pi/3)}}{2}+e^{j(8t+\pi/3)}+e^{-j(8t+\pi/3)}$$

$\omega_{01}=2$ $\omega_{02}=5$ $\omega_{03}=8$

$T_{01}=\frac{2\pi}{2}=\pi$ $=\frac{2\pi}{5}$ $\frac{2\pi}{8}$

π $\pi/4$

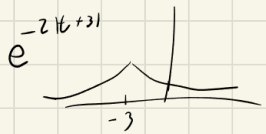
π 2π 3π
$2/5\pi$ $4/5\pi$ $6/5\pi$ $8/5\pi$ 2π
$\pi/4$ $\pi/2$ $3\pi/4$ π $5\pi/4$ $3\pi/2$ $7\pi/4$ 2π

} Lowest common denominator is 2π

Fundamental period is 2π

3.1-1 **(a)** Use direct integration to find the Fourier transform $G(f)$ of signal

$$g_a(t) = \Pi(t-1) - 5\exp(-2|t+3|)$$



$$e^{-2|t+3|} = \begin{cases} e^{-2(t+3)} & t > -3 \\ e^{-2(-t-3)} = e^{2(t+3)} & t < -3 \end{cases}$$

a) $g_a(t) = \Pi(t-1) - 5e^{(-2|t+3|)}$

$G_a(f) = \int_{-\infty}^{\infty} g_a e^{-j\omega t} dt$

$= \int_{-\infty}^{\infty} \Pi(t-1) e^{-j\omega t} dt - 5\int_{-\infty}^{\infty} e^{-2|t+3|} e^{-j\omega t} dt$

$\int_0^1 (1) e^{-j\omega t} dt - 5\left(\int_{-\infty}^{-3} e^{2(t+3)} e^{-j\omega t} dt + \int_{-3}^{\infty} e^{-2(t+3)} e^{-j\omega t} dt \right)$

$\frac{1}{-j\omega}\left(e^{-j\omega t}\right)\Big|_0^1$ $\quad -5\left(\int_{-\infty}^{-3} e^{2t+6} e^{-j\omega t} dt + \int_{-3}^{\infty} e^{-2t-6} e^{-j\omega t} dt \right)$

$\frac{1}{-j\omega}\left(e^{-j\omega} - 1\right)$ $\quad \int_{-\infty}^{-3} e^{-j\omega t + 2t + 6} dt + \int_{-3}^{\infty} e^{-j\omega t - 2t - 6} dt$

$\frac{1}{j\omega} - \frac{e^{-j\omega}}{j\omega}$ $\quad \frac{1}{-j\omega+2}\left(e^{-j\omega t + 2t + 6}\right)\Big|_{-\infty}^{-3} + \frac{1}{-j\omega-2}\left(e^{-j\omega t - 2t - 6}\right)\Big|_{-3}^{\infty}$

$\frac{1}{-j\omega+2}\left(e^{j\omega 3 - 6 + 6} - 1\right) + \frac{1}{-j\omega-2}\left(1 - e^{j\omega 3 + 6 - 6}\right)$

$\frac{1}{j\omega} - \frac{e^{-j\omega}}{j\omega}$ $\quad -5\left(\frac{1}{-j\omega+2}\left(e^{j\omega 3} - 1\right) + \frac{1}{-j\omega-2}\left(1 - e^{j\omega 3}\right) \right)$

$$G_a(f) = \frac{1}{j2\pi f} - \frac{e^{-j2\pi f}}{j2\pi f} - \frac{5}{-j2\pi f + 2}\left(e^{j6\pi f} - 1\right) - \frac{5}{-j2\pi f - 2}\left(1 - e^{j6\pi f}\right)$$

3.1-2 Consider the two signals shown in Fig. P3.1-2.

5) **(a)** From the definition in Eq. (3.1a), find the Fourier transforms of the signals.

$$G(F) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

$$g(t) = \begin{cases} 4, & 0 < t < 1 \\ 2, & 1 < t < 2 \end{cases}$$

2

(a)

$$\int_0^1 4e^{-j2\pi ft} dt + \int_1^2 2e^{-j2\pi ft} dt$$

$$\frac{4}{-j2\pi f}\left(e^{-j2\pi ft}\right)\Big|_0^1 + \frac{2}{-j2\pi f}\left(e^{-j2\pi ft}\right)\Big|_1^2$$

$$\frac{4}{-j2\pi f}\left(e^{-j2\pi f} - 1\right) + \frac{2}{-j2\pi f}\left(e^{-j4\pi f} - e^{-j2\pi f}\right)$$

$$\frac{2}{-j\pi f}\left(e^{-j2\pi f} - 1\right) + \frac{1}{-j\pi f}\left(e^{-j4\pi f} - e^{-j2\pi f}\right)$$

$$\frac{1}{-j\pi f}\left(2e^{-j2\pi f} - 2 + e^{-j4\pi f} - e^{-j2\pi f}\right)$$

$$G(f) = \frac{1}{j\pi f}\left(2 - e^{-j4\pi f} - e^{-j2\pi f}\right)$$