MATH 680 - Assignment #3

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Question 1

(a)

We know that the least squares criterion is a convex function. Also, if we were to find the gradient of the objective function, the k^{th} element of the second term in the objective function would be:

$$\frac{\partial}{\partial \tilde{\beta}_k}\left(\frac{\lambda}{\alpha}\sum_{j=1}^{p-1}|\tilde{\beta}_k|^\alpha\right) = \lambda|\tilde{\beta}_k|^{\alpha-1}\text{sign}(\tilde{\beta}_k),$$

if $\tilde{\beta}_k \neq 0$. Then the second partial derivative gives us:

$$\begin{aligned}\frac{\partial^2}{\partial \tilde{\beta}_k^2}\left(\frac{\lambda}{\alpha}\sum_{j=1}^{p-1}|\tilde{\beta}_k|^\alpha\right) &= \lambda\text{sign}^2(\tilde{\beta}_k)(\alpha-1)|\tilde{\beta}_k|^{\alpha-2} \\ &\geq 0,\end{aligned}$$

if $\alpha \geq 1$.