Kevin Mehall

Projectile motion

(1) $x(t) = x_0 + v_{0_x}t$

(2) $v_x(t) = v_{0_x}$

(3) $y(t) = y_0 + v_{0_y}t + \frac{1}{2}a_y t^2$

(4) $v_y(t) = v_{0_y} + a_y t$

(5) Time of peak = $t_p = \frac{v_0 \sin\alpha}{g}$ (solve (4) for 0)

(6) Height of peak = $h = \frac{(v_0 \sin\alpha)^2}{2g}$ (plug (5) into (3))

Distance traveled x$= \frac{v_0^2 \sin 2\alpha}{g}$

v_0 is squared because doubling the speed makes both the time in flight double and the horizontal velocity double

Max distance at $\alpha = 45°$ because $\sin 2 * 45° = 1$

30° and 60° will land at same place
(6) - $\sin(2 * 30°) = \sin(2 * 60°)$,
but take longer and go higher

Simple Harmonic Motion

Springs

(1) $T = 2\pi\sqrt{\frac{m}{k}}$

(2) $x = A\cos\omega t + \varphi$ where
A = amplitude
$\omega =$ angular frequency (rad/s)

Period = $T = \frac{2\pi}{\omega}$

(3) $-\omega^2 x + \frac{k}{m}x = 0$ - differential equation, must be true for any x, so $\omega^2 = \frac{k}{m}$

$\dot{x} = -A\omega\sin\omega t + \varphi$

accel = $a = \ddot{x} = -A\omega^2\cos\omega t + \varphi = -\omega^2 x$

(4) $\omega = \sqrt{\frac{k}{m}}$

Pendulum

$T = 2\pi\sqrt{\frac{l}{g}}$

Spring dependent on mass, pendulum not dependent on mass - spring provides fixed force proportional to distance, while pendulum is dependant on gravity, which is proportional to mass. Spring oscillations are used to weigh things in space.

Work and Energy

(1) $W = \int_A^B F\,dx$

$F = ma = m\frac{dv}{dt}$ and $dx = v\,dt$

(2) Substitute in (1): $W = \int_A^B mv\,dv = \frac{1}{2}mv^2$ (kinetic energy)

Conservative force: a force whose work is independent of path
ex: gravity, spring forces, but not friction

Potential energy = $U = mgh$

(3) $U_a + KE_a = U_b + KE_b$ (only true of conservative force)

$h > 2.5r$ for a ball to be dropped from height h and then go around a loop of radius r (due to the necessary centripetal acceleration $\frac{v^2}{r} > g$)
$gh = \frac{1}{2}v^2$
$g(h-2r) = \frac{1}{2}v^2$
$2g(h-2r) = v^2$
$g(2h-4r) > gr$
$2h - 4r > r$
$h > \frac{5}{2}r$

Gravitiational Potential Energy = $W = \int_\infty^R \frac{mMG}{r^2}dr = \frac{-mMG}{R}$

Drag and Resistive Forces

(1) $F_{res} = -(k_1 v + k_2 v^2)\hat{v}$

Spheres $|F_{res}| = C_1 r v + C_2 r^2 v^2$ where
1st term is viscous term - stickyness of medium (function of temperature) C_1 in $kgm^{-1}s^{-1}$
2nd term is pressure term - proportional to r^2 because of surface area of circle C_2 in kgm^{-3} (density)

When $F_{res} = mg$, constant velocity = terminal velocity

Critical velocity when $C_1 r v = C_2 r^2 v^2$;
$v_{crit} = \frac{c_1}{c_2 r}$

mass of a sphere: $m = \frac{4\pi}{3}\rho r^3$

Regime 1 when v << vcrit

$mg = C_1 r v_{term}$ therefore $v_{term} = \frac{mg}{C_1 r} \propto \sqrt{r}$

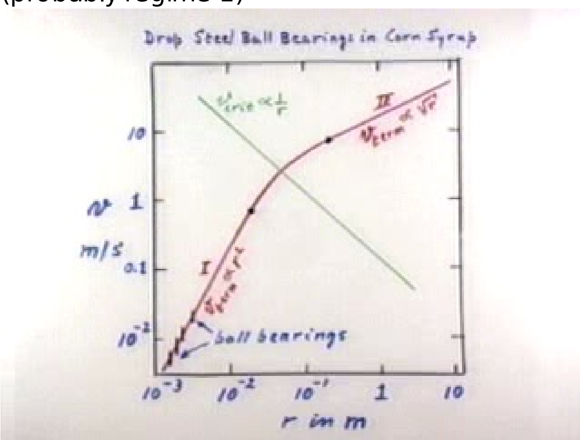
Regime 2 when v >> vcrit

$mg = C_2 r^2 v_{term}^2$ -> $v_{term} = \sqrt{\frac{mg}{c_2 r^2}} \propto \sqrt{r}$

Corn Syrup

Data [on OCW]

(probably regime 1)



Air

C1 = 3.1 e -4
C2 = 0.85
Vcrit = 3.7 e -4 / r
(probably regime 2)

Potential Energy and SHM

Negative work -> gravity does positive work (and vice versa)

Potential energy = u

Springs

$u = \int F\,dx = \int_b^a kx\,dx = \frac{1}{2}kx^2$

$\frac{du}{dx} = -F_x$

= Force is always in the direction opposite increasing potential energy

Simple gravity

$u = +mgy$

$\frac{du}{dy} = mg = -F_y$

General Gravity

$u = \frac{-mMG}{r}$

$\frac{du}{dr} = \frac{mMG}{r^2} = -F_g$

Stable / Unstable equilibrium

On ramps/hills with x position and height y:

Equilibrium points where $\frac{du}{dx} = 0$

Stable equilibrium where u'' is positive, unstable where negative

- Ball rolling on a track shaped as a circle segment has same equation as pendulum
- Pendulum: Force is always perpendicular to direction of motion, so no work is done total kinetic + potential energy constant

Circular Orbits, Power

(1) $E = \frac{1}{2}mv^2 - \frac{mMG}{r}$ total kinetic + potential energy (with PE negative; 0 at infinity)

(2) $v_{esc} = \sqrt{\frac{2M_eG}{R}}$

= 11.2 km/s on Earth

Orbit unbound when E >= 0 ; bound when E < 0

(3) $v_{orbit} = \sqrt{\frac{M_eG}{R}}$

(4) $T = \frac{2\pi R^{\frac{3}{2}}}{\sqrt{GM_e}}$

escape velocity is $\sqrt{2}$ times larger than orbital

(5) Substitute (3) into (1): $\frac{1}{2}m\frac{MG}{r} - \frac{mMG}{r} = \frac{1}{2}u = -KE$

Power

$P = \frac{dW}{dt}$

"We also know power in terms of political power. That's very different: Political power, you can do no work at all in a lot of time, and you have a lot of power. Here in physics, life is not that easy."

$dW = F\dot{d}s$ (s = displacement)

$P = F\dot{v}$
Dot product, so P is 0 if F and V are perpendicular

Electicity consumption per capita (US) = equivalent power output of 100 people

Momentum

$p = mv$ (kg m / s)

$F = ma = m\frac{dv}{dt} = \frac{d(mv)}{dt} = \frac{dp}{dt}$
-> if a particle changes its momentum, a force must have acted on it (and vice versa)

Conservation of momentum demo at ~ 30:00

Center of Mass

Break an object into i small pieces with mass m_i:
$M_{tot}\vec{r}_{cm} = \sum m_i\vec{r}_i$
$\vec{v}_{cm} = \frac{1}{M_{tot}}\sum m_i\vec{v}_t = \frac{1}{M_{tot}}\vec{P}_{tot}$

$\vec{P}_{tot} = M_{tot}\vec{v}_{cm}$
$\frac{dP_{total}}{dt} = F_{tot_{ext}} = M_{tot}\vec{a}_{cm}$

Same as F=ma : proves that an object can be treated as a point mass at its center of mass. Center of mass follows smooth motion, even if the object tumbles.

Demo at ~48:00

Collisions

(1) $K + Q = K'$
$K > 0$ superelastic
$K = 0$ Perfect elastic
$K < 0$ inelastic

Perfect elastic collisions with stationary object

$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1{v_1'}^2 + \frac{1}{2}m_2{v_2'}^2$
$v_1' = (\frac{m_1-m_2}{m_1+m_2})v_1$
$v_2' = (\frac{2m_1}{m_1+m_2})v_1$

$m_1 \gg m_2 : v_1' = v_1; v_2' = 2v_1$

$m_1 \ll m_2 : v_1' = -v_1; v_2 = 0$

$m_2 = m_1 : v_1' = 0; v_2' = v_1$ (Newton's cradle)

Elastic collision demo at ~15:00

Center of mass frame of reference

Total momentum from center of mass is 0

Velocity of center of mass is constant

Relative to the center of mass, after an elastic collision, velocities are the same and in the opposite direction.

$M_{tot}r_{cm} = m_1r_1 + m_2r_2$
derivative of r is v ->
$v_{cm} = \frac{m_1v_1+m_2v_2}{m_1+m_2}$

Inelastic collisions -> all kinetic energy lost in frame of center of mass (internal kinetic energy)

inelastic collisions demo at ~42:30

Ballistic Pendulum demo at ~9:00

(1) $\vec{I} = \int\limits_0^{\Delta t} F\,dt = \int\limits_0^{\Delta t} \frac{dp}{dt}dt = \int\limits_{p_i}^{p_f} dp = p_f - p_i$

$F = \frac{I}{\Delta t}$

Thrust = $F = \frac{dm}{dt}v$

Rocket Equation

u = velocity of exhaust
At time t: $p = mv$
At time $t + \Delta t : p = (m - \Delta m)(v + \Delta v) + \Delta m(v - u) = mv + m\Delta v - u\Delta m$
$\Delta p = 0 = m\Delta v - u\Delta m$
$\frac{dp}{dt} = 0 = ma - u\frac{dm}{dt}$
$ma = F_{thrust}$

... (more deriviation)

$v_f - v_i = -u\ln(\frac{m_f}{m_i}) - gt$

Rotating Rigid Bodies

ω = angular velocity (rad / s)

α = angular acceleration (rad/s^2)

I = moment of inerta (kg m^2)

$v_{tan} = \omega R = \dot{\theta}$

$a_{tan} = \dot{\omega} R = \ddot{\theta} R = \alpha R$

$x \Rightarrow \theta$
$v \Rightarrow \omega$
$a \Rightarrow \alpha$
$m \Rightarrow I$

e.g $\theta = \theta_0 + \omega_0 + \frac{1}{2}\alpha t^2$
$\omega = \omega_0 + \alpha t$

$K = \frac{1}{2} I \omega^2$

Given a point particle of mass m at position r away from center of disc:
$K_i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i \omega^2 r_i^2$

$K = \frac{w^2}{2} \sum m_i r_i^2 = I$ (moment of inertia)

Disk rotating around and perpendicular to center:
$I_c = \frac{1}{2} m R^2$

Solid sphere:
$I = \frac{2}{5} m R^2$

Rod rotating around center point perpendicular to length:
$I = \frac{1}{2} m l^2$

Parallel axis theorem:
When rotating around an axis $l' \parallel l$ distance d away from l $I' = I + md^2$

Rotational kinetic energy of sun = 1.5 x 10^35 J

If world power consumption came from slowing earth's rotation, the day would be 2.4 seconds longer after a year

Crab pulsar - high rotational kinetic energy turning into X rays

Angular Momentum and Torque

Angular momentum = L

At point r from origin q: $\vec{L}_q = \vec{r}_q \times \vec{p} = (\vec{r}_q \times \vec{v})m$

(1) $|L_q| = mvr_q sin\theta$

Relative to center, an object moving in a circle has constant angular momentum

(2) Torque = $\tau_Q = \frac{d\vec{L}}{dt} = \vec{r}_q \times \vec{F}$
Torque is change in angular momentum

No torque in circular motion because F is in opposite direction as r

$L = I_c\omega$

When there is rotation around center of mass, angular momentum will be constant no matter which origin point is chosen (called spin angular momentum)

When spinning, and mass is brought inwards, I decreases because radius falls. Therefore, to maintain constant angular momentum, velocity must increase. Demo at 24:45

In a closed system, all internal torques cancel out. Total angular momentum will not change unless acted upon by an external torque. (just like force and momentum)

(3) $\frac{d\vec{L}_q}{dt} = \tau_{Q_{ext}}$

When a star collapes, moment of inertia falls, and angular velocity goes up

$\alpha = \ddot{\theta}; \quad \omega = \dot{\theta}$

Rotation about Q

(4) $|\tau_Q| = I_Q \alpha_Q$

(5) $|L_q| = I_Q \omega_Q$

Rotation about any stationary axis through center of mass

(6) $|L_{cm}| = I_{cm} \omega_{cm}$

Circular motion (example: planets)

Use eq 1 or 5:
$|L_c| = r_c mv = mr_c^2 \omega = mR^2 \omega$

Torque relative to center is 0

Rod rotating around off-center point p

$|L_p| = I_p \omega = \frac{1}{12} Ml^2 + Md^2 \omega$

Force on point P, but 0 torque relative to (only) point p

Rod rotating around center point p

No net force at all, so no torque

Spin angular momentum, all torque equal relative to anywhere

$|L_c| = I_c \omega = \frac{1}{12} Ml^2 \omega$

Rod hit with off-center impulse on frictionless surface (Problem 7.9)

at 15:00

Will rotate about center of mass, because F=ma has to apply for center of mass, and spin around anywhere else would make the center of mass go in loops

$I = F\Delta t = Ma_{cm}\Delta t; a_{cm}\Delta t = v_{cm}$
$V_{cm} = \frac{I}{M}$
(normal physics holds for center of mass)

Choose center or point of impact as origin, and either will work

Oscillation of solid pendulum

Length = l; Hanging by point *P* a distance *b* away from center *C*

Use P as reference point so torque is 0

$|\tau_{pt}| = Mgb \sin\theta_t = \vec{r} \times \vec{F} = -I_p \alpha_t$

Small angle approximation: $sin \theta = \theta$

$Mgb\theta + I_p \ddot{\theta} = 0$

$\ddot{\theta} + \left(\frac{Mgb}{I_p}\right) \theta = 0$ -> SHO!

$\omega_t = \omega_{max}\cos(\omega t + y)$ this omega is angular frequency, not velocity!

$$\omega = \sqrt{\frac{mgb}{I_p}}$$

$$T = 2\pi\sqrt{\frac{I_p}{mgb}}$$

Still independent of mass because I includes a mass

$$I_p = \frac{1}{12}Ml^2 + Mb^2$$

$$T = 2\pi\sqrt{\frac{\frac{1}{12}l^2 + b^2}{gb}}$$

Oscillation of a hoop

Point P on hoop; radius R

$$|\tau_p| = MgR\sin\theta = -I_p\ddot{\theta}$$

All mass at radius R, so $I_{cm} = MR^2$

$$I_p = I_{cm} + MR^2 = 2MR^2$$

$$\ddot{\theta} + \frac{MgR}{I_p}\theta = 0$$

$$T = 2\pi\sqrt{\frac{2MR^2}{MgR}} = 2\pi\sqrt{\frac{2R}{g}}$$

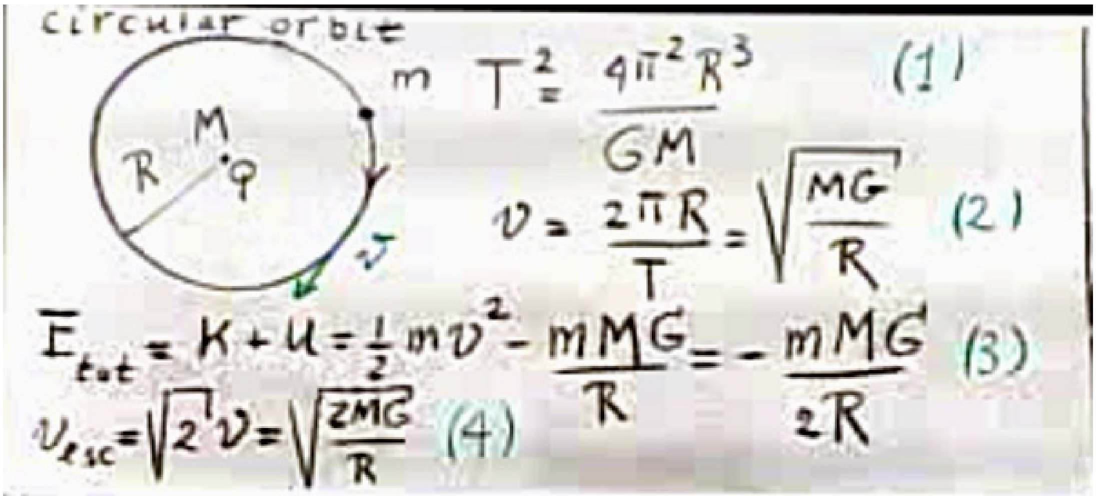
Same as pendulum with all mass at end!

Kevin Mehall

KEPLER'S LAWS

8.01 Lecture 22

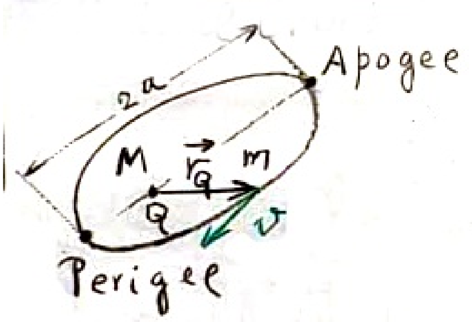
Circular Orbits



Kepler's Laws

1. Orbits are ellipses, with sun at one focus
2. Equal area in equal time
3. $T^2 \propto \bar{d}^3$

Elliptical Orbits



Apogee to Perigee = 2a, where a is semi-major axis

(5) $E_{tot} = K + U = \frac{1}{2}mv^2 - \frac{mMG}{r} = -\frac{mMG}{2a}$

(6) $T^2 = \frac{4\pi^2 a^3}{MG}$

(7) $v_{esc} = \sqrt{\frac{2MG}{r}}$ (set E = 0 in (5))

r and v change with time in an ellipse

Two orbits with the same a but different shape have the same energy and period

Everything can be found if you have M, v_0, r_o, Φ_0 at any point D

Find E_{tot} with (5)
Find T with (6)

Angular momentum must be conserved around point Q (the star)
Angular Momentum: $|L_q| = mv_D r_D sin\Phi_D = mv_p(QP)$
Energy: $E_{tot} = \frac{1}{2}mv_p^2 - \frac{mMG}{QP} - \frac{mMG}{2a}$(velocity perpendicular, so sin=1)

2 equations, 2 unknowns -> quadratic with 2 solutions - v_p, v_a

$|QP|v_p = |QA|v_a$ to conserve angular momentum

Orbital Change

Firing a rocket tangential to orbit (increasing velocity)

Total Energy increases -> elliptical orbit with $2a > 2R, T' > T$
(or opposite if you fire the other way and reduce energy)

Ham sandwich problem

2 people in circular orbit, distance $f 2\pi R$ apart

Trying to launch a ham sandwich from one to the other

$T_{sandwich} = (1-f)T_{astronaut}$, so the other astronaut is at the same place as the sandwich on astronaut's next orbit

$\sqrt{\frac{4\pi^2 a^3}{GM}} = (1-f)\sqrt{\frac{4\pi^2 R^3}{GM}}$
$a^{\frac{3}{2}} = (1-f)R^{\frac{3}{2}}$ $a = R(1-f)^{\frac{2}{3}}$
then use (5) to find velocity

Other solutions for other integer numbers of orbits before catch for both astronaut and sandwich

Doppler Effect and Astronomy

Sinusoidal frequency/time graph when sound source moves in a circle

$\frac{f_{max}}{f} = v_{transmitter}$

Period can be determined by timing on peak to the next

And solve for radius in $\frac{2\pi R}{T} = v_{tr}$

With light, whether source or receiver is moving doesn't matter

$f' = f\left(1 + \frac{v}{c} cos\theta\right)$

if $\theta < 90^\circ \quad f' > f$ (towards you)
if theta > 90^circ quad f' < f (away)

$\lambda = cT = \frac{c}{f}$

$\lambda' = \lambda\left(1 - \frac{v}{c} cos\theta\right)$

Absorption lines at precise frequencies, shifted by doppler effect

Can determine velocity, period, radius for binary stars

Use Kepler's law, and masses of both stars can be found

X-ray binaries

Binary system including neutron star - If inner lagrangian point (where gravity is balanced) is inside surface of big star, matter will fall towards neutron star, forming accretion disk

Speed at which matter hits neutron star from (close enough to) infinity is same as escape velocity

10g object => more kinetic energy than atomic bomb, falling at 0.8c

From high temperature, neutron star emits X rays

Magnetized, and gas is plasma, so it can ony fall at poles -> X ray hotspots at poles -> if it's spinning, we see pulses

Black Holes

because escape velocity at event horizon is speed of light, $R = \frac{2MG}{c^2}$

Doppler shift can't be observed directly, but mass can be found if you can estimate the mass of partner

Cygnus X-1 - first discovered black hole X ray binary

ROLLING MOTION AND GYROSCOPES

Object rolling down hill at angle β with friction

Pure Roll

- object not skidding or slipping
- after one rotation, center of mass has moved $2\pi R$
- $v_q = v_{circ} = \omega R$

normal force = $Mgcos\beta$

$a_{cm} = \dot{\omega} R = \alpha R$

N and mg go through center -> no effect on torque

$|\tau_{center}| = RF_{friction} = I_q\alpha = \frac{I_q a}{R}$ (1)

$Ma = Mg\sin\beta - F_{friction}$ (2)

$Ma = Mg\sin\beta - \frac{I_q a}{R^2}$

$a = \frac{MR^2 g\sin\beta}{MR^2 + I_q}$

Solid cylinder -> $I_q = \frac{1}{2}MR^2$

$a = \frac{2}{3}g\sin\beta$

Hollow Cylinder -> $I_q = MR^2$

$a = \frac{1}{2}g\sin\beta$

Does not depend on mass

Solid cylinder accelerates faster than hollow

Gyroscopes

Spin angular momentum will always move in the direction of torque applied

Precession

Angular momentum chases the torque

$\omega_{precession} = \frac{\tau}{L_{spin}} = \frac{rMg}{I_q\omega_s}$
only applies when angular momentum in wheel is much greater than angular momentum caused by precession around string axis

Increasing weight on the end will increase precession frequency

If you mount a gyroscope in a way that prevents any torque from being applied, spin angular momentum will always stay in the same direction

Kevin Mehall

STATIC EQUILIBRIUM

Static equilibrium: $\sum F = 0$ and $\sum \tau = 0$ around any point

Friction can be used to help balance:

Frictional force on a rope wrapped around a rod: integral of individual forces around angle

$\frac{T_2}{T_1} = e^{\mu\theta}$ where T2 is heavier than T1

Object hanging by point P other than center of mass: stable when P aligned vertically with center of mass so sine = 0 and torque = 0. Stable equilibrium when CM below P

Rope Walker - If you put the center of mass below the rope, it's very stable

ELASTICITY

$F \propto \Delta l$ (hooke's law)

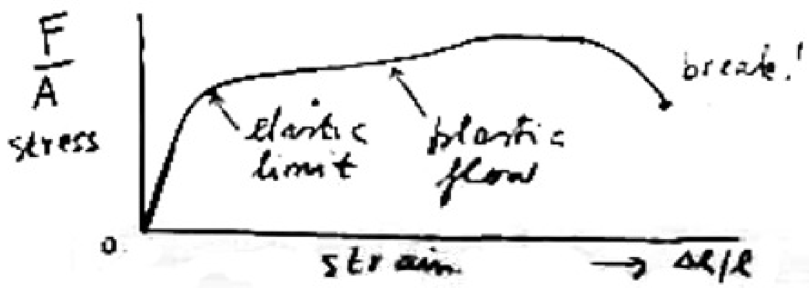
2 springs with same k in series: double extension at same force
in parallel: half the extension for same force

Same laws apply for solid rods

- $\Delta l \propto \frac{Fl}{a}$
- $\frac{F}{A} = Y\frac{\Delta l}{l}$ where:
    - $\frac{F}{A}$ = stress (same units as pressure)
    - $\frac{\Delta l}{l}$ = strain (dimensionless)
    - Y = young's modulus
        - Higher value of Y -> difficult to make it longer

Breaking point - maximum tensile strength (stress)

Elastic limit - Point where material deforms and Hooke's law no longer holds - will not return to original length when force is removed



Graph of $\frac{F}{A}$ (stress) vs $\frac{\Delta l}{l}$ (strain):

- Work beyond elastic limit goes into heat
- Horizontal component - plastic flow - continues to stretch with no additional force
- Then rod pinches and stress falls

Apparatus to measure Δl - A mirror allowed to pivot around bottom point is linked to wire. When the wire stretches, the mirror will tilt. Tilt is measured with laser bounced off mirror.

$F = \frac{-yA\Delta l}{l}$

($\frac{yA}{l}$ is like k)

Speed of sound depends on Young's modulus - $v_s = \sqrt{\frac{y}{density}}$ - because higher Young's modulus means material is stiffer so vibrations travel faster

FLUID MECHANICS

$P = \frac{F}{A}$

Pascal's Principle - in absence of gravity, pressure applied to an enclosed fluid is transmitted undiminished to every point in the fluid and to the walls of the container

Causes force perpendicular to container

Hydraulic Jack - $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ and $A_1 d_1 = A_2 d_2$

Hydrostatic pressure

Pressure increase at depth due to gravity

Evaluating a chunk of ocean with height y, surface area A, density ρ

$F_1 - F_2 - \Delta m g = 0$ to balance y forces
$P_y A - P_{y+\Delta y} a - A \Delta y \rho g = 0 \lim_{\Delta y \to 0} \frac{P_{y+\Delta y} - P_y}{\Delta y} = -\rho g = \frac{dP}{dy}$

$\int_{P_1}^{P_2} dP = -\rho g \int_{y_1}^{y_2}$
$P_2 - P_1 = -\rho g (y_2 - y_1)$
$P_1 - P_2 = \rho g (y_2 - y_1)$ (Pascal's law)

Barometric Pressure

Same is true for gases, but more complicated since density changes
10^5 Pa = 1 atm

Barometer - Column of liquid coming from larger container exposed to atmosphere. Liquid will only go up so far - $P = \rho g h$
760mm for mercury, ~10m for water at 1atm

Buoyancy

Buoyant force = $F_b = A\rho_{fluid}gh$ - same as weight of fluid displaced

Archimedes Principle - The buoyant force of an immersed body has the same magnitude as the weight of the fluid which is displaced by the body

Archimedes determined a crown was gold by weighing it in air ($W_1 = V\rho g$) and underwater ($V\rho g - V\rho_{water}g$)
$\frac{W_1}{W_1 - W_2} = \frac{\rho}{\rho_{water}}$

92% volume of iceberg is underwater because $Mg = V_{iceberg}\rho_{ice}g = V_{underwater}\rho water g$
$\frac{V_{underwater}}{V_{iceberg}} = \frac{\rho_{ice}}{\rho_{water}} = 0.92$

Floating - $\rho_{water} > \rho_{object}$

Helium balloon will still rise relative to perceived gravity caused by acceleration

Bernoulli's Equation

Pressure is energy per unit volume and ρgh is potential energy per volume

$\frac{1}{2}\rho v^2 + \rho gy + P_y = C$

Liquid travelling through an expanding pipe

$A_1v_1 = A_2v_2$ since quantity of water is conserved
$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2$
therefore $P_1 < P_2$

Siphon

Pressure = 1 atm on both sides of pipe

$v_2 = 0$ because it's such a small amount of water compared to area of top

$\frac{1}{2}\rho_{liquid}v_1^2 + \rho gy_1 = \rho gy_2$
$v_1 = \sqrt{2gh}$ (same if a solid were dropped from that height)

Solid pendulum

$\tau = I\alpha = -(r \times F)$

$\theta = \theta_{max}\cos(\omega r + \phi)$ [ω is ang. frequency, not velocity]

length of rod = b

$\omega = \sqrt{\frac{bMg}{I_p}}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I_p}{bMg}}$
independent of mass because there will be a mass in I

Rod

of mass M, length L

use parallel axis theorem to find I relative to pivot point

$T = 2\pi\sqrt{\frac{\frac{2}{3}L}{g}}$

Traditional Pendulum

$T = 2\pi\sqrt{\frac{L}{g}}$
(same result as before, but now we can do it in terms of torque)

Ring

$T = 2\pi\sqrt{\frac{2R}{g}}$
(derived in lec 21)

Disk

$T = 2\pi\sqrt{\frac{\frac{3}{2}R}{g}}$

Liquid pendulum

Tube bent into U shape

Mass M
density ρ
Area A
Length of liquid (along tube) l
Height of liquid on right side = y
y = u at equilibrium

M = A l ρ

Velocity always same because A is constant - $\dot{y}$

$\delta M = Ay\rho$

Assume conservation of energy - $\frac{1}{2}M\dot{y}^2 + \delta Mgy = Constant$
$\frac{1}{2}Al\rho\dot{y} = A\rho gy^2 = C$

$\frac{d}{dt} \Rightarrow \ddot{y} + \frac{2g}{l}y = 0$

$T = 2\pi\sqrt{\frac{l}{2g}}$
(same as pendulum with string length half of l)

Torsional Pendulum

κ is torsional spring constant (Nm / rad)

$\tau_p = -\kappa\theta = I_p\alpha$
$\ddot{\theta} + \frac{\kappa\theta}{I_p} = 0$

$t = 2\pi\sqrt{\frac{I_p}{\kappa}}$

κ is a function of cross-sectional area, length, and material

Forced Oscillations and Resonance

$ma = -kx + F_o cos(\omega t)$ ω is forced frequency

$\ddot{x} + \frac{k}{m}x = \frac{F_o}{m}cos(\omega t)$

Steady state: $x = Acos\omega t$

$A\left(\frac{k}{m} - \omega^2\right) = \frac{F_0}{m}$ and $\omega_0^2 = \frac{k}{m}$ (free oscillation) so
$A = \frac{\frac{F_0}{m}}{\omega_0^2 - \omega^2}$

if $\omega << \omega_0 \quad a = \frac{F_0}{k}$

if $\omega >> \omega_0 \quad A \to 0$

if $\omega = \omega_0 \quad A \to \infty$ (resonance)

A system has as many resonant frequencies as masses in the system
A string is equivalent to infinite number of masses

resonant frequencies - harmonics - $f_n = nf_1$

in a tube - $f_n = \frac{nv}{2L}$

Breaking a wine glass demo at ~40:00

Tacoma narrows footage at ~42:00

Kevin Mehall

Heat - Thermal Expansion

Celcius vs Fahrenheit vs Kelvin

Length Expansion proportional to delta temperature
$\Delta L = \alpha L \Delta T$ (α = linear expansion coefficient)

Bimetal strips

Volume increase = $(L + \Delta L)^3 = L^3(1 + \frac{\Delta L}{L})$

When x is small, $(1+x)^n$ can be approximated by $1 + nx$

$\Delta V = 3\Delta L L^2 = 3\alpha V \Delta T$
$\beta = 3\alpha$

Water expands when cooled 4°C - 0°C - negative β - and ice expands further

Kevin Mehall

Kinetic Gas Theory

$PV = nRT$
$R = 8.3 J/K$

1 mol = about 24 liters

$PV = NkT$
$k = \frac{R}{avogadro}$

Assuming temperature is constant (isothermal atmosphere) $P_h = P_o e^{\frac{-h}{H_o}}$
$H_0 = 8km$

QUANTUM MECHANICS

electron can only exist in well-defined energy levels

$E = hf$

$E = \frac{hc}{\lambda}$

emission spectrum

$\lambda = \frac{h}{p}$

Heisenberg Uncertainty Principle - the very concept of exact position of an object and its exact momentum together have no meaning in nature

$\Delta p \Delta x > h/2\pi = \hbar = 10^-34 Jsec$

Kevin Mehall

High-Energy Astrophysics

X-rays must be observed from above atmosphere

Sun-like stars don't emit many X-rays

[Slides on the balloons used to get above the atmosphere]

X-ray binaries: matter goes from donor to neutron star, falling at high velocity -> heats up star, so spectrum is primarily X-ray

Rotating; when x-ray beam hits earth, we see pulse

They now use satellites instead of balloons

X-ray bursts from thermonuclear fusion explosions on surfaces of neutron stars. Also causes visible flashes because x-rays heat surrounding gas. Delay between X-ray and optical flash allows measurement of gas disk.