1) Problem 2.9-2(a) (which means Fig. 2.1-10 (a)) (Plot only the Amplitude Spectrum)

2) Problem 2.9-2 (d)(which means Fig. 2.1-10 (d)) (Plot only the Amplitude spectrum)

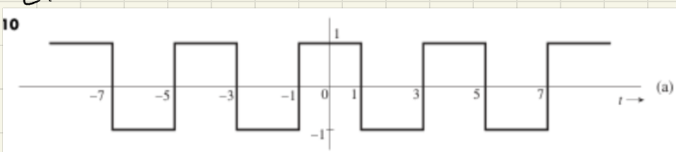
3) Problem 2.9.3 Write down the exponential Fourier series for the signal g(t) given in the problem. What is the fundamental period of this signal? (Ignore other parts of the problem, such as (a), (b) and (c))

4) Problem 3.1-1 (a)

5) Problem 3.1-2 (a): Do this problem only for Fig. 3.1-2(a)

2.9-2 For each of the periodic signals in Fig. P2.1-10, find exponential Fourier series and sketch the corresponding spectra.

1) 2.9-2 (a)



$T_0 = 4 \qquad \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$g(t) = \begin{cases} 1 & -1 < t < 1 \\ -1 & 1 < t < 3 \end{cases}$

$D_n = \frac{1}{T_0}\int_{T} g(t)\, e^{-jn\omega_0 t}\, dt$

$D_n = \frac{1}{4}\left(\int_{-1}^{1} 1\, e^{-j\frac{\pi}{2}nt}\, dt + \int_{1}^{3} (-1)\, e^{-j\frac{\pi}{2}nt}\, dt\right)$

$\frac{1}{4}\left(\left(-\frac{1}{j\frac{\pi}{2}n}\right) e^{-j\frac{\pi}{2}nt}\Big|_{-1}^{1} + \left(\frac{1}{j\frac{\pi}{2}n}\right) e^{-j\frac{\pi}{2}nt}\Big|_{1}^{3}\right)$

$\frac{1}{4}\left(\frac{2}{j\pi n}\right)\left(-e^{-j\frac{\pi}{2}nt}\Big|_{-1}^{1} + e^{-j\frac{\pi}{2}nt}\Big|_{1}^{3}\right)$

$\frac{1}{j\pi 2n}\left(-e^{-j\frac{\pi}{2}n} + e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n3} - e^{-j\frac{\pi}{2}n}\right)$

$D_n = \frac{1}{j\pi 2n}\left(-2e^{-j\frac{\pi}{2}n} + e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n3}\right)$

$\frac{1}{j2\pi n} e^{-j\frac{\pi}{2}n}\left(-2 + e^{j\pi n} + e^{-j\pi n}\right)$

$D_0 = \frac{1}{4}\left(\int_{-1}^{1} 1 e^{0} dt + \int_{1}^{3} -1 e^{0} dt\right)$

$\frac{1}{4}\left(t\Big|_{-1}^{1} + (-t)\Big|_{1}^{3}\right)$

$1/4\,(1-(-1)) + -3-(-1)$

$1/4\,(0)$

$D_0 = 0$

☆Calculator☆

$D_1 = 0.32 + 0j$

$D_2 = 0$

$|D_3| = 0.11$

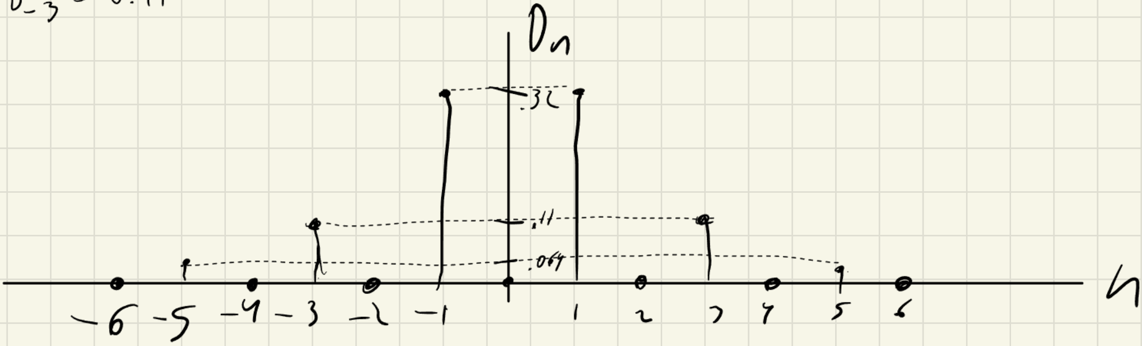
$D_4 = 0$

$D_5 = 0.064$

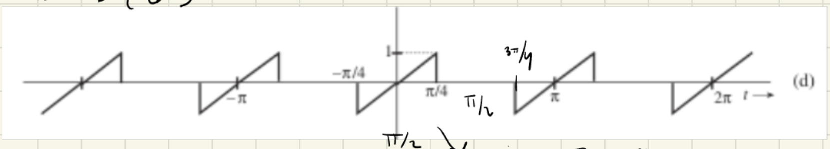
$D_6 = 0$

$D_{-1} = 0.32 \qquad D_{-5} = 0.064$

$D_{-3} = 0.11$



2) 2.9-2 (d)



$\searrow$ slope $= \frac{2}{\frac{\pi}{2}} = \frac{4}{\pi}$

$T_0 = \pi \quad \omega_0 = \frac{2\pi}{\pi} = 2$

$g(t) = \frac{4}{\pi} t \; ; \; -\pi/4 < t < \pi/4$

$$D_n = \frac{1}{T_0} \int_{T_0} g(t) e^{-j\omega_0 n t} dt \qquad \int u\,dv = uv - \int v\,du$$

$$= \frac{1}{\pi} \int_{-\pi/4}^{\pi/4} \frac{4}{\pi} t e^{-j2nt} dt$$

$$= \frac{4}{\pi^2} \int_{-\pi/4}^{\pi/4} t e^{-j2nt} dt \qquad v = -\frac{1}{j2n} e^{-j2nt} \quad du = 1dt \qquad = \frac{4}{\pi^2}\left(t\left(\frac{1}{j2n}\right)e^{-j2nt} + \int \frac{1}{j2n} e^{-j2nt} dt\right)$$

$$= \frac{4}{\pi^2}\left(t\left(\frac{1}{j2n}\right)e^{-j2nt} - \frac{1}{(j2n)^2} e^{-j2nt}\right)\Bigg|_{-\pi/4}^{\pi/4} = \frac{4}{\pi^2}\frac{1}{j2n}\left(t\, e^{-j2nt} - \frac{1}{j2n} e^{-j2nt}\right)\Bigg|_{-\pi/4}^{\pi/4}$$

$$= \frac{2}{\pi^2 jn}\left(\left(\frac{\pi}{4} e^{-jn\pi/2} - \frac{1}{j2n} e^{-jn\pi/2}\right) - \left(-\frac{\pi}{4} e^{jn\pi/2} - \frac{1}{j2n} e^{jn\pi/2}\right)\right)$$

$$\frac{2}{\pi^2 jn}\left(\frac{\pi}{4} e^{-jn\pi/2} - \frac{1}{j2n} e^{-jn\pi/2} + \frac{\pi}{4} e^{jn\pi/2} + \frac{1}{j2n} e^{jn\pi/2}\right)$$

$$D_n = \frac{2}{\pi^2 jn}\left(\frac{\pi}{2}\cos(n\pi/2) + \frac{1}{n}\sin(n\pi/2)\right)$$

$$D_0 = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{4}{\pi} t \cancel{e^{-j2nt}} dt = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{4}{\pi} t\, dt = \frac{4}{\pi^2}\left(\frac{t^2}{2}\right)\Bigg|_{-\pi/4}^{\pi/4} = \frac{2}{\pi^2} t^2\Bigg|_{-\pi/4}^{\pi/4}$$

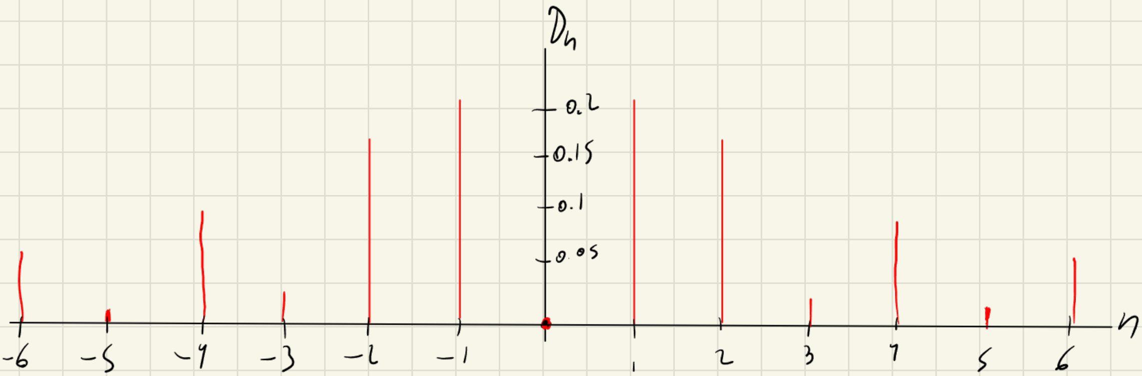
$$\frac{2}{\pi^2}\left(\left(\frac{\pi}{4}\right)^2-\left(-\frac{\pi}{4}\right)^2\right)=\frac{2}{\pi^2}\left(\frac{\pi^2}{16}-\frac{\pi^2}{16}\right)=0$$

$D_0=0 \qquad D_n=\frac{2}{\pi^2 j n}\left(\frac{\pi}{2}\cos(n\pi/2)+\frac{1}{n}\sin(n\pi/2)\right)$

* Calculator *

$|D_{-1}|=|D_1|=0.203 \qquad |D_{-2}|=|D_2|=0.16 \qquad |D_{-3}|=|D_3|=0.023$

$|D_{-4}|=|D_4|=0.080 \qquad |D_{-5}|=|D_5|=0.008 \qquad |D_{-6}|=|D_6|=0.05$



2.9-3 A periodic signal $g(t)$ is expressed by the following Fourier series:

$$g(t)=\sin 2t+\cos\left(5t-\frac{2\pi}{3}\right)+2\cos\left(8t+\frac{\pi}{3}\right)$$

(a) Sketch the amplitude and phase spectra for the trigonometric series.

(b) By inspection of spectra in part **(a)**, sketch the exponential Fourier series spectra.

(c) By inspection of spectra in part **(b)**, write the exponential Fourier series for $g(t)$.

$g(t)=\sin 2t+\cos(5t-2\pi/3)+2\cos(8t+\pi/3)$

$$\frac{e^{j2t}-e^{-j2t}}{2j}+\frac{e^{j(5t-2\pi/3)}-e^{-j(5t-2\pi/3)}}{2}+\frac{e^{j(8t+\pi/3)}+e^{-j(8t+\pi/3)}}{}$$

$\omega_{01}=2 \qquad \omega_{02}=5 \qquad \omega_{03}=8$

$T_{01}=\frac{2\pi}{2} \qquad =\frac{2\pi}{5} \qquad \frac{2\pi}{8}$

$=\pi$

$\pi \qquad \frac{\pi}{4}$

$\pi \quad 2\pi \quad 3\pi$

$2/5\pi \quad 4/5\pi \quad 6/5\pi \quad 8/5\pi \quad 2\pi$

$\pi/4 \quad \pi/2 \quad 3\pi/4 \quad \pi \quad 5\pi/4 \quad 3\pi/2 \quad 7\pi/4 \quad 2\pi$

$\boxed{\text{Fundamental period is } 2\pi}$

3.1-1 **(a)** Use direct integration to find the Fourier transform $G(f)$ of signal

$$g_a(t) = \Pi(t-1) - 5\exp(-2|t+3|)$$

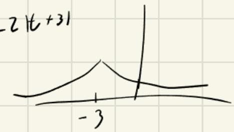
(b) Use direct integration to find the Fourier transform of

$$g_b(t) = \delta(2t-1) + 2e^{-t}u(t-1)$$

(c) Use direct integration to find the inverse Fourier transform of

$$G_c(f) = \delta(\pi f) - 2\delta(f-0.5)$$

$e^{-2|t+3|}$



$$e^{-2|t+3|} = \begin{cases} e^{-2(t+3)} & t > -3 \\ \underbrace{e^{-2(-(t+3))}}_{e^{2(t+3)}} & t < -3 \end{cases}$$

a) $g_a(t) = \Pi(t-1) - 5e^{(-2|t+3|)}$

$$G_a(f) = \int_{-\infty}^{\infty} g_a e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \Pi(t-1) e^{-j\omega t} dt - 5\int_{-\infty}^{\infty} e^{-2|t+3|} e^{-j\omega t} dt$$

$$\int_0^1 (1) e^{-j\omega t} dt \qquad -5\left(\int_{-\infty}^{-3} e^{2(t+3)} e^{-j\omega t} dt + \int_{-3}^{\infty} e^{-2(t+3)} e^{-j\omega t} dt\right)$$

$$\frac{1}{-j\omega}\left(e^{-j\omega t}\right)\Big|_0^1 \qquad -5\left(\int_{-\infty}^{-3} e^{2t+6} e^{-j\omega t} dt + \int_{-3}^{\infty} e^{-2t-6} e^{-j\omega t} dt\right)$$

$$\frac{1}{-j\omega}\left(e^{-j\omega} - 1\right) \qquad \int_{-\infty}^{-3} e^{-j\omega t + 2t + 6} dt + \int_{-3}^{\infty} e^{-j\omega t - 2t - 6} dt\Big)$$

$$\frac{1}{j\omega} - \frac{e^{-j\omega}}{j\omega} \qquad \frac{1}{-j\omega+2}\left(e^{-j\omega t + 2t + 6}\right)\Big|_{-\infty}^{-3} + \frac{1}{-j\omega-2}\left(e^{-j\omega t - 2t - 6}\right)\Big|_{-3}^{\infty}$$

$$\frac{1}{-j\omega+2}\left(e^{j\omega 3 - \cancel{6+6}^{0}} - 1\right) + \frac{1}{-j\omega-2}\left(1 - e^{j\omega 3 + \cancel{6-6}^{0}}\right)$$

$$\frac{1}{j\omega} - \frac{e^{-j\omega}}{j\omega} \quad -5\left(\frac{1}{-j\omega+2}\left(e^{j\omega 3} - 1\right) + \frac{1}{-j\omega-2}\left(1 - e^{j\omega 3}\right)\right)$$

$$G_a(f) = \frac{1}{j2\pi f} - \frac{e^{-j2\pi f}}{j2\pi f} - \frac{5}{-j2\pi f + 2}\left(e^{j6\pi f} - 1\right) - \frac{5}{-j2\pi f - 2}\left(1 - e^{j6\pi f}\right)$$

3.1-2 Consider the two signals shown in Fig. P3.1-2.
(a) From the definition in Eq. (3.1a), find the Fourier transforms of the signals.

$$G(F) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t}\, dt$$

$$g(t) = \begin{cases} 4, & 0 < t < 1 \\ 2, & 1 < t < 2 \end{cases}$$

$$\int_0^1 4e^{-j2\pi f t}\, dt + \int_1^2 2e^{-j2\pi f t}\, dt$$

$$\frac{4}{-j2\pi f}\left(e^{-j2\pi f t}\right)\Big|_0^1 + \frac{2}{-j2\pi f}\left(e^{-j2\pi f t}\right)\Big|_1^2$$

$$\frac{\cancel{4}\,2}{-j\cancel{2}\pi f}\left(e^{-j2\pi f} - 1\right) + \frac{\cancel{2}}{-j2\pi f}\left(e^{-j4\pi f} - e^{-j2\pi f}\right)$$

$$\frac{2}{-j\pi f}\left(e^{-j2\pi f} - 1\right) + \frac{1}{-j\pi f}\left(e^{-j4\pi f} - e^{-j2\pi f}\right)$$

$$\frac{1}{-j\pi f}\left(\cancel{2}e^{-j2\pi f} - 2 + e^{-j4\pi f} - \cancel{e^{-j2\pi f}}\right)$$

$$G(f) = \frac{1}{j\pi f}\left(2 - e^{-j4\pi f} - e^{-j2\pi f}\right)$$

2

(a)