Kevin Mueller
Homework 1

1a) $g_1, g_2 : \mathbb{R} \to \mathbb{R}$, $m : 2^V \to \mathbb{R}_+$

show that $h : 2^V \to \mathbb{R}$, $h(A) = \min(g_1(m(A)), g_2(m(A)))$

h is submodular if $g_1(m(A)) - g_2(m(A))$ is monotonically increasing or decreasing

We can then apply definition of concave $\mathbb{R}$ function.

For a concave function g it then follows

$g(m(A)) = \min(k, m(A)) = \min(k, a)$,

and $\min(k, a) \geq k$ for a non-decreasing function $m(A) = a$

Hence $g_1(a) - g_2(a)$ is monotonic.

1b) We need to show that $f(A) - f(V \setminus A)$ is monotonic

Let $B \subset A \subseteq V$ and f is monotonically increasing such that $f(B) \leq f(A)$

If we claim $f(\cdot) - f(V \setminus \cdot)$ is monotonically increasing then

$f(B) - f(V \setminus B) \leq f(A) - f(V \setminus A)$

By definition of complement we know that $f(V \setminus \cdot)$ must be monotonically decreasing if $f(A)$ is monotonically increasing, which implies

$\boxed{f(V \setminus B) \geq f(V \setminus A)}$

and since $f(B) \leq f(A)$ subtracting a larger quantity $f(V \setminus B)$ than $f(V \setminus A)$ on left hand side, guarantees the inequality holds, and $f(A) - f(V \setminus A)$ is monotonically increasing, which implies $h(A) = \min(f(A), f(V \setminus A))$ is submodular

1c)

	f	g
$\emptyset$	0	0
a	2	1
b	3	3
{a,b}	4	4

f-g not monotone.