MIT 8.02: Electricity and Magnetism

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Electric Charges

8.02 Lecture 1

Ben Franklin discovered that all objects have electric "fluid"; some with excess, some with deficiency. But didn't know about electrons - 50% chance of getting positive/negative right, but got it wrong.

glass - silk : positive charge
rubber - fur : negative

$F = \frac{q_1 q_2 K}{r^2} \hat{r}_{1,2}$

$q_p = -q_e = 1.6 \times 10^{-19} C$

$K = 9 \times 10^9 N m^2 C^- 2 = \frac{1}{4\pi\epsilon_0}$

Electricity is 10^{36} times stronger than gravity.
But large objects are typically neutral, so gravity dominates at large scales.

Electric Fields

8.02 Lecture 2

$\vec{E} = \frac{\vec{F}}{q} = \frac{Qk}{r^2} \hat{r}$ (NC^{-1})

Net field at a point is sum of all electric fields from all other charges.

Dipoles

Electric Flux

$d\Phi = \vec{E} \cdot \hat{n} dA = \vec{e} dA cos\theta$

Closed Surface
Sphere:

$\Phi = 4\pi R^2 E = \frac{Q}{\epsilon_0}$

Gauss's Law:

$\Phi = \oint \vec{E}.d\vec{A} = \frac{\sum Q_{inside}}{\epsilon_0}$

Sphere of radius R with charge Q uniformly distributed at surface:

Symmetry Argument 1:

all points at a given radius r must have equal field

Symmetry Argument 2:

field must point radially outward or inward, so angle is 0 or 180

At points $r < R$:

$4\pi R^2 E = \frac{Q_{inside}}{\epsilon_0}$

Q_{inside} is 0, so $E = 0$

no electric field inside charged sphere

At points $r > R$:

$4\pi R^2 E = \frac{Q_{inside}}{\epsilon_0}$

$Q_{inside} = Q$, so $E = \frac{Q}{4\pi r^2 \epsilon_0}$

from the outside, a charged sphere acts the same as if all charge were at center

Flat Horizontal Plane of infinite size:

Charge density

$= \sigma = \frac{Q}{A} (\frac{C}{m^2})$

Find electric field at point height d

Use for Gauss surface:

Cylinder with top and bottom parallel to plane, with top surface distance d above plane and bottom surface d below plane

No flux goes through sides of cylinder

$E = \frac{\sigma}{2\epsilon_0}$

Electric field does not fall off with distance

Two oppositely charged planes separated by distance d

Top plate has charge $+\sigma$, bottom $-\sigma$

Field above and below are 0, inside $E = \frac{\sigma}{\epsilon_0}$

Electrostatic Potential

U = Electrostatic Potential Energy

$$W = \int_{\infty}^{R} \vec{F_{push}} \cdot \vec{dr} = \int_{R}^{\infty} \vec{F}_{el} \cdot \vec{dr}$$

because forces are in opposite directions

$$W = \frac{q_1 q_2}{4\pi\epsilon_0} \int_{R}^{\infty} \frac{dr}{r^2}$$

$$\int_{R}^{\infty} \frac{dr}{r^2} = \frac{1}{R}$$

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 R}$$

sign-sensitive w.r.t. charges

V = Electric Potential

Work per unit charge to go from infinity to this location

$$V_p = \frac{Q}{4\pi\epsilon_0 R}$$

J / C = Volts

$$V = \int_{r}^{\infty} \frac{\vec{F}_{el}}{g} \cdot \vec{dr} = \int_{r}^{\infty} \vec{E} \cdot \vec{dr}$$

$$W = \Delta V q$$

Potential inside hollow metal sphere is constant = same as surface

Equipotential lines

Field lines always perpendicular

Never intersect

With multiple charges, add potential for all

Potential Difference

$$V_A = \int_{A}^{\infty} \vec{E} \cdot \vec{dl}$$

$$V_B = \int_{B}^{\infty} \vec{E} \cdot \vec{dl}$$

$$V_A - V_B \int_{A}^{B} \vec{E} \cdot \vec{dl}$$

More Electrostatic Potential

If a charge moves through a field and returns to its starting point

- W = 0
- $\oint$ means line is closed
- $\oint \vec{E} \cdot \vec{dl} = 0$
- $\frac{dV}{dr}\vec{r} = -E$

Equipotential lines always perpendicular to field lines because if you move only perpendicular to field lines, you do no work

Somewhere in space at point P, potential V_P

- step in x direction
  - $|E_x| = \frac{\Delta V}{\Delta x}|_{yz}$

Units for E: N/C = V/m

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right)$$

- $\vec{E} = -gradV$

Equipotential lines between oppositely charged parallel plates are parallel to plates

Electrostatic shielding - Faraday cage

- Charge in conductors re-arranges itself until potential is equal
  - Goes to outer surface
- Conducting sphere with charge +q somewhere inside
  - Gauss surface inside metal has 0 surface integral, therefore there must be 0 net charge inside
  - Negative charge moves to inside = -q, and +q on outside
    - Like induction
  - Uniform outside charge, independent of position of charge inside

High-Voltage Breakdown

Irregularly shaped object has higher charge density at more pointy parts

Proof:

Sphere A of radius R_A connected with wire to Sphere B of radius R_B
Both are equipotential
Charge Q_A on A, Q_B on B
If they weren't connected, they are far enough away to be independent

$V_A = \frac{Q_A}{4\pi\epsilon_0 R_A} = V_B = \frac{Q_B}{4\pi\epsilon_0 R_B}$

$\frac{Q_B}{R_B} = \frac{Q_A}{R_A}$

but charge density $\sigma = \frac{Q}{4\pi R^2}$
so smaller sphere has higher charge density

Electric field will be stronger there

$E = \frac{\sigma}{\epsilon_0}$

eV = 1.5 x 10^-16 V

Electric Breakdown when the field becomes too high

If the electron has enough energy to ionize oxygen/nitrogen, it knocks the electron off, and now there are 2, which then hit others
Dry air, room temperature
1 micron between collisions
To ionize oxygen = 12.5 eV
To ionize nitrogen = 15 eV
about 10^7 V/m (observed 3 x 10^6 V/m)

Current = charge/time [C/s = A]

Van de Graff Generator

$V = ER$; E < 3 x 10^6

Lightning

Top of cloud = +; Bottom = -
about 1km off ground, 10km tall, 5km wide
V = 3x10^9 V
Step leader from cloud to ground ionizes air, making conductive channel
Return stroke - most of the energy, light

Continuous discharge

Corona discharge / St. Elmo's Fire
Carbon-arc lamps

CAPACITANCE

Example

Top plate $+Q = +\sigma A$

Bottom Plate $-Q = -\sigma A$

$E = \frac{\sigma}{\epsilon_0}$

To move top plate from h to $h + x$, electric field is created, so work must be done.

$F = \frac{1}{2}QE$ because charge layer is halfway between field between plates and 0 field inside metal

$W = \frac{1}{2}QEx = \frac{1}{2}\epsilon_0 E^2 Ax$

$\frac{W}{Volume} = \frac{1}{2}\epsilon_0 E^2$ [J / m^3]

Field energy density

Potential Energy = $U = \int \frac{1}{2}\epsilon_0 E^2 dVolume$

Capacitance = Q/V [Farad] = charge / potential difference

Planes

$C = \frac{Q}{V} = \frac{\sigma A}{Ed} = \frac{E\epsilon_0}{d}$

smaller distance -> higher capacitance

$U = \frac{1}{2}CV^2$

The work done by moving charges plates apart (against electric field) goes into increasing the potential difference between them, since charge must remain constant

Dielectrics Polarization

Electric field can make electrons spend more time on one side of the atom, creating a dipole.

If you insert a material between the plates of a capacitor, a charge is induced in it. The induced charge lowers the electric field, lowering the potential difference.

$$E = \frac{E_{free}}{\kappa}$$

κ = Dielectric constant

$$E = \frac{\sigma_{free}}{\epsilon_0 \kappa}$$

$$V = Ed$$

$$C = \frac{Q_{free}}{V} = \frac{A\epsilon_0\kappa}{d}$$

Leyden Jars

Currents, Resistance

drift velocity = $v_d = \frac{eE}{m_e}\tau$

τ = time between collisions (due to thermal motion)

n = number of free electrons per cubic meter

about .5 cm/hr in copper at 10V

$$I = v_d A n e = \frac{e^2 n \tau}{m_e} A E$$

Conductivity = $\sigma = \frac{e^2 n \tau}{m_e}$

$\sigma_{Cu} \approx 10^8$

$\sigma_{glass} \approx 10^-12$

strong function of temperature -> hotter = higher resistance

$V = \frac{l}{\sigma A} I$

Resistivity = $\rho = \frac{1}{\sigma}$

Resistance = $R = \frac{l}{\sigma A} = l\rho$

$V = IR$

Ohm's law doesn't hold when current causes temperature change (thus changing resistance)

Adding ions to air or water increases its conductivity

Shoes have a resistance of about 4 billion ohms, but is still too high to prevent discharge of static electricity (high voltage, low charge).

Batteries and EMF

8.02 Lecture 10

In resistor part of circuit, current goes in the same direction as electric field, but in power source, electric field still goes + to -, so field is in opposite direction of current -- something must push the charge against the electric field.

Copper-Zinc battery: Cu + , Zn -

SO4- ion flows against electric field because when it reacts with Zn+, more energy is released than it costs to flow

$$V_b = IR - \mathcal{E} - Ir_i$$

$$P = IV = I^2R = \frac{V^2}{R}$$

Kirchhoff's Rules

1. $\oint Edl = 0$ - Around any closed loop there is no potential difference
2. Charge conservation
3. $\int_A^B Edl = V_{AB}$

<u>Kelvin Water Dropper</u>

Magnetic field

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Always dipoles

$$\hat{F} = \hat{I} \times \hat{B}$$
$$\vec{F_b} = q(\vec{v} \times \vec{B})$$

[N s / C m = T (Tesla)]

or G (Gauss) = 10^-4 T

$$\vec{F_{tot}} = q(\vec{E} + \vec{v} \times \vec{B})$$

Electric field can do work on a charge, but a magnetic field cannot - force is always perpendicular to motion

Motors

Moving Charges in Magnetic Fields

$qvB = \frac{mv^2}{R}; R = \frac{mv}{qB} = \sqrt{\frac{2mV}{qB^2}}$

Mass Spectrometer
- Heat to ionize
- accelerate over potential difference in magnetic field
- radius is different due to different mass
- separating U-238 and U-235

Cyclotron
- 2 D-shaped halves of ring, with magnetic field in perpendicular axis. Potential difference between halves is switched to increase proton's energy slightly with each rotation
- Time is independent of velocity = $\frac{2\pi m}{qB}$ - assuming no relativistic effects
- Synchrotron - changes switching frequency to compensate for relativistic effects

Modern accelerators are constant-diameter ring, so magnetic field must be increased as velocity increases to keep particle in ring

Cloud chamber
- Supercooled alcohol vapor condenses when hit by ion
- Spirals inward as velocity decreases and radius decreases

- Bubble chamber
  - Liquid hydrogen, vaporizes when hit
  - Higher density

Biot-Savart Law

$B \propto \frac{I}{R}$

Biot-Savart Law

$d\vec{B} = \frac{CId\vec{l}}{R} \times \hat{r}$

where C = 10^-7 = $\frac{\mu_0}{4\pi}$

Straight wire

$B = \frac{\mu_0 I}{2\pi R}$

Wire loop

$B = \frac{\mu_0 I}{2R}$

Maxwell's 2nd Equation

$\oint \vec{B}.d\vec{A} = 0$ unless you have a magnetic monopole

Power lines

$V_a - V_b = IR$

$V_b = V_a - IR$

$V_b I = V_a I - I^2 R$

$V_b I$ = Power consumed

$V_a I$ = Power produced

$I^2 R$ = Power lost

Making wires thicker lowers resistance, but is expensive

Heat loss $\propto I^2 R$ but power $\propto VI$, so higher voltage is more efficient.

But must avoid breakdown voltage / corona discharge

Leyden Jar

Can be charged, disassembled, all parts discharged, and when reassembled, still has a charge.

Happens because of corona discharge to the glass

Most power is stored on the surface of the glass

Ampere's Law, Solenoids

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Ampere's law: With any closed-loop surface penetrated by current-carrying wire

$$\oint \vec{B}.\vec{dl} = \mu_0 I_{penetrating}$$

Solenoids

$$B = \frac{\mu_0 IN}{L}$$

assumes length L substantially longer than radius

Revisiting the Kelvin Water Dropper

- Slight charge imbalance feeds on itself
  - Pulls ions to one side - attracted to oppositely charged ring - falls into bucket and increases charge
- Energy comes from gravity - Electric field opposes current (charged falling water)

Electromagnetic Induction; Faraday's Law

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Changing magnetic field causes current

- in direction that creates a magnetic field opposing change
- Lenz's law

$$\mathcal{E} = -\frac{d\Phi_b}{dt} = \int \vec{B}.\vec{dA}dt = \oint \vec{E}.\vec{dl}$$

- Faraday's Law

$$\oint \vec{E}.\vec{dl} = -\frac{d}{dt}\int \vec{B}.\vec{dA}$$

In the presence of a changing magnetic flux, Kirshoff's law no longer holds

- the electric field is non-conservative
  - it depends on the path
- Kirchhoff's law is a special case of Faraday's when there is no change in magnetic flux

- 2 voltmeters can be connected to the same place in the circuit and read different values

Motional EMF; Dynamos; Eddy Currents

$\Phi_B = \int BdA = xyBcos\theta$

current induced $\propto \frac{d\Phi}{dt}$ so changing area, angle, or magnetic field induces current

Spinning coil in magnetic field

$\omega = \frac{2\pi}{Period}$

$\theta = \omega t$

$\Phi_b = ABcos\omega t$

$-\frac{d\Phi}{dt} = AB\omega sinwt = \mathcal{E}(t)$

$I(t) = \frac{\mathcal{E}(t)}{R}$ - AC current

Dynamo

Changing area by sliding one side of loop

$\Phi_B = lxB$

$\frac{d\Phi_B}{dt} = lvB = |\mathcal{E}|$

$F_{lorentz} = IlB = -F_{applied}$

$P = IlBv = EI; E = lvB$

Eddy current

- Move conducting disk through magnetic field
  - current will be induced -> heat
  - force will be applied opposing motion (magnetic braking)

Displacement Current

Displacement Current

Consider a capacitor:

$$E = \frac{\sigma_{free}}{\kappa\epsilon_0} = \frac{Q_{free}}{\pi R^2 \kappa \epsilon_0}$$

$$I = \frac{dQ_{free}}{dt}$$

$$\frac{dE}{dt} = \frac{I}{\pi R^2 \kappa \epsilon_0}$$

In wire outside of capacitor

$$B = \frac{\mu_0 I}{2\pi r}$$

But inside of capacitor, or using surface that passes between capacitor plates

Current = 0 so magnetic field = 0 ?!?!

-> Ampere's law inadequate

In between capacitor plates, there is changing magnetic field

magnetic flux = $\Phi_E = \int EdA$

Amended Ampere's law: $\oint Bdl = \mu_0 \left(I + \epsilon_0 \kappa \frac{d}{dt} \int E.dA\right)$

Added $\mu_0 \epsilon_0 \kappa \Phi_E$ term is called displacement current

in the presence of dielectric (in capacitor), changing electric field will cause a current inside but in the vacuum, there is none

Maxwell thought a vacuum is a very special dielectric with $\kappa = 1$, but isn't technically true

Displacement current predicts radio waves

Multi-phase power

If multiple coils spin in B field at different angles, the AC current will be offset

If 3-phase power drives 3 coils offset by 120° (x 2 sides) the magnetic field inside will turn with the same frequency as the AC current

magnet inside will follow - synchronous motor

induction motor

conducting object inside will have eddy current that causes torque

APPLICATIONS OF ELECTROMAGNETISM

8.02 Lecture 19

The heart
- -80mV inside heart cells
  - Electric field across cell wall
- Goes to +20mV when activated
  - polarization wave travels downward, causing muscle to contract
  - then depolarization wave travels up, returning original charge and causing muscle to relax
- Wave transfers by moving ions in and out of cells
  - Creates dipole field

Electrocardiogram measures potential difference caused by this field

defribulators / pacemakers

Aurora Borealis
- Electrically charged particles spiral around magnetic field lines
- Solar wind contains charged particles, which are guided into north/south poles by earth's magnetic field
- Interact with nitrogen/oxygen to create light

Superconductivity
- Impossible to have an electric field in a superconductor
- When approached by a magnet, emf must remain 0, so eddy currents will flow to never allow magnetic flux
  - superposition of 2 fields squeezes the field around the superconductor
  - magnetic pressure $P = \frac{B^2}{2\mu_0}$ [N/m^2]
  - eddy current will always repel magnet, no matter what orientation

Maglev train
- magnet moving over conducting plate also creates similar magnetic pressure
- AC Current through coil over plate works similarly and doesn't have to move

INDUCTANCE

$L =$ self-inductance

Unit: Henry (H) = V sec A^-1

$phi_B = LI$

$emf_induced = \frac{-d\phi}{dt} = L\frac{dI}{dt}$

Solenoid

$B = \frac{\mu_0 IN}{l}$

$\phi_b = \pi r^2 NB = \pi r^2 N^2 \frac{\mu_0 I}{l}$

$L = \pi r^2 \frac{N^2}{l} \mu_0$

Every circuit has a finite self-inductance

$\oint Edl = \frac{-d\Phi}{dt} = L\frac{dI}{dT}$

$V - L\frac{dI}{dT} = IR$ - becomes ohm's law when current stops changing

$L\frac{dI}{dt} + IR - V = 0$ - not Kirshoff's rule!

Charging: $I = I_{max}\left(1 - e^{-\frac{R/L}{t}}\right)$

$I_{max} = \frac{V}{R}$

Discharging: $I = I_{max} e^{-\frac{R}{L}t}$

Energy stored in magnetic field

$$\int_0^\infty I^2 R dt = I_{max}^2 R \int_0^\infty e^{-\frac{2R}{L}t} dt = \frac{1}{2} L I_{max}^2$$

$\frac{1}{2} L I^2 = \frac{B^2}{2\mu_0} \pi r^2 l$

$\frac{B^2}{2\mu_0}$ = Magnetic field energy density = total energy / volume [J / m^3]

work to get a current going in a self-inductor (assuming superconductor)

AC Current

$V = V_0 cos\omega t$

$I = \frac{V_0}{\sqrt{R^2 + (\omega L)^2}} cos\omega t - \phi$

$tan\phi = \frac{wl}{R}$

Levitation in previous lecture depends on phase shift

Phase shift of superconductor is always 0

Magnetic materials

external magnetic field can induce magnetic dipoles at atomic level
- or can rotate
- easier at lower temperature

magnetic dipole moment $\mu = nIA$

$B = \kappa_m B_{vacuum}$
$\kappa_m = 1 + x_m$ - slightly change from vacuum field

diamagnetism
- dipole moment is induced to oppose magnetic field
- internal field is smaller than external
- repelled from field
- kappa_m < 1, x_m < 0
    - water x_m = -10^-5

paramagnetism
- randomized magnetic dipoles align with magnetic field
- dipoles can be modelled as flowing current - lorenz force component towards magnet
- or simply N/S / S/N
- causes material to be attracted to magnet
- returns to chaos when field is removed
- normally weaker than weight
    - exception is liquid oxygen
- $\kappa_m > 1$

ferromagentism
- dipoles in material
- domains where dipoles are aligned
- some domains may remain oriented when field is removed
- forces may be stronger than weight
- pulled towards stronger field
- κ_m 10^2 -> 10^5
- Curie temperature where domains cease to exist -> paramagnetic

Hysteresis; Bohr Magneton; Maxwell's Equations

κ_m because dipoles move with field

Magnetic dipole moment of an atom
- Modelling electron with classical mechanics

  $m_e = 9.1x10^{-31}kg$
  $e = 1.6x10^{-19}C$
  $Radius_{orbit} = 5x10^{11}m$ (Bohr radius)

- Electron orbiting atom = current (in opposite direction of electron)
  - creates magnetic field
  - $\mu = IA$
    - A = pi R^2 = 8 x 10^{-21} m^2
    - $F = \frac{e^2}{4\pi\epsilon_0 R^2} = \frac{mv^2}{R}$
      - $v = \sqrt{\frac{e^2}{m4\pi\epsilon_0 R}} = 2.3x10^6 ms^{-1}$
        - 5 millon mph - to the moon in 3minutes
      - $T = \frac{2\pi R}{v} = 1.4x10^{-16}s$
    - $I = \frac{e}{T} = 1.1x10^{-3}A$
    - $\mu = 9.3x10^{-24}Am^2$ Bohr magneton
      - QM says that magnetic moment of all electron in orbits must be a multiple of this number
- Electrons also have spin
- Most atoms/molecules have moment = 1, 2, 3 Bohr magneton

  $\vec{B} = \vec{B_{vac}} + \vec{B}'$
  - B' depends on strength of field and lower temperature

- if $B' \propto B_{vac}$
  - $B = (1 + X_m)B_{vac} = \kappa_m B_{vac}$
  - possibly not true for ferromagnetics

saturation is possible where all dipoles are aligned - ferromagnetics
- approx B' = 2.3T for example (2 bohr magnetons) and specific density
- graph of B vs B_vac flattens and no longer proportional to κ_m

if vacuum field is then reduced to 0
- B' does not fall to 0 - some domains remain aligned - metal has been magnetized
  - Hysteresis curve
    - for one value of B_vac, there are 2 possible values of B'
  - can be demagnetized by slowly decreasing amplitude of current

Ferromagnets change field configuration nearby - pulled towards metal and decreased elsewhere

Maxwell's equations revisited

Ampere's law has to be amended by a factor of κ_m

$$\oint B.dl = \kappa_m \mu_0 \left(I_{pen} + \epsilon_0 \kappa \frac{d\Phi_e}{dt} \right)$$

Transformers; Car Coils; RC Circuits

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RC Circuits

Resistor and capacitor in series

Charge phase

Voltage across capacitor asymtotically increases to battery voltage

$$I(t) = \frac{V_0}{R} r^{-t/RC}$$

Current asymtotically decreases

$$V(t) = V_0 \left(1 - e^{-t/RC}\right)$$

At time RC = time constant or decay time

$I = \frac{1}{e} \frac{V_0}{R}$ (~37% I_0)

$V = (1 - \frac{1}{e}) V_0$ (~63% V_0)

Discharge phase

$$I(t) = -\frac{V_0}{R} r^{-t/RC}$$

Transformers

primary and secondary coils with magnetic flux coupling

$V_1 = -L_1 \frac{dI_1}{dt} = \mathcal{E}_1 = -N_1 \frac{d\Phi_B}{dt}$ by Faraday's law

$$V_2 = -L_2 \frac{dI_2}{dt} = \mathcal{E}_1 = -N_2 \frac{d\Phi_B}{dt}$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

Assuming

R << omega L

No energy loss through eddy currents in core

Perfect flux coupling

then $I_1 V_1 = I_2 V_2$

Spark Plugs - Car coils

High voltage from 12V car battery

Run DC through a transformer where $N_2 >> N_1$

When circuit is broken, high voltage pulse in secondary coil

LRC Circuits

$\oint E.dl = V_c + 0 + IR - V_0 cos\omega t = -L\frac{dI}{dt}$
$I = \frac{dQ}{dt}$
$V_c = \frac{Q}{C}$
$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_0 cos\omega t$
$I = \frac{V_0}{Z}cos\omega t - \phi$
$tan\phi = \frac{x}{R}$

$\omega L - \frac{1}{\omega C} = x = reactance$ (ohms)

impendence $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ (ohms)
above is steady state solution

Resonance

x=0 thus $\omega L = \frac{1}{\omega C}$
$\omega_0 = \frac{1}{\sqrt{LC}}$
$Z = R$
$I_{max} = \frac{V_0}{R}$
$\phi = 0$

$\omega \to 0; Z \to \infty; I_{max} \to 0$ due to capacitor
$\omega \to \infty; Z \to \infty; I_{max} \to 0$ due to inductor

Below resonant frequency, C dominates
Above resonant frequency , L dominates

$\phi > 0$

$\Delta\omega = \frac{R}{L}$ = width of region greater than 70% of maximum current = half power

Quality = $\frac{\omega_0}{\Delta\omega} = \frac{1}{R}\sqrt{\frac{L}{C}}$ (narrowness of curve)

Used to tune radio receivers

usually variable capacitors

Metal detectors

set at resonance; metal nearby changes inductance
2 coils - metal changes magnetic coupling between them

Waves

$T = \frac{2\pi}{\omega}$

$\lambda = vT = \frac{v}{f}$

$f = \frac{v}{\lambda}$

$v = \frac{\omega}{k}$

Standing Waves

$$y_1 + y_2 = y_0 sin(kx - \omega t) + y_0 sin(kx + \omega t)$$
$$= y = 2y_0 sin(kx) cos(\omega t)$$

$$\lambda_n = \frac{2L}{n}$$
$$f_n = \frac{nv}{2L}$$

$v \propto \sqrt{\frac{Tension}{\frac{mass}{length}}}$

$v \propto \sqrt{\frac{temp}{molecular\ weight}}$

Resonance; Electromagnetic waves

Breaking a wine glass demo
Tacoma Narrows Bridge

Electromagnetic Waves

$\vec{E} = E_0\hat{x}cos(kz - \omega t)$

$\vec{B} = B_0\hat{y}cos(kz - \omega t)$

$B_0 = \frac{E_0}{c}$

$\frac{\omega}{k} = c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

$B_0 = \epsilon_0\mu_0 E_0 c$

then apply faraday's law to prove speed of light

Invariants

$\vec{E} \perp \vec{v}$

$\vec{B} \perp \vec{v}$

$\vec{E} \perp \vec{B}$

$\vec{E}$ and $\vec{B}$ are in phase

$\hat{E} \times \hat{B} = \hat{v}$

Electromagnetic wave energy

Energy density

electric: $U_E = \frac{1}{2}\epsilon_0 E^2$

magnetic: $U_B = \frac{1}{2\mu_0} B^2$

[J m^-3]

Energy density in the magnetic field of a travelling wave is the same as the energy density of the electric field

$$U_total = \epsilon_0 E^2 = \epsilon_0 EBc$$

Energy flux

$U_totc = \epsilon_0 EBc^2 = \frac{EB}{\mu_0}$ [J m-^2 sec]

Poynting vector

$\vec{S} = \frac{E \times B}{\mu_0}$

E and B are always perpendicular, so cross product is unnecessary, but predicts direction of wave's travel (direction of S)

Time average $\bar{S} = \frac{1}{2}\frac{E_0 B_0}{\mu_0} = \frac{1}{2}\frac{E_0^2}{\mu_0 c}$

Producing EM Waves

Accelerating charge

The change in electric field can not propagate faster than the speed of light, so the change ripples out as a wave

Maximum wave goes in directions perpendicular to charge motion

Momentum of photon $p = \frac{energy of one photon}{c}$

$\frac{S}{c} = \frac{energy}{cm^2 sec}$ is radiation pressure

$\frac{\bar{S}}{c}\alpha = Pressure$

$\alpha = 1$ Full absorption

$\alpha = 0$ Fully transparent

$\alpha = 2$ Full reflection

Solar sails

Second comet tail due to radiation pressure

blue is solar wind

white-yellow is radiation pressure

A charge q oscillates with acceleration $\vec{a}$. The electric charge $\vec{E}$ measured at a point P, a distance $\vec{r}$ and angle θ away from the direction of oscillation

$\vec{E} \perp \vec{r}$
$\vec{a}, \vec{r}, \vec{E}$ are in the same plane
$E \propto q\frac{a}{r}sin(\theta)$
$S \propto q^2\frac{a^2}{r^2}sin^2(\theta)$

Polarization

Refraction

8.02 Lecture 29

All beams in one plane

$\theta_3 = \theta_1$

$\frac{sin\theta_1}{sin\theta_2} = \frac{n_2}{n_1}$
Snell's Law

$n_{water} = 1.3$
$n_{glass} = 1.5$

Total reflection when $\theta_1 > Critical Angle$
$sin\theta_{cr} = \frac{n_2}{n_1}$ ($n_1 > n_2$)

Speed of light in non-vacuum
$c = \frac{1}{\sqrt{\epsilon_0\mu_0\kappa\kappa_M}} = \frac{c}{\sqrt{\kappa\kappa_M}} = \frac{c}{n}$
$n = \sqrt{\kappa\kappa_M}$

but dielectric constants depend on frequency- at high frequency, the diploles can't follow fast enough. lower at high frequencies
n_{water} for radio waves = 8.9

Human color vision

Problems with 3-color model
- Bentham top
  - different neuron phase delays between colors
- Edwin Land Slides
  - Black and white slides, when overlayed, one with a red filter, show multiple colors
  - http://people.msoe.edu/~taylor/eisl/land.htm

Polarization, Scattering

Malus's Law

$$I_1 = I_0 cos^2\theta$$

Reflected and refracted light become polarized

$$E_{par_{refl}} = E_{par_{inc}} = \frac{n_1 cos\theta_2 - n_2 cos\theta 1}{n_1 cos\theta_2 + n_2 cos\theta_1} = -E_{par} = \frac{tan(\theta_1 - \theta_2)}{tan(\theta_1 + \theta_2)}$$

when $\theta_1 + \theta_2 = 90^\circ$, $E_{par} = 0$ - 100% polarized light

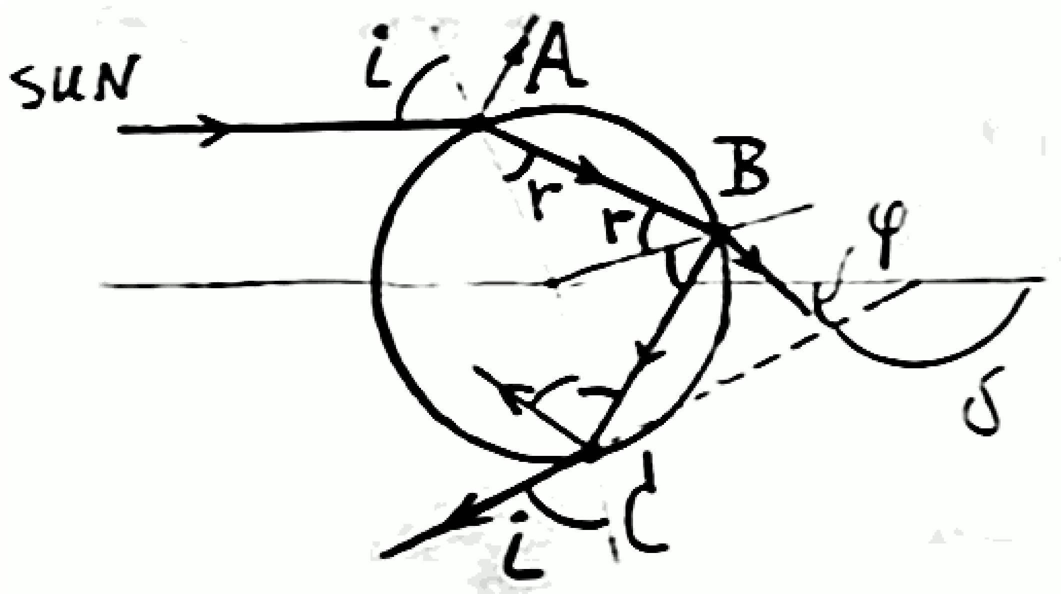
$$\frac{n_2}{n_1} = tan\theta_1$$

Brewster Angle

Only apples to dielectrics, not conductors

Light scattered off dust (<10 micron) at right angle becomes polarized
- oscillates electrons in particle; accelerated charged particles produce light
- particle re-radiates at same frequency, but direction changes, hence scattering
- At 90°
  - light has to be polarized because E-field must be perpendicular to r (line from dust to you), and in the plane of r and a
- Partially polarized at other angles
- Unpolarized at angle 0
- Probablitiy of scattering 10x higher for blue light than red light
  - Sky blue because of this phenomenon
    - linearly polarized at 90° away from sun
  - Sun rise and set are red because they are just above the horizon and blue light has already scattered out
  - Demo at 46:00

Rainbows



$\delta = 180 + 2i - 4r$
$\delta_{min} = 138$
$\phi_{max} = 42$
- 40.7 for blue
- 42.4 for red

Angle of reflection off back of raindrop is very close to Brewster angle
- polarized at tangent to bow

Creates a cone
- inside is white
- different maximum angle for each color - rainbow is seen on border
- raining in direction away from sun
  - 42 degrees away from your shadow line

Secondary rainbow
- radius 50.4°
- colors reversed

22 degreee halo
- ice crystals in high atmosphere
- also around moon

Glories
- radius depends on size of water drops
- shadow of airplanes
- result of diffraction
- fog bow

INTERFERENCE

8.02 Lecture 33

$E = amplitude^2$
$amplitude \propto \frac{1}{r}$

Destructive Interference

$r_2 - r_1 = (2n+1)\frac{\lambda}{2}$

sin theta_n = (2n+1) frac{lambda}{2d}

Constructive Interference

$r_2 - r_1 = (n)\lambda$

sin theta_n = frac{nlambda}{d}

Number of maxima / minima = $\frac{2d}{\lambda}$
Lines form hyperbola

Young Double-slit experiment

Multiple-slit interference, diffraction, resolution

Gratings

- Multiple slits
- All constructively interfere at the same points -> same as 2 slit
- Peaks are thinner, destructive troughs are larger (proportional to number of slits)
- Colors separate
- $d =$ distance between slits
- $\theta_n = \frac{n\lambda}{d}$
- N-1 minima between each maxima (N = number of slits)

Diffraction

- Single-slit interference
- narrower the slit, the wider the diffraction pattern
- $\theta = \lambda a$ where a = width of slit

Resolution

- Limited by diffraction
- Rayleigh criterion
  - $\theta = \frac{1.2\lambda}{a}$
- Human eye about 1 arcminute

Doppler Effect, Cosmology

Doppler effect

Sound

$$f' = f\frac{v_{sound} - v_{receiver}}{v_{sound} - v_{transmitter}}$$

Light

$$f' = f\left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}}; \beta = \frac{v_{relative}}{c}$$

Special relativity negates the difference between transmitter and receiver - only relative velocity matters

Used to determine velocity of stars by comparing known spectral lines

Hubble's law

v = H d

H = 72 km/s / Mpc

$$v = d\left(\frac{1}{R}\frac{dR}{dt}\right)$$

$$H = \left(\frac{1}{R}\frac{dR}{dt}\right)$$

Hubble's constant changes with time

Open/Closed/Flat