EE 496 PQ 2 (Individual) Fall 2025

1. For the function $h(t)$ shown in the figure, assume $A = 2$ and $B = -3$. The units are "volt" along the y-axis and "seconds" along the x-axis.
(a) Calculate the energy of $h(t)$. (worth 4 points)
(b) Suppose $h(t)$ is used to construct a periodic function $g(t)$ by repeating $h(t)$ on both sides with period $T_0 = 4$ seconds. In other words, $g(t) = h(t)$ for $-2 \le t \le 2$, and then $h(t)$ repeats on both sides. It is obvious that the fundamental frequency in $g(t)$ is $f_0 = 1/T_0 = 0.25$ Hz. The exponential Fourier series of $g(t)$ can be expressed as $g(t) = \sum_{n=-\infty}^{\infty} D_n \exp(j2\pi n f_0 t)$. Calculate D_n where $n = 0, \pm 1, \pm 2, \cdots$. You must evaluate the integral but there is no need to simplify to the most compact form. (worth 6 points)

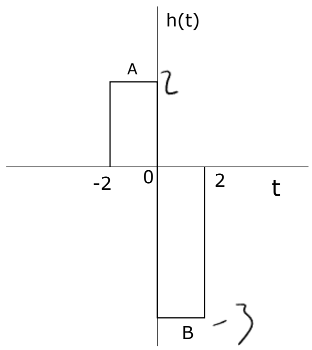

Figure 1: Function $h(t)$

$$E_h = \int_{-\infty}^{\infty} |h(t)|^2 dt$$
$$= \int_{-2}^{0} |2|^2 dt + \int_0^2 |-3|^2 dt$$
$$= \int_{-2}^{0} 4\, dt + \int_0^2 9\, dt$$
$$4t \Big|_{-2}^{0} + 9t \Big|_0^2$$
$$(4(0) - (-2)(4)) + (9(2) - 9(0))$$
$$8 \quad + \quad 18$$
$$E_h = 26$$

$$D_n = \frac{1}{T_0} \int_{T_0} h(t) e^{-jn2t} dt$$
$$D_n = \frac{1}{4} \int_{-2}^{2} h(t) e^{-j2nt} dt$$
$$= \frac{1}{4}\left[\int_{-2}^{0} 2 e^{-j2nt} dt + \int_0^2 -3 e^{-j2nt} dt \right]$$
$$\frac{1}{4}\left[\frac{2e^{-j2nt}}{-j2n}\Bigg|_{-2}^{0} + \frac{-3e^{-j2nt}}{-j2n}\Bigg|_{0}^{2} \right]$$
$$\frac{1}{4}\left[\frac{1}{-j2n}\left(1 - 2e^{j4n}\right) + \frac{1}{-j2n}\left(-3e^{-j4n} - 1\right) \right]$$
$$\frac{1}{j8n}\left(2e^{j4n} \cancel{-1} \cancel{+1} + 3e^{-j4n}\right)$$
$$D_n = \frac{1}{j8n}\left(2e^{j4n} + 3e^{-j4n}\right)$$