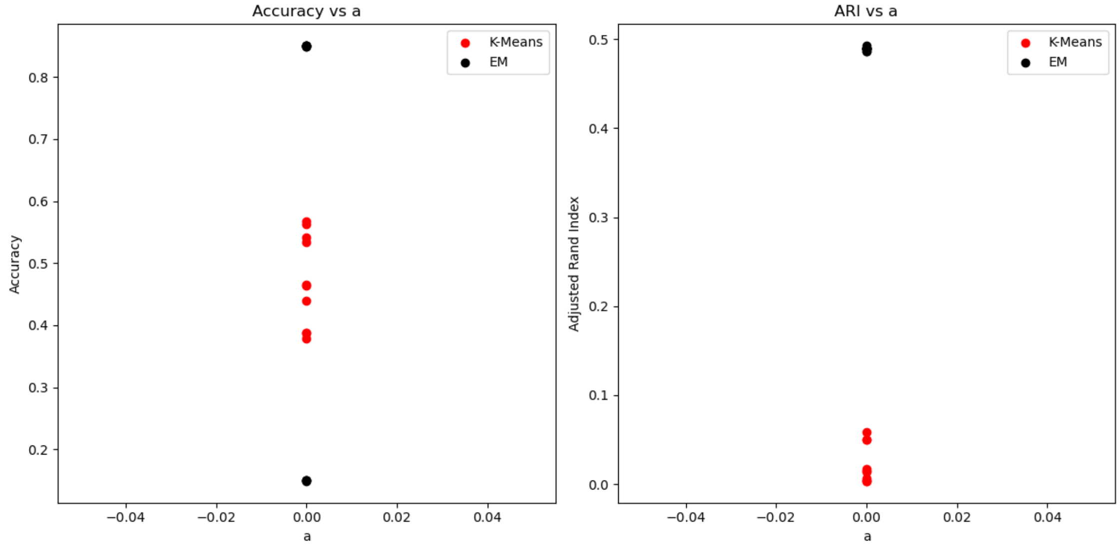
hw8

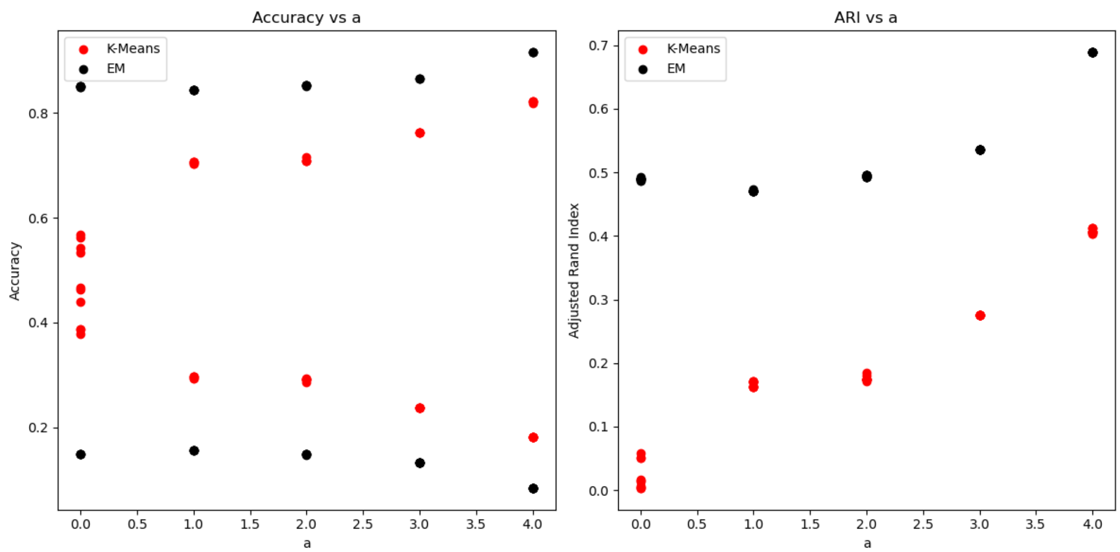
March 5, 2024

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.cluster import KMeans
from sklearn.mixture import GaussianMixture
from sklearn.metrics import accuracy_score, adjusted_rand_score, cluster
from numpy.linalg import svd
```

[ ]:
```
def generate_data(a, sigma=3, n=500):
    Xa = np.random.normal(loc=[a, 0], scale=1, size=(n, 2))
    Xq = np.random.normal(loc=[0, 0], scale=sigma, size=(n, 2))
    X = np.vstack((Xa, Xq))
    y = np.array([1] * n + [0] * n)
    return X, y

def cluster_and_evaluate(X, y, n_clusters=2, n_runs=10):
    accuracies_kmeans = []
    accuracies_em = []
    ari_kmeans = []
    ari_em = []

    for _ in range(n_runs):
        kmeans = KMeans(n_clusters=n_clusters, n_init=1).fit(X)
        em = GaussianMixture(n_components=n_clusters, n_init=1).fit(X)

        labels_kmeans = kmeans.labels_
        labels_em = em.predict(X)

        accuracies_kmeans.append(accuracy_score(y, labels_kmeans))
        accuracies_em.append(accuracy_score(y, labels_em))

        ari_kmeans.append(adjusted_rand_score(y, labels_kmeans))
        ari_em.append(adjusted_rand_score(y, labels_em))

    return accuracies_kmeans, accuracies_em, ari_kmeans, ari_em

def plot_results(a_values, accuracies_kmeans, accuracies_em, ari_kmeans,␣
 ↪ari_em):
```

```python
    plt.figure(figsize=(12, 6))

    # Repeat each value of a 10 times to match the number of runs
    if len(accuracies_em) == 5 :
        a_values_repeated = np.repeat(a_values, 10)
    else:
        a_values_repeated = np.repeat(0, 10)

    plt.subplot(1, 2, 1)
    plt.scatter(a_values_repeated, accuracies_kmeans, color='red',␣
 ↪label='K-Means')
    plt.scatter(a_values_repeated, accuracies_em, color='black', label='EM')
    plt.xlabel('a')
    plt.ylabel('Accuracy')
    plt.title('Accuracy vs a')
    plt.legend()

    plt.subplot(1, 2, 2)
    plt.scatter(a_values_repeated, ari_kmeans, color='red', label='K-Means')
    plt.scatter(a_values_repeated, ari_em, color='black', label='EM')
    plt.xlabel('a')
    plt.ylabel('Adjusted Rand Index')
    plt.title('ARI vs a')
    plt.legend()

    plt.tight_layout()
    plt.show()


a_values = [0, 1, 2, 3, 4]
accuracies_kmeans = []
accuracies_em = []
ari_kmeans = []
ari_em = []

for a in a_values:
    X, y = generate_data(a)
    acc_kmeans, acc_em, ari_k, ari_e = cluster_and_evaluate(X, y)
    accuracies_kmeans.append(acc_kmeans)
    accuracies_em.append(acc_em)
    ari_kmeans.append(ari_k)
    ari_em.append(ari_e)
    if a == 0:
        plot_results(a_values, accuracies_kmeans, accuracies_em, ari_kmeans,␣
 ↪ari_em)

plot_results(a_values, accuracies_kmeans, accuracies_em, ari_kmeans, ari_em)
```





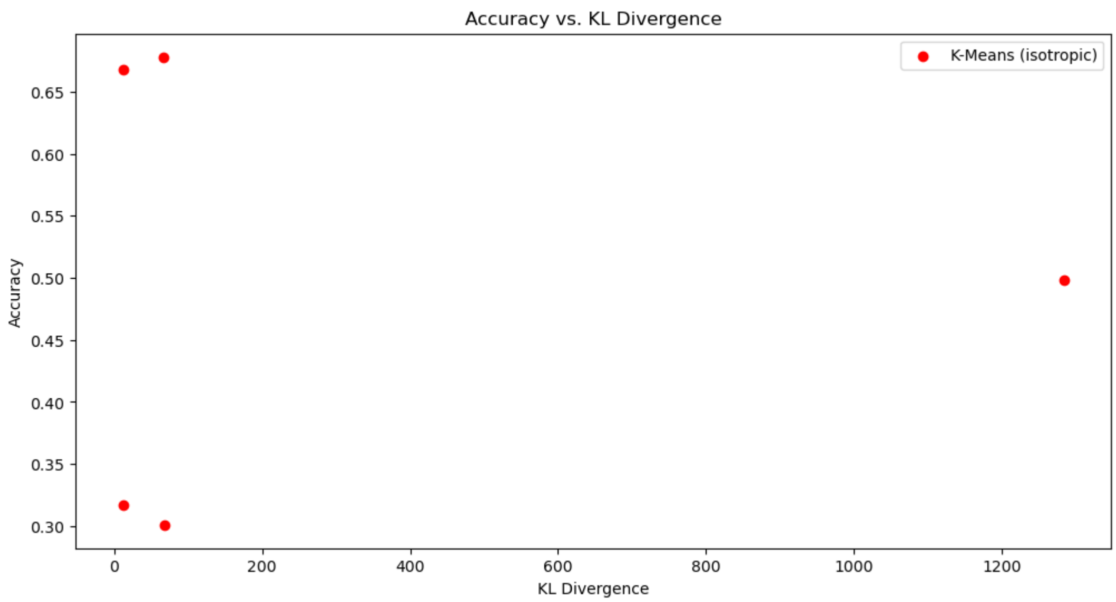

```
[ ]: def generate_random_rotation_matrix():
         M = np.random.normal(0, 1, (2, 2))
         U, _, _ = svd(M)
         return U

     def compute_kl_divergence(cov1, cov2):
         inv_cov2 = np.linalg.inv(cov2)
         return 0.5 * (np.log(np.linalg.det(cov2) / np.linalg.det(cov1))
                       - len(cov1) + np.trace(inv_cov2 @ cov1)
```

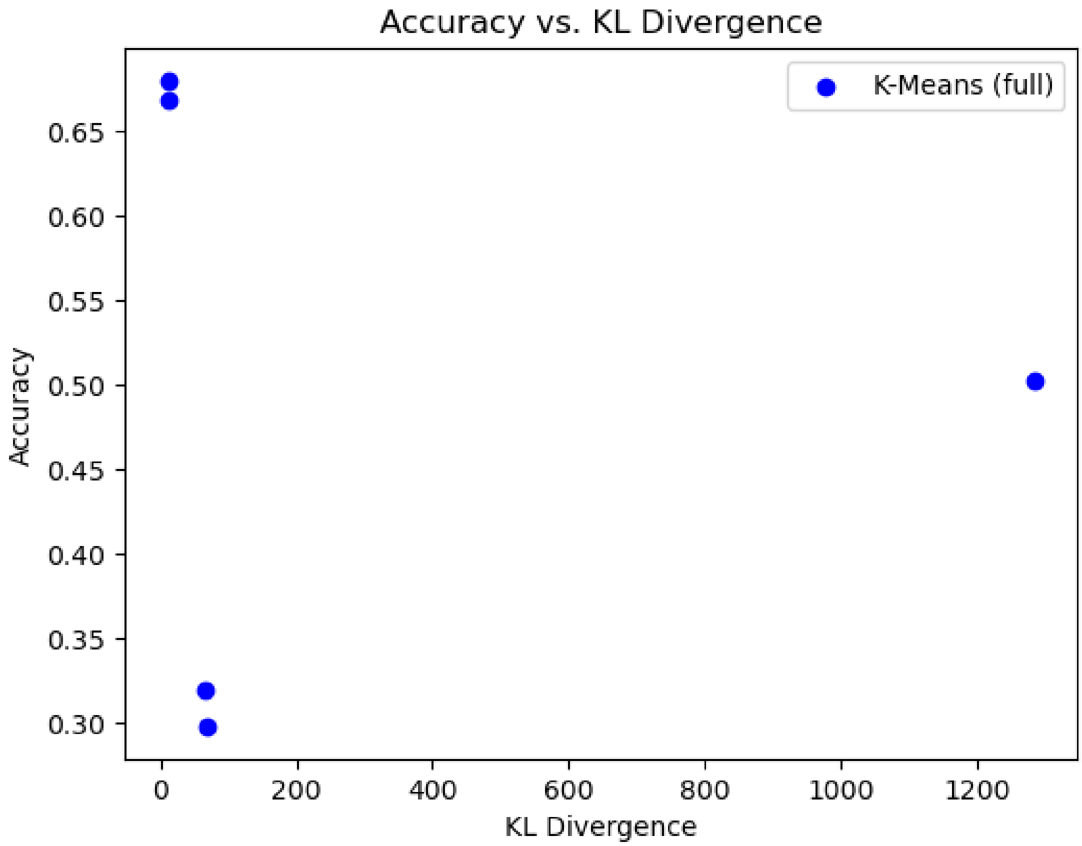
```python
                  + (np.mean(cov2, axis=0) - np.mean(cov1, axis=0)).T @␣
 ↪inv_cov2 @ (np.mean(cov2, axis=0) - np.mean(cov1, axis=0)))

def generate_dataset(U):
    cov = U @ np.diag([100, 1]) @ U.T
    Xq = np.random.multivariate_normal([0, 0], cov, 500)
    Xp = np.random.multivariate_normal([10, 0], cov, 500)
    X = np.vstack((Xq, Xp))
    y = np.array([0] * 500 + [1] * 500)
    return X, y, cov

def run_clustering(X, y):
    kmeans_iso = KMeans(n_clusters=2, n_init=10).fit(X)
    em_iso = GaussianMixture(n_components=2, covariance_type='spherical').fit(X)
    em_full = GaussianMixture(n_components=2, covariance_type='full').fit(X)

    acc_kmeans_iso = accuracy_score(y, kmeans_iso.labels_)
    acc_em_iso = accuracy_score(y, em_iso.predict(X))
    acc_em_full = accuracy_score(y, em_full.predict(X))

    ari_kmeans_iso = adjusted_rand_score(y, kmeans_iso.labels_)
    ari_em_iso = adjusted_rand_score(y, em_iso.predict(X))
    ari_em_full = adjusted_rand_score(y, em_full.predict(X))

    return acc_kmeans_iso, acc_em_iso, acc_em_full, ari_kmeans_iso, ari_em_iso,␣
 ↪ari_em_full


n_runs = 10
results = []
temp_results = []

for _ in range(n_runs):
    U = generate_random_rotation_matrix()
    X, y, cov = generate_dataset(U)
    acc_kmeans_iso, acc_kmeans_full, acc_em, ari_kmeans_iso, ari_kmeans_full,␣
 ↪ari_em = run_clustering(X, y)
    kl_divergence = compute_kl_divergence(np.diag([100, 1]), cov)
    results.append([acc_kmeans_iso, acc_kmeans_full, acc_em, ari_kmeans_iso,␣
 ↪ari_kmeans_full, ari_em, kl_divergence])
    if _ <= 4:
        temp_results.append([acc_kmeans_iso, acc_kmeans_full, acc_em,␣
 ↪ari_kmeans_iso, ari_kmeans_full, ari_em, kl_divergence])

# Convert results to a numpy array for easier slicing
results = np.array(results)
temp_results = np.array(temp_results)
```

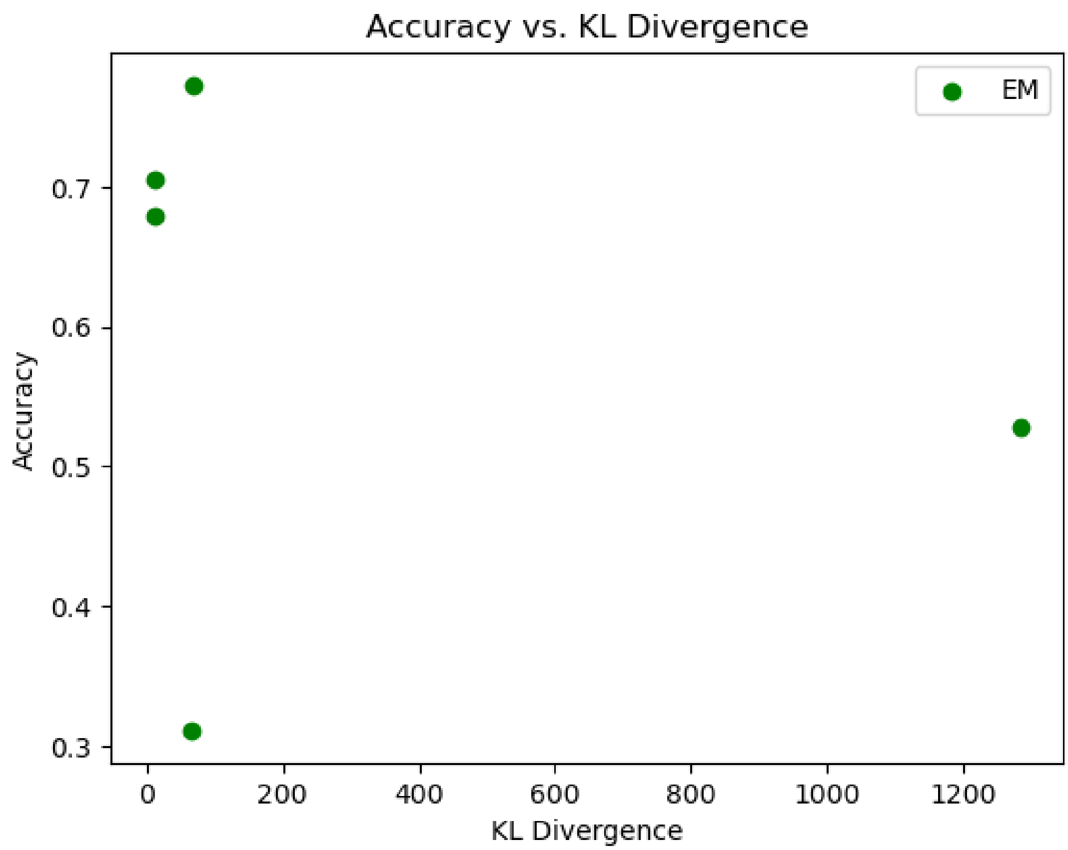
```python
# Plotting accuracy vs. KL divergence
plt.figure(figsize=(12, 6))
plt.scatter(temp_results[:, 6], temp_results[:, 0], color='red', label='K-Means␣
↪(isotropic)')
plt.xlabel('KL Divergence')
plt.ylabel('Accuracy')
plt.title('Accuracy vs. KL Divergence')
plt.legend()
plt.show()
plt.close()
plt.scatter(temp_results[:, 6],temp_results[:, 1], color='blue', label='K-Means␣
↪(full)')
plt.xlabel('KL Divergence')
plt.ylabel('Accuracy')
plt.title('Accuracy vs. KL Divergence')
plt.legend()
plt.show()
plt.close()
plt.scatter(temp_results[:, 6], temp_results[:, 2], color='green', label='EM')
plt.xlabel('KL Divergence')
plt.ylabel('Accuracy')
plt.title('Accuracy vs. KL Divergence')
plt.legend()
plt.show()
plt.close()


# Plotting ARI vs. KL divergence
plt.figure(figsize=(12, 6))
plt.scatter(temp_results[:, 6], temp_results[:, 3], color='red', label='K-Means␣
↪(isotropic)')
plt.xlabel('KL Divergence')
plt.ylabel('Adjusted Rand Index')
plt.title('ARI vs. KL Divergence')
plt.legend()
plt.show()
plt.close()
plt.scatter(temp_results[:, 6], temp_results[:, 4], color='blue',␣
↪label='K-Means (full)')
plt.xlabel('KL Divergence')
plt.ylabel('Adjusted Rand Index')
plt.title('ARI vs. KL Divergence')
plt.legend()
plt.show()
plt.close()
plt.scatter(temp_results[:, 6], temp_results[:, 5], color='green', label='EM')
```

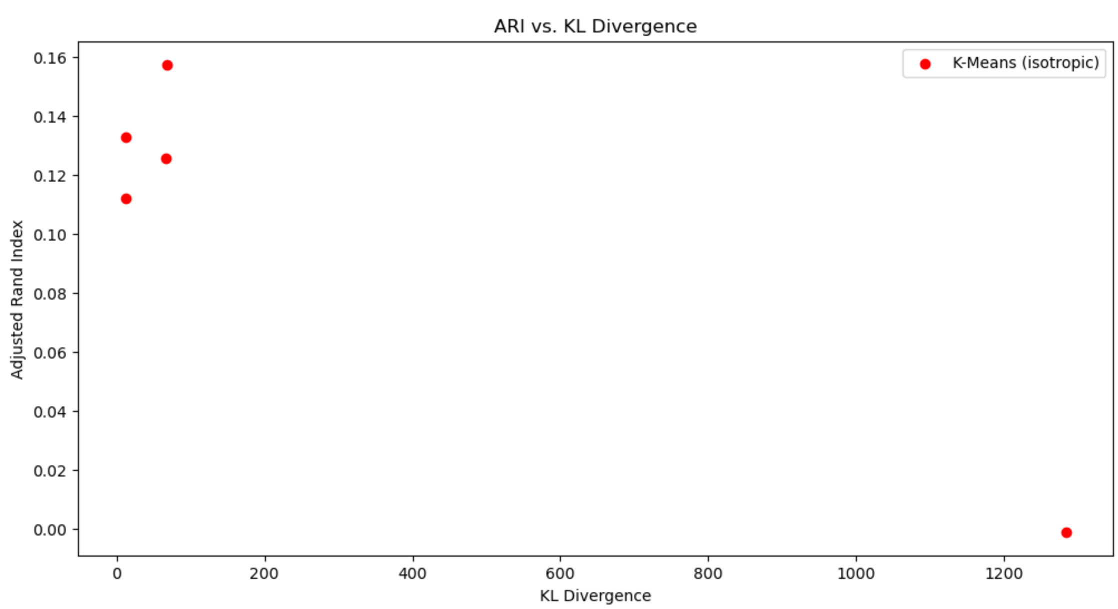
```
plt.xlabel('KL Divergence')
plt.ylabel('Adjusted Rand Index')
plt.title('ARI vs. KL Divergence')
plt.legend()
plt.show()
plt.close()

# Plotting accuracy vs. KL divergence
plt.figure(figsize=(12, 6))
plt.scatter(results[:, 6], results[:, 0], color='red', label='K-Means␣
↪(isotropic)')
plt.xlabel('KL Divergence')
plt.ylabel('Accuracy')
plt.title('Accuracy vs. KL Divergence')
plt.scatter(results[:, 6], results[:, 1], color='blue', label='K-Means (full)')
plt.xlabel('KL Divergence')
plt.ylabel('Accuracy')
plt.title('Accuracy vs. KL Divergence')
plt.scatter(results[:, 6], results[:, 2], color='green', label='EM')
plt.xlabel('KL Divergence')
plt.ylabel('Accuracy')
plt.title('Accuracy vs. KL Divergence')
plt.legend()
plt.show()
plt.close()


# Plotting ARI vs. KL divergence
plt.figure(figsize=(12, 6))
plt.scatter(results[:, 6], results[:, 3], color='red', label='K-Means␣
↪(isotropic)')
plt.xlabel('KL Divergence')
plt.ylabel('Adjusted Rand Index')
plt.title('ARI vs. KL Divergence')
plt.scatter(results[:, 6], results[:, 4], color='blue', label='K-Means (full)')
plt.xlabel('KL Divergence')
plt.ylabel('Adjusted Rand Index')
plt.title('ARI vs. KL Divergence')
plt.scatter(results[:, 6], results[:, 5], color='green', label='EM')
plt.xlabel('KL Divergence')
plt.ylabel('Adjusted Rand Index')
plt.title('ARI vs. KL Divergence')
plt.legend()
plt.show()
plt.close()
```

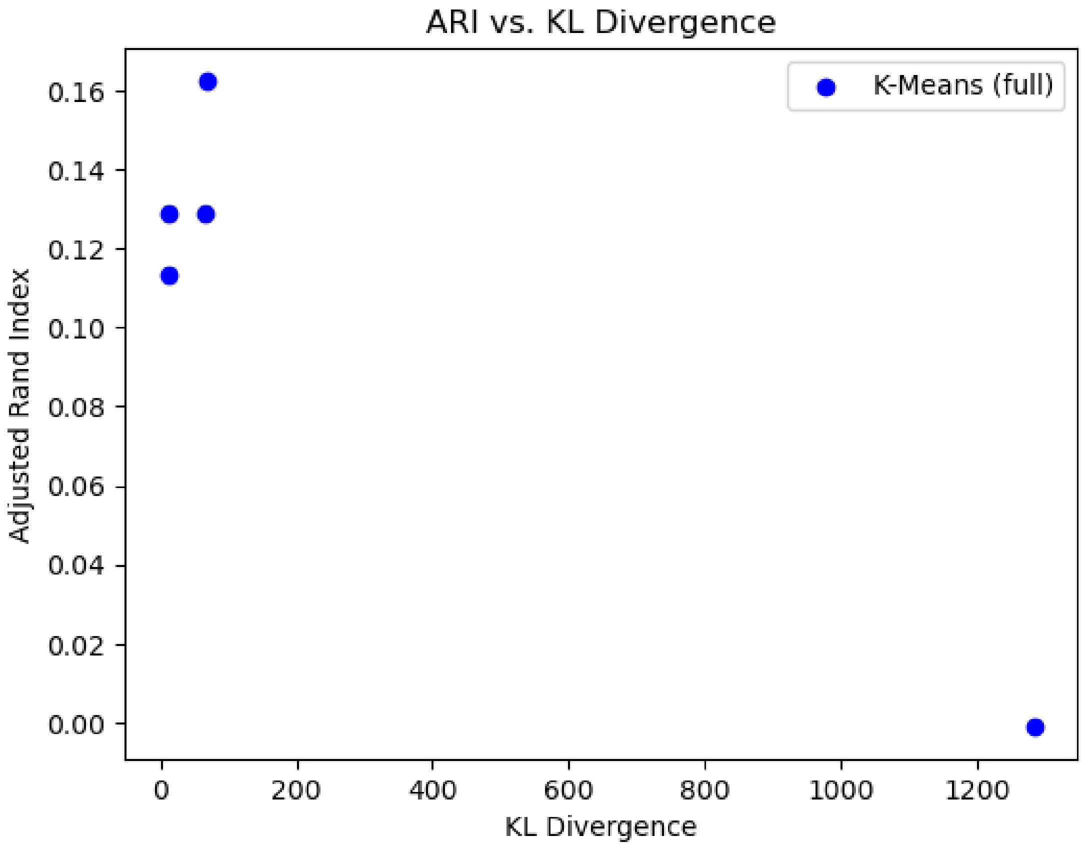
```
# Printing the results table
print("Results Table:")
print("Run\tKMeans (iso)\tKMeans (full)\tEM\t\tARI (iso)\tARI (full)\tARI␣
↪(EM)\tKL Div")
for i, result in enumerate(results):
    print(f"{i + 1}\t{result[0]:.3f}\t\t{result[1]:.3f}\t\t{result[2]:.
↪3f}\t\t{result[3]:.3f}\t\t{result[4]:.3f}\t\t{result[5]:.3f}\t\t{result[6]:.
↪3f}")
```

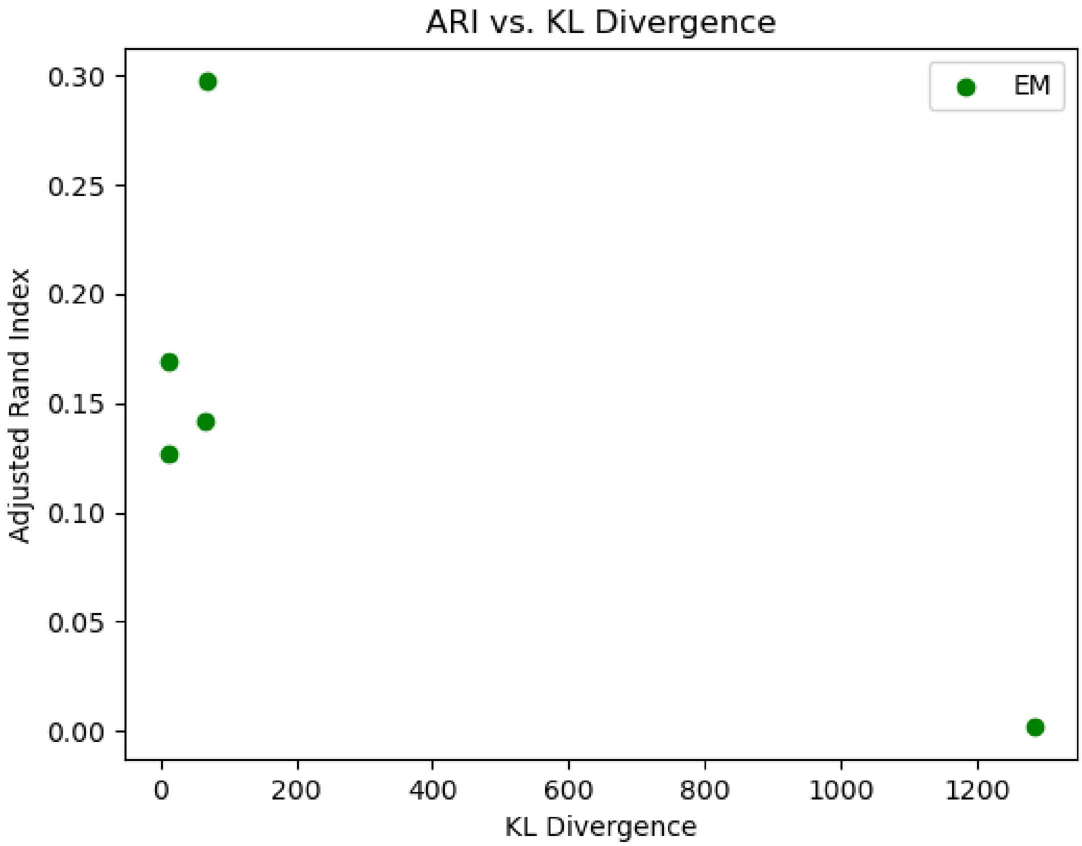


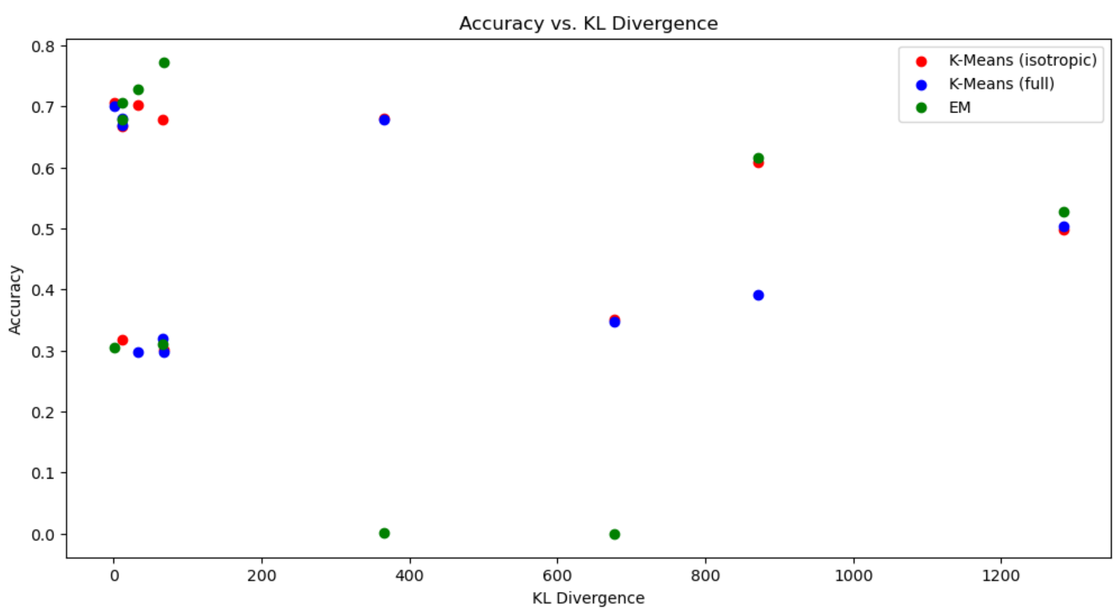

Accuracy vs. KL Divergence
K-Means (full)
Accuracy
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0
200
400
600
800
1000
1200
KL Divergence

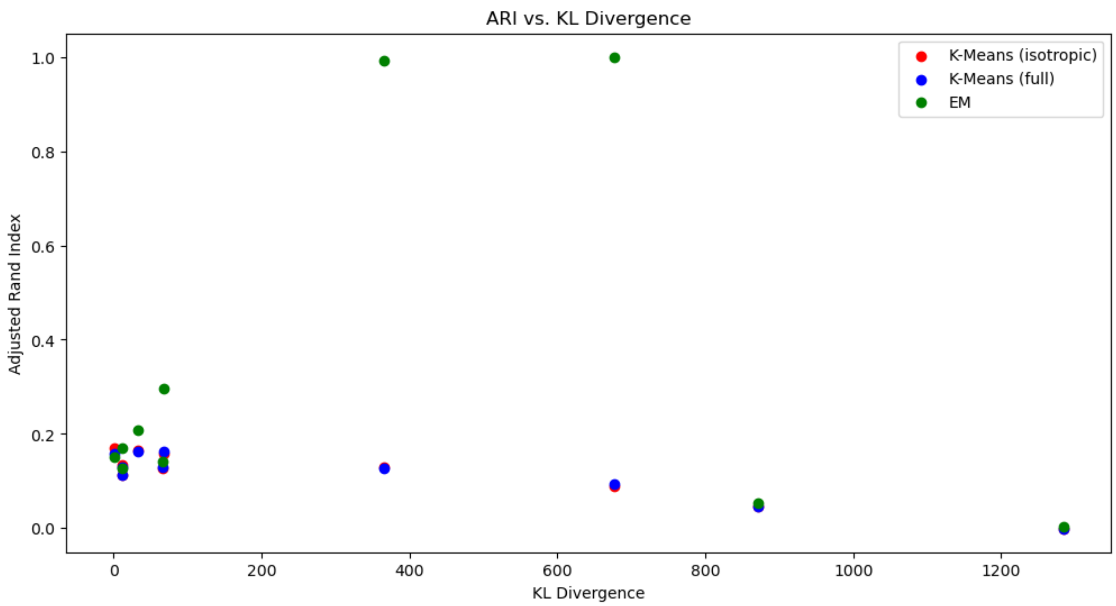

Accuracy vs. KL Divergence
EM
Accuracy
0.7
0.6
0.5
0.4
0.3
0
200
400
600
800
1000
1200
KL Divergence


ARI vs. KL Divergence
K-Means (isotropic)
Adjusted Rand Index
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
200
400
600
800
1000
1200
KL Divergence


ARI vs. KL Divergence
K-Means (full)
Adjusted Rand Index
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
200
400
600
800
1000
1200
KL Divergence


ARI vs. KL Divergence
EM
Adjusted Rand Index
KL Divergence
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
200
400
600
800
1000
1200


Accuracy vs. KL Divergence
K-Means (isotropic)
K-Means (full)
EM
Accuracy
KL Divergence
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
200
400
600
800
1000
1200




```
Results Table:
Run      KMeans (iso)     KMeans (full)    EM               ARI (iso)        ARI
(full)       ARI (EM)         KL Div
1        0.498            0.503            0.528            -0.001           -0.001
0.002            1284.698
2        0.668            0.669            0.679            0.112            0.113
0.127            11.779
3        0.317            0.680            0.706            0.133            0.129
0.169            11.442
4        0.678            0.320            0.311            0.126            0.129
0.142            66.019
5        0.301            0.298            0.773            0.158            0.162
0.297            67.987
6        0.351            0.347            0.000            0.088            0.093
1.000            677.060
7        0.706            0.700            0.305            0.169            0.159
0.151            0.382
8        0.680            0.679            0.002            0.129            0.127
0.992            366.010
9        0.703            0.298            0.728            0.164            0.162
0.207            33.449
10       0.609            0.391            0.616            0.047            0.047
0.053            871.523
```

[ ]: