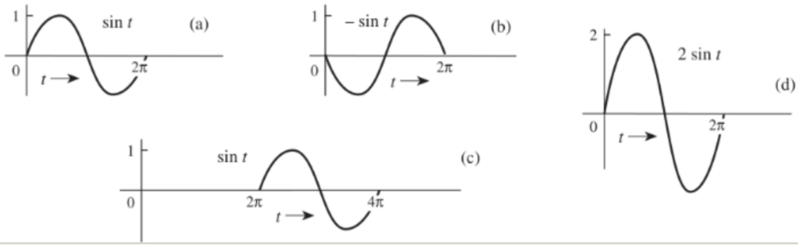
2.1-2 Find the energies of the signals shown in Fig. P2.1-2. Comment on the effect on energy of sign change, time shift, or doubling of the signal. What is the effect on the energy if the signal is multiplied by k?

Figure P2.1-2



a) $E = \int_0^{2\pi} |\sin(t)|^2 dt$

$= \int_0^{2\pi} \sin^2(t)\,dt \qquad \sin^2\theta = \frac{1-\cos(2\theta)}{2}$

$= \int_0^{2\pi} \frac{1-\cos(2t)}{2}\,dt = \frac{1}{2}\int_0^{2\pi} 1-\cos(2t)\,dt$

$= \frac{1}{2}\left(\int_0^{2\pi} dt - \int_0^{2\pi} \cos(2t)\,dt\right)$

$\frac{1}{2}\left(t - \frac{\sin(2t)}{2}\right)\Big|_0^{2\pi} \quad \sin 0 = 0, \ \sin 4\pi = 0$

$= \frac{1}{2}(2\pi - 0)$

$= \pi$

$\boxed{E = \pi}$

$u = 2t$
$du = 2dt$
$\frac{du}{2} = dt$

$\int_0^{2\pi} \frac{\cos(u)}{2}\,du$

$\frac{\sin(u)}{2} = \frac{\sin(2t)}{2}$

When a signal's sign is changed, or it's time shifted, Energy remains the same

When the signal is doubled energy is quadrupled

if $E_g = x$ watts
then $E_{2g} = 4x$ watts

If a signal is multiplied by K, the energy is multiplied by K^2

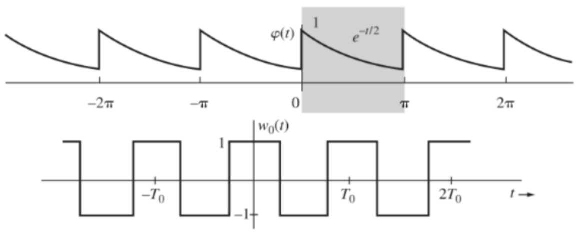
$$E_g = \int_{T_0} |g(t)|^2 \, dt = x$$

$$E_{Kg} = \int_{T_0} |K \cdot g(t)|^2 \, dt = K^2 \int_{T_0} |g(t)|^2 \, dt$$

$$\boxed{E_{Kg} = K^2 \cdot x}$$

2.1-3 Find the average power of the signals in Fig. P2.1-3.

Figure P2.1-3



$$P(\varphi(t)) = \frac{1}{\pi}\int_0^{\pi} |e^{-t/2}|^2 \, dt = \frac{1}{\pi}\int_0^{\pi} e^{-t} \, dt = -\frac{1}{\pi}\left(e^{-t}\right)\Big|_0^{\pi} = -\frac{1}{\pi}\left(e^{-\pi} - 1\right)$$

$$\boxed{P(\varphi) = \frac{1}{\pi}\left(1 - e^{-\pi}\right) \text{ Watts}}$$

$$P(W_0) = \int_0^{T_0} \frac{1}{T_0} |w_0(t)|^2 \, dt = \frac{1}{T_0}\left(\int_{-T_0/4}^{T_0/4} |1|^2 \, dt + \int_{T_0/4}^{3T_0/4} |-1|^2 \, dt\right)$$

$$= \frac{1}{T_0}\left(t\Big|_{-T_0/4}^{T_0/4} + t\Big|_{T_0/4}^{3T_0/4}\right) = \frac{1}{T_0}\left(T_0/2 + T_0/2\right) = \frac{1}{T_0}\left(T_0\right)$$

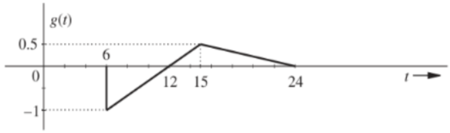
$$\boxed{P(w_0) = 1 \text{ Watt}}$$

2.3-3 For the signal $g(t)$ shown in Fig. P2.3-3,

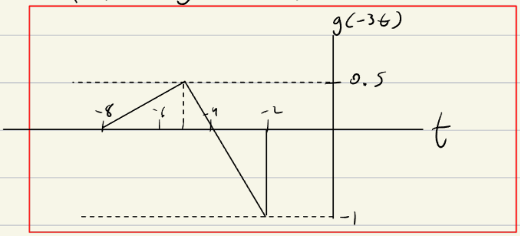
(a) Sketch signals **(i)** $g(-t)$; **(ii)** $g(t+2)$; **(iii)** $g(-3t)$; **(iv)** $g(t/3)$; **(v)** $g(2t+1)$; **(vi)** $g[2(t+1)]$.

(b) Find the energies of each signal in part **(a)**.

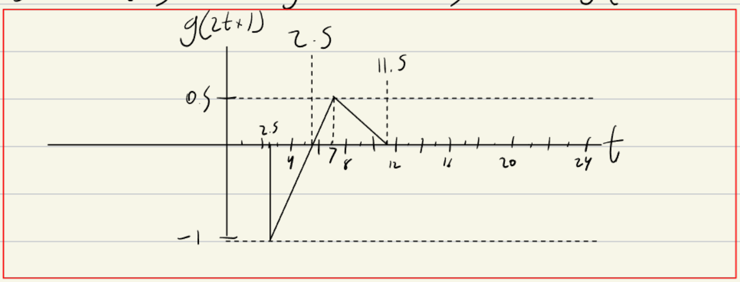
Figure P2.3-3



2.3-3 a (iii) $g(-3t)$



2.3-3 a (v) $g(2t+1) = g(2(t+1/2))$



2.4-4 Evaluate the following integrals:

(a) $\int_{-\infty}^{\infty} g(-3\tau+a)\delta(t-\tau)\,d\tau$

(b) $\int_{-\infty}^{\infty} \delta(\tau)g(t-\tau)\,d\tau$

(c) $\int_{-\infty}^{\infty} \delta(t+2)e^{-j\omega t}\,dt$

(d) $\int_{-\infty}^{1} \delta(t-2)\sin\pi t\,dt$

(e) $\int_{-2}^{\infty} \delta(2t+3)e^{-4t}\,dt$

(f) $\int_{-2}^{2} (t^3+4)\delta(1-t)\,dt$

(g) $\int_{-\infty}^{\infty} g(2-t)\delta(3-0.5t)\,dt$

(h) $\int_{-\infty}^{\infty} \cos\frac{\pi}{2}(x-5)\delta(3x-1)\,dx$

Hint: $\delta(x)$ is located at $x=0$. For example, $\delta(1-t)$ is located at $1-t=0$, that is, at $t=1$, and so on.

d) $\int_{-\infty}^{1} \delta(t-2)\sin\pi t\,dt$

$$\delta(t-2) = \begin{cases} \infty & t=2 \\ 0 & \text{else} \end{cases}$$

$$\boxed{\int_{-\infty}^{1} 0\cdot\sin(\pi t)\,dt = 0}$$

$$\delta(1-t) = \begin{cases} \infty, & t=1 \\ 0, & \text{else} \end{cases}$$

f) $\int_{-2}^{2} (t^3+4)\,\delta(1-t)\,dt = t^3+4$ @ $t=1$

$= 1+4$

$\boxed{= 5}$