MATHEMATICS IN ACTION

Prealgebra Problem Solving

3rd edition

The Consortium for Foundation Mathematics

MATHEMATICS IN ACTION

Prealgebra Problem Solving

This page intentionally left blank

MATHEMATICS IN ACTION

Prealgebra Problem Solving

THIRD EDITION

The Consortium for Foundation Mathematics

Ralph Bertelle	*Columbia-Greene Community College*
Judith Bloch	*University of Rochester*
Roy Cameron	*SUNY Cobleskill*
Carolyn Curley	*Erie Community College—South Campus*
Ernie Danforth	*Corning Community College*
Brian Gray	*Howard Community College*
Arlene Kleinstein	*SUNY Farmingdale*
Kathleen Milligan	*Monroe Community College*
Patricia Pacitti	*SUNY Oswego*
Rick Patrick	*Adirondack Community College*
Renan Sezer	*LaGuardia Community College*
Patricia Shuart	*Polk State College—Winter Haven, Florida*
Sylvia Svitak	*Queensborough Community College*
Assad J. Thompson	*LaGuardia Community College*

Addison-Wesley

Boston Columbus Indianapolis New York San Francisco Upper Saddle River
Amsterdam Cape Town Dubai London Madrid Milan Munich Paris Montréal Toronto
Delhi Mexico City São Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo

Editorial Director, Mathematics Christine Hoag
Editor in Chief Maureen O'Connor
Content Editor Courtney Slade
Assistant Editor Mary St. Thomas
Senior Managing Editor Karen Wernholm
Production Project Manager Beth Houston
Senior Designer/Cover Designer Barbara Atkinson
Interior Designer Studio Montage
Digital Assets Manager Marianne Groth
Production Coordinator Katherine Roz
Associate Producer Christine Maestri
Associate Marketing Manager Tracy Rabinowitz
Marketing Coordinator Alicia Frankel
Senior Author Support/Technology Specialist Joe Vetere
Rights and Permissions Advisor Michael Joyce
Senior Manufacturing Buyer Carol Melville
Production Management/Composition PreMediaGlobal
Cover photo Eric Michaud/iStockphoto

Library of Congress Cataloging-in-Publication Data
Mathematics in action : prealgebra problem solving / the Consortium for Foundation Mathematics.—3rd ed.
p. cm.
ISBN-13: 978-0-321-69859-9 (student ed.)
ISBN-10: 0-321-69859-2 (student ed.)
ISBN-13: 978-0-321-69282-5 (instructor ed.)
ISBN-10: 0-321-69282-9 (instructor ed.)
1. Mathematics. I. Consortium for Foundation Mathematics.

QA39.3.M384 2012
510—dc22 2009052324

Addison-Wesley
is an imprint of

PEARSON

www.pearsonhighered.com

1 2 3 4 5 6 7 8 9 10—EB—14 13 12 11 10

Contents

APPENDIX

Preface

Our Vision

Mathematics in Action: Prealgebra Problem Solving, Third Edition, is intended to help college mathematics students gain mathematical literacy in the real world and simultaneously help them build a solid foundation for future study in mathematics and other disciplines.

Our authoring team used the AMATYC *Crossroads* standards to develop a three-book series to serve a large and diverse population of college students who, for whatever reason, have not yet succeeded in learning mathematics. It became apparent to us that teaching the same content in the same manner to students who have not previously comprehended it is not effective, and this realization motivated us to develop a new approach.

Mathematics in Action is based on the principle that students learn mathematics best by doing mathematics within a meaningful context. In keeping with this premise, students solve problems in a series of realistic situations from which the crucial need for mathematics arises. *Mathematics in Action* guides students toward developing a sense of independence and taking responsibility for their own learning. Students are encouraged to construct, reflect on, apply, and describe their own mathematical models, which they use to solve meaningful problems. We see this as the key to bridging the gap between abstraction and application and as the basis for transfer learning. Appropriate technology is integrated throughout the books, allowing students to interpret real-life data verbally, numerically, symbolically, and graphically.

We expect that by using the *Mathematics in Action* series, all students will be able to achieve the following goals:

- Develop mathematical intuition and a relevant base of mathematical knowledge.
- Gain experiences that connect classroom learning with real-world applications.
- Prepare effectively for further college work in mathematics and related disciplines.
- Learn to work in groups as well as independently.
- Increase knowledge of mathematics through explorations with appropriate technology.
- Develop a positive attitude about learning and using mathematics.
- Build techniques of reasoning for effective problem solving.
- Learn to apply and display knowledge through alternative means of assessment, such as mathematical portfolios and journal writing.

We hope that your students will join the growing number of students using our approaches who have discovered that mathematics is an essential and learnable survival skill for the twenty-first century.

Pedagogical Features

The pedagogical core of *Mathematics in Action* is a series of guided-discovery activities in which students work in groups to discover mathematical principles embedded in realistic situations. The key principles of each activity are highlighted and summarized at the activity's conclusion. Each activity is followed by exercises that reinforce the concepts and skills revealed in the activity.

The activities are clustered within some of the chapters. Each cluster's activities all relate to a particular subset of topics addressed in the chapter. Chapter 7 and the Instructor's Resource Manual contain lab activities in addition to regular activities. The lab activities require more than just paper, pencil, and calculator—they often require measurements and data collection and are ideal for in-class group work. For specific suggestions on how to use the two types of activities, we strongly encourage instructors to refer to the *Instructor's Resource Manual with Tests* that accompanies this text.

Each cluster concludes with two sections: What Have I Learned? and How Can I Practice? The What Have I Learned? exercises are designed to help students pull together the key concepts of the cluster. The How Can I Practice? exercises are designed primarily to provide additional work with the mathematical skills of the cluster. Taken as a whole, these exercises give students the tools they need to bridge the gaps between abstraction, skills, and application.

Additionally, each chapter ends with a Summary that briefly describes key concepts and skills discussed in the chapter, plus examples illustrating these concepts and skills. The concepts and skills are also referenced to the activity in which they appear, making the format easier to follow for those students who are unfamiliar with our approach. Each chapter also ends with a Gateway Review, providing students with an opportunity to check their understanding of the chapter's concepts and skills, as well as prepare them for a chapter assessment.

Changes from the Second Edition

The Third Edition retains all the features of the previous edition, with the following content changes.

- All data-based activities and exercises have been updated to reflect the most recent information and/or replaced with more relevant topics.
- The language in many activities is now clearer and easier to understand.
- Chapters 3 and 4 have been reorganized so integers, fractions, and decimals are covered in three separate chapters: Chapter 3, *Problem Solving with Integers*, Chapter 4, *Problem Solving with Fractions*, and Chapter 5, *Problem Solving with Mixed Numbers and Decimals*.
- Chapters 5, 6, and 7 from the previous edition have been revised and renumbered as 6, 7, and 8.
- Activity 3.6, *Integers and Tiger Woods*, contains two additional objectives: combining like terms involving integers and solving equations of the form $ax + bx = c$, where $a + b \neq 0$.
- Activity 5.3, *Tiling the Bathroom*, contains an additional objective: solving equations of the form $ax + b = 0$, $a \neq 0$, that involve mixed numbers.
- Activity 5.8, *Four out of Five Dentists Prefer the Brooklyn Dodgers?*, which teaches proportional reasoning, is now the second activity in Chapter 6, *Problem Solving with Ratios, Proportions, and Percents*.
- An additional objective on using the distance formula to determine the distance between two points has been added to Lab Activity 7.7, *How About Pythagoras?*
- Several activities have moved to MyMathLab or the IRM to streamline the course without loss of content. This includes Activities 7.2, 7.4, and 7.6 from the second edition, as well as Activity 6.9 on similar triangles.
- Activity 7.1 in the second edition has been revised and renumbered as Activity 8.1.

Supplements

Instructor Supplements

Annotated Instructor's Edition

ISBN-10 0-321-69282-9
ISBN-13 978-0-321-69282-5

This special version of the student text provides answers to all exercises directly beneath each problem.

Instructor's Resource Manual with Tests

ISBN-10 0-321-69283-7
ISBN-13 978-0-321-69283-2

This valuable teaching resource includes the following materials:

- Sample syllabi suggesting ways to structure a course around core and supplemental activities
- Notes on teaching activities in each chapter
- Strategies for learning in groups and using writing to learn mathematics
- Extra practice worksheets for topics with which students typically have difficulty
- Sample chapter tests and final exams for in-class and take-home use by individual students and groups
- Information about technology in the classroom

TestGen®

ISBN-10 0-321-69285-3
ISBN-13 978-0-321-69285-6

TestGen enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson Education's online catalog.

Instructor's Training Video on CD

ISBN-10-0-321-69279-9
ISBN-13 978-0-321-69279-5

This innovative video discusses effective ways to implement the teaching pedagogy of the *Mathematics in Action* series, focusing on how to make collaborative learning, discovery learning, and alternative means of assessment work in the classroom.

Student Supplements

Worksheets for Classroom or Lab Practice

ISBN-10 0-321-73837-3
ISBN-13 978-0-321-73837-0

- Extra practice exercise for every section of the text with ample space for students to show their work.

- These lab- and classroom-friendly workbooks also list the learning objectives and key vocabulary terms for every text section, along with vocabulary practice problems.
- Concept Connection exercises, similar to the "What Have I Learned?" exercises found in the text, assess students' conceptual understanding of the skills required to complete each worksheet.

MathXL® Tutorials on CD

ISBN-10 0-321-69284-5
ISBN-13 978-0-321-69284-9

This interactive tutorial CD-ROM provides algorithmically generated practice exercises that are correlated at the objective level to the exercises in the textbook. Every practice exercise is accompanied by an example and a guided solution designed to involve students in the solution process. The software provides helpful feedback for incorrect answers and can generate printed summaries of students' progress.

InterAct Math Tutorial Website www.interactmath.com

Get practice and tutorial help online! This interactive tutorial Web site provides algorithmically generated practice exercises that correlate directly to the exercises in the textbook. Students can retry an exercise as many times as they like with new values each time for unlimited practice and mastery. Every exercise is accompanied by an interactive guided solution that provides helpful feedback for incorrect answers, and students can also view a worked-out sample problem that steps them through an exercise similar to the one they're working on.

Pearson Math Adjunct Support Center

The **Pearson Math Adjunct Support Center** (http://www.pearsontutorservices.com/mathadjunct.html) is staffed by qualified instructors with more than 100 years of combined experience at both the community college and university levels. Assistance is provided for faculty in the following areas:

- Suggested syllabus consultation
- Tips on using materials packed with your book
- Book-specific content assistance
- Teaching suggestions, including advice on classroom strategies

Supplements for Instructors and Students

MathXL® Online Course (access code required)

MathXL® is a powerful online homework, tutorial, and assessment system that accompanies Pearson Education's textbooks in mathematics or statistics. With MathXL, instructors can:

- Create, edit, and assign online homework and tests using algorithmically generated exercises correlated at the objective level to the textbook.
- Create and assign their own online exercises and import TestGen tests for added flexibility.
- Maintain records of all student work tracked in MathXL's online gradebook.

With MathXL, students can:

- Take chapter tests in MathXL and receive personalized study plans and/or personalized homework assignments based on their test results.
- Use the study plan and/or the homework to link directly to tutorial exercises for the objectives they need to study.
- Access supplemental animations and video clips directly from selected exercises.

MathXL is available to qualified adopters. For more information, visit our Web site at www.mathxl.com, or contact your Pearson representative.

MyMathLab® Online Course (access code required)

MyMathLab® is a text-specific, easily customizable online course that integrates interactive multimedia instruction with textbook content. MyMathLab gives you the tools you need to deliver all or a portion of your course online, whether your students are in a lab setting or working from home.

- **Interactive homework exercises,** correlated to your textbook at the objective level, are algorithmically generated for unlimited practice and mastery. Most exercises are free response and provide guided solutions, sample problems, and tutorial learning aids for extra help.
- **Personalized homework** assignments that you can design to meet the needs of your class, MyMathLab tailors the assignment for each student based on his or her test or quiz scores. Each student receives a homework assignment that contains only the problems he or she still needs to master.
- **Personalized Study Plan,** generated when students complete a test or quiz or homework, indicates which topics have been mastered and links to tutorial exercises for topics students have not mastered. You can customize the Study Plan so that the topics available match your course content.
- **Multimedia learning aids,** such as video lectures and podcasts, animations, and a complete multimedia textbook, help students independently improve their understanding and performance. You can assign these multimedia learning aids as homework to help your students grasp the concepts.
- **Homework and Test Manager** lets you assign homework, quizzes, and tests that are automatically graded. Select just the right mix of questions from the MyMathLab exercise bank, instructor-created custom exercises, and/or TestGen® test items.
- **Gradebook,** designed specifically for mathematics and statistics, automatically tracks students' results, lets you stay on top of student performance, and gives you control over how to calculate final grades. You can also add offline (paper-and-pencil) grades to the gradebook.
- **MathXL Exercise Builder** allows you to create static and algorithmic exercises for your online assignments. You can use the library of sample exercises as an easy starting point, or you can edit any course-related exercise.
- **Pearson Tutor Center** (www.pearsontutorservices.com) access is automatically included with MyMathLab. The Tutor Center is staffed by qualified math instructors who provide textbook-specific tutoring for students via toll-free phone, fax, email, and interactive Web sessions.

Students do their assignments in the Flash®-based MathXL Player, which is compatible with almost any browser (Firefox®, Safari™, or Internet Explorer®) on almost any platform (Macintosh® or Windows®). MyMathLab is powered by CourseCompass™, Pearson Education's online teaching and learning environment, and by MathXL®, our online homework, tutorial, and assessment system. MyMathLab is available to qualified adopters. For more information, visit www.mymathlab.com or contact your Pearson representative.

Acknowledgments

The Consortium would like to acknowledge and thank the following people for their invaluable assistance in reviewing and testing material for this text in the past and current editions:

Michele Bach, *Kansas City Community College*

Kathleen Bavelas, *Manchester Community College*

Vera Brennan, *Ulster County Community College*

Jennifer Dollar, *Grand Rapids Community College*

Kirsty J. Eisenhart, PhD, *Western Michigan University*

Marion Glasby, *Anne Arundel Community College*

Thomas J. Grogan, *Cincinnati State Technical and Community College*

Bob Hervey, *Hillsborough Community College*

Brian Karasek, *South Mountain Community College*

Ashok Kumar, *Valdosta State University*

Rob Lewis, *Linn Benton Community College*

Jim Matovina, *Community College of Southern Nevada*

Janice McCue, *College of Southen Maryland*

Kathleen Peters, *Manchester Community College*

Bobbi Righi, *Seattle Central Community College*

Jody Rooney, *Jackson Community College*

Janice Roy, *Montcalm Community College*

Andrew S. H. Russell, *Queensborough Community College*

Amy Salvati, *Adirondack Community College*

Carolyn Spillman, *Georgia Perimeter College*

Janet E. Teeguarden, *Ivy Technical Community College*

Sharon Testone, *Onondaga Community College*

Ruth Urbina-Lilback, *Naugatuck Valley Community College*

Cheryl Wilcox, *Diablo Valley College*

Jill C. Zimmerman, *Manchester Community College*

Cathleen Zucco-Teveloff, *Trinity College*

We would also like to thank our accuracy checkers, Shannon d'Hemecourt, Diane E. Cook, Jon Weerts, and James Lapp.

Finally, a special thank you to our families for their unwavering support and sacrifice, which enabled us to make this text a reality.

The Consortium for Foundation Mathematics

To the Student

The book in your hands is most likely very different from any mathematics book you have seen before. In this book, you will take an active role in developing the important ideas of arithmetic and beginning algebra. You will be expected to add your own words to the text. This will be part of your daily work, both in and out of class and for homework. It is our strong belief that students learn mathematics best when they are actively involved in solving problems that are meaningful to them.

The text is primarily a collection of situations drawn from real life. Each situation leads to one or more problems. By answering a series of questions and solving each part of the problem, you will be led to use one or more ideas of introductory college mathematics. Sometimes, these will be basic skills that build on your knowledge of arithmetic. Other times, they will be new concepts that are more general and far reaching. The important point is that you won't be asked to master a skill until you see a real need for that skill as part of solving a realistic application.

Another important aspect of this text and the course you are taking is the benefit gained by collaborating with your classmates. Much of your work in class will result from being a member of a team. Working in groups, you will help each other work through a problem situation. While you may feel uncomfortable working this way at first, there are several reasons we believe it is appropriate in this course. First, it is part of the learn-by-doing philosophy. You will be talking about mathematics, needing to express your thoughts in words—this is a key to learning. Secondly, you will be developing skills that will be very valuable when you leave the classroom. Currently, many jobs and careers require the ability to collaborate within a team environment. Your instructor will provide you with more specific information about this collaboration.

One more fundamental part of this course is that you will have access to appropriate technology at all times. Technology is a part of our modern world, and learning to use technology goes hand in hand with learning mathematics. Your work in this course will help prepare you for whatever you pursue in your working life.

This course will help you develop both the mathematical and general skills necessary in today's workplace, such as organization, problem solving, communication, and collaborative skills. By keeping up with your work and following the suggested organization of the text, you will gain a valuable resource that will serve you well in the future. With hard work and dedication you will be ready for the next step.

The Consortium for Foundation Mathematics

MATHEMATICS IN ACTION

Prealgebra Problem Solving

This page intentionally left blank

Chapter 1

Whole Numbers

Do you remember when you first started learning about numbers? From those early days, you went on to learn more about numbers—what they are, how they are related to one another, and how you operate with them.

Whole numbers are the basis for your further study of arithmetic and introductory algebra used throughout this book. In Chapter 1, we will see whole numbers in real-life use, clarify what you already know about them, and learn more about them.

Activity 1.1

Education Pays

Objectives

1. Read and write whole numbers.
2. Compare whole numbers using inequality symbols.
3. Round whole numbers to specified place values.
4. Use rounding for estimation.
5. Classify whole numbers as even or odd, prime, or composite.
6. Solve problems involving whole numbers.

The U.S. Bureau of the Census tracks information yearly about educational levels and income levels of the population in the United States (http://www.census.gov). In 2007, the bureau reported that more than one in four adults holds a bachelor's degree. The bureau also presented data on average 2007 earnings and education level for all workers, aged 18 and older. Some of the data is given in the table below.

1. a. What is the average income of those workers who had some college? An associate's degree? A high school graduate? A bachelor's degree?

   b. Which group of workers earned the most income in 2007? Which group earned the least income?

   c. What does the table indicate about the value of an education in the United States?

Learning to Earn

AVERAGE 2007 EARNINGS BY EDUCATIONAL LEVEL: WORKERS 18 YEARS OF AGE AND OLDER

Educational Level	Some high school	High school graduates	Some college	Associate's degree	Bachelor's degree	Master's degree	Doctorate degree	Professional degree
Average Income Level	$21,251	$31,286	$33,009	$39,746	$57,181	$70,186	$95,565	$120,978

d. How does the census information relate to your decision to attend college?

Whole Numbers

The earnings listed in the preceding table are represented by whole numbers.

The set of whole numbers consists of zero and all the counting numbers, 1, 2, 3, 4, and so on.

Whole numbers are used to describe "how many" (for example, the dollar values in Problem 1). Each **whole number** is represented by a **numeral**, which is a sequence of symbols called *digits*. The relative placement of the **digits** (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9) in our standard base-10 system determines the value of the number that the numeral represents.

Example 1 ***What does the numeral 3547 represent? What is the place value of 3 in the number 3547?***

SOLUTION

3547 is the numeral representing 3 thousands, 5 hundreds, 4 tens, and 7 ones. It is read and written in words as "three thousand five hundred forty-seven." In this number, the digit 3 has a **place value** of one thousand (1000). This means that the digit 3 represents 3000 of the units in the number 3547.

2. What are the place values of the other digits in the number 3547?

3. a. Write the number 30,928 in words.

b. What are the place values of the digits 8 and 9 in the number 30,928?

For ease in reading a number in the base-10 system, digits are grouped in threes with each grouping of three separated by a comma. The triples are named as shown in the following table. Beginning with the second triple from the right and moving to the left, the triples are named thousands, millions, billions, etc. For example, the number 548,902,473,150 is written in the following table.

Group	Billions			Millions			Thousands			Ones		
Triples	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
Example	5	4	8	9	0	2	4	7	3	1	5	0

The number in the preceding table is read "five hundred forty-eight billion, nine hundred two million, four hundred seventy-three thousand, one hundred fifty."

4. Write the earnings from Problem 1a in words.

5. The digit 0 occurs twice in the number in the preceding table. What is the place value of each occurrence?

Comparing Whole Numbers

The place value system of writing numbers makes it easy to compare numbers. For example, it is easy to see that an income of \$33,009 is less than an income of \$39,746 by comparing the values 3 and 9 in the thousands place. You can represent the relationship by writing $33{,}009 < 39{,}746$. Alternatively, you can say that \$39,746 is greater than \$33,009 and write $39{,}746 > 33{,}009$. Such statements involving the symbols $<$ (less than) and $>$ (greater than) are called **inequalities**.

6. In the 2007 census, Mississippi's population was counted as 2,918,785 and Iowa's was 2,988,046. Which state had the greater population?

Procedure

Comparing Two Whole Numbers

1. Use the symbol $>$ to write that one number is *greater than* another. For example, $8 > 3$ is read from left to right as "eight is greater than three."
2. Use the symbol $<$ to write that one number is *less than* another. For example, $3 < 8$ is read from left to right as "three is less than eight."
3. Use the symbol $=$ to write that two numbers *are equal*. For example, $2 + 1 = 3$ is read from left to right as "two plus one is equal to three."
4. Compare two numbers by reading each of them from left to right to find the first position where they differ. For example, 7,180,597 and 7,180,642 first differ in the hundreds place. Since $6 > 5$, write $7{,}180{,}642 > 7{,}180{,}597$. Alternatively, $7{,}180{,}597 < 7{,}180{,}642$ is also correct because $5 < 6$.

Rounding Whole Numbers

The U.S. Bureau of the Census provides United States population counts on the Internet on a regular basis. For example, the bureau estimated the population at 306,250,113 persons on April 19, 2009. Not every digit in this number is meaningful because the population is constantly changing. Therefore, a reasonable approximation is usually sufficient. For example, it would often be good enough to say the population is about 306,000,000, or three hundred six million.

One process of determining an approximation to a number is known as **rounding**.

Procedure

Rounding a Whole Number to a Specified Place Value

1. Underline the digit with the place value to which the number will be rounded, such as "to the nearest million" or "to the nearest thousand."
2. If the digit directly to its right is *less than 5*, keep the digit underlined in step 1 and replace all the digits to its right with zeros.
3. If the digit directly to its right is *5 or greater*, increase the digit underlined in step 1 by 1 unit and replace all the digits to its right with zeros.

Example 2 ***Round* 37,146 *to the nearest ten thousand.***

SOLUTION

The digit in the ten thousands place is 3. The digit to its right is 7. Therefore, increase 3 to 4 and insert zeros in place of all the digits to the right. The rounded value is 40,000.

7. On another date in 2007, the U.S. Bureau of the Census gave the U.S. population as 301,621,157 persons.

   a. Approximate this count by rounding to the nearest million.

   b. Approximate the count by rounding to the nearest ten thousand.

Classifying Whole Numbers: Even or Odd, Prime or Composite

At times, it can be useful to classify whole numbers that share certain common features. One way to classify whole numbers is as even or odd. **Even numbers** are those that are exactly divisible by 2. A whole number that is not even is **odd**.

8. When an odd number is divided by 2, what is its remainder? Give an example.

9. Is 6 an even number? Explain.

10. By examining the digits of a whole number, how can you determine if the number is even or odd?

Whole numbers can also be classified as prime or composite. Any whole number greater than 1 that is divisible *only* by itself and 1 is called **prime**. For example, 5 is divisible only by itself and 1, so 5 is prime. A whole number greater than 1 that is not prime is called **composite**. For example, 6 is divisible by itself and 1, but also by 2 and 3. Therefore, 6 is a composite number. Also, 1, 2, 3, and 6 are called the **factors** of 6.

11. Is 21 prime or composite? Explain.

12. Is 2 prime or composite? Explain.

13. **a.** List all the prime numbers between 1 and 30.

b. How many even numbers are included in your list?

c. How many even prime numbers do you think there are? Explain.

14. **a.** List all the factors of 24.

b. List the factors of 24 that are prime numbers.

Problem Solving with Whole Numbers

The U.S. Bureau of Labor Statistics provides data for various occupations and the education levels that they usually require. The Bureau's 2008–2009 Occupational Outlook Handbook lists nine occupations that require an associate's degree as the fastest growing and projected to have the largest numerical increases in employment between 2006 and 2016. The occupations are listed in the table below. The table also provides 2007 median annual earnings for these occupations. A median value of a data set is a value that divides the set into an upper half and a lower half of values. One-half of all the salaries for a given occupation are below the median salary and the other half are above the median salary. For example, one-half of workers employed as physical therapy assistants earned less than $44,140 and the other half earned more than $44,140 in 2007.

On the Job...

OCCUPATION	2007 MEDIAN ANNUAL SALARY, $
Veterinary technologists and technicians	27,980
Physical therapist assistant	44,140
Dental hygienists	64,730
Environmental science and protection technicians, including health	39,370
Cardiovascular technologists and technicians	44,950
Registered nurses	60,010
Computer support specialists	42,410
Paralegals and legal assistants	44,990
Legal secretaries	48,460

Source: Bureau of Labor Statistics

15. a. The salaries in the preceding table are rounded to what place value?

b. Is the median income of cardiovascular technologists and technicians more or less than paralegals and legal assistants? Explain how you obtained your answer.

c. Which two occupations are the farthest apart in median annual salaries?

d. If you rounded the median annual salary for dental hygienists to the nearest thousand dollars, how would you report the salary? Would you be overestimating or underestimating the salary?

e. If you rounded the median annual salary for environmental science and protection technicians to the nearest thousand dollars, how would you report the salary? Would you be overestimating or underestimating the salary?

SUMMARY: ACTIVITY 1.1

1. The set of whole numbers consists of 0 and all the counting numbers, 1, 2, 3, 4, and so on.
2. Each digit in a numeral has a **place value** determined by its relative placement in the numeral. Numbers are compared for size by comparing their corresponding place values. The symbols $<$ (read "less than") and $>$ (read "greater than") are used to compare the size of the numbers.
3. **Rounding** a given number to a specified place value is used to approximate its value. Rules for rounding are provided on page 4.
4. Whole numbers are classified as even or odd. **Even numbers** are whole numbers that are exactly divisible by 2. Any whole number that is not even is an **odd number**.
5. Whole numbers are also classified as **prime** or **composite**. A whole number is **prime** if it is greater than 1 and divisible only by itself and 1. A whole number greater than 1 that is not prime is called **composite**. The **factors** of a number are all the numbers that divide exactly into the given number.

EXERCISES: ACTIVITY 1.1

1. The sticker price of a new Lexus GS 350 AWD is fifty-three thousand four hundred seventy-two dollars. Write this number as a numeral.

2. The total population of the United States on December 21, 2009, was estimated at 307,886,149. Write this population count in words.

3. China has the largest population on Earth, with a January 2009 population of approximately one billion, three hundred twenty-five million, eighty-two thousand three hundred eighty. Write this population estimate as a numeral.

4. The average earned income for a person with a master's degree is $70,186. Round this value to the nearest thousand. To the nearest hundred.

5. One estimate of the world population toward the middle of 2009 was 6,774,451,418 people. Round this value to the nearest million. The nearest billion.

6. You use a check to purchase this semester's textbooks. The total is $343.78. How will you write the amount in words on your check?

7. Explain why 90,210 is less than 91,021.

8. Determine whether each of the following numbers is even or odd. In each case, give a reason for your answer.

   a. 22,225 **b.** 13,578 **c.** 1500

9. Determine whether each of the following numbers is prime or composite. In each case, give a reason for your answer.

   a. 35 **b.** 31 **c.** 51

Exercise numbers appearing in color are answered in the Selected Answers appendix.

10. Some of the fastest-growing jobs that earn the highest income and require a bachelor's degree are listed in the following table with their median annual salaries for 2007.

OCCUPATION	2007 MEDIAN ANNUAL SALARY, $
Network systems and data communications analysts	68,224
Computer software engineers, applications	83,138
Personal financial advisors	67,662
Financial analysts	70,408
Computer systems analysts	73,091

a. Round each median salary to the nearest hundred.

b. Round each median salary to the nearest thousand.

c. Is the median salary of personal financial advisors as much as that of network systems and data communications analysts? Justify your answer.

11. You are researching information on buying a new sports utility vehicle (SUV). A particular SUV that you are considering has a manufacturer's suggested retail price (MSRP) of $31,310. The invoice price to the dealer for the SUV is $28,707. Do parts a and b to estimate how much bargaining room you have between the MSRP and the dealer's invoice price.

a. Round the MSRP and the invoice price each to the nearest thousand.

b. Use the rounded values from part a to estimate the difference between the MSRP and invoice price.

Activity 1.2

Bald Eagle Population Increasing Again

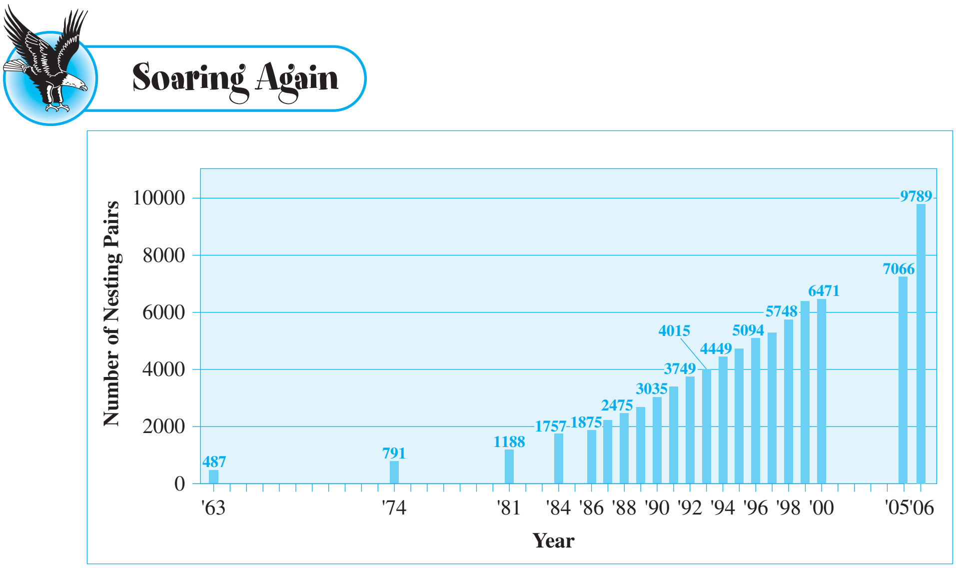
Objectives

1. Read tables.
2. Read bar graphs.
3. Interpret bar graphs.
4. Construct graphs.

The bald eagle has been the national symbol of the United States since 1782, when its image with outspread wings was placed on the country's Great Seal. Bald eagles were in danger of becoming extinct about forty years ago, but efforts to protect them have worked. On June 28, 2007, the Interior Department took the American bald eagle off the endangered species list.

Bar Graphs

The following bar graph displays the numbers of nesting bald eagle pairs in the lower 48 states for the years from 1963 to 2006. The horizontal direction represents the years from 1963 to 2006. The vertical direction represents the number of nesting pairs.



Source: U.S. Fish and Wildlife Service

1. a. What was the number of nesting pairs in 1963? In 1986? In 1998? In 2000? In 2006?

   b. Write the numbers from part a in words.

   c. Explain how you located these numbers on the bar graph.

2. a. Estimate the number of nesting pairs in 1987. In 1991. In 1997.

   b. Explain how you estimated the numbers from the bar graph.

   c. To what place value did you estimate the number of nesting pairs in each case?

   d. Compare your estimates with the estimates of some of your classmates. Briefly describe the comparisons.

3. a. Estimate the number of nesting pairs in 1985.

b. Explain how you determined your estimate from the graph.

c. Estimate to the nearest thousand the number of nesting pairs in 1977.

d. Compare the growth in the number of nesting pairs from 1963 to 1986 and from 1986 to 2006.

Graphing and Coordinate Systems

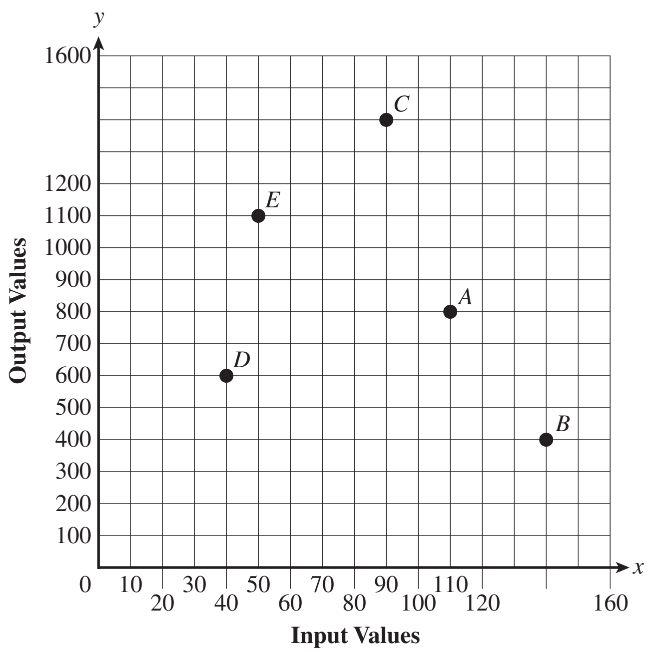
Note that you can represent the 791 nesting pairs in 1974 symbolically by (1974, 791). Two paired numbers listed in parentheses and separated by a comma are called an **ordered pair**. The first number in an ordered pair is always found or given along the horizontal direction on a graph and is called an **input** value. The second number is found or given in the vertical direction and is known as an **output** value.

4. a. Write the corresponding ordered pairs for the years 1988 and 1993, where the first number represents the year and the second number represents the number of nesting pairs.

b. What is the input value in the ordered pair (1989, 2680)? What does the value represent in this situation?

c. What is the output value in the ordered pair (1992, 3749)? What does the value represent in this situation?

The following is an example of a basic graphing grid used to display paired data values.



The horizontal line where the input values are referenced is called the **horizontal axis**. The vertical line where the output values are referenced is called the **vertical axis**. The scale (the number of units per block) in each direction should be appropriate for the given data. Each block in the grid on the previous page represents 10 units in the horizontal direction and 100 units in the vertical direction. It is important when making a graph to label the units in each direction.

5. Label the remaining three units on the horizontal axis on the graphing grid. Do the same for the vertical axis.

When units are given on the axes, you can determine the input and output values of a given point. For example, you can determine the input value of point *A* by following the vertical line straight down from point *A* to where the line crosses the horizontal axis. Read the input value 110 at the intersection. Similarly, read the output value 800 by following the horizontal line straight across from point *A* to the vertical axis.

6. Determine the input and output values of the points *B* and *C* on the graphing grid. Write each answer as ordered pairs on the grid next to its point.

The input and output values of an ordered pair are also referred to as the **coordinates** of the point that represents the pair on the graph. The letter x is frequently used to denote the input and y is used to denote the output. In such a case, the input value is called the x-coordinate and the horizontal axis is referred to as the x-axis. The output value is called the y-coordinate and the vertical axis is referred to as the y-axis.

7. a. What are the coordinates of the point *C* on the grid?

b. What is the x-coordinate of the point *D*?

c. What is the y-coordinate of the point *E*?

You can **plot**, or place, points on the grid after you determine the scale and label the units. For example, the point with coordinates (20, 300) is located as follows:

i. Start at the lower left-hand corner point labeled 0 (called the origin) and count 20 units to the right.

ii. Then count 300 units up and mark the spot with a dot.

8. a. Plot the point with coordinates (70, 900) on the graphing grid. Write the ordered pair next to the point.

b. The x-coordinate of a point is 150 and the y-coordinate is 100. Plot and label the point.

c. Plot and label the points (75, 350) and (122, 975).

d. Plot and label the points (0, 0), (0, 500), and (80, 0).

You may have noticed that a **bar chart** is a common variation of the basic grid that is used when inputs are **categories**. Categories on a bar chart are represented by intervals of equal length usually on the horizontal axis. The rectangular bars drawn from the horizontal axis have equal widths and vary in height according to their outputs.

9. Graph the data in the table on page 12 as a bar chart on the accompanying grid. Notice that the inputs are educational levels (categories).

a. Write the name of the output along the vertical axis and the name of the input along the horizontal axis.

b. List the input along the horizontal axis. The first two inputs are placed for you.

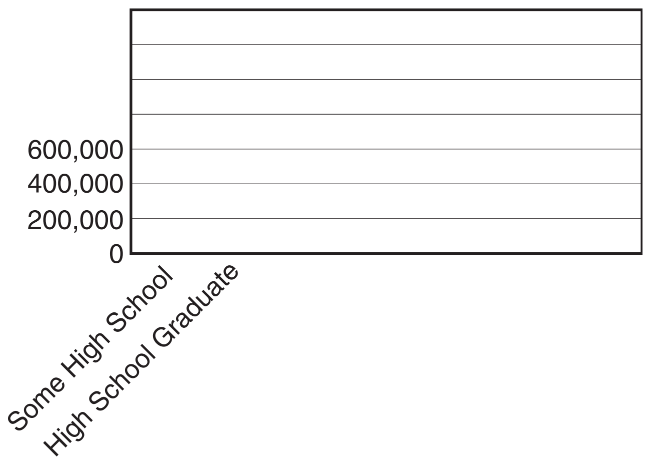
c. List the units along the vertical axis. The first three units are given.

d. Draw the bars corresponding to the given input/output pairs.

AVERAGE 2007 EARNINGS BY EDUCATIONAL LEVEL: FULL-TIME WORKERS 18 YEARS OF AGE AND OLDER

Educational Level	Some high school	High school graduate	Some college	Associate's degree	Bachelor's degree	Master's degree	Doctorate degree	Professional degree
Average Income Level	$29,120	$37,548	$43,554	$46,896	$66,689	$79,628	$106,014	$132,381

Source: U.S. Census Bureau

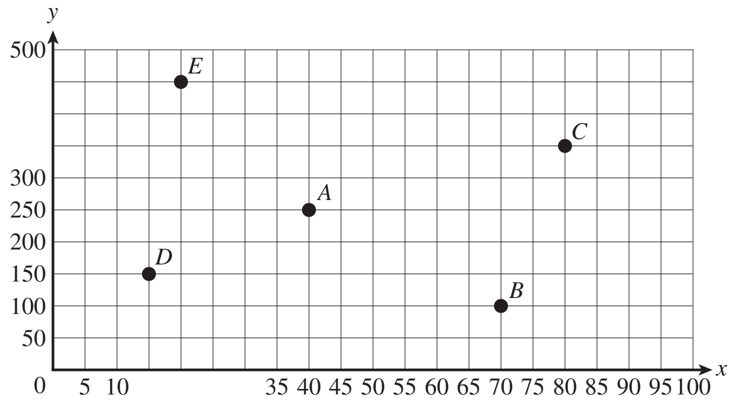


SUMMARY: ACTIVITY 1.2

1. A **graphing grid** displays paired data (input value, output value).
2. The horizontal direction is marked in units along a line called the **horizontal axis**. The vertical direction is marked in units along a line called the **vertical axis**.
3. Paired values are represented by points on the grid. The input value is read by following a vertical line down from the point to the horizontal axis. The output value is read by following a horizontal line from the point across to the vertical axis.
4. To graph a paired value (input, output), start at the point (0, 0) and move the given number of input units to the right and then move the given number of output units up. Mark the point.

EXERCISES: ACTIVITY 1.2

In Exercises 1–3, use the following grid to answer the questions.



1. a. What is the size of the input unit on the grid shown above? _______

 b. Write in the missing input units on the grid.

 c. What is the size of the output unit? _______

 d. Write in the missing output units on the grid.

2. a. What are the coordinates of the points A, B, and C on the given grid? Write your answers as ordered pairs next to the points on the grid.

 b. What is the x-coordinate of the point D?

 c. What is the y-coordinate of the point E?

3. a. Plot the point with coordinates (60, 100) on the given grid. Write the ordered pair next to the point.

 b. Plot the point with coordinates (10, 75) on the given grid. Write the ordered pair next to the point.

 c. The x-coordinate of a point is 95 and the y-coordinate is 100. Plot and label the point.

 d. The x-coordinate of a point is 55 and the y-coordinate is 225. Plot and label the point.

 e. Plot and label the points (0, 0), (0, 200), (40, 0), and (0, 325).

4. Graph the ordered pairs given in the table. Use the following grid as given; do not extend the graph in either direction.

Input	2	4	7	8	10	12	14	15
Output	5	10	17	20	25	30	35	38

 a. What size unit would be reasonable for the input?

 b. What size unit would be reasonable for the output?

c. List the units along the horizontal axis; along the vertical axis.

d. Now plot the points from the table. Label each point with its input/output value.

5. Graph the data in the following table as a bar chart on the accompanying grid.

a. Write the name of the output along the vertical axis and the name of the input along the horizontal axis.

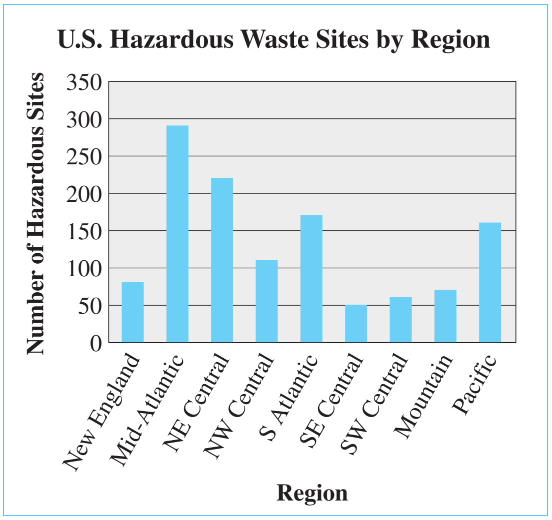
b. List the input categories along the horizontal axis.

c. List the units along the vertical axis.

d. Draw the bars corresponding to the given input/output pairs.

OCCUPATION	2007 MEDIAN ANNUAL SALARY ($)
Computer software engineer	85,660
Computer systems analyst	75,890
Database administrator	70,260
Physician assistant	77,800

6. In the twentieth century, the dumping of hazardous waste throughout the United States polluted waterways, soil, and air. Statistical evidence was one factor that led to federal laws aiming to protect human health in the environment. For example, the following chart presents some historical evidence on the number of abandoned hazardous waste sites found in various regions in the United States in the 1990s. Use the information in the chart to answer parts a–g that follow it.



a. How many regions are represented in the chart?

b. Which region has the greatest number of hazardous waste sites? Estimate the number.

c. Which region has the least number of hazardous waste sites? Estimate the number.

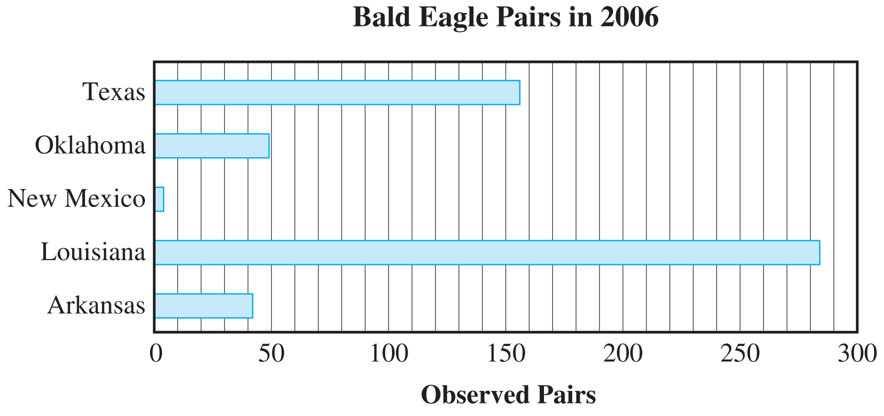
d. Which region would you say is in the middle in terms of waste sites?

e. What feature of this chart aids in estimating the number of waste sites for a given region?

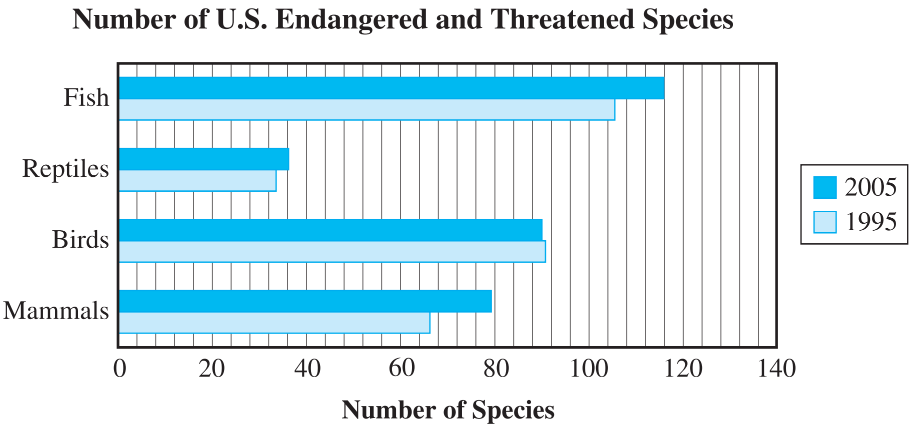
f. Based on this data, in what region(s) would you advise someone to live (or not live)?

g. From the chart, estimate the total number of waste sites found in the United States by the 1990s.

7. Bar graphs can be oriented either vertically or horizontally. Use the following chart to estimate how many more bald eagle pairs were observed in Louisiana compared to Texas in 2006.



8. Bar graphs can also show, for comparison, data collected at different times. The following chart shows data for four categories of animals: fish, reptiles, birds and mammals.



a. Which category of animal actually saw a drop in the number of endangered and threatened species between 1995 and 2005? Explain your answer.

b. Which category of animal showed the largest increase, and by approximately how many species?

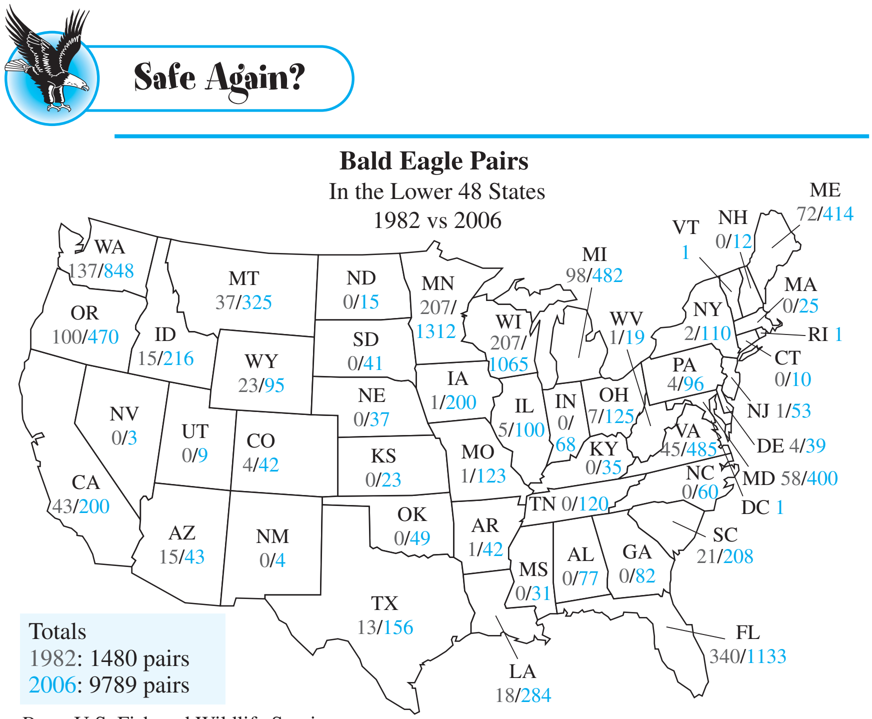
Activity 1.3

Bald Eagles Revisited

Objectives

1. Add whole numbers by hand and mentally.
2. Subtract whole numbers by hand and mentally.
3. Estimate sums and differences using rounding.
4. Recognize the associative property and the commutative property for addition.
5. Translate a written statement into an arithmetic expression.

In the 1700s, there were an estimated 25,000 to 75,000 nesting bald eagle pairs in what are now the contiguous 48 states. By the 1960s, there were less than 450 nesting pairs due to the destruction of forests for towns and farms, shooting, and DDT and other pesticides. In 1972, the federal government banned the use of DDT. In 1973, the bald eagle was formally listed as an endangered species. By the 1980s, the bald eagle population was clearly increasing. The following map of the contiguous 48 states displays the number of nesting pairs of bald eagles for each state. There are two numbers for each state. The first is for 1982, and the second is for 2006.



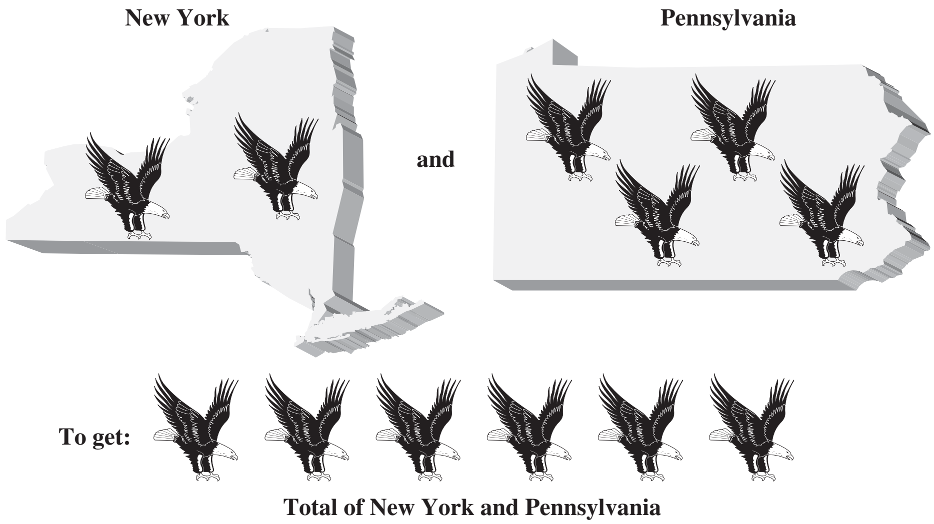
Data: U.S. Fish and Wildlife Service

Addition of Whole Numbers

Example 1 ***What was the total number of nesting bald eagle pairs in New York (NY) and Pennsylvania (PA) in 1982?***

SOLUTION

Calculate the total number of bald eagle nesting pairs in 1982 in New York and Pennsylvania by combining the two sets. Note that each eagle on the next page represents a bald eagle nesting pair.



In whole-number notation, $2 + 4 = 6$.

Example 2

Calculate the total number of bald eagle pairs in Maine (ME) and Kentucky (KY) in 2006.

SOLUTION

To calculate the total number of bald eagle pairs in Maine and Kentucky in 2006, combine the 414 pairs in Maine and 35 pairs in Kentucky using addition.

a. Set up the addition vertically so that the place values are aligned vertically.

b. Add all the digits in the ones place.

c. Add all the digits in the tens place.

Tens: Add the $1 + 3$ in tens place to get 4.

Hundreds: Bring down the 4.

Ones: Add the $4 + 5$ in the ones place to get 9.

$$\begin{array}{r} 414 \\ +\ 35 \\ \hline 449 \end{array}$$

The numbers that are being added, 414 and 35, are called **addends**. Their total, 449, is called the **sum**.

1. a. Calculate the total number of bald eagle pairs in New York (NY) and New Jersey (NJ) in 2006.

b. Calculate the total number of bald eagle pairs in Idaho (ID) and Missouri (MO) in 2006.

c. In part b, does it make a difference whether you set up the addition $216 + 123$ or $123 + 216$?

The order in which you add two numbers does not matter. This property of addition is called the **commutative property**. For example,

$$17 + 32 = 32 + 17$$

Example 3 ***Calculate the total number of bald eagle pairs in Lousiana (LA) and Indiana (IN) in 2006 by setting up the addition vertically.***

SOLUTION

Set up the addition vertically. There are two methods for determining the sum.

Method 1:

Sums of Digits in Place Value

$$\begin{array}{rl} 284 & \\ +\ 68 & \\ \hline 12 & \text{adding 4 ones + 8 ones} \\ 140 & \text{adding 8 tens + 6 tens} \\ 200 & \\ \hline 352 & \end{array}$$

Method 2:

Regroup to Next Higher Place Value

Regroup 1 to hundreds place.
Regroup 1 to tens place.

$$\begin{array}{r} {}^{1\,1} \\ 284 \\ +\ \ 68 \\ \hline 352 \end{array}$$

4 + 8 = 12, write 2 in ones place and regroup 1 to tens place.
1 + 8 + 6 = 15, write 5 in ones place and regroup 1 to hundreds place.

2. **a.** Calculate the total number of bald eagle pairs in Arizona (AZ) and Delaware (DE) in 2006 using method 1.

   b. Calculate the total number of bald eagle pairs in Arizona and Delaware in 2006 using method 2. Compare this result to the one obtained in part a.

   c. Calculate the total number of bald eagle pairs in Indiana (IN), Kansas (KS), and Kentucky (KY), in 2006 using method 1.

   d. Calculate the total number of bald eagle pairs in Indiana, Kansas, and Kentucky in 2006 using method 2. Compare this result to the one obtained in part c.

3. a. Mentally calculate the total number of bald eagle pairs in Idaho (ID), New Mexico (NM), and Arizona (AZ) in 2006 by determining the sum of two numbers and then adding the sum to the third number.

b. Does it make a difference which two numbers you add together first?

When calculating the sum of three whole numbers, it makes no difference whether the first two numbers or the last two numbers are added together first. This property of addition is called the **associative property**. For example,

$$(143 + 4) + 36 = 143 + (4 + 36).$$

4. a. Mentally calculate the total number of bald eagle pairs in Washington (WA), Oregon (OR), and California (CA) in 1982.

b. Mentally calculate the total number of bald eagle pairs in Washington, Oregon, and California in 2006.

c. Explain the process you used to add these numbers.

5. a. Choose five states and calculate the total number of nesting bald eagle pairs in 1982.

b. Calculate the total number of nesting bald eagle pairs in 2006 for the five states you chose in part a.

Estimating Sums of Whole Numbers

Estimation is useful for adding several numbers quickly and for checking that a calculated sum is reasonable. One way to estimate is to round each number (addend) to the same place value.

Example 4 ***Estimate the total number of nesting pairs in Florida (FL), South Carolina (SC), and Virginia (VA) in 2006. Then calculate the exact sum.***

SOLUTION

Estimated Sum:		*Exact Sum:*
1100 ←	Florida	→ 1133
200 ←	South Carolina	→ 208
+ 500 ←	Virginia	→ + 485
1800		1826

In Example 4, the estimate is lower than the exact sum. An estimate may be higher, lower, or occasionally equal to the exact sum. Notice that the exact sum 1826 is reasonable for the given data because it is close in value to the estimated sum 1800.

6. a. Estimate the number of nesting pairs in Louisiana (LA), Maine (ME), and Arizona (AZ) in 1982 by rounding each number to the tens place.

   b. Calculate the exact number of pairs in 1982.

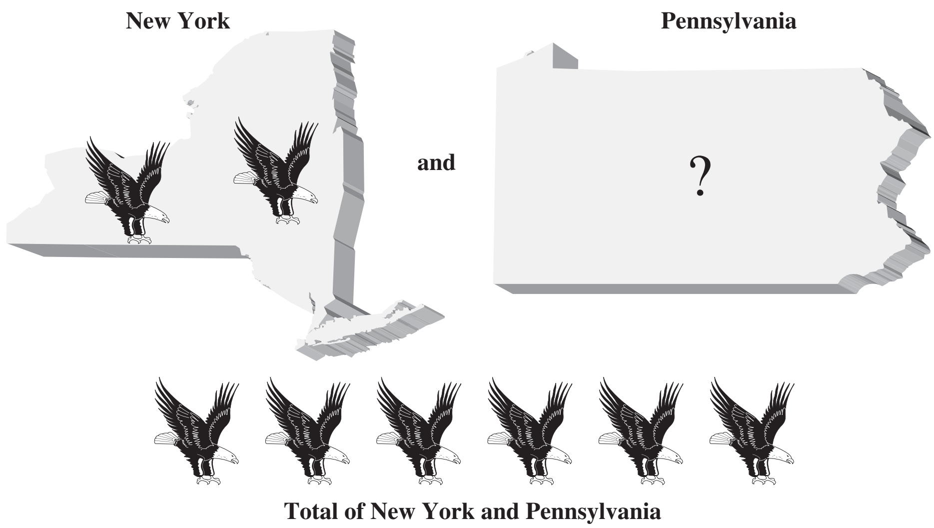
   c. Estimate the number of nesting pairs in Louisiana, Maine, and Arizona in 2006 by rounding each number to the tens place.

   d. Calculate the exact number of pairs in 2006.

   e. Compare the exact results to your estimates. State whether the estimates are higher, lower, or the same as the exact results. Do your estimates indicate that your exact results are reasonable sums for the data given?

Example 5 *Subtraction of Whole Numbers*

The number of bald eagle nesting pairs in 1982 for New York and the combined total for New York and Pennsylvania are given in the following graphic. Calculate the number of nesting pairs in Pennsylvania.



The picture suggests that the problem is to find the **missing addend**. In symbols, the calculation can be written in terms of addition as $\mathbf{2 + ? = 6}$. By thinking of a number that added to 2 gives 6, you see that the answer is 4 pairs of bald eagles.

Alternatively, by thinking of taking away **(subtracting)** 2 from 6, the answer is the same, 4 pairs of bald eagles. In symbols, the calculation is written as $\mathbf{6 - 2 = ?}$

Subtraction is finding the difference between two numbers. The operation of subtraction involves *taking away*. In Example 5, take 2 bald eagle nesting pairs away from 6 bald eagle nesting pairs to get the difference, 4 nesting pairs.

7. a. How many more bald eagle nesting pairs were there in 2006 than in 1982 in Oregon (OR)?

b. Set up the calculation for part a using the missing addendum approach.

c. Set up the calculation using subtraction.

Example 6

How many more nesting pairs were in Georgia (GA) than were in Oklahoma (OK) in 2006?

SOLUTION

$$\begin{array}{r} {\scriptstyle 7\ 12} \\ \cancel{8}\,2 \\ -\,4\,9 \\ \hline 3\,3 \end{array}$$

Convert 1 ten to 10 ones. Reduce 8 tens to 7 tens. Add the 10 ones to 2 to obtain 12 ones.

Subtract 9 from 12, and subtract 4 from 7.

Check using addition:

$$\begin{array}{r} 33 \\ +\ 49 \\ \hline 82 \end{array}$$

Formally, the number that is subtracted is called the **subtrahend**. The number subtracted from is called the **minuend**. The result is called the **difference**. To check subtraction, add the difference and the subtrahend. The result should be the minuend.

$$\begin{array}{rl} 82 & \text{minuend} \\ -\,49 & \text{subtrahend} \\ \hline 33 & \text{difference} \end{array}$$

Check using addition:

$$\begin{array}{r} 33 \\ +\ 49 \\ \hline 82 \end{array}$$

Here, addition is used to check subtraction. Addition is called the **inverse** operation for subtraction.

8. a. Determine the difference between the number of nesting pairs in Louisiana (LA) in 2006 and 1982.

b. What number is being subtracted? Why?

9. a. Calculate the increase in the population of nesting pairs in Michigan (MI) from 1982 to 2006.

b. Determine the difference between the number of nesting pairs in Wisconsin (WI) and in Minnesota (MN) in 2006.

c. Determine the difference between the number of nesting pairs in Wisconsin and Minnesota in 1982.

10. a. Estimate the difference between the number of nesting pairs in Florida (FL) and in Washington (WA) in 2006.

b. Determine the exact difference.

c. Was your estimate higher or lower than the exact difference?

11. a. To what place value should you round to estimate the difference between the number of nesting pairs in Louisiana (LA) and in Montana (MT) in 2006?

b. The highest place value is the hundreds. Would it make sense to round to the hundreds place? Why or why not?

Add or Subtract?

When you set up a problem, it is sometimes difficult to decide if you need to do addition or subtraction. It is helpful if you can recognize some key phrases so that you will write a correct arithmetic expression. An **arithmetic expression** consists of numbers, operation signs $(+, -, \cdot, \div)$, and sometimes parentheses.

The following tables contain some typical key phrases, examples, and corresponding arithmetic expressions for addition and subtraction.

ADDITION			SUBTRACTION		
KEY PHRASE	EXAMPLE	ARITHMETIC EXPRESSION	KEY PHRASE	EXAMPLE	ARITHMETIC EXPRESSION
sum of	sum of 3 and 5	$3 + 5$	difference of	difference of 12 and 7	$12 - 7$
increased by	7 increased by 4	$7 + 4$	decreased by	95 decreased by 10	$95 - 10$
plus	12 plus 10	$12 + 10$	minus	57 minus 26	$57 - 26$
more than	5 more than 6	$6 + 5$	less than	5 less than 23	$23 - 5$
total of	total of 13 and 8	$13 + 8$	subtracted from	12 subtracted from 37	$37 - 12$
added to	45 added to 50	$50 + 45$	subtract	8 subtract 5	$8 - 5$

12. Translate each of the following into an arithmetic expression.

a. 34 plus 42

b. difference of 33 and 22

c. 100 minus 25

d. total of 25 and 19

e. 17 more than 102

f. 14 subtracted from 28

g. 81 increased by 16

h. 50 less than 230

i. 250 decreased by 120

j. sum of 18 and 21

k. 101 added to 850

SUMMARY: ACTIVITY 1.3

1. Numbers that are added together are called **addends**. Their total is the **sum**.
2. **To add numbers:** Align them vertically according to place value. Add the digits in the ones place. If their sum is a two-digit number, write down the ones digit and carry the tens digit to the next column as a number to be added. Repeat with the next higher place value.
3. The order in which you add two numbers does not matter. This property of addition is called the **commutative property**.
4. When calculating the sum of three whole numbers, it makes no difference whether the first two numbers or the last two numbers are added together first. This property of addition is called the **associative property**.
5. **Estimation** is useful for adding or subtracting numbers quickly and to check the reasonableness of an exact calculation. One way to estimate is to round each number to its highest place value. In most cases, it may be better to round to a place value lower than the highest one.
6. **Subtraction** is used to find the difference between two numbers. The operation of subtraction involves *taking away.*

   The number that is subtracted is called the **subtrahend**. The number being subtracted from is called the **minuend**. The result is called the **difference**. To check subtraction, add the difference and the subtrahend. The result should be the minuend.

   To subtract: Align the subtrahend under the minuend according to place value. Subtract digits having the same place value. Regroup from a higher place value, if necessary.

EXERCISES: ACTIVITY 1.3

1. Determine the sum using method 1 (sum of the digits by place values) as shown in Example 3.

a. $\begin{array}{r} 256 \\ +\ 35 \\ \hline \end{array}$

b. $\begin{array}{r} 617 \\ +149 \\ \hline \end{array}$

c. $\begin{array}{r} 51 \\ 382 \\ +\ 77 \\ \hline \end{array}$

2. Determine the sum using method 2 (regroup to next higher place value) as shown in Example 3.

a. $\begin{array}{r} 159 \\ +\ 27 \\ \hline \end{array}$

b. $\begin{array}{r} 924 \\ +138 \\ \hline \end{array}$

c. $\begin{array}{r} 51 \\ 382 \\ +\ 77 \\ \hline \end{array}$

3. Determine the sum of 67 and 75.

a. $67 + 75$

b. $75 + 67$

c. Are the sums in parts a and b the same?

d. What property of addition is demonstrated?

4. Determine the sum. Do the addition in the parentheses first.

a. $34 + (15 + 71)$

b. $(34 + 15) + 71$

c. Are the sums in parts a and b the same?

d. What property of addition is demonstrated?

5. a. Estimate the sum: $171 + 90 + 226$

b. Determine the actual sum.

c. Was your estimate higher, lower, or the same as the actual sum?

6. a. Estimate the sum: $326 + 474$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

b. Determine the actual sum.

c. Was your estimate higher, lower, or the same as the actual sum?

7. Evaluate.

a. $123 - 91$ **b.** $543 - 125$ **c.** $78 - 49$

d. $1002 - 250$ **e.** $2001 - 1962$ **f.** $696 - 384$

8. a. Subtract the year in which you were born from this year.

b. Is the difference you obtain your age?

c. Have you had your birthday yet this year? Does this affect your answer in part b?

9. This week, you took home \$96 from your part-time job. You owe your mother \$39.

a. Estimate the amount of money that you will have after you pay your mother.

b. Determine the actual amount of money you will have after you pay your mother.

10. In 2006, there were 216 bald eagle nesting pairs in Idaho (ID), 284 in Louisiana (LA), and 325 in Montana (MT).

a. Estimate the total number of nesting pairs in all three states.

b. Determine the actual total.

c. Is your estimate higher or lower than the actual total?

11. In 2006, there were 125 nesting pairs in Ohio (OH) and 96 nesting pairs in Pennsylvania (PA).

a. Estimate the difference between the nesting pairs in Ohio and Pennsylvania by rounding both numbers to the hundreds place.

b. If you round both numbers to the tens place, what is your estimate?

c. Which is the better "estimate"? Explain.

12. Translate each of the following into an arithmetic expression.

a. 13 plus 23

b. 108 minus 15

c. difference of 70 and 58

d. total of 45 and 79

e. 7 more than 12

f. 13 subtracted from 28

g. 85 increased by 8

h. 52 less than 300

i. 25 decreased by 12

Activity 1.4

Summer Camp

Objectives

1. Multiply whole numbers and check calculations using a calculator.
2. Multiply whole numbers using the distributive property.
3. Estimate the product of whole numbers by rounding.
4. Recognize the associative and commutative properties for multiplication.

You accept a job working in the kitchen at a small, private summer camp in New England. One hundred children attend the 8-week program under the supervision of 24 staff members. The job pays well, and it includes room and board with every other weekend off. One of your responsibilities is to pick up supplies twice a week at a local wholesale food club. Some of the items listed on this week's order form appear in the following receipt from the food club.

QUANTITY	ITEM	UNIT PRICE ($)
8	1 CASE (24 BOTTLES) 10-OZ BOTTLES OF JUICE	11.15
20	36 1.55-OZ MILK CHOCOLATE BARS	16.08
12	36 1-OZ SERVINGS OF CREAM CHEESE	6.39
18	32 1.25-OZ GRANOLA BARS	7.97
6	1 BOX OF 15 CARTONS OF 1 DOZ. LARGE EGGS	20.24
10	4-LB PACKAGE HOT DOGS (40 COUNT)	9.99
12	10-LB PACKAGE HAMBURGER (40 COUNT)	17.88
16	24-PACK HOT DOG ROLLS	3.12
18	24-PACK HAMBURGER ROLLS	5.32
30	2-LOAF PACK 20-OZ BREAD (20 SLICES)	3.33
12	42-COUNT VARIETY PACKAGE OF CHIPS	10.08
8	250-COUNT PACKAGE 9" PLATES	9.08
1	1500-COUNT PACKAGE DISPENSER NAPKINS	15.57
4	600-COUNT PACKAGE SPOONS	9.19
2	600-COUNT PACKAGE KNIVES	9.19
3	600-COUNT PACKAGE FORKS	9.19

1. The total number of bottles of juice purchased can be represented by the following sum.

$$24 + 24 + 24 + 24 + 24 + 24 + 24 + 24 = \underline{\qquad\quad}$$

a. Calculate this sum directly.

b. Calculate this sum using your calculator.

c. Is there a more efficient (shorter) way to do this calculation?

Multiplication of whole numbers is repeated addition. The sum $24 + 24 + 24 + 24 + 24 + 24 + 24 + 24$ can be rewritten as the **product** $8 \cdot 24$ or 8×24. The operation sign "$\times$" is the multiplication symbol generally used in arithmetic courses, but the symbol "$\cdot$"

is more common in algebra. The whole numbers 8 and 24 are called **factors** of the product 192. To do the multiplication by hand, set up the calculation vertically as follows.

$$\begin{array}{rl}
24 & \\
\times\ \ 8 & \\
\hline
32 & \text{Multiply 8 times 4.} \\
160 & \text{Multiply 8 times 20.} \\
\hline
192 & \text{Add 32 and160.}
\end{array}$$

Multiplication works when set up vertically because 24 can be written as $20 + 4$ and the factor 8 multiplies both the 20 and the 4. Set up horizontally, the calculation is written as follows:

$$8 \cdot 24 = 8 \cdot (20 + 4) = 8 \cdot 20 + 8 \cdot 4 = 160 + 32 = 192$$

Rewriting $8 \cdot (20 + 4)$ as $8 \cdot 20 + 8 \cdot 4$ is an example of the **distributive property of multiplication over addition**.

2. a. Calculate the total number of cartons of eggs you are to purchase this week.

b. Calculate the number of 9-inch plates you will purchase.

The multiplication for the total number of hamburger rolls can be set up vertically or horizontally.

Vertically:

$$\begin{array}{rl}
24 & \\
\times\ 18 & \\
\hline
32 & \text{Multiply 8 times 4.} \\
160 & \text{Multiply 8 times 20.} \\
240 & \text{Multiply 10 times 24.} \\
\hline
432 &
\end{array}$$

Horizontally:

$$18 \cdot 24 = \underbrace{18 \cdot (20 + 4) = 18 \cdot 20 + 18 \cdot 4}_{\text{Distributive property}} = 360 + 72 = 432$$

To perform the same multiplication a different way using subtraction, you notice that $24 = 30 - 6$. Horizontally, we can write.

$$18 \cdot 24 = \underbrace{18 \cdot (30 - 6) = 18 \cdot 30 - 18 \cdot 6}_{\text{Distributive property}} = 540 - 108 = 432$$

Rewriting $18 \cdot (30 - 6)$ as $18 \cdot 30 - 18 \cdot 6$ is an example of the **distributive property of multiplication over subtraction**.

3. a. Use the distributive property of multiplication over addition to find the number of ounces of cream cheese.

b. Do the same calculation again, but use the distributive property of multiplication over subtraction.

c. Compare your two answers. Are they the same?

You would like to know the total number of servings of meat you will have on hand. The number of hot dogs is $40 \cdot 10$ and the number of hamburgers is $40 \cdot 12$. You notice that the number 40 is common to both products. The distributive property of multiplication over addition can be used in reverse:

$$40 \cdot 10 + 40 \cdot 12 = 40 \cdot (10 + 12) = 40 \cdot 22 = 880.$$

When the distributive property is used in this fashion, it can be a useful technique, usually called **factoring**.

4. Use the distributive property in reverse to find the total number of servings of bread rolls (both hot dog rolls and hamburger rolls).

5. a. How many ounces of juice are there in one case of juice from the wholesale club?

b. Use your answer in part a to calculate the total number of ounces of juice needed this week.

c. How many bottles of juice are needed?

d. Use your answer in part c to calculate the total number of ounces of juice needed this week.

e. Compare your answers for parts b and d. Explain why you think these answers should be the same.

The property illustrated in Problems 5b and 5d is the **associative property of multiplication**. For example,

$$5 \cdot (4 \cdot 7) = (5 \cdot 4) \cdot 7$$

6. Describe the associative property of multiplication in your own words.

7. Calculate the total number of eggs on the list in two ways using the associative property of multiplication.

8. The multiplication to determine the total number of hamburger rolls can be set up two ways.

$$\begin{array}{r} 24 \\ \times 18 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 18 \\ \times 24 \\ \hline \end{array}$$

a. Determine the product for each of the two multiplication problems.

b. Are the answers are the same?

The mathematical property illustrated in Problem 8a is the **commutative property of multiplication**. For example,

$$8 \cdot 4 = 4 \cdot 8$$

9. Describe the commutative property of multiplication in your own words.

10. a. Determine the total number of hamburgers and the total number of hamburger rolls you need to purchase.

b. Do you need to change the number of packages of hamburger rolls? Explain why or why not.

11. One 1500-count package of dispenser napkins is purchased.

a. Using multiplication, calculate the total number of napkins purchased.

b. If you multiply any whole number by 1, what is the result?

c. If you multiply any whole number by zero, what is the result?

Estimation

You may not have easy access to a calculator at summer camp. So you may need to multiply or check multiplication mentally or by hand. To calculate the total number of individual 1-ounce servings of cream cheese, you need to multiply 12 times 36. The product can be estimated by rounding.

- Round 12 down to 10.
- Round 36 up to 40.
- Multiply 10 times 40.
- The estimated product is ________.

12. a. Multiply 12 and 36 to determine the actual number of servings of cream cheese.

b. What is the difference between the actual number of servings and the estimated number of servings?

13. a. Estimate the total number of granola bars in the order.

b. Determine the actual number of granola bars.

c. What is the difference between the actual number and the estimated number of granola bars?

Procedure

Estimating Products

- Round each factor to a large enough place value so that you can do the multiplication mentally.
- There is no one correct answer when estimating, only a reasonable answer.

14. Estimate the total number of chocolate bars.

15. Estimate the total number of individual snack packs of chips.

SUMMARY: ACTIVITY 1.4

1. Multiplication properties of whole numbers

Any whole number multiplied by 1 remains the same.

Any whole number multiplied by 0 is 0.

2. Distributive property

A whole number placed in front of a set of parentheses containing a sum or difference of two numbers multiplies each of the inside numbers.

$$3 \cdot (7 + 2) = 3 \cdot 7 + 3 \cdot 2 \quad \text{or} \quad 5 \cdot (10 - 4) = 5 \cdot 10 - 5 \cdot 4$$

3. Associative property of multiplication

When multiplying three whole numbers, it makes no difference which two numbers are multiplied first.

$$(2 \cdot 5) \cdot 7 = 2 \cdot (5 \cdot 7)$$

4. Commutative property of multiplication

Changing the order of two whole numbers when multiplying them produces the same product.

$$3 \cdot 6 = 6 \cdot 3$$

5. Estimating products

Round each factor to a large enough place value so that you can do the multiplication mentally.

There is no one correct answer when estimating, only reasonable answers.

EXERCISES: ACTIVITY 1.4

1. Multiply vertically. Verify your answer using a calculator.

a. $\begin{array}{r} 34 \\ \times\ 4 \\ \hline \end{array}$

b. $\begin{array}{r} 529 \\ \times\ \ 8 \\ \hline \end{array}$

c. $\begin{array}{r} 67 \\ \times\ 5 \\ \hline \end{array}$

d. $\begin{array}{r} 807 \\ \times\ \ 9 \\ \hline \end{array}$

e. $\begin{array}{r} 125 \\ \times\ \ 8 \\ \hline \end{array}$

f. $\begin{array}{r} 2001 \\ \times\ \ 25 \\ \hline \end{array}$

g. $\begin{array}{r} 75 \\ \times 52 \\ \hline \end{array}$

h. $\begin{array}{r} 1967 \\ \times\ 105 \\ \hline \end{array}$

2. **a.** Multiply 8 and 47 by rewriting 47 as $40 + 7$ and use the distributive property to obtain the result.

Exercise numbers appearing in color are answered in the Selected Answers appendix.

b. Multiply 8 and 47 vertically.

3. a. Multiply 12 and 36 by rewriting 36 as $30 + 6$, and use the distributive property to obtain the result.

b. Multiply 12 and 36 vertically.

4. Three 600-count packages of forks are purchased.

a. Use addition to determine the total number of forks.

b. Use multiplication to determine the total number of forks.

5. Ten 40-count packages of hot dogs are purchased.

a. Determine the total number of hot dogs by calculating $10 \cdot 40$.

b. Calculate: $40 \cdot 10$.

c. What property of multiplication is demonstrated by the fact that the answers to parts a and b should be the same?

6. a. Evaluate: $72 \cdot 23$

b. Evaluate: $23 \cdot 72$

c. Are the answers to parts a and b the same?

d. What property do the results of this exercise demonstrate?

7. Evaluate by finding the product in parentheses first.

a. $7 \cdot (13 \cdot 20)$ **b.** $(7 \cdot 13) \cdot 20$

c. Are the answers to parts a and b the same?

d. What property do the results in parts a and b demonstrate?

8. You purchase eighteen 42-count packages of variety chips for the summer camp.

a. Estimate the total number of individual packages of chips purchased.

b. Determine the actual number of individual packages you purchased.

c. Is the estimated total higher or lower than the actual total?

9. You purchase sixteen 24-pack hot dog rolls this week.

a. Estimate the total number of hot dog rolls.

b. Determine the actual number of hot dog rolls.

c. Is the estimated total higher or lower than the actual total?

Activity 1.5

College Supplies

Objectives

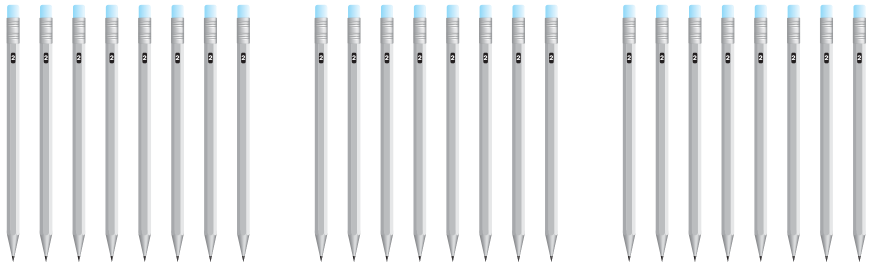
1. Divide whole numbers by grouping.
2. Divide whole numbers by hand and by calculator.
3. Estimate the quotient of whole numbers by rounding.
4. Recognize that division is not commutative.

School Supplies

It is the beginning of a new semester and time to purchase supplies. You and five fellow students decide to shop at a discount office supply store. The six of you purchase the following items.

QUANTITY	ITEM	UNIT PRICE ($)
3	8-PACK NUMBER 2 PENCILS	1.47
3	5-PACK MECHANICAL PENCILS	3.79
3	4-PACK REFILLABLE MECHANICAL PENCILS	5.79
2	10-PACK ASSORTED GEL RETRACTABLE PENS	10.99
3	2-PACK BALLPOINT PENS	3.99
2	4-PK LIQUID PAPER	7.79
3	10-PACK ASSORTED HIGHLIGHTERS	11.39
3	5-PACK PERMANENT MARKERS	5.29
6	400-COUNT PACKAGE 8.5" X 11" COLLEGE-RULED PAPER	4.99
20	POCKET FOLDERS	0.33
12	THREE-SUBJECT SPIRAL NOTEBOOK	4.99
4	5-COUNT PACK REPORT COVERS	5.49
3	500-COUNT PACK 3" X 5" INDEX CARDS	3.59

There are three 8-count packages of number-two pencils or a total of 24 pencils, illustrated below.



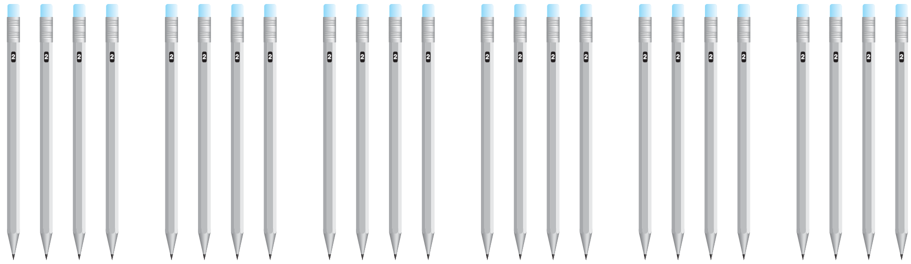
These 24 pencils need to be divided among the six of you. To set this up as a division calculation, write either $24 \div 6$, $24/6$, or $6\overline{)24}$.

Definition

The number *being divided* is called the **dividend**. The number that divides the dividend is called the **divisor**. Here, the number 6 is the divisor, and the number 24 is the dividend. The **quotient** is the result of the division. In this case, 4 is the quotient.

$$\begin{array}{rl} 4 & \leftarrow \text{quotient} \\ \text{divisor} \rightarrow 6\overline{)24} & \leftarrow \text{dividend} \end{array}$$

If 24 pencils are distributed equally among the six of you, you will each get 4 pencils.



You used division to distribute the 24 pencils equally among yourselves. The result was 4 pencils per student. In general, you use division to separate items into a specified number of equal groupings.

The two ways the division process can be stated:

$$\text{dividend} \div \text{divisor} = \text{quotient, with a possible remainder,}$$

or, in long division format,

$$\begin{array}{r} \text{quotient with a possible remainder} \\ \text{divisor}\overline{)\text{dividend}} \end{array}$$

Multiplication and division are **inverse operations**. This means that the division $24 \div 6 = 4$ can be written as the multiplication $4 \cdot 6 = 24$, and vice versa.

1. a. Your group buys three 5-packs of mechanical pencils. Distribute these 15 pencils equally among the six of you. How many pencils will each of you receive? Are there any pencils left over?

b. Identify the divisor and the dividend. What is the quotient? What is the remainder, if any?

2. a. How many gel pens are there in the two packages of 10-count assorted gel retractable pens?

b. Distribute the gel pens equally among the six of you. Set up as a division problem. What is the quotient? What is the remainder, if any?

To check division, multiply the quotient and the divisor, then add the remainder. The result should be the dividend.

$$(\text{quotient} \times \text{divisor}) + \text{remainder} = \text{dividend}$$

3. Check the division you did in Problem 2.

Division can be considered as repeated subtraction of the divisor from the dividend, with a remainder left over.

Example 1 ***You buy 20 pocket folders. Distribute them equally among the six of you.***

SOLUTION

a. Use long division.

$$\begin{array}{r} 3 \\ 6\overline{)20} \\ \underline{18} \\ 2 \end{array}$$

b. Use repeated subtraction.

$$\begin{array}{r} 20 \\ \underline{-\ 6} \\ 14 \\ \underline{-\ 6} \\ 8 \\ \underline{-\ 6} \\ 2 \end{array}$$

6 can be subtracted from 20 three times.

3 is the quotient.

2 is the remainder.

In general, when you divide, you are repeatedly subtracting multiples of the divisor from the dividend until no whole multiples remain.

Example 2 ***Your group purchased three 500-count packs of 3" × 5" index cards. Distribute these 1500 index cards equally among the six of you. How many index cards do you each get?***

SOLUTION

Use long division to determine that each of you receives 250 index cards with none left over.

$$\begin{array}{r} 250 \\ 6\overline{)1500} \\ \underline{-12\ \ } \\ 30\ \\ \underline{-30\ } \\ 00 \end{array}$$

6 does not divide into 1. 6 does divide into 15 two times.
Write down the product 2 · 6 = 12. Subtract 12 from 15.
Bring down 0, the next digit. 6 divides into 30 five times.
Write down the product 5 · 6 = 30. Subtract 30 from 30.
Bring down the last 0. 6 divides into 0 zero times.
The remainder is 0.

4. a. Your group purchased six 400-count packages of 8.5″ × 11″ college-ruled paper. If these packages are distributed equally among the six of you, how many packages will you each receive?

   b. Suppose 52 packages are distributed equally among 52 students. How many packages will each student receive?

   c. How many times can you subtract 52 from 52?

   d. What result do you get when you divide any nonzero number by itself?

5. There was a stapler in the shopping cart, but you returned it to the shelf because each of you already had one.

   a. How many staplers will you receive from this shopping expedition?

b. Use your calculator to divide 0 by 6. What is the result? Try dividing zero by another nonzero whole number. What is the result?

c. Use your calculator to divide 6 by 0. What happens?

If you try to divide by zero, a basic or scientific calculator will display the letter E to signify an error. Graphing calculators usually display the word ERROR with a message. The reason is that if you change $6 \div 0 = ?$ to a multiplication calculation, it becomes $0 \cdot ? = 6$. No whole number works because zero times any whole number is zero, not 6. This means that $6 \div 0$ has no answer. Another way to say this is that $6 \div 0$ is **undefined**.

d. Explain why 9 divided by 0 is undefined.

Definition

Division Properties Involving 0 and 1	Example:
• Any nonzero whole number divided by itself is 1.	$34 \div 34 = 1$
• Any whole number divided by 1 is itself.	$6 \div 1 = 6$
• 0 divided by any nonzero whole number is 0.	$0 \div 10 = 0$
• Division by 0 is undefined.	$7 \div 0$ is undefined.

6. Three 10-count packs of assorted highlighters contain a total of 30 highlighters.

a. By doing the division $30 \div 6$, determine the number of highlighters each student will receive.

b. Do you get the same result by doing the division, $6 \div 30$?

c. Does the commutative property hold for division? That is, does $6 \div 30 = 30 \div 6$?

Rent and Utilities

This year, you decide to rent an apartment near campus with three other students. The rent is \$1175 per month, high-speed Internet access is \$84 per month, and the average monthly utility bill is \$253.

To estimate your share of the monthly Internet access charge, you would round \$84 to the tens place and get \$80. Dividing \$80 by 4 (students), you estimate your share to be \$20 per month.

To estimate a quotient, first estimate the divisor and the dividend using numbers that allow for easier division mentally or by hand. For example, estimate $384 \div 6$ by rounding 384 to 400 and replacing 6 by 5. The estimate is $400 \div 5 = 80$, which is 16 more than the exact result, 64.

7. a. Estimate your share of the rent by rounding \$1175 to the hundreds place and then dividing by 4.

b. Is your estimate lower or higher than the actual amount that you owe?

8. a. Estimate your share of the utility bill by rounding \$253.

b. Is your estimate lower or higher than the actual amount that you owe?

9. Estimate your share of the monthly Internet access charge by rounding.

10. a. Add the monthly charges for rent, Internet access, and utilities. Calculate your actual share of this sum. Verify your calculation using your calculator.

b. Compare your actual share of the monthly expenses with the sum of the individual estimates for the monthly rent, Internet access, and utility costs.

11. a. Estimate: $19{,}500 \div 78$

b. Determine the exact answer.

c. Is your estimate lower, higher, or the same?

12. **a.** Estimate: $5880 \div 120$

b. Determine the exact answer.

c. Is your estimate lower, higher, or the same?

13. **a.** Estimate: $30{,}380 \div 490$

b. Determine the exact answer.

c. Is your estimate lower, higher, or the same?

SUMMARY: ACTIVITY 1.5

Division Properties of Whole Numbers

1. In general, use division to separate items into a specified number of equal groupings.
2. The numbers involved in the division process have specific names:
   dividend ÷ divisor = quotient with a possible remainder, or, in long division format,

$$\text{divisor}\overline{\smash{)}\text{dividend}}^{\text{quotient with a possible remainder}}$$

3. To check division, multiply the quotient by the divisor, then add the remainder. The result should be the dividend.

$$(\text{quotient} \times \text{divisor}) + \text{remainder} = \text{dividend}$$

4. Multiplication and division are **inverse operations**.

5. Division is *not* commutative. For example, $10 \div 5 \neq 5 \div 10$.

Division Properties Involving 0 and 1

6. Any nonzero whole number divided by itself is 1.

7. Any whole number divided by 1 remains the same.

8. 0 divided by any nonzero whole number is 0.

9. Division by 0 is undefined.

Estimating a Quotient

10. To estimate a quotient, first estimate the divisor and the dividend by numbers that provide a division easily done mentally or by hand.

EXERCISES: ACTIVITY 1.5

1. Calculate the following. As part of your answer, identify the quotient and remainder (if any).

a. $56 \div 7$

b. $112 \div 4$

c. $95 \div 3$

d. $222 \div 11$

e. $506 \div 13$

f. $587 \div 23$

g. $0 \div 15$

h. $15 \div 0$

2. Six students purchase seven 5-packs of report covers to distribute equally among themselves.

a. Determine the total number of report covers to be distributed.

b. How many report covers will each student receive?

Exercise numbers appearing in color are answered in the Selected Answers appendix.

c. Set the calculation up as a long division and divide.

d. Identify the divisor and the dividend. What is the quotient? What is the remainder, if any?

e. Are there any report covers left over?

f. Do this problem again using repeated subtraction.

3. Four students will share equally three 500-count packages of $3'' \times 5''$ index cards.

a. Determine the total number of index cards to be distributed.

b. How many index cards will each student receive?

c. Set up the calculation as a long division and divide.

d. Identify the divisor and the dividend. What is the quotient? What is the remainder, if any?

e. Are there any index cards left over?

4. a. Divide: $24 \div 8$.

   b. Do you get the same result by doing the division $8 \div 24$?

   c. Does the commutative property hold for division? That is, does $24 \div 8 = 8 \div 24$?

5. Your college campus has many more students who drive to campus than it has parking spaces. Even if you arrive early, it is difficult to find a parking space on any Monday, Wednesday, or Friday. The college is planning for future growth and in assessing the current parking problem estimates that 825 additional parking spaces will be needed. There are several parcels of land that will each accommodate a 180-car parking lot.

   a. Estimate the number of parking lots that are needed.

   b. Determine the actual number of parking lots that are needed by first dividing 825 by 180.

   c. Was your estimate too high or too low?

6. a. Estimate: $3850 \div 52$

   b. Find the exact answer.

   c. Is your estimate lower, higher, or the same?

7. a. Estimate: $28{,}800 \div 314$

   b. Find the exact answer.

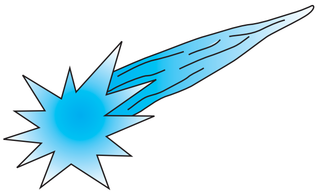
   c. Is your estimate lower, higher, or the same?

Activity 1.6

Reach for the Stars

Objectives

1. Use exponential notation.
2. Factor whole numbers.
3. Determine the prime factorization of a whole number.
4. Recognize square numbers and roots of square numbers.
5. Recognize cubed numbers.
6. Apply the multiplication rule for numbers in exponential form with the same base.



Astronomical Distances

On a clear night, thousands of stars are visible. Some stars appear larger and brighter than others. Their distances from Earth also vary greatly. For instance, the distance to the star Altair is about 100,000,000,000,000 miles. These distances are so large that they are unmanageable as written whole numbers. It is easier to write very large numbers using exponents. The distance from Earth to Altair is approximately one hundred trillion miles and can be written in exponential form as 10^{14} miles. The exponent, 14, means to use the base, 10, as a factor 14 times.

$$\underbrace{10}_{\text{base}}{}^{\overbrace{14}^{\text{exponent}}} = \underbrace{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}_{\text{10 is used as a factor 14 times.}}$$

$$= 100{,}000{,}000{,}000{,}000$$

A whole-number **exponent** indicates the number of times the **base** is used as a factor. An exponent is also called a **power**. Note that 10 can be written as 10^1. When there is no exponent, it is understood to be 1. A number such as 10^{14} is in **exponential form** and is read as "ten to the fourteenth power."

Because our number system uses 10 as its base, there is a shortcut to writing powers of ten. To write a power of ten as a whole number, write a 1 followed by as many zeros as the power of 10. For example, 10^3 is 1 followed by 3 zeros, or 1000.

1. The distance from Earth to the Great Whirlpool Galaxy is 10^{20} miles. Write this distance as a whole number.

2. a. Refer to the chart in Problem 3 and write the distance from Earth to Barnard's Galaxy as a whole number.

   b. Now write the distance to Barnard's Galaxy in base 10 using an exponent.

   c. Is it easier to write this distance as a whole number or as a number in base 10 using an exponent?

3. Fill in the missing whole numbers and powers of 10.

Learning to Earn

CELESTIAL BODY	DISTANCE FROM EARTH IN MILES	WRITTEN USING AN EXPONENT
Barnard's Galaxy, first known dwarf galaxy, discovered in 1882	10,000,000,000,000,000,000	10^{19}
Brightest quasar, 3C 273		10^{22}
Comet Hale-Bopp on April 6, 2091		10^{10}
Double star Shuart 1	1,000,000,000,000,000	10^{15}
First magnitude star, Altair	100,000,000,000,000	
First near-Earth asteroid, Eros, at its closest	10,000,000	
Great Whirlpool Galaxy	100,000,000,000,000,000,000	
Ionosphere		10^{2}
Mars on Nov. 2, 2001	100,000,000	
Million-star globular cluster Omega Centauri	100,000,000,000,000,000	
Nothing known about things at this distance		10^{12}
Russian *Molnyia* (Lightning) communications satellites at highest altitude		10^{4}
Saturn on Oct. 17, 2015	1,000,000,000	10^{9}
Space shuttle when you lose sight of it	1000	
Stratosphere	10	
Typical near-Earth asteroid when it flies by	1,000,000	

Example 1 ***A space shuttle is no longer visible to the naked eye when it is $1000 = 10^3$ miles away from an observer. Since 10 can be written as the product $2 \cdot 5$, you can rewrite 10^3 as $(2 \cdot 5)^3$. By the associative and commutative properties of multiplication $(2 \cdot 5)^3$ is equal to $2^3 5^3$. Therefore, 1000 can be written as***

$$1000 = 10^3 = (2 \cdot 5)^3 = (2 \cdot 5)(2 \cdot 5)(2 \cdot 5) = (2 \cdot 2 \cdot 2)(5 \cdot 5 \cdot 5) = 2^3 5^3$$

4. Use the associative and commutative properties of multiplication to show how or explain why $(3 \cdot 5)^2$ is equal to $3^2 \cdot 5^2$.

5. a. List the prime numbers that are less than 50.

b. What is the smallest prime number?

6. a. Determine the smallest prime number that divides 300 exactly. Identify the prime number and the quotient.

b. Determine the smallest prime number that divides exactly into the quotient from part a. Identify the resulting quotient.

c. Repeat the step in part b until the quotient is a prime number.

d. Rewrite the original number as a product of all the prime divisors and the final prime quotient.

e. Rewrite this product as the product of powers of distinct prime numbers.

The product in your answer to part e is known as the **prime factorization** of 300.

When a number is written as a product of its factors, it is called a **factorization** of the number. When all the factors are prime numbers, the product is called the **prime factorization** of the number. (Recall that a prime number is a whole number greater than 1 whose only whole-number factors are itself and 1.)

Fundamental property of whole numbers: For any whole number, there is only one prime factorization.

Notice that $2^3 5^3$ is a factorization of 1000 written in exponential form. It is also a prime factorization of 1000 because the factors, 2 and 5, which appear as base numbers, are both prime numbers.

7. a. $10 \cdot 10 \cdot 10 = 10^3$ is a factorization of 1000. Is 10^3 a prime factorization of 1000? Explain.

b. In Example 1, 10^3 was rewritten as $(2 \cdot 5)^3$. Is $(2 \cdot 5)^3$ a prime factorization of 1000?

c. If you ignore the order of the factors, how many prime factorizations of 1000 are there?

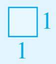
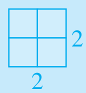
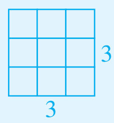
8. Determine the prime factorization of each number.

a. 45 **b.** 63 **c.** 90 **d.** 153

9. The distance to the double star Shuart 1 is 1,000,000,000,000,000, or 10^{15}, miles. Determine the prime factorization of this number. Write your result in both types of exponential form similar to those in Problem 5.

Example 2

The Square of a Number ***In geometry, a square is a rectangle in which all the sides have equal length. The area of a square is a product of two factors, each equal to the length of a side. In exponential form, you say that the length is squared.***

A square with sides 1 unit in length has an area equal to 1 square unit.		$1^2 = 1$
A square with sides 2 units in length has an area equal to 4 square units.		$2^2 = 4$
A square with sides 3 units in length has an area equal to 9 square units.		$3^2 = 9$

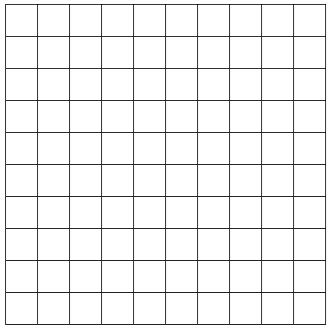
10. a. Determine the area of a square whose sides are 5 units in length.

b. Determine the area of a square whose sides are 11 units in length.

11. Explain how to determine the square of any number.

A whole number is a **square** (sometimes called a *perfect square*) if it is the product of a whole number times itself. For example, 36 and 100 are both square numbers because $36 = 6^2$ and $100 = 10^2$.

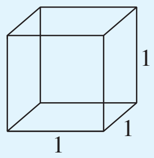
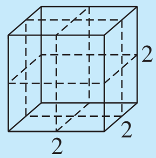
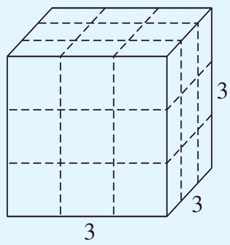
12. a. Draw a square that has an area of 49 square units on the grid.



b. What is the length of each side?

Example 3

The Cube of a Number ***In geometry, a cube is a box in which all the edges have equal length. The volume or space inside the cube is the product of three factors, each equal to the length of an edge. We say the volume is the length of the edge cubed. The following pictures illustrate this idea.***

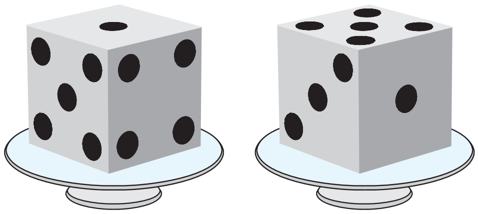
A cube with edges 1 unit in length has a volume equal to 1 cubic unit.		$1^3 = 1$
A cube with edges 2 units in length has a volume equal to 8 cubic units.		$2^3 = 8$
A cube with edges 3 units in length has a volume equal to 27 cubic units.		$3^3 = 27$

13. a. Determine the volume of a cube whose edges are 4 units in length.

b. Determine the volume of a cube whose edges are 8 units in length.

14. Explain how to determine the cube of any number.

15. You are a cake designer and have a client who is organizing a Monte Carlo Night for a charity benefit. The client wants several pairs of cakes that look like a pair of dice (cubes). You have 6-inch square pans. The square area of the bottom of the cake in each pan will be 36 square inches.



a. How high will a cake have to be to represent a die (cube)?

b. What will be the volume of the cube cake in cubic inches?

Perhaps you showed the calculation in Problem 15b as $6^2 \cdot 6^1 = 6 \cdot 6 \cdot 6 = 216$ cubic inches. Note that $6^2 \cdot 6^1$ is equivalent to 6^3 in value and that the sum of the exponents is $2 + 1 = 3$.

When multiplying two numbers written in exponential form that have the same base, add the exponents. This sum becomes the new exponent attached to the original base. For example,

$$5^4 \cdot 5^3 = 5^7,$$

since the product is the result of multiplying seven factors of 5.

$$(5 \cdot 5 \cdot 5 \cdot 5)(5 \cdot 5 \cdot 5) = 5^7$$

16. The Great Whirlpool Galaxy is one million times farther away from Earth than the first-magnitude star Altair. Use exponents to express this relationship (refer to chart in Problem 3).

17. Rewrite the following numbers using a single exponent. Check with your calculator.

a. $10^5 \cdot 10^4$ **b.** $4^7 \cdot 4^5$

18. Rewrite $3^5 \cdot 3^0$ using a single exponent.

Since $3^5 \cdot 1 = 3^5$ using the multiplication property of 1, and you found in Problem 18 that $3^5 \cdot 3^0 = 3^5$, it follows that $3^0 = 1$.

Any nonzero whole number raised to the zero power is equal to 1.

19. The National Collegiate Athletic Association (NCAA) Men's Basketball Tournament is a major TV event each year. Sixty-four colleges are invited to participate in the tournament based on their seasonal records and performance in their conference playoffs. The teams are then paired and half of the teams are eliminated after each round of play. After round one, there are 32 teams, then 16, 8, 4, 2, and, finally, 1.

a. Determine the prime factorization of the numbers 64, 32, 16, 8, 4, and 2 and write each number using an exponent. The first entry, 64, is done for you.

NUMBER OF TEAMS	PRIME FACTORIZATION	WRITTEN USING AN EXPONENT
64	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	2^6
32		
16		
8		
4		
2		
Winner		

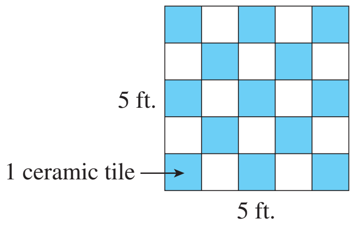
b. Describe the pattern for the exponents in the last column of this chart.

c. If you continue the pattern in the last column and write 1 in terms of base 2, what exponent would you attach to base 2?

Example 4

Ceramic Tile *Ceramic floor tiles can be square and measure 1 foot by 1 foot in size. If you want to tile a square space that is 5 feet by 5 feet as shown in the figure, you would need 25 ceramic tiles. The 5-foot sides are called the* **dimensions** *of the 25 square foot area.*

Numerically, you express the relationship between the dimensions and the area as $5 \cdot 5 = 5^2 = 25$. *The factor, 5, appears twice in this factorization of 25 and is called the square root of 25. Using symbols,* $\sqrt{25} = 5$, *which is read as "the square root of 25 is 5."*



20. a. Determine the dimensions of the square area that you could tile with 81 ceramic tiles.

b. Determine the dimensions of the square area that you could tile with 144 ceramic tiles.

21. a. Determine the square roots of 64 and 225.

b. If your calculator has a square root key, $\boxed{\sqrt{\ }}$, check your answers in part a.

22. a. Can you tile a square area with 100 of the 1-foot-square tiles? Explain.

b. Can you tile a square area with 24 of the 1-foot-square tiles? Explain.

SUMMARY: ACTIVITY 1.6

1. A whole-number **exponent** indicates the number of times to use the **base** as a factor. A number written as 10^{14} is in **exponential form**. The expression 10^{14} is called a **power** of 10.

2. Writing a number as a product of its factors is called **factorization**. When all the factors are prime numbers, the product is called the **prime factorization** of the number. A **prime number** is a whole number greater than 1 whose only whole number factors are itself and 1.

3. To determine the prime factorization of a number:

   a. Determine the smallest prime number that divides the number exactly. Write the prime number and the quotient.

   b. Determine the smallest prime number that divides exactly into the quotient from step a. Write the prime number and the resulting quotient.

   c. Repeat step b until the quotient is a prime number.

   d. Rewrite the original number as a product of all the prime divisors and the final prime quotient.

   e. Rewrite the product as the product of powers of distinct prime numbers.

4. **Fundamental property of whole numbers:** For any whole number, there is only one prime factorization.

5. Any nonzero whole number raised to the **zero power** equals 1.

6. When multiplying numbers written in exponential form that have the same base, add the exponents. This sum becomes the new exponent attached to the original base. For example,

$$9^4 \cdot 9^3 = 9^7.$$

7. A **square** is a rectangle in which all the sides have equal length. The area of a square is a product of two factors, each equal to the length of a side, that is, the length squared.

8. A whole number is a **perfect square** if it can be rewritten as the product of two whole-number factors that are equal, that is, as the square of a whole number. For example, 36 is a perfect square because $6^2 = 36$.

9. The **square root** of a whole number is one of the two equal factors whose product is the whole number. For example, 6 is the square root of 36 because $6^2 = 36$. Using symbols, $\sqrt{36} = 6$.

10. A **cube** is a box for which all of the edges have equal length. The volume of a cube is the product of three factors, each of which is the length of an edge, that is, the edge length cubed.

11. A whole number is a **perfect cube** if it can be written as the product of three whole-number factors that are equal, that is, the whole number raised to the third power. For example, 125 is a perfect cube because $5^3 = 125$.

EXERCISES: ACTIVITY 1.6

1. The distance from Earth to Alnilam, the center star in Orion's Belt, is 10^{16} miles. Write 10^{16} as a whole number.

2. The distance from Earth to the Large Magellanic Cloud, a satellite galaxy of the Milky Way, is 10,000,000,000,000,000,000 miles. Write this distance in exponential form.

3. Determine two prime numbers between 50 and 60.

4. How many prime numbers are there between 60 and 70? List them.

5. List all possible factorizations of the following numbers.

   a. 6 **b.** 15

   c. 35 **d.** 22

   e. How many different factorizations do each of these numbers have?

6. **a.** List all possible factorizations of 30 and 105.

   b. How many different factorizations do each of these numbers have?

7. **a.** List all possible factorizations of 4, 9, and 25.

   b. What do the factorizations of these three numbers share in common?

8. Determine the prime factorizations of each number.

   a. 12 **b.** 75

   c. 42 **d.** 96

9. **a.** Computers typically come with 1, 2, 4, or 8 GB of RAM (random access memory). Write 1, 2, 4, and 8 as powers of 2.

b. One GB equals 1024 MB. Older computers came with 512, 256, 128, or even 64 MB of RAM. Write 1024, 512, 256, 128, and 64 as powers of 2.

10. Write each exponential form as a whole number.

a. 3^0 **b.** 9^2 **c.** 5^4

d. 2^5 **e.** 12^2

11. Write each expression using a single exponent. Check your answers with a calculator.

a. $5^3 \cdot 5^8$ **b.** $9^2 \cdot 9^5$

c. $7^4 \cdot 7^7$ **d.** $7^5 \cdot 7^0$

12. Determine the square root of each number.

a. 64 **b.** 81 **c.** 121

d. 169 **e.** 225 **f.** 400

13. Determine if the given area is the area of a square that has a whole-number length.

a. 144 square feet **b.** 160 square feet

c. 664 square feet **d.** 256 square feet

Activity 1.7

You and Your Calculator

Objective

1. Use order of operations to evaluate arithmetic expressions.

A calculator is a powerful tool for problem solving. Calculators come in many sizes and shapes and with varying capabilities. Some calculators perform only basic operations such as addition, subtraction, multiplication, division, and square roots. Others also handle operations with exponents, perform operations with fractions, and do trigonometry and statistics. There are also calculators that graph equations and generate tables of values; some even manipulate algebraic symbols.

Unlike people, however, calculators do not think for themselves and can only perform tasks in the way that you instruct them (or program them). Therefore, if you understand the properties of numbers, you will understand how a calculator operates with numbers. In particular, you will learn the order in which your calculator performs the operations you request.

If you do not have a calculator for this course, perform the calculations using paper and pencil. There are many skills in this activity that are important to your understanding of whole numbers.

1. a. Use your calculator to determine the sum $126 + 785$.

   b. Now, input $785 + 126$ into your calculator and evaluate. How does this sum compare to the sum in Problem 1?

   c. If you use numbers other than 126 and 785, does reversing the order of the numbers change the result? Explain by giving examples.

   d. What property is demonstrated in this problem?

2. Is the commutative property true for the operation of subtraction? Multiplication? Division? Explain by giving examples for each operation.

Mental Arithmetic

It is sometimes necessary to do mental arithmetic (that is, *without* your calculator or paper and pencil). For example, to evaluate $3 \cdot 29$ without the aid of your calculator, think about the multiplication as follows: 29 can be written as $20 + 9$. Therefore, $3 \cdot 29$ can be written as $3 \cdot (20 + 9)$, which can be evaluated as $3 \cdot 20 + 3 \cdot 9$. The product $3 \cdot 29$ can now be thought of as $60 + 27$, or 87. To summarize,

$$3 \cdot 29 = 3 \cdot (20 + 9) = 3 \cdot 20 + 3 \cdot 9 = 60 + 27 = 87.$$

3. What property did the above calculation demonstrate?

4. Another way to express 29 is $25 + 4$ or $30 - 1$.

   a. Express 29 as $25 + 4$ and use the distributive property to multiply $3 \cdot 29$.

   b. Express 29 as $30 - 1$ and use the distributive property to multiply $3 \cdot 29$.

5. Evaluate mentally the following multiplication problems using the distributive property. Verify your answer using your calculator.

   a. $6 \cdot 72$

   b. $3 \cdot 109$

6. a. Evaluate $10 + 7 \cdot 3$ mentally and record the result. Verify your answer using your calculator.

   b. What operations are involved in the calculation in part a?

   c. In what order did you and your calculator perform the operations to get the answer?

   d. Evaluate $(10 + 7) \cdot 3$ and record your result. Verify using your calculator.

   e. Why is the result in part d different from the result in part a?

Order of Operations

Operations on numbers are performed in a universally accepted order. Scientific and graphing calculators are programmed to perform operations in this order. Part of the order of operations priority convention is as follows.

1. Perform multiplication and division before addition and subtraction.
2. If both multiplication and division are present, perform the operations in order, *from left to right*.
3. If both addition and subtraction are present, perform the operations in order, *from left to right*.

Example 1 ***Evaluate $12 - 2 \cdot 4 + 5$ without a calculator.***

SOLUTION

$12 - 2 \cdot 4 + 5$	**Do multiplication before addition and subtraction.**
$= 12 - 8 + 5$	**Subtract 8 from 12 since you encounter it first as you read from left to right.**
$= 4 + 5$	**Add.**
$= 9$	

7. Perform the following calculations *without* a calculator. Then use your calculator to verify your result.

   a. $24 \div 4 + 8$

   b. $24 \div 4 - 2 \cdot 3$

c. $6 + 24 - 4 \cdot 3 - 2$

d. $6 + 2 \cdot 9 - 16 \div 4$

Notice the importance of the "from left to right" rule for both multiplication/division and addition/subtraction. For example, $12 \div 4 \cdot 3 = 3 \cdot 3 = 9$ by performing the operations left to right. If multiplication is performed before division, the result is 1 (try it and see). This shows the need for a decision on which of these calculations is correct. All the arithmetic experience that people had over hundreds of years led to the decision to do multiplication/division from left to right. That decision became part of the order of operations agreement.

Check that $12 \div 4 \cdot 3 = 9$ by entering $12 \div 4 \cdot 3$ all at once on your calculator.

8. Perform the following calculations without a calculator. State which operation you must perform first, and why. Then use your calculator to verify your result.

a. $15 \div 5 \cdot 3$

b. $7 \cdot 8 \div 4$

c. $15 - 6 \div 2 + 4$

d. $20 + 3 - 5 + 8$

Some expressions involve parentheses. For example, the expression $15 \div (1 + 2)$ means 15 divided by the sum of 1 and 2. The calculation is $15 \div (1 + 2) = 15 \div 3 = 5$. This observation leads to a fourth convention for order of operations priorities.

Parentheses are grouping symbols that are used to override the standard order of operations. Operations contained in parentheses are performed first.

9. a. Evaluate $24 \div (2 + 6)$ without your calculator.

b. Use your calculator to evaluate $24 \div (2 + 6)$. Did you obtain 3 as a result? If not, then perhaps you entered the expression $24 \div 2 + 6$ and your answer is 18.

c. Explain why the result of $24 \div 2 + 6$ is 18.

Example 2

Evaluate $2 \cdot (3 + 4 \cdot 5)$ without a calculator.

SOLUTION

$2 \cdot (3 + 4 \cdot 5)$	**Evaluate the arithmetic expression in parentheses first using order of operations.**
$= 2 \cdot (3 + 20)$	**First do the multiplication inside the parentheses, then the addition.**
$= 2 \cdot 23$	
$= 46$	**Multiply the result by the 2 that was outside of the parentheses.**

10. Evaluate the following mentally and verify on your calculator.

a. $6/(3 + 3)$

b. $(2 + 8)/(4 - 2)$

c. $24 \div (4 + 8)$

d. $24 \div (4 - 2) \cdot 3$

e. $(6 + 24) \div (4 \cdot 3 - 2)$

f. $(6 + 2) \cdot 9 - 16 \div 4$

g. $5 + 2 \cdot (4 \div 2 + 3)$

h. $10 - (12 - 3 \cdot 2) \div 3$

Exponentiation

Recall that $5 \cdot 5$ can be written as 5^2 (read "5 squared"). Besides multiplying 5 times 5, there are two additional ways to square a number on your calculator. Try Problem 11 if you have a calculator. Otherwise, go to Example 3.

11. a. One way to evaluate x^2 is to use the $\boxed{x^2}$ key. Input 5 and then press the $\boxed{x^2}$ key. Do this now and record your answer.

b. Another way to evaluate 5^2 is to use the exponent key. Depending on your calculator, the exponent key may resemble $\boxed{x^y}$, $\boxed{y^x}$, or $\boxed{\wedge}$. To calculate 5^2, input 5, press the exponent key, then enter $\boxed{2}$ and press $\boxed{\text{ENTER}}$. Do this now and record your answer.

Example 3 *Order of Operations Involving Exponential Expressions*

a. Evaluate 5^3.

SOLUTION

5^3 can be written as $5 \cdot 5 \cdot 5 = 125$. Verify your answer using a calculator.

b. Evaluate the expression $20 - 2 \cdot 3^2$.

SOLUTION

To evaluate the expression $20 - 2 \cdot 3^2$, follow the steps:

$20 - 2 \cdot 3^2$	**Evaluate all exponents as you read the arithmetic expression from left to right.**
$= 20 - 2 \cdot 9$	**Do all multiplication and division as you read the expression from left to right.**
$= 20 - 18$	**Do all addition and subtraction as you read the expression from left to right.**
$= 2$	

An exponential expression such as 5^3 is called a **power** of 5, as you will recall from Activity 1.6. The **base** is 5 and the **exponent** is 3. When a power is contained in an expression, it is evaluated *before* any multiplication or division, but only after operations in parentheses.

12. If you have a calculator, enter the expression $20 - 2 \cdot 3^2$ into your calculator and verify the result in Example 3 on the previous page.

13. Evaluate the following numerical expressions by hand. Verify using a calculator.

a. $6 + 3 \cdot 4^3$

b. $2 \cdot 3^4 - 5^3$

c. $2^2 \cdot 3^2 \div 3 - 2$

d. $3^2 \cdot 2 + 3 \cdot 2^3$

14. Evaluate each of the following arithmetic expressions mentally or by hand. Perform the operations in the appropriate order and then use your calculator to check your results.

a. $18 - 2 \cdot (8 - 2 \cdot 3) + 3^2$

b. $3^4 + 5 \cdot 4^2$

c. $128/(16 - 2^3)$

d. $(17 - 3 \cdot 4)/5$

e. $5 \cdot 2^3 - 6 \cdot 2 + 5$

f. $5^2 \cdot 5^3$

g. $2^3 \cdot 3^2$

h. $4^2 + 4^3$

i. $500 \div 25 \cdot 2 - 3 \cdot 2$

j. $(3^2 - 6)^2$

SUMMARY: ACTIVITY 1.7

1. The **commutative property** states that the order in which you add or multiply two whole numbers gives the same result. The commutative property does *not* hold for subtraction or division.

2. $3(10 - 2) = 3 \cdot 10 - 3 \cdot 2$ is an example of the **distributive property**.

3. An exponential expression such as 5^3 is called a **power** of the base number 5. The **base** is 5 and the **exponent** is 3. The exponent indicates how many times the base is written as a factor. When a power is contained in an arithmetic expression, it is evaluated before any multiplication or division.

4. **Order of operations** for arithmetic expressions containing parentheses, addition, subtraction, multiplication, division, and exponentiation:

Operations contained within parentheses are performed *first* before any operations outside the parentheses. All operations are performed in the following order.

a. Evaluate all exponents as you read the expression from left to right.

b. Do all multiplication and division as you read the expression from left to right.

c. Do all addition and subtraction as you read the expression from left to right.

EXERCISES: ACTIVITY 1.7

1. Evaluate each expression. Check your answers with a calculator.

   a. $7(20 + 5)$

   b. $7 \cdot 20 + 5$

   c. $7 \cdot 20 + 7 \cdot 5$

   d. $20 + 7 \cdot 5$

   e. Which two of the preceding arithmetic expressions have the same answer?

   f. State the property that produces the same answer for that pair of expressions.

2. Evaluate each expression. Check your answers with a calculator.

   a. $20(100 - 2)$

   b. $20 \cdot 100 - 2$

   c. $20 \cdot 100 - 20 \cdot 2$

   d. $100 - 20 \cdot 2$

   e. Which two of the preceding arithmetic expressions have the same answer?

   f. State the property that produces the same answer for that pair of expressions.

3. Evaluate each expression using order of operations.

   a. $17 \cdot (52 - 2)$

   b. $(90 - 7) \cdot 5$

4. Evaluate each expression using the distributive property.

   a. $17 \cdot (52 - 2)$

   b. $(90 - 7) \cdot 5$

5. Perform the following calculations *without a calculator*. After solving, use your calculator to check your answer.

   a. $45 \div 3 + 12$

   b. $54 \div 9 - 2 \cdot 3$

   c. $12 + 30 \div 2 \cdot 3 - 4$

   d. $26 + 2 \cdot 7 - 12 \div 4$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

6. a. Explain why the result of $72 \div 8 + 4$ is 13.

b. Explain why the result of $72 \div (8 + 4)$ is 6.

7. Evaluate the following expressions. Check your answers with a calculator.

a. $48/(4 + 4)$

b. $(8 + 12)/(6 - 2)$

c. $120 \div (6 + 4)$

d. $64 \div (6 - 2) \cdot 2$

e. $(16 + 84) \div (4 \cdot 3 - 2)$

f. $(6 + 2) \cdot 20 - 12 \div 3$

g. $39 + 3 \cdot (8 \div 2 + 3)$

h. $100 - (81 - 27 \cdot 3) \div 3$

8. Evaluate the following expressions. Check your answers with a calculator.

a. $15 + 2 \cdot 5^3$

b. $5 \cdot 2^4 - 3^3$

c. $5^2 \cdot 2^3 \div 10 - 6$

d. $5^2 \cdot 2 - 5 \cdot 2^3$

9. Evaluate each of the following arithmetic expressions by performing the operations in the appropriate order. Check your answers with a calculator.

a. $37 - 2 \cdot (18 - 2 \cdot 5) + 1^2$

b. $3^5 + 2 \cdot 10^2$

c. $243/(36 - 3^3)$

d. $(75 - 2 \cdot 15)/9$

e. $7 \cdot 2^3 - 9 \cdot 2 + 5$

f. $2^5 \cdot 5^2$

g. $2^3 \cdot 2^5$

h. $5^3 + 2^6$

i. $1350 \div 75 \cdot 5 - 15 \cdot 2$

j. $(3^2 - 4)^2$

What Have I Learned?

Write your explanations in full sentences.

1. Would you prefer to win $1,050,000 or $1,005,000? Use the idea of place value to explain how you determined your answer.

2. Suppose you want to get a good deal on leasing a car for 3 years (36 months) and you do some checking. The following is some preliminary information that you found in car ads in your local newspaper. Note that these costs do not include other fees such as tax. Those fees are ignored in this problem.

CAR MODEL	DOWN PAYMENT*	MONTHLY FEE
Saturn VUE Hybrid	$2,863	$418
Toyota Tundra	$2,695	$397
Mercury Mountaineer	$3,503	$512
Ford Explorer	$3,599	$471
Dodge Durango	$2,599	$409

*The down payment includes the first monthly fee.

a. At first, you round the down payments to the nearest thousand and the monthly fee to the nearest hundred so you can mentally estimate the total payments for each car. Does your estimate allow you to say which car is the most expensive to lease and which is the least expensive to lease? Explain.

CAR MODEL	DOWN PAYMENT	DOWN PAYMENT ESTIMATE TO THE NEAREST THOUSAND	MONTHLY FEE	MONTHLY FEE ESTIMATE TO THE NEAREST HUNDRED	ESTIMATED TOTAL COST
Saturn VUE Hybrid	$2,863		$418		
Toyota Tundra	$2,695		$397		
Mercury Mountaineer	$3,503		$512		
Ford Explorer	$3,599		$471		
Dodge Durango	$2,599		$409		

Exercise numbers appearing in color are answered in the Selected Answers appendix.

b. What would be better choices for rounding the down payment costs and the monthly fee to estimate the leasing costs for each car? Use your choices to get new estimates.

CAR MODEL	DOWN PAYMENT	DOWN PAYMENT ESTIMATE TO THE NEAREST HUNDRED	MONTHLY FEE	MONTHLY FEE ESTIMATE TO THE NEAREST TEN	ESTIMATED TOTAL COST
Saturn VUE Hybrid	$2,863		$418		
Toyota Tundra	$2,695		$397		
Mercury Mountaineer	$3,503		$512		
Ford Explorer	$3,599		$471		
Dodge Durango	$2,599		$409		

c. Use your calculator to determine the actual total cost for each car from the actual down payment and the monthly fee. Do the actual costs show the same cars as most expensive and least expensive that you named in part b?

CAR MODEL	DOWN PAYMENT	MONTHLY FEE	TOTAL COST
Saturn VUE Hybrid	$2,863	$418	
Toyota Tundra	$2,695	$397	
Mercury Mountaineer	$3,503	$512	
Ford Explorer	$3,599	$471	
Dodge Durango	$2,599	$409	

d. From this exercise, what conclusions can you make about the usefulness of rounding numbers in calculations that you need to make so you can compare costs.

3. a. Explain a procedure you could use to determine whether 37 is a composite or prime number.

b. Check to see if your procedure works for a number greater than 100, say 101. Explain why it works or does not work.

c. What is the largest prime number that *you know*? Do you think it is the largest prime number there is? Give a reason for your answer.

d. How many prime numbers do you think there are in all? Give a reason for your answer. Compare your answer with those of your classmates.

4. a. For which two operations does the commutative property hold? Give an example in each case.

   b. For which two operations does the commutative property fail to hold? Give an example in each case.

5. a. Is it true that $(69 + 21) + 17 = 69 + (21 + 17)$? Justify your answer by calculating each side of the statement. In each case, add the numbers in the parentheses first.

   b. What arithmetic property did you demonstrate in part a?

   c. Does the same property hold true for $(69 - 21) - 17 = 69 - (21 - 17)$? Justify your answer.

6. Try this experiment: Ask a friend or classmate to calculate 9×999 by the usual vertical method. Then ask the person to calculate $9(1000 - 1)$ by using the distributive property.

   a. What is the correct answer in each case?

   b. Which calculation do you each think is "easier"? Why?

   c. Show how you would calculate 9×9990 by using the distributive property.

7. Division of whole numbers can be considered as repeated subtraction.

   a. Use this idea to divide 12 by 4. Show your calculation and the result.

   b. Does this idea work when you try to divide 12 by 0? Explain.

How Can I Practice?

1. You bought a laptop computer and wrote a check for one thousand one hundred six dollars. The price tag read $1016.

a. Write the amount of the check in numeral form.

b. Did you pay the correct amount, too much, or too little? Explain.

2. Currently, the disease diabetes affects an estimated 24,000,000 Americans, and about 1,600,000 new cases are diagnosed each year. What are the place values to which each estimate apparently is rounded?

3. In fiscal year 2006, the state of Oregon collected 395 million dollars in tax revenues from the sale of alcohol. The cost to Oregon's economy in 2006 from alcohol abuse (treatment costs, loss of employment, loss of life or productivity, etc.) was 3.24 billion dollars. How many times more was the cost in dollars than the amount collected in taxes? Write your answer to the nearest whole number.

4. In 2008, the U.S. Census Bureau indicated that the median household income level in the United States was $50,233. Round this amount to the nearest hundred dollars.

5. On a Web site, the distance from Earth to the Sun was given as 92,955,807 miles. Round this distance to

a. the nearest thousand miles.

b. the nearest million miles.

6. Calculate each of the following by hand. Check your answer with a calculator.

a. $\begin{array}{r} 523 \\ +108 \\ \hline \end{array}$

b. $\begin{array}{r} 1052 \\ +\ 957 \\ \hline \end{array}$

c. $\begin{array}{r} 3051 \\ 1282 \\ +\ 327 \\ \hline \end{array}$

7. Evaluate each of the following by hand. Check your answer with a calculator.

a. $283 - 95$

b. $233 - 145$

c. $67 - 39$

d. $1003 - 349$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

8. Translate each of the following into an arithmetic expression and calculate by hand. Check your answer with a calculator.

a. 67 plus 25

b. the difference between 24 and 18

c. 98 minus 15

d. total of 104 and 729

e. 25 more than 495

f. 33 subtracted from 67

g. 145 increased by 28

h. 34 less than 156

i. 95 decreased by 25

9. Multiply by hand. Check your answer with a calculator.

a. $\begin{array}{r} 25 \\ \times\ 9 \\ \hline \end{array}$

b. $\begin{array}{r} 347 \\ \times\ \ 6 \\ \hline \end{array}$

c. $\begin{array}{r} 167 \\ \times\ 17 \\ \hline \end{array}$

d. $\begin{array}{r} 227 \\ \times\ 109 \\ \hline \end{array}$

10. a. Multiply 3 times 45 in a vertical format.

b. Use the distributive property to calculate $3(40 + 5)$.

c. Use the distributive property to calculate $3 \cdot 49$.

11. Calculate. As part of your answer, identify the quotient and remainder (if any).

a. $126 \div 4$

b. $312 \div 4$

c. $195 \div 13$

d. $224 \div 12$

12. a. Estimate $3312 \div 414$.

b. Find the exact answer.

c. Is your estimate lower, higher, or the same?

13. Determine and list all the prime numbers between 30 and 50.

14. Determine and list all factors of the following numbers.

a. 12 **b.** 21

c. 71 **d.** 18

e. 24 **f.** 63

15. Determine the prime factorizations of the following numbers.

a. 12 **b.** 18

c. 24 **d.** 63

16. Determine the prime factorizations of the following numbers. Write your answers in exponent form.

a. 27 **b.** 125

c. What do the two prime factorizations have in common?

17. Write as whole numbers.

a. 8^0 **b.** 12^2 **c.** 3^4 **d.** 2^6

18. Write each numerical expression using a single exponent.

a. $7^3 \cdot 7^9$ **b.** $11^5 \cdot 11^7$ **c.** $13^2 \cdot 13^0$ **d.** $9 \cdot 9^2 \cdot 9^3$

19. Which of the following are perfect squares?

a. 160 **b.** 144

c. 900 **d.** 125

20. Determine the square root without using a calculator. Approximate if necessary. Check your answer with a calculator.

a. 4 **b.** 24 **c.** 225

d. 36 **e.** 90

21. Evaluate each of the following arithmetic expressions by performing the operations in the appropriate order. Use your calculator to check your results.

a. $7 - 3 \cdot (8 - 2 \cdot 3) + 2^2$

b. $3 \cdot 2^5 + 2 \cdot 5^2$

c. $144/(24 - 2^3)$

d. $(36 - 2 \cdot 9)/6$

e. $9 \cdot 5 - 5 \cdot 2^3 + 5$

f. $1^5 \cdot 5^1$

g. $2^3 \cdot 2^0$

h. $7^2 + 7^2$

i. $(3^2 - 4 \cdot 0)^2$

22. Determine the arithmetic property expressed by each numerical statement.

a. $25(30) = 30(25)$

b. $15(9) = 15(10 - 1) = 15 \cdot 10 - 15 \cdot 1$

c. $(11 + 21) + 39 = 11 + (21 + 39)$

23. Determine if each of the following numerical statements is true or false. In each case, justify your answer.

a. $7(20 + 2) = 7(22)$

b. $4(11)(10) = 10(11)4$

c. $25 - 10 - 4 = 25 - (10 - 4)$

d. $1/0 = 1$

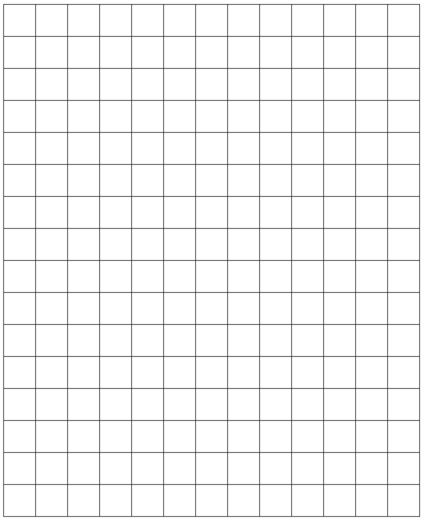
24. The Junior Tournament Bowlers Association of Ohio is a USBC Certified organization for bowlers aged 21 or younger. In the qualifying round of a scratch bowling tournament in August 2008, 24 females and 68 males bowled 6 games each. Here are scores for game 6 *only*, for those men and women who finished in the top three *or* the bottom three, overall in their gender group.

a. Round the scores to the nearest tens place to complete the table.

Ten Pins in Their Places!

BOWLER (INPUT)	GAME 6 SCORE	GAME 6 SCORE ROUNDED TO THE NEAREST TEN (OUTPUT)
1. Bobby	220	
2. Eric	297	
3. Mike	276	
4. Brian	174	
5. Kenny	135	
6. Nick	188	
7. Joy	202	
8. Heidi	186	
9. Danielle	211	
10. Tierra	161	
11. Katelyn	152	
12. Kelly	121	

b. Use the rounded numbers from the third column of your table to plot points that represent the scores for each of the 12 bowlers. Remember to label the units, the input along the horizontal axis, and the output along the vertical axis.

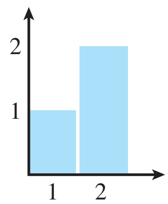


Chapter 1 Summary

The bracketed numbers following each concept indicate the activity in which the concept is discussed.

<table>
<tr><th>CONCEPT/SKILL</th><th>DESCRIPTION</th><th>EXAMPLE</th></tr>
<tr><td>Place value [1.1]</td><td colspan="2">
<table>
<tr><th colspan="3">Millions</th><th colspan="3">Thousands</th><th colspan="3">Ones</th></tr>
<tr><td>Hundreds</td><td>Tens</td><td>Ones</td><td>Hundreds</td><td>Tens</td><td>Ones</td><td>Hundreds</td><td>Tens</td><td>Ones</td></tr>
</table>
</td></tr>
<tr><td>Read and write whole numbers [1.1]</td><td>Start at the left and read each digit and its place value.</td><td>2345 is read "two thousand three hundred forty-five."</td></tr>
<tr><td>Round whole numbers to specified place value [1.1]</td><td>If the digit immediately to the right of the specified place value is 5 or more, add 1 to the digit in the specified place and change all the digits to its right to zero.

If the digit immediately to the right of the specified place value is less than 5, do not change the digit in the specified place but change all the digits to its right to zero.</td><td>157 rounded to the tens place yields 160.

152 rounded to the tens place yields 150.</td></tr>
<tr><td>Prime numbers [1.1], [1.6]</td><td>A prime number is a whole number greater than 1 whose only whole-number factors are itself and 1.</td><td>2, 3, 5, 7, 11, 13, 17, 19, 23,…</td></tr>
<tr><td>Read bar graphs [1.2]</td><td>Each bar along the horizontal axis is associated with a number along the vertical axis.</td><td>
</td></tr>
<tr><td>Interpret bar graphs [1.2]</td><td>Find and read input values along the horizontal axis and the corresponding output values along the vertical axis in context.</td><td></td></tr>
<tr><td>Addend, sum [1.3]</td><td>The numbers being added are called addends. Their total is called the sum.</td><td>$\underbrace{12}_{\text{addend}} + \underbrace{11}_{\text{addend}} = \underbrace{23}_{\text{sum}}$</td></tr>
<tr><td>Addition property of zero [1.3]</td><td>The sum of any whole number and zero is the same whole number.</td><td>$5 + 0 = 5$</td></tr>
<tr><td>Commutative property of addition [1.3]</td><td>Changing the order of the addends yields the same sum.</td><td>$9 + 8 = 8 + 9 = 17$</td></tr>
<tr><td>Associative property of addition [1.3]</td><td>Given three addends, it makes no difference whether the first two numbers or the last two numbers are added first.</td><td>$5 + (6 + 7)$
$= (5 + 6) + 7 = 18$</td></tr>
</table>

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Add whole numbers by hand and mentally [1.3]	***To add:*** Align numbers vertically according to place value. Add the digits in the ones place. If their sum is a two-digit number, write down the ones digit and regroup the tens digit to the next column as a number to be added. Repeat with next higher place value.	$\begin{array}{r} {}^{1}\\ 129 \\ +\ 17 \\ \hline 146 \end{array}$ **Regroup 1 to tens place.** **9 + 7 = 16; write 6 in ones place.**
Subtrahend, minuend, difference [1.3]	The number being subtracted is called the **subtrahend**. The number it is subtracted from is called the **minuend**. The result is called the **difference**.	$\underbrace{32}_{\text{minuend}} - \underbrace{18}_{\text{subtrahend}} = \underbrace{14}_{\text{difference}}$
Subtract whole numbers by hand and mentally [1.3]	***To subtract:*** Align the subtrahend under the minuend according to place value. Subtract digits having the same place value. Regroup from a higher place value, if necessary.	$\begin{array}{r} {}^{5\,12} \\ \cancel{6}2 \\ -\ 15 \\ \hline 47 \end{array}$ **Regroup from tens. 2 changes to 12, and 6 changes to 5.** **Subtract 5 from 12.**
Estimate sums and differences using rounding [1.3]	One way to estimate is to round each number (addend) to its highest place value. In some cases, it may be better to round to a place value lower than the highest one.	*Exact Sum:* $\begin{array}{r} 780 \\ 219 \\ +\ 164 \\ \hline 1163 \end{array}$ *Estimate:* $\begin{array}{r} 800 \\ 200 \\ +\ 200 \\ \hline 1200 \end{array}$
Missing addend approach to subtraction [1.3]	Minuend − subtrahend = difference can be written as subtrahend + difference = minuend.	10 − 6 = ? can be written 6 + ? = 10.
Key phrases for addition [1.3]	• Sum of • More than • Total of • Increased by • Plus • Added to	15 *increased by* 10: 15 + 10 3 *more than* 7: 7 + 3 Total of 10 and 25: 10 + 25
Key phrases for subtraction [1.3]	• Difference between • Decreased by • Subtracted from • Minus • Less than	50 *decreased by* 16: 50 − 16 6 *less than* 92: 92 − 6 7 *subtracted from* 15: 15 − 7
Factor, product [1.4]	**Factors** are numbers being multiplied. The result is called the **product**.	$\underbrace{12}_{\text{factor}} \cdot \underbrace{8}_{\text{factor}} = \underbrace{96}_{\text{product}}$
Multiplication as repeated addition [1.4]	4 times 30 can be thought of as 30 + 30 + 30 + 30.	$5 \cdot 9 = 9 + 9 + 9 + 9 + 9 = 45$ $9 \cdot 5 = 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 45$

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Multiply whole numbers [1.4]	***To multiply:*** Align numbers vertically by place value. Starting with the rightmost digit of the bottom factor, multiply each place value of the top factor. Repeat using the next rightmost digit of the bottom factor. Add the resulting products, to get the final product.	$\begin{array}{rl} 37 & \\ \times\ 4 & \\ \hline 28 & \text{product of 7 times 4} \\ 120 & \text{product of 30 times 4} \\ \hline 148 & \text{sum of 28 and 120} \end{array}$
Multiplication by 1 [1.4]	Any whole number multiplied by 1 remains the same.	$134 \cdot 1 = 134$
Multiplication by 0 [1.4]	Any whole number multiplied by 0 is 0.	$45 \cdot 0 = 0$
Distributive property of multiplication over addition [1.4]	A whole number placed immediately to the left or right of a set of parentheses containing the sum or difference of two numbers multiplies each of the inside numbers.	$\begin{array}{ll} \ 8 \cdot 24 & \text{or } 26 \cdot 5 \\ = 8 \cdot (20 + 4) & = (30 - 4) \cdot 5 \\ = 8 \cdot 20 + 8 \cdot 4 & = 30 \cdot 5 - 4 \cdot 5 \\ = 160 + 32 & = 150 - 20 \\ = 192 & = 130 \end{array}$
Commutative property of multiplication [1.4]	Changing the **order** of the factors yields the same product.	$5 \cdot 12 = 12 \cdot 5 = 60$
Associative property of multiplication [1.4]	When multiplying three factors, it makes no difference whether the first two numbers or the last two numbers are multiplied first. The same product results.	$\begin{array}{l} \ (5 \cdot 7) \cdot 12 \\ = 5 \cdot (7 \cdot 12) = 420 \end{array}$
Estimate products [1.4]	Round each factor to a large enough place value so that you can do the multiplication mentally. There is no one correct answer when estimating.	$\begin{array}{lr} 284 \longrightarrow & 300 \\ 525 \longrightarrow & \times\ 500 \\ \hline \text{estimate:} & 150{,}000 \end{array}$
Divisor, dividend [1.5]	The number being divided is called the **dividend**. The number that divides is called the **divisor**.	$\underbrace{3}_{\text{divisor}}\overline{)\underbrace{45}_{\text{dividend}}}$
Quotient [1.5]	A whole number representing the number of times the divisor can be subtracted from the dividend is called a **quotient**.	$\begin{array}{r} \overbrace{15}^{\text{quotient}} \\ \underbrace{3}_{\text{divisor}}\overline{)\underbrace{45}_{\text{dividend}}} \end{array}$
Remainder [1.5]	The **remainder** is the whole number left over after the divisor has been subtracted from the dividend as many times as possible.	$\begin{array}{rl} \overbrace{15}^{\text{quotient}} & \text{remainder } 2 \\ \underbrace{3}_{\text{divisor}}\overline{)47} & \longleftarrow \text{dividend} \\ 45 & \\ \hline 2 & \end{array}$

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Divide whole numbers [1.5]	$$\begin{array}{r} 17 \\ 6\overline{)104} \\ \underline{6} \\ 44 \\ \underline{42} \\ 2 \end{array}$$ 6 does not divide into 1. 6 does divide into 10 one time. Write down the product $1 \cdot 6 = 6$. Subtract 6 from 10. Bring down the next digit, 4. 6 divides into 44 seven times. Write down the product $7 \cdot 6 = 42$. Subtract 42 from 44. remainder	
Check division [1.5]	To check division, multiply the quotient times the divisor, then add the remainder. The result should be the dividend. (quotient × divisor) + remainder = dividend	$$\begin{array}{r} 3 \\ 7\overline{)25} \\ \underline{21} \\ 4 \end{array}$$ Check: $3 \cdot 7 + 4 = 25$
Inverse operations [1.5]	Multiplication and division are inverse operations	$3 \cdot 4 = 12$ $12 \div 4 = 3$
Division properties involving 0 [1.5]	0 divided by any nonzero whole number is 0. Division by 0 is undefined.	$0 \div 42 = 0$ $42 \div 0 =$ *undefined*!
Division properties involving 1 [1.5]	Any nonzero whole number divided by itself is equal to 1. Any whole number divided by 1 remains the same.	$71 \div 71 = 1$ $23 \div 1 = 23$
Division is not commutative [1.5]	The dividend and the divisor can *not* be interchanged without changing the quotient (unless the dividend and the divisor are the same whole number).	$9\overline{)81} \neq 81\overline{)9}$
Estimate a quotient [1.5]	To estimate a quotient, first estimate the divisor and the dividend by numbers that provide a division easily done mentally or by hand.	$18\overline{)680} \longrightarrow 20\overline{)700}$ with quotient $\overbrace{35}^{\text{estimate}}$
Exponent, base [1.6]	A whole-number **exponent** indicates the number of times to use the **base** as a factor.	$10^4 = 10 \cdot 10 \cdot 10 \cdot 10$ $= 10{,}000$
A whole number raised to the zero power [1.6]	Any nonzero whole number raised to the zero power equals 1.	$2^0 = 1, \quad 50^0 = 1, \quad 200^0 = 1$
Exponential form of a whole number [1.6]	A number written as 10^4 is in **exponential form**. 10 is the base and 4 is the exponent. The exponent indicates how many times the base appears as a factor.	$10^4 = \overbrace{10 \cdot 10 \cdot 10 \cdot 10}^{\text{10 is used as a factor 4 times.}}$ $= 10{,}000$

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Multiplication rule for numbers in exponential form [1.6]	When multiplying numbers written in exponential form with the *same base*, add the exponents. This sum becomes the new exponent attached to the original base.	$7^2 7^6 = 7^8$ $3^2 3^5 3^7 = 3^{14}$
Factor whole numbers [1.6]	Writing a number as a product of its factors is called **factorization.**	$12 = 1 \cdot 12 = 12 \cdot 1$ $12 = 2 \cdot 6 = 6 \cdot 2$ $12 = 4 \cdot 3 = 3 \cdot 4$
Prime factorization form of a whole number [1.6]	A number written as a product of its prime factors is called the **prime factorization** of the number.	$60 = 2 \cdot 2 \cdot 3 \cdot 5$
Recognizing whole-number perfect squares [1.6]	A whole number is a **perfect square** if it can be rewritten as the square of a whole number.	$36 = 6^2$ and $100 = 10^2$
Recognizing whole-number perfect cubes [1.6]	A whole number is a **perfect cube** if it can be written as the cube of a whole number.	$27 = 3^3$ and $1000 = 10^3$
Recognizing roots of square numbers [1.6]	The **square root** of a whole number is one of the two equal factors whose product is the whole number.	$\sqrt{25} = 5$ because $5^2 = 25$
Order of operations [1.7]	**Order of operations** for arithmetic expressions containing parentheses, addition, subtraction, multiplication, division, and exponentiation: Operations contained within parentheses are performed *first*—before any operations *outside* the parentheses. All operations are performed in the following order. a. Evaluate all exponents as you read the expression *from left to right.* b. Do all multiplication and division as you read the expression *from left to right.* c. Do all addition and subtraction as you read the expression from *left to right.*	$2 \cdot (3 + 4^2 \cdot 5) - 9$ $= 2 \cdot (3 + 16 \cdot 5) - 9$ $= 2 \cdot (3 + 80) - 9$ $= 2 \cdot (83) - 9$ $= 166 - 9$ $= 157$

Chapter 1 Gateway Review

1. When you move into a new apartment, you and your roommates stock up on groceries. The bill comes to \$243. How will you write this amount in words on your check?

2. The chance of winning in New York State Lotto is 1 out of 22,528,737. Write 22,528,737 in words.

3. Put the following whole numbers in order from smallest to largest: 108,901; 180,901; 108,091; 108,910; and 109,801.

4. What are the place values of the digits 0 and 9 in the number 40,693?

5. Determine whether each of the following numbers is even or odd. In each case, give a reason for your answer.

a. 333 **b.** 1378 **c.** 121

6. Determine whether each of the following numbers is prime or composite. In each case, give a reason for your answer.

a. 145 **b.** 61

c. 2 **d.** 121

7. a. Round 1,252,757 to the thousands place.

b. Round 899 to the tens place.

8. The following table provides the six most popular names for girls and for boys born in California in the year 2007. The names are listed in alphabetical order.

GIRL'S NAMES	NUMBER BORN IN YEAR 2007	BOY'S NAMES	NUMBER BORN IN YEAR 2007
Ashley	2504	Andrew	2986
Emily	2923	Angel	3457
Isabella	2899	Anthony	3725
Mia	2043	Daniel	3821
Samantha	2347	David	3069
Sophia	2570	Jacob	3072

a. What was the most popular girl's name?

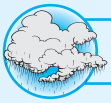
b. What was the most popular boy's name?

Answers to all Gateway exercises are included in the Selected Answers appendix.

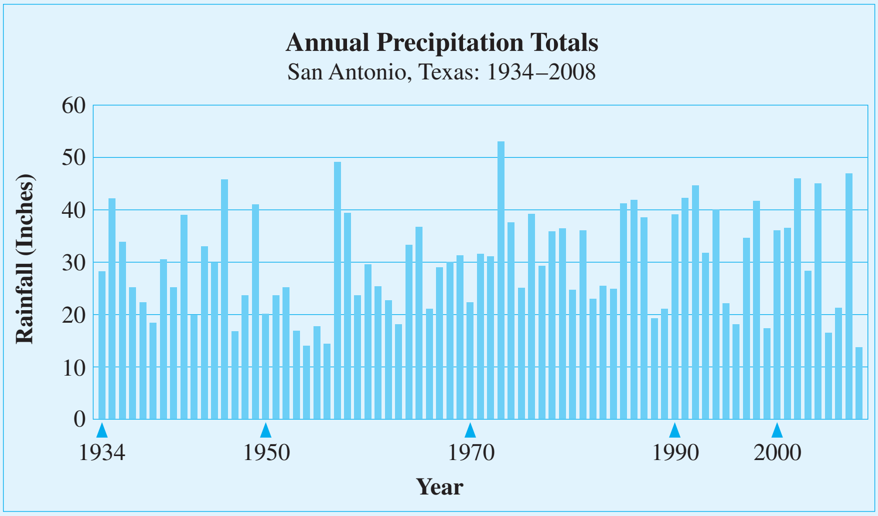
c. Which boy's name was the sixth most popular?

d. Which girl's name was the third most popular?

9. The following bar graph provides the annual rainfall in San Antonio, Texas, from 1934 to 2008.



When It Rains...



Source: National Oceanic and Atmospheric Administration's National Weather Service

a. In which year(s) was there the greatest rainfall? Estimate the number of inches.

b. In which year(s) was there the least rainfall? Estimate the number of inches.

c. How many years did San Antonio have 20 to 30 inches of rain per year? Is this more or less than the number of years that San Antonio had 30 to 40 inches of rain per year? Explain.

10. Use the following grid as given to plot the points listed in the accompanying table. Do not extend the grid in either direction.

a. What size unit would be reasonable for the input?

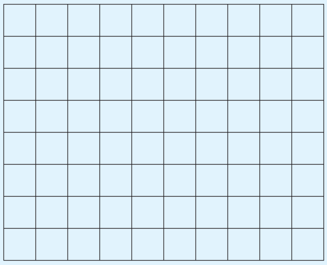
b. List the units along the horizontal axis.

c. What size unit would be reasonable for the output?

d. List the units along the vertical axis.

e. Plot the points from the following table on the grid that you labeled. Label each point with its input/output values.

Input	1	4	7	8	10	16	17	20
Output	5	10	20	15	25	30	5	40



11. Perform the indicated operations. Check your answers with a calculator.

a. $$\begin{array}{r} 7982 \\ +\ 969 \\ \hline \end{array}$$

b. $$\begin{array}{r} 235 \\ 45 \\ 59 \\ 210 \\ +347 \\ \hline \end{array}$$

c. $$\begin{array}{r} 231 \\ -\ 54 \\ \hline \end{array}$$

d. $$\begin{array}{r} 5344 \\ -4682 \\ \hline \end{array}$$

e. $654 - 179$

12. Rewrite the statement $113 - 41 = 72$ as a statement in terms of addition.

13. a. What property of addition does the statement $(8 + 5) + 5 = 8 + (5 + 5)$ demonstrate?

b. What property of addition does the statement $4 + (7 + 6) = 4 + (6 + 7)$ demonstrate?

c. Why is the expression $11 + 89 + 0$ equal to the expression $11 + 89$?

14. Are the numerical expressions $5 - 2$ and $2 - 5$ equal? What does this mean in terms of a commutative property for subtraction?

15. a. Estimate the sum: $510 + 86 + 120 + 350$.

b. Determine the actual sum.

c. Was your estimate higher or lower than the actual sum? Explain.

16. Translate each verbal expression into an arithmetic expression.

a. The sum of thirty and twenty-two

b. The difference between sixty-seven and fifteen

c. One hundred twenty-five decreased by forty-four

d. 175 less than 250

e. The product of twenty-five and thirty-six

f. The quotient of fifty-five and eleven

g. The square of thirteen

h. Two to the fifth power

i. The square root of forty-nine

j. The product of twenty-seven and the sum of fifty and seventeen

17. a. Write $5 \cdot 61$ as an addition calculation and find the sum.

b. How does the sum you obtained in part a compare to $5 \cdot 61$ when calculated as a product?

c. Write 61 as $60 + 1$ and use the distributive property to evaluate the product of 5 and 61.

18. a. Determine the product of $4 \cdot 29$ by rewriting 29 as $30 - 1$ and then using the distributive property.

b. Is it faster to mentally multiply $4 \cdot 29$ or $4 \cdot 30 - 4 \cdot 1$?

c. Multiply $6 \cdot 98$ by using the distributive property.

19. a. What property of multiplication does the statement $8 \cdot (6 \cdot 12) = (8 \cdot 6) \cdot 12$ demonstrate?

b. What property of multiplication does the statement $4 \cdot (7 \cdot 6) = 4 \cdot (6 \cdot 7)$ demonstrate?

c. Why is the expression $1 \cdot (4 + 5)$ equivalent to the expression $(4 + 5)$?

20. a. Determine the quotient of $72 \div 12$ by repeatedly subtracting 12 from 72.

b. Determine the quotient and remainder of $86 \div 16$ by repeatedly subtracting 16 from 86.

21. Calculate $12 \div 6$ and then $6 \div 12$. Use the results to determine if the commutative property holds for division.

22. In each case, perform the indicated operation. Check your answer with a calculator.

a. $\begin{array}{r} 2096 \\ \times \quad 87 \\ \hline \end{array}$

b. $15\overline{)231}$

c. $(41 \cdot 3) \cdot 7$

d. $41 \cdot (3 \cdot 7)$

e. $27 \div 27$

f. $0 \div 27$

g. $27 \div 0$

h. $24\overline{)6572}$

23. **a.** Estimate $329 \cdot 75$.

b. Determine the exact answer.

c. Is your estimate lower, higher, or the same?

24. **a.** Estimate $1850 \div 42$.

b. Determine the exact answer.

c. Is your estimate lower, higher, or the same?

25. In converting 63 feet into yards, suppose you divided 63 by 3 and obtained 20 yards. Check to see if you calculated correctly.

26. Explain the difference between the expressions $1 \div 0$ and $0 \div 1$.

27. **a.** If you divide any whole number by 1, what is the result? Give an example.

b. If you divide the number 5634 by itself, what is the result? Does this property apply to any whole number you choose? Explain.

28. List all whole-number factors of each of the following numbers.

a. 350

b. 81

c. 36

29. Determine the prime factorization of each of the following numbers.

a. 350 **b.** 81 **c.** 36

30. Write each number as a whole number in standard form.

a. 6^0 **b.** 7^2

c. 2^5 **d.** 25^0

31. **a.** Write 81 as a power of 9. **b.** Write 81 as a power of 3.

c. Write 625 as a power of 25. **d.** Write 625 as a power of 5.

32. Write each number using a single exponent.

a. $3^3 \cdot 3^7$ **b.** $5^{33} \cdot 5^{12}$ **c.** $21^0 \cdot 21^{19}$

33. Determine which of the following numbers are perfect-square numbers. In each case, explain your answer.

a. 25 **b.** 125

c. 121 **d.** 100

e. 200

34. Determine the square root of each of the following numbers.

a. 16 **b.** 36 **c.** 289

35. Which of the following numbers are perfect squares, perfect cubes, or both?

a. 40 **b.** 400 **c.** 10

d. 10,000 **e.** 1000 **f.** 64

36. Evaluate each of the following arithmetic expressions by performing the operations in the appropriate order. Verify using your calculator.

a. $48 - 3 \cdot (18 - 2 \cdot 7) + 3^2$

b. $2^4 + 4 \cdot 2^2$

c. $243/(35 - 2^3)$

d. $(160 - 2 \cdot 25)/10$

e. $7 \cdot 2^3 - 9 \cdot 2 + 5$

f. $2^3 \cdot 3^2$

g. $6^2 + 2^6$

h. $(3^2 - 2^3)^2$

37. Do the expressions $6 + 10 \div 2$ and $(6 + 10) \div 2$ have the same result when calculated? Explain.

This page intentionally left blank

Chapter 2

Variables and Problem Solving

In Chapter 1, you saw how the properties of whole numbers and arithmetic are used to solve problems.

In Chapter 2, you will be introduced to basic algebra, where the properties of arithmetic are extended to solve a wider range of problems.

Activity 2.1

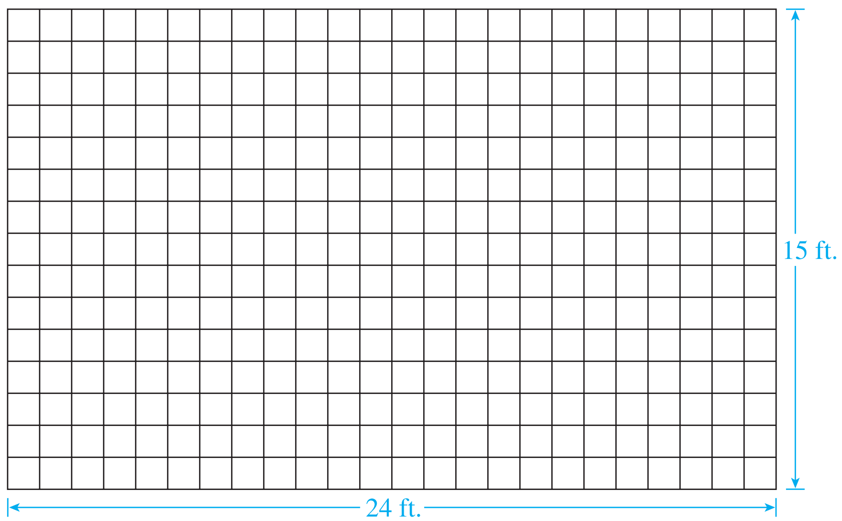
How Much Do I Need to Buy?

Objectives

1. Recognize and understand the concept of a variable in context and symbolically.
2. Translate a written statement (verbal rule) into a statement involving variables (symbolic rule).
3. Evaluate variable expressions.
4. Apply formulas (area, perimeter, and others) to solve contextual problems.

To earn money for college, you and a friend run a home-improvement business during the summer. Most of your jobs involve interior work, such as painting walls and ceilings, and installing carpeting, floor tiles, and floor molding. One of the first tasks in any job is to calculate the quantity of material to buy.

Your newest clients need floor molding installed around the edges of their living room. You measure the floor of the rectangular room. It is 24 feet long and 15 feet wide. The grid below represents a floor plan of the room.



1. How much floor molding will you need for the living room? (You may ignore any door openings for now.) Describe or show the calculation you used.

Definition

The distance around the edge of a rectangle is the **perimeter** of the rectangle. Since perimeter is a distance, it is measured in feet, meters, miles, etc.

2. a. For any rectangle, describe in your own words how to determine the perimeter if you know its length and width.

A description of a rule in words is called a **verbal rule**. A verbal rule can be translated into an equation that uses letters to represent the quantities. For example, you can represent the **perimeter** of the rectangle with the letter ***P***, the **length** with the letter ***l***, and the **width** with the letter ***w***.

b. Write an equation that translates your verbal rule in part a into an equation using the letters ***P***, ***l***, and ***w***. The equation is called a **symbolic rule**.

Since quantities like width, length, and perimeter may **vary**, or change in value, from one situation to another, they are called variables. A **variable** is usually represented by a letter (or some other symbol) as a simple shorthand notation. In Problem 2, P, l, and w are shorthand notations for the variables perimeter, length, and width, respectively.

3. The clients have a rectangular dining room, where they also want floor molding installed. The dining room measures 16 feet long by 12 feet wide. List the variables and state their values in this situation.

The equation $P = 2l + 2w$ connecting the variables perimeter, length, and width is an example of a formula. A **formula** is an equation or symbolic rule that represents the relationship between **input variable(s)** and an **output variable**. In this case, the formula is a method for calculating the perimeter of any rectangle, once you know the length and width. The expression $2l + 2w$, which is on the right-hand side of the equals sign, is called a **variable expression**. Such an expression is a symbolic code of instructions for performing arithmetic operations with variables and numbers. The numbers, specific numerical values that do not change, are called **constants**.

4. Use the formula for perimeter to calculate the amount of floor molding you will need for the following rooms. Show the calculations in each case. (All measurements are in feet.)

Do You Measure Up?

ROOM	LENGTH, l (feet)	WIDTH, w (feet)	CALCULATIONS: $2l + 2w = P$	PERIMETER, P (feet)
Living	24	15	$2(24) + 2(15) = 48 + 30 = 78$	78
Dining	16	12		
Family	30	16		
Bedroom 1	18	15		
Bedroom 2	14	14		
Bedroom 3	13	13		

In Problem 4, you determined the perimeter, P, by replacing l and w with the numerical values in the formula $P = 2l + 2w$. You then performed the arithmetic operations to determine the perimeter. This process is called **evaluating a variable expression** for the given input values. Note that replacing the variables (l and w in this case) with numerical values is also referred to as **substituting** given values for the variables.

Determining an output value in a formula often requires using the order of operations procedure, which you learned in Activity 1.7 and used in Problem 4 of this activity. You first substitute the known values for the variables in the expression and then use the order of operations procedure to evaluate the expression. This method works for formulas in general.

Example 1 ***Evaluate the formula $h = 160t - 16t^2$ for $t = 2$.***

SOLUTION

$$h = 160t - 16t^2$$
$$h = 160 \cdot 2 - 16 \cdot 2^2$$
$$h = 160 \cdot 2 - 16 \cdot 4$$
$$h = 320 - 64$$
$$h = 256$$

5. Your client decides to replace the molding in the den with a more expensive oak molding. The floor of the den is a square, measuring 12 feet by 12 feet.

a. Write a verbal rule to describe how to determine the perimeter of a square if you know the length of one of the sides.

b. Let s represent the length of one side of a square. Translate the verbal rule in part a into a symbolic rule (a formula) for the perimeter of a square.

c. Use the formula to determine the perimeter of the square den.

6. Evaluating expressions is an essential skill in algebra. Evaluate each of the following expressions if the value of x is known to be 8 and the value of y is known to be 5.

a. $5x - 17$

b. $3y - x + 14$

c. $2x + 3y^2$

d. $2(x - y) + x^2$

7. You find a deal on floor molding at Home Depot. The molding costs \$2.00 for a 10-foot-long piece. In the rooms you have measured, there are a total of 10 doorways, each 3 feet wide. How much will the molding cost for all six rooms?

Area of a Rectangle

8. For the living room in Problem 1, your client wants you to buy enough carpeting to cover the entire floor (wall to wall).

   a. How many square feet (squares that measure 1 foot by 1 foot) will it take to cover the living room floor? You may want to refer to the floor plan on page 83.

   b. Describe or show the calculation you used.

Definition

The **area** of a geometric figure is the measure of the size of the region bounded by the sides of the figure. Area is measured in square units, such as square feet, square meters, square miles, etc.

   c. Write a verbal rule to describe how to determine the area of a rectangle if you know the length and width of the rectangle.

   d. Represent the area by the letter A. Translate the verbal rule in part c into a formula for the area of a rectangle. Use l to represent length and w to represent width.

The expression lw means l times w, also written as $l \cdot w$ or $(l)(w)$. Generally, you do not use the multiplication symbol $\times$ in a variable expression, since it might be confused with the letter x.

   e. Use the formula in part d to verify your result in part a.

9. Use the formula for the area of a rectangle to calculate the number of square feet of carpeting you will need for the following rooms. Show the calculations in each case.

Wide Open Spaces

ROOM	LENGTH, l (feet)	WIDTH, w (feet)	CALCULATIONS: $lw = A$	AREA, A (square feet)
Living	24	15	$24 \cdot 15 = 360$	360
Dining	16	12		
Family	30	16		
Bedroom 1	18	15		
Bedroom 2	14	14		
Bedroom 3	13	13		

10. Carpeting is usually measured by the square foot but is sold by the square yard. Therefore, you have to convert your measurements from square feet to square yards.

a. How many feet are in 1 yard?

b. How many square feet are in 1 square yard? (*Hint*: Draw a 3-foot by 3-foot square, and count the number of 1-foot by 1-foot squares.)

c. Calculate the number of square yards of carpeting you will need for the living room.

11. Use s to represent the length of one side of a square and write the formula for the area, A, of a square. Use your formula to determine the area of the den in Problem 5.

12. You belong to a food-purchasing co-op and buy everything in bulk. You must buy a case containing 12 packages of any item you wish to purchase. The total price of a case depends on the price of each package.

a. What is the price of a case of organic broccoli costing $2 per package?

b. Because you are interested in determining the case price given the price of an individual package for several items, you create a formula for the case price. What is your input variable?

c. What is your output variable?

d. Write a verbal rule that describes how to determine the case price for a given price of an item.

e. Translate the verbal rule in part d into a symbolic rule with i representing the input (individual package price) and c representing the output (case price).

f. Use your formula from part e to determine the cost of a case of each organic item in the following table.

ITEM	INDIVIDUAL PRICE ($)	CASE PRICE ($)
Tower Farms cauliflower	3	
Ezekiel bread	5	
Dark chocolate (88%)	4	

13. Translate the following verbal rules into symbolic rules using x as the input and y as the output variable.

a. The output is six more than the input.

b. The output is 40 less than the input.

c. The output is twice the input.

d. The output is six more than twice the input.

e. The output is twice the sum of the input and six.

f. The output is five more than the square of the input.

SUMMARY: ACTIVITY 2.1

1. A **variable** is a quantity that changes (or varies) in value from one situation to another. A single letter usually represents a variable.
2. A **variable expression** is a symbolic code giving instructions for performing arithmetic operations with variables and specific numerical values called **constants**.
3. A **formula** is an equation (symbolic rule) consisting of an output variable, usually on the left-hand side of the equals sign, and an algebraic expression of input variables, usually on the right-hand side of the equals sign.
4. In a formula, the **input variable** is found in the expression. Values for input variables are usually given first and used to determine output values.
5. In a formula, output values depend on values of the input variables. Values for an **output variable** are determined by evaluating expressions for given input values.
6. Multiplication is always the operation to perform when you see a number next to a variable or two variables next to each other.

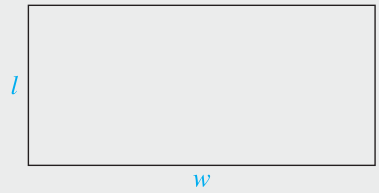
7. Some formulas from geometry:

a. perimeter of a rectangle

$P = 2l + 2w$

b. area of a rectangle

$A = lw$

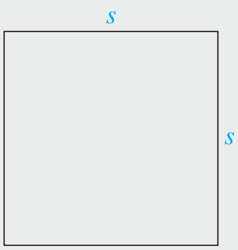


c. perimeter of a square

$P = 4s$

d. area of a square

$A = s^2$



EXERCISES: ACTIVITY 2.1

1. Use the formulas for perimeter and area to answer the following questions.

a. The side of a square is 5 centimeters. What is its perimeter?

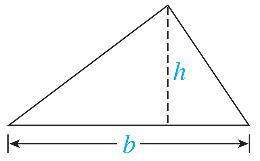
b. A rectangle is 23 inches long and 35 inches wide. What is its area?

c. The adjacent sides of a rectangle are 6 and 8 centimeters, respectively. What is the perimeter of the rectangle?

d. A square measures 15 inches on each side. What is the area of this square?

Exercise numbers appearing in color are answered in the Selected Answers appendix.

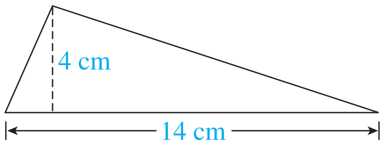
2. The area of a triangle can be found using the formula $A = b \cdot h \div 2$. In this formula, b represents the base of the triangle and h represents the height.



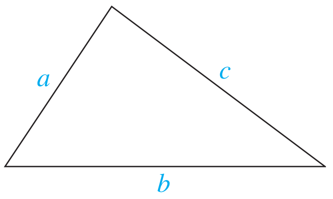
Use this formula to determine the areas of the following triangles.

a. The base of a triangle is 10 inches and its height is 5 inches.

b.



3. Let a, b, and c be the lengths of the sides of a triangle, as shown.



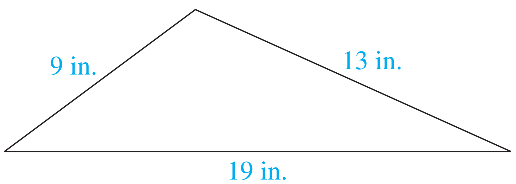
a. Write a formula for the perimeter of a triangle.

b. Use the formula you wrote in part a to determine the perimeter of the following triangles.

i. The sides of the triangle are 4 feet, 7 feet, and 10 feet.

ii. $a = 24$ cm, $b = 31$ cm, and $c = 47$ cm

iii.



4. You are coating your rectangular driveway with blacktop sealer. The driveway is 60 feet long and 10 feet wide. One can of sealer costs $15 and covers 200 square feet.

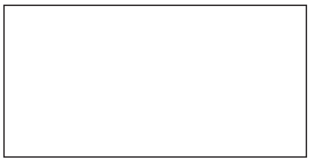
a. Determine the area of your driveway.

b. Determine how many cans of blacktop sealer you need.

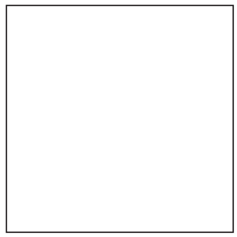
c. Determine the cost of sealing the driveway.

5. Identify each geometric figure as a square, rectangle, or triangle. Use a metric ruler to measure the lengths that you need to determine the perimeter and area of each figure. Measure each length to the nearest whole centimeter; then calculate the perimeter and area.

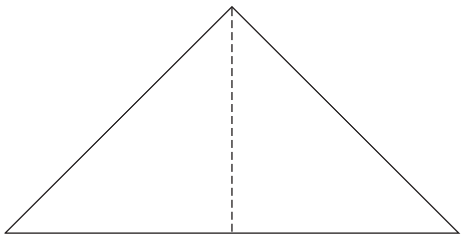
a.



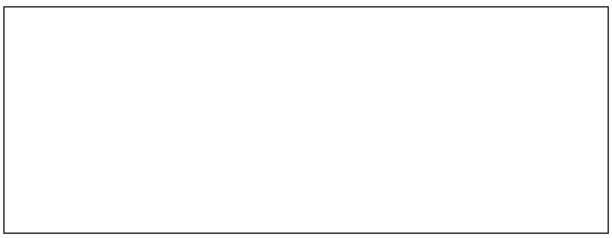
b.



c.

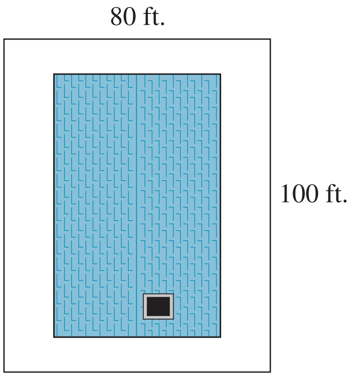


d.



6. You built your house on a rectangular piece of land with dimensions of 80 feet by 100 feet.

a. What is the area of the lot?



b. The rectangular house measures 50 feet by 79 feet. You wish to lay sod on the remainder of your property. How many square feet of sod are needed?

c. Sod is sold by the square yard. How many square yards do you have to buy?

Formulas are useful in many business applications. For Exercises 7–9,

a. *determine what variable quantity or quantities represent input and what variable quantity represents the output;*

b. *choose appropriate letters to represent each variable quantity and write what each letter represents;*

c. *use the letters to translate each verbal rule into a symbolic formula;*

d. *use the formula to determine the result.*

7. Profit is equal to the total revenue from selling an item minus the cost of producing the item. Determine the profit if the total revenue is $400,000 and the cost is $156,800.

8. Net pay is the difference between a worker's gross income and his or her deductions. A person's gross income for the year was $65,000 and the total deductions were $12,860. What was the net pay for the year?

9. Annual depreciation equals the difference between the original cost of an item and its remaining value, divided by its estimated life in years. Determine the annual depreciation of a new car that costs \$25,000, has an estimated life of 10 years, and has a remaining value of \$2000.

Formulas are also found in the sciences. In Exercises 10–11, use the given formulas to determine the results.

10. In general, temperature is measured using either the Celsius (C) or Fahrenheit (F) scales. To convert from degrees Celsius to degrees Fahrenheit, use the following formula.

$$F = (9C \div 5) + 32$$

   a. Use the formula to convert 20° Celsius into degrees Fahrenheit.

   b. 100°C is equal to how many degrees Fahrenheit?

11. To convert from degrees Fahrenheit to degrees Celsius, use the following formula.

$$C = 5(F - 32) \div 9$$

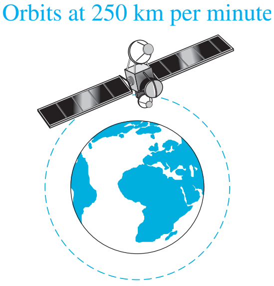
   a. Convert 86°F into degrees Celsius.

   b. What is a temperature of 41°F equal to in degrees Celsius?

12. The distance an object travels is determined by how fast it goes and for how long it moves. If an object's speed remains constant, the formula is $d = rt$, where d is the distance, r is the speed, and t is the time.

a. If you drive at 50 miles per hour for 5 hours, how far will you have traveled?

b. A satellite is orbiting Earth at the rate of 250 kilometers per minute. How far will it travel in 2 hours?



13. Translate the following verbal rules into symbolic rules using x as the input and y as the output variable.

a. The output is ten more than the input.

b. The output is the sum of the input and 12.

c. The output is five less than the input.

d. The output is six times the input.

e. The output is three less than six times the input.

f. The output is six times the difference of the input and three.

g. The output is 25 less than the input squared.

Activity 2.2

How High Will It Go?

Objectives

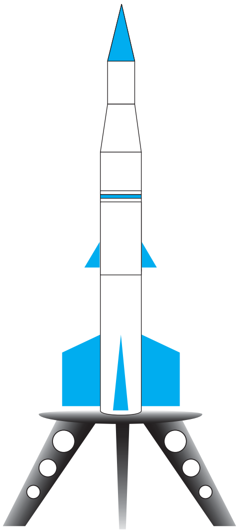
1. Recognize the input/output relationship between variables in a formula or equation (two variables only).
2. Evaluate variable expressions in formulas and equations.
3. Generate a table of input and corresponding output values from a given equation, formula, or situation.
4. Read, interpret, and plot points in rectangular coordinates that are obtained from evaluating a formula or equation.

You gave your niece a model rocket for her birthday. The instruction booklet states that the rocket will have a speed of 160 feet per second at launch. The instructions also provide a formula that determines the height of the rocket after launching. The variables are time, t, measured in seconds after launch, and height, h, measured in feet above the ground. Using these variables, the formula is

$$h = 160t - 16t^2.$$

Note that the formula contains only one input variable, namely, time, t.

Use this formula to answer Problems 1–8 in this activity.



Example 1 ***Determine the rocket's height above the ground at 5 seconds after launch, based on the formula given above.***

SOLUTION

The rocket's height above the ground at 5 seconds after launch is determined by replacing the variable t in the formula with the number 5 and evaluating the expression. Recall that a number next to the variable in a formula means to multiply. Also, remember to follow the order of operations when doing the arithmetic.

Substituting 5 for t, the formula becomes

$$h = 160t - 16t^2$$
$$h = 160 \cdot 5 - 16 \cdot 5^2$$
$$h = 160 \cdot 5 - 16 \cdot 25$$
$$h = 800 - 400$$
$$h = 400 \text{ ft.}$$

The rocket's height above the ground at 5 seconds after launch is 400 feet.

1. How high will the rocket be after 1 second?

2. How high will the rocket be after 2 seconds? After 3 seconds?

Input/Output Table

In Problems 1 and 2, you started with values for the input variable time, t, and calculated corresponding values for the output variable height, h. A table is useful to show and record a set of such calculations.

Each row in a **table of values** displays a value for the **input variable** and the corresponding value for the **output variable**. Keep in mind that in a formula, the variable involved in the variable expression is the input and the variable by itself on the other side of the equals sign is the output.

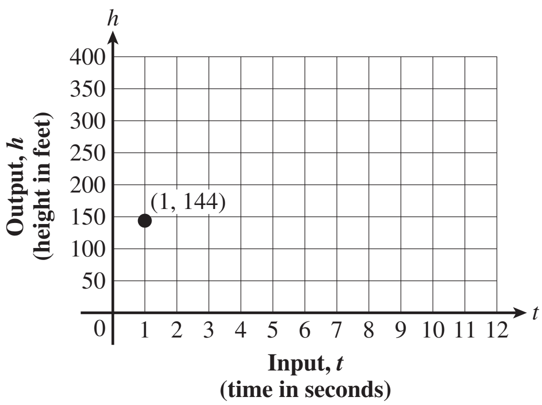
3. Record your results from Problems 1 and 2 in the following table. Then calculate the remaining heights to complete the table of values.

3, 2, 1 Blastoff

INPUT, t (time in seconds)	CALCULATIONS $h = 160 \cdot t - 16 \cdot t^2$	OUTPUT, h (height in feet)
0	$h = 160(0) - 16(0)^2$	0
1		
2		
3		
4		
5		

Visual Display of Input/Output Pairs

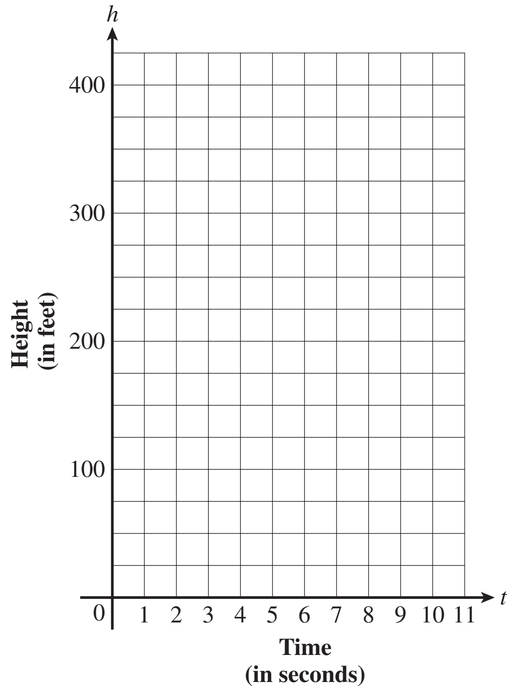
As discussed in Activity 1.2, Bald Eagle Population Increasing Again, a useful way to display the information in your table is visually with a graph. Each pair of input/output numbers is represented by a plotted point on a rectangular grid. The input value is referenced on a horizontal number line, the horizontal axis. The output value is referenced on a vertical number line, the vertical axis. For example, in the grid, one of the (input, output) pairs of numbers from Problem 3, $(1, 144)$, is indicated by a small labeled dot.



When plotted as a point, a pair of input/output numbers is called the **coordinates** of the point and can be written next to the point in parentheses, separated by a comma. Notice that the numbers on each axis, the **scales**, are different. The choice of scale for an axis is determined by the range of the coordinate values the axis represents.

4. a. Use the table of values from Problem 3 to plot the points that represent each input/output pair of values. The grid has been scaled for you. Estimate the heights on the vertical axis as best you can.

b. Notice that the scale on each axis is uniform. This means, for example, that on the time axis the distance between adjacent gridlines always represents 1 second. What distance is represented between adjacent gridlines on the height axis?



5. Extend the table in Problem 3 for the times given below and plot the additional points on the graph in Problem 4.

Getting Grounded

INPUT, t (time in seconds)	CALCULATIONS $h = 160t - 16t^2$	OUTPUT, h (height in feet)
6		
7		
8		
9		
10		

6. a. Use the output values in the table (Problems 3 and 5) to determine how high the rocket goes.

b. How long does it take to reach this height?

7. What happens at 10 seconds?

8. Draw a curved line to connect the points on the graph in Problem 4. How might this be useful?

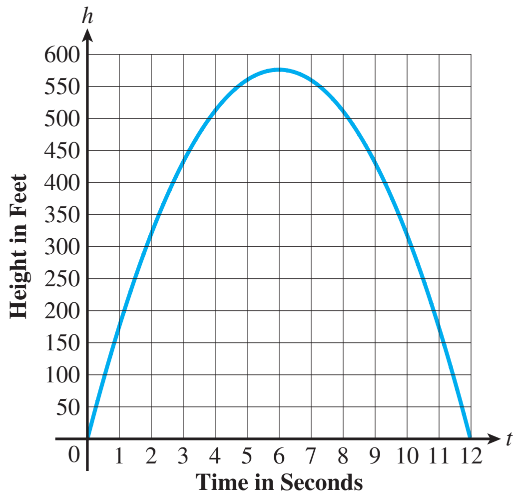
A Second Rocket Launch

To visually see the points determined by the rocket height formula, you created a table of input/output values, selected some of the ordered pairs of numbers and plotted them on a grid (see Problems 4 and 5). However, a formula usually determines infinitely many points. By connecting the selected points you plotted on the grid with a smooth curve, you can estimate many other points given by the formula.

Definition

The collection of all the points determined by a formula results in a curve called the **graph** of the formula or equation.

The following graph shows the height of a different rocket after launch, given by the formula $h = 192t - 16t^2$. Use the graph below to answer Problems 9–14.



9. What is the input variable and the output variable?

10. Use the graph to estimate the height of the rocket 3 seconds after launch.

11. At what time(s) is the rocket approximately 200 feet above the ground?

12. Approximately how high above the ground does the rocket get, and how many seconds after launch is this highest point reached?

13. How many seconds after launch will the rocket hit the ground?

14. Give the coordinates of the points you used to answer Problems 10–13.

15. a. Identify the input variable and output variable in the formula for the second rocket launch, $h = 192t - 16t^2$.

b. Check the accuracy of your estimates of the input/output pairs you listed in Problem 14 by substituting the input values into the formula and calculating the corresponding output values. Use the following table to show and record your calculations.

INPUT, t (time in seconds)	CALCULATIONS $h = 192t - 16t^2$	OUTPUT, h (height in feet)	PROBLEM 14 ESTIMATE
3			
1			
11			
6			
12			

c. How close were your estimates compared to the calculated output values?

SUMMARY: ACTIVITY 2.2

1. A **formula** can be used to represent the relationship between a single **input variable** and an **output variable**. For example, h is the output variable and t is the input variable in the formula $h = 160t - 16t^2$.

2. A vertical **table of values** is a listing of **input values** with their corresponding **output values** in the same row. Alternatively, a horizontal table of values is a listing of input values with their corresponding output values in the same column.

3. An input value and its corresponding output value can be written as an **ordered pair** of numbers within a set of parentheses, separated by a comma. By tradition, the input value is always written first and the output value is second. For example, $(6, 576)$ represents an input of 6 with a corresponding output of 576.

4. An ordered pair of input/output values from a formula can be represented as a point on a rectangular coordinate grid. If x represents the input, then on the grid, the input value refers to the horizontal or x-axis and is called the ***x*-coordinate** of the point. Similarly, if the output is represented by y, then the output value refers to the vertical or y-axis and is called the ***y*-coordinate** of the point. The point can be labeled on the grid using the ordered-pair notation.

5. When selected points from a formula or equation are plotted on a grid and then smoothly connected by a curve, the result is called a **graph** of the formula or equation.

EXERCISES: ACTIVITY 2.2

1. Use the formulas for the perimeter and area of a square from Activity 2.1 to complete the input/output tables for squares of varying sizes.

s, LENGTH OF SIDE	*P*, PERIMETER OF SQUARE
1	
2	
3	
4	
5	
6	
7	

s, LENGTH OF SIDE	*A*, AREA OF SQUARE
1	
2	
3	
4	
5	
6	
7	

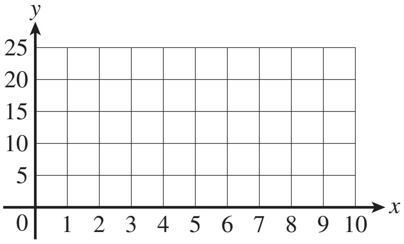
2. Consider the formula $y = 2x + 1$.

a. Complete the table of values for this formula.

b. Which variable is the input variable?

x	*y*
0	
2	
4	
6	
8	
10	

c. Plot the points that represent the input/output pairs in your table on the coordinate system. Label each point with its coordinates.



3. Consider the formula $y = x^2 + 3x + 2$.

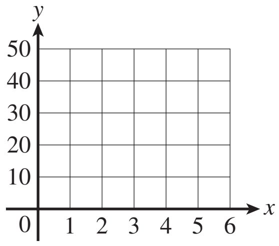
a. Complete the table of values for this formula.

b. Which variable is the output variable?

x	*y*
0	
1	
2	
3	
4	
5	

Exercise numbers appearing in color are answered in the Selected Answers appendix.

c. Plot the points that represent the input/output pairs of your table on the coordinate system.

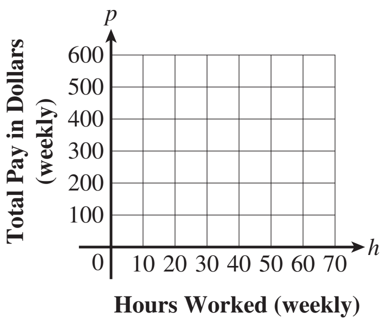


4. You work at a local restaurant. You are paid \$8.00 per hour up to the first 40 hours per week, and \$12.00 per hour for any hours over 40 (overtime hours).

a. Let h represent the hours you work in one week and p your total pay for one week. Complete the input/output table.

Input, *h*, in hours	20	25	30	35	40	45	50	55	60
Output, *p*, in dollars									

b. Plot the input/output pairs from the table in part a on the following coordinate system.

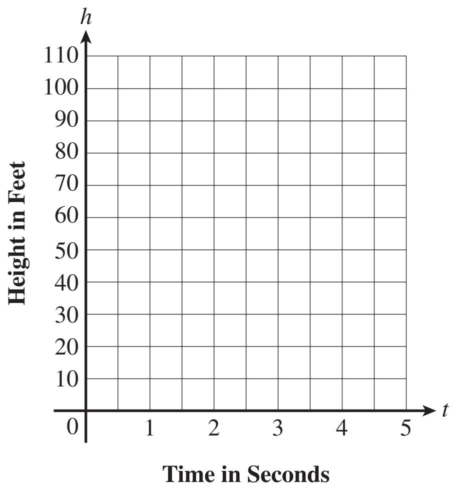


5. The formula $h = 80t - 16t^2$ gives the height above the ground at time t for a ball that is thrown straight up in the air with an initial velocity of 80 feet per second.

a. Complete a table of values for the following input values: $t = 0, 1, 2, 3, 4,$ and 5 seconds.

t, Time in Seconds	0	1	2	3	4	5
h, Height in Feet						

b. Plot the input/output pairs from the table in part a on the coordinate system, and connect them in a smooth curve.



c. Use the graph to estimate how high above the ground the ball will get.

d. From the graph, when will the ball be 100 feet above the ground?

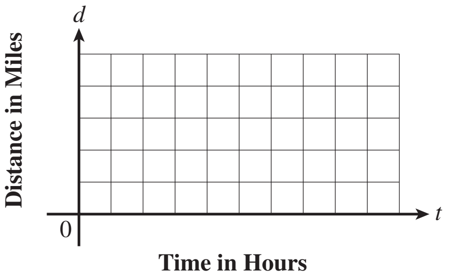
e. When will the ball hit the ground? Give a reason for your answer.

6. The formula $d = 50t$ determines the distance, in miles, that a car travels in t hours at a steady speed of 50 miles per hour.

a. Complete a table of values for the input values: $t = 0, 1, 2, 3, 4$, and 5 hours.

t, Time in Hours	0	1	2	3	4	5
d, Distance in Miles						

b. Plot the input/output pairs from the table in part a on the following grid. Choose uniform scales on each axis that will allow you to clearly plot all your points.



c. How long does it take to travel 200 miles?

7. a. Use the following formulas to fill in the table indicating which letter represents the input variable and which letter represents the output variable.

	INPUT VARIABLE	OUTPUT VARIABLE
$T = 40x$		
$F = 2x^2 + 4x - 5$		
$C = 5(F - 32)/9$		
$A = 13s + 7$		

b. Choose a value for each input variable and calculate the corresponding value of the output variable.

Activity 2.3

Are You Balanced?

Objectives

1. Translate contextual situations and verbal statements into equations.
2. Apply the fundamental principle of equality to solve equations of the forms $x + a = b$, $a + x = b$ and $x - a = b$.

Keeping track of your growing DVD collection can be a big job. You would like to know how close you are to owning 200 DVDs. Counting them shows you have 117 DVDs.

1. a. How many DVDs do you need to reach your goal of 200 DVDs?

   b. State the arithmetic operation you performed.

Problem 1 is an introduction to one of the most powerful tools in mathematics, solving equations that represent real-world problems.

You can model the DVDs problem by the verbal rule:

> The number of DVDs you have plus the number of DVDs you need is equal to the number of DVDs in your goal.

Each quantity in the verbal rule can be replaced by either a known number that is given in the problem or by a letter that represents an unknown quantity. If you let N represent the number of DVDs you need, the verbal rule becomes the equation

$$117 + N = 200.$$

The value of N that makes this equation a true statement is called the **solution** of the equation.

2. Verify that your answer to Problem 1 is the solution to the equation. To do this, begin to copy the equation, but when you reach N, write the number you found in Problem 1 *instead* of the letter N. In other words, you replace N with your answer from Problem 1. Next, perform the addition, and see if you have a true statement.

Keep in mind the meaning of the equals symbol, =. In any equation, the symbol = indicates that the quantity on the left side of the equals symbol must have *exactly the same value* as the quantity on the right side of the equals symbol, although the two sides may look quite different.

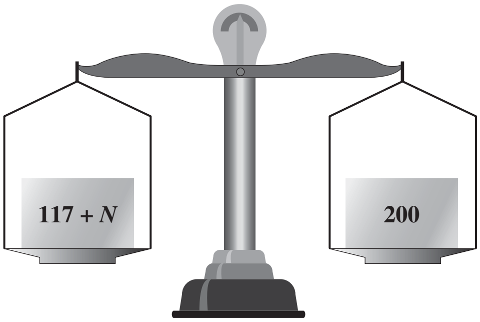
Solving Equations of the Form $x + a = b$ and $a + x = b$

In the equation $117 + N = 200$, the letter N represents an unknown quantity. To solve the equation means to determine the value of the unknown quantity, N. The strategy is to perform one or more operations on *both sides of the equation* in order to isolate the letter N. The goal is to have an equation with N on only one side, which might look like $N =$ *the value you determined*. We read this as "N is equal to *the value you determined*." Now that we know a value of N that makes the equation a true statement, that value is considered a solution of the original equation.

> The **fundamental principle of equality** is a powerful guide to solving equations. It states that in performing the same operation on both sides of the equals sign in a true equation, the resulting equation is still true.

For example, $13 = 13$ is a true statement. If you add 12 to both sides of $13 = 13$, you get $13 + 12 = 13 + 12$, which is $25 = 25$, which is still a true statement.

This fundamental principle can be viewed visually. Think of a true equation as a scale in balance. A balanced scale will be horizontal as long as both sides are equal in weight.



The fundamental principle states that if an equal amount is added to or subtracted from both sides, the scale will remain in balance.

Example 1 ***Apply the fundamental principle of equality to solve the equation $117 + N = 200$ for N.***

SOLUTION

When solving equations, it is good practice to record your work very carefully. To isolate the variable N, you need to undo the addition of 117. This can be accomplished by subtracting 117 from both sides of the equation.

$$\begin{array}{rcr} 117 + N &=& 200 \\ -117 & & -117 \\ \hline 0 + N &=& 83 \\ N &=& 83 \end{array}$$

To isolate N, subtract 117 from both sides of the equation. Note that $117 - 117 = 0$.

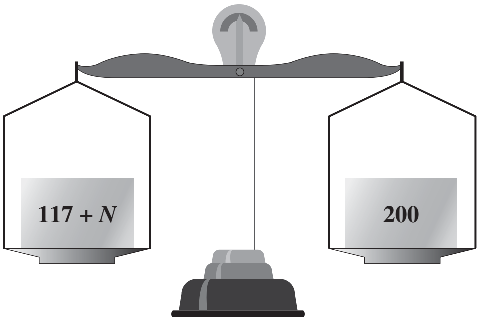
Check: $117 + 83 = 200$

$200 = 200$

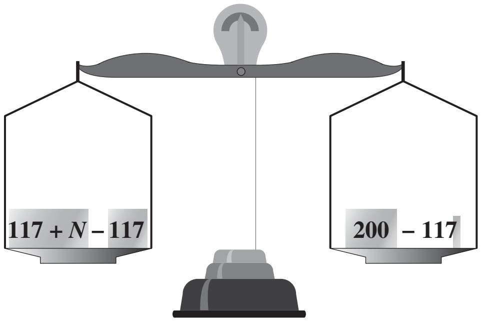
To check your solution, use your original equation, substitute the value of N, and perform the addition.

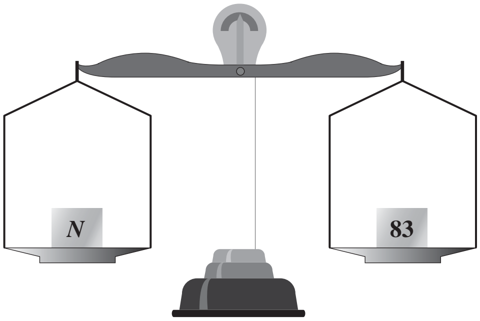
The fundamental principle of equality is the algebraic tool, or approach, that we use to solve equations.

In solving $117 + N = 200$, you can imagine the following series of scales.



Subtract 117 from both the left and right sides to keep the scale in balance. The end result gives the solution.





3. Solve the equation $N + 129 = 350$.

a. To isolate the variable N on one side of the equation, what operation must you perform on both sides of $N + 129 = 350$?

b. Use the result from part a to solve the equation. Check your solution.

Solving Equations of the Form $x - a = b$

4. Solve the equation $x - 35 = 47$ for x.

a. To isolate the variable x, you need to undo the subtraction of 35 from x. What operation must you perform on each side of the equation to accomplish this?

b. Perform this calculation on both sides of the equation to keep the equation in balance and obtain a solution for x.

c. Use the original equation to check your solution from part b.

5. Suppose you have a $15 coupon that you can use on any purchase over $100 at your favorite store.

   a. Write a verbal rule to determine the amount you will pay on a purchase of DVDs worth more than $100.

   b. Let x represent the cost (amount is over $100) of the DVDs you are buying. Let p represent the amount you pay (ignore sales tax). Translate the verbal rule in part a into an equation.

   c. If you purchase $167 worth of DVDs, determine how much you pay using your coupon.

   d. Suppose you pay $205 at the register. Write an equation that can be used to determine the value of the DVDs (before the discount coupon is applied).

   e. Solve the equation in part d using an algebraic approach.

In general, you can use an algebraic approach to solve equations of the form: $x + a = b$ or $x - a = b$ where x is the unknown quantity and a and b are numbers. In each case, the strategy is to isolate the variable on one side of the equation by undoing the given operation. To solve $x + a = b$, subtract a from both sides. To solve $x - a = b$, add a to both sides. Operations that undo each other, like addition and subtraction, are called **inverse operations**.

6. Use an algebraic approach to solve each of the following equations for x.

   a. $100 = x + 27$

   b. $16 = x - 7$

   c. $x + 4 = 53$

   d. $x - 13 = 45$

SUMMARY: ACTIVITY 2.3

1. An **equation** is a statement that two expressions are equal. In the equation $10 + x = 25$, $10 + x$ is an expression that is stated to be equal to the expression 25.
2. The **fundamental principle of equality** states that performing the same operation on both sides of a true equation will result in an equation that is also true.
3. The **solution** to an **equation** is a number that when substituted for the unknown quantity in the equation will result in a true statement when all the arithmetic has been performed.
4. An **equation** is **solved** when the unknown quantity is determined. When the equation $10 + x = 25$ is solved, the solution is $x = 15$.
5. **Expressions** are **evaluated.** For example, to evaluate the expression $12 + x$, replace x with any numerical value and perform the addition. If $x = 30$, then $12 + 30 = 42$.
6. To solve equations of the form $x + a = b$ or $a + x = b$ (where a and b are numbers and x is the unknown quantity), subtract a from both sides of the equation to "undo" the addition.
7. To solve equations of the form $x - a = b$ (where a and b are numbers and x is the unknown quantity), add a to both sides of the equation to "undo" the subtraction.
8. Because addition and subtraction "undo" each other, they are called **inverse operations** of each other.

EXERCISES: ACTIVITY 2.3

1. Have you ever seen the popular TV game show *The Price Is Right*? The idea is to guess the price of an item. You win the item by coming closest to guessing the correct price of the item without going over. If your opponent goes first, a good strategy is to overbid her regularly by a small amount, say \$20. Then your opponent can win only if the price falls in that \$20 region between her bid and yours.

   a. Write a verbal rule to determine your bid if your opponent bids first.

   b. Which bid is the input variable and which is the output variable in the verbal rule? Choose letters to represent your bid and your opponent's bid.

   c. Translate your verbal rule into an equation, using the letters you chose in part b to represent the input and output variables.

   d. Use your equation to determine your bid if your opponent bid \$575 on an item.

Exercise numbers appearing in color are answered in the Selected Answers appendix.

e. If you bid \$995 on a laptop computer, use your equation to determine your opponent's bid if he went first.

2. You are looking for a new car and the dealer claims the price of the model you like has been reduced by \$1150. The window sticker shows the new price as \$12,985.

   a. Represent the original price by a letter and then write an equation for the new price.

   b. Solve your equation to determine the original price. (Remember to check your result as a last step.)

3. Use an algebraic approach to solve each of the following equations.

   a. $x + 43 = 191$
   b. $315 + s = 592$
   c. $w - 37 = 102$
   d. $428 = y + 82$
   e. $t + 100 = 100$
   f. $31 = y - 14$
   g. $643 + x = 932$
   h. $z - 56 = 244$
   i. $W + 2495 = 8340$
   j. $y - 259 = 120$
   k. $4200 + x = 6150$
   l. $251 = t - 250$
   m. $y + 788 = 788$
   n. $32{,}900 = z + 29{,}240$
   o. $x - 648 = 0$

4. Your goal is to save enough money to purchase a new portable DVD player costing \$190. In the past 5 weeks, you have saved \$40, \$25, \$50, \$35, and \$20.

   a. Write an equation that relates your savings to the cost of the DVD player, using a letter for the dollar amount you still need to save.

   b. Solve your equation to determine the amount you still need to save.

5. Monthly salaries for the managers in your company have been increased by \$1500.

   a. Write a verbal statement to determine a manager's new salary after the \$1500 increase.

b. Translate your verbal statement in part a into an equation. State what each letter in your equation represents.

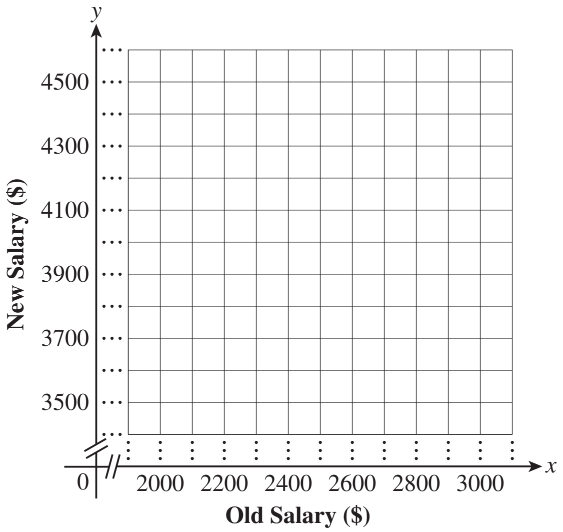
c. Which variable in your equation in part b is the input variable? Which is the output variable?

d. Complete this input/output table.

OLD SALARY x (in dollars)	NEW SALARY y (in dollars)
2200	
2400	
2600	
2800	
3000	

e. Use the results from the table to solve $x + 1500 = 4100$.

f. Plot the input/output pairs from your table on the coordinate system below.



g. What pattern would the points make if you were to connect them?

Activity 2.4

How Far Will You Go? How Long Will It Take?

Objectives

1. Apply the fundamental principle of equality to solve equations in the form $ax = b, a \neq 0$.
2. Translate contextual situations and verbal statements into equations.
3. Use the relationship rate · time = amount in various contexts.

Imagine that you are about to take a trip and plan to drive at a constant speed of 60 miles per hour (mph).

1. How far will you go in 2 hours? In 5 hours?

The answers to Problem 1 are based on an important basic relationship:

rate · time = distance

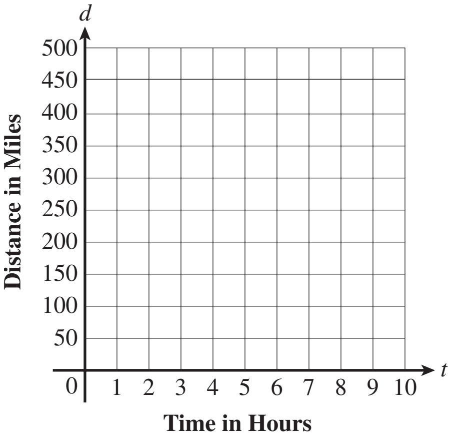
In this situation, the speed of 60 mph is the constant rate or how fast you are driving, and 2 hours is the time or how long you drove. Therefore, the distance you traveled in 2 hours is 60 mph · 2 hours = 120 miles. In 5 hours you will travel 60 mph · 5 hours = 300 miles.

2. a. Let t represent the time traveled (in hours) and d the distance (in miles). Write a formula to determine the distance traveled for various times at an average speed of 60 mph.

b. Use the formula from part a to complete an input/output table. Note that t is the input variable and d is the output variable.

t, TIME IN HOURS	*d*, DISTANCE IN MILES
2	
3	
5	
6	
8	

3. Plot the points that represent the input/output pairs of the table in Problem 2b. Connect the points. What pattern do the points make?



4. Now suppose you are traveling a constant 45 miles per hour.

a. Write a formula to calculate the distance traveled in this case.

b. How far will you go in 3 hours? 4 hours? 6 hours? 10 hours? Summarize your results in the following table.

t, TIME IN HOURS	*d*, DISTANCE IN MILES
3	
4	
6	
10	

c. Plot the points from this table on the same coordinate system as used for Problem 3. Connect the points.

d. How does the pattern of points from part c compare to that for the points in Problem 3, driving at 60 mph?

Now, instead of asking the question, "How far will you go?", you might ask the question, "How long will it take?" For example, suppose you are driving down the highway at 45 miles per hour, and you ask, "How long will it take to go 225 miles?" To answer this question, you can again use the formula: $d = 45t$.

5. Use the formula $d = 45t$ to write an equation to answer the question, How long will it take to travel 225 miles if you drive at an average speed of 45 miles per hour?

To solve the equation in Problem 5 for the variable t, apply the fundamental principle you used in Activity 2.3. That is, if you perform the same operation to both sides of a true equation, the result will still be a true equation.

Example 1 ***Solve the equation that you obtained in Problem 5, $45t = 225$, for t.***

SOLUTION

$45t = 225$

$45t \div 45 = 225 \div 45$ To isolate variable t, divide both sides of the equation by 45, since division will undo the multiplication. Note that $45 \div 45 = 1$.

$t = 5$ hr.

Check: $45 \cdot 5 = 225$ The solution checks.

$225 = 225$

Like addition and subtraction, multiplication and division are inverse operations because they undo each other.

6. Solve each equation for the given variable. Check your answer in each case.

a. $10t = 220$ **b.** $4x = 12$ **c.** $3q = 27$

There are other situations that are modeled by the relationship

$$\text{rate} \cdot \text{time} = \text{amount}.$$

7. Suppose you are pumping gasoline into your car's gas tank. Assume that the gas pump delivers 5 gallons per minute.

a. How much gasoline will you pump in 2 minutes?

b. Why did you multiply to get the correct answer?

c. How much gasoline will be pumped in 3 minutes? In 5 minutes?

d. Let g represent the number of gallons of gasoline pumped in t minutes and write a formula that determines g when you pump at a rate of 5 gallons per minute.

As in Problem 4, you can turn things around to ask, "How long will it take?" rather than, "How much is pumped?"

8. a. Assume that you continue to pump the gas at 5 gallons per minute. Write an equation to determine how long it will take to pump 20 gallons into your SUV's gas tank.

b. Solve the equation you wrote in part a.

9. a. Write a formula for the number of gallons of water, w, drawn into a bathtub in t minutes, when the water is flowing in at 3 gallons per minute.

b. How much water is in the bathtub after 9 minutes?

c. How long does it take to fill the bathtub with 48 gallons? First write an equation and then solve it.

10. **a.** Write a formula for the number of rabbits, R, added to a population after t months if the rabbit population grows at a rate of 150 rabbits per month.

b. How many rabbits are added after 4 months?

c. How long will it take to add 900 rabbits? First write an equation and then solve it.

11. Many conversion formulas are of the form $ax = b$. For example, to convert from feet to inches, you use

12 inches per foot multiplied by the number of feet = the number of inches.

a. Use i for the distance in inches and f for the number of feet to write the formula for converting feet into inches.

b. To convert 15 feet to inches, substitute 15 for f in the formula from part a, and evaluate the resulting numerical expression.

c. To convert 156 inches to feet, substitute 156 for i in the formula from part a, and solve the resulting equation.

SUMMARY: ACTIVITY 2.4

1. To solve an equation, apply the fundamental principle of equality, which states: Performing the same operation on *both sides* of a true equation will result in a true equation.

2. To solve equations of the form $ax = b$ (where a is a nonzero number and x is the unknown quantity), "undo" the multiplication by dividing both sides of the equation by a, thus isolating x.

3. Because division "undoes" multiplication and multiplication "undoes" division, the two operations are inverse operations.

EXERCISES: ACTIVITY 2.4

1. Use the fundamental principle of equality to solve each of the following equations.

a. $6x = 54$ b. $12t = 156$ c. $240 = 3y$ d. $15w = 660$

e. $169 = 13z$ f. $5x = 285$ g. $374w = 748$ h. $2y = 5874$

i. $382t = 0$ j. $381 = 3x$ k. $8y = 336$ l. $875 = 25x$

2. a. Write a formula for the area of grass, A, you can mow in t seconds if you can mow 12 square feet per second.

b. How many square feet can be mowed in 30 seconds?

c. How long will it take to mow 600 square feet? Write an equation and then solve.

3. One gallon of paint will cover 400 square feet. (This is the rate of coverage, 400 square feet per gallon.) Use g for the number of gallons of paint and A for the area to be covered to write an equation for each of the following problems. Determine the unknown in each equation.

a. How much area can be covered with 5 gallons of paint?

b. How many gallons would be needed to cover 2400 square feet?

c. All the walls in your house are rectangular and 8 feet high. The table on the next page shows the size of each room you wish to paint. How many gallons of paint will you need? (Ignore all window and door openings.)

Painting by Numbers

ROOM	LENGTH (feet)	WIDTH (feet)	HEIGHT (feet)	WALL AREA (square feet)
Living	24	15	8	624
Dining	16	12	8	
Family	30	16	8	
Bedroom 1	18	15	8	
Bedroom 2	14	14	8	
Bedroom 3	13	13	8	
Total				

4. For each of the following conversions, write an equation of the form $ax = b$. Use letters of your own choosing. Determine the unknown in each equation to answer each question.

   a. There are 5280 feet in one mile. How many feet are in 4 miles?

   b. There are 16 ounces in 1 pound. How many pounds are in 832 ounces?

   c. There are 100 centimeters in 1 meter. How many centimeters are in 49 meters?

   d. Computers store data in units called bits and bytes. There are 8 bits in 1 byte. How many bytes are in 4272 bits?

5. Solve the following equations by applying the fundamental principle of equality. Remember to check your solution.

a. $35 + x = 167$

b. $14v = 700$

c. $w - 27 = 101$

d. $1350 = 15y$

e. $s + 66 = 91$

f. $33 = p - 11$

g. $325g = 650$

h. $0 = d - 42$

i. $15r = 555$

j. $3721 = 1295 + x$

k. $477 = 9y$

l. $826 = z - 284$

m. $16w = 192$

n. $x + 439 = 1204$

o. $32{,}065 = 5t$

Activity 2.5

Web Devices for Sale

Objectives

1. Identify like terms.
2. Combine like terms using the distributive property.
3. Solve equations of the form $ax + bx = c$.

In your new Web device business, a particular model has been a hot item. You sold 45 of them in January and 75 in February. You know that the total revenue for these sales was $24,000, but you need the retail price of this particular model, which was not written in your record of monthly sales.

To determine the retail price, assuming it did not change, you can write and solve an equation that includes the revenue in January and in February.

1. a. Use p to represent the unknown retail price and write an equation for the combined revenue in January and February.

The expression $45p + 75p$ in the equation has two terms, $45p$ and $75p$, which are separated by an addition sign. The expression $45p$ means 45 times p, and 45 is the **coefficient** of the variable p.

b. Name the coefficient for each variable.

i. $75p$

ii. $10x$

iii. $6a + 3b$

In the case of $45p + 75p$, the two terms are called **like terms** because they contain exactly the same unknown quantity, p. It is important that the variables are *exactly* the same, including any exponents, before we can group them together.

c. You also sell protective cases for your web devices. You sold 35 of them in January. Use q to represent the unknown price of the cases. Write an expression for the combined January revenue of the devices and the cases.

Assuming that the price of the device and the price of the case are different, we cannot combine $45p$ and $35q$ in a simpler expression, because p and q are not the same unknown quantity. That is, $45p$ and $35q$ are *not* like terms.

May we combine $3p + 5p^2$ to form a simpler expression? No, because p and p^2 are not the same unknown quantity. They are *not* like terms.

We can use the distributive property as shown in Activity 1.4 to write the expression

$$45p + 75p$$

as

$$(45 + 75)p.$$

We apply the order of operations, as discussed in Activity 1.7, to add the numbers in the parentheses first. So,

$$(45 + 75)p = 120p.$$

This use of the distributive property is called **combining like terms**. To combine like terms, we add (or subtract) the coefficients of the common variable.

Example 1 ***The expression $13a - 7a + 5a$ has three like terms since each term contains the same variable factor a. To combine like terms, use the distributive property to write $13a - 7a + 5a$ as $(13 - 7 + 5)a$. Use the order of operations to combine the coefficients $13 - 7 + 5$ to obtain 11. Therefore, $13a - 7a + 5a = 11a$.***

2. Do the following expressions contain like terms? Give a reason for your answer.

a. $5a + 3a$

b. $4x + 5y + 7y^2$

c. $2q + 3q - 9q$

d. $7p + 3q - 2r - 5p$

3. Combine like terms.

a. $4x + 2x$

b. $9t - 5t$

c. $5b - 2b + 10b$

d. $9x + 3y + 5x - 2y$

4. For each of the following expressions, combine like terms, if possible.

a. $6t + t$

b. $3a + 5a - 6a$

c. $8x - 3x + 10y - 6y + 2$

d. $5a + 6a - 4c - 3$

5. We can now solve the combined revenue equation from Problem 1a for the price, p, of our web devices by replacing $45p + 75p$ with $120p$.

a. Solve the equation $120p = 24,000$.

b. Check your answer in the original equation.

6. Solve the following equations by first combining like terms. Check each of your solutions by substituting into the original equation.

a. $3x + 9x = 72$

b. $16y - 4y = 132$

c. $25t - 15t = 4200$

d. $326s + 124s = 3600$

e. $6n + 2n - 7n = 15$

7. a. Suppose 90 Web devices of the same model were sold in March and 120 were sold in April. If the total revenue from the sale of this device in March and April was $40,950, write an equation to determine the price, p, of this Web device.

b. Solve the equation in part a. (*Hint*: Combine like terms before solving the equation.)

c. Did the price change from February to March?

8. There was a price change for the Web device starting in May. A total of 140 devices were sold in May, 160 were sold in June, and the total revenue for both months was $51,000.

a. Write an equation for total revenues in May and June.

b. Solve your equation to determine the new price of a Web device.

c. How much revenue would you have in August if you sold 200 devices?

9. You put a new model of the Web device on sale during April. You sold 135 of these models, but 15 devices were returned and you refunded the cost. The net revenue for this model in April was $25,200.

a. Write an equation for the net revenue from the model on sale in April.

b. Solve the equation to determine the sale price of a Web device.

c. How much did you refund for the returned devices?

SUMMARY: ACTIVITY 2.5

1. **Like terms** are terms with exactly the same variable factor raised to the same power. The numeric factor of the term is called the **coefficient** of the variable factor.

   For example, $3x$ and $8x$ are like terms. 3 and 8 are coefficients of x. $3x$ and $3y$ are *not* like terms. $3x^2$ and $7x^2$ are like terms. $9x$ and $5x^2$ are *not* like terms.

2. **Combining like terms** into a single term means applying the distributive property to add or subtract the coefficients of the like terms.

   For example, $3x + 8x = (3 + 8)x = 11x$ and $9x - 8x = (9 - 8)x = 1x = x$.

3. To solve an equation like $3x + 8x = 22$, first combine like terms to get $11x = 22$. Then apply the fundamental principle of equality by dividing each side of the equation by the coefficient of x. In this example,

$$11x \div 11 = 22 \div 11$$
$$x = 2.$$

EXERCISES: ACTIVITY 2.5

1. a. How many terms are in the expression $5t + 3s + 2t$?

   b. What are the coefficients?

   c. What are the like terms?

   d. Simplify the expression by combining like terms.

2. Rewrite each expression by combining like terms.

   a. $14x - 8x + 7y + 5y$

   b. $2s + 11t + 9s + 3t + 14$

   c. $10x + 8y + 2x - 6y$

   d. $5b + 12t + t + 9b - 6$

   e. $27p + 39n + 12p + 2n + 5n$

   f. $252w - 181w + 130z + 65z + 175y - 88y$

3. Solve the following equations. Check your answers in the original equation.

   a. $3x + 15x = 720$

   b. $14y - 7y = 49$

   c. $320 = 21s - 13s$

   d. $0 = 34w + 83w$

   e. $5x + 17x - 4x = 270$

   f. $32 = 4t - 3t + 7t$

   g. $y + 3y - 2y = 178$

   h. $13x + 24x - 31x = 144$

   i. $2368 = 41t + 58t - 95t$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

j. $3w + 2w + 5w - 4w + w - 3w + w = 1635$

k. $36 = 14x - 13x + 8$

l. $65y - 64y - 51 = 13$

4. In your part-time job, your hours vary each week. Last month you worked 22 hours the first week, 25 hours the second week, 14 hours during week 3, and 19 hours in week 4. Your gross pay for those 4 weeks was $960.

a. Let p represent your hourly pay rate. Write the equation for your gross pay over these 4 weeks.

b. Solve this equation to determine your hourly pay rate.

c. How much will you earn in gross pay this week if you work 20 hours?

5. Over the first 3 months of the year you sold 12, 15, and 21 SuperPix digital cameras. The total in sales was $9360.

a. Let p represent the price of one SuperPix digital camera. Write the equation for your total in sales over these 3 months.

b. Solve the equation in part a to determine the retail price of the SuperPix digital camera.

c. This month you sell 10 of these cameras. What is your total in sales?

Activity 2.6

Make Me an Offer

Objectives

1. Use the basic steps for problem solving.
2. Translate verbal statements into algebraic equations.
3. Use the basic principles of algebra to solve real-world problems.

Many students believe "I can do math. I just can't do word problems." In reality, you learn mathematics so you can solve practical problems in everyday life and in science, business, technology, medicine, and most other fields.

On a personal level, you solve problems every day while balancing your checkbook, remembering an anniversary, or figuring out how to get more exercise. Most everyday problems don't require algebra, or even arithmetic, to be solved. But the basic steps and methods for solving any problem can be discussed and understood. In the process you will, with practice, become a better problem solver.

Solve the following problem any way you can, making notes of your thinking as you work. In the space provided, record your work on the left side. On the right side, jot down in a few sentences what you are thinking as you work on the problem.

"Night Train" Henson's Contract

1. "Night Train" Henson is negotiating a new contract with his team. He wants $800,000 for the year with an additional $6000 for every game he starts. His team offered $10,000 for every game he starts, but only $700,000 for a base salary. How many games would he have to start to make more with the team's offer?

SOLUTION TO THE PROBLEM	YOUR THINKING

2. Talk about and compare your methods for solving the problem above with classmates or your group. As a group, write down the steps you all went through from the very beginning to the end of your problem solving.

Experience has shown that there is a set of basic steps that serve as a guide to solving problems.

Procedure

Basic Steps for Problem Solving Solving any problem generally requires the following four steps.

1. Understand the problem.
2. Develop a strategy for solving the problem.
3. Execute your strategy to solve the problem.
4. Check your solution for correctness.

In Problems 3–6 that follow, compare your group's strategies for solving "Night Train" Henson's problem with each of the four basic steps for problem solving.

Let's consider each of the steps 1 through 4.

Step 1. Understand the problem. This step may seem obvious, but many times this is the most important step, and is the one that commonly leads to errors. Read the problem carefully, as many times as necessary. Draw some diagrams to help you visualize the situation. Get an explanation from available resources if you are unsure of any details.

3. List some strategies you could use to make sure you understand the "Night Train" Henson problem.

Step 2. Develop a strategy for solving the problem. Once you understand the problem, it may not be at all clear what strategy is required. Practice and experience is the best guide. Algebra is often useful for solving a math problem but is not always required.

4. How many different strategies were tried in your group for solving the "Night Train" problem? Describe one or two of them.

Step 3. Execute your strategy to solve the problem. Once you have decided on a strategy, carrying it out is sometimes the easiest step in the process.

5. When you decided on a strategy, how confident did you feel about your solution? Give a reason for your level of confidence.

Step 4. Check your solution for correctness. This last step is *critically important*. Your solution must be reasonable, it must answer the question, and most importantly, it must be correct.

6. Are you absolutely confident your solution is correct? If so, how do you know?

Solution: "Night Train" Henson's Contract

As a beginning problem solver, you may find it useful to develop a consistent strategy. Examine the following way you can solve "Night Train's" problem by applying some algebra tools as you follow the four basic steps of problem solving.

Step 1. Understand the problem. If you understand the problem, you will realize that "Night Train" needs to make \$100,000 more with the team's offer $(800{,}000 - 700{,}000)$ to make up for the difference in base salary. So he must start enough games to make up for this difference. Otherwise, he should not want the team's offer.

Step 2. Develop a strategy for solving the problem. With the observation above, you could let the variable x represent the number of games "Night Train" has to start to make the same amount of money with either offer. Then we note that he would make \$4000 more $(10{,}000 - 6000)$ for each game with the team's offer. So, \$4000 for each game started times the number of games started must equal \$100,000.

Step 3. Execute your strategy for solving the problem. You can translate the statement above into an equation, then apply the fundamental principle of equality to obtain the number of games started. Let x represent the number of games started. \$4000 times the number of games started equals \$100,000.

$$\begin{aligned} 4000x &= 100{,}000 \\ 4000x \div 4000 &= 100{,}000 \div 4000 \\ x &= 25 \end{aligned}$$

The solution to your equation says that "Night Train" must start in 25 games to make the same money with each deal.

Your answer would be: "Night Train" needs to play in more than 25 games to make more with the team's offer.

Step 4. Check your solution for correctness. This answer seems reasonable, but to check it you should refer back to the original statement of the problem. If "Night Train" plays in exactly 25 games his salary for the two offers can be calculated.

$$\begin{aligned} \text{"Night Train's" demand:} \quad & \$800{,}000 + \$6000 \cdot 25 = \$950{,}000 \\ \text{Team's offer:} \quad & \$700{,}000 + \$10{,}000 \cdot 25 = \$950{,}000 \end{aligned}$$

If he starts in 26 games, the team's offer is \$960,000, but "Night Train's" demand would give him only \$956,000. These calculations confirm the solution.

There are other ways to solve this problem. But the preceding solution gives you an example of how algebra can be used. Use algebra in a similar way to solve the following problems. Be sure to follow the four steps of problem solving.

Additional Problems to Solve

7. You need to open a checking account and decide to shop around for a bank. Acme Bank has an account with a \$10 monthly charge, plus a 25-cent charge per check. Farmer's Bank will charge you \$12 per month, with a 20-cent charge per check. How many checks would you need to write each month to make Farmer's Bank the better deal?

Step 1. Understand the problem.

Step 2. Develop a strategy for solving the problem.

Step 3. Execute your strategy to solve the problem.

Step 4. Check your solution for correctness.

8. The perimeter of a rectangular pasture is 3600 feet. If the width is 800 feet, how long is the pasture?

Step 1. Understand the problem.

Step 2. Develop a strategy for solving the problem.

Step 3. Execute your strategy to solve the problem.

Step 4. Check your solution for correctness.

Translating Statements into Algebraic Equations

The key to setting up and solving many word problems is recognizing the correct arithmetic operations. If there is a known formula, like the perimeter of a rectangle, then the arithmetic is already determined. Sometimes there is *only* the language to guide you.

In Problems 9–12, translate each statement into an equation. In each statement, let the unknown number be represented by the letter x.

9. The sum of 35 and a number is 140.

10. 144 is the product of 36 and a number.

11. What number times 7 equals 56?

12. The difference between the regular price and sale price of $245 is $35.

13. In Problems 9, 10, and 12, the verb "is" translates into what part of the equation?

14. Solve each of the equations you wrote in Problems 9–12.

a.

b.

c.

d.

SUMMARY: ACTIVITY 2.6

The Four Steps of Problem Solving

Step 1. Understand the problem.

a. Read the problem completely and carefully.

b. Draw a sketch of the problem, if possible.

Step 2. Develop a strategy for solving the problem.

a. Identify and list everything you know about the problem, including relevant formulas. Add labels to the diagram if you have one.

b. Identify and list what you want to know.

Step 3. Execute your strategy to solve the problem.

a. Write an equation that includes the known quantities and the unknown quantity.

b. Solve the equation.

Step 4. Check your solution for correctness.

a. Is your answer reasonable?

b. Is your answer correct? (Does it answer the original question? Does it agree with all the given information?)

EXERCISES: ACTIVITY 2.6

In Exercises 1–10, translate each statement into an equation and then solve the equation for the unknown number.

1. An unknown number plus 425 is equal to 981.

2. The product of some number and 40 is 2600.

3. A number minus 541 is 198.

4. The sum of an unknown number and 378 is 2841.

Exercise numbers appearing in color are answered in the Selected Answers appendix.

5. Fifteen subtracted from a number is 45.

6. The prime factors of 237 are 79 and an unknown number.

7. 13 plus an unknown number is equal to 51.

8. The difference between a number and 41 is 105.

9. 642 is the product of 6 and an unknown number.

10. The sum of an unknown number and itself is 784.

In Exercises 11–17, solve each problem by applying the four steps of problem solving. Use the strategy of solving an algebraic equation for each problem.

11. In preparing for a family trip, your assignment is to make the travel arrangements. You can rent a car for $75 per day with unlimited mileage. If you have budgeted $600 for car rental, how many days can you drive?

12. You need to drive 440 miles to get to your best friend's wedding. How fast must you drive to get there in 8 hours?

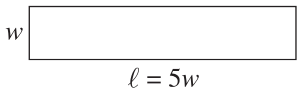
13. Your goal is to save $1200 to pay for next year's books and fees. How much must you save each month if you have 5 months to accomplish your goal?

14. You have enough wallpaper to cover 240 square feet. If your walls are 8 feet high, how wide a wall can you paper?

15. In your part-time job selling kitchen knives, you have two different sets available. The better set sells for $35; the cheaper set, for $20. Last week you sold more of the cheaper set, in fact twice as many as the better set. Your receipts for the week totaled $525. How many of the better sets did you sell?

16. A rectangle that has an area of 357 square inches is 17 inches wide. How long is the rectangle?

17. A rectangular field is 5 times longer than it is wide. If the perimeter is 540 feet, what are the dimensions (length and width) of the field?



What Have I Learned?

Write your responses in complete sentences.

1. What is the difference between an expression and an equation?

2. Describe each step required to evaluate the expression $5t^2 - 3t$ for $t = 2$.

3. In your own words, describe the fundamental principle of equality.

4. Describe the step required to solve the equation $6x = 96$.

5. Describe the step required to solve the equation $w - 38 = 114$.

6. Generate an input/output table for $y = x^2 - 5$, using input values $x = 3, 4, 5,$ and 6.

x	$y = x^2 - 5$	y
3		
4		
5		
6		

7. Which of the following are like terms: $4y, x, 7x, 4w, 3, 3y, y^2, 5x^2$? Why?

8. Give an example of how the distributive property allows you to combine like terms.

9. What are the four steps of problem solving?

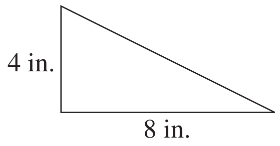
10. What would you do to check your answer after solving a problem?

How Can I Practice?

1. Determine the area and perimeter of a rectangle that is 13 centimeters wide and 25 centimeters long.

2. What is the perimeter of a triangle, in inches, that has sides that are 6 inches, 9 inches, and 1 foot long?

3. What is the area of the triangle shown?



4. You need to buy grass seed for a new lawn. The lawn will cover a rectangular plot that is 60 feet by 90 feet. Each ounce of grass seed will cover 120 square feet.

   a. How many ounces of grass seed will you need to buy?

   b. If the grass seed you want comes in 1-pound bags, how many bags will you need to buy?

5. Translate each statement into a formula. State what each letter in your fomula represents.

   a. The total weekly earnings equal \$12 per hour times the number of hours worked.

   b. The total time traveled equals the distance traveled divided by the average speed.

c. The distance between two cities, measured in feet, is equal to the distance measured in miles times 5280 feet per mile.

6. Use your formulas in Exercise 5 to answer the following questions. State your final answer in a complete sentence.

a. If you make \$12 per hour, what will your earnings be in a week when you work 35 hours?

b. If you earned \$336 last week (still assuming \$12 per hour), how many hours did you work?

c. If you travel 480 miles at an average speed of 40 miles per hour, for how many hours were you traveling?

d. The distance between Dallas and Houston is 244 miles. How far is the distance in feet?

7. What will the temperature be in degrees Fahrenheit when it is 40°C outside? Recall the formula $F = (9C \div 5) + 32$.

8. In the following formulas, which letter represents the input variable and which letter represents the output variable?

a. $w = 13x + 5$

b. $6(9 - 2x) = S$

9. Translate the following verbal rules into symbolic rules using x as the input and y as the output variable.

a. The output is five more than the input.

b. The output is 50 less than twice the input.

c. The output is three times the input squared.

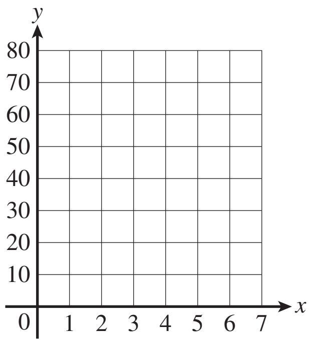
d. The output is the sum of ten and four times the input.

10. **a.** Complete the table for the formula $y = 2x^2 + x - 2$.

b. Which variable is the output variable?

x	*y*
1	
2	
3	
4	
5	
6	

c. Plot the points that represent the input/output pairs in the table. Label each point with its coordinates.



11. For each of the following expressions, combine like terms, if possible.

a. $8a + 5a + 10w - 6w$

b. $7w + 9w - 4w + 2w^2$

c. $5x + 7y + 3x - 4y + 9$

d. $8x^2 + 4y + 2x + 3y$

12. Solve the following equations for the unknown value.

a. $42x = 462$

b. $x + 33 = 214$

c. $518 = 14s$

d. $783 = 5y + 4y$

e. $6t - 5t - 14 = 90$

f. $413 = w - 46$

g. $39x - 24x = 765$

h. $340 = 2x + 3x$

13. Translate each statement into an equation. In each statement, let the unknown number be represented by x. Then solve the equation for the unknown number.

a. The product of some number and 30 is 1350.

b. A number minus 783 is 1401.

c. The sum of twice an unknown number and 5 times the unknown number is 56.

14. **a.** Determine a formula for the height of a tree, H, that is growing at the rate of 3 feet per year, where x is the number of years since planting.

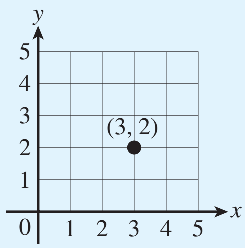
b. How tall is the tree after 10 years?

c. How long will it take the tree to grow to 75 feet?

15. At your town's community college, tuition is $105 per credit for less than 12 credits. Your aunt decided to register for classes on a part-time basis and paid a tuition bill of $945. Describe how you will determine from this information how many credits your aunt is taking.

Chapter 2 Summary

The bracketed numbers following each concept indicate the activity in which the concept is discussed.

<table>
<tr><th>CONCEPT/SKILL</th><th>DESCRIPTION</th><th>EXAMPLE</th></tr>
<tr><td>Variable [2.1]</td><td>A quantity that varies from one situation to another, usually represented by a letter.</td><td>P, l, and w can be shorthand notation for the variables perimeter, length, and width of a rectangle.</td></tr>
<tr><td>Formula [2.1]</td><td>An equation or symbolic rule that is a recipe for performing a specific calculation.</td><td>The formula for the perimeter of a rectangle, $P = 2w + 2l$, is a recipe for calculating the perimeter of any rectangle, once you know the length and width.</td></tr>
<tr><td>Variable expression [2.1, 2.2]</td><td>A symbolic code of instructions for performing arithmetic operations with variables and numbers.</td><td>The right side of the above formula, $2w + 2l$, is an algebraic expression.</td></tr>
<tr><td>Evaluating an expression [2.1, 2.2]</td><td>Replacing the variable in the expression with a numerical value and then performing all the arithmetic.</td><td>If $w = 10$ and $l = 15$, then $2w + 2l = 2(10) + 2(15)$ $= 20 + 30 = 50$.</td></tr>
<tr><td>Input variable [2.2]</td><td>In a formula, a variable found in the expression. Values for input variables are usually given first and used to determine output values.</td><td>In the formula $d = 25t$, t is typically the input variable.</td></tr>
<tr><td>Output variable [2.2]</td><td>In a formula, values that depend on values of the input variables. Values for an output variable are determined by evaluating expressions for given input values.</td><td>In the formula $d = 25t$, d is typically the output variable.</td></tr>
<tr><td>Table of values [2.2]</td><td>A listing of input variable values with their corresponding output variable values in the same row.</td><td><table><tr><th>t (input)</th><th>d (output)</th></tr><tr><td>2</td><td>50</td></tr><tr><td>4</td><td>100</td></tr></table></td></tr>
<tr><td>Rectangular coordinate system [2.2]</td><td>A grid scaled by two coordinate axes (number lines) drawn at right angles to each other. The horizontal axis corresponds to the input variable and the vertical axis corresponds to the output variable.</td><td>
</td></tr>
</table>

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Coordinates of a point [2.2]	Identify the location of an input/output ordered pair in a rectangular coordinate system. Written as (input, output).	See (3, 2) on the graph at the bottom of previous page.
Solution of an equation [2.3]	A value for the unknown quantity that makes the equation a true statement.	$x = 5$ is a solution to $3x = 15$ because $3 \cdot 5 = 15$ is true.
Fundamental principle of equality [2.3]	In general, performing the same operation on both sides of the equals sign in a true equation results in a true equation.	Adding 7 to both sides of $3 = 3$, you get $3 + 7 = 3 + 7$, which is $10 = 10$, still a true equation.
Solving equations of the form $a + x = b$ or $x + a = b$ [2.3]	To solve equations of the form $x + a = b$ or $a + x = b$, (where a and b are numbers and x is the unknown quantity), "undo" the addition by subtracting a from both sides of the equation.	If $x + 14 = 29$ is true, then subtracting 14 from both sides of the equation will result in a true equation. $$\begin{array}{rcr} x + 14 &=& 29 \\ -\ 14 && -14 \\ \hline x &=& 15 \end{array}$$
Solving equations of the form $x - a = b$ [2.3]	To solve equations of the form $x - a = b$ (where a and b are numbers and x is the unknown quantity), "undo" the subtraction by adding a to both sides of the equation.	If $x - 23 = 31$ is true, then adding 23 to both sides of the equation will result in a true equation. $$\begin{array}{rcr} x - 23 &=& 31 \\ +\ 23 && +23 \\ \hline x &=& 54 \end{array}$$
Solving equations of the form $ax = b$ [2.4]	To solve equations of the form $ax = b$ (where a and b are numbers, $a \neq 0$, and x is the unknown quantity), undo the multiplication by dividing both sides of the equation by a, the factor that multiplies the variable.	If $9x = 144$ is true, then dividing both sides of the equation by 9 will result in a true equation. $$\begin{aligned} 9x &= 144 \\ 9x \div 9 &= 144 \div 9 \\ x &= 16 \end{aligned}$$
Terms of an expression [2.5]	Parts of an expression that are separated by addition or subtraction signs are called **terms**.	The expression $45p + 75p$ has two terms, $45p$ and $75p$, which are separated by an addition sign.
Like terms [2.5]	Terms in an expression with exactly the same variable factor raised to the same power.	$3x$ and $8x$ are like terms. 5 and $5x$ are *not* like terms. $2x^2$ and $7x^2$ are like terms. $9x$ and $3x^2$ are *not* like terms. $3x$ and $5y$ are *not* like terms.
Coefficient [2.5]	The factor that multiplies the variable portion of a term.	3 is the coefficient of x in the term $3x$.

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Combining like terms [2.5]	Use the distributive property to add or subtract the coefficients of like terms to obtain a single term.	$3x + 8x = (3 + 8)x = 11x$ and $9x - 8x = (9 - 8)x = 1x = x$
The four steps of problem solving [2.6]	**Step 1.** Understand the problem.	Read the problem completely and carefully, draw a diagram, ask questions.
	Step 2. Develop a strategy for solving the problem.	Identify and list every quantity, known and unknown. Write an equation that relates the known quantities and the unknown quantity.
	Step 3. Execute your strategy to solve the problem.	Solve the equation.
	Step 4. Check your solution for correctness.	Is your answer reasonable? Is your answer correct?

Chapter 2 Gateway Review

1. Use the appropriate formulas for perimeter and area to answer the following questions.

a. The side of a square is 24 inches. What is its perimeter?

b. A rectangle is 41 centimeters long and 28 centimeters wide. What is its area?

c. The adjacent sides of a rectangle are 3 feet and 11 feet, respectively. What is the perimeter of the rectangle?

d. A square measures 33 centimeters on each side. What is the area of this square?

2. Recall the formula for calculating the temperature in degrees Fahrenheit if you know the temperature in degrees Celsius: $F = (9C \div 5) + 32$.

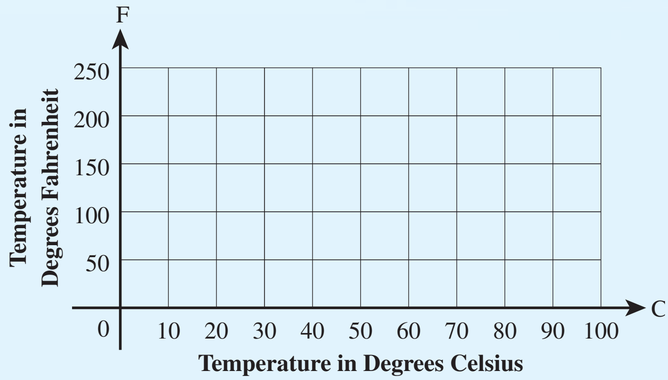
a. Which variable is the input variable?

b. Complete the following table of values.

TEMPERATURE (degrees Celsius)	TEMPERATURE (degrees Fahrenheit)
0	
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	

Answers to all Gateway exercises are included in the Selected Answers appendix.

c. Plot the points that represent the input/output pairs of your table. Label each point with its coordinates.

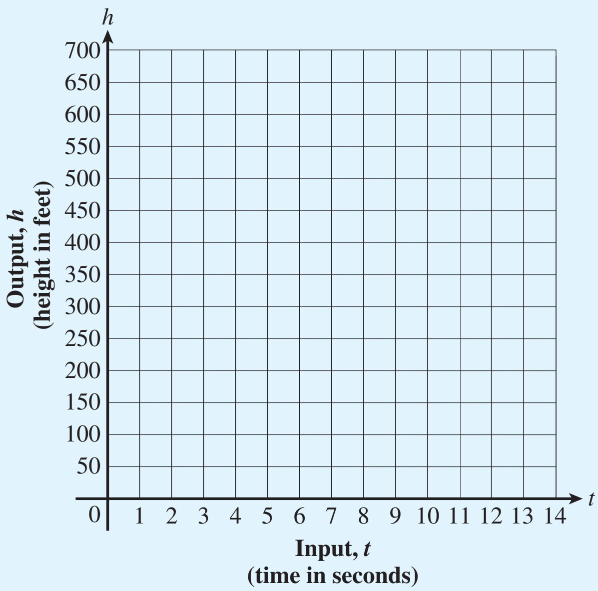


3. The formula $h = 200t - 16t^2$ gives the height of a ball that is tossed in the air with an initial velocity of 200 feet per second.

a. Complete a table of values for the following input values: $t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$, and 10 seconds.

t											
h											

b. Plot the points that represent the input/output pairs of your table and label each point with its coordinates. Then connect the points with a smooth curve.



c. When will the ball be 400 feet above the ground?

d. Approximately how high do you think the ball will get?

e. Approximately when will the ball hit the ground?

4. Solve each of the following equations for the unknown value.

a. $35 + x = 121$

b. $25y = 1375$

c. $603 = 591 + t$

d. $14x + 7x - 17x = 216$

e. $58 + x = 63$

f. $504 = 3w$

g. $91 = 3y + 12y - 8y$

h. $3w - 2w + 11 = 58$

5. You have a goal to buy a used car for \$2500. If you have saved \$240, \$420, and \$370 in the previous 3 months, how much more must you save to reach your goal? Write and solve an equation for the cost of the used car.

6. Your study group can review 20 pages of text in 1 hour. Use the relationship rate $\cdot$ time $=$ amount to write and solve an equation to answer the following question. How long will it take your group to review 260 pages of text?

7. You ordered 3 pizzas this week, 6 pizzas last week, and 5 pizzas the week before that. Assuming all these pizzas were the same price, how much did each pizza cost if your total bill for all the pizzas came to \$126?

8. Translate each statement into an equation, then solve. Let x represent the unknown number.

a. The sum of 351 and some number is 718.

b. 624 is the product of 48 and an unknown number.

c. The product of 27 and what number is equal to 405?

d. The difference between some number and 38 is 205.

e. The sum of an unknown number and twice itself is 273.

9. Solve the following problem by applying the four steps of problem solving. Use the strategy of solving an algebra equation.

Your average reading speed is 160 words per minute. There are approximately 800 words on each page of the textbook you need to read. Approximately how long will it take you to read 50 pages of your textbook?

Chapter 3

Problem Solving with Integers

Chapters 1 and 2 deal with whole numbers, which include zero and quantities greater than zero. Whole numbers greater than zero are known as counting numbers and also as *positive numbers*. However, quantities like debits in business or electrical charges in physics and chemistry require numbers that represent negative quantities. These numbers are called negative numbers. Positive and negative counting numbers and zero, taken together, form a number system called the *integers*. In Chapter 3, you will solve problems using integers.

Activity 3.1

On the Negative Side

Objectives

1. Recognize integers.
2. Represent quantities in real-world situations using integers.
3. Represent integers on the number line.
4. Compare integers.
5. Calculate absolute values of integers.

The Hindus introduced negative numbers to represent debts, and in 628 C.E. the great Indian mathematician and astronomer Brahmagupta introduced arithmetic rules for operating with positive and negative numbers. In his major text, *The Opening of the Universe,* he gave these rules in terms of fortunes and debts. He also recognized zero as a number and his arithmetic rules included operations involving zero. However, the history of mathematics shows that it took a long time for negative numbers to be accepted by everyone, including mathematicians, and that didn't happen until the 1600s.

1. Describe with an example how you can represent a temperature colder than 0°F.

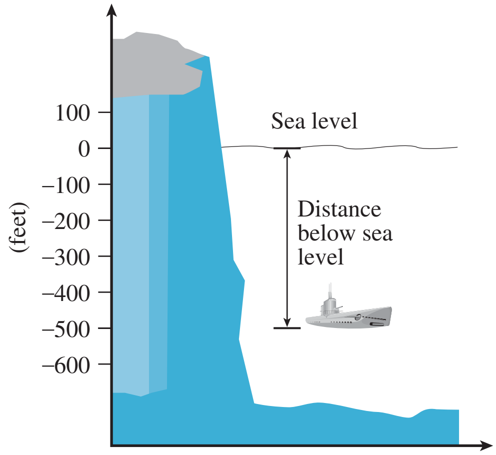
2. Can a number be used to represent a checking account balance less than $0? Explain.

3. In locating how far objects are above or below Earth's surface (for example, an airplane or a submarine), sea level is usually taken as the starting point.

   a. What number would you use to represent sea level?

   b. How would you represent an object's location above sea level (altitude)? Describe an example.

c. How would you represent an object's location below sea level (depth)? Describe an example.



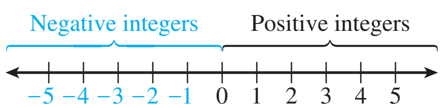
4. If you gain 10 pounds you can represent your change in weight by the number 10. What number represents a loss of 10 pounds?

As a whole number, zero represents the absence of quantity. Numbers greater than zero are positive; numbers less than zero are negative. Negative numbers are indicated by a dash, $-$, called a negative sign (or minus sign), to the left of the number, such as -20, -100, and -625. In a similar way, a positive number, for example positive three, can be written as $+3$. However, most of the time, a positive three is simply written as 3. This is true for positive numbers in general.

Integers and the Number Line

The collection of all of the counting numbers, zero, and the negatives of the counting numbers is called the set of **integers**: $\{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$. The positive counting numbers are called *positive integers* and the negative counting numbers are called *negative integers*. Note that the terms *counting numbers* and *positive whole numbers* mean the same collection of numbers.

A good technique for visualizing integers is to use a number line. A number line is a line with evenly spaced tick marks. A tick mark is a small line segment perpendicular to the number line. Each tick mark on a number line represents an integer. The most common number line is drawn horizontally, as shown here. Notice that the negative integers are to the left of zero and the positive integers are to the right of zero. The numbers increase as you read from left to right.

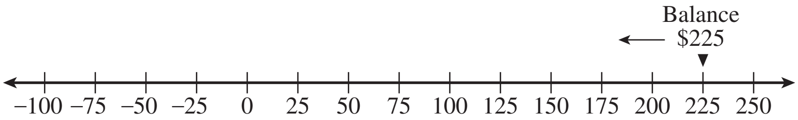


It is also common to represent a number line vertically, as in Problem 3. On a vertical number line, negative numbers are below zero and positive numbers are above zero. The numbers increase as you read from bottom to top.

5. a. Give an example of a positive integer, a negative integer, and an integer that is neither negative nor positive, and place these integers on a number line.

b. Describe a situation in which integers might be used. What would zero represent in the situation you describe?

6. You start the month with \$225 in your checking account with no hope of increasing the balance during the month. You place the balance on the following number line.



a. During the first week, you write checks totaling \$125. What is the new balance in your checking account after week 1? Place that balance on the preceding number line.

b. During week 2, your checks added up to \$85. Compute the amount left in your checking account after 2 weeks and place that amount on the number line.

c. During week 3, you wrote a check for \$15. What is the new balance in your checking account after 3 weeks? Place that amount on the number line.

d. During the last week in the month your car is towed and you must write a check for \$90 to get it back. What is the new balance in your checking account after 4 weeks? Place that amount on the number line.

Comparing Integers

Sometimes it is important to decide if one integer is greater than or less than another integer. The number line is one way to compare integers. Recall that on a number line as you move from *left to right* integers *increase* in value.

7. **a.** Which is warmer: $-10°F$ or $-16°F$? Use a number line to help explain your answer.

b. Which checking account balance would you prefer, $-\$60$ or $-\$100$? Explain.

c. Which is closer to sea level, a depth of 20 feet or 80 feet? Explain.

d. On a number line the larger number is to the ________ (right/left) of the smaller number.

Symbols for Comparing Integers

The symbol for "less than" is $<$. The statement $3 < 5$ is read as "3 is less than 5" or "3 is smaller than 5." The statement $-10 < -2$ is read as "-10 is less than -2," and the statement is true because -10 is to the left of -2 on the number line.

The symbol for "greater than" is $>$. The statement $6 > 4$ is read "6 is greater than 4" or "6 is larger than 4." The statement $-7 > -12$ is read "-7 is greater than -12," and the statement is true because -7 is to the right of -12 on the number line.

The symbols $<$ and $>$ are called **inequality symbols.**

8. Identify which of the following statements are true and which are false.

a. $-3 > -5$

b. $20 < -100$

c. $0 > -40$

d. $-30 < -50$

Absolute Value on the Number Line

The absolute value of a number is a useful idea in working with positive and negative numbers. You may have noticed that on the number line an integer has two parts: its distance from zero and its direction (to the right or left of 0). For example, the 7 in -7 represents the number of units (distance) that -7 is from 0 and the negative sign $(-)$ represents the direction of -7 to the left of 0. Note that $+7$ or 7 is also a distance of 7 units from 0, but to the right. The distance 7 is positive and is called the absolute value of the integers -7 and $+7$.

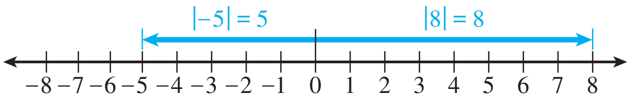
9. **a.** What is the increase in temperature (in °F) if the temperature moves from 0°F to 8°F? How many degrees does the temperature decrease if the temperature moves from 0°F to $-10°F$?

b. Which is further away from a \$0 balance in a checking account, \$150 or $-\$50$? Explain.

c. How far from sea level (0 feet altitude) is an altitude of 200 feet? How far from sea level (0 feet altitude) is an altitude of -20 feet?

On the number line, the **absolute value** of a number is represented by the distance the number is from zero. Since distance is always considered positive or zero, the absolute value of a number is *always* positive or zero.

For example, the absolute value of -5 is 5 since -5 is 5 units from zero on the number line. The absolute value of 8 is 8 since 8 is 8 units from zero on the number line.



Note that this distance from zero is always positive or zero whether the number is to the left or to the right of zero on the number line.

10. Determine the absolute value of each of the following integers.

a. 47

b. -30

c. -64

d. 56

To represent the absolute value of a number in symbols, enclose the number in vertical lines. For example, $|7|$ represents the absolute value of 7, which is 7; $|-32|$ represents the absolute value of -32, which is 32.

11. Determine the following.

a. $|-12|$

b. $|127|$

c. $|0|$

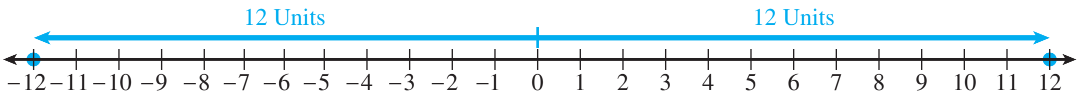
12. **a.** What number has the same absolute value as -12?

b. What number has the same absolute value as 45?

c. Is there a number whose absolute value is zero?

d. Can the absolute value of a number be negative?

Two numbers that are the same distance from zero but on opposite sides of zero are called **opposites**. For example, in Problem 12a and b, -12 and 12 are opposites and 45 and -45 are opposites. 0 is its own opposite.



13. a. What number is the opposite of 22?

b. What number is the opposite of -15?

c. What number is the opposite of $|-7|$?

SUMMARY: ACTIVITY 3.1

1. **Positive numbers** are numbers greater than zero and **negative numbers** are numbers that are less than zero. Zero is neither positive nor negative and separates the positive numbers from the negative numbers.

2. The set of **integers** includes all the whole numbers (the counting numbers and zero) and their opposites (the negatives of the counting numbers).

3. If $a < b$, then a is to the left of b on a number line. If $a > b$, then a is to the right of b on a number line.

4. The **absolute value** of a number a, written as $|a|$, is its value without its sign, so it is always positive or zero. On the number line, the absolute value of a number indicates the distance of the number from zero, which is always positive or zero.

5. Two numbers that are the same distance from zero on the number line but are on opposite sides of zero are called **opposites**. Zero is its own opposite. More generally speaking, two numbers that have the same absolute value but have different signs are called opposites.

EXERCISES: ACTIVITY 3.1

1. In France, the number zero is used in hotel elevators to represent the ground floor.

   a. Describe the floor numbered -2.

   b. If you are on the floor numbered -3, would you take the elevator up or down to get to the floor labeled -1?

2. Express the quantities in each of the following as an integer.

a. The Dow-Jones stock index lost 120 points today.

b. The scuba diver is 145 feet below the surface of the water.

c. You made a deposit of $50 into your checking account.

d. The Oakland Raiders lost 15 yards on a penalty.

e. You made a withdrawal of $75 from your checking account.

3. Write $<$ or $>$ between each of the following to make the statement true.

a. $7 \; \square \; 5$ b. $-5 \; \square \; 4$ c. $-10 \; \square \; -15$

d. $0 \; \square \; -2$ e. $-4 \; \square \; -3$ f. $-50 \; \square \; 0$

4. Determine the value of each of the following.

a. $|4|$ b. $|-13|$ c. $|32|$

d. $|-7|$ e. $|0|$

5. a. What number is the opposite of -6?

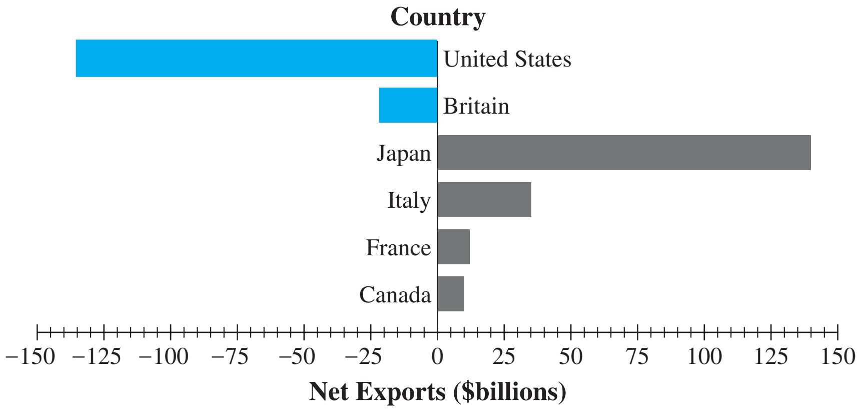
b. What number is the opposite of 8?

6. a. Which is warmer: -3°F or 0°F?

b. Which is farther below sea level: -20 feet or -100 feet?

7. Data on exports and imports by countries around the world are often shown by graphs for comparison purposes. The following graph is one example. The number line in the graph represents net exports in billions of dollars for six countries. Net exports are obtained by subtracting total imports from total exports; a negative net export means the country imported more goods than it exported.

The World of Exports



a. Estimate the net amount of exports for Japan. Is your answer a positive or negative integer? Explain what this number tells you about the imports and exports of Japan.

b. Estimate the net amount of exports for the United States. Is your answer a positive or negative integer? Explain what this number tells you about the imports and exports of the United States.

c. Determine the absolute value of the net amount of exports for Britain.

d. What is the opposite of the net amount of exports for Canada?

Activity 3.2

Maintaining Your Balance

Objectives

1. Add and subtract integers.
2. Identify properties of addition and subtraction of integers.

This activity guides you to learn the addition and subtraction of integers using concise and efficient rules. Through practice, you will make these rules your own.

Adding Integers

Suppose you use your checking account to manage all your expenses. You keep very accurate records. There are times when you must write a check for an amount that is greater than your balance. When you record this check, your new balance is represented by a negative number.

CHECK NUMBER	DATE	DESCRIPTION	SUBTRACTIONS (–)		ADDITIONS (+)		BALANCE	
							64	
	3/2/10	Paycheck			120			
381	3/4/10	Car repairs	167					
382	3/15/10	Clothes	47					

1. a. You deposit a paycheck for $120 from your part-time job. If the balance in your account before the deposit was $64, what is the new balance? Write the new balance in the check record.

   b. Determine the balance after making your car repair payment and record it in the check record.

2. a. After writing a check in the amount of $47 for clothing, you realize that your account is overdrawn by $30. Explain why.

   b. What integer would represent the balance at this point? Record that integer in the check record.

   c. Because you have a negative balance in your account, you decide to borrow $20 from a friend for expenses until you get an additional paycheck. What integer represents what you owe your friend?

   d. What is the total of your checking account balance and the money you owe your friend?

In Problem 1a, your balance and paycheck accumulate to a total of $184.

$$64 + 120 = 184$$

In Problem 2d, your negative balance and debt to your friend accumulate to $-\$50$.

$$-30 + (-20) = -50$$

These examples show that adding two positive integers results in a positive integer and adding two negative integers results in a negative integer. Note that in Problem 2d, you obtain the answer by adding 30 to 20 before attaching the negative sign to the result. This is the same as first adding the absolute values of -30 and -20.

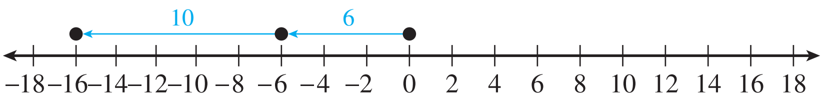
These observations lead to the following rule.

Rule 1: To add two numbers with the same sign, add the absolute values of the numbers. The sign of the sum is the same as the sign of the numbers being added.

3. Calculate the following.

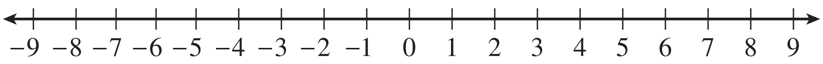
a. $-18 + (-26)$ **b.** $17 + 108$ **c.** $-48 + (-9)$

The addition of integers can be viewed in several ways. Consider the sum $-6 + (-10) = -16$. You can think of the sum as accumulating debts of \$6 and \$10, yielding a total debt of \$16, similar to Problem 2d. You can also think of the sum as moving on a number line. Starting at 0, move 6 units to the left and then another 10 units to the left, for a total distance of 16 units to the left of 0.



4. On the following number line, demonstrate the following calculations.

a. $-4 + (-3)$ **b.** $5 + 2$



5. a. You finally get paid. You cash the check for \$120 and give \$10 to your friend. Now, what do you owe your friend?

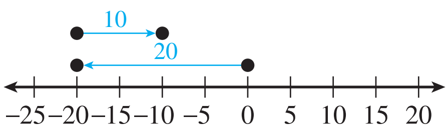
b. You deposit the remaining \$110 into your checking account. What is the new balance? Recall that you were overdrawn by \$30.

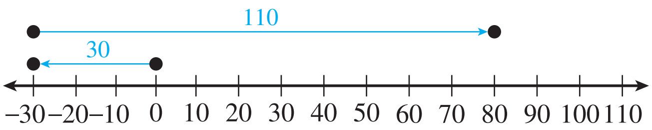
In Problem 5a and 5b, the integers being added have different signs.

Problem 5a: $-20 + 10 = -10$

Problem 5b: $-30 + 110 = 80$

The calculation $-20 + 10 = -10$ can also be viewed on a number line. Starting at 0, move left 20 units to the location -20 and then move right 10 units to the final location, -10. Similarly, the calculation $-30 + 110 = 80$ can be demonstrated by starting at 0, moving 30 units to the left, and then moving 110 units to the right to the final location, 80 units.





The number lines suggest that when you add two numbers with different signs, such as -30 and 110, the sign of the result will be the same as the sign of the number with the greater absolute value. Because 110 is larger in absolute value than -30, the result will be positive. To calculate the number part of the answer, ignore the signs of the numbers and subtract the smaller absolute value from the larger one. Therefore, $-30 + 110 = 110 - 30 = 80$.

6. Explain why the sum $-20 + 10$ is -10.

7. Calculate the following.

a. $40 + (-10)$ **b.** $-60 + 85$

c. $-50 + 30$ **d.** $25 + (-35)$

This discussion leads to the following rule.

Rule 2: To add two integers with different signs, determine their absolute values and then subtract the smaller from the larger. The sign of the result is the sign of the number with the larger absolute value.

8. Calculate the following.

a. $4 + 8$ **b.** $4 + (-8)$ **c.** $-3 + (-6)$

d. $10 + (-8)$ **e.** $(-7) + (4)$ **f.** $-3 + (+8)$

Subtracting Integers

The current balance in your checking account is \$195. You order merchandise for \$125 and record this amount as a debit. That is, you enter the amount as $-\$125$. The check is returned because the merchandise is no longer available. Adding the \$125 back to your checking account balance of \$70 brings the balance back to \$195.

$$70 + 125 = \$195$$

You also figure that if you subtract the debit from the \$70, you will get the same result. That means you reason that

$$70 - (-125) = 70 + 125 = \$195.$$

Of course, you prefer the simpler method of balancing your checkbook by adding the opposite of the debit, that is, adding the credit to your account. But subtracting the debit shows an important mathematical fact. The fact is that *subtracting a negative number is always equivalent to adding its opposite*. In the expression $70 - (-125)$, the term $-(-125)$ is replaced by $+125$ and $70 - (-125)$ becomes $70 + 125$.

9. Calculate the following.

a. $7 - (-3)$ **b.** $-8 - (-12)$ **c.** $1 - (-7)$ **d.** $-5 - (-2)$

10. A friend had \$150 in her checking account and wrote a check for \$120. She calculated her checkbook balance as $\$150 - (+\$120) = \$30$. You pointed out that she might as well have written $\$150 - \$120 = \$30$. Explain why.

11. Calculate the following.

a. $11 - (+7)$

b. $5 - (+8)$

c. $-7 - (+9)$

d. $-3 - (+2)$

The preceding discussion and problems lead to the following rule.

Rule 3: To subtract an integer b from another integer a, change the subtraction to the addition of integer b's opposite. Then follow either rule 1 or rule 2 for adding integers.

12. Perform the following subtractions and state what rule(s) you used in each case.

a. $-8 - 2$

b. $10 - (-3)$

c. $-4 - (-6)$

d. $5 - 15$

e. $-5 - (+8)$

f. $+2 - (+10)$

Procedure

The subtraction sign and the sign of the number being subtracted can be replaced by a single sign.

$$a - (+b) = a - b$$

$$a - (-b) = a + b$$

For example, $-6 - (+2) = -6 - 2 = -8$ and $9 - (-4) = 9 + 4 = 13$.

Adding or Subtracting More Than Two Integers

When adding or subtracting more than two numbers, add or subtract from left to right.

Example 1 *To compute* **4 + 6 − 3,** *first add* **4 + 6 = 10.** *Then subtract* **3** *from* **10** *to obtain the answer of* **7.**

Example 2 $-3 - 7 + 5 = -10 + 5 = -5$

13. Perform the following calculations. Verify with your calculator.

a. $-3 + 7 - 5$ **b.** $5 + 3 + 10$ **c.** $-2 - 4 + 5$

d. $-4 - 2 - 4$ **e.** $3 - 7 - 6$ **f.** $-6 + 7 + 3 - 5$

Properties of Addition and Subtraction of Integers

14. a. Add: $9 + (-4)$

b. Add: $-4 + 9$

c. What property of addition is illustrated by comparing parts a and b?

15. a. Subtract: $5 - (-2)$

b. Subtract: $-2 - (+5)$

c. Is the operation of subtraction commutative? Explain.

16. a. Add the integers $5, -3, -9$ by first adding 5 and -3, and then adding -9.

b. Add the integers given in part a: $5, -3, -9$, but this time first add -3 and -9, and then add 5 to the sum.

c. Does it matter which two integers you add together first?

d. What property is being demonstrated in part c?

SUMMARY: ACTIVITY 3.2

1. Rules for adding integers:

   To add two numbers with the same sign, add the absolute values of the numbers. The sign of the sum is the same as the sign of the numbers being added.

   For example,

$$7 + 9 = 16,$$
$$-6 + (-5) = -11.$$

   To add two integers with different signs, determine their absolute values and then subtract the smaller from the larger. The sign of the sum is the sign of the number with the larger absolute value.

   For example,

$$-5 + 9 = 4,$$
$$10 + (-13) = -3.$$

2. Rules for subtracting integers:

   To subtract an integer b from an integer a, change the subtraction to the addition of the opposite of b, then follow the rules for adding integers.

   For example,

$$5 - (-7) = 5 + 7 = 12,$$
$$-5 - (+3) = -5 + (-3) = -8.$$

3. Procedure for simplifying the subtraction of an integer from another integer:

   The subtraction sign and the sign of the number that follows it can be replaced by a single sign.

$$a - (+b) = a - b$$
$$a - (-b) = a + b$$

   For example,

$$-3 - (+6) = -3 - 6 = -9 \quad \text{and} \quad 7 - (-8) = 7 + 8 = 15.$$

4. Commutative property of addition:

   For all integers a and b, $a + b = b + a$.

   For example,

$$6 + (-9) = (-9) + 6, \quad \text{since } -3 = -3.$$

   Note: The commutative property is not true in general for subtraction.

   For example,

$$3 - 4 \neq 4 - 3, \quad \text{because } -1 \neq 1.$$

5. Associative property of addition:

For all integers a, b, and c, $(a + b) + c = a + (b + c)$.

For example,

$$(4 + 5) + (-3) = 4 + [5 + (-3)]$$
$$9 - 3 = 4 + 2$$
$$6 = 6.$$

Note: The associative property is not true in general for subtraction.

For example,

$$(7 - 5) - 1 \neq 7 - (5 - 1)$$
$$2 - 1 \neq 7 - 4$$
$$1 \neq 3.$$

EXERCISES: ACTIVITY 3.2

1. Perform the indicated operation.

a. $5 + (-7)$

b. $13 + (-17)$

c. $8 + (-8)$

d. $15 + (-6)$

e. $-2 + 9$

f. $-11 + 17$

g. $-20 + 4$

h. $-10 + (+2)$

i. $-5 + -1$

j. $-9 - 5$

k. $-12 + (-12)$

l. $-6 + (-17)$

m. $-14 - (-15)$

n. $-16 - (-12)$

o. $-21 - (-18)$

2. Perform the indicated operations.

a. $3 + (-4) - 10$

b. $-5 - 6 + 3$

c. $-4 - 5 - 3$

d. $-1 - 2 - 3$

e. $4 + (-6) + 6$

f. $9 + 8 + (-3)$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

g. $1 - (-1) - 1$

h. $7 - (-3) - 7$

i. $-4 - (-5) + (-6)$

j. $-2 - (-7) + (-5)$

3. The temperature increased by 8°F from -10°F. What is the new temperature?

4. The temperature was -7°F in the afternoon. That night, the temperature dropped by 5°F. What was the nighttime temperature?

5. The temperature increased 4°F from -1°F. What is the new temperature?

6. The temperature was 6°F in the morning. By noon, the temperature rose by 5°F. What was the noon temperature?

7. You began a diet on November 1 and lost 5 pounds. Then you gained 2 pounds over the Thanksgiving break. By how much did your weight change in November?

8. While descending into a valley in a hot-air balloon, your elevation decreased by 500 feet from an elevation of -600 feet (600 feet below sea level). What is your new elevation?

9. **a.** You are a first-year lifeguard at Rockaway Beach and earn \$603 per week. On August 28 you deposited a week's pay into your checking account. The balance of your account before the deposit was \$39. What is the new balance?

b. Your \$660 monthly rent is due on September 1. So on September 1 you write and record a \$660 check. What is the new balance?

c. On September 4 you deposit your \$603 paycheck. What is the new balance?

d. You write a \$600 check on September 5 for your first installment on your college bill for the fall term. What is your new balance?

e. You write a check for groceries on September 10 in the amount of \$78. What is your new balance?

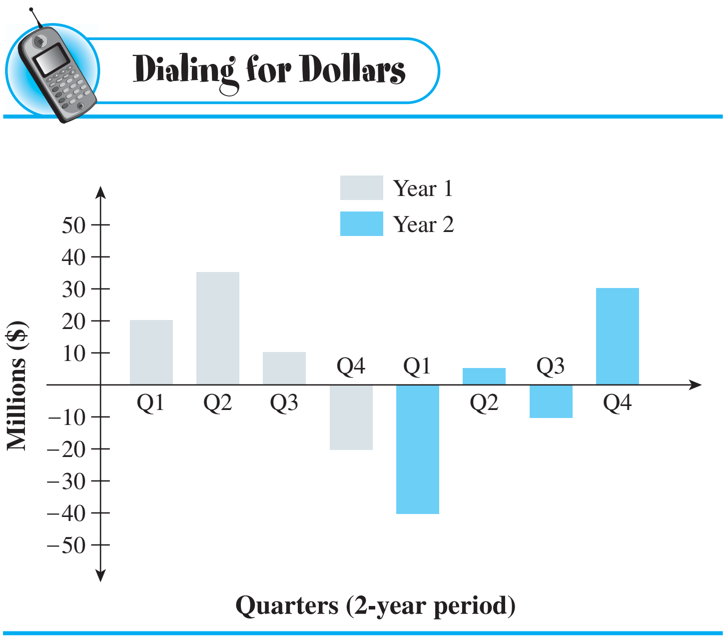
f. You deposit your last lifeguard paycheck on September 11. What is your new balance?

10. The highest point in Asia is at the top of Mount Everest, which is 8850 meters above sea level. The lowest point in Asia is the Dead Sea at 408 meters below sea level.

a. Write the elevation level of the Dead Sea, including the appropriate sign.

b. What is the difference between the highest and the lowest points in Asia?

11. Profits and losses in millions of dollars per quarter over a 2-year period for a telecommunications company are shown in the bar graph. Determine the total profit or loss for the company over the 2-year period.



Activity 3.3

What's the Bottom Line?

Objectives

1. Write formulas from verbal statements.
2. Evaluate expressions in formulas.
3. Solve equations of the form $x + b = c$ and $b - x = c$.
4. Solve formulas for a given variable.

You run a retail shop in a tourist town. To determine the selling price of an item, you add the profit you want to what the item cost you. For example, if you want a profit of \$4 on a decorated coffee mug that cost you \$3, you would sell the mug for \$7.

1. a. Write a formula for the selling price of an item in terms of your cost for the item and the profit you want. In your formula, let C represent your cost; P, the profit on the item; and S, its selling price.

   b. You make a profit of \$8 on a wall plaque that costs you \$12. Use your formula from part a to determine the selling price.

2. A particular style of picture frame has not sold well and you need space in the store to make room for new merchandise. The frames cost you \$14 each and you decide to sell them for \$2 less than your cost.

   a. Write the value of your profit, P, as an integer.

   b. Substitute the values for C and P in the equation $S = C + P$ and determine S, the selling price.

Problems 1 and 2 demonstrate the general method of determining an output from inputs that was shown in Chapter 2. In the equation $S = C + P$, the inputs are C and P in the expression on the right side of the equation, and S is the output variable. Replacing C and P with their values in the expression and evaluating their sum determines the value of S.

3. A wristwatch costs you \$35 and you sell it for \$60. Substitute these values of C and S in the equation $C + P = S$ and solve the equation for P, the profit.

In Problem 3, the unknown value, P, is part of the expression on the left side of the equation. The output value of the equation, S, is known. As shown in Chapter 2, determining the unknown input value in the equation is called *solving the equation*.

4. You make a profit of \$11 on a ring you sell for \$25. Replace P and S with these values in the equation $C + P = S$ and solve for C, the cost.

5. You sell a large candle for $16 that cost you $19.

a. Use the formula $C + P = S$ to determine your profit or loss.

b. Comment on your profit for this sale.

To solve the equations in Problems 3–5, you subtracted a given number from each side of the equation to isolate the unknown variable. This is the same as adding the opposite of the given number to each side.

6. Substitute the given values into the equation $c = a + b$ and then determine the value of the remaining unknown variable.

a. $a = 5, b = -17$

b. $a = 12, c = -30$

c. $b = -4, c = -9$

d. $b = 7, a = -23$

e. $c = 49, a = -81$

f. $a = -34, b = -55$

7. Substitute the given values into the equation $c = a - b$ and then determine the value of the remaining unknown variable.

a. $a = 12, b = -19$

b. $b = 7, c = -42$

c. $c = 36, b = 40$

d. $a = -17, b = 29$

Solving Formulas for a Given Variable

The process for solving formulas for a given variable is the same as the process for solving an equation. For example, to solve the formula $S = C + P$ for P, subtract C from both sides of the equation and simplify as demonstrated in Example 1.

Example 1

$$S = C + P$$
$$S - C = C - C + P$$
$$S - C = 0 + P$$
$$S - C = P$$
$$P = S - C$$

8. The formula $r_1 + r_2 = r$ is used in electronics. Solve for r_1.

9. The formula $p + q = 1$ is used in the study of probability. Solve for q.

10. The formula $P = R - C$ is used in business, where P represents profit; R, revenue or money received; and C, cost of doing business. Solve for revenue, R.

Look again at the formula in Problem 10. What if you need a formula for cost based on profit and revenue? You will need to get C alone on one side of the equation. The process of solving the formula $P = R - C$ is the same as the process for solving an equation such as $7 - x = 10$, as shown in Example 2.

Example 2 ***Solve* $7 - x = 10$ *for* x.**

SOLUTION

$$7 - x = 10$$
$$7 - x + x = 10 + x$$

Since x is subtracted from 7, undo the subtraction by performing the opposite operation, that is, add x *to both sides.*

$$7 = 10 + x$$
$$-3 = x$$
$$x = -3$$

Example 3 ***Solve $y = b - x$ for x. (The goal is to get x alone on one side of the equation.)***

SOLUTION

Since x is subtracted from b, undo that subtraction by performing the opposite operation: Add x to both sides.

$$y = b - x$$
$$y + x = b - x + x$$
$$y + x = b$$
$$-y + y + x = -y + b$$
$$x = -y + b \quad \text{or} \quad x = b - y$$

11. Solve $P = R - C$ for C. (*Hint:* Add C to each side of the equation, simplify, and continue to solve, as demonstrated in Example 3.)

12. In retail sales, the relationship between an item's cost, C, the amount of the markup in price, M, and the selling price, S, is $M = S - C$. Suppose the selling price of an item is $21.

a. Write a formula for the markup on the item.

b. Solve the formula for the cost, C.

SUMMARY: ACTIVITY 3.3

1. To solve for x in formulas or equations of the forms $x + b = c$ or $b + x = c$, add the opposite of b to both sides of the equation to obtain $x = c - b$.

2. To solve the equation $b - x = c$ for x, add x to both sides of the equation to obtain $b = c + x$. Then subtract c from each side to obtain $b - c = x$.

EXERCISES: ACTIVITY 3.3

1. When you began your diet, you weighed 154 pounds. After 2 weeks, you weighed 149 pounds. You can express the relationship between your initial weight, final weight, and the change between them by the equation

$$154 + x = 149,$$

where x represents your change in weight in the 2 weeks.

a. Solve the equation for x.

b. Was your change in weight a loss or a gain? Explain.

2. a. Another way to write an equation expressing your change in weight during the first 2 weeks of your diet is to subtract the initial weight, 154, from your final weight, 149. Then the equation is $x = 149 - 154$, where x represents the change in weight. Determine the value of x.

b. Write a formula to calculate the amount a quantity changes when you know the final value and the initial value. Use the words *final value*, *initial value*, and *change in quantity* to represent the variables in the formula. (*Hint:* Use the equation from part a as a guide.)

3. Suppose you lost 7 pounds during August. At the end of that month, you weighed 139 pounds. Let I represent your initial weight at the beginning of August.

a. Write an equation representing this situation. (*Hint:* Use your formula from Problem 2b as a guide.)

b. Solve this equation for I to determine your weight at the beginning of the month.

4. a. The following table gives information about your diet for a 6-week period. For each week, write an equation to solve for the unknown amount using the variables indicated. Enter the equations into the table.

Exercise numbers appearing in color are answered in the Selected Answers appendix.

Losing Proposition

Week	1	2	3	4	5	6
Initial Weight, I	154			150	146	144
Ending Weight, E		149	150	146		
Weight Change, C	-3	-2	1		-2	-5
Equation						

b. Solve each equation in part a and complete the table. State whether you are evaluating an expression or solving the equation.

5. The sale price, S, of an item is the difference between the regular price, P, and the discount, D.

a. Write a formula for S.

b. If a video game is discounted \$8 and sells for \$35, use the formula in part a to determine the regular price.

6. You write a check for \$57 to pay for a laboratory manual that you need for chemistry class.

   a. If the new balance in your checking account is \$314, write an equation to determine the original balance.

   b. Solve the equation to determine the original balance.

7. Solve each of the following formulas for the given variable.

   a. $K = C + 273$, for C

   b. $S = P - D$, for D

   c. $A = B - C$, for B

   d. $C + M = S$, for M.

8. In each of the following, let x represent an integer. Then translate the given verbal expression into an equation and solve for x.

   a. The sum of an integer and 10 is -12.

   b. 9 less than an integer is 16.

   c. The result of subtracting 17 from an integer is -8.

   d. If an integer is subtracted from 10, the result is 6.

Activity 3.4

Riding in the Wind

Objectives

1. Translate verbal rules into equations.
2. Determine an equation from a table of values.
3. Use a rectangular coordinate system to represent an equation graphically.

Windchill

Bicycling is enjoyable in New York State all year around, but it can get very cold in the winter. If there is a wind, it feels even colder than the actual temperature. This effect is called the windchill temperature, or windchill for short. The following table gives the actual temperature, T, and the windchill, W, for a 10-mile-per-hour wind. Both are given in degrees Fahrenheit.

Blowin' in the Wind

Actual Temperature, T (°F)	-15	-10	-5	0	5	10
Windchill, W, for a 10 mph Wind (°F)	-20	-15	-10	-5	0	5

1. a. Write a verbal rule that describes how to determine the windchill for a 10-mile-per-hour wind if you know the actual temperature.

   b. Translate the verbal rule in part a into an equation where T represents the actual temperature and W represents windchill.

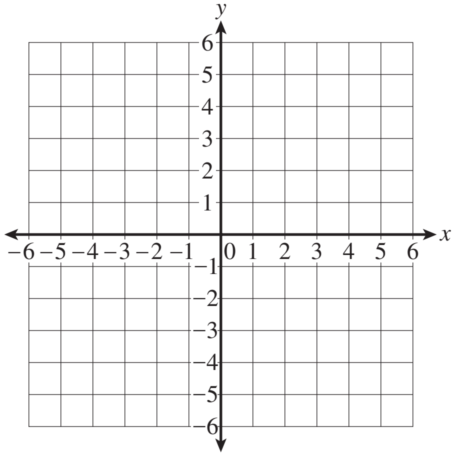
   c. Use the equation in part b to determine the windchill, W, for a temperature of -3°F.

   d. Use the equation in part b to determine the actual temperature, T, for a windchill of -17°F.

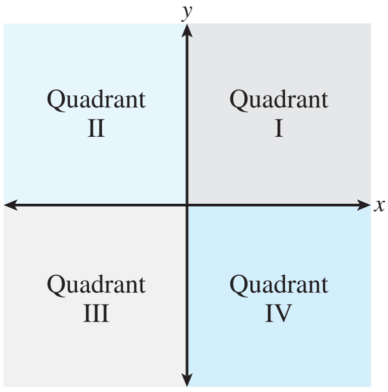
Rectangular Coordinate System Revisited

To obtain a graphical view of the windchill data, you can plot the values in the table above on a rectangular coordinate grid. Notice that the table contains both positive and negative values for the actual temperature and the windchill.

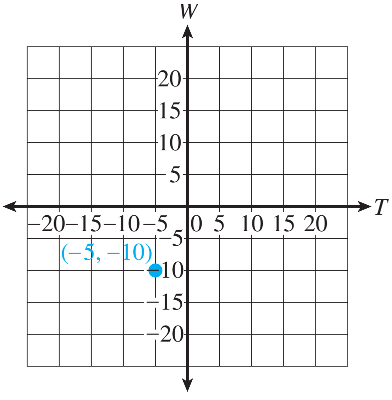
Until now in this textbook, the horizontal (input) axis and the vertical (output) axis contained only nonnegative values. To plot the windchill data, you need to extend each axis in the negative direction.



Now the grid has two complete number lines (axes) that intersect at a right $(90°)$ angle at the **origin**, $(0, 0)$. In this rectangular coordinate system, the two axes together divide the plane into four parts called **quadrants**. The quadrants are numbered as follows.



To locate the point with coordinates $(-5, -10)$, start at the origin $(0, 0)$ and move 5 units left on the horizontal axis. Then, move 10 units down, parallel to the vertical axis. The point $(-5, -10)$ is located in quadrant III.



2. a. Plot the remaining values from the windchill table on page 167 on the preceding grid.

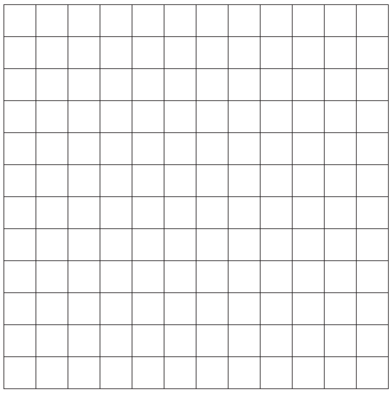
   b. The points you graphed were determined by the equation $W = T - 5$. The points and their pattern are a graphical representation of the equation. What pattern do the graphed points suggest? Connect the points on the graph in the pattern you see.

3. Scale and label the following grid and plot the following points.

   a. $(2, -5)$ b. $(-1, 3)$

   c. $(4, 2)$ d. $(0, -4)$

   e. $(5, 0)$ f. $(-2, -6)$



4. Without plotting the points, identify the quadrant in which each of the following points is located.

   a. $(-12, 24)$ b. $(35, -26)$

   c. $(56, 48)$ d. $(-28, -34)$

5. Identify the quadrant of each point whose coordinates have the given signs.

   a. $(-, +)$ b. $(-, -)$

   c. $(+, +)$ d. $(+, -)$

The Bicycle Shop

Your local sporting goods store sells a wide variety of bicycles priced from \$80 to \$500. The store sells bikes assembled or unassembled. The charge for assembly is \$20 regardless of the price of the bike.

6. a. What is the cost of a \$100 bicycle with assembly?

b. What is the cost of a \$250 bicycle with assembly?

7. a. Write a verbal rule that describes how to determine the cost of a bicycle with assembly.

b. Translate the verbal rule in part a into an equation where A represents the cost of the bicycle with assembly and U represents the price of the unassembled bicycle.

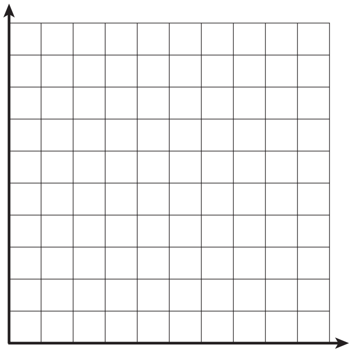
c. Use the equation in part b to determine the price, A, if the price of the bicycle is \$160 without assembly.

d. Use the equation in part b to determine the price, U, of the unassembled bicycle if it costs \$310 assembled.

8. Complete the following table.

PRICE WITHOUT ASSEMBLY, U, (\$)	COST WITH ASSEMBLY, A, (\$)
80	
	140
240	
	320
360	
460	480

9. a. Plot the values from the table in Problem 8 on an appropriately labeled and scaled coordinate system. Let U represent the input values on the horizontal axis. Let A represent the output values on the vertical axis.

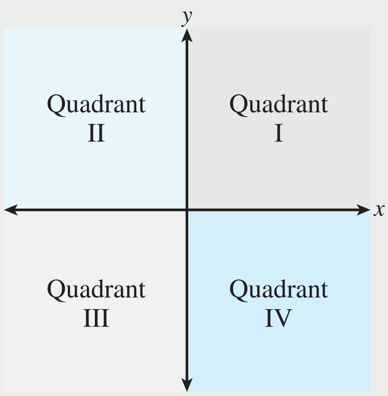


b. Connect the points on the graph to obtain a graphical representation of the equation in Problem 7b.

SUMMARY: ACTIVITY 3.4

Points on a rectangular coordinate system.

1. A point on a rectangular coordinate grid is written as an ordered pair in the form (x, y), where x is the input (horizontal axis) and y is the output (vertical axis).
2. Ordered pairs are plotted as points on a rectangular grid that is divided into four quadrants by a horizontal (input) axis and a vertical (output) axis.



3. The signs of the coordinates of a point determine the quadrant in which the point lies.

x-COORDINATE	y-COORDINATE	QUADRANT
+	+	I
−	+	II
−	−	III
+	−	IV

EXERCISES: ACTIVITY 3.4

1. You make and sell birdhouses to earn some extra money. The following table lists the cost of materials and the total cost, both in dollars, for making the birdhouses.

Out on a Limb

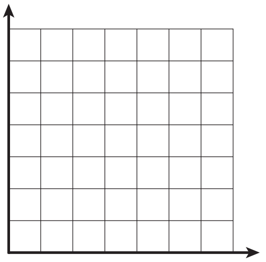
Number of Birdhouses	1	2	3	4
Material Cost ($)	6	12	18	24
Total Cost ($)	8	14	20	26

a. Write a verbal rule that describes how to determine the total cost if you know the cost of the materials.

b. Translate the verbal rule in part a into an equation where T represents the total cost and M represents the cost of the materials.

c. Use the equation in part b to determine the total cost if materials cost $36.

d. Plot the values from the table above on an appropriately labeled and scaled rectangular coordinate system.



e. Connect the points on the graph to obtain a graphical representation of the equation in part d.

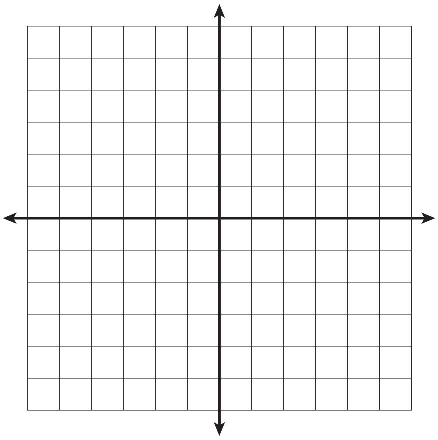
2. The sum of two integers is 3.

a. Translate this verbal rule into an equation where x represents one integer and y the other.

b. Use the equation in part a to complete the following table.

x	y
4	
	−3
−3	
	4
0	
	1

c. Plot the values from the table in part b on an appropriately labeled and scaled coordinate system.



d. Connect the points on the graph to obtain a graphical representation of the equation in part a.

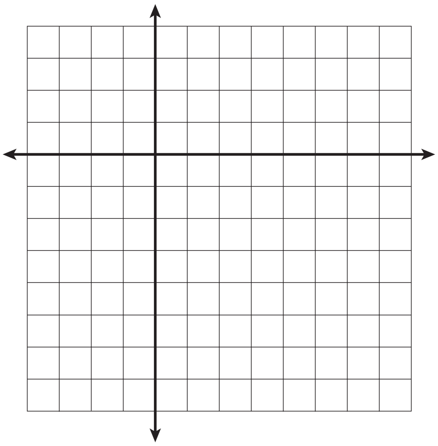
3. The difference of two integers is 5.

a. Translate this verbal rule into an equation where x represents the larger integer and y the smaller.

b. Use the equation in part a to complete the following table.

x	y
5	
	−6
−3	
	1
0	
	−1

c. Plot the values from the table in part b on an appropriately labeled and scaled coordinate system.



d. Connect the points on the graph to obtain graphical representation of the equation in part a.

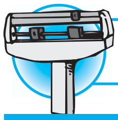
Activity 3.5

Are You Physically Fit?

Objectives

1. Multiply and divide integers.
2. Perform calculations that involve a sequence of operations.
3. Apply exponents to integers.
4. Identify properties of calculations that involve multiplication and division with zero.

As a member of a health-and-fitness club, you have a special diet and exercise program developed by the club's registered dietitian and your personal trainer. Your weight gain (positive value) or weight loss (negative value) over the first 6 weeks of the program is recorded in the following table.



Weight and See

Number of Weeks	1	2	3	4	5	6
Change in Weight (lb.)	-3	-3	4	-3	-3	4

1. a. Counting only those weeks in which you gained weight, what was your weight gain during the 6-week period? Write your answer as an integer and in words.

   b. Counting only those weeks in which you lost weight, what was your weight loss during the 6-week period? Write your answer as an integer and in words.

   c. Explain how you calculated the answers to parts a and b.

   d. At the end of the first six weeks, what is the total change in your weight?

There are two ways to determine the answers to parts a and b of Problem 1. One way to determine the increase in weight in part a is by repeated addition:

$$4 + 4 = 8 \text{ lb.}$$

A second way uses the fact that multiplication is repeated addition. So, the increase in weight is also determined by the product:

$$2(4) = 8 \text{ lb.}$$

Similarly in part b, the weight loss is determined by repeated addition:

$$-3 + (-3) + (-3) + (-3) = -12.$$

Again similar to part a, multiplication can be used to determine the weight loss:

$$4(-3) = -12.$$

Because positive integers are also whole numbers, the commutative property of multiplication is also true for positive integers. Therefore, $4(2) = 2(4) = 8$. The **commutative property of multiplication** is also true for the product of a negative integer and a positive integer. The product $4(-3)$ may be interpreted as adding four instances of -3 (in the negative direction), resulting in a negative product. Therefore, $4(-3) = -3(4) = -12$.

2. Multiply each of the following; then check by using your calculator.

   a. $5(-2)$ b. $(-2)(5)$ c. $7(-8)$

   d. $-8(7)$ e. $6(-4)$ f. $-4(6)$

The integers 1 and -1 play major roles in multiplication and division. The integer 1 has the property that every number can be written as the product of itself and 1. For example, the number 5 can be written as $5 \cdot 1$; the number 23 can be written as $23 \cdot 1$.

3. Write each of the following integers as a product of itself and 1.

a. 7 **b.** -5 **c.** 0

Multiplication of any number by -1 always produces the opposite number. For example, $-1 \cdot 3 = -3$, the opposite of 3, $-1 \cdot -7 = 7$, the opposite of -7, and $-1 \cdot -1 = 1$. The integer -1 is also a building block of all other negative numbers. Every negative number can be written as the product of its opposite (a positive number) and -1. For example, $-8 = (-1) \cdot 8$ and $-101 = (-1) \cdot 101$.

4. Calculate the following products.

a. $-1 \cdot 9$ **b.** $-1(11)$ **c.** $-1 \cdot 25$ **d.** $-1(25)$ **e.** $25(-1)$

5. Calculate the following products.

a. $(-1) \cdot (-9)$ **b.** $(-1)(-11)$ **c.** $-1 \cdot (-4)$

d. $(-6)(-1)$ **e.** $-1(-2)$

Multiplication of integers is a **binary operation**; that is, the operation is done with two integers at a time. For example, $-1 \cdot 3 \cdot 4 = -3 \cdot 4 = -12$. Note that you can also calculate the product in the following order, $-1 \cdot 12 = -12$, demonstrating that the **associative property of multiplication** is true for integers. These properties work for any number of factors. For example, $-1 \cdot 2 \cdot 3 \cdot 4$ is equal to $-2 \cdot 12 = -24$ or to $-1 \cdot 6 \cdot 4 = -1 \cdot 24 = -24$.

6. Calculate the following products, showing steps.

a. $-1 \cdot (-1) \cdot 4$

b. $-2 \cdot (-1) \cdot (-1) \cdot (-3)$

c. $5 \cdot (-1) \cdot (-2) \cdot (-3)$

In Problem 6b, the product is 6. If you calculate the product by first multiplying the middle two integers, you obtain $-2 \cdot 1 \cdot (-3) = -2 \cdot (-3)$. The answer is 6, so $-2 \cdot (-3)$ must be 6. This example demonstrates that the product of two negative integers is positive.

7. In each of the following, multiply the two integers. Then check your answer using your calculator.

a. $(-2)(-4)$ **b.** $-6(-7)$ **c.** $(-1)(4)$

d. $5(-8)$ **e.** $(-3)(-9)$ **f.** $(-12)(3)$

Product of Two Integers

Rule 1: The product of two integers with the same sign is positive.

$$(+)(+) = (+) \rightarrow (4)(5) \quad = 20$$
$$(-)(-) = (+) \rightarrow (-4)(-5) = 20$$

Rule 2: The product of two integers with opposite signs is negative.

$$(-)(+) = (-) \rightarrow (-4)(5) = -20$$
$$(+)(-) = (-) \rightarrow (4)(-5) = -20$$

Division of Integers

Your friend gained 15 pounds over a 5-week period. If his weight gain was the same each week, then the calculation $15 \div 5 = 3$ or $\frac{15}{5} = 3$ shows that he gained 3 pounds each week. You can check that a gain of 3 pounds per week is correct by multiplying 5 by 3 to obtain 15. In other words, the quotient $\frac{15}{5}$ is 3 because $5 \cdot 3 = 15$ by a multiplication check.

Another friend lost 15 pounds over the same 5-week period, losing the same amount each week. The calculation $-15 \div 5 = -3$ or $\frac{-15}{5} = -3$ shows that she lost 3 pounds each week. The quotient $\frac{-15}{5}$ is -3. The product $5(-3) = -15$ shows the quotient $\frac{-15}{3}$ has to be -3.

8. Evaluate each of the following. Use the multiplication check to verify your answers.

a. $27 \div 9$ **b.** $-32 \div 8$ **c.** $18 \div (-2)$

d. $\frac{-10}{5}$ **e.** $\frac{21}{-7}$ **f.** $\frac{38}{2}$

9. **a.** Use the multiplication check for division to obtain the only reasonable answer for $(-22) \div (-11)$.

b. What rule does part a suggest for dividing a negative integer by a negative integer?

c. Evaluate each of the following and verify your answer by the multiplication check.

i. $(-42) \div (-7)$ **ii.** $\frac{-9}{-3}$ **iii.** $(-7) \div (-1)$

10. **a.** The sign of the quotient of two integers with the same sign is ______________.

b. The sign of the quotient of two integers with opposite signs is ______________.

Quotient of Two Integers

Rule 1: The quotient of two integers with the same sign is positive.

$$\frac{(+)}{(+)} = (+) \longrightarrow \frac{12}{4} = 3$$

$$\frac{(-)}{(-)} = (+) \longrightarrow \frac{-12}{-4} = 3$$

Rule 2: The quotient of two integers with opposite signs is negative.

$$\frac{(-)}{(+)} = (-) \longrightarrow \frac{-12}{4} = -3$$

$$\frac{(+)}{(-)} = (-) \longrightarrow \frac{12}{-4} = -3$$

Calculations Involving a Combination of Operations

11. A friend joins you on your health-and-fitness program. The following table lists his weight loss and gain during a 5-week period.

Week	1	2	3	4	5
Weight Change (lb.)	−4	−3	2	−3	−2

Your friend computes his average weight change by adding the weekly changes in weight and then dividing the result by the number of weeks. You volunteer to check his results.

a. What is his change in weight for the 5 weeks?

b. What is your friend's average weight change for the 5-week period?

c. Did your friend have an average gain or an average loss of weight during the 5-week period? Explain.

12. The following table gives weight gains and losses for 16 persons during a given week.

NUMBER OF PERSONS	WEIGHT CHANGE (lb.)
2	−5
3	−4
4	−3
2	−2
1	0
3	1
1	3

a. What is the weight change of the group during the given week?

b. What is the average weight change for the group of 16 persons?

c. Does this average change represent a weight gain or loss?

13. a. What is the sign of the product $(-3)(-5)(-7)(-2)$? Explain.

b. What is the sign of the product of $(-1)(-1)(-1)(-1)(-1)(-1)(-1)$? Explain.

c. If you multiplied 16 factors of -1, what would be the sign of the product?

d. What can you conclude about the sign of a product with an odd number of factors with negative signs?

e. What can you conclude about the sign of a product with an even number of factors with negative signs?

14. Perform the indicated operations. Use a multiplication or division check to verify your answers.

a. $-5 \cdot (-4)$

b. $-50 \div (-10)$

c. $2 \cdot 6$

d. $6 \div 3$

e. $-3(-6)(-2)$

f. $5(-3)(-4)$

g. $-15 \div -3$

h. $2(-3)(5)(-1)$

i. $(-2)(6)$

j. $60 \div (-5)$

k. $(-1)(-1)(-1)(-1)(-1)(-1)$

l. $(-2)(-3)(-4)(5)(-6)$

Exponents and Negative Integers

The product $(-3)(-3)$ can be written as $(-3)^2$. The integer -3 is the factor that appears two times in the product and is called the **base**. The integer 2 represents the number of factors of -3 in the product and is called the **exponent**.

15. a. Evaluate $(-3)(-3)$.

b. Evaluate $(-3)^2$ using the exponent key on your calculator. Be sure to key in the parentheses when you do the calculation.

c. Suppose, when using your calculator, you omit the parentheses in $(-3)^2$ and instead you calculate the expression -3^2. Does -3^2 have the same value as $(-3)^2$?

You should think of -3^2 as the opposite of 3^2. Therefore,

$$-3^2 = -(3^2) = -(3)(3) = -9.$$

16. Evaluate the following expressions by hand. Use your calculator to check the results.

a. -5^2 **b.** $(-5)^2$ **c.** $(-3)^3$

d. -1^4 **e.** $2 - 4^2$ **f.** $(2 - 4)^2$

g. $-5^2 - (-5)^2$ **h.** $(-1)^8$ **i.** $-5^2 + (-5)^2$

Multiplication and Division Involving Zero

17. a. Recall that multiplication is repeated addition. For example, $4 \cdot 3 = 3 + 3 + 3 + 3$. Use this fact to write $6 \cdot 0$ as an addition problem.

b. Compute the answer in part a.

c. What is 0 times any integer? Explain.

18. a. Recall the multiplication check for division. For example, $8 \div 2 = 4$ because $2 \cdot 4 = 8$. Use the multiplication check to compute the answer to the division problem $0 \div 7$.

b. What is 0 divided by any nonzero integer? Explain.

c. Use the multiplication check to explain why $7 \div 0$ has no answer.

d. Can any integer be divided by 0? Explain.

19. Evaluate each of the following, if possible. If not possible, explain why not.

a. $0 \cdot (-8)$ **b.** $12 \cdot 0$ **c.** $0 \div 4$

d. $-6 \div 0$ **e.** $0 \div (-6)$ **f.** $0 \div 0$

When multiplying any integer by 0, the answer is 0. When dividing 0 by any nonzero integer, the answer is 0. No integer can be divided by 0.

SUMMARY: ACTIVITY 3.5

1. Product of Two Integers

Rule 1: The product of two integers with the same sign is positive.

$$(+)(+) = (+) \rightarrow \quad (4)(5) = 20$$
$$(-)(-) = (+) \rightarrow (-4)(-5) = 20$$

Rule 2: The product of two integers with opposite signs is negative.

$$(-)(+) = (-) \rightarrow (-4)(5) = -20$$
$$(+)(-) = (-) \rightarrow (4)(-5) = -20$$

2. Quotient of Two Integers

Rule 1: The quotient of two integers with the same sign is positive.

$$\frac{(+)}{(+)} = (+) \rightarrow \frac{12}{4} = 3$$
$$\frac{(-)}{(-)} = (+) \rightarrow \frac{-12}{-4} = 3$$

Rule 2: The quotient of two integers with opposite signs is negative.

$$\frac{(-)}{(+)} = (-) \rightarrow \frac{-12}{4} = -3$$
$$\frac{(+)}{(-)} = (-) \rightarrow \frac{12}{-4} = -3$$

3. Commutative Property of Multiplication

   For all integers a and b, $a \cdot b = b \cdot a$.

4. Associative Property of Multiplication

   For all integers a, b and c, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

5. **a.** The base for $(-2)^4$ is -2; $(-2)^4 = (-2)(-2)(-2)(-2) = 16$.

   b. The base for -2^4 is 2; $-2^4 = -(2)(2)(2)(2) = -16$ because the expression -2^4 represents the opposite of 2^4.

6. **a.** The product of an integer and 0 is always 0.

   b. 0 divided by any nonzero integer is 0.

   c. No integer may be divided by 0.

EXERCISES: ACTIVITY 3.5

1. Determine the product or quotient. Use a multiplication or division check to verify your answers.

   a. $-6 \cdot 7$ **b.** $(3)(-8)$ **c.** $8(-20)$

   d. $(-1)(7)$ **e.** $8 \cdot 0$ **f.** $(-3)(-25)$

   g. $-3(6)$ **h.** $5(-4)$ **i.** $-9(-6)$

   j. $-6(-40)$ **k.** $-30 \div 6$ **l.** $(-45) \div (-9)$

   m. $-3 \div 0$ **n.** $-25 \div (-1)$ **o.** $0 \div (-6)$

   p. $\frac{-64}{16}$ **q.** $\frac{-56}{-8}$ **r.** $\frac{0}{-167}$

2. Perform the following calculations. Check the results using your calculator.

   a. $(-2)(-4)(1)(-5)$ **b.** $(3)(-4)(2)(-4)(2)$ **c.** $(-1)(-1)(-1)(-1)$

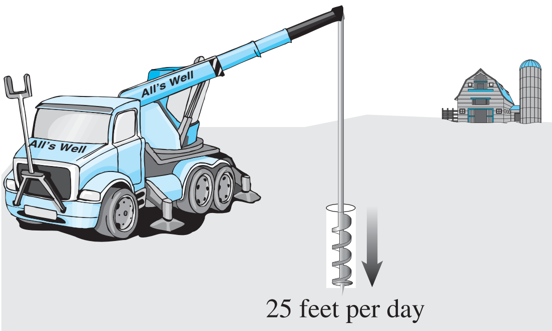
   d. $(-1)(-5)(-11)$ **e.** -6^2 **f.** $(-6)^2$

   g. $(-4)^3$ **h.** -4^3

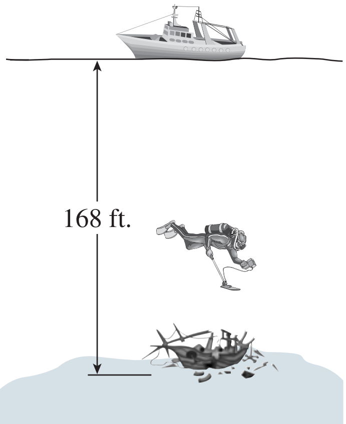
Exercise numbers appearing in color are answered in the Selected Answers appendix.

3. The daily low temperature in Lake Placid, New York, dropped 5°C per day for 6 consecutive days. Use a negative integer to represent the drop in temperature each day and calculate the total drop over the 6-day period.

4. You recently bought a piece of property in a rural area and need a well for your water supply. The well-drilling company you hire says they can drill down about 25 feet per day. It takes the company approximately 5 days to reach water. Represent the company's daily drilling rate by a negative integer and calculate the approximate depth of the water supply.



5. You plan to dive to a shipwreck located 168 feet below sea level. You can dive at the rate of approximately 2 feet per second. Use negative integers to represent the distance below sea level in the following calculations.



a. Calculate the depth you dove in 1 minute.

b. Calculate where you are in relationship to the shipwreck after 1 minute.

6. You are one of three equal partners in a company. Your company experienced a loss of \$150,000 in 2009. What was your share of the loss? Write the answer in words and as a signed number.

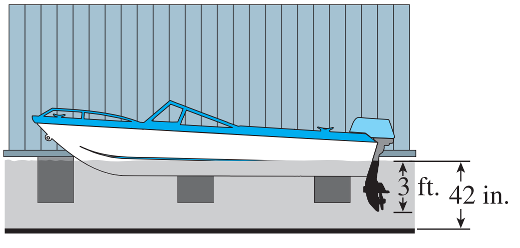
7. The running back of your favorite football team lost 6 yards per carry for a total of a 30-yard loss. Use negative integers to represent a loss in yardage and calculate the number of plays that were involved in this yardage loss.

8. To discourage random guessing on a multiple-choice exam, the instructor assigns 5 points for a correct response, -2 points for an incorrect answer, and 0 points for leaving the question blank. What is the score for a student who had 22 correct answers, 7 incorrect answers, and left 6 questions blank?

9. You are interning at the weather station. This week the low temperatures were 4°C, -8°C, 4°C, -1°C, -2°C, -2°C, -2°C. Determine the average low temperature for the week.

10. It has not rained in northern New York State for a month. You are worried about the depth of the water in your boathouse. You need 3 feet of water so that your boat motor does not hit bottom and get stuck in the mud. Each week you measure the water depth. The initial measurement was 42 inches. The following table tells the story of the change in depth each week.

Week	1	2	3	4
Change in Water Level (inches)	-2	-2	-2	-3



a. What was the depth of the water in your boathouse after week 1?

b. Determine the total change in the water level for the first 3 weeks.

c. Include the fourth week in part b. Determine the total change in the water level.

d. Determine the depth of the water in your boathouse after the fourth week.

e. Was your motor in the mud? Explain.

11. Each individual account in your small business has a current dollar balance, which can be either positive (a credit) or negative (a debit). Your records show the following balances.

−$230	−$230	$350	−$230	−$230	$350

What is the net balance for these six accounts?

12. You wrote four checks, each in the amount of $23, for your daughters to play summer league soccer. You had $82 in your checking account and thought you had just enough to cover the checks.

a. Write an arithmetic expression and simplify to determine if you will have enough money.

b. What possible mathematical error made you believe you could write the four checks?

Activity 3.6

Integers and Tiger Woods

Objectives

1. Use order of operations with expressions that involve integers.
2. Apply the distributive property.
3. Evaluate algebraic expressions and formulas using integers.
4. Combine like terms.
5. Solve equations of the form $ax = b$, where $a \neq 0$, that involve integers.
6. Solve equations of the form $ax + bx = c$, where $a + b \neq 0$, that involve integers.

The popularity of golf has increased greatly, due in large part to the achievements of Tiger Woods. In the 1997 Masters Tournament, one of golf's most prestigious events, Woods won by an amazing 12 strokes, the widest margin of victory the tournament has ever seen. At 21 years of age, he became the youngest golfer to win the Masters Tournament, and the first of African or Asian descent. He went on to win three more Masters Tournaments and, by July 2009, had been ranked as the number 1 golfer for a record 556 weeks of his career.

In the game of golf, the object is to hit a ball with a club into a hole in the ground with as few strokes (or swings of the club) as possible. The golfer with the lowest total number of strokes is the winner. A standard game consists of 18 different holes laid out in a parklike setting. Each hole, depending upon its length and difficulty, is assigned a number of strokes that represents the average number of strokes a very good player is expected to need to get the ball into the hole. This number is called *par* (you may know the cliché, "that's par for the course").

Most golfers refer to how their score relates to par; they give a score as the number of strokes above or below par. In this way, par acts like zero. Scoring at par can be represented by zero, scoring below par by a negative integer, and scoring above par by a positive integer. Golfers also use special terminology for the number of strokes above or below par, as summarized in the following table.

Getting the Swing of It . . .

TERM	MEANING	DIFFERS FROM PAR
Double eagle	3 strokes less than par	-3
Eagle	2 strokes less than par	-2
Birdie	1 stroke less than par	-1
Par	Expected number of strokes	0
Bogey	1 stroke more than par	1
Double bogey	2 strokes more than par	2

In the Masters, par for a game (round) of 18 holes is 72. In the last round of the 2001 Masters Tournament, Tiger Woods had a score of 68. This meant that he completed the course in 4 fewer strokes than was expected. That is, he was 4 under par.

There is no need to keep a tally of the actual number of strokes in a game. Instead, the numbers of birdies, pars, bogeys, and so on, are recorded. If a Masters golfer pars every hole, his score would be 72. If he bogeys 5 times and pars the rest, 5 would be added to 72, for a score of 77.

1. a. If a Masters player bogeys 7 holes and pars the rest, what would his score be? Explain.

b. Determine a player's score if he eagles 3 holes and pars the rest.

c. A golfer double-eagles 5 holes at the Masters and double-bogeys 13 holes. What is his score?

In Problem 1b, you first multiplied -2 by 3 and then added the product, -6, to 72 to obtain a score of 66. Symbolically, the steps can be written as $72 + (-2)(3) = 66$, or $72 + (-6) = 66$. This calculation indicates that the **order of operations** for whole numbers, discussed in Chapter 1, is also valid for integers.

Example 1 ***Rewrite the calculation from Problem 1c symbolically and evaluate using order of operations.***

SOLUTION

Write the calculation symbolically as $72 + (-3)(5) + (2)(13)$.

Evaluating,

$$
\begin{aligned}
& 72 + (-3)(5) + (2)(13) & \\
&= 72 + (-15) + 26 && \text{Multiplication before addition} \\
&= 72 - 15 + 26 && \text{Addition of a negative integer is subtraction of its opposite} \\
&= 57 + 26 && \text{Addition/subtraction from left to right} \\
&= 83. &
\end{aligned}
$$

The **order of operations** first introduced in Chapter 1 for whole numbers also applies to integers.
Operations contained within parentheses are performed *first* before any operations outside parentheses. All operations are performed in the following order.

1. Apply *all exponents* as you read the expression from left to right.
2. Perform all *multiplications and divisions* from left to right.
3. Perform all *additions and subtractions* from left to right.

2. The Corning Country Club is the site of the LPGA Corning Classic Golf Tournament for professional women golfers. The course consists of 18 holes and par is 70. One golfer summarized her scores for the first round by the following table. Calculate her score for the first round.

Join the Club

SCORE ON A HOLE	NUMBER OF TIMES SCORE OCCURRED
Eagle: -2	1
Birdie: -1	5
Par: 0	9
Bogey: $+1$	2
Double bogey: $+2$	1

3. Evaluate the following expressions using the order of operations. Then check the answer with your calculator.

a. $6 + 4 \cdot (-2)$

b. $-6 + 2 - 3$

c. $(6 - 10) \div 2$

d. $-3 + (2 - 5)$

e. $(2 - 7) \cdot 5$

f. $-3 \cdot (-3 + 4)$

g. $-2 \cdot 3^2 - 15$

h. $(2 + 3)^2 - 10$

i. $-7 + 8 \div (5 - 7)$

j. $-15 \div (-4 + 1) \cdot 5$

k. $(16 - 7) - 3^2 + 2(4 - 9)$

4. a. Evaluate the expression $-2(-3 + 7)$ by following the order of operations.

b. Evaluate the expression $-2(-3 + 7)$ by using the distributive property. Recall that $a(b + c) = ab + ac$.

c. Compare the results from parts a and b.

Your results from Problem 4 show that the distributive property of multiplication over addition with integers produces the same result as using the order of operations, just as it did with whole numbers.

Evaluating Algebraic Expressions

The American Century Celebrity Golf Championship at Lake Tahoe uses a rather unusual scoring system, called modified Stableford scoring. Rather than counting strokes, players are awarded points on each hole depending on how well they played the hole. The points are summarized in the following table.

Stroke of Luck

SCORE	POINTS
Double eagle	10
Hole in one	8
Eagle	6
Birdie	3
Par	1
Bogey	0
Double bogey or worse	−2

A player's score can be determined by the following expression:

$$10 \cdot a + 8 \cdot b + 6 \cdot c + 3 \cdot d + 1 \cdot e + 0 \cdot f + (-2) \cdot g,$$

where a = number of double eagles; b = number of holes in one; c = number of eagles; d = number of birdies; e = number of pars; f = number of bogeys; and g = number of double bogeys or worse.

5. Determine the score of golfer John Elway in the second round if his round included no double eagles, no holes in one, 1 eagle, 6 birdies, 9 pars, 2 bogeys, and 3 double bogeys.

The process in Problem 5 is an example of evaluating an expression involving integers. Recall from Chapter 2 that you can evaluate an algebraic expression when you know the value of each variable. To evaluate, replace each variable with its given value, then calculate using the order of operations.

Example 2 ***Evaluate the expression $3cd - 4d^2$ for $c = -2$ and $d = -5$.***

$3cd - 4d^2 = 3(-2)(-5) - 4(-5)^2$	Substitute values for c and d.
$= 30 - 4(25)$	Simplify using order of operations.
$= 30 - 100$	Simplify using order of operations.
$= -70$	

6. Evaluate each of the following expressions for the given values.

a. $6a + 3b - a^2$ for $a = -5$ and $b = 4$

b. $-4xy + 3x - 6y$ for $x = -4$, $y = -5$

c. $\dfrac{7c - 2d^2}{-3w}$ for $c = -10$, $d = -4$ and $w = 2$

Solving Equations of the Form $ax = b$ that Involve Integers

7. a. You join the fitness center and lose weight at the rate of 2 pounds per week for 10 weeks. Represent the rate of weight loss as a negative number and use it to determine your total weight loss.

b. Write a verbal rule that expresses your total weight loss in terms of the rate of weight loss per week and the number of weeks.

c. Let t represent the total weight loss, r the rate of weight loss per week, and w the number of weeks. Translate the verbal statement from part b into an equation.

d. One of the members of the Fitness Center lost a total of 42 pounds, represented by -42, at the rate of 3 pounds per week, represented by -3. Use the formula in part c to write an equation to determine the number of weeks it took the member to lose the weight.

e. Solve the equation in part d.

8. Solve the following equations for the given variables. Check your results in the original equation.

a. $-4x = 36$ **b.** $6s = -48$ **c.** $-32 = -4y$

Combining Like Terms Revisited

The process of combining like terms is the same for integers as it is for whole numbers. Thus, you can rewrite a subtracted term as the addition of the opposite and use the commutative property of addition to group the like terms together.

Example 3 ***Combine like terms:* $8x + 6y - 5x$.**

$8x + 6y - 5x = 8x + 6y + (-5x)$	**Rewrite "subtract $5x$" as "add $-5x$."**
$= 8x + (-5x) + 6y$	**Use the commutative property of addition.**
$= 3x + 6y$	**Combine like terms.**

9. Combine like terms.

a. $3x + 7y - 5x$ **b.** $9x - 4y - 12x + 10y$

c. $2x - 4y + 8x + 7y$ **d.** $10x + 15y - 14x - 9y + 3x$

10. Solve the following equations by first combining like terms.

a. $3x - 8x = -35$

b. $-11y - 9y = 260$

c. $-30x + 18x = 36$

d. $7x - 15x = 32$

SUMMARY: ACTIVITY 3.6

1. The order of operations first introduced in Chapter 1 for whole numbers also applies to integers.

Operations contained within parentheses are performed *first* before any operations outside the parentheses. All operations are peformed in the following order.

a. Apply *all exponents* as you read the expression from left to right.

b. Perform all *multiplications and divisions* from left to right.

c. Perform all *additions and subtractions* from left to right.

2. To evaluate formulas or expressions involving integers, substitute for the variables and evaluate using the order of operations.

3. To solve the equation $ax = c$ for x, where $a \neq 0$, divide each side of the equation by a to obtain $x = \frac{c}{a}$.

EXERCISES: ACTIVITY 3.6

1. In 2005, Tiger Woods again won the Masters Tournament. His performance in the last round (all 18 holes) is given in the following table.

Mastering the Game

SCORE ON A HOLE	NUMBER OF TIMES SCORE OCCURRED
Birdie: -1	5
Par: 0	9
Bogey: $+1$	4

Exercise numbers appearing in color are answered in the Selected Answers appendix.

If par for the 18 holes is 72, determine Tiger's score in the last round of the tournament.

2. You own 24 shares of stock in a high-tech company and have been following this stock over the past 4 weeks.

WEEK	GAIN OR LOSS PER SHARE ($)
1	2
2	−3
3	4
4	−5

a. What is the net gain or loss in total value of the stock over the 4-week period?

b. If your stock was worth $18 per share at the beginning of the first week, how much was your stock worth at the end of the fourth week?

c. Let the variable *x* represent the cost per share of your stock at the beginning of the first week. Write an expression to determine the cost per share at the end of the fourth week.

d. Use the expression in part c to determine the value of your stock at the end of the fourth week if it was worth $27 per share at the beginning of the first week.

3. Evaluate the following. Check your results by using your calculator.

a. $3 + 2 \cdot 4$

b. $3 - 2(-4)$

c. $-2 + (2 \cdot 4 - 3 \cdot 5)$

d. $4 \cdot 3^2 - 50$

e. -6^2

f. $(-6)^2$

g. $(-2)^2 + 10 \div (-5)$

h. $12 - 3^2$

i. $-12 \div 3 - 4^2$

j. $(2 \cdot 3 - 4 \cdot 5) \div (-2)$

k. $(-13 - 5) \div 3 \cdot 2 \cdot 1$

l. $-5 + 9 \div (8 - 11)$

m. $14 - (-4)^2$

n. $3 - 2^3$

o. $3^2(-5)^2 \div (-1)$

4. Use the distributive property to evaluate the following.

a. $5(-3 + 4)$

b. $-2(5 - 7)$

c. $4(-6 - 2)$

5. Evaluate the following expressions using the given values.

a. $ab^2 + 4c$ for $a = 3, b = -5, c = -3$

b. $(5x + 3y)(2x - y)$ for $x = -4, y = 6$

c. $-5c(14cd - 3d^2)$ for $c = 3, d = -2$

d. $\dfrac{3ax - 4c}{4ac}$ for $a = -2, c = 4, x = -8$

6. You are on a diet to lose 15 pounds in 8 weeks. The first week you lost 3 pounds, and then you lost 2 pounds per week for each of the next 3 weeks. The fifth week showed a gain of 2 pounds, but the sixth and seventh each had a loss of 2 pounds. You are beginning your eighth week. How many pounds do you need to lose in the eighth week in order to meet your goal?

7. Translate each of the following into an equation and solve. Let x represent the number.

a. The product of a number and -6 is 54.

b. -30 times a number is -150.

c. -88 is the product of a number and 8.

d. Twice a number is -28.

8. Solve each equation. Check your result.

a. $3x = 27$ b. $9s = -81$ c. $-24 = -3y$

d. $-5x = 45$ e. $-6x = -18$ f. $-x = -5$

For Exercises 9–11, write an equation to represent the situation and solve.

9. On average, you can lose 5 pounds per month while dieting. Your goal is to lose 65 pounds. Represent each weight loss by a negative number. How many months will it take you to do this?

10. As winter approached in Alaska, the temperature dropped each day for 19 consecutive days. The total decrease in temperature was 57°F, represented by -57. What was the average drop in temperature per day?

11. Over a 10-day period the low temperature for International Falls, Minnesota, was recorded as -18°F on 3 days, -15°F on 2 days, -10°F on 2 days, and -8°F, -5°F, and -3°F on the remaining 3 days. What was the average low temperature over the 10-day period?

12. Combine like terms.

a. $6y + 2x - 9y$ b. $5x - 3y - 11x + 13y$

c. $x - 2y + 6x + 10y$ d. $12x + 13y - 16x - 4y + 2x$

13. Solve the following equations by first combining like terms.

a. $15x - 21x = -30$ b. $-13y - 7y = 180$

c. $-14x + 7x = 42$ d. $6y - 10y = 64$

What Have I Learned?

1. Which number is always greater, a positive number or a negative number? Give a reason for your answer.

2. A number and its opposite are equal. What is the number?

3. Two numbers, x and y, are negative. If $|x| > |y|$, which number is smaller? Give an example to illustrate.

4. In each of the following, fill in the blank with *positive, negative*, or *zero* to make the statement true.

   a. When you add two positive numbers, the sign of the answer is always ____________.

   b. When you add two negative numbers, the sign of the answer is always ____________.

   c. The absolute value of zero is _______.

   d. When you subtract a negative number from a positive number, the answer is always ____________.

   e. When you subtract a positive number from a negative number, the answer is always ____________.

5. Describe in your own words how to add a positive number and a negative number. Use an example to help.

6. Describe in your own words how to add two negative numbers. Use an example to help.

7. Describe in your own words how to subtract two positive numbers. Use an example to help.

8. Describe in your own words how to subtract a positive number from a negative number. Use an example to help.

Exercise numbers appearing in color are answered in the Selected Answers appendix.

9. Describe in your own words how to subtract a negative number from a positive number. Use an example to help.

10. Describe in your own words how to subtract two negative numbers. Use an example to help.

11. Draw a rectangular coordinate grid and plot the following points: $(8, 90)$, $(6, -50)$, $(-4, 60)$, $(0, 0)$, $(-5, -50)$, $(-7, 0)$.

12. You process concert ticket orders for a civic center in your hometown. There is a \$3 processing fee for each order. Write a formula to represent the calculations you would do to determine the total cost of an order. State what the variables are in this situation and choose letters to represent them.

13. When you multiply a positive integer, n, by a negative integer, is the result greater or less than n? Give a reason for your answer.

14. **a.** When you multiply a negative integer, n, by a second negative integer, is the result greater or less than n? Explain your answer.

b. Does your answer to part a depend on the absolute value of the second negative integer? Why or why not?

15. **a.** Complete the following table to list the rules for the multiplication and division of two integers.

MULTIPLICATION	DIVISION
$(+) \cdot (+) =$	$(+) \div (+) =$
$(+) \cdot (-) =$	$(+) \div (-) =$
$(-) \cdot (+) =$	$(-) \div (+) =$
$(-) \cdot (-) =$	$(-) \div (-) =$

b. Use the table you completed above to show how the rules for division are related to the rules for multiplication of two integers.

16. How will you remember the rules for multiplying and dividing two integers?

17. When you multiply several integers, some positive and some negative, how do you determine the sign of the product?

18. When a negative number is raised to a power, is the result negative? Illustrate your answer with examples.

19. Explain why -7^2 is not the same as $(-7)^2$.

How Can I Practice?

1. Determine the opposite of each of the following.

a. 3 **b.** -5 **c.** 0

2. Evaluate.

a. $|-13|$ **b.** $|15|$ **c.** $-|-39|$ **d.** $-|39|$

3. Calculate.

a. $15 + 36$ **b.** $-21 + 9$ **c.** $18 + (-35)$

d. $-5 + (-21)$ **e.** $-17 + 5$ **f.** $-16 + 20$

g. $8 + (-13)$ **h.** $8 + (-3)$ **i.** $-6 + -8$

j. $-5 + (-9)$ **k.** $-7 + 2 + (-3)$

l. $-6 + (-4) + 8$ **m.** $-3 + 7 + (-9)$

n. $-5 + 8 + (-11) + (-2)$ **o.** $-10 + (-3) + 6 + (-2)$

4. Calculate.

a. $16 - 62$ **b.** $36 - 82$ **c.** $-14 - (-28)$

d. $-18 - (-6)$ **e.** $24 - (-48)$

f. $45 - 54$ **g.** $-4 - (-12) - 15$

h. $-11 - (-28) - 33 - (-12)$ **i.** $3 - (-12) - 20 - (-13) - 25$

5. Calculate the following products and quotients. Use your calculator to check the results.

a. $-2 \cdot 8$ **b.** $(5)(-6)$ **c.** $-8(-20)$

d. $-9 \cdot 0$ **e.** $-3\,(10)$ **f.** $-4(-12)$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

g. $45 \div (-9)$ **h.** $-36 \div 9$ **i.** $(-54) \div (-6)$

j. $-8 \div 0$ **k.** $\frac{72}{-12}$ **l.** $\frac{-24}{-3}$

m. $(-2)(-1)(3)(-2)(-1)$ **n.** $(5)(-1)(2)(-3)(-1)$

6. Evaluate by performing the given operations.

a. $-9 - 1 + 5 - 11$

b. $-17 + 3 - (-8) + 21$

c. $13 - 9 + (-18) - (-12)$

d. $-51 + 27 - (-21) + 42$

e. $19 - 31 + (-42) + (-5)$

f. $-14 + 6 - (-20) - 24 - (-21)$

g. $7 - 10 + (-9) - 3 - (-4)$

7. Evaluate each expression. Check the results using your calculator.

a. $-6 + 4(-3)$ **b.** $10 - 5(-2)$

c. $(6 - 26 + 5 - 10) \div -5^2$ **d.** $40 + 8 \div (-4) \cdot 2$

e. $-14 + 8 \div (10 - 12)$ **f.** $-27 \div 3 - 6^2$

g. $18 - 6 + 3 \cdot 2$ **h.** $-7^2 - 7^2$ **i.** $(-7)^2 - 7 \cdot 2 + 5$

8. Evaluate the expression $x + y$, for each set of given values.

a. $x = -18$ and $y = 7$ **b.** $x = 13$ and $y = 8$

c. $x = -21$ and $y = -12$ **d.** $x = -17$ and $y = 5$

9. Evaluate the expression $x - y$, for each set of given values.

a. $x = 14$ and $y = 6$ **b.** $x = -16$ and $y = 9$

c. $x = 23$ and $y = -62$ **d.** $x = -27$ and $y = -32$

10. Evaluate each expression using the given values of the variables.

a. $4x - 3(x - 2)$, where $x = -5$

b. $x^2 - xy$, where $x = -6$ and $y = 4$

c. $2(x + 3) - 5y$, where $x = -2$ and $y = -4$

11. Solve the following equations and check your answer in the original equation.

a. $x + 21 = -63$

b. $x + 18 = 49$

c. $32 + x = 59$

d. $41 + x = -72$

e. $x - 17 = 19$

f. $x - 35 = 42$

g. $-22 + x = 16$

h. $-28 + x = 11$

i. $10 - x = -3$

j. $-12 - x = 7$

k. $-3x = 96$

l. $2y = -78$

m. $-7x = -63$

12. Let x represent an integer. Translate the following verbal expression into an equation and solve for x.

a. The sum of an integer and 5 is 12.

b. 7 less than an integer is 13.

c. The difference of an integer and 8 is 6.

d. The result of subtracting 3 from an integer is 12.

e. The difference of an integer and -2 is -3.

f. The result of subtracting -6 from an integer is -2.

g. The opposite of an integer is 9.

h. Three times an integer is -15.

i. -54 is the product of -3 and what integer?

j. The product of an integer and -4 is 48.

k. 90 is the product of -3, -6, and an integer.

13. Combine like terms.

a. $8x + 10y - 6x$

b. $7x - 3y - 13x + 9y$

c. $8x - 5y + 2x + 12y$

d. $12x + 16y - 10x - 9y - 3x$

14. Solve the following equations by first combining like terms.

a. $4x - 9x = -65$

b. $-11y - 7y = 360$

c. $-30x + 42x = -84$

d. $3x - 11x = 56$

15. The temperature in the morning was $-9°F$. The weather report indicated that by noon the temperature would be 4°F warmer. Determine the noontime temperature.

16. The temperature in the morning was $-13°F$. It was expected that the temperature would fall 5°F by midnight. What would be the midnight temperature?

17. The average temperature in July in town is 88°F. The average temperature in January in the same town is $-5°F$. What is the change in average temperature from July to January?

18. The temperature on Monday was $-8°F$. By Tuesday the temperature was $-13°F$. What was the change in temperature?

19. The temperature at the beginning of the week was $-6°F$, and at the end of the week it was 7°F. What was the change in the temperature?

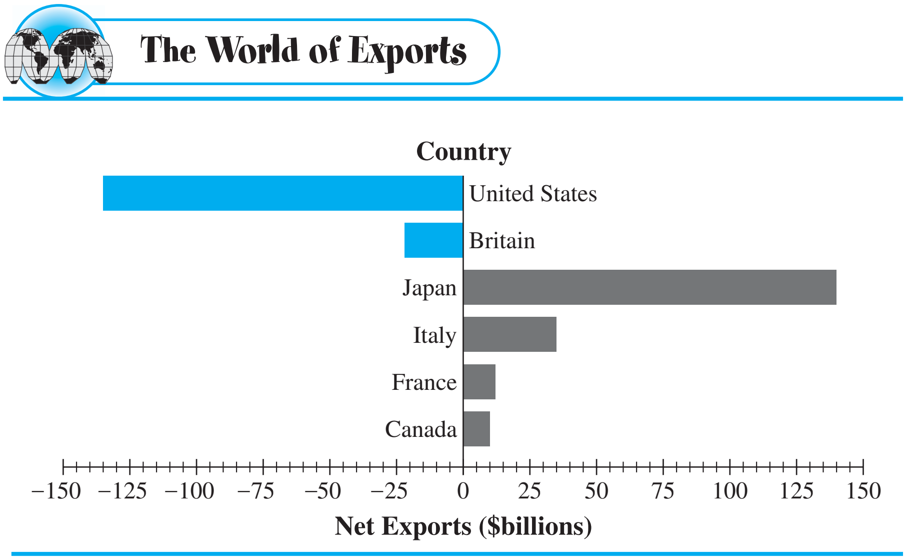
20. **a.** Your favorite stock opened the day at \$29 per share and ended the day at \$23 per share. Let x represent the amount of your gain or loss per share. An equation that represents the situation is $29 + x = 23$. Solve for x.

b. The following table represents the beginning and ending values of your stock for 1 week. Complete the table indicating the amount of your gain or loss per share, each day.

Opening Value	23	21	21	18	20
Closing Value	21	21	18	20	23
Gain or Loss, x		0			

c. What is the total change in your stock's value for this week?

21. The following graph represents net exports in billions of dollars for six countries represented on the number line. (Net exports are obtained by subtracting total imports from total exports.)



a. What is the difference between the net exports of Japan and Italy?

b. What is the difference between the net exports of Britain and France?

c. What is the total sum of net exports for the six countries listed?

22. A contestant on the TV quiz show *Jeopardy!* had $1000 at the start of the second round of questions (double jeopardy). She rang in on the first two questions, incorrectly answering a $1200 question but giving a correct answer to an $800 question. What was her score after answering the two questions?

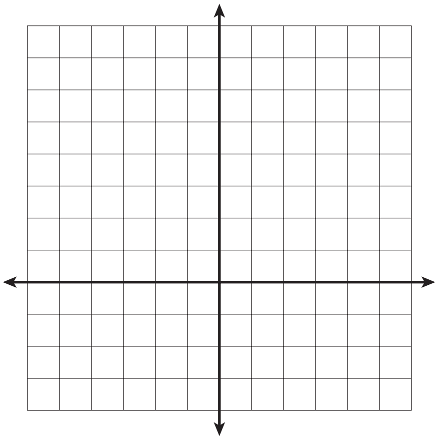
23. The difference of two integers is -3.

a. Translate this verbal rule into an equation, where x represents the smaller integer and y the larger.

b. Use the equation you wrote in part a to complete the following table.

x	y
0	
	0
5	
	−2
−2	
	6

c. Plot the values from the table in part b on an appropriately labeled and scaled coordinate system.



d. Connect the points on the graph to obtain the graphical representation of the equation in part a.

24. In hockey, a defenseman's plus/minus record (+/− total) determines his success. If he is on the ice for a goal by the opposing team, he has a −1. If he is on the ice for a goal by his team, he gets a +1. A defenseman for a professional hockey team has the following on-ice record for five games.

Shot . . . Score!

GAME	YOUR TEAM'S GOALS	OPPOSITION GOALS	+/− TOTAL
1	4	1	$1(4) + (-1)(1) = +3$
2	3	5	
3	0	3	
4	3	0	
5	1	6	

For example, an expression that represents his +/− total for game 1 is

$$1(4) + (-1)(1).$$

Evaluating this expression, he was a +3 for game 1.

a. Write an expression for each game and use it to determine the +/− total for that game. Enter the expression and +/− total for each game in the table.

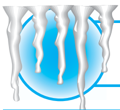
b. Write an expression and use it to determine the defenseman's +/− total for the five games.

25. At the end of a hockey season a defenseman summarized his +/− total in the following chart. For example: In the first column, in three games his +/− total was +6. His total +/− for the 3 games was +18.

No. of Games	3	4	12	8	6	11	3	15	9	6	3
+/− Total	+6	+4	+3	+2	+1	0	−1	−2	−3	−4	−5

Write an expression that will determine the +/− total for the season. What is his +/− total for the season?

26. The following table contains the daily midnight temperatures in Buffalo for a week in January.



A Balmy −12°

Day	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
Temp. (°F)	−3°	6°	−3°	−12°	6°	−3°	−12°

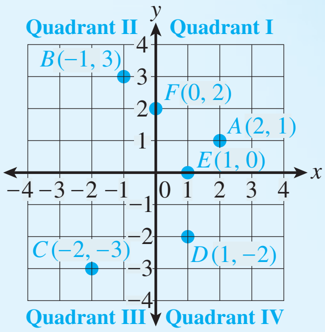
a. Write an expression to determine the average daily temperature for the week.

b. What is the average daily temperature for the week?

Chapter 3 Summary

The bracketed numbers following each concept indicate the activity in which the concept is discussed.

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Comparing integers [3.1]	The numbers increase from left to right on a number line. The further to the left a number is, the smaller it is.	$-18 < -3$ $-32 < 1$
Absolute value [3.1]	The distance of a number from zero on the number line represents its absolute value. The absolute value of a number is always nonnegative.	$\|0\| = 0$ $\|-17\| = 17$ $\|13\| = 13$
Adding/subtracting integers [3.2]	1. Rules for adding integers: a. To add two numbers with the same sign, add the absolute values of the numbers. The sign of the sum is the same as the sign of the numbers being added.	$4 + (+7) = 4 + 7 = 11$ $-3 + (-5) = -8$
	b. To add two numbers with different signs, find their absolute values and then subtract the smaller from the larger. The sign of the sum is the sign of the number with the larger absolute value.	$-5 + (+7) = 2$ $+10 + (-13) = -3$
	2. Rules for subtracting integers: To subtract two integers, change the operation of subtraction to addition and change the number being subtracted to its opposite; then follow the rules for adding integers.	$-13 - (-5) =$ $-13 + 5 = -8$ $-11 - (+7) =$ $-11 + (-7) = -18$
Evaluating expressions [3.3]	To evaluate an expression, substitute the given number for the letter. Perform the arithmetic, using the order of operations.	Evaluate $a - b$, where $a = 15$, $b = -7$. $15 - (-7) = 15 + 7 = 22$
Solving equations of the form $x + b = c$, $x - b = c$, and $b = c - x$ [3.3], [3.4]	1. For $x + b = c$, add the opposite of b to both sides of the equation to obtain $x = c - b$.	$x + 3 = 19$ $x + 3 + (-3) = 19 + (-3)$ $x = 16$
	2. For $x - b = c$, add b to both sides of the equation to obtain $x = c + b$.	$x - 4 = -2$ $x - 4 + 4 = -2 + 4 = 2$ $x = 2$
	3. For $b = c - x$, add x to both sides of the equation and subtract b from both sides to obtain $x = c - b$.	$5 = 8 - x$ $5 + x = 8 - x + x$ $5 - 5 + x = 8 - 5$ $x = 3$

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Points plotted on a rectangular coordinate system [3.4]	An ordered pair is always given in the form of (x, y), where x is the input and y is the output. The x-axis (horizontal) and the y-axis (vertical) determine a coordinate plane, divided into four quadrants numbered counterclockwise from the upper right.	 A: $(2, 1)$ B: $(-1, 3)$ C: $(-2, -3)$ D: $(1, -2)$ E: $(1, 0)$ F: $(0, 2)$

QUADRANT	x-COORDINATE	y-COORDINATE
I	+	+
II	−	+
III	−	−
IV	+	−

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Multiply/divide integers with the same sign [3.5]	To multiply or divide two integers with the same sign: 1. Multiply or divide their absolute values. 2. The product or quotient will always be positive.	$(-6) \cdot (-7) = 42$ $4 \cdot 3 = 12$ $8 \div 2 = 4$ $(-20) \div (-4) = 5$
Multiply/divide integers with opposite signs [3.5]	To multiply or divide two integers with opposite signs: 1. Multiply or divide their absolute values. 2. The product or quotient will always be negative.	$(-42) \div 14 = -3$ $11 \cdot (-4) = -44$
Product of an even number of negative factors [3.5]	The product will be positive if you are multiplying an even number of negative integers.	$(-3) \cdot (-4) \cdot (-2) \cdot (-1) = 24$
Product of an odd number of negative factors [3.5]	The product will be negative if you are multiplying an odd number of negative integers.	$(-3) \cdot (-4) \cdot (-2) = -24$
Multiplications and divisions that involve zero [3.5]	1. Any number times 0 is 0. 2. 0 divided by any nonzero integer is 0. 3. No integer may be divided by 0.	$3 \cdot 0 = 0$ $0 \div 6 = 0$ $6 \div 0$ is not possible.

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Negation of a squared number [3.5]	Negation, or determining the opposite, always follows exponentiation in the order of operations.	$-5^2 = -(5^2)$ $= -1 \cdot 25$ $= -25$
The distributive properties [3.6]	The distributive properties, $a(b + c) = ab + ac$ $a(b - c) = ab - ac,$ hold for integers.	$-2(5 - 9)$ $= (-2) \cdot 5 - (-2) \cdot 9$ $= -10 + 18$ $= 8$
Order of operations [3.6]	The order of operation rules for integers are the same as those applied to whole numbers.	$9 + 3(4) - (2)^2$ $= 9 + 12 - 4$ $= 21 - 4$ $= 17$
Evaluate expressions or formulas that involve integers [3.6]	To evaluate formulas or expressions that involve integers, substitute for the variables and evaluate using the order of operations.	Evaluate $6xy - y^2$, where $x = 1$ and $y = 3$. $6xy - y^2 = 6(1)(3) - 3^2$ $= 18 - 9$ $= 9$
Solving equations of the form $ax = b$, $a \neq 0$, that involve integers [3.6]	To solve the equation $ax = b, \quad a \neq 0,$ divide each side of the equation by a to obtain the value for x.	Solve $-3x = -216$. $\frac{-3x}{-3} = \frac{-216}{-3}$ $x = 72$
Solving equations of the form $ax + bx = c$, where $a + b \neq 0$ [3.6]	To solve, combine like terms with the distributive property. Then solve as you did with the form $ax = b$.	$3x + 4x = 21$ $(3 + 4)x = 21$ $7x = 21$ $x = \frac{21}{7} = 3$

Chapter 3 Gateway Review

1. Determine the absolute value for each of the following.

a. $|15|$ **b.** $|23|$ **c.** $|0|$ **d.** $|-42|$ **e.** $|67|$

2. Perform the indicated operations.

a. $4 - (+13)$ **b.** $11 - (+17)$ **c.** $5 - (-11)$

d. $-2 + (-12)$ **e.** $27 + (-15)$ **f.** $15 + (-23)$

g. $-18 + (-35)$ **h.** $-27 + (+21)$ **i.** $-4 \cdot 38$

j. $-3 \cdot -9$ **k.** $-42 \div -6$ **l.** $-9(-6)$

m. $52 \div -2$ **n.** $(-2)(-3)(-4)$ **o.** $(-3)^2$

p. -4^2 **q.** $-(-7)^2$

3. Determine the product of 13 factors of -1.

4. Explain the difference between -5^2 and $(-5)^2$.

5. Evaluate each expression.

a. $x - y$, where $x = 3$ and $y = -3$

b. $-x + y$, where $x = 5$ and $y = -2$

c. $-x - y$, where $x = -3$ and $y = -7$

d. $x + y$, where $x = -2$ and $y = -5$

e. $-x \cdot y$, where $x = -2$ and $y = 5$

f. $-x \cdot -y$, where $x = 3$ and $y = -5$

g. $x \div -y$, where $x = -20$ and $y = -5$

Answers to all Gateway exercises are included in the Selected Answers appendix.

h. $2x - 6(x - 3)$, where $x = 4$

i. $-3xy - x^2$, where $x = -2$ and $y = -6$

6. Evaluate each expression.

a. $-2 \cdot 6 + 81 \div (-9)$

b. $10 - 6 \cdot 4 - 3^2$

7. Combine like terms.

a. $6y + 7x - 2y$

b. $9x - 3y - 11x + 8y$

8. Solve for x.

a. $x - 15 = -17$

b. $x + 7 = -12$

c. $x + 18 = 3$

d. $-11 + x = 32$

e. $-23 + x = -61$

f. $-12x = -72$

g. $-6x = 30$

h. $-42 = -7x$

i. $2x + 3x = 25$

j. $12x - 17x = -30$

9. Translate each of the following verbal statements into an equation where x represents the number. Then solve for x.

a. A number increased by eighteen is negative seven.

b. The sum of a number and eleven is twenty-nine.

c. Fifteen increased by a number is negative twenty-eight.

d. The difference of a number and twenty is thirty-nine.

e. A number subtracted from eight is twelve.

f. Seventeen subtracted from a number is forty-two.

g. Thirteen less than a number is negative thirty-four.

h. The product of a number and -15 is 30.

i. 3 times a number is -15.

j. 18 is the product of a number and 2.

10. a. The temperature in the morning was -9°F and by noon the temperature increased by 3°F. Determine the temperature at noon.

b. The temperature at noon was 3°F and by 7 P.M. the temperature dropped 5°F. Determine the 7 P.M. temperature.

c. The temperature in the morning was -8°F and by noon it was -3°F. Determine the change in the temperature.

d. The temperature in the morning was -9°F and the evening temperature was -14°F. What was the change in the temperature?

11. You are babysitting three children who have very strict ideas about sharing candy. The bag of Gummi Bears has 28 pieces. You quickly eat one when they aren't looking. How many Gummi Bears can you give to each child?

12. You get a new job with direct deposit of a weekly paycheck. You open a savings account and sign up for weekly automatic transfer from your checking account.

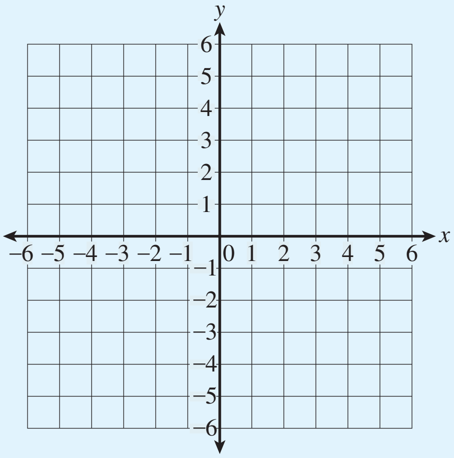
a. $5 per week is going into your savings. How much money will you have saved in 12 weeks?

b. You spend all your savings on a present for your mom and begin saving again, but you increase the amount of the automatic transfer. After 12 more weeks you have \$240. How much are you saving each week?

13. Plot the following points on the coordinate system:

a. $(4, 3)$ **b.** $(-3, 4)$ **c.** $(0, -3)$

d. $(-2, -3)$ **e.** $(-2, 0)$ **f.** $(6, -1)$



Chapter 4

Problem Solving with Fractions

Chapter 3 deals with integers, which include positive and negative counting numbers and zero. In some situations, a quantity is divided into parts; in other situations, two quantities are compared. These situations require numbers called fractions. Another name for a fraction is a *rational number*. In this chapter, you will solve problems involving fractions.

Activity 4.1

Are You Hungry?

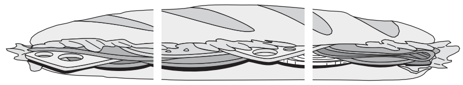
Objectives

1. Identify the numerator and the denominator of a fraction.
2. Determine the greatest common factor (GCF).
3. Determine equivalent fractions.
4. Reduce fractions to equivalent fractions in lowest terms.
5. Determine the least common denominator (LCD) of two or more fractions.
6. Compare fractions.

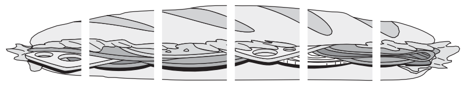
You decide to have some friends over to watch movies. After the first movie, you call the local sub shop to order three giant submarine sandwiches. When the subs are delivered, you pay the bill and everyone agrees to reimburse you, depending on how much they eat.

Because some friends are hungrier than others, you cut one sub into three equal (large) pieces, a second sub into six equal (medium) parts, and the third sub into twelve equal (small) parts.

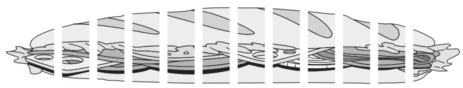
Sub 1: Large pieces



Sub 2: Medium pieces



Sub 3: Small pieces



What Is a Fraction?

1. a. Because the first sub is divided into three large equal pieces, each piece represents a fractional part of the whole sub. If you are served a large piece, what fraction of the sub do you get?

b. Your friend Elsa has a medium-size piece. What fraction of the sub does each medium piece represent? Explain.

c. Girmay already ate, but he takes a small piece. What fraction of the sub does each small piece represent? Explain.

Definitions

The bottom number of a fraction is called the **denominator**. The denominator indicates the total number of equal parts into which a whole unit is divided.

The top portion of a fraction is called the **numerator**. The numerator indicates the number of equal parts that the fraction represents out of the total number of parts.

2. a. If you eat 5 small pieces, what fractional part of the whole sub did you eat? Explain.

   b. What is the denominator of this fraction? What does the denominator represent in this context?

   c. What is the numerator of this fraction? What does this number represent?

Note that the fraction line, which separates the numerator and denominator, represents "out of" or "divided by." So, $\frac{1}{3}$ can be thought of as 1 out of 3, or 1 divided by 3.

3. Suppose a friend brought 2 jumbo-sized chocolate chip cookies to a party and 3 guests wanted an equal share of the cookies. What fraction represents the amount served to each guest? Explain.

A fraction that has an integer numerator and a nonzero integer denominator is a **rational number**.

Definition

A **rational number** is a number that can be written in the form $\frac{a}{b}$, where a and b are integers and b is not zero. Every integer is also a rational number, since any integer a can be written as $\frac{a}{1}$.

Note that the denominator of a fraction cannot be zero. It would indicate that the whole is divided into zero equal parts, which makes no sense.

4. a. Suppose a friend is very hungry and he eats all three large pieces of the sub. What fraction represents the amount he ate?

b. What is the value of the fraction in part a? Explain.

c. If $\frac{6}{6}$ represents the amount of the medium sub that was eaten, what is the value of the fraction?

d. In general, what is the value of a fraction whose numerator and denominator are equal but nonzero?

5. a. If you did not eat any of the large pieces of the first sub, what fraction represents the amount of the first sub that you ate?

b. What is the value of the fraction in part a?

c. If $\frac{0}{12}$ represents the amount that you ate of the third sub, what does the numerator indicate?

d. What is the value of the fraction $\frac{0}{12}$?

e. In general, what is the value of a fraction whose numerator is zero?

Equivalent Fractions

6. As you are cutting the subs, you notice that two medium pieces placed end-to-end measure the same as one large piece. Therefore, $\frac{2}{6}$ of the sub represents the same portion as $\frac{1}{3}$.

a. How many small pieces represent the same portion as one large piece?

b. What fraction of the sub does the number of small pieces in part a represent?

In Problem 6, three different fractions were used to represent the same portion of a whole sub. These fractions, $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{4}{12}$, are called **equivalent fractions**. They represent the same quantity.

Procedure

Writing Equivalent Fractions

To obtain a fraction equivalent to a given fraction, multiply or divide the numerator and the denominator of the given fraction by the same nonzero number.

7. **a.** Divide the numerator and the denominator of $\frac{2}{6}$ by 2 to obtain an equivalent fraction.

b. Multiply the numerator and denominator of $\frac{1}{3}$ by 4 to obtain an equivalent fraction.

8. Write the given fraction as an equivalent fraction with the given denominator.

a. $\frac{3}{10} = \frac{?}{20}$

b. $\frac{4}{5} = \frac{?}{30}$

c. $\frac{7}{11} = \frac{?}{33}$

d. $\frac{7}{9} = \frac{?}{72}$

Reducing a Fraction to Lowest Terms

9. **a.** What is the largest whole number that is a factor of both the numerator and denominator of $\frac{6}{8}$?

b. What is the largest whole number that is a factor of both the numerator and denominator of $\frac{8}{12}$?

c. What whole number is a factor of both the numerator and denominator of $\frac{1}{3}$?

Definitions

If the largest factor of both the numerator and denominator of a fraction is 1, the fraction is said to be in **lowest terms**. For example, $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ are written in lowest terms.

The **greatest common factor** (GCF) of two numbers is the largest factor common to both numbers. For example, the GCF of 8 and 12 is 4.

Procedure

Reducing a Fraction to Lowest Terms

To reduce a fraction to lowest terms, divide the numerator and denominator by their greatest common factor (GCF). For example, to reduce the fraction $\frac{8}{12}$ to an equivalent fraction in lowest terms, divide the numerator and denominator by 4.

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

10. Reduce the following fractions to equivalent fractions in lowest terms.

a. $\frac{6}{8}$

b. $\frac{6}{15}$

c. $\frac{3}{8}$

d. $\frac{15}{25}$

Comparing Fractions: Determining Which Fraction Is Larger

11. Suppose one group of friends ate 4 medium pieces and another group ate 10 small pieces.

a. What fractional part of the sub did the first group eat?

b. What fractional part of the sub did the second group eat?

c. Use the graphics display of the subs at the beginning of this activity to help determine which group ate more.

It is much easier to compare fractions that have the same denominator.

12. Write $\frac{4}{6}$ as an equivalent fraction with a denominator of 12.

It is easy to see that $\frac{10}{12}$ is larger than $\frac{8}{12}$ since they have the same denominator and $10 > 8$.

13. Which is the larger fraction, $\frac{5}{8}$ or $\frac{3}{4}$?

Problems 11, 12, and 13 show that two or more fractions are easily compared if they are expressed as equivalent fractions having the same (common) denominator. Then the fraction with the largest numerator is the largest fraction.

Definitions

A **common denominator** of two or more fractions is a number that is a multiple of each denominator. For example, $\frac{3}{4}$ and $\frac{5}{6}$ have common denominators of 12, 24, 36, ...

The **least common denominator (LCD)** of two fractions is the smallest common denominator. For example, the LCD of $\frac{3}{4}$ and $\frac{5}{6}$ is 12.

Procedure

Determining the Least Common Denominator

To determine the LCD, identify the largest denominator of the fractions involved. Look at multiples of the largest denominator. The smallest multiple that is divisible by the other denominator(s) is the LCD.

Example 1

Determine the least common denominator of $\frac{1}{6}$ and $\frac{3}{8}$.

SOLUTION

In the case of $\frac{1}{6}$ and $\frac{3}{8}$, the larger denominator is 8.

The smallest multiple of the denominator 8 that is exactly divisible by the denominator 6 is the LCD, determined as follows:

$8 \cdot 1 = 8$ **8 is not a multiple of 6.**

$8 \cdot 2 = 16$ **16 is not a multiple of 6.**

$8 \cdot 3 = 24$ **24 is a multiple of 6, therefore 24 is the LCD.**

14. a. Write equivalent fractions for $\frac{1}{6}$ and $\frac{3}{8}$ so that each has a denominator of 24.

b. Use the result from part a to compare $\frac{1}{6}$ and $\frac{3}{8}$.

15. Use the inequality symbols $<$ or $>$ to compare the following fractions.

a. $\frac{3}{5}$ and $\frac{1}{2}$

b. $\frac{3}{8}$ and $\frac{1}{3}$

c. $\frac{1}{4}$ and $\frac{3}{16}$

SUMMARY: ACTIVITY 4.1

1. The **greatest common factor (GCF)** of two or more numbers is the largest number that is a factor of each of the given numbers.
2. To write an **equivalent fraction**, multiply or divide both the numerator and denominator of a given fraction by the same number.
3. To reduce a fraction to an equivalent fraction in lowest terms, divide the numerator and the denominator by their GCF.
4. To determine the **least common denominator (LCD)** of two or more fractions, identify the largest denominator and look at multiples of it. The smallest multiple that is divisible by the other denominator(s) is the LCD.
5. To compare two or more fractions, express them as equivalent fractions with a common positive denominator. The fraction with the largest numerator is the largest fraction.

EXERCISES: ACTIVITY 4.1

Determine the missing number.

1. $\frac{2}{3} = \frac{?}{12}$

2. $\frac{3}{4} = \frac{?}{20}$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

Reduce the given fraction to an equivalent fraction in lowest terms.

3. $\frac{9}{15}$

4. $\frac{8}{28}$

Use the inequality symbols < or > to compare the following fractions.

5. $\frac{5}{12}$ and $\frac{1}{3}$

6. $\frac{3}{7}$ and $\frac{2}{5}$

7. You and your sister ordered two individual pizzas. You ate $\frac{3}{4}$ of your pizza and your sister ate $\frac{5}{8}$ of her pizza. Who ate more pizza?

8. You and your friend are painting a house. While you painted $\frac{2}{12}$ of the house, your friend painted $\frac{3}{18}$ of the house. Who painted more?

9. In a basketball game, you made 4 baskets out of 12 attempts. Your teammate made 3 baskets out of 8 attempts.

a. What fraction of your attempted shots did you make?

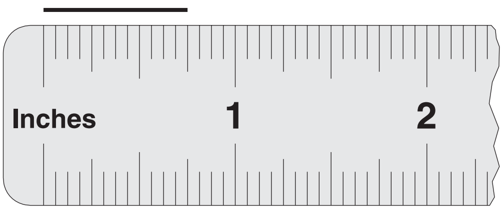
b. What fraction of your teammate's attempted shots did she make?

c. Who was the more accurate shooter?

10. An Ivy League college accepts 5 students for every 100 that apply. What fraction of applicants is accepted? Write your result in lowest terms.

11. Measuring with rulers provides an opportunity to test your understanding of fractions. Use the graphic of a ruler to answer the following questions about measuring the line segment drawn above the ruler.

a. Look at the ruler and determine the number of $\frac{1}{4}$ inch segments in 1 inch.



b. How many $\frac{1}{4}$-inch segments did the given line measure?

c. How long is the given line segment (in inches)?

d. Measure the given line segment in $\frac{1}{8}$-inch units.

e. Finally, measure the given line segment in $\frac{1}{16}$-inch units.

f. Explain why the three fractional answers in parts c, d, and e are equivalent.

g. Another line segment measures $\frac{2}{3}$ inch. Is it longer or shorter than the given line segment? Explain.

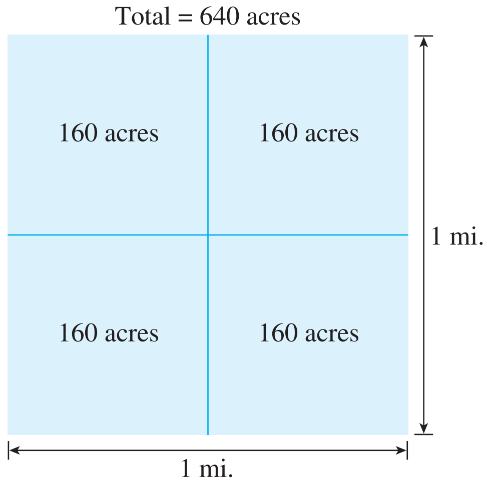
Activity 4.2

Get Your Homestead Land

Objectives

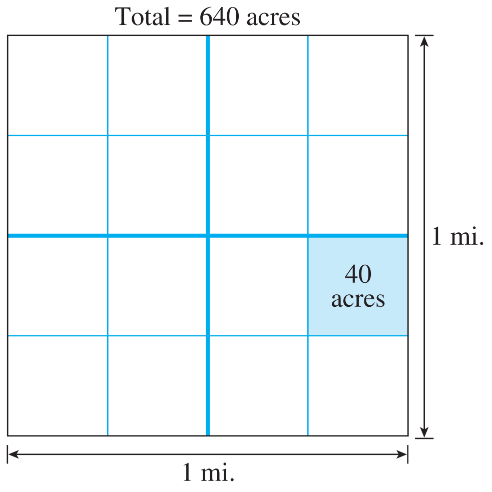
1. Multiply and divide fractions.
2. Recognize the sign of a fraction.
3. Determine the reciprocal of a fraction.
4. Solve equations of the form $ax = b, a \neq 0$, that involve fractions.

In 1862, Congress passed the Homestead Act. It provided for the transfer of 160 acres of unoccupied public land to each homesteader who paid a nominal fee and lived on the land for 5 years. As part of an assignment, you are asked to determine the size of 160 acres, in square miles. Your research reveals that there are 640 acres in 1 square mile, so you draw the following diagram, dividing the square mile into four equal portions (quarters).



1. Use the diagram to determine the fractional part of the original square mile that the 160 acres represents.

You decide to investigate further and divide each 160-acre homestead into four equal squares. Each *new square* represents 40 acres, as illustrated.



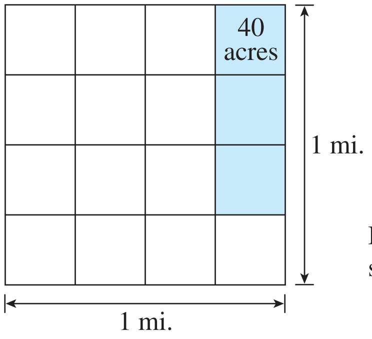
2. Use the diagram to determine the fractional part of the original square mile that 40 acres represents.

3. a. Use the diagram on the previous page to write the dimensions of a 40-acre square in terms of miles.

b. Use the diagram to determine the area of a 40-acre square in square miles.

4. Since the area of a square is determined by multiplying the lengths of two sides, you can combine the results of Problem 3a and b as follows:

$$\left(\frac{1}{4}\text{ mile}\right) \cdot \left(\frac{1}{4}\text{ mile}\right) = \frac{1}{16}\text{ sq. mi.}$$



a. Suppose someone is able to acquire three adjacent lots of 40 acres along one side of the square mile. What fractional part of the square mile did he acquire?

b. What are the overall dimensions of this combined lot, in miles?

Because he acquired 3 of the 16 lots, the area in Problem 4a is $\frac{3}{16}$ square mile. The dimensions of his lot are $\frac{1}{4}$ mile by $\frac{3}{4}$ mile. So the area of his lot is

$$\frac{1}{4} \cdot \frac{3}{4} = \frac{3}{16}\text{ sq. mi.}$$

5. In Problems 3 and 4, you determined that $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$ and $\frac{1}{4} \cdot \frac{3}{4} = \frac{3}{16}$. Describe in your own words a rule for multiplying fractions.

Procedure

Multiplying Fractions

To multiply fractions, multiply the numerators to obtain the new numerator and multiply the denominators to obtain the new denominator. Stated symbolically:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

The rules for multiplying integers also apply to positive and negative fractions.

Example 1 ***Multiply*** $-\frac{2}{3} \cdot \frac{5}{7}$.

SOLUTION

$$-\frac{2}{3} \cdot \frac{5}{7} = -\frac{2 \cdot 5}{3 \cdot 7} = -\frac{10}{21}$$

6. Multiply $-\frac{7}{8} \cdot \frac{3}{5}$.

Example 2 *Multiply* $\mathbf{28 \cdot \frac{3}{4}}$.

SOLUTION

Rewrite the whole number 28 as the fraction $\frac{28}{1}$; then multiply: $\frac{28}{1} \cdot \frac{3}{4} = \frac{84}{4} = 21$.

7. Multiply $21 \cdot \frac{2}{7}$.

Example 3 *Multiply* $\mathbf{\frac{3}{8} \cdot \frac{1}{6}}$.

SOLUTION

Multiply the fractions, $\frac{3}{8} \cdot \frac{1}{6} = \frac{3 \cdot 1}{8 \cdot 6} = \frac{3}{48}$.

Then reduce to lowest terms: $\frac{3}{48} = \frac{3 \cdot 1}{3 \cdot 16} = \frac{3}{3} \cdot \frac{1}{16} = 1 \cdot \frac{1}{16} = \frac{1}{16}$.

In Example 3, the common factor 3 in the numerator and denominator can be divided out, because, in effect, it amounts to multiplication by 1. It is actually preferable to remove (divide out) the common factor of 3 from both numerator and denominator before the final multiplication.

$$\frac{3}{8} \cdot \frac{1}{6} = \frac{\overset{1}{\cancel{3}} \cdot 1}{8 \cdot \underset{1}{\cancel{3}} \cdot 2} = \frac{1}{8 \cdot 2} = \frac{1}{16}$$

This will guarantee that your final answer will be written in lowest terms.

Example 4 *Multiply* $\mathbf{-\frac{12}{25} \cdot -\frac{10}{9}}$.

SOLUTION

$$-\frac{12}{25} \cdot -\frac{10}{9} = \frac{\overset{1}{\cancel{3}} \cdot 4 \cdot 2 \cdot \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \cdot 5 \cdot \underset{1}{\cancel{3}} \cdot 3} = \frac{4 \cdot 2}{5 \cdot 3} = \frac{8}{15}$$

Note that the common factors of 3 and 5 were divided out. This procedure is sometimes called *canceling common factors.*

If you don't divide out the common factors first, you will produce larger numbers that will need to be reduced. In this case, $-\frac{12}{25} \cdot -\frac{10}{9} = \frac{120}{225}$, where it is not as easy to determine the GCF and reduce to lowest terms.

8. Multiply $-\frac{15}{16} \cdot -\frac{8}{3}$.

Sign of a Fraction

Every fraction is a quotient that represents the division of two integers. For example, a 3-inch string divided into four equal lengths will result in strings of length $\frac{3}{4}$ inch $\left(3 \div 4 = \frac{3}{4}\right)$. As with integers, fractions may be positive or negative. For example, consider $-\frac{3}{4}$. Usually the negative sign precedes the fraction, indicating that the value of the fraction is less than zero. Since the quotient of two integers having opposite signs represents a negative fraction, dividing -3 by 4 will result in a negative fraction, as will 3 divided by -4.

$$\frac{-3}{4} = -\frac{3}{4} \quad \text{and} \quad \frac{3}{-4} = -\frac{3}{4}$$

These are three different ways to express exactly the same fraction.

9. Evaluate each of the following. Check your answers by using the fraction feature on your calculator.

a. $-\frac{2}{7} \cdot -\frac{4}{7}$

b. $\frac{4}{5} \cdot -\frac{8}{9}$

c. $-\frac{1}{4} \cdot \frac{-8}{9}$

Dividing Fractions

To understand division by a fraction, consider dividing a pie into three equal pieces. Each piece is equal to $1 \div 3 = \frac{1}{3}$ of the pie. But $\frac{1}{3}$ is also the same as $1 \cdot \frac{1}{3}$. This means that $1 \div 3 = 1 \cdot \frac{1}{3}$; that is, dividing 1 by 3 is equivalent to multiplying 1 by $\frac{1}{3}$. The number $\frac{1}{3}$ is called the **reciprocal** of 3. Note that the product $3 \cdot \frac{1}{3} = 1$. This example leads to the general definition for reciprocals.

Definition

Two numbers are **reciprocals** if their product is 1. Reciprocal pairs are either both positive or both negative. To obtain the reciprocal of the fraction $\frac{a}{b}$, interchange the numerator and the denominator to obtain the fraction $\frac{b}{a}$.

Example 5 ***Determine the reciprocal of the given number. Multiply the number and its reciprocal in the third column.***

SOLUTION

NUMBER	RECIPROCAL	PRODUCT
$\frac{2}{3}$	$\frac{3}{2}$	$\frac{2}{3} \cdot \frac{3}{2} = \frac{6}{6} = 1$
$-\frac{1}{4}$	$-\frac{4}{1} = -4$	$-\frac{1}{4} \cdot -\frac{4}{1} = \frac{4}{4} = 1$
-12	$-\frac{1}{12}$	$-\frac{12}{1} \cdot -\frac{1}{12} = \frac{12}{12} = 1$

10. a. Determine the reciprocal of each number. Show your calculation of the product of the number and its reciprocal in the third column.

NUMBER	RECIPROCAL	PRODUCT
6		
$-\frac{3}{5}$		
$\frac{1}{7}$		

b. Does 0 have a reciprocal? Explain.

11. Suppose you own a 4-acre plot of land and wish to subdivide it into plots that are each $\frac{2}{5}$ of an acre. How many such smaller plots would you have? Use repeated subtraction.

Your answer to Problem 11 was determined by repeatedly subtracting $\frac{2}{5}$ from the original 4 acres. ($4 - \frac{2}{5} = 3\frac{3}{5}$, $3\frac{3}{5} - \frac{2}{5} = 3\frac{1}{5}$, etc.) You could subtract $\frac{2}{5}$ ten times, since $\frac{2}{5} \cdot 10 = \frac{20}{5} = 4$ acres. Hence you could make ten $\frac{2}{5}$-acre plots.

A much more efficient way to determine the number of such plots is to divide 4 by $\frac{2}{5}$. (How many times does $\frac{2}{5}$ go into 4?) From Problem 11, you know $4 \div \frac{2}{5} = 10$. By changing this division to $4 \cdot \frac{5}{2}$ you will get the same result since $4 \cdot \frac{5}{2} = \frac{20}{2} = 10$. Note that $\frac{5}{2}$ is the *reciprocal* of $\frac{2}{5}$. This shows the way to a procedure for dividing a number by a fraction.

$$4 \div \frac{2}{5} = \frac{\overset{2}{\cancel{4}}}{1} \cdot \frac{5}{\underset{1}{\cancel{2}}} = \frac{10}{1} = 10$$

Procedure

Dividing by Fractions

To divide one fraction by another fraction, multiply the dividend fraction by the reciprocal of the divisor fraction. The method can be stated algebraically.

$$\underbrace{\frac{a}{b}}_{\text{dividend}} \div \underbrace{\frac{c}{d}}_{\text{divisor}} = \frac{a}{b} \cdot \underbrace{\frac{d}{c}}_{\text{reciprocal}} = \frac{ad}{bc}$$

Example 6

Divide $\frac{3}{8} \div \frac{15}{16}$.

SOLUTION

$$\frac{3}{8} \div \frac{15}{16} = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{8}}} \cdot \frac{\overset{2}{\cancel{16}}}{\underset{5}{\cancel{15}}} = \frac{2}{5}$$

12. Evaluate the following. Check your answers by using the fraction feature on your calculator.

a. $\frac{7}{9} \div \frac{14}{15}$

b. $\frac{6}{13} \div -24$

c. $-\frac{18}{23} \div \frac{30}{-23}$

d. $\dfrac{\frac{8}{15}}{\frac{-36}{45}}$

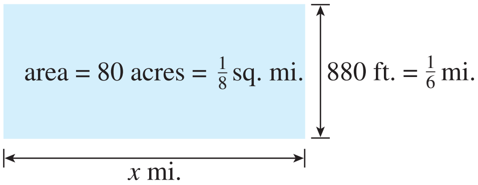
Solving Equations of the Form $ax = b, a \neq 0$, that Involve Fractions

13. Suppose you acquire a rectangular plot that contains 80 acres.

a. What fractional part of a square mile (640 acres) does 80 acres represent?

b. You measure one side of the 80-acre plot and obtain 880 feet. What fractional portion of a mile is 880 feet? (There are 5280 feet in one mile.)

The following diagram summarizes what you determined in Problem 13.



14. Let x represent the length of the unknown side of the rectangular plot. Using the formula for the area of a rectangle, write an equation relating $\frac{1}{8}$ square mile, $\frac{1}{6}$ mile, and x miles.

You can solve the equation in Problem 14 for x, because it is of the form $ax = b$. What is new here is that a and b are fractions.

Example 7

Solve $\frac{1}{6}x = \frac{1}{8}$ to determine the length of the unknown side of the rectangular plot in Problem 14.

SOLUTION

Dividing both sides of the equation by $\frac{1}{6}$ is the same as multiplying both sides by $\frac{6}{1}$, the reciprocal of $\frac{1}{6}$.

$$\frac{6}{1} \cdot \frac{1}{6}x = \frac{1}{8} \cdot \frac{6}{1}$$

$$x = \frac{6}{8} = \frac{3}{4}$$

The unknown side of the rectangular plot in Problem 14 is $\frac{3}{4}$ mile.

15. Solve the following equations for the given variable. Check your result.

a. $\frac{2}{3}x = \frac{4}{9}$

b. $\frac{2}{3}y = \frac{-4}{15}$

c. $-\frac{1}{2}w = \frac{3}{8}$

d. $\frac{x}{5} = \frac{3}{25}$ $\left(\mathit{Hint}: \frac{x}{5} = \frac{1}{5}x\right)$

SUMMARY: ACTIVITY 4.2

1. To **multiply fractions**,
   a. Divide out common factors between the numerator and denominator.
   b. Multiply the remaining numerators to obtain the new numerator and multiply the remaining denominators to obtain the new denominator.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

2. Two numbers are reciprocals of each other if their product is 1. Reciprocal pairs are either both positive or both negative. To obtain the reciprocal of the fraction $\frac{a}{b}$, switch the numerator and the denominator to obtain the fraction $\frac{b}{a}$. Zero does not have a reciprocal.

3. To **divide by a fraction**, multiply the dividend by the reciprocal of the divisor.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

4. There are three different ways to place the sign for a negative fraction. For example: $-\frac{2}{5} = \frac{-2}{5} = \frac{2}{-5}$ are all the same number. $-\frac{2}{5}$ is the preferred notation.

5. To solve an equation of the form $\frac{a}{b}x = \frac{c}{d}$ for x, multiply each side by the reciprocal of $\frac{a}{b}$ to obtain

$$x = \frac{c}{d} \div \frac{a}{b} = \frac{c}{d} \cdot \frac{b}{a} = \frac{cb}{da}.$$

EXERCISES: ACTIVITY 4.2

1. Determine the reciprocal of each of the given numbers.

NUMBER	RECIPROCAL
$\frac{4}{5}$	
$-\frac{8}{3}$	
-7	

Exercise numbers appearing in color are answered in the Selected Answers appendix.

2. Multiply each of the following and express each answer in simplest form.

a. $\frac{2}{7} \cdot \frac{3}{5}$ **b.** $\frac{5}{8} \cdot \frac{3}{7}$ **c.** $\frac{-5}{12} \cdot \frac{3}{-25}$ **d.** $\frac{-7}{12} \cdot \frac{15}{11}$

e. $\frac{16}{27} \cdot \frac{-18}{24}$ **f.** $\frac{10}{7} \cdot \frac{3}{5}$ **g.** $\frac{17}{26} \cdot \frac{12}{18}$ **h.** $\frac{-4}{15} \cdot \frac{40}{-14}$

3. Divide each of the following and express each answer in simplest form.

a. $\frac{1}{5} \div \frac{1}{10}$ **b.** $\frac{-5}{9} \div \frac{25}{-21}$ **c.** $-8 \div \frac{8}{10}$

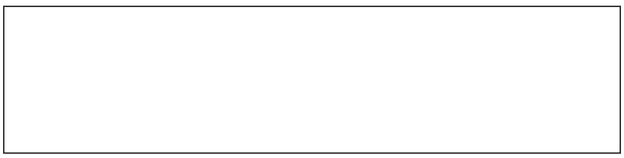
d. $-\frac{12}{26} \div \frac{8}{15}$ **e.** $\dfrac{\frac{-3}{10}}{\frac{-5}{9}}$ **f.** $\dfrac{\frac{23}{19}}{-46}$

4. a. As part of a science project, you do a study on the number of beans that actually sprout. You determine that approximately 9 out of 10 seeds germinate. Write this relationship as a fraction.

b. Your uncle has ordered 2500 bean seeds for his commercial garden. Assuming that 9 out of every 10 seeds sprout, how many bean plants can he expect to sprout?

5. a. You have purchased some land to start a game farm. Two-thirds of the rectangular piece of property will be used for the animals and three-fourths of that piece will be left as wilderness. Using the diagram of the farm shown here, mark off the part of the land that will be for the animals.

b. Shade the area that will be left as wilderness.



c. What fractional part of the whole piece of land will be left as wilderness?

6. You are doing a survey for your statistics class and discover that $\frac{3}{4}$ of the students at the college take a math class; $\frac{1}{6}$ of these students take statistics. What fraction of the students at the college take statistics?

7. In your budget you have $\frac{1}{10}$ of your annual salary saved for your college tuition. If your annual salary is $32,450, how much will you have for your tuition?

8. You and five of your friends go back to your place for lunch between classes. Your roommate has eaten approximately $\frac{1}{4}$ of an apple pie you made. Your friends all want pie. What fractional part of the remaining pie can each of them have, if you decide not to have any?

9. How many $\frac{5}{8}$-inch-long pieces of wire can be made from a wire that is 20 inches long?

10. Translate each of the following into an equation and solve.

a. The product of a number and 3 is $\frac{6}{5}$.

b. $\frac{2}{3}$ times a number is -4.

c. A number divided by 7 is 9.

d. The quotient of a number and 2 is $\frac{3}{8}$.

11. Solve the following equations for the given variable.

a. $\dfrac{15}{16}x = \dfrac{3}{8}$

b. $-\dfrac{4}{15}x = \dfrac{1}{10}$

c. $\dfrac{2}{9}w = -\dfrac{3}{20}$

d. $-\dfrac{15}{32}y = \dfrac{5}{24}$

e. $-\dfrac{4}{25}t = -\dfrac{12}{15}$

f. $\dfrac{13}{27}x = \dfrac{20}{45}$

Activity 4.3

On the Road with Fractions

Objectives

1. Add and subtract fractions with the same denominator.
2. Add and subtract fractions with different denominators.
3. Solve equations in the form $x + b = c$ and $x - b = c$ that involve fractions.

You often encounter fractions when following a recipe or on a trip. For example, a recipe may call for $\frac{1}{2}$ teaspoon of salt and a car's gas gauge may show that $\frac{3}{4}$ of the tank is empty. Changing a recipe or determining how much gas to buy often requires some basic calculations with fractions.

Adding and Subtracting Fractions with Same Denominators

Example 1 ***A cross-country car trip to see college friends will take you several days, and you want to take some food with you. You decide to bake zucchini bread and cookies for snacking. The recipes call for $\frac{1}{4}$ teaspoon of baking powder for the bread and $\frac{3}{4}$ teaspoon of baking powder for the cookies. How much baking powder do you need?***

SOLUTION

Add the amounts of baking powder to obtain the total amount you will need.

$$\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

You will need 1 teaspoon of baking powder.

Example 1 illustrates a procedure for adding or subtracting fractions with the same denominator. Fractions with the same denominators are sometimes referred to as **like fractions**.

Procedure

Adding and Subtracting Fractions with the Same Denominator

To add two like fractions, add the numerators and keep the common denominator:

$$\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}$$

To subtract one like fraction from another, subtract one numerator from the other and keep the common denominator:

$$\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}$$

If necessary, reduce the resulting fraction to lowest terms.

1. For each of the following, perform the indicated operation. Write the result in lowest terms.

   a. $\frac{1}{3} + \frac{1}{3}$

   b. $\frac{3}{10} + \frac{4}{10}$

   c. $\frac{3}{17} + \frac{9}{17}$

d. $\frac{3}{8} + \frac{5}{8}$ **e.** $\frac{6}{7} - \frac{4}{7}$ **f.** $\frac{7}{8} - \frac{3}{8}$

g. $\frac{1}{4} - \frac{3}{4}$ **h.** $\frac{3}{18} - \frac{5}{18}$

You are ready to start your cross-country trip to see your friends. You expect to drive a total of 2400 miles to get to your destination and to make stops along the way to do some sightseeing.

2. The first day, you drive 600 miles and stop to see statues of famous historical figures in a Wax Museum. What fraction of the 2400 miles do you complete on the first day? Write the fraction in lowest terms.

3. The second day, you drive only 400 miles so you can visit an art gallery that features the works of M. C. Escher. What fraction of the 2400 miles do you complete on the second day? Write the fraction in lowest terms.

4. a. You complete $\frac{1}{4}$ of the total 2400-mile mileage on the first day and $\frac{1}{6}$ of the total mileage on the second day. Write a numerical expression that can be used to determine what fraction of the 2400-mile trip you complete on the first 2 days.

b. How is this addition problem different from those in Problem 1?

To add or subtract two or more fractions, they all must have the same denominator. Recall (Activity 4.1) that a common denominator for a set of fractions can be determined by finding a number that is a multiple of each of the denominators in the fractions.

5. In Problem 4 you can choose any number that is a multiple of both 4 and 6 as the common denominator. List at least four numbers that are multiples of both 4 and 6.

Is 12 one of the numbers that you listed in Problem 5? Notice that 12 is divisible by both 4 and 6 and that it is the smallest such number. Therefore, 12 is the **least common denominator (LCD)** of $\frac{1}{4}$ and $\frac{1}{6}$.

6. a. Write $\frac{1}{4}$ as an equivalent fraction with a denominator of 12.

b. Write $\frac{1}{6}$ as an equivalent fraction with a denominator of 12.

c. Add the two like fractions to determine the fraction of the trip you complete on the first 2 days.

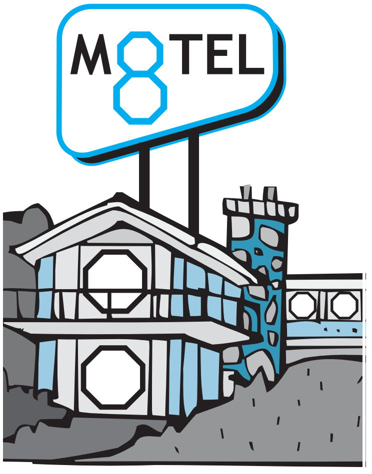
7. On the third day, you drive 800 miles and stop to see a flower garden where the leaf and petal arrangements form mathematical patterns. What fraction of the 2400 miles do you complete on the third day? Write the fraction in lowest terms.

8. a. You complete $\frac{5}{12}$ of the trip on the first two days and $\frac{1}{3}$ of the trip on the third day. What is the LCD for the two fractions?

b. What fractional part of the trip do you complete on the first 3 days?

9. During the fourth day, after completing $\frac{1}{12}$ more of the trip, you stay at Motel 8, where the windows are all regular octagons. How much of the trip do you finish in the first 4 days?

10. What fraction of the trip do you have to complete to get to your destination on the fifth day?



11. Determine the LCD for each set of fractions.

a. $\frac{1}{3}, \frac{1}{2}$ **b.** $\frac{2}{7}, \frac{1}{3}$ **c.** $\frac{3}{10}, \frac{1}{4}$

d. $\frac{3}{8}, \frac{1}{20}$ **e.** $\frac{1}{4}, \frac{1}{6}, \frac{3}{8}$ **f.** $\frac{5}{12}, \frac{1}{8}, \frac{3}{16}$

g. $\frac{2}{11}, \frac{1}{2}, \frac{1}{4}$ **h.** $\frac{1}{5}, \frac{1}{4}, \frac{1}{3}$ **i.** $\frac{1}{7}, \frac{1}{6}, \frac{1}{3}$

12. Use the LCDs from Problem 11 to find the following sums.

a. $\frac{1}{3} + \frac{1}{2}$ **b.** $\frac{2}{7} + \frac{1}{3}$ **c.** $\frac{3}{10} + \frac{1}{4}$

d. $\frac{3}{8} + \frac{1}{20}$ **e.** $\frac{1}{4} + \frac{1}{6} + \frac{3}{8}$

f. $\frac{5}{12} + \frac{1}{8} + \frac{3}{16}$ **g.** $\frac{2}{11} + \frac{1}{2} + \frac{1}{4}$

h. $\frac{1}{5} + \frac{1}{4} + \frac{1}{3}$ **i.** $\frac{1}{7} + \frac{1}{6} + \frac{1}{3}$

Solving Equations

Problem 10 can also be solved using algebra. If x represents the fractional part of the trip to be completed on the fifth day, then

$$x + \frac{5}{6} = 1.$$

Recall that this equation may be solved for x by *subtracting* $\frac{5}{6}$ from both sides of the equation.

$$x + \frac{5}{6} - \frac{5}{6} = 1 - \frac{5}{6}$$

$$x = \frac{6}{6} - \frac{5}{6} = \frac{1}{6}$$

So, you have $\frac{1}{6}$ of the trip to complete on the fifth day.

13. Once you arrive, you and your friends compare college experiences, grades and the different methods professors use to determine grades. For example, the final grade in one of your courses is determined by quizzes, exams, a project, and class participation. Quizzes count for $\frac{1}{4}$ of the final grade, exams $\frac{1}{3}$, and the project $\frac{1}{4}$ of the final grade.

a. If x represents the fractional part of the final grade for class participation, write an equation relating $x, \frac{1}{4}, \frac{1}{3}, \frac{1}{4}$, and 1.

b. Solve the equation for x.

14. Solve each of the following equations for the unknown quantity. Check your result in the original equation.

a. $x + \dfrac{3}{10} = \dfrac{47}{100}$

b. $y - \dfrac{3}{5} + \dfrac{1}{2} = \dfrac{1}{6}$

SUMMARY: ACTIVITY 4.3

1. To add or subtract fractions with the same denominator, add or subtract the numerators and write the sum or difference over the given denominator. Reduce to lowest terms if necessary.

2. To add or subtract fractions with different denominators,

a. Find the LCD and convert each fraction to an equivalent fraction that has the LCD you found.

b. Add or subtract the numerators of the equivalent fractions to obtain the new numerator, leaving the LCD in the denominator.

c. If necessary, reduce the resulting fraction to lowest terms.

EXERCISES: ACTIVITY 4.3

Perform the indicated operation. Write all results in lowest terms.

1. $\frac{3}{8} + \frac{3}{8}$

2. $\frac{4}{5} - \frac{1}{5}$

3. $\frac{5}{12} + \frac{1}{12}$

4. $\frac{5}{8} - \frac{3}{8}$

5. $\frac{2}{3} - \frac{2}{3}$

6. $\frac{7}{9} - \frac{1}{9}$

7. $\frac{1}{24} + \frac{5}{24}$

8. $-\frac{3}{7} - \frac{2}{7}$

9. $-\frac{3}{16} - \frac{3}{16}$

10. $-\frac{7}{18} + \frac{1}{18}$

11. $\frac{1}{6} + \frac{1}{2}$

12. $\frac{3}{8} - \frac{1}{4}$

13. $-\frac{2}{5} + \frac{9}{10}$

14. $\frac{1}{5} - 1$

15. $\frac{1}{14} - \frac{1}{2}$

16. $\frac{1}{3} - \frac{1}{8}$

17. $-\frac{1}{24} - \frac{1}{12}$

18. $-\frac{2}{5} + \left(-\frac{1}{4}\right)$

19. $-\frac{1}{6} + \frac{1}{10}$

20. $-\frac{2}{7} - \frac{1}{21}$

21. $-\frac{5}{6} + \frac{2}{5}$

22. $-\frac{1}{6} - \frac{1}{8}$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

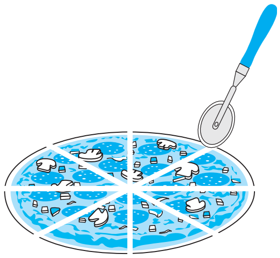
23. $\frac{1}{4} + \frac{4}{9} - \frac{1}{9}$

24. $\frac{1}{2} - \frac{1}{3} - \frac{1}{4}$

25. A pizza is cut into 8 equal slices. You eat 2 slices and your friend eats 3 slices.

a. What fraction of the pizza did you and your friend eat?

b. What fraction of the pizza is left?



26. You are at a restaurant with your spouse and four other couples. The group decides to split the check evenly. What fractional portion of the check will each couple pay?

27. You have three 2-ounce slices of turkey on your plate. You feed one slice to your puppy. What fraction of the turkey is left for you?

a. Calculate your fractional portion of the turkey by the number of slices.

b. To verify your answer for part a, calculate your fraction of the turkey by ounces.

28. You take home \$400 a month from your part-time job as a cashier. Each month you budget \$120 for car expenses, \$160 for food, and the rest for entertainment.

a. What fraction of your take-home pay is budgeted for car expenses?

b. What fraction of your take-home pay is budgeted for food?

c. What fraction of your take-home pay is budgeted for entertainment?

29. You check the oil level in your car, and it is down a quart. You dutifully add an estimated $\frac{1}{3}$ quart because that is all you have on hand.

a. Later, you buy another quart of oil and add about $\frac{1}{2}$ quart before you stop and ask yourself this question: How much more oil will it take without overfilling?

b. Estimate the amount wasted if you pour in the remaining half quart, assuming that anything over the fill line spills out.

30. A student spends $\frac{1}{3}$ of a typical day sleeping, $\frac{1}{6}$ of the day in classes, and $\frac{1}{8}$ of the day watching TV.

a. If x represents the fraction of the rest of the day available for study, write an equation describing the situation.

b. Solve the equation for x.

31. The final grade in one of your friend's courses is determined by a term paper, exams, quizzes, and class participation. The term paper is worth $\frac{1}{5}$, the exams $\frac{1}{3}$, and the quizzes $\frac{1}{4}$ of the final grade.

a. If x represents the fractional part of the final grade for class participation, write an equation describing the situation.

b. Solve the equation for x.

In Exercises 32–37, solve each equation for the unknown quantity. Check your answers.

32. $\frac{3}{10} = c - \frac{1}{5}$

33. $\frac{5}{6} = \frac{1}{3} - x$

34. $x - \frac{2}{3} = \frac{1}{15}$

35. $\frac{1}{2} + b = \frac{2}{3}$

36. $\frac{2}{3} + x = -\frac{1}{7}$

37. $-\frac{4}{9} = -\frac{5}{6} - x$

Activity 4.4

Hanging with Fractions

Objectives

1. Calculate powers and square roots of fractions.
2. Evaluate equations that involve powers.
3. Evaluate equations that involve square roots.
4. Use order of operations to calculate numerical expressions that involve fractions.
5. Evaluate algebraic expressions that involve fractions.
6. Use the distributive property with fractions.
7. Solve equations of the form $ax + bx = c$ with fraction coefficients.

Many scientific applications of mathematics use formulas that involve exponents and/or square roots and also use fractions as input and output values. Well-known examples in physics are equations that determine the distance an object falls due to the force of gravity.

Gravity, Exponents, and Fractions

For objects falling from very near Earth's surface, the simplest equation is

$$s = 16t^2.$$

This equation does not take into account other forces, such as wind resistance, air pressure, and so on.

The output variable s represents the distance measured in feet that an object falls to Earth. The input variable t represents the time measured in seconds that the object takes to reach Earth. The integer 16 represents the constant value of gravity's force on Earth.

You can evaluate the factor t^2 when t is a rational number just as you do when t is an integer. For example,

$$\left(\frac{1}{2}\right)^2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

1. You dropped a penny from the railing of a platform and it took $\frac{1}{2}$ second to reach the ground. Use the equation for gravity given above to determine the distance from the railing to the ground.

Procedure

Raising Fractions to a Power

To raise a fraction to a power, raise the numerator to the power and the denominator to the power. Symbolically, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$.

2. Dice for board games are in the shape of a cube. A die for a child's board game measures $\frac{7}{10}$ inch on each side. Use the volume formula for a cube, $V = s^3$, to determine the volume of the die.

3. Evaluate the following expressions. Write your final fraction in lowest terms.

   a. $-\frac{1}{2}\left(\frac{3}{5}\right)^2$

   b. $50\left(\frac{1}{4}\right)^3$

c. $\left(\frac{1}{10}\right)^4$

d. $200\left(\frac{-1}{10}\right)^3$

Square Roots and Fractions

The term *hang time* was made popular by sports reporters following Michael Jordan during the 1990s. Hang time is the time of a basketball player's jump, measured from the instant he or she leaves the floor until he or she returns to the floor. Hang time, t, depends on the height of the jump, s. These quantities are related by the equation

$$t = \frac{1}{2}\sqrt{s},$$

where s is the input variable measured in feet and t is the output variable measured in seconds.

The equation shows that to determine the hang time of any jump, you need to take square roots. You can calculate the square root of a fraction by taking the square root of the numerator and denominator separately. Symbolically, this property of square roots is written as

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, \quad b \neq 0.$$

Example 1 ***Determine*** $\sqrt{\frac{25}{49}}$.

SOLUTION

$$\sqrt{\frac{25}{49}} = \frac{\sqrt{25}}{\sqrt{49}} = \frac{5}{7}$$

Procedure

Determining the Square Root of a Fraction

1. Reduce the fraction under the radical, if possible.
2. Determine the square root of the numerator.
3. Determine the square root of the denominator.
4. The square root of the fraction is the quotient $\frac{\text{square root of the numerator}}{\text{square root of the denominator}}$.

4. Calculate the following square roots.

a. $\sqrt{\frac{1}{4}}$ **b.** $\sqrt{\frac{81}{100}}$ **c.** $\sqrt{\frac{4}{49}}$ **d.** $\sqrt{\frac{25}{64}}$

e. $\sqrt{\frac{121}{144}}$ **f.** $\sqrt{\frac{36}{64}}$ **g.** $\sqrt{\frac{25}{100}}$ **h.** $\sqrt{\frac{36}{81}}$

Example 2 ***Determine the hang time, t, of a $\frac{64}{100}$ -foot jump.***

SOLUTION

$$t = \frac{1}{2}\sqrt{\frac{64}{100}} = \frac{1}{2}\sqrt{\frac{16}{25}} \qquad \text{Reduce the fraction.}$$

$$= \frac{1}{2} \cdot \frac{\sqrt{16}}{\sqrt{25}} \qquad \text{Apply the property } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}.$$

$$= \frac{1}{2} \cdot \frac{4}{5} \qquad \text{Determine each of the square roots.}$$

$$= \frac{2}{5} \qquad \text{Multiply fractions; reduce to lowest terms.}$$

A $\frac{64}{100}$-foot jump has a hang time of $\frac{2}{5}$ seconds.

5. a. Use the hang time formula to determine the hang time of a $\frac{9}{16}$ -foot jump.

b. Determine the hang time of a $\frac{36}{49}$-foot jump.

Order of Operations Involving Fractions

When you evaluate a numerical expression that involves rational numbers, you follow the same order of operations you used with integers and whole numbers.

6. Calculate the value of each of the following numerical expressions.

a. $\frac{3}{4} - \frac{5}{8} \cdot \frac{4}{5}$

b. $\frac{5}{7} + \frac{1}{4} \div \frac{7}{8}$

c. $\frac{1}{2} \div \frac{1}{4} \cdot \frac{2}{5}$

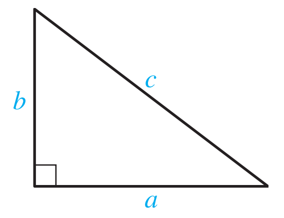
d. $\frac{5}{6}\left(\frac{3}{4} + \frac{2}{5}\right)$

e. $-\frac{7}{10}\left(\frac{3}{10}\right)^2$

f. $\frac{1}{5} + \left(\frac{1}{2} - \frac{1}{4}\right)^2 \div \frac{5}{16}$

Evaluating Formulas and Expressions Involving Fractions

In geometry, the well-known Pythagorean Theorem, $a^2 + b^2 = c^2$, relates the lengths of the sides of a right triangle. The shorter sides, a and b, of a right triangle are called the *legs*. The longest side, c, is called the *hypotenuse*.



If the lengths of sides a and b are known, the formula can be used to determine c^2. For example, if $a = 3$ and $b = 4$, then $c^2 = 3^2 + 4^2 = 9 + 16 = 25$. The formula is valid for rational numbers as well.

7. If the sides, a and b, of a right triangle measure $\frac{1}{2}$ in. and $\frac{2}{3}$ in., what is the value of c^2?

8. Notice that if the two known sides of a right triangle are c and b, then the third side a can be calculated by the formula $a^2 = c^2 - b^2$. Suppose $c = \frac{1}{2}$ and $b = \frac{2}{5}$. Find the value of a^2.

In Problem 7, c is the square root of c^2, so $c = \sqrt{c^2} = \sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$.

9. Determine the value of a in Problem 8.

You can replace c^2 in the equation $c = \sqrt{c^2}$ by its equivalent expression from the Pythagorean theorem, namely, $a^2 + b^2$. Then the formula $c = \sqrt{c^2}$ becomes $c = \sqrt{a^2 + b^2}$.

Example 3 ***Use the formula $c = \sqrt{a^2 + b^2}$ to determine c if $a = \frac{3}{5}$ and $b = \frac{4}{5}$.***

SOLUTION

$$c = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} \qquad \text{Replace } a \text{ and } b \text{ with their given values.}$$

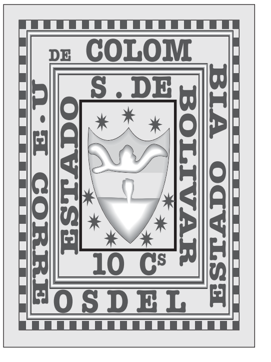
$$c = \sqrt{\frac{9}{25} + \frac{16}{25}} \qquad \text{Evaluate each term.}$$

$$c = \sqrt{\frac{16 + 9}{25}} = \sqrt{\frac{25}{25}} \qquad \text{Sum the two fractions.}$$

$$c = 1 \qquad \frac{25}{25} = 1 \text{ and } \sqrt{1} = 1$$

10. In a right triangle, $a = \frac{3}{8}$ and $b = \frac{1}{2}$. Determine the value of c.

Fractions and the Distributive Property



Bolivar, a province of Columbia, is reported to have issued the world's smallest stamp in 1863. Called the Bolivar 10c green, the stamp measures $\frac{8}{25}$ inch by $\frac{19}{50}$ inch. The inner border of the stamp measures $\frac{7}{25}$ inch by $\frac{11}{50}$ inch.

If a stamp collector wants to know the perimeter of the inner border, he or she might think of the formulas $P = 2l + 2w$ or $P = 2(l + w)$. In both formulas, P represents the perimeter, l, the length, and w, the width. For example, if $l = 5$, and $w = 7$, then $P = 2 \cdot 5 + 2 \cdot 7 = 24$ and $P = 2(5 + 7) = 2 \cdot 12 = 24$. The formulas demonstrate the distributive property, which was first presented in Chapter 1 for whole numbers. The distributive property extends to all numbers, including integers and rational numbers. Symbolically, it is stated as

$$a(b + c) = a \cdot b + a \cdot c$$

The process of combining like terms, which uses the distributive property, is the same for fractions as it is for integers and whole numbers.

11. Calculate the perimeter of the inner border by both formulas and compare the results to verify the distributive property for rational numbers.

The process of combining terms is also essential for solving equations.

Example 4 *Solve $\frac{1}{2}x + \frac{1}{4}x = \frac{3}{5}$ for x.*

SOLUTION

$$\left(\frac{1}{2} + \frac{1}{4}\right)x = \frac{3}{5}$$ Use the distributive property.

$$\left(\frac{2}{4} + \frac{1}{4}\right)x = \frac{3}{5}$$ Find a common denominator for combining the fractions.

$$\frac{3}{4}x = \frac{3}{5}$$ Add the fractions on left side of equation.

$$\frac{\cancel{4}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{4}}x = \frac{\cancel{3}}{5} \cdot \frac{4}{\cancel{3}}$$ Multiply each term in the equation by the reciprocal of $\frac{3}{4}$ and cancel common terms.

$$x = \frac{4}{5}$$

12. Solve the following equations for the unknown values. Check each of your results in the original equation.

a. $\frac{1}{3}x + \frac{1}{5}x = \frac{4}{9}$

b. $\frac{2}{3}x - \frac{9}{10}x = \frac{1}{6}$

c. $-\frac{1}{2}x - \frac{1}{6}x = \frac{3}{7}$

d. $-\frac{7}{8}x + \frac{3}{4}x = -\frac{1}{3}$

SUMMARY: ACTIVITY 4.4

1. To raise a fraction to a power, raise the numerator to the power and the denominator to the power. Symbolically,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0. \qquad \text{Example: } \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} = \frac{8}{27}.$$

2. To determine the square root of a fraction

a. Reduce the fraction under the radical, if possible.

b. Determine the square root of the numerator.

c. Determine the square root of the denominator.

d. The square root of the fraction is the quotient $\dfrac{\text{square root of the numerator}}{\text{square root of the denominator}}$.

$$\text{Symbolically, } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, b \neq 0. \qquad \text{Example: } \sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}.$$

3. Order of Operations and Fractions

The order of operations applies to whole numbers, integers, and fractions and to all rational numbers.

4. Distributive Property and Fractions

The distributive property applies to whole numbers, integers, and, fractions and to all rational numbers.

EXERCISES: ACTIVITY 4.4

1. Evaluate and reduce to lowest terms.

a. $\left(\frac{2}{9}\right)^2$ **b.** $\left(-\frac{3}{5}\right)^3$ **c.** $\left(-\frac{4}{7}\right)^2$ **d.** $\left(\frac{2}{3}\right)^5$

e. $\sqrt{\frac{9}{81}}$ **f.** $\sqrt{\frac{64}{100}}$ **g.** $\sqrt{\frac{60}{15}}$ **h.** $\sqrt{0}$

i. $\sqrt{\frac{0}{4}}$ **j.** $\sqrt{\frac{25}{0}}$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

2. Evaluate each expression.

a. $2x - 4(x - 3)$, where $x = \frac{1}{2}$

b. $a(a - 2) + a(3a - 2)$, where $a = \frac{1}{3}$

c. $2x + 6(x + 2)$, where $x = \frac{3}{4}$

d. $6xy - y^2$, where $x = \frac{5}{16}$ and $y = \frac{1}{8}$

e. $4x^3 + x^2$, where $x = \frac{1}{2}$

3. The measurement of one side of a square stamp is $\frac{3}{4}$ inch.

a. Determine the area of the stamp in square inches (use $A = s^2$).

b. If you had 16 of these stamps, what would be their total area in square inches?

c. Calculate your answer to part b in a different way. (*Hint:* One arrangement is to put all the stamps side by side and then use $A = l \cdot w$.)

4. Solve the following equations.

a. $\frac{6}{7}x = \frac{1}{2}$

b. $\frac{3}{11}x = -\frac{3}{13}$

c. $-\frac{5}{6}t = -\frac{10}{21}$

d. $-\frac{4}{5}y = \frac{24}{35}$

5. Due to the curvature of Earth's surface, the maximum distance, d, in kilometers that a person can see from a height h meters above the ground is given by the formula

$$d = \frac{7}{2}\sqrt{h}.$$

a. Use the formula to determine the maximum distance a person can see from a height of 64 meters.

b. Use the formula to determine the maximum distance a person can see from a height of 100 meters.

6. An object dropped from a height falls a distance of d feet in t seconds. The formula that describes this relationship is $d = 16t^2$. A math book is dropped from the top of a building that is 400 feet high.

a. How far has the book fallen in $\frac{1}{2}$ second?

b. After $\frac{3}{4}$ seconds, how far above the ground is the book?

7. Recall from the activity that hang time t for a jump is given by $t = \frac{1}{2}\sqrt{s}$, where s is the height of the jump in feet, and t is measured in seconds. Determine the hang time for the following heights.

a. $s = 1$ foot

b. $s = 3$ inches

c. $s = \frac{4}{9}$ foot

8. Solve the following equations for x.

a. $\frac{2}{3}x + \frac{1}{5}x = \frac{8}{15}$

b. $\frac{1}{16}x - \frac{3}{4}x = \frac{1}{2}$

c. $-\frac{1}{9}x - \frac{1}{3}x = \frac{1}{6}$

d. $-\frac{1}{12}x - \frac{3}{4}x = -\frac{5}{36}$

What Have I Learned?

1. **a.** Describe in your own words how to add or subtract fractions that have the same denominator.

b. Give an example.

2. **a.** Describe in your own words how to determine the LCD of two or more fractions.

b. Give an example.

3. When do you need to use the LCD of two or more fractions?

4. **a.** Describe in your own words how to add or subtract fractions that have different denominators.

b. Give an example.

5. Determine whether the following statements are true or false. Give a reason for each answer.

a. The LCD of two denominators must be greater than or equal to either of those denominators.

b. $\frac{2}{7} + \frac{4}{7} = \frac{6}{14}$

c. $\frac{2}{7} \cdot \frac{3}{7} = \frac{6}{7}$

6. What would be the advantage of reducing fractions to lowest terms before multiplying fractions?

7. Will a fraction that has a negative numerator and a negative denominator be positive or negative?

8. What is the sign of the product when you multiply a positive fraction by a negative fraction?

9. What is the sign of the quotient when you divide a negative fraction by a negative fraction?

10. Explain, by using an example, how to divide a positive fraction by a negative fraction.

11. Explain, by using an example, how to multiply a negative fraction by a negative fraction.

12. Why do you have to know the meaning of the word *reciprocal* in this chapter?

13. Explain how you would solve the following equation:

    one-fifth times a number equals negative four.

14. Explain the difference between squaring $\frac{1}{4}$ and taking the square root of $\frac{1}{4}$.

How Can I Practice?

1. Determine the missing numerators.

a. $\frac{3}{11} = \frac{?}{33}$

b. $\frac{5}{7} = \frac{?}{42}$

c. $\frac{8}{9} = \frac{?}{81}$

d. $\frac{7}{12} = \frac{?}{36}$

e. $\frac{13}{18} = \frac{?}{72}$

2. Reduce the following fractions to lowest terms.

a. $\frac{12}{18}$

b. $\frac{8}{72}$

c. $\frac{36}{39}$

d. $\frac{42}{48}$

e. $\frac{54}{82}$

3. Rearrange the following fractions in order from smallest to largest.

$$\frac{1}{6}, \frac{3}{4}, \frac{5}{8}, \frac{11}{12}, \frac{3}{24}$$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

4. When fog hit the New York City area, visibility was reduced to $\frac{1}{16}$ mile at JFK Airport, $\frac{1}{8}$ mile at La Guardia Airport, and $\frac{1}{2}$ mile at Newark Airport.

   a. Which airport had the best visibility?

   b. Which airport had the worst visibility?

5. Add the following and write the result in lowest terms.

   a. $\frac{4}{9} + \frac{3}{9}$

   b. $\frac{3}{17} + \frac{5}{17}$

   c. $\frac{5}{12} + \frac{1}{4}$

   d. $-\frac{5}{6} + \frac{11}{24}$

   e. $\frac{5}{12} + \frac{7}{18}$

   f. $\left(-\frac{17}{24}\right) + \frac{31}{48}$

   g. $-\frac{3}{10} + \frac{7}{30}$

   h. $\frac{5}{9} + \left(-\frac{6}{7}\right)$

   i. $-\frac{3}{7} + \left(-\frac{5}{13}\right)$

   j. $-\frac{3}{14} + \left(-\frac{6}{35}\right)$

6. Subtract and write the result in lowest terms.

   a. $\frac{9}{13} - \frac{2}{13}$

   b. $\frac{28}{54} - \frac{11}{54}$

   c. $\frac{13}{48} - \frac{5}{24}$

   d. $\frac{8}{11} - \frac{15}{44}$

   e. $\frac{15}{56} - \frac{9}{28}$

   f. $\frac{5}{24} - \frac{4}{9}$

g. $-\frac{7}{12} - \frac{1}{6}$

h. $-\frac{11}{42} - \frac{2}{7}$

i. $-\frac{4}{5} - \left(-\frac{11}{15}\right)$

j. $\frac{3}{8} - \left(-\frac{5}{12}\right)$

k. $\frac{8}{21} - \left(-\frac{5}{14}\right)$

l. $-\frac{3}{22} - \left(-\frac{5}{6}\right)$

m. $-\frac{9}{14} - \left(-\frac{4}{21}\right)$

n. $-\frac{14}{15} - \left(-\frac{11}{12}\right)$

7. Multiply and write each answer in lowest terms. Use a calculator to check answers.

a. $\frac{1}{9} \cdot \frac{2}{9}$

b. $\frac{10}{11} \cdot \frac{3}{13}$

c. $\frac{14}{15} \cdot \frac{-3}{28}$

d. $\frac{30}{35} \cdot \frac{10}{25}$

e. $\frac{-3}{10} \cdot \frac{-4}{9}$

f. $-20 \cdot \frac{2}{45}$

g. $\frac{10}{12} \cdot \frac{87}{100}$

h. $\frac{3}{16} \cdot \frac{2}{-3}$

8. Divide and write each answer in lowest terms. Use a calculator to check answers.

a. $\frac{-3}{7} \div \frac{9}{7}$

b. $\frac{8}{33} \div \frac{12}{11}$

c. $-\frac{4}{9} \div -\frac{20}{21}$

d. $\frac{24}{45} \div \frac{15}{18}$

e. $-\frac{\frac{9}{14}}{\frac{6}{7}}$

f. $-\frac{35}{54} \div -\frac{7}{5}$

9. Evaluate.

a. $-\left(\frac{4}{9}\right)^2$

b. $\sqrt{\frac{81}{121}}$

c. $\left(-\frac{3}{5}\right)^3$

d. $\sqrt{\frac{49}{64}}$

10. Calculate the value of each of the following numerical expressions.

a. $\frac{2}{3} - \frac{3}{4} \cdot \frac{1}{2}$

b. $\frac{2}{3}\left(-\frac{3}{4} + \frac{1}{2}\right)$

c. $\frac{2}{3} \div \frac{3}{4} \cdot \frac{5}{8}$

d. $-\frac{2}{5}\left(\frac{5}{8}\right)^2$

e. $-\frac{1}{4} + \left(\frac{1}{3} - \frac{5}{6}\right)^2 \div \frac{3}{8}$

11. Solve each of the following equations for x.

a. $\frac{3}{7} + x = -\frac{1}{7}$

b. $\frac{3}{5} + x = \frac{4}{5}$

c. $-\frac{3}{7} + x = -\frac{4}{5}$

d. $x - \frac{2}{9} = -\frac{4}{5}$

e. $-25x = 6$

f. $-5 = 42x$

g. $-63x = 0$

h. $\frac{-1}{6}x = -12$

i. $\frac{2}{5} = -8x$

j. $\frac{-3}{4}x = \frac{-9}{16}$

k. $\frac{2}{7}x + \frac{1}{3}x = -\frac{26}{49}$

l. $\frac{1}{4}x - \frac{3}{10}x = -5$

12. Translate the following into equations where x represents the unknown number. Then solve the equation for x.

a. The sum of a number and $\frac{2}{3}$ is $\frac{5}{6}$.

b. A number subtracted from $\frac{11}{15}$ is $\frac{3}{5}$.

c. $\frac{4}{13}$ times a number is -20.

d. $\frac{3}{5}$ is the product of a number and $\frac{9}{10}$.

e. The quotient of a number and -7 is -25.

13. While testing a new drug, doctors found that $\frac{1}{2}$ of the patients given the drug improved, $\frac{2}{5}$ showed no change in their condition, and the remaining patients got worse. What fraction of the patients taking the new drug got worse?

14. On your last math test, you answered $\frac{7}{8}$ of the questions. Of the questions that you answered, you got $\frac{4}{5}$ of them correct. What fraction of the questions on the test did you get right?

15. Your town has purchased a rectangular piece of land to develop a park. Three-fourths of the property will be used for the public. The rest of the land will be for park administration. The land will be divided so that $\frac{5}{6}$ of the public land will be used for picnicking and the remainder will be a swimming area.

a. Divide the rectangle into fourths and separate the public part from the park administration.

b. Shade the area that will be used for picnics.

c. Determine from the picture what fractional part of the whole piece of property will be used for picnics?

d. Write an expression and evaluate to show that part c is correct.

e. What fractional part of the property will be used by the park administration?

f. What fractional part of the property will be used for swimming?

g. Show that the fractional parts add up to the whole.

16. You drop your sunglasses out of a boat. Your depth finder on the boat measures 27 feet of water. You jump overboard with your goggles to see if you can rescue the glasses before they land on the bottom. You return for a gulp of air with your sunglasses in hand. You think they were about $\frac{2}{3}$ of the way down.

a. Estimate at what depth from the surface you found your glasses.

b. Write an expression that will show the distance from the surface of the water.

c. Evaluate the expression.

17. You decide to drop a coin into the gorge at Niagara Falls. The distance the coin will fall, in feet s, and in time t, in seconds, is given by the formula $s = 16t^2$.

a. After $\frac{1}{2}$ second how far has the coin fallen?

b. How far is the coin from the top of the gorge after $\frac{3}{4}$ seconds?

18. As a magician, you ask a friend to do this problem with you.

a. Ask your friend to select a number.

Then he should add three to the number.

Now have your friend multiply the result by $\frac{1}{3}$.

Next have him subtract 4.

Then, multiply by 6.

Finally, multiply by $\frac{1}{2}$, and ask your friend to tell you the answer.

Add 9 to the number your friend gives you. It will be the original number your friend selected.

b. Repeat part a with several numbers.

c. To determine the relationship between the original number selected and the final answer, let x represent the original number and write an expression to determine the final answer.

d. Simplify the expression.

Chapter 4 Summary

The bracketed numbers following each concept indicate the activity in which the concept is discussed.

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Greatest common factor (GCF) [4.1]	The greatest common factor of two numbers is the largest factor of both numbers.	GCF of 18 and 12 is 6.
Equivalent fractions [4.1]	To write an equivalent fraction, multiply or divide both the numerator and the denominator of a given fraction by the same nonzero number.	$\frac{4}{5} = \frac{?}{25}$ $\frac{4 \cdot 5}{5 \cdot 5} = \frac{20}{25}$
Reducing fractions [4.1]	To reduce a fraction to lowest terms, divide the numerator and denominator by their greatest common factor (GCF).	$\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$
Least common denominator (LCD) [4.1]	To determine the LCD, identify the largest denominator of the fractions involved. Look at multiples of the largest denominator. The smallest multiple that is divisible by all the denominators involved is the LCD.	Find the LCD of $\frac{1}{12}, \frac{5}{18}$: The largest denominator is 18. Multiples of 18 are 18, 36, 54, ... So the LCD is 36.
Comparing fractions [4.1]	Write the fractions as equivalent fractions with a common denominator. The larger fraction is the one with the largest numerator.	Compare $\frac{6}{7}, \frac{8}{9}$. $\frac{6}{7} = \frac{54}{63}$; $\frac{8}{9} = \frac{56}{63}$ $56 > 54$ $\frac{8}{9} > \frac{6}{7}$
Multiplying fractions [4.2]	To multiply fractions, multiply the numerators together and the denominators together. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	$\frac{3}{5} \cdot \frac{2}{7} = \frac{3 \cdot 2}{5 \cdot 7} = \frac{6}{35}$
The negative sign for a negative fraction [4.2]	There are three different ways to place the sign for a negative fraction.	$-\frac{2}{5} = \frac{-2}{5} = \frac{2}{-5}$ are all the same number.
Reciprocals [4.2]	The reciprocal of a nonzero number, written in fraction form, is the fraction obtained by interchanging the numerator and denominator of the original fraction. The product of a number and its reciprocal is always 1.	The reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$, because $\frac{5}{7} \cdot \frac{7}{5} = 1$.

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Dividing by fractions [4.2]	To divide a number by a fraction: 1. Multiply the dividend by the reciprocal of the divisor. 2. Reduce to lowest terms. $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$	$\frac{3}{8} \div \frac{5}{7} = \frac{3}{8} \cdot \frac{7}{5} = \frac{21}{40}$
Solving equations of the form $ax = b$ that involve fractions [4.2]	To solve the equation $$ax = b, a \neq 0,$$ divide each side of the equation by a to obtain the value for x.	Solve $\frac{-4}{15}x = \frac{1}{7}$ $\frac{-15}{4} \cdot \frac{-4}{15}x = \frac{1}{7} \cdot \frac{-15}{4}$ $x = -\frac{15}{28}$
Adding and subtracting fractions with the same denominator [4.3]	• If you have two fractions with the same denominator, add or subtract them by adding or subtracting the numerators, leaving the denominator the same. • If the resulting fraction is not in lowest terms, reduce it.	$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$ $\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$
Adding and subtracting fractions with different denominators [4.3]	Find the LCD, then convert each fraction to an equivalent fraction, using the LCD you found. Add or subtract the numerators, leaving the LCD in the denominator. If the resulting fraction is not in lowest terms, reduce it.	Add: $\frac{3}{4} + \frac{1}{7}$ $\frac{21}{28} + \frac{4}{28} = \frac{25}{28}$
Raising fractions to a power [4.4]	To raise a fraction to a power, raise the numerator to the power and raise the denominator to the power. The result is the quotient of the powers. Symbolically, $$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0.$$	$\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$
Square roots of fractions [4.4]	To determine the square root of a fraction: 1. Reduce the fraction under the radical, if possible. 2. Determine the square root of the numerator. 3. Determine the square root of the denominator. 4. The square root of the fraction is the quotient of the two square roots.	$\sqrt{\frac{25}{144}} = \frac{\sqrt{25}}{\sqrt{144}} = \frac{5}{12}$

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Order of operations [4.4]	The order of operation rules for rational numbers are the same as those applied to whole numbers.	$\frac{1}{9} + \frac{1}{3}\left(\frac{1}{4}\right) - \left(\frac{1}{2}\right)^2$ $= \frac{1}{9} + \frac{1}{12} - \frac{1}{4}$ $= \frac{4}{36} + \frac{3}{36} - \frac{9}{36}$ $= -\frac{2}{36} = -\frac{1}{18}$
Evaluating expressions or formulas that involve fractions [4.4]	To evaluate formulas or expressions involving rational numbers, substitute for the variables and evaluate using the order of operations.	Evaluate $6xy - y^2$, where $x = \frac{1}{3}$ and $y = 3$. $6xy - y^2 = 6\left(\frac{1}{3}\right)(3) - 3^2$ $= 6 - 9 = -3$
The distributive properties [4.4]	The distributive properties, $a(b + c) = ab + ac$ $a(b - c) = ab - ac,$ hold for rational numbers.	$\frac{3}{4}\left(\frac{1}{2} + \frac{1}{2}\right) = \frac{3}{4}\cdot\frac{1}{2} + \frac{3}{4}\cdot\frac{1}{2}$ $= \frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$ $\frac{1}{2}\left(\frac{3}{5} - \frac{2}{5}\right) = \frac{1}{2}\cdot\frac{3}{5} - \frac{1}{2}\cdot\frac{2}{5}$ $= \frac{3}{10} - \frac{2}{10} = \frac{1}{10}$
Solving equations of the form $ax + bx = c$ that involve fractions [4.4]	To solve, combine like terms with the distributive property. Then, solve in the same way as the form $ax = b$.	$\frac{1}{3}x + \frac{1}{4}x = \frac{1}{2}$ $\left(\frac{1}{3} + \frac{1}{4}\right)x = \frac{1}{2}$ $\frac{7}{12}x = \frac{1}{2}$ $x = \frac{1}{2}\left(\frac{12}{7}\right) = \frac{12}{14} = \frac{6}{7}$

Chapter 4 Gateway Review

1. Use $>$ or $<$ to compare the following fractions.

a. $\frac{3}{7} \square \frac{2}{5}$

b. $\frac{4}{11} \square \frac{1}{3}$

c. $\frac{4}{7} \square \frac{5}{8}$

d. $\frac{6}{11} \square \frac{1}{12}$

e. $\frac{7}{10} \square \frac{5}{8}$

f. $\frac{8}{11} \square \frac{3}{4}$

2. Perform the indicated operations. Write your answer in lowest terms.

a. $\frac{4}{7} + \frac{2}{7}$

b. $-\frac{13}{15} + \frac{7}{15}$

c. $\frac{9}{11} - \frac{2}{11}$

d. $\frac{3}{8} + \frac{2}{7}$

e. $\frac{4}{13} + \frac{14}{39}$

f. $-\frac{5}{12} + \left(-\frac{7}{18}\right)$

g. $-\frac{4}{5} - \left(-\frac{13}{15}\right)$

h. $\frac{6}{11} - \left(+\frac{20}{33}\right)$

i. $\frac{1}{9} + \frac{5}{9}$

j. $-\frac{3}{20} + \left(-\frac{13}{20}\right)$

k. $-\frac{3}{20} + \left(-\frac{3}{5}\right)$

l. $-\frac{7}{10} - \frac{2}{15}$

m. $\frac{3}{16} - \frac{15}{16}$

n. $\frac{5}{9} - \frac{7}{12}$

o. $\frac{4}{5} - \frac{9}{10}$

p. $-\frac{3}{8} - \frac{5}{12}$

q. $\frac{2}{9} - \left(-\frac{3}{5}\right)$

r. $\frac{7}{18} - \left(-\frac{10}{27}\right)$

s. $-\frac{4}{5} + \frac{3}{10}$

t. $-\frac{5}{7} + \frac{3}{4}$

u. $\frac{4}{9} \cdot -\frac{15}{14}$

Answers to all Gateway exercises are included in the Selected Answers appendix.

v. $\frac{1}{6} \div \frac{3}{8}$

w. $\frac{9}{20} \cdot \left(-\frac{4}{15}\right)$

x. $-\frac{2}{15} \div \frac{6}{35}$

3. Perform the indicated operations.

a. $\left(-\frac{8}{13}\right)^2$

b. $-\left(-\frac{3}{4}\right)^2$

c. $\sqrt{\frac{121}{144}}$

4. Evaluate each expression.

a. $-\frac{5}{9} \div \frac{20}{27} - \frac{2}{9} \cdot \left(-\frac{15}{16}\right)$

b. $\frac{6 - (2)^2}{10 - 6 \cdot 4 - 3^2}$

5. Evaluate each expression.

a. $2x - (x - y)^2 \div (xy)$, where $x = -\frac{1}{3}$ and $y = -\frac{1}{4}$

b. $-3xy - x^2$, where $x = -\frac{1}{2}$ and $y = -\frac{1}{3}$

6. Solve for the unknown in each of the following.

a. $x - \frac{5}{8} = \frac{1}{6}$

b. $x + \frac{7}{9} = \frac{3}{7}$

c. $\frac{3}{11} + x = -\frac{5}{22}$

d. $-\frac{2}{3} + x = -\frac{3}{4}$

e. $x + \frac{5}{14} = -\frac{3}{7}$

f. $x - \frac{4}{9} = -\frac{2}{5}$

g. $-12x = -72$

h. $\frac{-x}{6} = 14$

i. $-\frac{4}{9} = -18x$

j. $-\frac{5}{9} = \frac{20}{33}s$

k. $-\frac{7}{8}x + \frac{5}{6}x = \frac{3}{8}$

l. $-\frac{2}{5}x - \frac{3}{7}x = -\frac{2}{7}$

7. Translate each of the following verbal statements into an equation and solve for the unknown value. Let x represent the unknown number.

a. The sum of a number and $\frac{1}{2}$ is $\frac{5}{6}$.

b. A number increased by $\frac{2}{3}$ is $\frac{1}{4}$.

c. The difference of $\frac{3}{7}$ and a number is $\frac{2}{5}$.

d. The quotient of a number and -15 is -7.

e. $-\frac{3}{11}$ times a number is 18.

f. $\frac{2}{3}$ is the product of a number and $\frac{8}{9}$.

8. A game of Scrabble has 100 letter tiles, 42 of which are vowels.

a. What fraction of Scrabble's letter tiles are vowels?

b. Super Scrabble has 200 letter tiles, 75 of which are vowels. Which game has a greater fraction of vowels?

9. Your 20-ounce bottle of Coca-Cola states that a standard serving is 8 ounces.

a. What portion of the bottle should you drink for one serving? (Write as a fraction and reduce, if possible.)

b. If you drink the whole 20-ounce bottle, you would consume $\frac{11}{50}$ of the carbohydrates that a person should consume in one day. Use your answer from part a to help you determine what part of a day's carbohydrates you would get in only one serving.

10. Inspired by artist Nathan Sawaya's museum exhibit *The Art of the Brick*, you decide to make your own life-size sculpture completely of Legos. You read that the 29 figures in the exhibit used about a million bricks.

a. Assuming each figure takes about the same number of bricks, write a fraction to represent the portion of the Legos used for one figure.

b. Use your answer from part a to estimate the number of Lego bricks for one figure.

c. You have $3000 saved to use for this massive summer project. A standard Lego brick may be purchased for 26 cents, or $\frac{26}{100}$ of a dollar. Write a fraction to represent the portion of the necessary cash you have.

d. To have a better understanding of where you are with your budget, round the amount of cash needed to the nearest thousand, and redo part c.

11. You need to use $\frac{1}{2}$ your next paycheck for rent, and $\frac{1}{8}$ of it for two new tires. How much of your paycheck will be left for other purposes?

12. From the beginning of the H1N1 flu pandemic until the end of 2009, the U.S. Center for Disease Control estimates that about 50 million people in the United States had been infected with the virus, 200,000 had been hospitalized as a result of the virus, and 10,000 had died.

a. Write a fraction to represent the estimated fraction of those infected who died due to H1N1.

b. Assume that all those who died from H1N1 were first hospitalized. What fraction of persons hospitalized from H1N1 died from the virus?

Chapter 5 Problem Solving with Mixed Numbers and Decimals

Rational numbers and fractions can be expressed verbally and symbolically in several different formats. In Chapter 4, fractions were written in the form $\frac{a}{b}$, where a and b are integers, $b \neq 0$. Sometimes rational numbers are represented by the sum of an integer and a proper fraction in a format called a mixed number; for example, $3\frac{1}{2}$ is read "three and one-half." More frequently, a fraction is represented in decimal format, where the denominator of the fraction is 10 or a power of 10. For example, the decimal 0.3 represents the fraction $\frac{3}{10}$, which is read "three-tenths." The decimal number 0.17 represents the fraction $\frac{17}{100}$ and is read "seventeen-hundredths."

In Chapter 5 you will study the properties of rational numbers in these various formats and use them to solve problems. In particular, you will study how the metric system, used in many parts of the world and in the sciences everywhere, takes advantage of the special properties of decimal numbers.

Cluster 1 Mixed Numbers and Improper Fractions

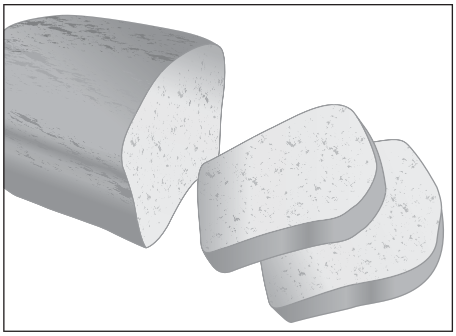
Activity 5.1

Food for Thought

Objectives

1. Determine equivalent fractions.
2. Add and subtract fractions and mixed numbers with the same denominator.
3. Convert mixed numbers to improper fractions and improper fractions to mixed numbers.

Bread has been a food staple for people everywhere since recorded time. It comes in many shapes, sizes, textures, tastes, and shelf lives, reflecting the influence of culture throughout the world. East Indians bake their naan bread daily and eat it immediately. Germans are well known for their dark pumpernickel that can last for months when properly stored.

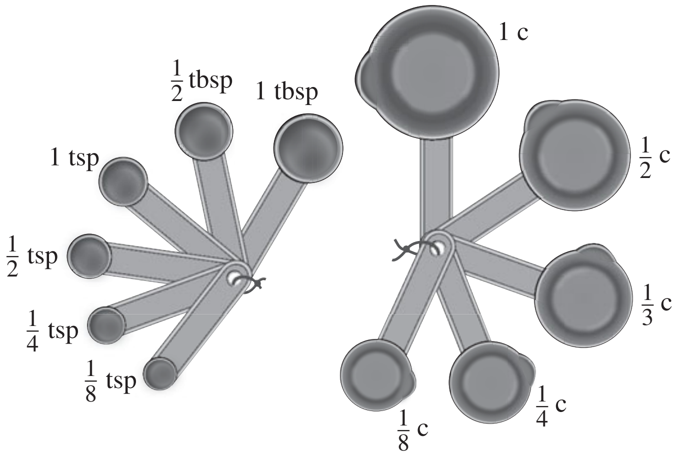


Bread recipes, like many other recipes, include ingredients measured in fractions and mixed numbers. For example, a bread recipe may call for $2\frac{1}{2}$ cups of flour, $\frac{1}{2}$ teaspoon of baking powder, and so on. Changing recipes requires basic calculations, usually involving fractions and mixed numbers.

Bread, Mixed Numbers, and Improper Fractions

Many people who bake yeast bread at home these days use a bread machine to shorten preparation time. Here is a basic recipe for whole wheat bread baked in a machine.

WHOLE WHEAT BREAD (makes 1 loaf)	
$1\frac{1}{3}$ cup warm water	$2\frac{1}{4}$ cups bread flour
3 tbsp. honey plus $1\frac{3}{4}$ tsp. honey	$1\frac{3}{4}$ cups whole wheat flour
$\frac{1}{2}$ tsp. salt	$2\frac{1}{3}$ tsp. vital wheat gluten
2 tbsp. plus $1\frac{1}{4}$ tsp. canola oil	$2\frac{1}{4}$ tsp. active dry yeast
Abbreviations: tsp = teaspoon; tbsp = tablespoon	



1. The recipe calls for $1\frac{3}{4}$ cups of whole wheat flour. A standard set of 6 measuring cups includes $\frac{1}{4}$-, $\frac{1}{3}$-, $\frac{1}{2}$-, $\frac{2}{3}$-, $\frac{3}{4}$-, and 1-cup sizes. Explain how you would measure out $1\frac{3}{4}$ cups of whole wheat flour using only two cups from the set.

Problem 1 illustrates the fact that $1\frac{3}{4}$ is a shorthand way to write the sum $1 + \frac{3}{4}$. Because $1\frac{3}{4}$ is a combination of an integer and a fraction, it is called a **mixed number**.

2. Explain how you would measure $1\frac{3}{4}$ cups whole wheat flour using only the $\frac{1}{4}$-cup size.

Problem 2 shows that you will need to use the $\frac{1}{4}$-cup size seven times to measure out the whole wheat flour, leading to the calculation $7 \cdot \frac{1}{4} = \frac{7}{4}$. The calculation shows that amount of whole wheat flour required can be expressed as $\frac{7}{4}$ cups. The fraction $\frac{7}{4}$ is called an **improper fraction** because the numerator 7 is greater than the denominator 4.

3. Why is the mixed number $1\frac{3}{4}$ in Problem 1 equivalent to the improper fraction $\frac{7}{4}$ in Problem 2?

Definitions

An **improper fraction** is a fraction in which the absolute value of the numerator is greater than or equal to the absolute value of the denominator.

A **mixed number** represents the **sum** of an integer and a proper fraction. This sum is implied, meaning that the sum sign, $+$, is not written between the integer and the fraction. For example, the positive mixed number $2\frac{3}{5}$ represents $2 + \frac{3}{5}$, and the negative mixed number $-2\frac{3}{5}$ represents the sum of a negative integer and a negative proper fraction, $-2 + \left(-\frac{3}{5}\right)$, or $-2 - \frac{3}{5}$.

It is often helpful to convert mixed numbers into improper fractions and vice versa.

Example 1

Convert $2\frac{11}{12}$ to an improper fraction.

SOLUTION

$$2\,\frac{11}{12} = 2 + \frac{11}{12} = \frac{2 \cdot 12}{1 \cdot 12} + \frac{11}{12} = \frac{24}{12} + \frac{11}{12} = \frac{35}{12}$$

Note in Example 1 that the integer 2 was rewritten as an equivalent fraction with denominator 12 so it could be added to the fractional part $\frac{11}{12}$. This observation leads to a shortcut method to convert a mixed number to an improper fraction:

$$2\frac{11}{12} = \frac{2 \cdot 12 + 11}{12} = \frac{35}{12}$$

Improper fractions, like proper fractions, can be negative. A mixed number is negative when it is equivalent to a negative improper fraction. The integer part and the fraction part are both negative. For example, $-1\frac{1}{2} = -\frac{3}{2}$, where $-1\frac{1}{2}$ means $-1 + \left(-\frac{1}{2}\right)$, or, $-1 - \frac{1}{2}$.

4. a. To verify that $-1\frac{1}{2} = -\frac{3}{2}$, carry out the addition: $-1 + \left(-\frac{1}{2}\right)$ and write your result as an improper fraction.

b. Verify $-1\frac{1}{2} = -\frac{3}{2}$ by using the shortcut method for converting a mixed number into an improper fraction. *Hint:* Recall from Chapter 3 that every negative number can be written as the product of its opposite (a positive number) and -1; that is, $-1\frac{1}{2}$ can be written as $-1 \cdot \left(1\frac{1}{2}\right)$.

c. Compare your answers from part a and part b and write what you observe.

Procedure

Converting a Mixed Number to an Improper Fraction

1. To convert a positive mixed number to an improper fraction, multiply the whole-number part by the denominator and add the numerator. The result is the numerator of the improper fraction. The denominator is unchanged.
2. To convert a negative mixed number $-a\frac{b}{c}$ to an improper negative fraction:
   a. Rewrite the mixed number as $-1 \cdot \left(a\frac{b}{c}\right)$.
   b. Convert $a\frac{b}{c}$ to an improper fraction, following the procedure for positive mixed numbers.
   c. Write the final result as a negative improper fraction.

5. Write the following mixed numbers as improper fractions in lowest terms.

   a. $1\frac{5}{8}$

   b. $4\frac{9}{12}$

   c. $-5\frac{6}{10}$

   d. $-3\frac{2}{7}$

Example 2

Convert $-\frac{35}{12}$ to a mixed number, in lowest terms.

SOLUTION

Rewrite $-\frac{35}{12}$ as $-1 \cdot \frac{35}{12}$.

$$\begin{array}{r} 2 \\ 12\overline{)35} \\ -24 \\ \hline 11 \end{array}$$

$$-1 \cdot \left(2\frac{11}{12}\right) = -2\frac{11}{12}$$

Procedure

Converting an Improper Fraction to a Mixed Number

1. To convert a positive improper fraction to a mixed number, divide the numerator by the denominator. The quotient is the whole-number part of the mixed number, the remainder is the numerator of the fractional part, and the divisor is the denominator of the fractional part.
2. To convert a negative improper fraction to a negative mixed number:
   a. Write the improper fraction $-\frac{a}{b}$ as the product $-1 \cdot \frac{a}{b}$, where a and b are positive integers.
   b. Follow the procedure for converting a positive improper fraction to a mixed number.
   c. Affix the negative sign for the final result.

6. Write the given improper fractions as mixed numbers in lowest terms.

a. $\frac{5}{2}$ **b.** $\frac{24}{10}$ **c.** $-\frac{29}{7}$ **d.** $-\frac{48}{5}$

Adding Mixed Numbers with Like Denominators

Example 3 ***The whole wheat bread recipe calls for $2\frac{1}{4}$ cups bread flour and $1\frac{3}{4}$ cups whole wheat flour. What is the total amount of flour for the whole wheat bread recipe?***

SOLUTION

$$2\frac{1}{4} = 2 + \frac{1}{4}$$ Definition of mixed number.

$$+1\frac{3}{4} = 1 + \frac{3}{4}$$

$$= 3 + \frac{4}{4}$$ Sum of integer and fraction part.

$$= 3 + 1 = 4$$ Sum in simplest form.

The total amount of flour in the whole wheat bread recipe is 4 cups.

Procedure

Adding Mixed Numbers

To add mixed numbers with the same denominators, add the integer parts and the fraction parts separately. If the sum of the fraction parts is an improper fraction, convert it to a mixed number and add the integer parts. Reduce the fraction part to lowest terms if necessary.

7. For each of the following, perform the indicated operation. Write the fractional part of the mixed number in lowest terms.

a. $10\frac{3}{7} + 15\frac{2}{7}$ **b.** $1\frac{1}{6} + 9\frac{5}{6}$

c. $12\frac{1}{8} + 48\frac{5}{8}$ **d.** $1\frac{13}{16} + 4\frac{5}{16}$

Subtracting Mixed Numbers with Like Denominators

Example 4 ***To replace some old electrical wiring in your house, you used $35\frac{7}{10}$ feet of electrical wire from the $62\frac{4}{10}$ feet that you had. How much wire do you have left after completing the repair?***

SOLUTION

Subtract $35\frac{7}{10}$ from $62\frac{4}{10}$ to determine the amount of wire left over.

$$\begin{array}{rcll} 62\frac{4}{10} & = & 62 + \frac{4}{10} & \text{Definition of mixed number.} \\ -35\frac{7}{10} & = & -35 - \frac{7}{10} & \\ \hline & = & 27 - \frac{3}{10} & \text{Differences of integer and fraction parts.} \\ & = & 26 + \frac{10}{10} - \frac{3}{10} & \text{Split 27 into 26 + 1 and convert 1 to fraction form.} \\ & = & 26\frac{7}{10} & \text{Difference in simplest form.} \end{array}$$

I have $26\frac{7}{10}$ feet of wiring left over.

Procedure

Subtracting Mixed Numbers

To subtract one mixed number from another, subtract the integer parts and the fraction parts separately. If the resulting integer and fraction parts have opposite signs as in Example 4, remove 1 from the integer part to complete the subtraction. Reduce the fraction part to lowest terms if necessary.

8. Perform each subtraction. Write the fractional part of the mixed number in lowest terms.

a. $13\frac{5}{7} - 9\frac{4}{7}$ **b.** $9\frac{5}{6} - 8\frac{1}{6}$

c. $45\frac{9}{13} - 17\frac{5}{13}$

d. $14\frac{11}{16} - 4\frac{5}{16}$

Some subtractions of mixed numbers may require additional steps. The following problems involve extra steps. In each problem, identify the extra step(s).

9. You need $5\frac{1}{3}$ tablespoons of butter to bake a small apple tart. You have one stick of butter, which is 8 tablespoons. How much butter will you have left after you make the tart?

10. The purpose of baseboard molding in a room is to finish the area where the wall meets the floor. A carpenter got a good deal on the molding he wanted because he was willing to buy the last $112\frac{3}{10}$ feet of it that the store had. If he used $96\frac{9}{10}$ feet to finish two rooms, how much did he have left over?

11. Use the following beef stew recipe to solve the problems in parts a–d.

Basic Beef Stew

$2\frac{1}{8}$ cups of chunked, cooked beef	$1\frac{5}{8}$ cups chopped onions
$3\frac{3}{8}$ cups of broth	2 cups chunked carrots
$\frac{1}{8}$ cup of garlic salt	$1\frac{7}{8}$ cups chopped potatoes

a. You need to mix all these ingredients together in a large mixing bowl. To determine what size mixing bowl to use, add all ingredient amounts listed in the recipe. What is the total number of cups of ingredients in the stew?

b. Your mixing bowls come in 10-cup, 15-cup, and 20-cup sizes. Which mixing bowl should you use? Explain.

c. How much space do you have for mixing the ingredients in the bowl you have chosen?

d. A friend who does not like potatoes is coming to dinner. So you decide to take the potatoes out of the recipe. Use subtraction to determine the new total number of cups of ingredients in the potato-free version of the stew.

12. Perform the indicated operations for each of the following.

a. $-4\frac{3}{7} + 2\frac{2}{7}$

b. $-2\frac{3}{8} - \left(-5\frac{2}{8}\right)$

c. $-4\frac{2}{9} - 7\frac{8}{9}$

SUMMARY: ACTIVITY 5.1

Mixed Numbers and Improper Fractions

1. An **improper fraction** is a fraction in which the absolute value of the numerator is greater than or equal to the absolute value of the denominator. For example, $\frac{7}{3}$ and $\frac{-4}{4}$ are improper fractions.

2. A **mixed number** represents the **sum** of an integer and a proper fraction. This sum is implied, meaning that the sum sign, "+," is not written between the integer and the fraction. For example, the positive mixed number $4\frac{3}{7}$ represents $4 + \frac{3}{7}$ and the negative mixed number $-4\frac{3}{7}$ represents the sum of a negative integer and a negative proper fraction, $-4 + \left(-\frac{3}{7}\right)$ or $-4 - \frac{3}{7}$.

3. To convert a positive mixed number to an improper fraction, multiply the whole number part by the denominator and add the numerator. The result is the numerator of the improper fraction; the denominator remains unchanged.

4. To convert a negative mixed number $-a\frac{b}{c}$ to an improper negative fraction:

   a. Rewrite the mixed number as $-1 \cdot \left(a\frac{b}{c}\right)$.

   b. Convert $a\frac{b}{c}$ to an improper fraction, following the procedure for positive mixed numbers.

   c. Write the final result as a negative improper fraction.

5. To convert a positive improper fraction to a mixed number, divide the numerator by the denominator. The quotient is the whole number part of the mixed number and the remainder becomes the numerator of the fraction part; the denominator remains unchanged.

6. To convert a negative improper fraction to a negative mixed number:

   a. Write the improper fraction $-\frac{a}{b}$ as the product $-1 \cdot \frac{a}{b}$, where a and b are positive integers.

   b. Follow the procedure for converting a positive improper fraction to a mixed number.

   c. Affix the negative sign for the final result.

7. To add mixed numbers that have the same denominator, add the integer parts and the fraction parts separately. If the sum of the fraction parts is an improper fraction, convert it to a mixed number and add the integer parts. Reduce the fraction part to lowest terms if necessary.

8. To subtract one mixed number from another, subtract the integer parts and the fraction parts separately. If the resulting integer and fraction parts have opposite signs, remove 1 from the integer part to complete the subtraction. Reduce the fraction part to lowest terms if necessary.

9. A fraction or the fractional part of a mixed number in an answer should always be written in lowest terms.

EXERCISES: ACTIVITY 5.1

Reduce the following improper fractions to lowest terms and convert to mixed numbers.

1. $\frac{5}{3}$

2. $-\frac{21}{14}$

Convert the following mixed numbers to improper fractions in lowest terms.

3. $3\frac{3}{8}$

4. $-5\frac{6}{8}$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

5. You have two pieces of lumber. One piece measures 6 feet long and the second piece measures $\frac{25}{4}$ feet long.

a. Write 6 as an improper fraction with a denominator of 4.

b. Compare the lengths of the two pieces of lumber using improper fractions. Which piece is longer? Explain.

Perform the indicated operation. Write all results as mixed numbers in simplest form.

6. $\frac{2}{3} + \frac{2}{3}$

7. $\frac{1}{4} + \frac{5}{4}$

8. $\frac{20}{12} - \frac{4}{12}$

9. $2\frac{2}{5} + 5\frac{1}{5}$

10. $4\frac{5}{7} - 3\frac{2}{7}$

11. $3\frac{1}{4} + 2\frac{1}{4}$

12. $8\frac{2}{5} - 5\frac{4}{5}$

13. $-7\frac{4}{9} + 5\frac{2}{9}$

14. $6\frac{11}{17} - 8\frac{15}{17}$

15. $8\frac{7}{11} + \left(-13\frac{5}{11}\right)$

16. $-14\frac{7}{8} - \left(-3\frac{3}{8}\right)$

17. $-12\frac{5}{13} - 8\frac{9}{13}$

18. Your bedroom measures $12\frac{1}{8}$ feet by $10\frac{3}{8}$ feet. You want to put a wallpaper border around the perimeter of the room. How much wallpaper border do you need? Remember, the perimeter of a rectangle is the sum of the lengths of the four sides.

Activity 5.2

Mixing with Denominators

Objectives

1. Determine the least common denominator (LCD) for two or more mixed numbers.
2. Add and subtract mixed numbers with different denominators.
3. Solve equations of the form $x + b = c$ and $x - b = c$ that involve mixed numbers.

You are going to do some baking and have chosen a rye bread recipe. For a $1\frac{1}{2}$-pound loaf, it calls for $2\frac{7}{8}$ cups of white flour and $1\frac{1}{2}$ cups of rye flour. You are a little short on the white flour, but you figure it will be fine as long as you use the correct total number of cups. How many cups of flour will you need? In Activity 4.3, you learned to find a common denominator to add or subtract fractions with unlike denominators. A similar process will be needed to add the fractional parts of the mixed numbers to solve the rye flour problem.

1. a. Add the integer parts of the cups needed.

   b. Add the fractional parts of the cups needed, and write the answer as a mixed number.

   c. How many total cups of flour do you need for the recipe? Write as a mixed number.

   d. Explain how you obtained your answer in part c.

Adding or Subtracting Two or More Mixed Numbers with Unlike Denominators

Example 1 ***Calculate*** $\mathbf{1\frac{2}{3} + 4\frac{5}{6}}$.

SOLUTION

$1\frac{2}{3} + 4\frac{5}{6} = 1 + 4 + \frac{2}{3} + \frac{5}{6}$	**A mixed number is a sum of an integer and a proper fraction.**
$= 5 + \frac{4}{6} + \frac{5}{6}$	**Rewrite proper fractions in terms of the LCD.**
$= 5 + \frac{9}{6}$	**Calculate the sum of the proper fractions.**
$= 5 + 1\frac{3}{6}$	**Convert the improper fraction to a mixed number.**
$= 6\frac{1}{2}$	**Add and write final result as a mixed number in lowest terms.**

2. In the following expressions, add the mixed numbers and write the final result as a single mixed number in lowest terms.

   a. $5\frac{1}{4} + 9\frac{3}{5}$

b. $8\frac{5}{6} + 7\frac{3}{8}$

c. $-2\frac{1}{6} - 4\frac{1}{4}$

d. $1\frac{1}{2} + 6\frac{7}{10} + 4\frac{1}{3}$

Example 2 ***Calculate*** $\boldsymbol{4\frac{2}{3} - 1\frac{2}{5}}$**.**

SOLUTION

$$4\frac{2}{3} - 1\frac{2}{5} = 4 + \frac{2}{3} - 1 - \frac{2}{5}$$ A mixed number is a sum of an integer and a proper fraction.

$$= 4 - 1 + \frac{2}{3} - \frac{2}{5}$$ Separate the integer parts from the fraction parts.

$$= 3 + \frac{10}{15} - \frac{6}{15}$$ Determine the LCD and rewrite the fractions in terms of the LCD.

$$= 3 + \frac{4}{15}$$ Calculate the difference of the proper fractions.

$$= 3\frac{4}{15}$$ Write the result as a mixed number.

Procedure

Adding or Subtracting Mixed Numbers with Unlike Denominators

Add or subtract the integer parts and the fraction parts separately. To combine the fraction parts, determine the LCD and rewrite them as fractions with like denominators; then add or subtract and write the result in lowest terms. If the sum of the fractions is an improper fraction, rewrite it as a mixed number and combine the integer parts, with careful attention to the signs of the numbers.

3. In the following expressions, subtract the mixed numbers and write the final result as a single mixed number in lowest terms.

a. $9\frac{5}{7} - 2\frac{3}{5}$ **b.** $10\frac{5}{6} - 7\frac{3}{8}$ **c.** $1\frac{1}{2} + 6\frac{1}{4} - 1\frac{1}{5}$

Pay close attention to the next example. It shows how to handle the situation in which the final integer and fraction parts have opposite signs.

Example 3

Calculate $\mathbf{9\frac{1}{2} - 4\frac{2}{3}}$.

SOLUTION

$9\frac{1}{2} - 4\frac{2}{3} = 9 - 4 + \frac{1}{2} - \frac{2}{3}$ **Separate the integer and fraction parts.**

$= 5 + \frac{3}{6} - \frac{4}{6}$ **Determine the LCD and rewrite fractions in terms of LCD.**

$= 5 - \frac{1}{6}$ **Calculate the difference of the fractions.**

$= 4 + \frac{6}{6} - \frac{1}{6} = 4 + \frac{5}{6}$ **Rewrite 5 as 4 + 1 in the form $4 + \frac{6}{6}$ and subtract.**

$= 4\frac{5}{6}$ **Write final result as a mixed number.**

4. Calculate the following numerical expressions and write the final result as a single mixed number in lowest terms.

a. $5\frac{1}{4} - 2\frac{1}{3}$

b. $-2\frac{5}{6} + 7\frac{3}{8}$

c. $1\frac{1}{2} - 3\frac{4}{5}$

d. $6\frac{1}{4} - 3\frac{1}{4} - 5\frac{1}{3}$

5. You need $10\frac{2}{3}$ yards of linen fabric to make curtains for your bedroom. While shopping, you find a remnant that is $15\frac{3}{4}$ yards long and you purchase it because you get a deep discount if you buy the whole piece. How many yards will you have left over?

6. A triangle has a perimeter of $7\frac{3}{5}$ inches. Two of the sides measure $2\frac{1}{8}$ inches and $2\frac{3}{8}$ inches, respectively. Determine the length of the third side.

Solving Equations Involving Mixed Numbers

When **solving equations** that include fractions or mixed numbers, several techniques are available, and each is worth knowing. The following examples and problems show why.

Example 4 ***Solve the equation $x + 1\frac{1}{3} = 3\frac{1}{2}$.***

SOLUTION

$x + 1\frac{1}{3} - 1\frac{1}{3} = 3\frac{1}{2} - 1\frac{1}{3}$ — Subtract $1\frac{1}{3}$ from each side of the equation to isolate x.

$x = 3\frac{1}{2} - 1\frac{1}{3}$ — x is now isolated on the left-hand side of the equation.

$x = 3 - 1 + \frac{1}{2} - \frac{1}{3}$ — Separate the integer from the fraction in each mixed number.

$x = 2 + \frac{3}{6} - \frac{2}{6}$ — Find the LCD of the fractions; determine equivalent fractions in terms of the LCD.

$x = 2 + \frac{1}{6}$ — Calculate the difference between the fractions and simplify.

$x = 2\frac{1}{6}$ — Write the result as a mixed number.

7. You decide to make lunch for your fraternity picnic and discover that you all like salad. Your favorite recipe for a simple salad dressing is made of oil and vinegar. You need a total of $2\frac{1}{2}$ cups of salad dressing, of which $1\frac{7}{8}$ cups is oil. Determine the amount of vinegar you need to add.

a. If x represents the amount of vinegar to be used in the salad dressing, write an equation relating the amounts of oil and vinegar for the $2\frac{1}{2}$ cups of salad dressing.

b. Solve the equation for x using the method shown in Example 4.

8. Solve the following equations using the method shown in Example 4.

a. $x + 4\frac{3}{8} = 5\frac{1}{2}$

b. $x - 6\frac{2}{5} = 9\frac{3}{10}$

c. $x + 5\frac{1}{2} = 2\frac{1}{3}$

Note that the method of adding or subtracting mixed numbers shown in Example 4 worked easily in Problems 7 and 8. The following example illustrates another approach for handling mixed numbers in solving an equation.

Recall the Fundamental Principle of Equality (Chapter 2), which states that performing the same operation on each term in a true equation will result in a true equation. Example 5 uses this principle.

Example 5

Solve the equation $x - 1\frac{1}{2} = 2\frac{1}{3}$ by using the LCD of the fraction parts to apply the Fundamental Principle of Equality. Write the answer as a mixed number.

SOLUTION

$x - \frac{3}{2} = \frac{7}{3}$	Convert mixed numbers to improper fractions.
$x = \frac{7}{3} + \frac{3}{2}$	Add $\frac{3}{2}$ to both sides.
$\text{LCD} = 2 \cdot 3 = 6$	Determine LCD of the fraction.
$6x = 6\left(\frac{7}{3}\right) + 6\left(\frac{3}{2}\right)$	Multiply each term by 6.
$6x = 14 + 9$	Carry out the calculations to simplify the equation.
$6x = 23$	
$x = \frac{23}{6} = 3\frac{5}{6}$	To solve for x, divide by 6. Convert the improper fraction to a mixed number.

9. Solve the following equations using the method in Example 5.

a. $x - 4\frac{1}{2} = 8\frac{3}{4}$

b. $x + 6\frac{2}{3} = 9\frac{8}{10}$

c. $x + 3\frac{1}{2} = 2\frac{1}{3}$

10. a. Solve the equation in Problem 7 using the method shown in Example 5.

b. Compare the answer you obtained here with the answer you obtained in Problem 7. State what you observed.

SUMMARY: ACTIVITY 5.2

1. To determine the **least common denominator (LCD)** of mixed numbers:

a. Identify the largest denominator of the fraction parts of the mixed numbers involved.

b. Look at multiples of the largest denominator. The smallest multiple that is divisible by the smaller denominator(s) is the LCD.

2. To add or subtract mixed numbers with different denominators:

a. Separate the fraction parts from the integer parts and add or subtract the integer parts.

b. Find the LCD of the fraction parts and convert each fraction to an equivalent fraction in terms of the LCD.

c. Add or subtract the numerators of the equivalent fractions to obtain the new numerator, leaving the LCD in the denominator.

d. If necessary, reduce the resulting fraction to lowest terms.

e. If the fraction part is an improper fraction, convert it to a mixed number and combine it with the integer part obtained in part a.

3. To solve equations of the forms $x + a = b$ and $x - a = b$ that involve mixed numbers:

a. Use the Fundamental Principle of Equality to isolate the unknown value, x.

b. Calculate the sum or difference of the mixed numbers and simplify if necessary. The result is the value of the unknown, x.

EXERCISES: ACTIVITY 5.2

In Exercises 1–17, perform the indicated operation.

1. $3\frac{1}{6} + 5\frac{1}{2}$

2. $7\frac{3}{8} - 4\frac{1}{4}$

3. $-1\frac{2}{5} + 2\frac{9}{10}$

4. $3\frac{1}{3} + 1\frac{1}{6}$

5. $5\frac{7}{12} + 8\frac{13}{16}$

6. $12\frac{3}{4} + 6\frac{2}{5}$

7. $5\frac{2}{12} + 3\frac{7}{18}$

8. $12\frac{1}{4} - 7\frac{1}{8}$

9. $14\frac{3}{4} - 6\frac{5}{12}$

10. $11 - 6\frac{3}{7}$

11. $12\frac{1}{4} - 5$

12. $8\frac{5}{6} - 3\frac{11}{12}$

13. $4\frac{3}{11} - 2\frac{9}{22}$

14. $-7\frac{5}{8} + 2\frac{1}{6}$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

15. $-9\frac{3}{5} + \left(-3\frac{4}{15}\right)$

16. $2\frac{2}{7} - \left(-3\frac{3}{8}\right)$

17. $-5\frac{2}{5} + \left(-6\frac{4}{9}\right)$

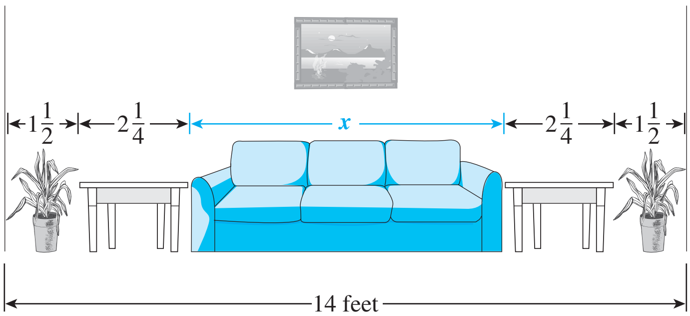
18. You are considering learning to play the piano and check with a friend about practice time. For 3 consecutive days before a recital, she practiced $1\frac{1}{4}$ hours, $2\frac{1}{2}$ hours, and $3\frac{2}{3}$ hours. What was her total practice time for the 3 days?

19. A favorite muffin recipe calls for $2\frac{2}{3}$ cups of flour, 1 cup of sugar, $\frac{1}{2}$ cup of crushed cashews, and $\frac{5}{8}$ cup of milk, plus assorted spices. How many cups of batter does the recipe make?

20. A cake recipe calls for $3\frac{1}{2}$ cups of flour, $1\frac{1}{4}$ cups of brown sugar, and $\frac{5}{8}$ cup of white sugar. What is the total amount of dry ingredients?

21. To create a table for a report, you need two columns, each $1\frac{1}{2}$ inches wide, and five columns, each $\frac{3}{4}$ inch wide. Will your table fit on a piece of paper $8\frac{1}{2}$ inches wide?

22. Your living room wall is 14 feet long. You want to buy a couch and center it on the wall. You have two end tables, each $2\frac{1}{4}$ feet long, that will be on each side of the couch. You plan to leave $1\frac{1}{2}$ feet next to each end table for floor plants. Determine how long a couch you can buy.



a. If x represents the length of the couch, write an equation describing the situation.

b. Solve the equation for x.

In Exercises 23–26, solve each equation for the unknown quantity. Check your answers.

23. $x - 3\frac{1}{2} = 6\frac{3}{4}$

24. $x + 4 = 2\frac{1}{2}$

25. $2\frac{1}{3} + x = 5\frac{5}{6}$

26. $x + 5\frac{1}{4} = -7\frac{1}{3}$

Activity 5.3

Tiling the Bathroom

Objectives

1. Multiply and divide mixed numbers.
2. Evaluate expressions with mixed numbers.
3. Calculate the square root of a mixed number.
4. Solve equations of the form $ax + b = 0$, $a \neq 0$, that involve mixed numbers.

You have decided it is time for a new floor in the bathroom. You tear up the old tile and repair the subfloor. You then measure the floor and determine the dimensions are 8 feet 8 inches by 12 feet 9 inches.

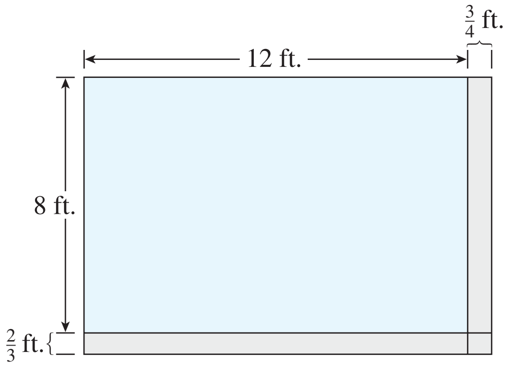
1. Use mixed numbers to express the dimensions of the bathroom in feet.

2. Round each dimension of the bathroom to the nearest foot, and then estimate the area of the bathroom.

3. Now use the dimensions in Problem 1 to determine the actual area two ways.

   a. First, change the mixed numbers to improper fractions and multiply.

   b. Second, consider the following diagram.



Calculate the area of each of the four regions and determine their sum.

4. Your answers to Problem 3a and b should agree. Based upon your estimate in Problem 2, are these answers reasonable? Explain.

Procedure

Multiplying or Dividing Mixed Numbers

1. Change the mixed numbers to improper fractions.
2. Multiply or divide as you would proper fractions.
3. If the product is an improper fraction, convert it to a mixed number in reduced form.

When a problem is given in mixed number form, the end result is also expressed in mixed number form.

5. Multiply each of the following.

a. $4\frac{2}{5} \cdot 3$

b. $3\frac{1}{3} \cdot 1\frac{1}{5}$

c. $-5\frac{5}{8} \cdot 5\frac{1}{3}$

6. Divide each of the following.

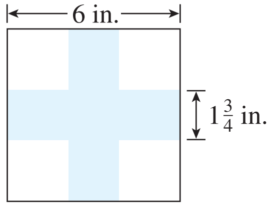
a. $5\frac{1}{4} \div 3$

b. $-5\frac{5}{6} \div -1\frac{3}{4}$

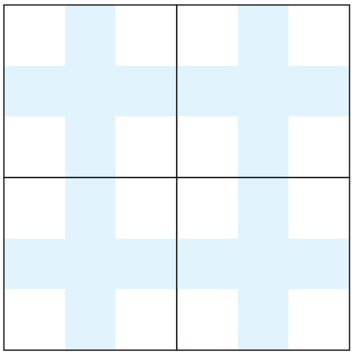
c. $8\frac{2}{5} \div -2\frac{2}{9}$

Powers of Mixed Numbers

7. Ceramic tiles for home decorating come in a large variety of shapes and sizes. The new tile you have chosen has a symmetrical design. Each tile is a 6-inch-by-6-inch square containing a pink "plus" sign of width $1\frac{3}{4}$ inches, as illustrated here.



When the tiles are laid, a large white square is formed where four tiles meet as shown in the diagram.



Studying the pattern formed by the four tiles makes you wonder whether there is more white than pink on a single tile.

a. What are the dimensions of each small white square on a single tile?

b. What is the total area of white on each tile? Express your answer as a mixed number.

c. What fraction of a single tile is white? Is there more pink or white on a single tile?

You can also determine the white area of an individual tile by thinking of the four small white squares as a single larger square, as seen in the center of the four-tile drawing. From Problem 7a, each single white square on a single tile measures $2\frac{1}{8}$ inches on a side. The larger white square measures $s = 2 \cdot \left(2\frac{1}{8}\right)$ inches on a side. Using the formula $A = s^2$, the area of the larger white square is

$$A = \left(2 \cdot 2\tfrac{1}{8}\right)^2.$$

Recalling the order of operations, you need to first multiply within the parentheses before applying the exponent. To multiply a mixed number by a whole number, one approach is to convert $2\frac{1}{8}$ to an improper fraction, $\frac{17}{8}$, and multiply by $\frac{2}{1}$.

$$\frac{17}{8} \cdot \frac{2}{1} = \frac{17 \cdot 2}{4 \cdot 2 \cdot 1} = \frac{17}{4}$$

Then, square to obtain

$$\left(\frac{17}{4}\right)^2 = \frac{17 \cdot 17}{4 \cdot 4} = \frac{289}{16} = 18\frac{1}{16}.$$

Caution: When multiplying with mixed numbers, be very careful not to confuse the addition implied by a mixed number with multiplication. A dot, or other multiplication symbol, *must* be present to show multiplication of numbers. For example,

$$6 \cdot \frac{1}{2} = 3, \text{ but } 6\frac{1}{2} = 6 + \frac{1}{2}.$$

Example 1 ***Evaluate*** $\left(1\frac{2}{3}\right)^3$.

SOLUTION

$$\left(1\frac{2}{3}\right)^3 = \left(\frac{5}{3}\right)^3$$ **Convert the mixed number to an improper fraction.**

$$= \frac{5}{3} \cdot \frac{5}{3} \cdot \frac{5}{3} = \frac{125}{27}$$ **Apply the exponent and multiply.**

$$= 4\frac{17}{27}$$ **Convert back to a mixed number.**

8. Evaluate $\left(3 \cdot 2\frac{1}{3}\right)^3$.

Evaluating Expressions that Involve Mixed Numbers

To finish the bathroom floor, you decide to install baseboard molding around the base of each wall. You need the molding all the way around the room except for a 39-inch gap at the doorway. Recall the dimensions of the room are $8\frac{2}{3}$ feet by $12\frac{3}{4}$ feet. Converting 39 inches to $\frac{39}{12} = 3\frac{1}{4}$ feet, you determine that the number of feet of molding, M, that you need is given by the formula

$$M = 2l + 2w - 3\frac{1}{4},$$

where l is the length and w is the width of the room.

9. Explain why this formula makes sense.

10. Determine the amount of molding you need by evaluating the formula's expression for the given room dimensions.

When working with formulas, you frequently evaluate expressions. Here are more examples.

Example 2 ***Evaluate $3xy - y$, where $x = -\frac{3}{4}$ and $y = -2\frac{1}{2}$.***

SOLUTION

$3\left(-\frac{3}{4}\right)\left(-2\frac{1}{2}\right) - \left(-2\frac{1}{2}\right)$ Substitute the given values.

$= 3\left(-\frac{3}{4}\right)\left(-\frac{5}{2}\right) - \left(-\frac{5}{2}\right)$ Convert to improper fractions.

$= \frac{45}{8} + \frac{5}{2} = \frac{45}{8} + \frac{20}{8}$ Perform the calculations.

$= \frac{65}{8}$

$= 8\frac{1}{8}$ Convert back to mixed number form.

11. Evaluate $a - b(6 - c)$, where $a = -\frac{1}{4}, b = \frac{2}{3}, c = 1\frac{1}{2}$.

12. The heights of three brothers were measured as 6 feet 2 inches, 5 feet 10 inches, and 5 feet 9 inches. Use the formula

$$\mathrm{A} = (a + b + c)/3$$

to determine the average height of the brothers.

In future algebra classes you will often need to evaluate expressions containing exponents.

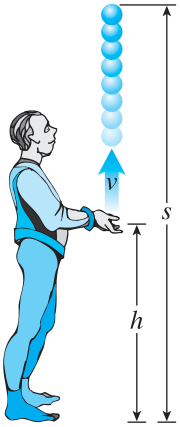
Example 3 ***Evaluate $xy - y^2$, where $x = \frac{5}{6}$ and $y = -\frac{2}{3}$.***

SOLUTION

Substituting the given values,

$$\begin{aligned}\left(\frac{5}{6}\right)\left(-\frac{2}{3}\right) - \left(-\frac{2}{3}\right)^2 &= -\frac{5 \cdot 2}{6 \cdot 3} - \frac{2 \cdot 2}{3 \cdot 3} \\ &= -\frac{5}{9} - \frac{4}{9} \\ &= -\frac{9}{9} \\ &= -1\end{aligned}$$

13. Evaluate $a^2 - b(c - 2)^2$, where $a = \frac{1}{6}, b = \frac{5}{4}, c = \frac{8}{3}$.



14. A ball is thrown straight up in the air. Let h be the height, in feet, of the ball above the ground when it is thrown. Let v be the initial velocity, in feet per second (that is, the velocity of the ball when it is thrown). Let t be the elapsed time, in seconds, since the ball was thrown. Then the height of the ball above the ground in feet, s, can be determined using the formula

$$s = h + vt - 16t^2.$$

Determine the height of a ball above the ground at $2\frac{1}{2}$ seconds, if it starts from a height of $5\frac{1}{2}$ feet and its initial velocity is 44 feet per second.

Hang Time and Square Roots

Recall from Chapter 4 that hang time is the time of a basketball player's jump, measured from the instant he or she leaves the floor until he or she returns to the floor. Hang time, t, depends on the height of the jump, s. These quantities are related by the equation

$$t = \frac{1}{2}\sqrt{s},$$

where s is the input variable measured in feet and t is the output variable measured in seconds. To determine the hang time of any jump, you need to take square roots. You can calculate the square root of a mixed number by converting the mixed number to an improper fraction and taking the square root of the numerator and denominator separately.

Example 4 ***Simplify*** $\sqrt{2\frac{14}{25}}$.

SOLUTION

$\sqrt{2\frac{14}{25}} = \sqrt{\frac{64}{25}}$ Change the mixed number to a fraction.

$= \frac{\sqrt{64}}{\sqrt{25}}$ Take the square root of the numerator and denominator separately.

$= \frac{8}{5}$ Calculate the square roots.

$= 1\frac{3}{5}$ Convert back to a mixed number.

15. Use the hang-time formula to determine the hang time of a $2\frac{1}{4}$-foot jump.

Procedure

Determining the Square Root of a Fraction or a Mixed Number

1. To determine the square root of a mixed number, change it to an improper fraction.

2. The square root of the fraction is the quotient, $\frac{\text{square root of the numerator}}{\text{square root of the denominator}}$.

3. Write the result as a mixed number if necessary.

16. Calculate the following square roots.

a. $\sqrt{\frac{9}{100}}$ **b.** $\sqrt{\frac{64}{49}}$ **c.** $\sqrt{\frac{81}{16}}$

To solve equations involving mixed numbers, change the mixed numbers to improper fractions.

17. Solve the following equations.

a. $1\frac{2}{3}x = 20$

b. $3\frac{1}{2}n = -1\frac{1}{6}$

c. $1\frac{1}{7}x = -1\frac{3}{14}$

SUMMARY: ACTIVITY 5.3

1. To multiply or divide mixed numbers:

a. Change the mixed numbers to improper fractions.

b. Multiply or divide as you would proper fractions.

c. If the product is an improper fraction, convert it to a mixed number in reduced form.

2. The rules for order of operations when evaluating expressions that involve improper fractions and/or mixed numbers are the same as for integers and proper fractions.

3. To determine the square root of a fraction or a mixed number:

a. Convert the mixed number to an improper fraction.

b. The square root of the fraction is the quotient, $\dfrac{\text{square root of the numerator}}{\text{square root of the denominator}}$.

c. Write the result as a mixed number if necessary.

EXERCISES: ACTIVITY 5.3

1. Multiply and write your answer as a mixed number, if necessary.

a. $3\frac{2}{3} \cdot 1\frac{4}{5}$

b. $-2\frac{2}{15} \cdot 4\frac{1}{2}$

c. $2\frac{4}{11} \cdot 4\frac{2}{5}$

d. $\left(-5\frac{3}{8}\right) \cdot (-24)$

2. Divide and write your answer as a mixed number, if necessary.

a. $3\frac{2}{7} \div 1\frac{9}{14}$

b. $-1\frac{7}{8} \div \frac{3}{4}$

c. $4\frac{2}{9} \div 8\frac{1}{3}$

d. $\dfrac{-12\frac{2}{3}}{-3\frac{1}{6}}$

3. Simplify.

a. $\left(\frac{2}{9}\right)^2$

b. $\left(-\frac{3}{5}\right)^3$

c. $\left(-\frac{4}{7}\right)^2$

d. $\left(\frac{2}{3}\right)^5$

e. $\sqrt{\frac{36}{81}}$

f. $\sqrt{\frac{64}{100}}$

g. $\sqrt{\frac{144}{49}}$

h. $\sqrt{\frac{375}{15}}$

4. Evaluate each expression.

a. $2x - 4(x - 3)$, where $x = 1\frac{1}{2}$

b. $a(a - 2) + 3a(a + 4)$, where $a = \frac{2}{3}$

c. $2x + 6(x + 2)$, where $x = 2\frac{1}{4}$

d. $6xy - y^2$, where $x = 1\frac{5}{16}$ and $y = \frac{1}{8}$

e. $x^3 + x^2$, where $x = 1\frac{1}{2}$

5. a. The length of one side of a square tile is $1\frac{1}{2}$ feet. Determine the area of the tile in square feet $\left(\text{use } A = s^2\right)$.

b. How many tiles would you need to cover a 9-foot by 10-foot bathroom floor, assuming no errors or waste?

6. As an apprentice at your uncle's cabinet shop, you are cutting out pieces of wood for a dresser. You need pieces that are $5\frac{1}{4}$ inches long. Each saw cut wastes $\frac{1}{8}$ inch of wood. The board you have is 8 feet long. How many pieces can you make from this board?

7. Solve the following equations.

a. $2\frac{3}{4}x = 22$

b. $6\frac{1}{8}w = -24\frac{1}{2}$

c. $-1\frac{4}{5}t = -9\frac{9}{10}$

d. $5\frac{1}{3}x = \frac{-5}{6}$

8. Due to Earth's curvature, the maximum distance, d, in kilometers that a person can see from a height h meters above the ground is given by the formula

$$d = \frac{7}{2}\sqrt{h}.$$

Use the formula to determine the maximum distance a person can see from a height of $20\frac{1}{4}$ meters.

9. An object dropped from a height falls a distance of d feet in t seconds. The formula that describes this relationship is $d = 16t^2$. A math book is dropped from the top of a building that is 400 feet high.

a. How far has the book fallen in $1\frac{1}{2}$ seconds?

b. After $3\frac{1}{2}$ seconds, how far above the ground is the book?

c. Another book was dropped from the top of a building across the street. It took $6\frac{1}{2}$ seconds to hit the ground. How tall is that building?

Cluster 1 What Have I Learned?

1. a. How do you convert a mixed number to an improper fraction?

   b. Give an example.

2. a. How do you convert an improper fraction to a mixed number?

   b. Give an example.

3. Determine whether the following statements are true or false. Give a reason for each answer.

   a. For a fraction to be called improper, the absolute value of the numerator has to be greater than or equal to the absolute value of the denominator.

   b. The LCD of two denominators must be greater than or equal to either of those denominators.

   c. $-6\frac{1}{4} = -6 + \frac{1}{4}$

   d. $7\frac{2}{9} = \frac{65}{9}$

   e. $\frac{2}{7} + \frac{4}{7} = \frac{6}{14}$

4. a. Outline the general steps in adding mixed numbers. Be sure to allow for differences when the denominators are the same or different.

Exercise numbers appearing in color are answered in the Selected Answers appendix.

b. Give an example of two or more mixed numbers with same denominators. Then determine their sum.

c. Give an example of two or more mixed numbers with different denominators. Then determine their sum.

5. a. Outline the general steps in subtracting mixed numbers. Be sure to indicate how you make a decision to regroup from the integer part.

b. Give an example of subtracting two mixed numbers with the same denominator. Then determine their difference.

c. Give an example of subtraction in which two mixed numbers have different denominators and you do *not* need to regroup. Then determine the difference.

d. Give an example of subtraction in which two mixed numbers have different denominators and you *do* need to regroup. Then determine the difference.

6. To multiply or divide mixed numbers, what must be done first?

Cluster 1 How Can I Practice?

1. Convert the following mixed numbers into improper fractions in lowest terms.

a. $3\frac{7}{9}$

b. $5\frac{5}{8}$

c. $4\frac{3}{12}$

d. $-10\frac{8}{13}$

e. $-1\frac{27}{31}$

2. Convert the following improper fractions to mixed numbers in lowest terms.

a. $\frac{71}{32}$

b. $\frac{28}{13}$

c. $\frac{89}{12}$

d. $-\frac{45}{5}$

e. $-\frac{92}{14}$

3. Perform the following operations and write the result in lowest terms.

a. $1\frac{3}{4} + 3\frac{3}{4}$

b. $8\frac{6}{7} + 4\frac{5}{9}$

c. $5\frac{5}{12} + 3\frac{11}{18}$

d. $13 + 4\frac{9}{13}$

e. $13\frac{28}{54} - 2\frac{11}{54}$

f. $11\frac{4}{9} - 7\frac{5}{24}$

g. $46\frac{13}{28} - 34\frac{10}{56}$

h. $74\frac{3}{8} - 61$

Exercise numbers appearing in color are answered in the Selected Answers appendix.

i. $-8\frac{4}{6} + \left(-25\frac{7}{24}\right)$

j. $-6\frac{17}{24} + \left(-12\frac{31}{48}\right)$

k. $-13\frac{5}{6} - 8\frac{6}{7}$

l. $-9\frac{3}{14} - 11\frac{6}{35}$

m. $48 - 21\frac{5}{9}$

n. $72 - 13\frac{7}{12}$

o. $13\frac{2}{15} - 8\frac{4}{5}$

p. $29\frac{7}{12} - 21\frac{15}{18}$

q. $16\frac{5}{8} - \left(-3\frac{7}{12}\right)$

r. $-6\frac{4}{5} - \left(-7\frac{11}{15}\right)$

s. $-17\frac{9}{14} + 9\frac{8}{21}$

t. $-14\frac{1}{36} + 7\frac{1}{6}$

4. Multiply and write each answer in simplest form. Use a calculator to check answers.

a. $\frac{45}{14} \cdot \frac{35}{18}$

b. $-27 \cdot -\frac{25}{21}$

c. $\frac{-9}{10} \cdot \frac{-4}{3}$

d. $-20 \cdot \frac{2}{35}$

5. Divide and write each answer in simplest form. Use a calculator to check answers.

a. $\frac{7}{2} \div \frac{7}{3}$

b. $-\frac{15}{8} \div \frac{35}{36}$

c. $\frac{\frac{6}{7}}{-\frac{9}{14}}$

d. $\frac{-\frac{35}{10}}{-3}$

6. Multiply or divide and write each answer as a mixed number in simplest form. Use a calculator to check answers.

a. $\frac{-1}{2} \cdot 5\frac{5}{6}$

b. $-2\frac{5}{6} \div \frac{3}{7}$

c. $-37\frac{1}{2} \cdot -1\frac{3}{5}$

d. $7\frac{1}{5} \div 2\frac{2}{5}$

e. $2\frac{1}{3} \cdot 1\frac{1}{2}$

f. $-8\frac{1}{4} \div -1\frac{1}{2}$

7. Simplify.

a. $-\left(\frac{8}{7}\right)^2$

b. $\sqrt{\frac{16}{9}}$

c. $\left(-\frac{3}{2}\right)^3$

d. $\sqrt{\frac{100}{81}}$

8. Evaluate each expression.

a. $3x - 5(x - 6)$, where $x = -1\frac{2}{3}$

b. $x^2 - xy$, where $x = \frac{1}{4}$ and $y = -1\frac{1}{3}$

9. Solve each of the following equations for x. Check your answers by hand or by calculator.

a. $14\frac{3}{7} + x = 21$

b. $-17\frac{3}{5} + x = -12\frac{4}{5}$

c. $x - 13\frac{5}{6} = 17\frac{4}{7}$

d. $x + 8\frac{4}{9} = 19\frac{2}{3}$

e. $-18\frac{2}{11} + x = -23\frac{1}{3}$

f. $-5x = 38$

g. $-64 = 6x$

h. $-\frac{7}{4}x = -42$

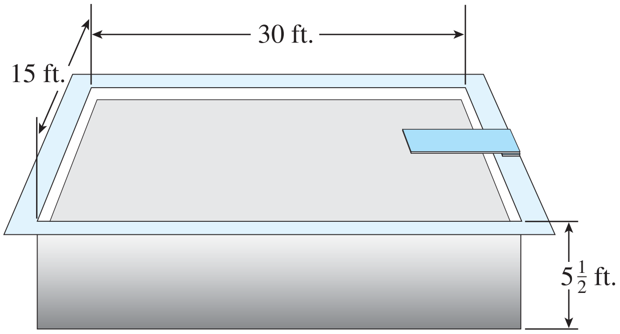
10. You purchased a roll of wallpaper that is $30\frac{1}{2}$ yards long. You used $26\frac{7}{8}$ yards to wallpaper one room. Is there enough wallpaper left on the roll for a job that requires 4 yards of wallpaper?

11. According to an ad on television, a man lost 30 pounds over $6\frac{1}{4}$ months. How much weight loss did he average per month?

12. A large-quantity recipe for pea soup calls for $64\frac{1}{2}$ ounces of split peas. You decide to make $\frac{1}{3}$ of the recipe. How many ounces of split peas will you need?

13. A large corporation recently reported that 16,936 employees were paid at an hourly rate. Approximately 1 out of every 5 of these workers earned \$15 or more per hour. How many workers earned \$15 or more per hour?

14. a. Your swimming pool is 15 feet by 30 feet and will be filled to a height of $5\frac{1}{2}$ feet. Use $V = lwh$ (volume equals length times width times height) to determine the volume of the water in the pool.



b. If 1 cubic foot of water weighs $62\frac{2}{5}$ pounds, determine the weight of the water in your pool.

Cluster 2 Decimals

Activity 5.4

What Are You Made of?

Objectives

1. Identify place values of numbers written in decimal form.
2. Convert a decimal to a fraction or a mixed number, and vice versa.
3. Classify decimals.
4. Compare decimals.
5. Read and write decimals.
6. Round decimals.

Have you ever wondered about the chemical elements that are contained in your body? According to the *Universal Almanac*, a 150-pound person is made up of the following elements.

Your Body's Table of Elements

ELEMENT	WEIGHT IN POUNDS	ELEMENT	WEIGHT IN POUNDS
Oxygen	97.5	Cobalt	0.00024
Carbon	27.0	Copper	0.00023
Hydrogen	15.0	Manganese	0.00020
Nitrogen	4.5	Iodine	0.00006
Calcium	3.0	Zinc	Trace
Phosphorus	1.8	Boron	Trace
Potassium	0.3	Aluminum	Trace
Sulfur	0.3	Vanadium	Trace
Chlorine	0.3	Molybdenum	Trace
Sodium	0.165	Silicon	Trace
Magnesium	0.06	Fluorine	Trace
Iron	0.006	Chromium	Trace
		Selenium	Trace

Source: Universal Almanac

Note that the weights are expressed as **decimals**. A number written as a decimal has an integer part to the left of the decimal point, and a fractional part that is to the right of the decimal point. Decimals extend the place value system for integers to include fractional parts. Also, decimals, like fractions, are used to express portions of a whole, that is, numbers less than 1.

Interpreting and Classifying Decimals

As you recall from Chapter 1, the place values for the integer part, to the left of the decimal point, are powers of 10: 1, 10, 100, 1000, and so on. The place values for the fractional part of a decimal, to the *right* of the decimal point, are powers of 10 in the denominator: $\frac{1}{10} = \frac{1}{10^1}, \frac{1}{100} = \frac{1}{10^2}, \frac{1}{1000} = \frac{1}{10^3}$, and so on.

The exponent of 10 in the denominator indicates the number of places to the right of the decimal point that the fractional part of the decimal requires. For example, $\frac{1}{10}$ requires one decimal place to the right, written as, 0.1, and read as "one-tenth." Similarly, the decimal equivalent of $\frac{1}{100} = \frac{1}{10^2}$ requires two decimal places to the right and is 0.01, read as "one-hundredth." The zero to the left of the decimal point in 0.01 indicates that the integer part is zero.

1. Write the decimal equivalent of the following fractions.

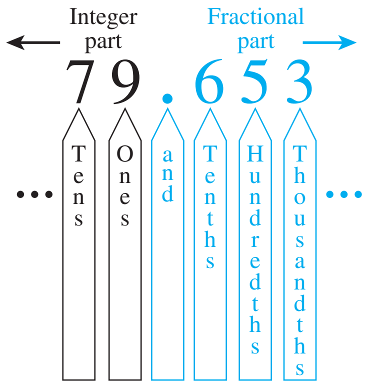
a. $\frac{1}{1000}$ **b.** $\frac{1}{10{,}000}$ **c.** $\frac{7}{1000}$ **d.** $\frac{13}{100}$

The fractions in Problem 1 are all proper fractions and are equivalent to decimals whose integer part is always 0. Decimals whose integer parts are nonzero are equivalent to mixed numbers.

The amount of nitrogen in the body of a 150-pound person is $4.5 = 4 + \frac{5}{10} = 4\frac{5}{10}$ pounds. The decimal 4.5 is read as "four and five-tenths," 4 representing the integer part and $\frac{5}{10}$ the fractional part.

Example 1 ***Rewrite the decimal 79.653 in terms of its place values.***

SOLUTION



Therefore, $79.653 = 70 + 9 + \frac{6}{10} + \frac{5}{100} + \frac{3}{1000}$.

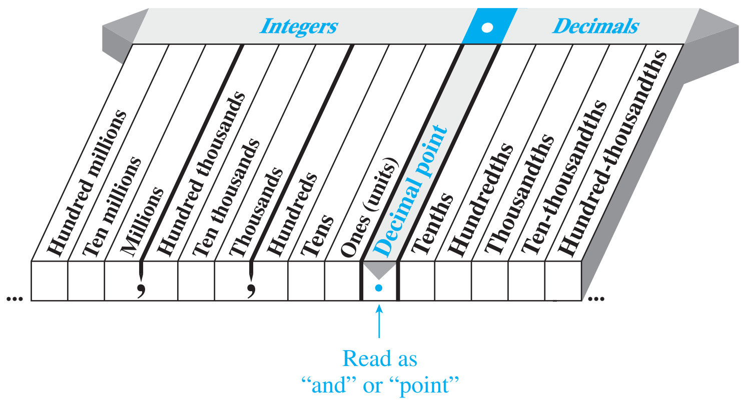
The fractional part of the number 79.653 is $\frac{6}{10} + \frac{5}{100} + \frac{3}{1000}$. Notice that the least common denominator of these three fractions is 1000. So,

$$\frac{6}{10} + \frac{5}{100} + \frac{3}{1000} = \frac{600}{1000} + \frac{50}{1000} + \frac{3}{1000} = \frac{653}{1000}.$$

Therefore, 79.653 can be written as $79\frac{653}{1000}$, which is read as "79 and 653 thousandths."

Reading and Writing Decimals

It is important to know how to name, read, and write decimals as fractions. For example, 0.165 is named as "one hundred sixty-five thousandths." More generally, an extension of the place value system to the right of the decimal point is found in the following chart.



To read a decimal, read the digits to the right of the decimal point as though they were *not* preceded by a decimal point, and then attach the place value of its rightmost digit. Use the word "and" to separate the integer part from the decimal or fractional part.

Example 2 ***The decimal 251.737 is read "two hundred fifty-one and seven hundred thirty-seven thousandths."***

2. a. Write in words the weight of potassium in a 150-pound body.

   b. Write in words the weight of the amount of copper in a 150-pound body.

   c. Which element would have a weight of six hundred-thousandths of a pound?

3. a. Write the number 9,467.00624 in words.

   b. Write the number 35,454,666.007 in words.

   c. Write the following number using decimal notation: four million sixty-four and seventy-two ten-thousandths.

   d. Write the following number using decimal notation: seven and forty-three thousand fifty-two millionths.

Converting Decimals to Fractions

Note that each of the decimals you have encountered thus far in this activity end after a few digits. Decimals such as these are referred to as **terminating decimals**. When a terminating decimal is written as a fraction or mixed number, the denominator corresponds to the place value of the rightmost digit in the decimal. This leads to the following procedure to convert a decimal to an equivalent fraction or mixed number.

Procedure

Converting a Terminating Decimal to a Fraction or Mixed Number

1. Write the nonzero integer part of the number and drop the decimal point.
2. Write the fractional part of the number as the numerator of a fraction. The denominator is determined by the place value of the rightmost digit.
3. Reduce the fraction if necessary.

Example 3 ***Write 0.2035 as a fraction reduced to lowest terms.***

SOLUTION

Drop the leading zero and decimal point and write 2035 as the numerator. The rightmost digit, 5, is in the ten-thousandths place. Thus the denominator of the fraction is 10,000.

$$0.2035 = \frac{2035}{10{,}000} = \frac{407}{2000}$$

4. Write each of the following decimals as an equivalent fraction or mixed number. Reduce the fraction, if possible.

a. 0.776 **b.** 2.0025 **c.** -5.67 **d.** 109.01

Some decimals have decimal parts that repeat indefinitely. One such common decimal is $0.66666... = 0.\overline{6}$, representing $\frac{2}{3}$. These are referred to as **nonterminating, repeating** decimals. Terminating decimals can also be written as repeating decimals where the repeating part is all zeros. For example, $0.25 = 0.250000...$. Every repeating decimal can be written as a fraction or integer. Thus, they are rational numbers.

Decimals that are nonterminating and nonrepeating represent **irrational numbers**. One irrational number with which you might be familiar is represented by the Greek letter π, pi. You will encounter π in Chapter 6.

5. a. Write the weight of sodium given in the table as a fraction with denominator 1000.

b. Write the weight of potassium given in the table as a fraction with denominator 1000.

c. Compare the weights of potassium and sodium in the fraction form that you determined in parts a and b. Which one is larger?

There is a faster and easier method for comparing decimals. Suppose you want to compare 2.657 and 2.68. The first step is to line up the decimal points as shown here. Ensure that all the decimals have the same number of decimal places by attaching zeros at the end of the "shorter" ones. This is equivalent to writing the decimals as fractions with the same denominators.

2.657
2.680

Zero since no digit is present.

Now, moving left to right, compare digits that have the same place value. The digit in the ones and tenths places is the same in both numbers. In the hundredths place, 8 is greater than 5, and it follows that $2.68 > 2.657$.

Procedure

Comparing Decimals

1. First compare the integer parts (to the left of the decimal point). The number with the larger integer part is the larger number.
2. If the integer parts are the same, compare the fractional parts (to the right of the decimal point).
   a. Write the decimal parts as fractions with the same denominator and compare the fractions.

   or

   b. Line up the decimal points and compare the digits that have the same place value, moving from left to right. The first number with the larger digit is the larger number.

6. a. Use the method just described to determine if an average person has more sodium or potassium in his body.

   b. Does the average person have more cobalt or iron in their body?

Rounding Decimals

In Chapter 1 you estimated whole numbers by a technique called rounding off, or rounding for short. Decimals can also be rounded to a specific place value.

Example 4 ***Round the weight of sodium in a 150-pound body to the tenths place.***

SOLUTION

Observe that 0.165 has a 1 in the tenths place and that $0.165 > 0.1$. So the task is to determine whether 0.165 is closer to 0.1 or 0.2.

0.15 is exactly halfway between 0.1 and 0.2.

$0.165 > 0.15$, so 0.165 must be closer to 0.2.

There is approximately 0.2 pound of sodium in a 150-pound body.

Procedure

Rounding a Decimal to a Specified Place Value

1. If the specified place value is 10 or higher, drop the fractional part and round the resulting whole number.
2. Otherwise, locate the digit with the specified place value.
3. If the digit directly to the right is less than 5, keep the digit in step 2 and delete all the digits to the right.
4. If the digit directly to the right is greater than or equal to 5, increase the digit in step 2 by 1 and delete all the digits to the right.

Example 5 ***Round 279.583 to the ones place.***

SOLUTION

The digit 9 in the ones place will either stay the same or increase by 1. Basically, you want to know whether the number is closer to 279 or 280. Since the tenths digit is 5, the number rounds to 280 (following step 4 in the procedure).

7. Copper's weight of 0.00023 pound, rounded to the nearest ten-thousandth place, is 0.0002 pound. Explain how this answer is obtained.

8. a. What is the weight of sodium, rounded to the nearest hundredth?

   b. What is cobalt's weight, rounded to the nearest ten-thousandth?

Converting Fractions to Decimals

9. Seven-tenths of the students in your class have a family dog or cat at home. Write $\frac{7}{10}$ as a decimal.

In Problem 9, the fraction $\frac{7}{10}$ was written as a decimal by reading the number aloud and writing its decimal form. All fractions can be written as decimals, even if their denominator is not a power of 10.

Procedure

Converting a Fraction to a Decimal

1. Using the long division format, divide the denominator into the numerator.
2. Place a decimal point to the right of the ones place in the dividend and a decimal point directly above it in the quotient.
3. Divide, adding as many zeros to the right of the decimal as necessary.
4. Round to the desired place value.

Example 6 *Convert the fraction $\frac{4}{7}$ to a decimal rounded to the hundredths place.*

SOLUTION

$$7\overline{)4} = 7\overline{)4.000}$$

$$\begin{array}{r} 0.571 \\ 7\overline{)4.000} \\ -35 \\ \hline 50 \\ -49 \\ \hline 10 \\ -7 \\ \hline 3 \end{array}$$

$\frac{4}{7} = 0.57$ rounded to the hundredths place.

10. Write each of the following fractions as a terminating decimal or, if a repeating decimal, rounded to the hundredths place.

a. $\frac{2}{3}$

b. $\frac{7}{25}$

c. $\frac{7}{24}$

d. $\frac{9}{40}$

SUMMARY: ACTIVITY 5.4

- **Reading and Writing Decimals**

  To name a decimal number, read the digits to the right of the decimal point as though they were not preceded by a decimal point, and then attach the place value of its rightmost digit. Use the word *and* to separate the integer part from the decimal or fractional part.

- **Converting a Decimal to a Fraction or a Mixed Number**

  1. Write the nonzero integer part of the number and drop the decimal point.

  2. Write the fractional part of the number as the numerator of a new fraction, whose denominator is determined by the place value of the rightmost digit.

  3. Reduce the fraction if necessary.

- **Comparing Decimals**

  Align the decimal points vertically and compare the digits that have the same place value, moving from left to right. The first number with the larger digit in the same value place is the larger number.

- **Rounding a Decimal to a Specified Place Value**

  1. If the specified place value is 10 or more, drop the fractional part and round the resulting whole number.

  2. Otherwise, locate the digit with the specified place value.

3. If the digit directly to the right is less than 5, keep the digit in step 2 and delete all the digits to the right.
4. If the digit directly to the right is greater than or equal to 5, increase the digit in step 2 by 1 and delete all the digits to the right.

- **Converting a Fraction to a Decimal**

  Divide the denominator into the numerator. If the decimal part does not terminate, round to the desired number of places.

EXERCISES: ACTIVITY 5.4

1. Using the table on page 305, list the chemical substances chlorine, calcium, iron, magnesium, iodine, cobalt, and manganese in the order in which their amounts are in your body from largest to smallest.

2. a. The euro, the basic unit of money in the European Union, was recently worth $1.3256 U.S. dollars. Round to estimate the worth of 1 euro in cents.

   b. To what place did you round in part a?

3. a. The Canadian dollar was recently worth $0.9549. Round this value to the nearest cent.

   b. Round the value of the Canadian dollar to the nearest dime.

4. a. The Mexican peso was recently worth $0.0765. Round this value to the nearest cent.

   b. Round the value of the peso to the nearest dime.

5. a. Write 0.052 in words.

   b. Write 0.00256 in words.

   c. Write 3402.05891 in words.

   d. Write 64 ten-thousandths as a decimal.

   e. Write one hundred twenty-five thousandths as a decimal.

   f. Write two thousand forty-one and six hundred seventy-three ten-thousandths as a decimal.

Exercise numbers appearing in color are answered in the Selected Answers appendix.

6. The *National Institutes of Health* reported that U.S. alcohol consumption from 1940 to 2006 (in gallons of ethanol, per person) was as follows.

	GALLONS PER PERSON			
YEAR	BEER	WINE	SPIRITS	ALL BEVERAGES
1940	0.73	0.16	0.67	1.56
1950	1.04	0.23	0.77	2.04
1960	0.99	0.22	0.86	2.07
1970	1.14	0.27	1.11	2.52
1980	1.38	0.34	1.04	2.76
1990	1.34	0.33	0.77	2.45
2000	1.22	0.31	0.65	2.18
2006	1.19	0.37	0.71	2.27

Source: http://www.niaaa.nih.gov/Resources/DatabaseResources/QuickFacts/AlcoholSales/consum01.htm

a. In which year was beer consumption the highest?

b. Round the amount of all beverage consumption in 2000 to the nearest tenth and write your answer as a mixed number.

c. In which year was the least amount of spirits consumed?

d. Write in words the amount of spirits consumption in 1970.

e. Round the amount of wine consumption to the nearest tenth in the years 1970, 1980, 1990, and 2000. Can you use the rounded values to determine when the most wine was consumed? Explain.

7. Circle either true or false for each of the following statements and explain your choice.

a. True or false: $456.77892 < 456.778902$

b. True or false: $0.000501 > 0.000510$

c. True or false: 7832.00375 rounded to the nearest thousandth is 7832.0038.

d. True or false: Seventy-three and four hundred-thousandths is written 73.0004.

8. **a.** Express the number of pounds of sulfur in a 150-pound person's body as a fraction and in words.

b. Express the number of pounds of iron in a 150-pound person's body as a fraction and in words.

9. In what was supposed to be a great game against Arizona, there were only 6 minutes left in the game, but your fantasy football team's wide receiver, Seattle Seahawk's T. J. Houshmandzadeh, had only 1.7 fantasy points. Even worse, your quarterback, Seattle Seahawk's Matt Hasselbeck, had -0.13 fantasy points!

a. Express T. J.'s points as a fraction and in words.

b. Express your quarterback's points as a fraction and in words.

10. Write each of the following fractions as a terminating decimal or, if a repeating decimal, rounded to the thousandths place.

a. $\frac{6}{5}$ **b.** $\frac{9}{25}$ **c.** $\frac{5}{24}$

d. $\frac{1}{8}$ **e.** $\frac{3}{20}$ **f.** $2\frac{5}{6}$

g. $\frac{3}{7}$ **h.** $\frac{11}{8}$

Activity 5.5

Dive into Decimals

Objectives

1. Add and subtract decimals.
2. Compare and interpret decimals.
3. Solve equations of the type $x + b = c$ and $x - b = c$ that involve decimals.

In Olympic diving, a panel of judges awards a score for each dive. There is a preliminary round and a final round of dives. The preliminary round total and the scores of the five dives in the final round are given below for the top six divers in the women's 10-meter platform event at the 2008 Beijing Olympics. The total of the five final dives determines who wins the gold, silver, and bronze medals for first, second, and third place.

1. Based *only* on the preliminary round, which of these six divers would have won the gold, silver, and bronze medals?

Going for Gold

DIVER	SCORE FOR PRELIMINARY ROUND	SCORES FOR THE FINAL ROUND					FINAL ROUND TOTAL
		DIVE 1	DIVE 2	DIVE 3	DIVE 4	DIVE 5	
Ruolin Chen (China)	428.80	85.5	84.8	88.0	89.1	100.3	
Paola Espinosa (Mexico)	343.6	72.0	84.15	68.0	75.2	81.6	
Emilie Heymans (Canada)	403.85	83.3	86.4	84.15	95.2	88.0	
Tatiana Ortiz (Mexico)	345.7	63.0	40.0	72.6	86.4	81.6	
Xin Wang (China)	420.3	79.5	84.8	86.4	89.1	90.1	
Melissa Wu (Australia)	340.35	79.5	75.2	72.0	24.75	86.7	

Adding Decimals

To add integers, add the digits in each place value position and regroup where needed. Add decimal numbers in a similar way.

2. **a.** Determine the total of the five dives in the final round for Emilie Heymans.

b. Describe in words how to line up the digits when adding decimals.

c. How do you locate the decimal point in your answer?

3. Determine the total final round score for each of the six divers listed in the table and record your answer in the appropriate place in the table.

4. a. Which diver won the gold medal?

b. Which diver won the silver medal?

c. Which diver won the bronze medal?

5. Explain why Xin Wang did not win the silver medal, as predicted in Problem 1.

Procedure

Adding Decimals

1. Line up the place values vertically with decimal points as a guide.
2. Add as usual.
3. Insert the decimal point in the answer directly in line with the decimal points of the numbers being added.

Subtracting Decimals

6. By how many points did the gold medalist win?

The answer to Problem 6 involves the subtraction of decimals. The procedure for subtracting decimals is very similar to the procedure for adding decimals.

7. Use the procedure for adding decimals as a model to write a three-step procedure for subtracting decimals.

8. For each diver, determine the difference between her highest and lowest scores in the final round.

a. Ruolin Chen

b. Paolo Espinosa

c. Emilie Heymans

d. Tatiana Ortiz

e. Xin Wang

f. Melissa Wu

9. Based on the results in Problem 8, which diver was most consistent? Explain.

Adding and Subtracting Negative Decimals

Just as water can freeze to become a solid or boil to become a gas, the elements of the Earth can exist in solid, liquid, or gaseous form. For example, the metal mercury is a liquid at room temperature. Mercury boils at 673.9°F and freezes at −38.0°F.

10. What is the difference between the freezing point and the boiling point of mercury?

11. You have some mercury stored at a temperature of 75°F.

a. By how many degrees Fahrenheit must the mercury be heated for it to boil?

b. By how many degrees Fahrenheit must the mercury be cooled to freeze it?

12. The use of the element krypton in energy-efficient windows helps meet new energy conservation guidelines. Krypton has a boiling point of −242.1°F. This means that krypton boils and becomes a gas at temperatures above −242.1°F. Is krypton a solid, liquid, or gas at room temperature?

13. You have some krypton stored at −250°F in a special container in your laboratory.

a. If you heat the krypton by 7.5°F, does it boil? If not, how much more must it be heated to become a gas?

b. Krypton freezes and becomes a solid below −249.9°F. What is the difference between krypton's boiling and freezing points?

14. The melting points of various elements are given in parts a–d. In each part, circle the element with the higher melting point, then determine the difference between the higher and the lower melting points.

a. radon: −96°F, xenon: −169.4°F

b. fluorine: −363.3°F, nitrogen: −345.8°F

c. bromine: −7.2°C, cesium: 28.4°C

d. argon: −308.6°F, hydrogen: −434.6°F

15. Complete the table to indicate whether each element is solid (below its freezing point), liquid (between its freezing and boiling points), or gas (above its boiling point) at 82°F.

How Low Can It Go?

ELEMENT	FREEZING POINT	BOILING POINT	STATE AT 82°F
Neon	−416.7°F	−411°F	
Lithium	356.9°F	2248°F	
Francium	80.6°F	1256°F	
Bromine	19.0°F	137.8°F	

Solving Equations Involving Decimal Numbers

In preparing the chemicals for a lab experiment in chemistry class, precise measurements are required. In Problems 16, 17, and 18, represent the unknown quantity with the variable x, and write an equation that accurately describes the situation. Solve the equation to get your answer.

16. You need to have precisely 1.250 grams of sulfur for your experiment. Your lab partner has measured 0.975 gram.

a. How much more sulfur is needed?

17. You also need 2.120 grams of copper. This time your lab partner measures 2.314 grams of copper.

    a. How much of this copper needs to be removed for your experiment?

18. The acidic solution needed for your experiment must measure precisely 0.685 liter.

    a. How much water must be added to 0.325 liter of sulfuric acid and 0.13 liter of hydrochloric acid to result in the desired volume?

19. Solve each of the following equations for the unknown variable. Check your answers.

    a. $x + 51.78 = 548.2$

    b. $y - 14.5 = 31.75$

    c. $1.637 - z = 9.002$

    d. $0.1013 + t = 0$

SUMMARY: ACTIVITY 5.5

- Adding or subtracting decimals
  1. Line up the numbers vertically on their decimal points.
  2. Add or subtract as usual.
  3. Insert the decimal point in the answer directly in line with the decimal points of the numbers being added.
- To solve equations of the form $x + b = c$, add the opposite of b to both sides of the equation to obtain $x = c - b$.
- To solve equations of the form $x - b = c$, add b to both sides of the equation to obtain $x = b + c$.

EXERCISES: ACTIVITY 5.5

1. Perform the indicated operations. Verify by using your calculator.

a. $30.6 + 1.19$ **b.** $5.205 + 5.971$ **c.** $1.78 - 1.865$

d. $0.298 + 0.113$ **e.** $2.242 - 1.015$ **f.** $-18.27 + 13.45$

g. $-20.11 - 0.294$ **h.** $-10.078 - 46.2$ **i.** $15.59 - 0.374$

j. $-876.1 - 73.63$ **k.** $0.0023 - 0.00658$ **l.** $-0.372 + 0.1701$

2. Use the table on page 318 to answer the following questions.

a. Determine the difference between the boiling point and freezing point of neon.

b. Determine the difference between the freezing points of lithium and neon.

c. Determine the difference between the boiling points of lithium and neon.

3. You have a $95.13 balance in your checking account. You will deposit your paycheck for $125.14 as soon as you get it 3 days from now. You need to write some checks now to pay some bills. You write checks for $25.48, $69.11, and $33.15. Unfortunately, the checks are cashed before you deposit your paycheck. The bank pays the $33.15 check but charges a $15.00 fee. What is your account balance after you deposit your paycheck?

4. Preparing a meal, you use several cookbooks, some with metric units and others with English units. To make sure you do not make a mistake in measuring, you check a book of math tables for the following conversions.

Measure for Measure

ENGLISH SYSTEM	METRIC SYSTEM
1 fluid ounce	0.02957 liters
1 cup	0.236588 liters
1 pint	0.473176 liters
1 quart	0.946353 liters
1 gallon	3.78541 liters

Exercise numbers appearing in color are answered in the Selected Answers appendix.

a. One recipe calls for mixing 1 pint of broth, 1 cup of cream, and 1 ounce of vinegar. What is the total amount of liquid, measured in liters?

b. Another recipe requires 1 gallon of water, 1 quart of milk, 1 one pint of cream. You have a 6-liter pan to cook the liquid ingredients. How much space (in liters) is left in the pan after you have added the three ingredients?

c. You also notice in the book of tables that there are 2 cups in a pint. Is this fact consistent with the information in the table on page 320?

d. Based on the table, how many pints are there in 1 quart? Explain.

5. The results for the top eight countries in the women's gymnastics competition at the 2008 Beijing Olympics are summarized in the following table. Each team's final score is the total of the scores for each of the four events: the vault, uneven bars, balance beam, and the floor exercise.

A Balancing Act

COUNTRY	VAULT	UNEVEN BARS	BEAM	FLOOR	TOTAL
Australia	44.150	43.575	45.900	42.900	
Brazil	44.300	43.700	43.900	42.975	
China	46.350	49.625	47.125	45.800	
France	44.050	44.175	43.725	43.325	
Japan	43.550	45.525	44.950	42.675	
Romania	45.275	45.000	46.175	45.075	
Russian Federation	45.425	46.950	44.900	43.350	
United States	46.875	47.975	47.250	44.425	

a. In each event, determine the order of the top three finishers.

b. Determine the totals for each country and record them in the table. What countries won the gold, silver, and bronze medals?

c. Consider the four event scores for each country. Which team had the largest difference between their lowest- and highest-scoring events? Which team had the smallest difference?

6. Solve each equation for the given variable. Check your answers by hand or by calculator.

a. $1.04 + 0.293 + x = 5.3$

b. $0.01003 - x = 0.0091$

c. $y - 7.623 = 84.212$

d. $1.626 + b = 14.503$

e. $x + 8.28 = 3.4$

f. $0.0114 + z = 0.005 + 0.0101$

7. A dental floss package you have contains 91.4 meters of cinnamon-flavored dental floss. You also have a small package of dental floss that you notice contains only 11 meters of floss. Let x be the difference in length for the two kinds. Write an equation and solve for x.

8. Arm & Hammer baking soda deodorant weighs 56.7 grams. Sure Clear Dry deodorant weighs 45.0 grams. Let x be the difference in weight. Write an equation and solve for x.

9. You need precisely 1.25 grams of mercury for a lab experiment. If you have 0.76 gram of mercury in your test tube, how much more mercury do you need? Write an equation to solve for this unknown amount.

10. The human body requires many minerals and vitamins. The minimum daily requirement of magnesium has been set at 0.4 grams. If your intake today has been 0.15 grams, how many grams of magnesium do you still need to meet the minimum daily requirement? Write an equation to solve for this unknown amount.

Activity 5.6

Quality Points and GPA: Tracking Academic Standing

Objectives

1. Multiply and divide decimals.
2. Estimate products and quotients that involve decimals.

Your college tracks your academic standing by an average called a *grade point average* (GPA). Each semester the college determines two grade point averages for you. One average is for the semester and the other includes all your courses up to the end of your current semester. The second grade point average is called a *cumulative GPA*. At graduation, your cumulative GPA represents your final academic standing.

At most colleges the GPA ranges from 0 to 4.0. A minimum GPA of 2.0 is required for graduation. You can track your own academic standing by calculating your GPA yourself. You may find this useful if you apply for internships or jobs at your college and elsewhere where a minimum GPA is required.

Quality Points and Multiplication

To calculate a GPA, you must first determine the number of points (known as quality points) that you earn for each course you take. The following problems will show you why.

1. Suppose that your schedule this term includes a four-credit literature class and a two-credit photography class. Is it fair to say that the literature course should be worth more towards your academic standing than the photography course? Explain.

The GPA gives more weight to courses with a higher number of credits than to ones with lower credit values. For example, a B in literature and an A in photography means that you have four credits each worth a B but only two credits that are each worth an A. To determine how much more the literature course is worth in the calculation of your GPA, you must first change the letter grades into quality point numerical equivalents. You then multiply the number of credits by the numerical equivalents of the letter grade to obtain the quality points for the course.

The following table lists the numerical equivalents of a typical set of letter grades.

Numerical Equivalents for Letter Grades

Letter Grade	A	A−	B+	B	B−	C+	C	C−	D+	D	D−	F
Quality Points (Numerical Equivalents)	4.00	3.67	3.33	3.00	2.67	2.33	2.00	1.67	1.33	1.00	0.67	0.00

2. a. The table shows that each B credit is worth 3 quality points. How many quality points is your B worth in the 4-credit literature course?

   b. How many quality points did you earn in your 2-credit photography course if your grade was an A?

3. a. Your grade was a B+ in a 3-credit psychology course. How many quality points did you earn?

b. Complete the following table for each pair of grades and credits. For example, in the row labeled 4 credits and the column labeled A, the number of quality points is 4×4.00 or 16.00. Similarly, the number of quality points for a grade of C– in a 2-credit course is 3.34.

QUALITY POINTS FOR GRADE AND CREDIT PAIRS												
Letter Grades	A	A−	B+	B	B−	C+	C	C−	D+	D	D−	F
Numerical Equivalents	4.00	3.67	3.33	3.00	2.67	2.33	2.00	1.67	1.33	1.00	0.67	0.00
Course Credits												
4 Credits	16.00											0.00
3 Credits			9.99									
2 Credits								3.34				
1 Credits										1.00		

4. A community college currently offers an internship course on geriatric health issues. The course, which includes one lecture hour and four fieldwork hours, is worth 1.5 credits.

a. Calculate the number of quality points for a grade of C in this course.

b. Estimate the number of quality points for a grade of C−.

c. Determine the exact number of quality points for a grade of C− by first multiplying 15 times 167, ignoring the decimal point. Then, use your estimate in part b to determine where the decimal point should be placed.

Notice in Problem 4c that there are three digits to the right of the decimal point in your answer. This is the sum total of digits to the right of the decimal point in both factors. Example 1 illustrates how this pattern occurs.

Example 1

***Show by converting the decimals to fractions that* $0.3 \cdot 0.5 = 0.15$.**

SOLUTION

Since $0.3 = \dfrac{3}{10}$ and $0.5 = \dfrac{5}{10}$, you have

$$0.3 \cdot 0.5 = \frac{3}{10} \cdot \frac{5}{10} \qquad \text{Convert decimals to fractions.}$$

$$= \frac{15}{100} \qquad \text{Multiply fractions.}$$

$$= 0.15 \qquad \text{Convert fraction back to a decimal.}$$

Example 1 shows that the number of decimal places to the right of the decimal point in the product 0.15 has to be two, one from each of the factors.

Procedure

Multiplying Two Decimals

1. Multiply the two numbers as if they were whole numbers, ignoring decimal points for the moment.
2. Add the number of digits to the right of the decimal point in each factor, to obtain the number of digits that must be to the right of the decimal point in the product.
3. Place the decimal point in the product by counting digits from right to left.

Example 2 ***Multiply* 8.731 *by* 0.25.**

SOLUTION

8.731	3 decimal places
× 0.25	2 decimal places
43655	
17462	
2.18275	5 decimal places

In Example 2, since 0.25 is $\frac{1}{4}$, and $\frac{1}{4}$ of 8.731 is approximately 2, you see that the general rule for multiplying two decimals placed the decimal point in the correct position.

5. Calculate the following products. Check the results using your calculator.

   a. $0.008 \cdot 57.2$

   b. $0.0201 \cdot -27.8$

   c. $-0.45 \cdot -3.12$

GPA and Division

Once you calculate the quality points you earned in a semester, you are ready to determine your GPA for that semester.

6. Suppose that in addition to the credit courses in literature, photography, and psychology that you took this term, you also enrolled in a 1-credit weight training class. Using the following table to record your results, calculate the quality points for each course. Then determine the total number of credits and the total number of quality points. The calculation for literature has been done for you.

	CREDITS	GRADE	NUMERICAL EQUIVALENT	QUALITY POINTS
Literature	4	B	3.00	12.00
Photography	2	A		
Psychology	3	B+		
Weight Training	1	B−		
Totals		n/a	n/a	

You earned 32.66 quality points for 10 credits this semester. To calculate the average number of quality points per credit, divide 32.66 by 10 to obtain 3.266. Note that dividing by 10 moves the decimal point in 32.66 one place to the left.

GPAs are usually rounded to the nearest hundredth, so your GPA is 3.27.

Procedure

Determining a GPA

1. Calculate the quality points for each course.
2. Determine the sum of the quality points.
3. Determine the total number of credits.
4. Divide the total quality points by the total number of credits.

7. Your cousin enrolled in five courses this term and earned the grades listed in the following table. Determine her GPA to the nearest hundredth.

	CREDITS	GRADE	NUMERICAL EQUIVALENT	QUALITY POINTS
Poetry	3	C+		
Chemistry	4	B		
History	3	A−		
Math	3	B+		
Karate	1	B−		
Totals		n/a	n/a	
			GPA	

In Problem 7, note that $42.66 \div 14$ is approximately $45 \div 15 = 3$. Therefore, the decimal point in 3.05 is correctly placed.

Example 3

In Problem 4, you read that a community college offers a 1.5-credit course to investigate issues in geriatric health. A coworker told you he earned 4.5 quality points in that course. What grade did he earn?

SOLUTION

To determine your coworker's grade, divide the 4.5 quality points by 1.5 credits, written as $4.5 \div 1.5$. The division can be written as a fraction, $\frac{4.5}{1.5}$, where the numerator is *divided* by the denominator.

$$4.5 \div 1.5 = \frac{4.5}{1.5}$$ **Write division as a fraction.**

$$= \frac{4.5}{1.5} \cdot \frac{10}{10}$$ **Multiply by $\frac{10}{10} = 1$ to obtain an equivalent fraction.**

$$= \frac{45}{15}$$ **Multiply fractions to obtain a whole-number denominator.**

$$= 3$$ **Divide the numerator by the denominator.**

He earned a grade of B.

Example 3 leads to a general method for dividing decimals.

$$\begin{array}{r} 3. \\ 1.5\overline{)4.5} \end{array}$$

Procedure

Dividing Decimals

1. Write the division in long division format.
2. Move the decimal point the same number of places to the right in both divisor and dividend so that the divisor becomes a whole number. Append zeros to the dividend, if needed.
3. Place the decimal point in the quotient directly above the decimal point in the dividend and divide as usual.

Example 4

a. *Divide* $\underbrace{\mathbf{92.4}}_{\text{dividend}}$ *by* $\underbrace{\mathbf{0.25}}_{\text{divisor}}$**.**

b. *Divide* $\underbrace{\mathbf{0.00052}}_{\text{dividend}} \div \underbrace{\mathbf{0.004}}_{\text{divisor}}$**.**

SOLUTION

a. $0.25\overline{)92.40}$ becomes $25\overline{)9240.}$

$$\begin{array}{r} 369.6 \\ 25\overline{)9240.0} \\ \underline{-75} \\ 174 \\ \underline{-150} \\ 240 \\ \underline{-225} \\ 150 \\ \underline{-150} \\ 0 \end{array}$$

b. $0.004\overline{)0.00052}$ becomes $4\overline{)0.52}$.

$$\begin{array}{r} 0.13 \\ 4\overline{)0.52} \\ \underline{-4} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

8. Determine the following quotients. Check the results using your calculator.

a. $11.525 \div 2.5$

b. $-45.525 \div 0.0004$

c. Calculate $-3.912 \div (-0.13)$ and round the result to the nearest ten-thousandth.

9. You need to calculate your GPA to determine if you qualify for a summer internship that requires a cumulative GPA of at least 3.3. You had 14 credits and 44.5 quality points in the fall term. In the spring you earned 16 credits and 55.2 quality points. Did you qualify for the internship?

Estimating Products and Quotients

Example 5 ***You need to drive to campus, but your gas tank is just about empty. The tank holds about*** **18.5** ***gallons of gas. Today, gas is selling for*** **$2.489** ***per gallon in your town. Estimate how much it will cost to fill the tank.***

SOLUTION

Note that 18.5 gallons is about 20 gallons and $2.489 is about $2.50. Therefore, $20 \cdot 2.5 = \$50$. You will need approximately $50 to fill the tank.

Procedure

Estimating Products of Decimals

1. Round each factor to one or two nonzero digits (numbers that multiply easily).
2. Multiply the results of step 1. This is the estimate.

10. Estimate the following products.

a. $57.3 \cdot 615.3$

b. $-0.031 \cdot 322.76$

c. $7893.65 \cdot 0.0016$

d. If you multiply a positive number by a decimal between 0 and 1, is the product greater or smaller than the original number?

11. Your uncle teaches at a college in San Francisco where gas costs $3.259 a gallon. His car holds 17.1 gallons of gasoline.

a. Estimate how much he would spend to fill an empty tank.

b. Calculate the amount he would actually spend to fill an empty tank.

Example 6 ***In June of* 2009*, IBM's Blue Gene/P computer, known as JUGENE, made third place on the list of fastest supercomputers in the world. Its speed was* 825.5 *teraflops (trillions of mathematical operations per second). The IBM BladeCenter QS22/LS21 Cluster Roadrunner system at the U.S. Department of Energy's Los Alamos Laboratory in New Mexico was the world's fastest supercomputer. Its speed was* 1.105 *petaflops, or quadrillions of mathematical operations per second. This is equivalent to* 1105 *teraflops. Estimate how many times faster the Roadrunner is than JUGENE.***

SOLUTION

1105 is approximately 1000 and 825.5 is approximately 800.

$$\frac{1105}{825.5} \approx \frac{1000}{800} = \frac{5}{4} = 1.25$$

Therefore, Roadrunner is approximately 1.25 times as fast as JUGENE.

Procedure

Estimating Quotients of Decimals

1. Think of the division as a fraction with the dividend as the numerator and the divisor as the denominator.
2. Round the numerator and denominator to one or two nonzero digits (numbers that divide easily).
3. Divide the results of step 2. This is the estimate.

12. Estimate the following quotients.

a. $857.3 \div 61.53$

b. $-0.0315 \div 32.2$

c. $8934.65 \div 0.0018$

d. If you divide a positive number by a decimal fraction between 0 and 1, is the quotient greater than or smaller than the original number?

SUMMARY: ACTIVITY 5.6

1. Multiplying two decimals:

   a. Multiply the two numbers as if they were whole numbers, ignoring decimal points for the moment.

   b. Add the number of digits to the right of the decimal point in each factor, to obtain the number of digits that must be to the right of the decimal point in the product.

   c. Place the decimal point in the product by counting digits from right to left.

2. Dividing decimals:

   a. Write the division in long division format.

   b. Move the decimal point the same number of places to the right in both divisor and dividend so that the divisor becomes a whole number.

   c. Place the decimal point in the quotient directly above the decimal point in the dividend and divide as usual.

3. Estimating products of decimals:

   a. Round each factor to one or two nonzero digits (numbers that multiply easily).

   b. Multiply the results of step a. This is the estimate.

4. Estimating quotients of decimals:

   a. Think of the division as a fraction, with the dividend as the numerator and the divisor as the denominator.

   b. Round the numerator and denominator to one or two nonzero digits (numbers that divide easily).

   c. Divide the results of step b. This is the estimate.

EXERCISES: ACTIVITY 5.6

1. You drive your own car while delivering pizzas for a local pizzeria. Your boss said she would reimburse you $0.47 per mile for the wear and tear on your car. The first week of work you put 178 miles on your car.

   a. Estimate how much you will receive for using your own car.

   b. Explain how you determined your estimate.

   c. How much did you actually receive for using your own car?

2. Perform the following calculations. Use your calculator to check your answer.

 a. $-12.53 \cdot -8.2$

 b. $115.3 \cdot -0.003$

 c. $14.62 \cdot -0.75$

 d. Divide 12.05 by 2.5.

 e. Divide 18.99729 by 78.

 f. Divide 14.05 by 0.0002.

 g. $150 \div 0.03$

 h. $0.00442 \div 0.017$

 i. $69.115 \div 0.0023$

3. You and some college friends go out to lunch to celebrate the end of midterm exams. There are six of you, and you decide to split the $53.86 bill evenly.

 a. Estimate how much each person owed for lunch.

 b. Explain how you determined your estimate.

 c. How much was each person's share of the bill? Round your answer to the nearest cent.

4. You have just purchased a new home. The town's assessed value of your home (which is usually much *lower* than the purchase price) is $84,500. For every $1000 of assessed value, you will pay $16.34 in taxes. How much do you pay in taxes?

5. Ryan Howard of the Philadelphia Phillies led the National League in home runs with 48 in the 2008 season. He also made 153 hits in 610 times at bat. What was his batting average? (Batting average = number of hits ÷ number of at bats.) Use decimal notation and round to the nearest thousandth.

6. The top-grossing North American concert tour between 1985 and 1999 was the Rolling Stones tour in 1994. The tour included 60 shows and sold $121.2 million worth of tickets.

 a. Estimate the average amount of ticket sales per show.

 b. What was the actual average amount of ticket sales per show?

 c. The Stones 2005 tour broke their previous record, selling $162 million worth of tickets in 42 performances. What was the average amount of ticket sales per show in 2005, rounded to the nearest $1000?

7. In 1992, the NEC SX-3/44 supercomputer had a speed of 0.02 teraflops (trillions of mathematical operations per second). In 2009, IBM announced that it was building a new supercomputer, dubbed the Sequoia, for the U.S. Department of Energy's National Nuclear Security Administration to simulate nuclear tests and explosions. The Sequoia will be able to achieve processing speeds close to 20 petaflops, or quadrillions of mathematical operations per second. Twenty petaflops are equivalent to 20,000 teraflops. How many times faster will the Sequoia be than the NEC SX-3/44?

8. Your friend tracked his GPA for his first three semesters as shown in the following table.

a. Calculate your friend's GPA for each of the semesters listed in the table. Record the results in the table.

	CREDITS	QUALITY POINTS	GPA
Semester 1	14	36.54	
Semester 2	15	49.05	
Semester 3	16	45.92	

b. Determine his cumulative GPA for the first two semesters.

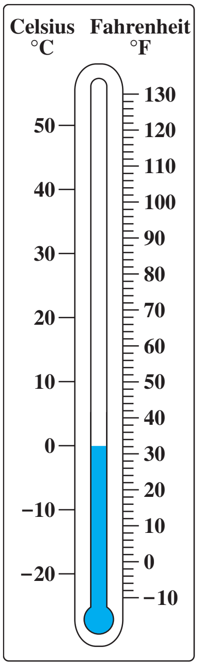
c. Determine his cumulative GPA for all three semesters.

Activity 5.7

Tracking Temperature

Objectives

1. Use the order of operations to evaluate expressions that include decimals.
2. Use the distributive property in calculations that involve decimals.
3. Evaluate formulas that include decimals.
4. Solve equations of the form $ax = b$ and $ax + bx = c$ that involve decimals.



Temperature measures the warmth or coolness of everything around us—of lake water, the human body, the air we breathe. The thermometer is Alaska's favorite scientific instrument. During the winter, says Ned Rozell at the Geophysical Institute of the University of Alaska at Fairbanks, "we check our beloved thermometers thousands of times a day."

Thermometers have scales whose units are called *degrees*. The two commonly used scales are Fahrenheit and Celsius. The Celsius scale is the choice in science. Most English-speaking countries began changing to Celsius in the late 1960s, but the United States is still the outstanding exception to this changeover.

Each scale was determined by considering two reference temperatures, the freezing point and the boiling point of water. On the Fahrenheit scale, the freezing point of water is taken as 32°F and the boiling point is 212°F. On the Celsius scale, the freezing point is taken as 0°C and the boiling point is 100°C. This leads to a formula that converts Celsius degrees to Fahrenheit degrees,

$$y = 1.8x + 32,$$

where y represents degrees Fahrenheit and x represents degrees Celsius.

1. Each year, the now-famous Iditarod Trail Sled Dog Race in Alaska starts on the first Saturday in March. Temperatures on the trail can range from about 5°C down to about −50°C.

   a. Use the formula $y = 1.8x + 32$ to convert 5°C to degrees Fahrenheit.

   b. Convert −50°C to degrees Fahrenheit.

   c. In a sentence, describe the temperature range in degrees Fahrenheit.

2. The Tour de France bicycle race is held in July in France. The race ends in Paris where July's temperature ranges from 57°F to 77°F. Use the formula $x = (y - 32) \div 1.8$ to convert these temperatures to degrees Celsius.

The previous problems show that expressions that involve decimals are evaluated by the usual order of operations procedures.

3. Evaluate the following expressions using order of operations. Use your calculator to check your answer.

   a. $100 + 100(0.075)(2.5)$

   b. $2(8.4 + 11.7)$

   c. $0.5(9.16 + 6.38) \cdot 4.21$

Solving Equations of the Form $ax = b$ and $ax + bx = c$ that Involve Decimals

4. The Tour de France is typically held in 21 stages of varying distances. One stage in the 2009 race ran from Marseille to Grande-Motte, a distance of 196.5 kilometers. The best time for completing this stage, won by Mark Cavendish of Great Britain, was 5.0233 hours.

a. Use the formula $d = rt$ to determine the average speed of the cyclist with the best time for this stage. Recall that d represents distance, t, time, and r, average speed.

b. The distance in miles for this stage is 122.1 miles. Determine Mark Cavendish's average speed for this stage in terms of miles per hour (mph).

The formula $d = rt$ is an example of an equation that is of the form $b = ax$. Problem 4 illustrates the fact that equations of the form $b = ax$ (or $ax = b$) that involve decimals are solved in the same way as those with whole numbers, integers, or fractions.

5. a. In the 2009 Iditarod Trail Sled Dog Race, the winner, Lance Mackey of Fairbanks, Alaska, and his dog team ran the last leg of the race from Safety to Nome in 2.83 hours. The distance between these two towns is about 22.0 miles. What was the average speed of Mackey's team?

b. Lance Mackey finished the entire 2009 race in 9 days, 21 hours, 38 minutes, and 46 seconds. In terms of hours, this is 237.646 hours. The race followed the northern route of about 1131 miles. What was his team's average speed over the entire race?

Solving equations of the form $ax + bx = c$ with decimals as coefficients is accomplished in the same way as with whole numbers, integers, or fractions. First, combine like terms and then solve the simpler equation of the form $ax = b$.

6. Solve the following equations for the unknown. Round the result to the nearest hundredth, if necessary.

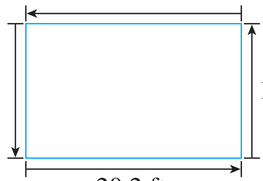
a. $0.6x + 2.1x = 54$

b. $2.4x - (-3.05)x = 6.54$

c. $3.21x - 5.46x = -742.5$

Distributive Property

Example 1 ***The vegetable garden along the side of your house measures* 20.2 *feet long by* 12.6 *feet wide. You want to enclose it with a picket fence. How many feet of fencing will you need?***



The diagram indicates that you can add the length to the width and then double it to determine the amount of fencing. Numerically, you write

$$2(20.2 + 12.6) = 2 \cdot 32.8 = 65.6 \text{ ft.}$$

Note that another way to solve the problem is to add *twice* the length to *twice* the width. Numerically,

$$2 \cdot 20.2 + 2 \cdot 12.6 = 40.4 + 25.2 = 65.6 \text{ ft.}$$

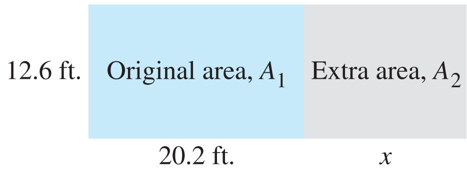
In either case, the amount of fencing is 65.6 feet. This means that the expression $2(20.2 + 12.6)$ is equal to the expression $2 \cdot 20.2 + 2 \cdot 12.6$. This example illustrates that the distributive property you learned for whole numbers, integers, and fractions also holds for decimal numbers.

As discussed in Chapter 2, the amount of fencing can be determined by using a formula for the perimeter of a rectangle. If P represents the perimeter, l, the length, and w, the width, then

$$P = 2(l + w) \quad \text{or} \quad P = 2l + 2w.$$

7. Your neighbor also has a rectangular garden, but its length is 10.5 feet by 13.8 feet wide. Use a perimeter formula to determine the distance around your neighbor's garden.

8. Next year you plan to make your garden longer and keep the width the same to gain more area for growing vegetables. You have not decided how much you want to increase the length. Call the extra length x. As you look at the diagram, note that a formula for the area of the new garden can be determined in two ways.



Method 1

a. Determine an expression for the new length in terms of x.

b. Multiply the expression in part a by 12.6 to obtain the new area, A. Recall that area equals the product of the length and width, or $A = lw$.

Method 2

c. Determine an expression for the extra area, A_2.

d. Add the extra area, A_2, to the original area, A_1, to obtain the new area A.

9. Explain how you may obtain the expression in Problem 8d directly from the expression in Problem 8b by the distributive property.

10. Use the distributive property to simplify the following.

a. $0.2(4x - 0.15) =$

b. $3.4(2.05x - 0.1) =$

11. Simplify each of the following expressions. Check the results using your calculator.

a. $0.2 - 0.42(5.4 - 6)^2$

b. $0.24 \div 0.06 \cdot 0.2$

c. $4.96 - 6 + 5.3 \cdot 0.2 - (0.007)^2$

12. Evaluate the following.

a. $2x - 4(y - 3)$, where $x = 0.5$ and $y = 1.2$

b. $b^2 - 4ac$, where $a = 6$, $b = 1.2$, and $c = -0.03$

c. $\dfrac{2.6x - 0.13y}{x - y}$, where $x = 0.3$ and $y = 6.8$

13. Evaluate the following using the order of operations. Check the results using a calculator.

a. $(-0.5)^2 + 3$

b. $(2.6 - 1.08)^2$

c. $(-2.1)^2 - (0.07)^2$

SUMMARY: ACTIVITY 5.7

1. The rules that apply to whole numbers, integers, and fractions, *also apply to decimals.* They are:

- the order of operations procedure
- the distributive property
- the evaluation of formulas

EXERCISES: ACTIVITY 5.7

In Exercises 1–4, calculate using the order of operations. Check the results using a calculator.

1. $48.5 - 10.34 + 4.66$

2. $0.56 \div 0.08 \cdot 0.7$

3. $10.31 + 8.05 \cdot 0.4 - (0.08)^2$

4. $-2.6 \cdot 0.01 + (-0.01) \cdot (-8.3)$

In Exercises 5–7, evaluate the expression for the given values.

5. $\frac{1}{2}h(a + b)$, where $h = 0.3$, $a = 1.24$, and $b = 2.006$

6. $3.14r^2$, where $r = 0.25$ inches

7. $A \div B - C \cdot D$, where $A = 10.8$, $B = 0.12$, $C = 2.4$, and $D = 6$

8. You earn \$8.15 per hour and are paid once a month. You record the number of hours worked each week in a table.

Week	1	2	3	4
Hours Worked	35	20.5	12	17.75

a. Write a numerical expression using parentheses to show how you will determine your gross salary for the month.

b. Use the expression in part a to calculate your monthly salary.

c. You think you are going to get a raise but don't know how much it might be. Write an expression to show your new hourly rate. Represent the amount of the raise by x.

d. Calculate the total number of hours worked for the month. Use it with your result in part c to write an expression to represent your new gross salary after the raise.

e. Use the distributive property to simplify the expression in part d.

f. Your boss tells you that your raise will be \$0.50 an hour. How much will you make next month if your hours are the same?

9. You have been following your stock investment over the past 4 months. Your friend tells you that his stock's value has doubled in the past 4 months. You know that your stock did better than that. Without telling your friend how much your original investment was, you would like to do a little boasting. The following table shows your stock's activity each month for 4 months.

Up with the Dow-Jones

Month	1	2	3	4
Stock Value	decreased \$75	doubled	increased \$150	tripled

a. Represent your original investment by x dollars and write an algebraic expression to represent the value of your stock after the first month.

b. Use the result from part a to determine an expression for the value at the end of the second month. Simplify the expression if possible. Continue this procedure until you determine the value of your stock at the end of the fourth month.

c. Explain to your friend what has happened to the value of your stock over 4 months.

d. Instead of simplifying after each step, write a single algebraic expression that represents the changes over the 4-month period.

e. Simplify the expression in part d. How does the simplified algebraic expression compare with the result in part b?

f. Your stock's value was $4678.75 four months ago. What is your stock's value today?

10. Translate each of the following verbal statements into equations and solve. Let x represent the unknown number.

a. The quotient of a number and 5.3 is -6.7.

b. A number times 3.6 is 23.4.

c. 42.75 is the product of a number and -7.5.

d. A number divided by 2.3 is 13.2.

e. -9.4 times a number is 47.

11. Solve each of the following equations. Round the result to the nearest tenth if necessary.

a. $3x = 15.3$

b. $-2.3x = 10.35$

c. $-5.2a = -44.2$

d. $15.2 = 15.2y$

e. $98.8 = -4y$

f. $4.2x = -\sqrt{64}$

g. $-0.6x + 2.1x = 54$

h. $4.675x - (-3.334x) = -18.50079$

i. $2.8x - 3.08x = -0.294$

12. You and your best friend are avid cyclists. You are planning a bike trip from Corning, New York, to West Point. On the Internet, you find that the distance from Corning to West Point is approximately 236.4 miles. On a recent similar trip, you both averaged about 15.4 miles per hour. You are interested in estimating the riding time for this trip to the nearest tenth of an hour.

a. Reread the problem and list all the numerical information you have been given.

b. What are you trying to determine? Give it a letter name.

c. What is the relationship between the length of your intended trip, the rate at which you travel, and what you want to determine?

d. Write an equation for the relationship in part c using the information in this problem.

e. Solve the equation in part d.

f. Is your solution reasonable? Explain.

13. Your Durango's gas tank holds 25 gallons. The last time you put in fuel, the gauge read empty, but it took only 22.5 gallons to fill it. What portion of the gas in the tank remains when the gauge reads empty?

Activity 5.8

Think Metric

Objectives

1. Know the metric prefixes and their decimal values.

2. Convert measurements between metric quantities.

The shop where you have a new job has just received a big contract from overseas. A significant part of your job will involve converting measurements in the English system to the metric system, used by most of the world. To find out more about changing measurements to the metric system, visit *www.metrication.com*.

The Old English system of measurement (for example, feet for length, pounds for weight) used throughout most of the United States was not designed for its computational efficiency. It evolved over time before the decimal system was adopted by Western European cultures. The metric system of measurement, which you may already know something about, was devised to take advantage of our decimal numeration system.

Example 1

A foot is divided into 12 equal parts, 12 inches. But as a fraction of a foot, $\frac{1}{12}$ is not easily expressed as a decimal number. The best you can do is approximate. One inch equals $\frac{1}{12}$ of a foot, or approximately $1 \div 12 \approx 0.08333$ feet. In the metric system, each unit of measurement is divided into 10 equal parts. So 1 meter (a little longer than 3 feet) is divided into 10 decimeters, each decimeter equaling $\frac{1}{10} = 0.1$ meter. In this case, the fraction can be expressed exactly as a decimal.

1. How many decimeters are in 1 meter?

2. If the specifications for a certain part is 8 decimeters long, what is the length in meters?

In general, the metric system of measurement relies upon each unit being divided into 10 equal subunits, which in turn are divided into 10 equal subunits, and so on. The prefix added to the base unit signifies how far the unit has been divided. Starting with the meter, as the basic unit for measuring distance, the following smaller units are defined by Latin prefixes.

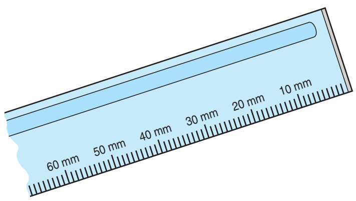
UNIT OF LENGTH	FRACTIONAL PART OF A METER
meter	1 meter
decimeter	0.1 meter
centimeter	0.01 meter
millimeter	0.001 meter

3. Complete each statement to show how many of each unit are in 1 meter.

_______ decimeters equal 1 meter.

_______ centimeters equal 1 meter.

_______ millimeters equal 1 meter.

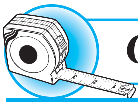


In Latin, the prefixes mean exactly what you found in Problem 3: deci- is 10, centi- is 100, and milli- is 1000.

Example 2 ***Any measurement in one metric unit can be converted to another metric unit by relocating the decimal point. 235 centimeters = 2.35 meters since 200 centimeters is the same as 2 meters and 35 centimeters is $\frac{35}{100}$ of 1 meter.***

4. Express a distance of 480 centimeters in meters.

Abbreviations are commonly used for all metric units:



Going the Distance

METRIC UNIT OF DISTANCE	ABBREVIATION
meter	m
decimeter	dm
centimeter	cm
millimeter	mm

5. How would you express 2.5 meters in centimeters?

6. Since there are 1000 millimeters in 1 meter, how many meters are there in 4500 millimeters?

7. How many millimeters are there in 1 centimeter?

8. Which distance is greater, 345 centimeters or 3400 millimeters?

In the metric system, each place value is equivalent to the unit of measurement. Each digit can be interpreted as a number of units.

Example 3 ***The distance 6.358 meters can be expanded to 6 meters + 3 decimeters + 5 centimeters + 8 millimeters.***

9. Expand 7.25 meters, as in Example 3.

Larger metric units are defined using Greek prefixes.

UNIT OF LENGTH	ABBREVIATION	DISTANCE IN METERS
meter	m	1 meter
decameter	dam	10 meters
hectometer	hm	100 meters
kilometer	km	1000 meters

In practice, the decameter and hectometer units are rarely used.

10. How many meters are in a 10-kilometer race?

11. Convert 3450 meters into kilometers.

The metric system is also used to measure the mass of an object and volume. Grams are the basic unit for measuring the mass of an object (basically, how heavy it is). Liters are the basic unit for measuring volume (usually of a liquid). The same prefixes are used to indicate larger and smaller units of measurement.

12. Complete the following tables.

UNIT OF MASS	ABBREVIATION	MASS IN GRAMS
milligram	mg	
centigram	cg	
decigram	dg	
gram	g	1 g
decagram	dag	
hectogram	hg	
kilogram	kg	

UNIT OF VOLUME	ABBREVIATION	VOLUME IN LITERS
milliliter	mL	
centiliter	cL	
deciliter	dL	
liter	L	1 L
decaliter	daL	
hectoliter	hL	
kiloliter	kL	

SUMMARY: ACTIVITY 5.8

1. The basic metric units for measuring distance, mass, and volume are meter, gram, and liter, respectively.

2. Larger and smaller units in the metric system are based on the decimal system. The prefix before the basic unit determines the size of the unit, as shown in the table.

UNIT PREFIX (BEFORE METER, GRAM, OR LITER)	ABBREVIATION	SIZE IN BASIC UNITS (METERS, GRAMS, OR LITERS)
milli-	mm, mg, mL	0.001
centi-	cm, cg, cL	0.01
deci-	dm, dg, dL	0.1
deca-	dam, dag, daL	10
hecto-	hm, hg, hL	100
kilo-	km, kg, kL	1000

EXERCISES: ACTIVITY 5.8

1. Convert 3.6 meters into centimeters.
2. Convert 287 centimeters into meters.
3. Convert 1.9 meters into millimeters.
4. Convert 4705 millimeters into meters.
5. Convert 4675 meters into kilometers.
6. Convert 3.25 kilometers into meters.
7. Convert 42.55 grams into centigrams.
8. Convert 236 grams into kilograms.
9. Convert 83.2 liters into milliliters.
10. Convert 742 milliliters into liters.
11. Which is greater, 23.67 cm or 237 mm?
12. Which is smaller, 124 milligrams or 0.15 grams?
13. Which is greater, 14.5 milliliters or 0.015 liters?
14. You have 1000 millimeters of string licorice, and your friend has 100 centimeters.
    a. Who has more licorice?
    b. You eat 200 millimeters of your candy. How many centimeters of licorice do you have?

Exercise numbers appearing in color are answered in the Selected Answers appendix.

c. Your friend gives you half of her candy. How many millimeters do you have now?

d. How many meters of licorice do you have?

15. Adults need to consume about 0.8 grams of protein per day for each kilogram of body weight. Children need varying amounts; for example, a child between the ages of 1 and 3 years old will need about 13 grams of protein per day.

a. How much daily protein is recommended for a 70-kilogram adult?

b. At what body weight would an adult require 80 grams of protein per day?

c. One large egg provides about 6.5 grams of high quality protein, with appropriate proportions of amino acids that our bodies need. How many eggs would provide enough daily protein for a child between 1 and 3 years old?

Cluster 2 What Have I Learned?

1. How do you determine which of two decimal numbers is the larger?

2. Why can zeros written to the right of the decimal point and after the rightmost nonzero digit be omitted without changing its value?

3. a. Round 0.31 to the nearest tenth.

   b. Determine the difference between 0.31 and 0.3.

   c. Determine the difference between 0.4 and 0.31.

   d. Which of the two differences you calculated is smaller, the one in part b or in part c?

   e. In the light of your answer to part d, give a justification for the rounding procedure.

4. Which step in the process of adding or subtracting decimals must be done first and very carefully?

5. Are the rules for addition or subtraction of decimals different from those of whole numbers?

6. When multiplying two decimals, how do you decide where to place the decimal point in the product? Illustrate with an example.

7. Why must you line up the decimal point when adding or subtracting decimals?

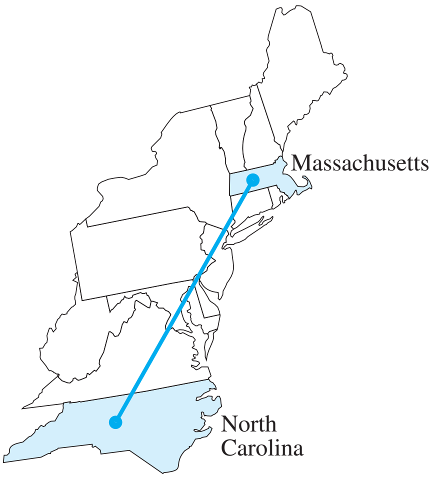
8. a. When faced with a division problem such as $2.35\overline{)1950.5}$, what would you do first?

b. Explain how you would estimate this quotient.

9. How do the rules for order of operations and the distributive property apply to decimals?

10. Explain the steps you would take to determine the value for I from the formula $I = P + Prt$ if $P = 1500$, $r = 0.095$, and $t = 2.8$.

11. **a.** Each day, you estimate the average speed at which you are traveling on a car trip from Massachusetts to North Carolina. You keep track of the hours and mileage. Explain how you would use the formula $d = rt$ to determine your average speed.



b. Explain how the two formulas, $d = rt$ and $I = Pr$ are mathematically similar.

12. **a.** Give at least one reason why estimating a product or quotient would be beneficial.

b. Give a reason(s) for the steps you would take to estimate $15{,}776 \div 425$.

13. If you divide a negative number by a number between 0 and 1, is the quotient greater than or less than the original number? Give an example to illustrate your answer.

Cluster 2 How Can I Practice?

1. Write in words the value of the given decimal number.

   a. 3495.065

   b. 71,008.0049

   c. 13,053.3891

   d. 6487.08649

2. Write the following in decimal notation.

   a. Eighty-two thousand, seventy-six and four hundred eight ten-thousandths

   b. Six hundred eleven thousand, seven hundred twelve and sixty-eight hundred-thousandths

   c. Three million, five thousand, ninety-two and forty-one thousandths

   d. Forty-nine thousand, eight hundred nine and six thousand four hundred thirteen hundred-thousandths

   e. Nine billion, five and twenty-eight hundredths

   f. Ninety-three million, five hundred sixty-four thousand, seven hundred and forty-nine hundredths

3. Rearrange the following decimals from smallest to largest.

   a. 0.9 0.909 0.099 0.0099 0.9900

   b. 0.384 0.0987 0.392 0.561 0.0983

   c. 1.49 1.23 1.795 0.842 0.01423

   d. 0.0838 0.8383 0.3883 0.00888 0.3838

Exercise numbers appearing in color are answered in the Selected Answers appendix.

4. a. Round 639.438 to the nearest hundredth.

   b. Round 31.2695 to the nearest thousandth.

   c. Round 182.09998 to the nearest ten-thousandth.

   d. Round 59.999 to the nearest tenth.

5. Estimate the products and quotients.

   a. $36.42 \cdot (-4.2)$

   b. $-226.90 \cdot (-0.005)$

   c. $14.62 \div (-0.75)$

   d. Divide 43,096.48 by 78.4.

6. Perform the additions and subtractions. Use a calculator to verify.

   a. $3.28 + 17.063 + 0.084$

   b. $628.13 + 271.78 + 68.456$

   c. $9628.13 + 271.78 + 6814.56 + 13.91$

   d. $171.004 + 18.028 + 51.68 + 4.5$

   e. $-69.73 + (-198.32)$

   f. $-131.02 + (-7.689)$

   g. $18.62 - 29.999$

   h. $-13.58 + 6.729$

   i. $258.1204 + 49.0683 + 71.099$

   j. $100 - 71.998$

   k. $212.085 - 63.0498$

   l. $329.79 - 84.6591$

   m. $8000.49 - 6583.725$

   n. $678.146 - 39.08$

   o. $-72.43 - 8.058$

   p. $49.62 - (-6.088)$

   q. $-18.173 - 12.59$

7. Calculate the products and quotients. Check your result with a calculator.

a. $0.108 \cdot 0.9$

b. $15.6 \cdot 2.05$

c. $36.42 \cdot (-4.2)$

d. $-226.90 \cdot (-0.005)$

e. $0.108 \div 0.9$

f. $40.368 \div 0.08$

g. $-2147.04 \div (-6.3)$

h. $14.625 \div (-0.75)$

8. Perform the following operations.

a. $0.63 \div 0.09 \cdot 0.5$

b. $30.38 + 4.05 \cdot 0.6 - (0.03)^2$

c. $-6.2 \cdot 0.03 + (-0.01) \cdot (-6.3)$

9. Evaluate.

a. $x + y$, where $x = -7.29$ and $y = 4.8$

b. $h + k$, where $h = -6.13$ and $k = -5.017$

c. $x - y$, where $x = -13.478$ and $y = 7.399$

d. $p - r$, where $p = -4.802$ and $r = -19.99$

e. $\frac{1}{2}h(a + b)$, where $h = 0.7$, $a = 3.45$, and $b = 5.00$

f. πr^2, where $r = 0.84$ inches; use 3.14 for π and round to the hundredths place.

g. $A \div B - C \cdot D$, where $A = 9.9$, $B = 0.33$, $C = 2.7$, and $D = 4$

10. Solve each of the equations for x. Round answer to nearest hundredth if necessary.

a. $x + 23.49 = -71.11$

b. $x - 13.14 = 69.17$

c. $-28.99 + x = -55.03$

d. $35.17 + x = 12.19$

e. $7x = 5.81$

f. $-2.3x = 58.65$

g. $-5.2a = -57.72$

h. $0.5x - 2.16 = -5.3$

i. $5.02x + 26.3 = -4.824$

11. Translate each of the following verbal statements into equations and solve. Let x represent the unknown number.

a. A number increased by 4.3 is 6.28.

b. The sum of a number and -1.32 is 8.6.

c. Eighteen subtracted from a number is -14.76.

d. The difference of 14.5 and a number is -6.34.

e. The quotient of a number and -7.4 is 13.5.

f. A number divided by -9.5 is 78.3.

g. The product of a number and -9.76 is 678.32.

12. Your hourly salary was increased from \$6.75 to \$7.04. How much was your raise?

13. To prepare for your new job, you spent \$25.37 for a shirt, \$39.41 for pants, and \$52.04 for shoes. How much did you spend?

14. The gross monthly income from your new job is \$2105.96. You have the following monthly deductions: \$311.93 for federal income tax, \$64.72 for state income tax, and \$161.11 for Social Security. What is your take-home pay?

15. You buy a Blu-ray disc player at a discount and pay \$136.15. This discounted price represents seven-tenths of the original price. Answer the following questions to determine the original price.

a. List all of the information you have been given.

b. What quantity are you trying to determine?

c. Write a verbal statement to express the relationship between the price you paid, the discount rate, and what quantity you want to determine.

d. Select a letter to represent the original price. Then write an equation based on the verbal statement in part c.

e. Solve the equation in part d to obtain the original price.

f. Is your solution reasonable? Explain.

16. George Herman "Babe" Ruth was baseball's first great batter and is one of the most famous players of all time.

a. In 1921, Babe Ruth played for the New York Yankees, hitting 59 home runs and making 204 hits out of 540 at bats. What was his batting average? Recall that batting averages are recorded to the nearest thousandth.

b. In 2001, Barry Bonds of the San Francisco Giants broke the single-season record for most home runs (73). During that season, he made 156 hits out of 476 at bats. What was his batting average?

c. Which of the two batting averages is the highest?

17. A student majoring in science earned the credits and quality points listed in the following table. Complete the table by determining the student's GPA for each semester and his cumulative GPA at the end of the fourth semester.

Majoring in Science

	CREDITS	QUALITY POINTS	GPA
Semester 1	14	35.42	
Semester 2	15	44.85	
Semester 3	16	55.20	
Semester 4	16	51.68	
Cumulative GPA			

18. A sophomore at your college took the courses and earned the grades that are listed in the following table. Complete the table and determine the GPA to the nearest hundredth.

	CREDITS	GRADE	NUMERICAL EQUIVALENT	QUALITY POINTS
English	3	C−	1.67	
Physics	4	B	3.00	
History	3	C+	2.33	
Math	4	A−	3.67	
Modern Dance	2	B+	3.33	
Totals		n/a	n/a	
GPA	n/a	n/a	n/a	

19. For Independence Day weekend, you and a Canadian friend are traveling to Philadelphia, PA, to tour U.S. historical sites. The temperature on the Fourth of July is forecast to range from a low of 66°F to a high of 82°F. Determine these temperatures in degrees Celsius for your friend coming from Canada to do the tour. Recall the formula $x = (y - 32) \div 1.8$, where x is the temperature in Celsius and y is the temperature in Farenheit.

20. Perform the following conversions.

a. Convert 3872 milligrams to grams.

b. Convert 7.34 kilometers to meters.

c. Convert 392 milliliters to liters.

d. Convert 0.64 kilograms to grams.

e. Convert 4.5 kilometers to millimeters.

f. Convert 0.014 liter to milliliters.

21. The boundaries around a parcel of land measure 378 m, 725 m, 489 m, and 821 m. What is the perimeter, measured in kilometers?

22. You have 670 grams of carbon but need precisely 2.25 kg for your experiment. How much more carbon do you need?

23. You start with 1.275 liters of solution in a beaker. Into four separate test tubes you pour 45 milliliters, 75 milliliters, 90 milliliters, and 110 milliliters of the solution. How much solution is left in your original beaker?

24. In your chemistry class, you need to weigh a sample of pure carbon. You will weigh the amount that you have three times and take the average (called the *mean*) of the three weighings. The average weight is the one you will report as the actual weight of the carbon. The three weights you obtain are 12.17 grams, 12.14 grams, and 12.20 grams.

a. Explain how you will calculate the average.

b. Determine the average weight of the carbon to the nearest hundredth of a gram.

25. Write each of the following decimals as an equivalent reduced fraction or mixed number.

a. 0.576

b. 6.075

c. -7.56

d. 113.02

26. Write each of the following fractions as a terminating decimal or, if a repeating decimal, rounded to the thousandths place.

a. $\frac{3}{5}$

b. $\frac{11}{25}$

c. $\frac{7}{22}$

d. $\frac{9}{20}$

Chapter 5 Summary

The bracketed numbers following each concept indicate the activity in which the concept is discussed.

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Converting mixed numbers to improper fractions [5.1]	To convert a mixed number to an improper fraction, multiply the integer part by the denominator and add the numerator. The result is the numerator of the improper fraction. The denominator remains unchanged.	$6\frac{2}{3} = \frac{6 \cdot 3 + 2}{3}$ $= \frac{20}{3}$
Converting improper fractions to mixed numbers [5.1]	To convert an improper fraction to a mixed number, divide the numerator by the denominator. The quotient becomes the integer part of the mixed number. The remainder becomes the numerator of the fraction part, and the denominator is unchanged.	$\frac{27}{4}$; $\begin{array}{r} 6 \\ 4\overline{)27} \\ -24 \\ \hline 3 \end{array}$ $\frac{27}{4} = 6\frac{3}{4}$
Adding mixed numbers with the same denominator [5.1]	To add mixed numbers that have the same denominator, add the integer parts and the fraction parts separately. If the sum of the fraction parts is an improper fraction, convert it to a mixed number and add the integer parts. Write the fraction part in lowest terms if necessary.	$2\frac{13}{15} + 1\frac{7}{15} = 2 + 1 + \frac{13}{15} + \frac{7}{15}$ $= 3\frac{20}{15} = 3 + 1\frac{5}{15}$ $= 4\frac{5}{15} = 4\frac{1}{3}$
Subtracting mixed numbers with the same denominator [5.1]	To subtract one mixed number from another, subtract the integer parts and the fraction parts separately. If the resulting integer and fraction parts have opposite signs, remove 1 from the integer part to complete the subtraction. Write the fraction part in lowest terms if necessary.	$7\frac{4}{9} - 4\frac{7}{9} = 7 + \frac{4}{9} - 4 - \frac{7}{9}$ $= 7 - 4 + \frac{4}{9} - \frac{7}{9}$ $= 3 - \frac{3}{9} = 2 + 1 - \frac{3}{9}$ $= 2 + \frac{9}{9} - \frac{3}{9} = 2\frac{6}{9} = 2\frac{2}{3}$
Adding and Subtracting Mixed Numbers with Different Denominators [5.2]	Add or subtract the integer parts and the fraction parts separately. Determine the LCD and rewrite both fractions with the LCD as the denominator; then add or subtract and write the result in lowest terms. If the sum or difference of the fractions is an improper fraction, rewrite it as a mixed number and combine the integer parts.	$2\frac{3}{4} + 6\frac{5}{7} = 2\frac{21}{28} + 6\frac{20}{28}$ $= 2 + 6 + \frac{21}{28} + \frac{20}{28}$ $= 8\frac{41}{28} = 8 + 1\frac{13}{28} = 9\frac{13}{28}$

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Solving equations of the form $x + b = c$ and $x - b = c$ that involve mixed numbers [5.2]	• For $x + b = c$, add the opposite of b to both sides of the equation to obtain $x = c - b$. • For $x - b = c$, add b to both sides of the equation to obtain $x = c + b$.	$x - 3\frac{1}{4} = 9\frac{1}{2}$ $\underline{+3\frac{1}{4} = 3\frac{1}{4}}$ $x = 9\frac{1}{2} + 3\frac{1}{4}$ $x = 9 + 3 + \frac{1}{2} + \frac{1}{4}$ $x = 12 + \frac{2}{4} + \frac{1}{4} = 12\frac{3}{4}$
Multiplying mixed numbers [5.3]	To multiply mixed numbers: 1. Change the mixed numbers to improper fractions. 2. Multiply as you would proper fractions. 3. If the product is an improper fraction, convert it to a mixed number.	$3\frac{1}{2} \cdot 4\frac{3}{8} = \frac{7}{2} \cdot \frac{35}{8}$ $= \frac{245}{16} = 15\frac{5}{16}$
Dividing mixed numbers [5.3]	To divide mixed numbers: 1. Change the mixed numbers to improper fractions. 2. Divide as you would proper fractions. 3. If the quotient is an improper fraction, convert it to a mixed number.	$4\frac{5}{8} \div 2\frac{1}{3} = \frac{37}{8} \div \frac{7}{3}$ $= \frac{37}{8} \cdot \frac{3}{7} = \frac{111}{56}$ $= 1\frac{55}{56}$
Reading and writing decimals [5.4]	To name a decimal number, read the digits to the right of the decimal point as an integer, and then attach the place value of its rightmost digit. Use "and" to separate the integer part from the decimal or fractional part.	• Write 3.42987 in words. *Answer:* three and forty-two thousand nine hundred eighty-seven hundred-thousandths • Write seven and sixty-nine thousandths in decimal form. *Answer:* 7.069
Converting a decimal to a fraction or a mixed number [5.4]	1. Write the nonzero integer part of the number and drop the decimal point. 2. Write the fractional part of the number as the numerator of a new fraction, whose denominator is the same as the place value of the rightmost digit. If possible, reduce the fraction.	3.0025 $= 3\frac{25}{10{,}000}$ $= 3\frac{1}{400}$
Comparing decimals [5.4]	To compare two or more decimals, line up the decimal points and compare the digits that have the same place value, moving from left to right. The first number with the largest digit in the same "value place" is the largest number.	Compare 0.035 and 0.038 0 . 0 3 5 0 . 0 3 8 same same same 8 > 5 $0.038 > 0.035$

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Rounding decimals [5.4]	1. Locate the digit with the specified place value. 2. If the digit directly to its right is less than 5, keep the digit that is in the specified place and delete all the digits to the right. 3. If the digit directly to its right is 5 or greater, increase this digit by 1 and delete all the digits to the right. 4. If the specified place value is located in the integer part, proceed as instructed in step 2 or step 3. Then insert trailing zeros to the right and up to the decimal point as place holders. Drop the decimal point.	Round 0.985 to the nearest hundredth. *Answer:* 0.99 Round 0.653 to the nearest hundredth. *Answer:* 0.65 Round 424.6 to the nearest ten. *Answer:* 420
Adding and subtracting decimals [5.5]	1. Rewrite the numbers vertically, lining up the decimal points. 2. Add or subtract as usual. 3. Insert the decimal point in the answer directly in line with the decimal points of the numbers being added.	Add: $3.689 + 41 + 12.07$ $\begin{array}{r} 3.689 \\ 41.000 \\ +12.070 \\ \hline 56.759 \end{array}$ **Assume zeros for any digits not shown.** **Decimal point carries down.**
Multiplying decimals [5.6]	To multiply decimals: 1. Multiply the decimal numbers as if they were whole numbers, ignoring decimal points for the moment. 2. Add the number of digits to the right of the decimal point from both factors to obtain the number of digits that must be to the right of the decimal point in the product. 3. Place the decimal point in the product by counting digits from right to left.	$\begin{array}{r} 23.4 \\ \times\ 2.45 \\ \hline 1170 \\ 936 \\ 468 \\ \hline 57.330 \end{array}$
Dividing decimals [5.6]	To divide decimals: 1. Write the division in long division format. 2. Move the decimal point the same number of places to the right in both divisor and dividend so that the divisor becomes a whole number. 3. Place the decimal point in the quotient directly above the decimal point in the dividend and divide as usual.	$\begin{array}{r} 56.7 \\ 2.3\overline{)130.41} \\ 115 \\ \hline 154 \\ 138 \\ \hline 161 \\ 161 \\ \hline 0 \end{array}$
Estimating products [5.6]	To estimate products: 1. Round each factor to one or two nonzero digits. 2. Multiply the rounded factors. This is the estimate.	Estimate 279×52. $300 \times 50 = 15{,}000$

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Estimating quotients [5.6]	To estimate quotients: 1. Write the division as a fraction. 2. Round the numerator and denominator to one or two nonzero digits. 3. Divide the results of step 2. This is the estimate.	Estimate $93.4\overline{)345.26}$. $\frac{345.26}{93.4} \approx \frac{350}{100} = 3.5$
Order of operations [5.7]	The order of operation rules for decimals are the same as those applied to integers.	$30.38 + 4.5 \cdot 0.6 - (0.3)^2$ $= 30.38 + 4.5 \cdot 0.6 - 0.09$ $= 30.38 + 2.7 - 0.09$ $= 33.08 - 0.09$ $= 32.99$
Evaluating expressions or formulas that involve decimals [5.7]	To evaluate formulas or expressions that involve decimals, substitute the desired values for each of the variables and evaluate using the order of operations.	Evaluate $7 - x^2$, where $x = 0.5$. $7 - x^2 = 7 - (0.5)^2$ $= 7 - 0.25$ $= 6.75$
Solving equations of form $ax + b = c$ with decimals [5.7]	Combine like terms; then solve the simpler equation of the form $ax = b$.	$3.2x + 0.9 = 19.46$ $3.2x = 18.56$ $x = 5.8$
The metric system [5.8]	Basic metric units for measuring: distance : meter (m) mass : gram (g) volume : liter (L) Larger and smaller units in the metric system are based on the decimal system. The prefix before the basic unit determines the size of the unit, as shown in the table.	1000 mm = 1 m 100 cm = 1 m 10 dm = 1 m 0.001 m = 1 mm 0.01 m = 1 cm 0.1 m = 1 dm 1000 g = 1 kg 0.001 kg = 1 g 100 mL = 0.1 L 243 mL = 0.243 L 6.8 L = 6800 mL 735 cm = 7.35 m 3.92 km = 3920 m

UNIT PREFIX	ABBR.	SIZE IN BASIC UNITS
milli-	m-	0.001
centi-	c-	0.01
deci-	d-	0.1
deca-	da-	10
hecto-	h-	100
kilo-	k-	1000

Chapter 5 Gateway Review

1. Convert each to a mixed number.

a. $\frac{63}{5}$

b. $\frac{63}{7}$

c. $\frac{77}{15}$

d. $\frac{77}{13}$

2. Convert each to an improper fraction.

a. $4\frac{3}{5}$

b. $2\frac{5}{7}$

c. $5\frac{2}{11}$

d. $10\frac{3}{8}$

3. Write the following numbers in words.

a. 849,083,659.0725

b. 32,004,389,412.23418

c. 235,000,864.587234

d. 784,632,541.00819

4. Write the following numbers in decimal notation.

a. Sixty-five million, seventy-three thousand, four hundred twelve and six hundred eighty-two ten-thousandths

b. Eighty-nine billion, five hundred forty-nine thousand, six hundred thirteen and forty-eight thousandths

c. Seven million, six hundred twelve thousand, eleven and five thousand, six hundred one hundred-thousandths

Answers to all Gateway exercises are included in the Selected Answers appendix.

5. Convert the following decimals to mixed numbers and reduce to lowest terms.

a. 0.0085

b. 3.834

c. 4.25

d. 15.0125

e. 7.12

6. Write each of the following fractions as a terminating decimal or, if a repeating decimal, rounded to the hundredths place.

a. $\frac{5}{8}$

b. $\frac{47}{1000}$

c. $\frac{20}{33}$

d. $-\frac{19}{25}$

7. Round 692,895.098442 to the nearest specified place values.

a. hundredth

b. thousandth

c. tenth

d. whole number

e. hundreds

f. ten thousands

8. Rearrange the following groups of decimal numbers in order from smallest to largest:

a. 4.078 4.78 4.0078 4.0708 4.00078 4.07008

b. 3.00805 3.085 3.0085 3.00085 3.0805 3.85

c. 8.046 8.0046 8.46 8.0406 8.00046 8.00406

9. Perform the indicated operations. Leave your answer in lowest terms.

a. $14\frac{5}{7} + 10\frac{2}{7}$

b. $12\frac{1}{9} + 7\frac{5}{9}$

c. $13\frac{5}{6} + 10\frac{7}{8}$

d. $11\frac{1}{9} + 8\frac{5}{6}$

e. $3\frac{5}{8} + 7\frac{3}{4}$

f. $23\frac{11}{13} - 12\frac{9}{13}$

g. $4\frac{1}{3} - 2\frac{1}{6}$

h. $10\frac{4}{5} - 4\frac{9}{10}$

i. $8 - 3\frac{2}{3}$

j. $5\frac{3}{7} - 2$

k. $-3\frac{3}{10} + 7\frac{4}{5}$

l. $-5\frac{5}{9} + \left(-8\frac{5}{6}\right)$

m. $6\frac{3}{8} - \left(+8\frac{1}{6}\right)$

n. $10\frac{5}{9} - \left(-4\frac{3}{5}\right)$

o. $-11\frac{5}{9} + \left(-1\frac{7}{12}\right)$

p. $-15\frac{4}{9} - \left(-3\frac{1}{6}\right)$

q. $-7 - \frac{4}{5}$

r. $\frac{4}{9} \cdot -\frac{15}{14}$

s. $2\frac{4}{5} \div 2\frac{1}{10}$

t. $-3\frac{1}{8} \cdot -2\frac{2}{7}$

u. $-4\frac{2}{3} \div 1\frac{1}{6}$

v. $-\frac{5}{8} \div -\frac{1}{2}$

10. Perform the indicated operations.

a. $-16.78 + 4.29 + (-5.13)$

b. $-15.79 + (-18.63) + (+63.49)$

c. $-23.19 - (+8.93)$

d. $-42.78 - (-19.41)$

e. $-23.58 - (+17.39)$

f. $-4 \cdot 38$

g. $-42 \div -6$

h. $30.8 \div .002$

i. $-4.89(-.0008)$

11. Evaluate each expression.

a. $-7.2 \cdot 0.06 + .81 \div (-0.9)$

b. $-\frac{5}{6} \div \frac{2}{3} \cdot -2\frac{1}{4} - \left(1\frac{3}{4}\right)^2$

12. Evaluate.

a. $L + W$, where $L = 12.68$ and $W = 7.05$

b. $P - D$, where $P = 98.99$ and $D = 14.85$

c. $K + L$, where $K = -17.32$ and $L = -4.099$

d. $S - T$, where $S = 3.98$ and $T = 0.125$

e. $2x - 6(x - 3)$, where $x = -1\frac{1}{2}$

f. $\pi r^2 h$, where $r = 2.4$ and $h = 0.9$

13. Solve for x.

a. $x - 2\frac{5}{8} = 7\frac{1}{6}$

b. $x + 13\frac{7}{9} = 6\frac{3}{7}$

c. $-18\frac{3}{4} + x = -21\frac{1}{3}$

d. $16\frac{3}{11} + x = -17\frac{5}{22}$

e. $x + 17\frac{9}{14} = -6\frac{3}{7}$

f. $x - 3\frac{7}{12} = 5\frac{5}{18}$

g. $x + 23.14 = 48.69$

h. $17.32 + x = -28.73$

i. $x - 16.39 = 32.11$

j. $x - 22.03 = -41$

k. $-18.79 + x = -11.32$

l. $-2\frac{1}{4} = -\frac{x}{18}$

m. $\frac{6}{5}x = -72$

n. $-\frac{33}{20}w = \frac{9}{5}$

o. $8n = 73.84$

p. $-1.24x + 3.8x = -51.2$

14. Translate each of the following verbal statements into an equation and solve for the unknown value. Let x represent the unknown value.

a. The quotient of a number and -15 is -0.7.

b. $-\frac{11}{12}$ times a number is $2\frac{1}{16}$.

c. 1.08 is the product of a number and 0.2.

15. You want to install a wallpaper border in your bedroom. The bedroom is rectangular, with length $12\frac{5}{12}$ and width $8\frac{1}{4}$ feet. How much wallpaper border would you use?

16. On Sunday morning you went for your usual run. On the route you stopped and bought a newspaper and bagels. The newsstand is $\frac{1}{3}$ mile from your house. The distance from the newsstand to your favorite bagel shop is approximately $1\frac{3}{5}$ miles. How many miles did you run each way?

17. Your living room wall is 14 feet wide. You have a couch that is $6\frac{1}{3}$ feet long that is placed against the wall. You also have two end tables $2\frac{1}{8}$ feet wide on each side of the couch. You saw a bookcase that is $2\frac{5}{6}$ feet wide that you would like to buy, but you are not sure if it would fit in the remaining wall space. Would it?

18. Your new TV cost $179.99. There was a sales tax of $14.85. What was the final price of the TV?

19. You buy $18.35 worth of groceries and pay for them with a $20 bill. How much change would you receive?

20. A dress that usually sells for $67.95 is marked down by $10.19. What is the sale price of the dress?

21. A cook making $1504.75 a month has deductions of $157.32 for federal income tax, $115.11 for Social Security, and $45.12 for state income tax. What is the cook's take-home pay?

22. You had $62.05 in your bank account on Monday morning. On Tuesday, you wrote two checks for $25.12 and $13.59. On Wednesday, your friend who owes you $40 returned the money, and you deposited it in your bank account. On Thursday, knowing that you would deposit your paycheck on Friday, you wrote two checks for $117.50 and $85.38. On Friday, you deposited your paycheck for $359.13. When all your checks clear, how much money will you have in your bank account?

23. You have prepared $9\frac{1}{2}$ gallons of fruit punch for your friend's bridal shower. How many $\frac{3}{4}$-cup servings will you have for the guests?

24. According to the U.S. Bureau of the Census, the population of North Dakota in 2000 was 642,195. In 1930, North Dakota's population was 680,845. Determine the average population decrease per year since 1930.

25. There are 47,214 square miles of land in New York State. The population of the state is 19,409,297. If the land were divided equally among all people in New York State, approximately what part of a square mile would each person get? Round your answer to the nearest thousandth.

26. In 1820, there were approximately 2.9 million workers in the United States. Approximately 7 out of 10 of these workers were employed in farm-related occupations. How many people had farm occupations in 1820? (*Source*: U.S. Dept. of Agriculture)

27. You are trying to understand your electric bill. It looks like there are delivery charges for the electricity as well as supply charges. The basic charge for electricity for May 6 to June 7 was $16.21. In addition, you used 300 kWh of electricity. The delivery charge for 300 kWh was $0.060447 per kWh. The supply charge was $0.04751 per kWh. Determine your total electric bill before taxes.

28. There are 7318 air miles between London and Honolulu. If you are a passenger on a commercial plane, what is the average speed of the plane if the trip takes approximately $11\frac{3}{4}$ hours? Estimate your answer and then solve the equation.

29. You are making crafts to sell at an arts festival. There are two different types. The ceramic bowl sells for \$15.85 and the candle holder sells for \$9.95. Let *b* represent the number of bowls that you sell and *h* represent the number of candle holders.

a. Write an expression that will determine the revenue from the sale of your crafts at the festival.

b. If you sell 26 bowls and 13 holders, what would be the total revenue?

30. To help pay tuition, you take a part-time job at a fast-food restaurant. The job pays \$7.25 per hour. Since you are going to a community college full-time, you need to determine how many hours you will work at the restaurant so you can balance your schoolwork and your job.

a. Use the input/output table to calculate the total amount of salary (output) for different numbers of hours (input) worked.

No. of Hours	12	18	24	30
Total Salary				

b. Explain how you determined the total salary in the table.

c. Write an equation to determine the number of hours you should work to receive \$200 per week and solve.

31. You are planning a trip with your best friend from college. You have 4 days for your trip, and you plan to travel *x* hours per day. On the first day you drove 2 extra hours; the second day you doubled the number of hours you planned; on the third day you lost 4 hours due to fog; and on the fourth day you traveled only one-fourth of the planned time due to sightseeing. The table indicates the number of hours traveled per day in relationship to *x* hours per day.

Day	1	2	3	4
No. of Hours	gained 2	double time	lost 4	$\frac{1}{4}$ of planned time

a. Write an expression to show how many hours you traveled the first day.

b. Write an expression to show the number of hours you traveled for the first 2 days.

c. Write an expression to show the number of hours you traveled for the 4 days. Write in simplest form.

d. If you averaged 52 miles per hour over the 4 days, write an equation to determine the total distance traveled for the 4 days.

e. If 7 hours per day was your anticipated time traveling per day, determine the number of miles you would have traveled on your trip.

32. Perform the following conversions.

a. Convert 795 milligrams into grams.

b. Convert 2.75 kilometers into meters.

c. Convert 25 milliliters into liters.

d. Convert 1.05 kilograms into grams.

e. Convert 2.35 kilometers into millimeters.

f. Convert 0.085 liters into milliliters.

33. The boundaries around a parcel of land measure 455 meters, 806 meters, 423 meters, and 795 meters. What is the perimeter, measured in kilometers?

34. You have 285 milligrams of sulfur but need precisely 1.25 grams for your experiment. How much more sulfur do you need?

35. You start with 1.92 liters of solution in a beaker. Into three separate test tubes you pour 95 milliliters, 150 milliliters, and 180 milliliters of the solution. How much solution is left in your original beaker? Let x represent the number of milliliters left in the original beaker.

Chapter 6

Problem Solving with Ratios, Proportions, and Percents

When you deal with sales tax, mortgage and car payments, sales and discounts, salaries, and sports statistics, you use proportional reasoning.

When you work with drug doses in medicine, liquid solutions in chemistry, weights and volumes in physics, species identification in biology, and similar triangles in geometry, you use proportional reasoning. The list goes on and on.

Proportional reasoning is based on the idea of relative comparison of quantities. In this chapter you will learn about relative comparisons and how such comparisons lead to the use of fractions, decimals, and percents. When you have completed this chapter, you will have the basic mathematical tools you need for solving problems that require proportional reasoning.

Activity 6.1

Everything Is Relative

Objectives

1. Understand the distinction between actual and relative measure.
2. Write a ratio in its verbal, fraction, decimal, and percent formats.

In the 2009 National Basketball Association finals, the Los Angeles Lakers defeated the Orlando Magic. Among the outstanding players during the series were Kobe Bryant and Pau Gasol of the Lakers and Dwight Howard and Hedo Turkoglu of the Magic. The data in the table represent each man's field goal totals for the five-game championship series.

PLAYER	FIELD GOALS MADE	FIELD GOALS ATTEMPTED
Bryant	58	135
Gasol	36	60
Howard	21	43
Turkoglu	30	61

1. Using *only* the data in column 2, Field Goals Made, rank the players from best to worst according to their field goal performance.

2. You can also rank the players by using the data from both column 2 and column 3. For example, Kobe Bryant made 58 field goals out of the 135 he attempted. The 58 successful baskets can be compared to the 135 attempts by *dividing* 58 *by* 135. You can represent that comparison numerically as the fraction $\frac{58}{135}$, or, equivalently, as the decimal 0.430 (rounded to thousandths).

   Complete the following table and use your results to determine another ranking of the four players from best to worst performance based on the ratio of goals made to goals attempted.

Key Players

PLAYER	FIELD GOALS MADE	FIELD GOALS ATTEMPTED	EXPRESSED		
			VERBALLY	AS A FRACTION	AS A DECIMAL
Bryant	58	135	58 out of 135	$\frac{58}{135}$	0.430
Gasol	36	60			
Howard	21	43			
Turkoglu	30	61			

Actual and Relative Measure

The two sets of rankings in Problems 1 and 2 were based on two different points of view. The ranking in Problem 1 is from an **actual** viewpoint in which you just count the *actual* number of field goals made. The ranking in Problem 2 is from a **relative** perspective in which you take into account the number of successes relative to the number of attempts.

3. Which measure best describes field-goal performance: actual or relative? Explain.

4. Identify which statements refer to an actual measure or a relative measure. Explain your answers.

a. I got 7 answers wrong.

b. I guessed on 4 answers.

c. Two-thirds of the class failed.

d. I saved $10.

e. I saved 40%.

f. 4 out of 5 students work to help pay tuition.

g. Derek Jeter's career batting average is 0.316.

h. Barry Bonds hit 73 home runs, the major league record, in 2001.

i. In 2008, 21 million Americans owned a Blackberry.

j. By 2010, 70% of all physicians relied on a personal digital assistant or smartphone.

5. What mathematical notation or verbal phrases in Problem 4 indicated a relative measure?

Definitions

Relative measure is a term used to describe the comparison of two similar quantities.

Ratio is the term used to describe relative measure as a quotient of **two similar quantities**, often a "part" and a "total," where "part" is *divided by* the "total."

6. Use the free-throw statistics from the 2009 championship games to express each player's relative performance as a ratio in verbal, fraction, and decimal form. The ratios for Kobe Bryant are completed for you. Round decimals to the nearest thousandth.

Nothing but Net

PLAYER	FREE THROWS MADE	FREE THROW ATTEMPTED	EXPRESSED		
			VERBALLY	AS A FRACTION	AS A DECIMAL
Bryant	37	44	37 out of 44	$\frac{37}{44}$	0.841
Gasol	21	27			
Howard	35	58			
Turkoglu	23	31			

7. a. Three friends, shooting baskets in the schoolyard, kept track of their performance. Andy made 9 out of 15 shots, Pat made 28 out of 40, and Val made 15 out of 24. Rank their relative performance.

b. Which ratio form (fraction, decimal, or other) did you use to determine the ranking?

8. A local animal rescue group limits its rescues to dogs, cats, and birds. Currently it has 6 dogs, 15 cats, and 8 birds available for adoption. Write each of the following ratios verbally and as a fraction.

a. The ratio of dogs to cats.

b. The ratio of birds to cats.

c. The ratio of birds to four-legged animals.

d. The ratio of dogs to the total number of animals.

9. Your mathematics class is composed of 35 students, 12 of whom are males. Write each of the following ratios verbally and as a fraction.

a. The ratio of males to the total.

b. The ratio of males to females.

10. Describe how to determine if the two ratios "12 out of 20" and "21 out of 35" are equivalent?

11. a. Match each ratio from column A with the equivalent ratio in column B.

Column A	Column B
15 out of 25	84 out of 100
42 out of 60	65 out of 100
63 out of 75	60 out of 100
52 out of 80	70 out of 100

b. Which set of ratios, those in column A or those in column B, is more useful in comparing and ranking the ratios? Why?

Percents

Relative measure based on 100 is familiar and natural. There are 100 cents in a dollar and 100 points on many tests. You most likely have an instinctive understanding of ratios relative to 100. A ratio such as 40 out of 100 can be expressed as 40 **per** 100, or, more commonly, as 40 **percent**, 40%. Percent *always* indicates a ratio "out of 100."

12. Express each ratio in column B of Problem 11 as a percent, using the symbol "%."

Since each ratio in Problem 12 is already a ratio "out of 100," you replaced the phrase "out of 100" with the % symbol. But suppose you need to write a ratio such as 21 out of 25 in percent format. You may recognize that the denominator, 25, is a factor of 100 $(25 \cdot 4 = 100)$. Then the fraction $\frac{21}{25}$ can be written equivalently as

$$\frac{21 \cdot 4}{25 \cdot 4} = \frac{84}{100}, \text{ which is precisely } 84\%.$$

A more general method to convert the ratio 21 out of 25 into percent format is to first calculate the quotient, 21 divided by 25, by calculator or by long division.

On a calculator:

Key in [2] [1] [÷] [2] [5] [=] to obtain 0.84.

Using long division:

$$\begin{array}{r} 0.84 \\ 25\overline{)21.00} \\ \underline{200} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

Next, you convert the decimal form of the ratio into percent format by multiplying the decimal by $\frac{100}{100} = 1$.

$$0.84 = 0.84 \cdot \frac{100}{100} = (0.84 \cdot 100) \cdot \frac{1}{100} = \frac{84}{100} = 84\%$$

Note that multiplying the decimal by 100 moves the decimal point in 0.84 two places to the right. This leads to a shortcut for converting a decimal to a percent.

Procedure

Converting a Fraction or Decimal to a Percent

1. Convert the fraction to decimal form by dividing the numerator by the denominator.
2. Move the decimal point in the quotient from step 1 two places to the right, inserting placeholder zeros if necessary.
3. Then attach the % symbol to the right of the number.

$$\text{Example: } \frac{4}{5} = 5\overline{)4} = 0.80 = 80\%$$

13. Convert each of the following ratios to a percent.

a. 35 out of 100

b. 16 out of 50

c. 8 out of 20

d. 7 out of 8

In many applications, you will need to convert a percent to decimal form. The following examples demonstrate the process.

Example 1

Convert 73% to a decimal.

SOLUTION

First locate the decimal point.

$$73.\%$$

Next, since percent means "out of 100,"

$$73.\% = \frac{73}{100}.$$

Finally, since division by 100 results in moving the decimal point two places to the left,

$$\frac{73.}{100} = 0.73$$

Therefore, $73\% = 0.73$.

Example 2

Convert 4.5% to a decimal.

SOLUTION

$$4.5\% = \frac{4.5}{100} = 0.045$$

Note that it was necessary to insert a placeholding zero.

Procedure

Converting a Percent to a Decimal

1. Locate the decimal point in the number attached to the % symbol.
2. Move the decimal point two places to the left, inserting placeholding zeros if needed, and write the decimal number without the % symbol.

14. Convert the following percents to a decimal.

a. 75%

b. 3.5%

c. 200%

d. 0.75%

15. Use the three-point shot statistics from the 2009 NBA finals to express each player's relative performance as a ratio in verbal, fraction, decimal, and percent form. Round the decimal to the nearest hundredth. The ratios for the Laker's Kobe Bryant are completed for you.

PLAYER	THREE-POINT SHOTS MADE	THREE-POINT SHOTS ATTEMPTED	EXPRESSED			
			VERBALLY	AS A FRACTION	AS A DECIMAL	AS A PERCENT
K. Bryant (LAL)	9	25	9 out of 25	$\frac{9}{25}$	0.36	36%
D. Ariza (LAL)	10	24				
R. Lewis (ORL)	16	40				
H. Turkoglu (ORL)	7	16				

SUMMARY: ACTIVITY 6.1

1. **Relative measure** is a term used to describe the comparison of two similar quantities.
2. **Ratio** is the term used to describe relative measure as a quotient of **two similar quantities**, often a "part" and a "total," where "part" is *divided by* the "total."
3. Ratios can be expressed in several forms: **verbally** (4 out of 5), as a **fraction** $\left(\frac{4}{5}\right)$ as a **decimal** (0.8), or as a **percent** (80%).
4. **Percent** always indicates a ratio out of 100.
5. **Converting a fraction or decimal to a percent**
   - **i.** Convert the fraction to decimal form by dividing the numerator by the denominator.
   - **ii.** Move the decimal point two places to the right, inserting placeholding zeros if needed, and then attach the % symbol.
6. **Converting a percent to a decimal**
   - **i.** Locate the decimal point in the number attached to the % symbol.
   - **ii.** Move the decimal point two places to the left, inserting placeholding zeros if needed, and write the decimal number without the % symbol.
7. Two ratios are **equivalent** if their decimal or reduced fraction forms are equal.

EXERCISES: ACTIVITY 6.1

1. Complete the following table by representing each ratio in all four formats. Round decimals to the thousandths place and percents to the tenths place.

Numerically Speaking...

VERBAL	REDUCED FRACTION	DECIMAL	PERCENT
1 out of 3			
2 out of 5			
18 out of 25			
8 out of 9			
3 out of 8			
25 out of 45			
120 out of 40			
3 out of 4			
27 out of 40			
3 out of 5			
2 out of 3			
4 out of 5			
1 out of 200			
2 out of 1			

2. **a.** Match each ratio from column 1 with the equivalent ratio in column 2.

Column 1	**Column 2**
12 out of 27	60 out of 75
28 out of 36	25 out of 40
45 out of 75	21 out of 27
64 out of 80	42 out of 70
35 out of 56	20 out of 45

b. Write each matched pair of equivalent ratios as a percent. If necessary, round to the nearest tenth of a percent.

3. Your biology instructor returned three quizzes today. On which quiz did you perform best? Explain how you determined the best score.

Quiz 1: 18 out of 25 **Quiz 2:** 32 out of 40 **Quiz 3:** 35 out of 50

4. A baseball batting average is the ratio of hits to the number of times at bat. It is reported as a three-digit decimal. Determine the batting averages of three players with the given records.

   a. 16 hits out of 54 at bats

   b. 25 hits out of 80 at bats

   c. 32 hits out of 98 at bats

5. There are 1720 females among the 3200 students at the local community college. Express this ratio in each of the following forms.

   a. fraction
   b. reduced fraction
   c. decimal
   d. percent

6. At the state university campus near the community college in Exercise 5, there are 2304 females and 2196 males enrolled. In which school, the community college or university, is the relative number of females greater? Explain your reasoning.

7. In the 2008 Major League Baseball season, the world champion Philadelphia Phillies ended their regular season with a 92–70 win-loss record. During their playoff season, their win-loss record was 11–3. Did the Phillies play better in the regular season or in the playoff season? Justify your answer mathematically.

8. a. A random check of 150 Southwest Air flights last month identified 127 of them that arrived on time. What on-time percent does this represent?

b. Write the ratio of late arrivals to on-time arrivals verbally and as a fraction.

9. A consumer magazine reported that of the 13,350 subscribers who owned brand A dishwasher, 2940 required a service call. Only 730 of the 1860 owners of brand B needed repairs. Which brand of dishwasher has the better repair record?

10. The admissions office in a local college has organized data in the following table listing the number of men and women who are currently enrolled. Admissions will use the data to help recruit students for the next academic year. Note that a student is matriculated if he or she is enrolled in a program that leads to a college degree.

School-Bound

	FULL-TIME MATRICULATED STUDENTS (≥12 Credits)	PART-TIME MATRICULATED STUDENTS (< 12 Credits)	PART-TIME NONMATRICULATED STUDENTS (< 12 Credits)
Men	214	174	65
Women	262	87	29

a. How many men attend the college?

b. What percent of the men are full-time students?

c. How many women attend the college?

d. What percent of the women are full-time students?

e. Write the ratio of men to women verbally and as a fraction.

f. How many students are enrolled full-time?

g. How many students are enrolled part-time?

h. Write the ratio of part-time students to full-time students verbally and as a fraction.

i. What percent of the full-time students are women?

j. What percent of the part-time students are women?

k. How many students are nonmatriculated?

l. Write the ratio of nonmatriculated men to matriculated men verbally and as a fraction.

m. What percent of the student body is nonmatriculated?

Activity 6.2

Four out of Five Dentists Prefer the Brooklyn Dodgers?

Objectives

1. Recognize that equivalent fractions lead to a proportion.
2. Use a proportion to solve a problem that involves ratios.

Manufacturers of retail products often conduct surveys to see how well their products are selling compared to competing products. In some cases, they use the results in advertising campaigns.

One company, Proctor and Gamble, ran a TV ad in the 1970s claiming that "four out of five dentists surveyed preferred Crest toothpaste over other leading brands." The ad became a classic, and the phrase, *four out of five prefer*, has become a popular cliché in advertising, in appeals, and in one-line quips.

What does "four out of five prefer" mean in a survey? How does that lead to solving an equation called a proportion? Continue in this activity to find out.

1. Suppose Proctor and Gamble asked 250 dentists what brand toothpaste he or she preferred, and 200 dentists responded that they preferred Crest.

   a. What is the ratio of the number of dentists who preferred Crest to the number who were asked the question? Write your answer in words, and as a fraction.

   b. Reduce the ratio in part a to lowest terms and write the result in words.

   c. Rewrite $\frac{4}{5}$ as an equivalent fraction by multiplying its numerator and denominator by 50.

Note that parts b and c of Problem 1 both show that the fraction $\frac{200}{250}$ is equivalent to the fraction $\frac{4}{5}$, or $\frac{200}{250} = \frac{4}{5}$. This is an example of a **proportion**.

Definition

The mathematical statement that two ratios are equivalent is called a **proportion**. In fraction form, a proportion is written $\frac{a}{b} = \frac{c}{d}$.

If one of the four numbers, a, b, c, or d, in a proportion is unknown and the other three are known, the equation can be used to determine the unknown number. The next example shows how.

Example 1 ***Suppose that Proctor and Gamble had interviewed 600 dentists and reported that $\frac{4}{5}$ of the 600 dentists preferred Crest. However, the report did not say how many dentists in this survey preferred Crest. Show how this number can be determined from the given information.***

SOLUTION

The comparison ratio for all the dentists interviewed is $\frac{x}{600}$, where the variable x represents the unknown number of dentists who preferred Crest and 600 is the total number of dentists surveyed. According to the report, this ratio has to be equal to $\frac{4}{5}$, that is,

$$\frac{x}{600} = \frac{4}{5}.$$

Since $\frac{x}{600}$ can be written as $\frac{1}{600}x$, the equation is solved for x by multiplying each fraction by 600 and simplifying:

$$\frac{600}{1} \cdot \frac{x}{600} = \frac{4}{5} \cdot \frac{600}{1}$$

$$\frac{\cancel{600}}{1} \cdot \frac{x}{\cancel{600}} = \frac{4}{\cancel{5}} \cdot \frac{\overset{120}{\cancel{600}}}{1}$$

$$x = 4 \cdot 120 = 480$$

Therefore, 480 dentists out of 600 dentists in the survey preferred Crest.

2. Suppose Proctor and Gamble surveyed another group of 85 dentists in Idaho. If $\frac{4}{5}$ of the group preferred Crest, how many of this group preferred Crest? Let x represent the number of dentists who preferred Crest.

3. The best season for the New York Mets baseball team was 1986, when they won $\frac{2}{3}$ of the games they played and won the World Series, beating the Boston Red Sox. If the Mets played 162 games, how many did they win?

Sometimes, the unknown in a proportion is in the denominator of one of the fractions. The next example shows a method of solving such a proportion.

Example 2

The Los Angeles Dodgers were once the Brooklyn Dodgers who made it to the World Series seven times from 1941 to 1956, each time opposing their archrivals, the New York Yankees. The Brooklyn Dodgers won only one World Series Championship against the Yankees and that was in 1955.

In the 1955 season, the Dodgers won about 16 out of every 25 games that they played. They won a total of 98 games. How many games did they play during the season?

SOLUTION

Let x represent the total number of games that the Dodgers played in the 1955 season. To write a proportion, the units of the numerators must be the same. Similarly, the units of the denominators must be the same. In this case, the proportion is

$$\frac{16 \text{ games won}}{25 \text{ total games played}} = \frac{98 \text{ games won}}{x \text{ total games played}} \quad \text{or} \quad \frac{16}{25} = \frac{98}{x}$$

This equation can be solved in three steps that will lead to a shortcut method. First, follow the three steps.

Step 1. Multiply each fraction by x and simplify.

$$\frac{16}{25} \cdot x = \frac{98}{\cancel{x}} \cdot \cancel{x}$$

$$\frac{16x}{25} = 98$$

Step 2. Multiply each term of the equation by 25.

$$\frac{16x}{\cancel{25}} \cdot \cancel{25} = 98 \cdot 25$$

$$16x = 98 \cdot 25$$

Step 3. Divide each term of the equation by 16 and calculate the value for x.

$$\frac{\cancel{16}x}{\cancel{16}} = \frac{98 \cdot 25}{16}$$

$$x = \frac{98 \cdot 25}{16} \approx 153.1$$

The Dodgers played 153 games in the 1955 season.

A Shortcut Method for Solving a Proportion

In steps 1 and 2 of the solution in Example 2, the proportion $\frac{16}{25} = \frac{98}{x}$ was rewritten as $16x = 98 \cdot 25$. Observe that the denominator of each fraction is multiplied by the numerator of the other.

$$\frac{16}{25} = \frac{98}{x}$$

$$16x = 98 \cdot 25$$

Divide both terms in the equation by 16 to obtain the value for x.

$$x = \frac{98 \cdot 25}{16} \approx 153$$

The shortcut process just described is called **cross multiplication** because the numerator of the first fraction multiplies the denominator of the second, and vice versa.

Procedure

Solving Proportions by Cross Multiplication

Rewrite the proportion $\frac{a}{b} = \frac{c}{d}$ as $a \cdot d = b \cdot c$ and solve for the unknown quantity.

4. a. Solve the proportion $\frac{7}{10} = \frac{x}{24}$.

b. Solve the proportion $\frac{9}{x} = \frac{6}{50}$.

5. As a volunteer for a charity, you were given a job stuffing envelopes for an appeal for donations. After stuffing 240 envelopes, you were informed that you are two-thirds done. How many envelopes in total are you expected to stuff? Let x represent the number of envelopes to be stuffed.

6. Solve each proportion for x.

a. $\frac{2}{3} = \frac{x}{48}$

b. $\frac{5}{8} = \frac{120}{x}$

c. $\frac{3}{20} = \frac{x}{3500}$

SUMMARY: ACTIVITY 6.2

1. The mathematical statement that two ratios are equivalent is called a **proportion**. In fraction form, a proportion is written as $\frac{a}{b} = \frac{c}{d}$.

2. Shortcut (cross multiplication) procedure for solving a proportion:

Rewrite the proportion $\frac{a}{b} = \frac{c}{d}$ as $a \cdot d = b \cdot c$ and solve for the unknown quantity.

EXERCISES: ACTIVITY 6.2

1. A company that manufactures optical products estimated that three out of five people in North America wear eyeglasses. The population of North America was projected to be about 540 million in 2010. Use a proportion to estimate the number of people in North America who were expected to wear eyeglasses in 2010.

2. Solve each proportion for x.

a. $\frac{2}{9} = \frac{x}{108}$ **b.** $\frac{8}{7} = \frac{120}{x}$ **c.** $\frac{x}{20} = \frac{70}{100}$

3. The Center for Education Reform was founded in 1993 to improve education opportunities in U.S. communities. According to Charity Navigator, an independent charity watchdog, the Center for Education Reform used 10.5 cents out of every dollar in its spending budget for administrative expenses in 2007. That year, the center's spending budget was 2.2 million dollars.

a. Use a proportion to estimate the administrative expenses for 2007.

b. Out of every dollar spent on administrative expenses in 2007, 79.6 cents went to the Center's president as compensation. Use a proportion to estimate her pay for that year to the nearest thousand dollars.

Exercise numbers appearing in color are answered in the Selected Answers appendix.

4. You earned a score of 16 on a quiz that was worth 20 points. Show why your score of 16 is equivalent to a score of 80 on a quiz worth 100 points.

5. You invested \$5000 in a mutual fund and earned \$250 last year. Your friend invested \$3500 in the same mutual fund at the same interest rate. What did she earn?

6. People starting on a weight-loss program are advised to set realistic weight loss goals. One method is to set a first goal of losing 5% to 10% of their starting weight.

a. You are starting a weight loss program. You weigh 170 pounds and aim to lose 10% of your body weight. Use a proportion to determine how many pounds you hope to lose.

b. Your friend is on a weight-loss program and wants to reach a final goal weight of 160 pounds. His starting weight was 180 pounds. If he has lost 8% of his body weight so far, how many pounds does he still have to lose to reach his final goal?

7. Tealeaf, a company that offers management solutions to companies that sell online, announced in a 2008 consumer survey that nearly 9 out of 10 customers reported problems with transactions online. The survey sampled 2010 adults in the United States, 18 years and older, who had conducted an online transaction in the past year. Estimate the number of adults who reported problems with transactions online.

8. Barack Obama won 365 electoral votes out of a total of 538 electoral votes in the 2008 presidential election. What percent of the total electoral votes did he win? Write your result to the nearest tenth of a percent.

Activity 6.3

The Devastation of AIDS in Africa

Objectives

1. Use proportional reasoning to apply a known ratio to a given piece of information.
2. Write an equation using the relationship ratio · total = part and then solve the resulting equation.

International public health experts are desperately seeking to stem the spread of the AIDS epidemic in sub-Saharan Africa, especially in the small nation of Botswana. According to a recent United Nations report, almost 24% of Botswana's 1,250,000 adults (ages 15–49) were infected with the AIDS virus.

The data in the preceding paragraph is typical of the kind of numerical information you will find in reading virtually any printed document, report, or article.

1. What relative data (ratio) appears in the opening paragraph? What phrase or symbol identifies it as a relative measure?

2. Express the ratio in Problem 1 in fraction, decimal, and verbal form.

3. The opening paragraph contains a second piece of information, namely, 1,250,000. What does this number represent?

Proportional Reasoning

From the information given and your answers to Problems 1, 2, and 3, you know that the adult population of Botswana is 1,250,000 and that 24% is the percentage of the adult population that has AIDS. The unknown quantity in this case is the number of adults in the total population that have AIDS. To determine the unknown quantity, take 24% of 1,250,000 as follows:

$$0.24 \cdot 1{,}250{,}000 = 300{,}000$$

Therefore, 300,000 adults in Botswana are infected with AIDS. Because the ratio $\frac{300{,}000}{1{,}250{,}000} = 0.24$, or 24%, you know that the calculation is correct.

This example illustrates a process called **proportional reasoning**. It shows that the relationship between three values can be expressed in two equivalent ways,

$$\textit{ratio} \cdot \textit{total} = \textit{part}$$

and

$$\textit{ratio} = \frac{\textit{part}}{\textit{total}}$$

In each case, if two of the three values are known, the third value can be determined. The first equation was used to determine the number of adults out of the total population that has AIDS. The second equation was used to show the percent of the population that has AIDS if you know the number of adults who had AIDS and the total number of adults in the population.

The following describes the direct method of applying a known ratio to a given piece of information.

Procedure

Setting Up and Solving Ratio · Total = Part When the Ratio Is Known

Step 1. Identify whether the given piece of information represents a *part* or a *total*.

Step 2. Substitute all the known information into the formula *ratio* · *total* = *part* to form an equation. The unknown may be represented by a letter.

Step 3. Solve the equation.

4. You do further research about this disturbing health problem and discover that by 2007 there were approximately 22,000,000 adults and children living in sub-Saharan Africa with HIV/AIDS. This represented 2.8% of the total population there.

a. The ratio of the total population infected with AIDS is given in the preceding information. What is it? In what form is the ratio expressed?

b. Identify the other given information.

c. Identify the unknown piece of information.

d. Use the formula $ratio \cdot total = part$ to estimate the total population.

5. It was estimated that 300,000 adults from Botswana were infected with HIV/AIDS in 2007, and that 170,000 of them were women. What percent of those infected were women? Use the formula $ratio = \frac{part}{total}$.

6. Africa has lost two-thirds of the 34 million people worldwide who have died from AIDS since the beginning of the AIDS epidemic. Calculate how many people from Africa have died from AIDS.

7. In the year 2007, 1.8 million sub-Saharan African children under the age of 15 were living with AIDS. This represents 90% of all children worldwide who were infected in 2007. Use the following steps to estimate the total number of all children worldwide who were infected.

a. Identify the known ratio.

b. Identify the given piece of information. Does this number represent a part or a total?

c. Identify the unknown piece of information. Does this number represent a part or a total?

d. Use the formula *ratio* $\cdot$ *total* $=$ *part* to determine your estimate. Let the unknown quantity be represented by x.

8. Of the 9.7 million people in developing countries who are in immediate need of lifesaving AIDS drugs, only 2.99 million are receiving the drugs. What percent are receiving the drugs?

a. Identify the given piece of information that represents a total.

b. Identify the given piece of information that represents a part.

c. Write the unknown percent as a ratio out of 100.

d. Set up and solve a proportion to determine the percent who are receiving the needed AIDS drugs.

9. In 2007, 2.7 million adults and children became newly infected with HIV. Seventy percent of these new cases occurred in sub-Saharan Africa and 14% of the new cases occurred in Asia.

a. Determine how many of these new cases of AIDS occurred in sub-Saharan Africa.

b. How many of these new cases of AIDS occurred in Asia?

10. Of the world's 6.7 billion people, 3.1 billion live on less than \$2 per day. Calculate the percent of the world's population who live on less than \$2 per day.

SUMMARY: ACTIVITY 6.3

Proportional reasoning is the process that uses the equation *ratio* · *total* = *part* to determine the unknown quantity when two of the quantities in the equation are known. This relationship is equivalently written as $ratio = \dfrac{part}{total}$.

Direct approach to solve for an unknown part or total:

Step 1. Identify whether the given piece of information represents a part or a total.

Step 2. Substitute all the known information into the formula *ratio* · *total* = *part* to form an equation. The unknown may be represented by a letter.

Step 3. Solve the equation.

Note that sometimes the unknown piece of information is the ratio. In this case, use the formula, $ratio = \dfrac{part}{total}$ to calculate the ratio directly.

EXERCISES: ACTIVITY 6.3

1. Determine the value of each expression.

a. $12 \cdot \dfrac{2}{3}$

b. $18 \div \dfrac{2}{3}$

c. $\dfrac{3}{5}$ of 25

d. $24 \div \dfrac{3}{4}$

e. 30% of 60

f. $150 \div 0.60$

g. $45 \cdot \dfrac{4}{9}$

h. 25% of 960

i. $5280 \div 75\%$

j. $2000 \div \dfrac{4}{5}$

k. 80% of 225

l. $30{,}000 \div 50\%$

2. Sub-Saharan Africa accounted for 71% of the 2.2 million adults and children who died of AIDS in the year 2007. How many sub-Saharan adults and children died of AIDS that year?

a. Identify the known ratio.

b. Identify the given piece of information. Does this number represent a part or a total?

c. How many sub-Saharan adults and children died of AIDS in 2007?

Exercise numbers appearing in color are answered in the Selected Answers appendix.

3. The 22 million sub-Saharan people (adults and children) currently living with AIDS account for 67% of the entire world population infected with the disease. How many people worldwide are infected with AIDS?

4. The customary tip on waiter service in New York City restaurants is approximately 20% of the food and beverage cost. This means that your server is counting on a tip of \$0.20 for each dollar you spend on your meal. What is the expected tip on a dinner for two costing \$65?

5. Your new car cost \$22,500. The state sales tax rate is 8%. How much sales tax will you be paying on the car purchase?

6. You happened to notice that the 8% sales tax that your uncle paid on his new luxury car came to \$3800. How much did the car itself cost him?

7. In your recent school board elections only 45% of the registered voters went to the polls. If 22,000 votes were cast, approximately how many voters are registered in your school district?

8. a. In 2008, new car and truck sales totaled approximately 13,250,000 in the United States. The breakdown among the leading automobile manufacturers was

General Motors 22.3%
Ford 15.2%
Chrysler 11%

Approximately how many new cars did each U.S. company sell that year?

b. Toyota sold 2,217,660 vehicles in 2008. What was its market share?

9. In a very disappointing season, your softball team won only 40% of the games it played this year. If you won 8 games, how many games did you play? How many did you lose?

10. In a typical telemarketing campaign, 5% of the people contacted purchase the product. If your quota is to sign up 50 people, approximately how many phone calls do you anticipate making?

11. Approximately three-fourths of the teacher-education students in your college pass the licensing exam on their first attempt. This year, 92 students will sit for the exam. How many are expected to pass?

12. You paid \$90 for your economics textbook. At the end of the semester, the bookstore will buy back your book for 20% of the original purchase price. The book sells used for \$65. How much money does the bookstore make on the resale?

Activity 6.4

Who Really Did Better?

Objectives

1. Define actual and relative change.
2. Distinguish between actual and relative change.

After researching several companies, you and your cousin decided to purchase your first shares of stock. You bought $200 worth of stock in one company and your cousin invested $400 in another. At the end of a year, your stock was worth $280 and your cousin's stock was valued at $500.

1. How much money did you earn from your purchase?

2. How much did your cousin earn?

3. Who earned more money?

Your answers to Problems 1 and 2 represent **actual change**, the actual numerical value by which a quantity has changed. When a quantity increases in value, such as your stock, the actual change is positive. When a quantity decreases in value, the actual change is negative.

4. Keeping in mind the amount each of you originally invested, whose investment do you believe did better? Explain your answer.

In answering Problem 4, you may have considered the actual change *relative* to the original amount invested. This comparison leads to the mathematical idea of relative change.

Definition

The ratio formed by comparing the actual change to the original value is called the **relative change**. Relative change is written in fraction form with the actual change placed in the numerator, and the original (or earlier) amount placed in the denominator.

$$\text{relative change} = \frac{\text{actual change}}{\text{original value}}$$

Since *relative change* (increase or decrease) is frequently reported as a percent, it is often called *percent change*. The two terms are interchangeable.

5. Determine the relative change in your $200 investment. Write your answer as a fraction and as a percent.

6. Determine the relative change in your cousin's $400 investment. Write your answer as a fraction and as a percent.

7. In relative terms, whose stock performed better?

8. Determine the actual change and the percent change of the following quantities.

a. You paid \$20 for a rare baseball card that is now worth \$30.

b. Last year's graduating class had 180 students; this year's class size is 225.

c. Ann invested \$300 in an energy stock, and now it is worth \$540.

d. Bob invested \$300 in a technology stock, but now it is worth only \$225.

e. Pat invested \$300 in a risky dot-com venture, and his stock is now worth \$60.

When speaking about percent change, a percent greater than 100 makes sense. For example, suppose a \$50 investment tripled to \$150. The actual increase in the investment is

$$\$150 - \$50 = \$100.$$

Then the relative change is $\dfrac{\text{actual increase}}{\text{original value}} = \dfrac{\$100}{\$50} = 2.00$. Therefore, the investment increased by 200%.

9. Determine the actual change and the percent change of the following quantities.

a. Your parents purchased their home in 1970 for \$40,000. They sold it recently for \$200,000.

b. After building a new convention center, the number of hotel rooms in a city increased from 1500 to 6000.

Actual Change vs. Relative Change

Actual change and relative (percent) change provide two perspectives to understanding change. Actual change looks only at the *change itself*, ignoring the original value. Relative change indicates the significance or importance of the change by *comparing it to the original value*.

The following problems will show you how relative change is different from actual change.

10. You and your colleague are bragging about your respective stock performance. Each of your stocks earned $1000 last year. Then it slipped out that your original investment was $2000, while your colleague had invested $20,000 in her stock.

a. What was the actual increase in each investment?

b. Calculate the relative increase (in percent format) for each investment.

c. Whose investment performance was more impressive? Why?

11. During the first week of classes, campus clubs actively recruit members. The Ultimate Frisbee Club signed up 8 new members, reaching an all-time high membership of 48 students. The Yoga Club also attracted 8 new members and now boasts 18 members.

a. What was the membership of the Ultimate Frisbee Club before recruitment week?

b. By what percent did the Frisbee Club increase its membership?

c. What was the membership of the Yoga Club before recruitment week?

d. By what percent did the Yoga Club increase its membership?

SUMMARY: ACTIVITY 6.4

1. The ratio formed by comparing the actual change to the original value is called the **relative change**. Relative change is written in fraction form with the actual change placed in the numerator and the original (or earlier) amount placed in the denominator.

$$\text{relative change} = \frac{\text{actual change}}{\text{original value}}$$

Since *relative change* (increase or decrease) is frequently reported as a percent, it is often called *percent change*. The two terms are interchangeable.

EXERCISES: ACTIVITY 6.4

1. The full-time enrollment in your college last year was 3200 students. This year there are 3560 full-time students on campus.

   a. Determine the actual increase in full-time enrollment.

   b. Determine the relative increase (as a percent) in full-time enrollment.

2. In 2008, the price of gasoline rose dramatically. The average price for regular gas in the United States began the year at \$3.05 per gallon. By July, the price had reached a high of \$4.11 per gallon.

   a. Determine the actual increase in gasoline price.

   b. Determine the relative increase (as a percent) in gasoline price.

3. This year, the hourly wage from your part-time job is \$13.00, up from last year's \$12.50. What percent increase does this represent?

4. In June 2008, Borders Group Inc. reduced its number of corporate jobs by 275, including about 150 jobs at its Ann Arbor headquarters. After the layoffs, the number of corporate employees at Ann Arbor was 1000.

   a. What percent of corporate employees at the Ann Arbor headquarters were laid off?

b. Borders employed a total of 30,000 employees throughout its offices and stores. What percent of total employees were laid off in June 2008?

c. What percent of all corporate layoffs occurred at the Ann Arbor headquarters?

5. You are now totally committed to changing your eating habits for better health. You have decided to cut back your daily caloric intake from 2400 to 1800 calories. By what percent are you cutting back on calories?

6. a. You paid $10 per share for a promising technology stock. Within a year, the stock price skyrocketed to $50 per share. By what percent did your stock value increase?

b. You held on to the stock for a bit too long. The price has just fallen from its $50 high back to $10. By what percent did your stock decrease from $50?

7. Perform the following calculations.

a. Start with a value of 10, double it, and then calculate the percent increase.

b. Start with a value of 25, double it, and then calculate the percent increase.

c. Start with a value of 40, double it, and then calculate the percent increase.

d. When any quantity doubles in size, by what percent has it increased?

8. Perform a similar set of calculations to the ones you did in Exercise 7 to help determine the percent increase of any quantity that triples in size.

Activity 6.5

Don't Forget the Sales Tax

Objectives

1. Define and determine growth factors.
2. Use growth factors in problems that involve percent increases.

In order to collect revenue (income), many state and local governments require merchants to collect sales tax on the items they sell. In a number of localities in the United States, the sales tax is as much as 8% of the selling price, or even higher. The purchaser pays both the selling price and the sales tax.

1. Determine the total cost (including the 8% sales tax) to the customer of the following items. Include each step of your calculation in your answer.

   a. A greeting card selling for \$1.50

   b. A shirt selling for \$35

   c. A vacuum cleaner priced at \$300

   d. A car priced at \$25,000

Many people will correctly determine the total costs for Problem 1 in two steps.

- They will first compute the sales tax on the item.
- Then they will add the sales tax to the selling price to obtain the total cost.

It is also possible to compute the total cost in one step by using the idea of a **growth factor**. If you use the growth factor, you won't need to calculate the sales tax. Later in the activity you will be asked to determine the selling price of an item for which you know the total checkout price. You will see that the *only* direct way to answer this "reverse" question is through use of a growth factor. The problems that follow will introduce you to the growth factor concept and how to use it.

Growth Factors

2. If a quantity increases by 50%, how does its new value compare to its original value? That is, what is the ratio of new value to original value? Complete the second and third rows of the table to confirm/discover your answer. The first row is completed for you.

On the Up and Up

ORIGINAL VALUE	NEW VALUE (Increased by 50%)	RATIO OF NEW VALUE TO ORIGINAL VALUE		
		FRACTION FORM	DECIMAL FORM	PERCENT FORM
10	15	$\frac{15}{10} = \frac{3}{2}$	1.50	150%
50				
100				

3. Use the results from the table in Problem 2 to answer the question, What is the ratio of new value to original value of any quantity that increases by 50%? Write the ratio as a reduced fraction, as a decimal, and as a percent.

The ratio, $\frac{3}{2} = \frac{\text{new value}}{\text{original value}}$, that you calculated in the table and observed in Problem 3 is called the **growth factor** for a 50% increase in any quantity. The definition of a growth factor for any percent increase is given in the following.

Definition

For a specified percent increase, no matter what the original value may be, the ratio of the new value to the original value is always the same. This ratio is called the **growth factor** associated with the specified percent increase. The formula is

$$\text{growth factor} = \frac{\text{new value}}{\text{original value}}.$$

The growth factor is most often written in decimal form. As you determined in Problems 2 and 3, the growth factor, in decimal form, associated with a 50% increase is 1.50. Since 150% is the same as 100% + 50%, the growth factor is the sum of 100% and 50%. Since 150% = 1.50 in decimal form, the growth factor is also the sum of 1.00 and 0.50. This leads to a second way to determine a growth factor.

Procedure

Determining a Growth Factor from a Percent Increase

The growth factor is determined by adding the specified percent increase to 100% and then changing this percent into its decimal form.

4. In each of the following, determine the growth factor of any quantity that increases by the given percent. Write your result as a percent and then as a decimal.

a. 30%

b. 75%

c. 15%

d. 7.5%

5. A growth factor can always be written in decimal form. Explain why this decimal will always be greater than 1.

Using Growth Factors in Percent Increase Problems

In Problems 2 and 3, you saw that a growth factor is given by the formula,

$$\text{growth factor} = \frac{\text{new value}}{\text{original value}}.$$

This formula is equivalent to the formula

$$\text{original value} \cdot \text{growth factor} = \text{new value}.$$

For example, $1.5 = \frac{75}{50}$ is the same as $50 \cdot 1.5 = 75$. This observation leads to a procedure for determining a new value using growth factors.

Procedure

Determining the New Value Using Growth Factors

When a quantity increases by a specified percent, its new value can be obtained by multiplying the original value by the corresponding growth factor. That is,

$$\text{original value} \cdot \text{growth factor} = \text{new value}.$$

6. a. Use the growth factor 1.50 to determine the new price of a stock portfolio that increased by 50% from its original value of $400.

b. Use the growth factor 1.50 to determine the population of a town that has grown by 50% from its previous population of 120,000 residents.

7. a. Determine the growth factor for any quantity that increases by 20%.

b. Use the growth factor to determine this year's budget, which has increased by 20% over last year's budget of $75,000.

c. Use the growth factor to determine the enrollment of an organization that has grown by 20% from its original size of 250 members.

8. a. Determine the growth factor for any quantity that increases by 8%.

b. Use the growth factor to determine the total cost of each item from Problem 1.

In the previous problems you were given an original (earlier) value and a percent increase and were asked to determine the new value. Suppose you are asked a "reverse" question: Sales in your small retail business have increased by 20% over last year. This year you had gross sales receipts of $75,000. How much did you gross last year?

Using the growth factor 1.20, you can write

$$\text{original value} \cdot 1.20 = 75{,}000.$$

Let x represent the original value (last year's gross sales). This equation can now be written as

$$x \cdot 1.20 = 75{,}000.$$

To solve for x, divide both sides by 1.20.

$$x = \frac{75{,}000}{1.20} = 62{,}500$$

So, last year's gross sales were \$62,500. This example leads to a general procedure for determining an original value from a new value and the growth factor.

Procedure

Determining the Original Value Using Growth Factors

When a quantity has *already* increased by a specified percent, its original value is obtained by dividing the new value by the corresponding growth factor.

$$\text{new value} \div \text{growth factor} = \text{original value}$$

9. You determined a growth factor for an 8% increase in Problem 8. Use it to answer the following questions.

a. The cash register receipt for your new coat, which includes the 8% sales tax, totals \$243. What was the ticketed price of the coat?

b. The credit-card receipt for your new sofa, which includes the 8% sales tax, came to \$1620. What was the ticketed price of the sofa?

SUMMARY: ACTIVITY 6.5

1. When a quantity increases by a specific percent, the ratio of the new value to the original value is called the **growth factor** associated with the specified percent increase. The growth factor is also formed by *adding* the specified percent increase to 100% and then changing the resulting percent into decimal form.

2. The equation that defines the growth factor as the ratio of the new value to the original value is

$$\text{growth factor} = \frac{\text{new value}}{\text{original value}}$$

3. When the growth factor equation is multiplied on both sides of the equals sign by the original value, an equivalent equation is obtained:

$$\text{original value} \cdot \text{growth factor} = \frac{\text{new value}}{\cancel{\text{original value}}} \cdot \cancel{\text{original value}}$$

Therefore, when a quantity increases by a specified percent, its new value can be obtained by multiplying the original value by the corresponding growth factor.

$$\text{new value} = \text{original value} \cdot \text{growth factor}$$

4. When the equation, new value = original value · growth factor is divided on both sides by the growth factor, another equivalent equation is obtained

$$\frac{\text{new value}}{\text{growth factor}} = \frac{\text{original value} \cdot \cancel{\text{growth factor}}}{\cancel{\text{growth factor}}}$$

Therefore, when a quantity has *already* increased by a specified percent, its original value can be obtained by dividing the new value by the corresponding growth factor.

$$\text{original value} = \text{new value} \div \text{growth factor}$$

EXERCISES: ACTIVITY 6.5

1. Complete the following table.

% Increase	5%		15%		100%	300%	
Growth Factor		1.35		1.045			11

2. You are purchasing a new car. The price you negotiated with a car dealer in Staten Island, New York, was $17,944 excluding sales tax. The sales tax rate for Staten Island is 8.375%.

a. What growth factor is associated with the sales tax rate?

b. Use the growth factor to determine the total cost of the car.

3. The cost of first-class postage in the United States increased by approximately 5% on May 11, 2009. If the cost is now 44¢, how much did first-class postage cost in April 2009? Round your answer to the nearest penny.

4. A Florida state university reported that its enrollment increased from 10,935 students in spring 2008 to 11,300 students in spring 2009.

a. What is the ratio of the number of students in spring 2009 to the number of students in spring 2008?

b. From your result in part a, determine the growth factor. Write it in decimal form to the nearest hundredth.

c. Write the growth factor in percent form.

5. The U.S. Census Bureau reports that from the 2000 U.S. Census to July 2008, the population of the state of Arizona increased by approximately 26.7%. The population in 2008 was 6,500,180. What was Arizona's population in the 2000 census?

6. Your grandparents bought a house in 1967 for $26,000. The United States Bureau of Labor Statistics reports that due to inflation, the cost of the same house in 2009 would be $184,984.

   a. What was the inflation growth factor for housing from 1967 to 2009?

   b. What was the inflation rate (to the nearest whole-number percent) for housing from 1967 to 2009?

   c. Your grandparents sold their house in 2009 for $465,000. What is the profit they made in terms of 2009 dollars?

7. Your friend plans to move from Richmond, Virginia, to Anchorage, Alaska. She earns $60,000 a year in Richmond. How much must she earn in Anchorage to maintain the same standard of living if the cost of living increases from Richmond to Anchorage by 19%?

8. You decide to invest $3000 in a 1-year certificate of deposit (CD) that earns an annual percentage yield (APY) of 2.4%. How much will your investment be worth in a year?

9. Your company is moving you from New York City, United States, to one of the other cities listed in the following table. For each city, the table lists the cost of a basket of goods and services that would cost $100 in New York City.

   In this problem, you will compare the cost of the same basket of goods in each of the cities listed to the $100 cost in New York City. The comparison ratio in each case will be a growth factor relative to the cost in New York City.

Living Abroad

CITY	COST OF BASKET	GROWTH FACTOR IN COST OF LIVING	PERCENT INCREASE IN SALARY
New York, NY, United States	100		
Brussels, Belgium	103		
Tokyo, Japan	126		
Sydney, Australia	107		
Paris, France	112		
Oslo, Norway	124		
Zurich, Switzerland	116		

a. Determine the growth factor for each cost in the table and list it in the third column.

b. Determine the percent increase in your salary that would allow you the same cost of living as you have in New York City. List the percent increase in the fourth column of the table.

c. You earn a salary of $45,000 in New York City. What would your salary have to be in Tokyo, Japan, to maintain the same standard of living?

10. Since 2007, the International Data Corporation (IDC) has been keeping track of the amount of digital information created and replicated on computers and the Internet. IDC calls this set of information the digital universe. In 2009, IDC reported that the size of the digital universe in 2008 was 487 billion gigabytes. In 2007, that number was 281 billion gigabytes.

a. What was the growth factor in the amount of digital data from 2007 to 2008? Round your answer to the nearest hundredth.

b. Use the growth factor from part a to predict the size of the digital universe in 2009, assuming the growth factor remains the same. Round your answer to the nearest billion.

Activity 6.6

It's All on Sale!

Objectives

1. Define and determine decay factors.
2. Use decay factors in problems that involve percent decreases.

It's the sale you have been waiting for all season. Everything in the store is marked 40% off the original ticket price.

1. Determine the discounted price of the following items. Include each step of your calculation in your answer.

a. A pair of sunglasses originally selling for \$20

b. A shirt originally selling for \$35

c. A bathing suit originally priced at \$80

d. A dress originally priced at \$250

Many people will correctly determine the discounted prices for Problem 1 in two steps.

- They will first compute the actual dollar discount on the item.
- Then they will subtract the dollar discount from the original price to obtain the sale price.

It is also possible to compute the sale price in one step by using the idea of a **decay factor**. If you use a decay factor, you will not have to calculate the actual dollar discount separately. Later in the activity you will determine the original price of an item for which you know the discounted sale price. You will see that the *only* direct way to answer this "reverse" question is through use of a decay factor. The problems that follow will introduce you to the decay factor concept and how to use it.

Decay Factors

2. If a quantity decreases by 20%, how does its new value compare to its original value? That is, what is the ratio of new value to original value? Complete the second and third rows in the following table. The first row has been done for you. Notice that the new values are less than the originals, so the ratios of new values to old will be less than 1.

A Matter of Values

ORIGINAL VALUE	NEW VALUE (Decreased by 20%)	RATIO OF NEW VALUE TO ORIGINAL VALUE		
		FRACTION FORM	DECIMAL FORM	PERCENT FORM
20	16	$\frac{16}{20} = \frac{4}{5}$	0.80	80%
50				
100				

3. What is the ratio of the new value to the original value of any quantity that decreases by 20%? Use the results from the table in Problem 2 to answer this question. Express the ratio as a reduced fraction, a decimal, and as a percent.

The ratio, $\frac{4}{5} = \frac{\text{new value}}{\text{original value}}$, that you calculated in the table and observed in Problem 3 is called the **decay factor** for a 20% decrease in any quantity. The definition of a decay factor for any percent decrease is given in the following.

Definition

For any specified percent *decrease*, no matter what the original value may be, the ratio of the new value to the original value is always the same. This ratio is called the **decay factor** associated with the specific percent decrease. The equation is

$$\text{decay factor} = \frac{\text{new value}}{\text{original value}}.$$

It is important to remember that a percent decrease describes the percent that has been *removed* (such as the discount taken off an original price). Therefore, the corresponding decay factor always represents the percent remaining (the portion that you still must pay).

The decay factor is most often written in decimal form. As you determined in Problems 2 and 3, the decay factor associated with a 20% decrease is 80%. Since 80% = 100% − 20%, the decay factor is the difference of 100% and 20%. Since 80% is 0.80 in decimal form, the decay factor is also the difference of 1.00 and 0.20. This leads to a second way to determine a decay factor.

Procedure

Determining a Decay Factor from a Percent Decrease

The decay factor is determined by *subtracting* the specified percent decrease from 100% and changing the resulting percent into decimal form.

4. In each of the following, determine the decay factor of a quantity that decreases by the given percent. Write your result as a percent and then as a decimal.

a. 30%

b. 75%

c. 15%

d. 7.5%

e. 5%

5. Explain why every decay factor will have a decimal value less than 1.

Using Decay Factors in Percent Decrease Problems

In Problems 2 and 3, you saw that a decay factor is given by the formula

$$\text{decay factor} = \frac{\text{new value}}{\text{original value}}.$$

This formula is equivalent to the formula

$$\text{original value} \cdot \text{decay factor} = \text{new value}.$$

For example, $0.80 = \frac{16}{20}$ is the same as $20 \cdot 0.8 = 16$. This observation leads to a procedure for determining a new value using decay factors.

Procedure

Determining the New Value Using Decay Factors

When a quantity decreases by a specified percent, its new value can be obtained by multiplying the original value by the corresponding decay factor. That is,

$$\text{original value} \cdot \text{decay factor} = \text{new value}.$$

6. a. Use the decay factor 0.80 to determine the new value of a stock portfolio that has decreased by 20% from its original value of $400.

b. Use the decay factor 0.80 to determine the population of a town that has decreased by 20% from its previous size of 300,000 residents.

In the previous questions you were given an original value and a percent decrease and were asked to determine the new (smaller) value. Suppose you are asked a "reverse" question: Voter turnout in the local school board elections declined by 40% from last year. This year only 3600 eligible voters went to the polls. How many people voted in last year's school board elections?

The decay factor is $100\% - 40\% = 60\%$. Using the decay factor 0.60 and the new value 3600, you can write

$$\text{original value} \cdot 0.60 = 3600.$$

If you let x represent the original value (last year's voter turnout), you can rewrite the equation as

$$x \cdot 0.60 = 3600.$$

Now you divide this year's turnout by the decay factor to obtain the original value. In the equation, this means divide both sides by 0.60. So,

$$x = \frac{3600}{0.60} = 6000.$$

Therefore, last year 6000 people voted in the school board elections.

The example just given leads to a general procedure for determining an original value from a new value and the decay factor.

Procedure

Determining the Original Value Using Decay Factors

When a quantity has *already* decreased by a specified percent, its original value is obtained by dividing the new value by the corresponding decay factor.

$$\text{new value} \div \text{decay factor} = \text{original value}$$

7. You purchased a two-line answering machine and telephone on sale for \$150. The discount was 40%. What was the original price?

8. a. Assume that all merchandise in a store has been reduced by 25%. Determine the decay factor for any price that decreases by 25%.

Use the decay factor to determine the prices requested in parts b through e.

b. The sale price of a blouse that originally sold for \$80

c. The sale price of a suit that originally sold for \$360

d. The original price of a designer jacket that is on sale for \$1620

e. The original price of a Nordic Track treadmill that is on sale for \$1140

SUMMARY: ACTIVITY 6.6

1. When a quantity decreases by a specific percent, the ratio of the new value to the original value is called the decay factor associated with the specified percent decrease. The decay factor is formed by *subtracting* the specified percent decrease from 100% and changing the resulting percent into decimal form.

2. The formula that defines the decay factor as the ratio of the new value to the original value is

$$\text{decay factor} = \frac{\text{new value}}{\text{original value}}.$$

3. When the decay factor equation is multiplied on both sides of the equals sign by the original value, an equivalent formula is obtained.

$$\text{original value} \cdot \text{decay factor} = \frac{\text{new value}}{\cancel{\text{original value}}} \cdot \cancel{\text{original value}}$$

Therefore, when a quantity decreases by a specified percent, its new value can be obtained by multiplying the original value by the corresponding decay factor.

$$\text{new value} = \text{original value} \cdot \text{decay factor}$$

4. When the formula new value = original value · decay factor is divided on both sides by the decay factor, another equivalent formula is obtained.

$$\frac{\text{new value}}{\text{decay factor}} = \frac{\text{original value} \cdot \cancel{\text{decay factor}}}{\cancel{\text{decay factor}}}$$

Therefore, when a quantity has *already* decreased by a specified percent, its original value can be obtained by dividing the new value by the corresponding decay factor.

$$\text{original value} = \text{new value} \div \text{decay factor}$$

EXERCISES: ACTIVITY 6.6

1. Complete the following table.

% Decrease	5%		15%		12.5%
Decay Factor		0.45		0.94	

2. You wrote an 8-page article that will be published in a journal. The editor asked you to revise the article and to reduce the number of pages to 6. By what percent must you reduce the length of your article?

3. In 2000, there were 7,325 multiple births (triplets, quadruplets, or higher order) recorded in the U.S. In 2005, the number of multiple births dropped to 6,694. What was the decay factor for multiple births for the 5-year period from 2000 to 2005? What was the percent decrease?

4. A car dealer will sell you the car you want for \$18,194, which is just \$200 over the dealer's invoice price (the price the dealer pays the manufacturer for the car). You tell him that you will think about it. The dealer is anxious to meet his monthly quota of sales, so he calls the next day to offer you the car for \$17,994 if you agree to buy it tomorrow, October 31. You decide to accept the deal.

a. What is the decay factor associated with the decrease in the price to you? Write your answer in decimal form to the nearest thousandth.

Exercise numbers appearing in color are answered in the Selected Answers appendix.

b. What is the percent decrease of the price, to the nearest tenth of a percent?

c. The sales tax is 6.5%. How much did you save on sales tax by taking the dealer's second offer?

5. Your company is moving you from New York City, United States, to one of the other cities listed in the following table. For each city, the table lists the cost of a basket of goods and services that would cost $100 in New York City.

In this problem, you will compare the cost of the same basket of goods in each of the cities listed to the $100 cost in New York City. The comparison ratio in each case will be a decay factor relative to the cost in New York City.

Over There

CITY	COST OF BASKET	DECAY FACTOR IN COST OF LIVING	PERCENT DECREASE IN COST OF LIVING
New York, NY, United States	100		
Lisbon, Portugal	93		
Berlin, Germany	97		
Jakarta, Indonesia	82		
Melbourne, Australia	96		
Mumbai, India	76		
Montreal, Canada	84		

a. Determine the decay factor for each cost in the table and list it in the third column.

b. Determine the percent decrease in the cost of living for each country compared to living in New York City. List the percent decrease in the fourth column of the table.

c. You earn a salary of $45,000 in New York City. If you move to Jakarta, Indonesia, how much of your salary will you need for the cost of living there?

d. How much of your salary can you save if you move to Lisbon, Portugal?

6. You needed a fax machine but planned to wait for a sale. Yesterday, the model you wanted was reduced by 30%. It originally cost $129.95. Use the decay factor to determine the sale price of the fax machine.

7. Your doctor advises you that losing 10% of your body weight will significantly improve your health.

   a. Before you start dieting, you weigh 175 pounds. Determine and use the decay factor to calculate your goal weight to the nearest pound.

   b. After 6 months, you reach your goal weight. However, you still have more weight to lose to reach your ideal weight range, which is between 125 and 150 pounds. If you lose 10% of your new weight, from part a, will you be in your ideal weight range? Explain.

8. The World Wildlife Fund (WWF) estimated that in 2001 the world tiger population was between 5000 and 7000. Accurate data is difficult to obtain but one estimate is that this is a 95% reduction from the tiger population in 1900.

   a. What is a lower-range estimate for the tiger population in 1900?

   b. What is an upper-range estimate for the tiger population in 1900?

9. Airlines often encourage their customers to book online by offering a 5% discount on their ticket. You are traveling from Kansas City to Denver. A fully refundable fare is $558.60. A restricted nonrefundable fare is $273.60. Determine and use the decay factor to calculate the cost of each fare if you book online.

Activity 6.7

Take an Additional 20% Off

Objective

1. Apply consecutive growth and/or decay factors to problems that involve two or more percent changes.

Your friend Maureen arrives at your house after shopping for a jacket. She explains that today's newspaper contained a 20% off coupon at Old Navy. The $100 jacket she had been eyeing all season was already reduced by 40%. She clipped the coupon, drove to the store, selected her jacket and walked up to the register. The cashier brought up a price of $48; Maureen thought that the price should have been only $40. The store manager arrived, reentered the transaction and, again, the register displayed $48. Maureen decided not to purchase the jacket and drove to your house to get your advice.

1. How do you think Maureen calculated a price of $40?

2. You grab a pencil and start your own calculation. You determine the decay factor and use it to calculate the ticketed price that reflects the 40% reduction. Explain how to calculate this price.

3. To what price does the 20% off coupon apply?

4. Use the decay factor associated with the 20% discount to determine the final price of the jacket.

You are now curious if you could justify a better price by applying the discounts in the reverse order. You start a new set of calculations, again using the decay factor approach.

5. Starting with the list price, determine the sale price after taking the 20% reduction.

6. Apply the 40% discount to the intermediate sale price.

7. Which sequence of discounts gives a better sale price?

Consecutive Decay Factors

The important point to understand from Problems 1–7 is that multiple discounts are always applied sequentially, one after the other. That is, discounts are *never* added together first before calculating the discounted price. With this in mind, you can determine the sale price simply and quickly by using the associated decay factors.

The following example shows how applying multiple discounts as decay factors can be done in a single chain of multiplications.

Example 1 ***A stunning* $2000 *gold-and-diamond necklace was far too expensive to consider. However, over the next several weeks you tracked the following successive discounts:***

20% off list; 30% off marked price; an additional 40% off every item

Determine the selling price after each of the discounts is taken.

SOLUTION

Step 1. Calculate the first reduced price by applying the decay factor corresponding to the 20% discount to the original price.

$$\text{Decay factor: } 100\% - 20\% = 80\% = 0.80$$

$$\text{First sale price: } \$2000 \cdot 0.80 = \$1600$$

Step 2. Next, determine the decay factor corresponding to the 30% discount and apply it to the first sale price to obtain the second sale price.

$$\text{Decay factor: } 100\% - 30\% = 70\% = 0.70$$

$$\text{Second sale price: } \$1600 \cdot 0.70 = \$1120$$

Step 3. Finally, determine the decay factor corresponding to the 40% discount and apply it to the second sale price to get the final sale price.

$$\text{Decay factor: } 100\% - 40\% = 60\% = 0.60$$

$$\text{Final sale price: } \$1120 \cdot 0.60 = \$672$$

The final cost after the three consecutive discounts is $672.

Note that the three steps can be combined into a single step once you know each decay factor. Then the final price is calculated from the original price by using a single "chain" of multiplications by the decay factors.

$$\text{Final sale price: } 2000 \cdot 0.80 \cdot 0.70 \cdot 0.60 = \$672$$

Check this calculation using your calculator.

8. a. Use the chain of multiplications approach shown in Example 1 to determine the final sale price of the necklace if the discounts had been taken in the reverse order (40%, 30%, and 20%).

b. Did you obtain the same final price in Problem 8a as in Example 1? Why?

Forming a Single Decay Factor for Consecutive Percent Decreases

The single chain of multiplication approach in Example 1 can be used to produce a single decay factor to represent two or more consecutive percent decreases. The single decay factor is the *product* of the consecutive decay factors.

In Example 1, the single decay factor is given by the product $0.80 \cdot 0.70 \cdot 0.60$, which equals .336 (or 33.6%). The associated single discount is calculated by subtracting the decay factor (in percent form) from 100%, to obtain 66.4%. Therefore, the effect of applying 20%, 30%, and 40% consecutive discounts is identical to a single discount of 66.4%. The single discount is called the *effective discount* and the associated single decay factor is called the *effective decay factor.*

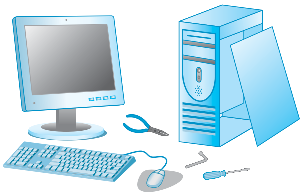
9. a. Determine the effective decay factor that represents the cumulative effect of consecutively applying Old Navy's 40% and 20% discounts.

b. Use this decay factor to determine the effective discount on Maureen's jacket.

10. a. Determine the effective decay factor that represents the cumulative effect of consecutively applying discounts of 40% and 50%.

b. Use this decay factor to determine the effective discount.

Consecutive Growth Factors



You can also use the single chain of multiplications approach to apply multiple percent increases. In this case, you will multiply by growth factors. The following problem will guide you in using this method for such growth.

11. Your computer repair business is growing faster than you had ever imagined. Last year, you had 100 employees statewide. This year, you opened several additional locations and increased the number of workers by 30%. With demand so high, next year you will be opening new stores nationwide and plan to increase your employee roll by an additional 50%.

To determine the projected number of employees next year, you can use growth factors to simplify the calculations.

a. Determine the growth factor corresponding to a 30% increase.

b. Apply this growth factor to calculate your current workforce.

c. Determine the growth factor corresponding to an additional 50% increase.

d. Apply this growth factor to your current workforce (130 employees) to determine the projected number of employees next year.

e. Write a single chain of multiplications to calculate the projected number of employees from last year's workforce of 100. Compare your answer to the result in part d.

You can form a single growth factor to represent two or more consecutive percent increases. The single growth factor is the product of the associated growth factors. In Problem 11, the single growth factor is given by the product $1.30 \cdot 1.50$, which equals 1.95. This shows that the number of employees will nearly have doubled since last year. Note also that the associated percent increase is calculated by subtracting 100% from the single growth factor 195% to obtain 95%. Therefore, the effect of applying consecutive 30% and 50% increases is identical to a single increase of 95%. The single percent increase is called the *effective increase* and the associated single growth factor is called the *effective growth factor.*

12. A college laboratory technician started his job at $30,000 and expects a 3% increase in his salary each year.

a. What is the growth factor associated with the percent increase expected?

b. What is the technician's salary after 3 years?

Forming Single Factors for Consecutive Percent Increases and Decreases

The next problem will now guide you in applying consecutive growths and decays by using growth and decay factors together.

13. You purchased \$1000 in a recommended stock last year and gleefully watched as it rose quickly by 30%. Unfortunately, the economy turned downward and you discover that your stock has recently fallen 30% from last year's high. The question is, have you made or lost money on your investment? You might be surprised by the answers obtained in the following.

a. What is the growth factor corresponding to a 30% increase?

b. What was the stock worth after the 30% increase?

c. Form the decay factor corresponding to a 30% decrease.

d. What was the stock worth after the 30% decrease?

Problem 13 shows that you can form a single factor that represents the cumulative effect of applying the consecutive percent increase and decrease—the single factor is the *product* of the growth and decay factors. In Problem 13, the effective factor is given by the product $1.30 \cdot 0.70$, which equals 0.91.

14. a. Does 0.91 in Problem 13 represent a growth factor or a decay factor? How can you tell?

b. Use the single factor from part a to determine the current value of your stock. What is the cumulative effect (as a percent change) of applying a 30% increase followed by a 30% decrease?

c. What is the cumulative effect if the 30% decrease had been applied first, followed by the 30% increase?

SUMMARY: ACTIVITY 6.7

1. The cumulative effect of a sequence of percent changes is the *product* of the associated growth or decay factors. For example,

a. To calculate the effect of consecutively applying 20% and 50% increases, determine the respective growth factors and multiply. The effective growth factor is

$$1.20 \cdot 1.50 = 1.80,$$

representing an effective increase of 80%.

b. To calculate the effect of applying a 30% *decrease* followed by a 40% *increase*, determine the respective growth and decay factors and then multiply. The effective factor is

$$0.70 \cdot 1.40 = 0.98,$$

representing an effective *decrease* of 2%.

c. To calculate the effect of applying a 25% *increase* followed by a 20% *decrease*, determine the respective growth and decay factors and then multiply. The effective factor is

$$1.25 \cdot 0.80 = 1.00,$$

indicating neither growth nor decay. That is, the quantity has returned to its original value.

2. The cumulative effect of a sequence of percent changes is the same regardless of the order in which the changes are applied. That is, the cumulative effect of applying a 20% *increase* followed by a 50% *decrease* is equivalent to first having applied the 50% *decrease* followed by the 20% *increase*.

3. It is important to remember that you do not add the individual percent changes when applying the percent changes consecutively. That is, a 20% increase followed by a 50% increase is *not* equivalent to a 70% increase.

EXERCISES: ACTIVITY 6.7

1. A $300 suit is on sale for 30% off. You present a coupon at the cash register for an additional 20% off.

a. Form the decay factor corresponding to each percent decrease.

b. Use these decay factors to determine the price you paid for the suit.

2. Your union has just negotiated a 3-year contract containing annual raises of 3%, 4%, and 5% during the term of contract. Your current salary is $42,000. What will you be earning in 3 years?

3. You had anticipated a large demand for a popular toy and increased your inventory of 1600 by 25%. You were able to sell 75% of your inventory. How many toys remain?

4. You deposit $2000 in a 5-year certificate of deposit that pays 4% interest compounded annually. To the nearest dollar, what balance will the account show when your certificate comes due?

Exercise numbers appearing in color are answered in the Selected Answers appendix.

5. Budget cuts have severely crippled your department over the last few years. Your operating budget of $600,000 has decreased by 5% each of the last 3 years. What is your current operating budget?

6. When you became a manager, your $60,000 annual salary increased by 25%. You found the new job too stressful and requested a return to your original job. You resumed your former duties at a 20% reduction in salary. How much are you making at your old job due to the transfer and return?

7. A coat with an original price tag of $400 was marked down by 40%. You have a coupon good for an additional 25% off.

   a. What is its final cost?

   b. What is the effective percent discount?

8. A digital camera with an original price tag of $500 was marked down by 40%. You have a coupon good for an additional 30% off.

   a. What is the decay factor for the first discount?

   b. What is the decay factor for the additional discount?

   c. What is the effective decay factor?

   d. What is the final cost of the camera?

   e. What is the effective percent discount?

9. Your friend took a job 3 years ago that started at $30,000 a year. Last year, she got a 5% raise. She stayed at the job for another year but decided to make a career change and took another job that paid 5% less than her current salary. Was the starting salary at the new job more, or less, or the same as her starting salary at her previous job?

10. You wait for the price to drop on a diamond-studded watch at Macy's. Originally, it cost $2500. The first discount was 20% and the second discount is 50%.

a. What are the decay factors for each of the discounts?

b. What is the effective decay factor for the two discounts?

c. Use the effective decay factor in part b to determine how much you will pay for the watch.

Activity 6.8

Fuel Economy

Objectives

1. Apply rates directly to solve problems.
2. Use proportions to solve problems that involve rates.
3. Use unit analysis or dimensional analysis to solve problems that involve consecutive rates.

You are excited about purchasing a used car for commuting to college. Concerned about the cost of driving, you did some research on the Internet and came across the Web site, *http://www.fueleconomy.gov*. You found that this Web site listed fuel efficiency, in miles per gallon (mpg), for five cars that you are considering. You recorded the miles per gallons for city and highway driving in the table after Problem 1.

1. **a.** For each of the cars listed in the table, how many city miles can you travel per week on 5 gallons of gasoline? Explain the calculation you will do to obtain the answers. Record your answers in the third column of the table.

   b. The daily round-trip drive to your college is 32 city miles, which you do 4 days per week. Which of the cars would get you to school each week on 5 gallons of gas?

Every Drop Counts

FUEL ECONOMY GUIDE: MODEL YEAR 2006					
MAKE/MODEL	CITY MPG	CITY MILES ON 5 GALLONS OF GAS	HIGHWAY MPG	GALLONS NEEDED TO DRIVE 304 HIGHWAY MILES	FUEL TANK CAPACITY IN GALLONS
Chevrolet Aveo5	25		34		12.0
Ford Focus	24		33		13.5
Honda Civic	25		36		13.2
Hyundai Accent	26		35		11.9
Toyota Yaris	29		35		11.1

2. Suppose you plan to move and your round-trip commute to the college will be 304 highway miles each week. How many gallons of gas would each of the cars require? Explain the calculation you will do to obtain the answers. Record your answers to the nearest tenth in the fifth column of the table.

Miles per gallon (mpg) is an example of a **rate**. Mathematically, a rate is a comparison by division of two quantities that have different units of measurement. A rate such as 26 miles per gallon expresses the number of miles that can be traveled per 1 gallon of gas. The word *per* signifies division.

Solving problems involving rates is similar to solving problems using ratios. The difference is that to solve rate problems you will use the units of measurement as a guide to set up the appropriate calculations. The problems that follow will guide you to do this efficiently.

Definition

A **rate** is a comparison, expressed as a quotient, of two quantities that have different units of measurement.

Unit analysis (sometimes called *dimensional analysis*) uses units of measurement as a guide in setting up a calculation or writing an equation involving one or more rates. When the calculation or equation is set up properly, the result is an answer with the appropriate units of measurement.

There are two common methods for solving problems involving rates. One method is to apply the known rate directly by multiplication or division. The second method is to set up and solve a proportion. In each, you use the units of measurement as a guide in setting up the calculation or writing the appropriate equation.

Applying a Known Rate Directly by Multiplication/ Division to Solve a Problem

In Problem 1, the known rate is city miles per gallon for several cars. You can write the miles per gallon in fraction form, $\frac{\text{number of miles}}{\text{1 gallon}}$. In the case of the Chevrolet Aveo5, the city miles per gallon is $\frac{\text{25 mi.}}{\text{1 gal.}}$. To determine how many miles the Aveo5 can travel on 5 gallons, you notice that the units in the answer should be miles. So, you multiply the rate by 5 gallons.

$$5\ \cancel{\text{gal.}} \cdot \frac{25\text{ mi.}}{1\ \cancel{\text{gal.}}} = 125\text{ mi.}$$

Notice that gallon occurs in both a numerator and a denominator. You divide out common measurement units in the same way that you reduce a fraction by dividing out common numerical factors. Therefore, you can drive for 125 miles in the Chevrolet Aveo5 on 5 gallons of gas.

Procedure

Multiplying and Dividing Directly to Solve Problems That Involve Rates

- Identify the measurement unit of the answer to the problem.
- Set up the calculation so the appropriate units will divide out, leaving the measurement unit of the answer.
- Multiply or divide the numbers as usual to obtain the numerical part of the answer.
- Divide out the common units to obtain the measurement unit of the answer.

3. Use the direct method to solve each of the following problems involving rates.

a. The gas tank of a Honda Civic holds 13.2 gallons. How many highway miles can you travel on a full tank of gas?

b. The Hyundai Accent gas tank holds 11.9 gallons. Is it possible to travel as far in this car as in the Honda Civic on the highway?

The Proportion Method for Solving Problems That Involve Rates

In Activity 6.2, you set up and solved proportions $\frac{a}{b} = \frac{c}{d}$. You can extend this method to solving problems with rates. The first fraction, $\frac{a}{b}$, is the known rate. Its numerator represents one measurement unit and its denominator, the other unit. In Problem 1, the known gas mileage rate for the Chevrolet Aveo5 is $\frac{25 \text{ mi.}}{1 \text{ gal.}}$. To determine the second fraction, $\frac{c}{d}$, the measurement unit of the numerator c must have the same unit as the numerator of the known rate, miles. The denominator d must have the same measurement unit as the denominator of the known rate, gallons.

You know that you have 5 gallons of gas. Therefore, 5 gallons is the denominator d. The unknown is the number of miles you can drive on 5 gallons. Represent the unknown in the numerator by a variable such as x. The numerator, c, is x miles.

Therefore, the fraction $\frac{c}{d}$ is $\frac{x \text{ mi.}}{5 \text{ gal.}}$ and the proportion becomes

$$\frac{25 \text{ mi.}}{1 \text{ gal.}} = \frac{x \text{ mi.}}{5 \text{ gal.}}.$$

Solving $\frac{25}{1} = \frac{x}{5}$, you obtain $x = 125$ miles.

Therefore, in the Chevrolet Aveo5 you can drive 125 miles on 5 gallons of gasoline.

Procedure

Using Proportions to Solve Problems That Involve Rates

- Identify the known rate and write it in fractional form.
- Identify the given information and the quantity to be determined.
- Write a second fraction, placing the given information and the unknown quantity x in the same positions as their units in the known rate.
- Equate the two fractions to obtain a proportion.
- Solve the equation for x. Affix the correct unit to the numerical result.

4. Solve the following problems by both methods and compare the results. They should be the same.

a. The gas tank of a Ford Focus holds 13.5 gallons. How many city miles can you travel on a full tank of gas?

b. The Toyota Yaris's gas tank holds 11.1 gallons. Is it possible to drive as far in the city in this car as in the Ford Focus?

Solving Rate Problems Using Measurement Units as a Guide

In Problem 2, you were asked to determine how many gallons of gas were required to drive 304 highway miles. Using the direct method, you may have known to divide the total miles by the miles per gallon. For example, in the case of the Honda Civic, $\frac{304}{36} \approx 8.4$ gallons.

One way to be sure the problem is set up correctly is to use the units of measurement. In Problem 2, you are determining the number of gallons. Set up the calculation so that miles will divide out and the remaining measurement unit will be gallons.

$$304\ \cancel{\text{mi.}} \cdot \frac{1\text{ gal.}}{36\ \cancel{\text{mi.}}} = \frac{304}{36} \approx 8.4\text{ gal.}$$

This example shows that a rate may be expressed in two ways. For miles and gallons, the rate can be expressed as $\frac{\text{gallons}}{\text{miles}}$ or as $\frac{\text{miles}}{\text{gallons}}$.

Alternatively, you may have set up a proportion.

$$\frac{36\text{ mi.}}{1\text{ gal.}} = \frac{304\text{ mi.}}{x\text{ gal.}}$$

$$36x = 304$$

$$x = \frac{304}{36} \approx 8.4\text{ gal.}$$

5. After you purchase your new car, you would like to take a trip to see a good friend in another state. The highway distance is approximately 560 miles.

Solve the following problems by both the direct and proportion methods and compare the results.

a. If you bought the Hyundai Accent, how many gallons of gas would you need to make the round trip?

b. How many tanks of gas would you need for the trip?

6. You decide to read a 15-page article that provides advice for negotiating the lowest price for a car. You read at the rate of 10 pages an hour. How long will it take you to read the article?

7. You are feeding a group of six friends who are joining you to watch a figure-skating competition, and everyone wants soup. The information on a can of soup states that one can of soup contains about 2.5 servings. Use a proportion to determine the number of cans of soup to open so that all seven of you have one serving of soup. Let x represent the number of cans to be opened.

Unit Conversion

In many countries, distance is measured in kilometers (km) and gasoline in liters (L). A kilometer is equivalent to 0.6214 miles and 1 liter is equivalent to 0.264 gallons. In general, the equivalence of two units can be treated as a rate written in fraction form. For example, the fact that a kilometer is equivalent to 0.6214 miles is written as

$$\frac{1 \text{ km}}{0.6214 \text{ mi.}} \quad \text{or} \quad \frac{0.6214 \text{ mi.}}{1 \text{ km}}$$

Using the fraction form, you can convert one unit to another by multiplying directly.

8. Your friend joins you on a trip through Canada where gasoline is measured in liters and distance in kilometers.

a. Write the equivalence of liters and gallons in fraction form.

b. If you bought 20 liters of gas, how many gallons did you buy?

Note that conversion equivalents are given on the inside back cover of this textbook.

9. To keep track of mileage and fuel needs in Canada your friend suggests that you convert your car's miles per gallon into kilometers per liter. Your car's highway fuel efficiency is 45 miles per gallon.

a. Convert your car's fuel efficiency to *miles per liter.*

b. Use the result you obtained in part a to determine your car's fuel efficiency in *kilometers per liter* (km/L).

Using Unit Analysis to Solve a Problem Involving Consecutive Rates

Problem 8 involved a one-step rate problem. In Problem 9, to convert the car's fuel efficiency from miles per gallon (mpg) to kilometers per liter you were guided to do two one-step conversions. You may have observed that the two calculations could be done in a single chain of multiplications.

$$\frac{45\ \cancel{\text{mi.}}}{1\ \cancel{\text{gal.}}} \cdot \frac{0.264\ \cancel{\text{gal.}}}{1\text{ L}} \cdot \frac{1\text{ km}}{0.6214\ \cancel{\text{mi.}}} = 19.1\text{ km/L}$$

Note how the units given in the problem guide you to choose the appropriate equivalent fractions so the method becomes a single chain of multiplications.

Procedure

Applying Consecutive Rates

- Identify the measurement unit of the answer.
- Set up the sequence of multiplications so the appropriate units divide out, leaving the desired measurement unit.
- Multiply and divide the numbers as usual to obtain the numerical part of the answer.
- Check that the appropriate measurement units divide out, leaving the expected measurement unit for the answer.

10. You plan to buy 3 gallons of juice to serve guests at a breakfast rally that your local political group is hosting. How many cups would you have?

11. You are on a 1500-mile trip where gas stations are far apart. Your car is averaging 40 miles per gallon and you are traveling at 60 miles per hour (mph). The fuel tank holds 12 gallons of gas and you just filled the tank. Use a chain of multiplications to determine how long it will be before you have to refill the tank.

12. You have been driving for several hours and notice that your car's 13.2-gallon fuel tank registers half empty. How many more miles can you travel if your car is averaging 30 miles per gallon?

SUMMARY: ACTIVITY 6.8

1. A **rate** is a comparison, expressed as a quotient, of two quantities that have different units of measurement.

2. **Unit analysis** (sometimes called *dimensional analysis*) uses units of measurement as a guide in setting up a calculation or writing an equation involving one or more rates. When the calculation or equation is set up properly, the result is an answer with the appropriate units of measurement.

3. Common methods used to solve problems that involve rates:

Direct method: Multiply or divide directly by rates to solve problems

a. Identify the measurement unit of the result.

b. Set up the calculation so that the appropriate measurement units will divide out, leaving the unit of the result.

c. Multiply or divide the numbers as indicated to obtain the numerical part of the result.

d. Divide out the common measurement units to obtain the unit of the answer.

Proportion method: Set up and solve a proportion equation

a. Identify the known rate, and write it in fractional form.

b. Identify the given information and the unknown quantity to be determined.

c. Write a second fraction, placing the given information and unknown quantity x in the same position as the measurement units in the known rate.

d. Equate the two fractions to obtain a proportion equation.

e. Solve the equation for x. Affix the correct unit to the numerical result.

4. Applying several rates consecutively:

a. Identify the measurement unit of the result.

b. Set up the sequence of multiplications so the appropriate measurement units divide out, leaving the unit of the result.

c. Multiply and divide the numbers as usual to obtain the numerical part of the result.

d. Check that the appropriate measurement units divide out, leaving the expected unit of the result.

EXERCISES: ACTIVITY 6.8

Use the conversion tables or formulas on the inside back cover of the textbook for conversion equivalencies.

1. The length of a football playing field is 100 yards between the opposing goal lines. What is the length of the football field in feet?

2. The distance between New York City, NY, and Los Angeles, CA, is approximately 4485 kilometers. What is the distance between these two major U.S. cities in miles?

3. As part of your job as a quality-control worker in a factory you can check 16 parts in 3 minutes. How long will it take you to check 80 parts?

4. Your car averages about 27 miles per gallon on highways. With gasoline priced at $2.74 per gallon, how much will you expect to spend on gasoline during your 500-mile trip?

5. You currently earn $11.50 per hour. Assuming that you work fifty-two 40-hour weeks per year with no raises, what total gross salary will you earn over the next 5 years?

6. The aorta is the largest artery in the human body. In the average adult, the aorta attains a maximum diameter of about 1.18 inches where it adjoins the heart. What is the maximum diameter of the aorta in terms of centimeters? In millimeters?

7. Mount Everest in the Himalaya mountain range along the border of Tibet and Nepal reaches upward to a record height of 29,035 feet. How high is Mt. Everest in miles? In kilometers? In meters?

Exercise numbers appearing in color are answered in the Selected Answers appendix.

8. The average weight of a mature human brain is approximately 1400 grams. What is the equivalent weight in kilograms? In pounds?

9. Approximately 4.5 liters of blood circulates in the body of the average human adult. How many quarts of blood does the average person have? How many pints?

10. How many seconds are in a day? In a week? In a non–leap year?

11. The following places are three of the wettest locations on Earth. Determine the annual rainfall for each site in centimeters.

LOCATION	ANNUAL RAINFALL (Inches)	ANNUAL RAINFALL (Centimeters)
Mawsynram, Meghala, India	467	
Tutenendo, Columbia	463.5	
Mt. Waialeali, Kauai	410	

12. The mass of diamonds is commonly measured in carats. Five carats is equivalent to 1 gram. How many grams are in a 24-carat diamond? How many ounces?

13. The tissue of Earth organisms contains carbon molecules known as proteins, carbohydrates, and fats. A healthy human body is approximately 18% carbon by weight. Determine how many pounds and kilograms of carbon your own body contains.

14. Powdered skim milk sells for \$6.99 a box in the supermarket. The amount in the box is enough to make 8 quarts of skim milk. What is the price for 12 quarts of skim milk prepared in this way?

15. A person who weighs 120 pounds on Earth would weigh 42.5 pounds on Mars. What would be a person's weight on Mars if she weighs 150 pounds on Earth?

16. You want to make up a saline (salt) solution in chemistry lab that has 12 grams of salt per 100 milliliters of water. How many grams of salt would you use if you needed 15 milliliters of solution for your experiment?

17. In 2007, the birth rate in the United States was estimated to be 14.3 births per 1000 persons. If the population estimate for the United States was 306 million, how many births were expected that year?

What Have I Learned?

1. On a 40-question practice test for this course, you answered 32 questions correctly. On the test itself, you correctly answered 16 out of 20 questions. Does this mean that you did better on the practice test than you did on the test itself? Explain your answer using the concepts of actual and relative comparison.

2. a. Ratios are often written as a fraction in the form $\frac{a}{b}$. List the other ways you can express a ratio.

   b. Thirty-three of the 108 colleges and universities in Michigan are 2-year institutions. Write this ratio in each of the ways you listed in part a.

3. Florida Legislature's Office of Economic and Demographic Research reports that the population of Florida in 2007 was 18,614,712 persons, and approximately 171 out of every 1000 residents were 65 years or older. Determine the number of Florida residents who were 65 years or older in 2007.

4. Explain how you would determine the percent increase in the cost of a cup of coffee at your local diner if the price increased from \$1.30 to \$1.50 last week.

5. a. Think of an example and use it to show how to determine the growth factor associated with a percent increase. For instance, you might refer to growth factors associated with yearly interest rates for a savings account.

Exercise numbers appearing in color are answered in the Selected Answers appendix.

b. Show how you would use the growth factor to apply the percent increase twice, then three times.

6. Current health research shows that losing just 10% of body weight produces significant health benefits, including a reduced risk for a heart attack. Suppose a relative who weighs 199 pounds begins a diet to reach a goal weight of 145 pounds.

a. Use the idea of a decay factor to show your relative how much he will weigh after losing the first 10% of his body weight.

b. Explain to your relative how he can use the decay factor to determine how many times he has to lose 10% of his body weight to reach his goal weight of 145 pounds.

7. a. In conversion tables, conversions between two measurement units are usually given in an equation format. For example, 1 quart = 0.946 liters. Write this conversion in two different fraction formats.

b. Write the conversion 1 liter = 1.057 quarts in two different fraction formats.

c. Are the two conversions given in parts a and b equivalent? Explain.

How Can I Practice?

1. Write the following percents in decimal format.

   a. 25%
   b. 87.5%
   c. 6%
   d. 3.5%
   e. 250%
   f. 0.3%

2. An error in a measurement is the difference between the measured value and the true value. The relative error in measurement is the ratio of the absolute value of the error to the true value. That is,

$$\text{relative error} = \frac{|\text{error}|}{\text{true value}} = \frac{|\text{measured value} - \text{true value}|}{\text{true value}}$$

Determine the actual and relative errors in the following measurements. The first row is done for you.

MEASUREMENT	TRUE VALUE	ERROR	RELATIVE ERROR (as a percent)
107 inches	100 inches	7 inches	$7 \div 100 = 0.07 = 7\%$
5.7 ounces	5.0 ounces		
4.3 grams	5.0 grams		
11.5 cm	12.5 cm		

3. You must interpret a study of cocaine-addiction relapse after three different treatment programs. The subjects were treated with the standard therapy, with an antidepressant, or with a placebo. They were tracked for 3 years.

TREATMENT	SUBJECTS WHO RELAPSED	SUBJECTS STILL SUBSTANCE FREE
Standard therapy	36	20
Antidepressant therapy	27	18
Placebo	30	9

Which therapy has proved most effective? Explain. Use the table to record your calculations.

	TOTAL NUMBER OF SUBJECTS	PERCENT OF SUBJECTS WHO ARE SUBSTANCE FREE
Standard therapy:		
Antidepressant therapy:		
Placebo:		

Exercise numbers appearing in color are answered in the Selected Answers appendix.

4. The 2619 female students on campus comprise 54% of the entire student body. What is the total enrollment of the college?

5. The New York City Board of Education recently announced plans to eliminate 1500 administrative staff jobs over the next several years, reducing the number of employees at its central headquarters by 40%. What is the current size of its administrative staff (prior to any layoffs)?

6. Hurricane Katrina hit the U.S. Gulf Coast on August 29, 2005. According to a U.S. Census press release on July 1, 2006, the population of Louisiana was 4.3 million, down 220,000 from a year before. What percent decrease does this represent?

7. A suit with an original price of $400 was marked down by 30%. You have a coupon good for an additional 20% off.

a. What is the suit's final cost?

b. What is the effective percent discount?

8. In recent contract negotiations between the Transit Workers Union and the New York City Metropolitan Transit Authority, the workers agreed to a 3-year contract, with 5%, 4%, and 3% wage increases, respectively, over each year of the contract. At the end of this 3-year period, what will be the salary of a motorman who had been earning $48,000 at the start of the contract?

9. You are moving into a rent-controlled apartment and the landlord is raising the rent by the maximum allowable, 20%. You will be paying $900. What was the former tenant's rent?

10. Gasoline prices increased from $2.25 per gallon to a price of $2.75. What percent increase does this represent?

11. In 2007, the Ford Motor Company sold 137,817 new Explorers, a decrease of approximately 23% from 2006. Approximately how many Explorers were sold in 2006?

12. The Mariana Trench near the Mariana Islands in the South Pacific Ocean extends down to a maximum depth of 35,827 feet. How deep is this deepest point in miles? In kilometers? In meters?

13. Under the current union contract, you earn $22.50 per hour as a college laboratory technician. The contract is valid for the next 2 years. Assuming that you stay in this position and work 40 hours per week, 52 weeks a year, what will your total gross earnings be over the next 2 years?

14. As a nurse in a rehabilitation center, you have received an order to administer 60 milligrams of a drug. The drug is available at a strength of 12 milligrams per milliliter. How many milliliters would you administer?

15. Solve each proportion for x.

a. $\frac{x}{8} = \frac{120}{320}$ **b.** $\frac{6}{13} = \frac{42}{x}$ **c.** $\frac{24}{x} = \frac{6}{5}$

16. A three-pound bag of rice costs $2.67. At that rate, what would a 20-pound bag of rice cost?

Chapter 6 Summary

The bracketed numbers following each concept indicate the activity in which the concept is discussed.

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Ratio [6.1]	A quotient that compares two similar numerical quantities, such as part to total.	4 out of 5 is a ratio. It can be expressed as a fraction, $\frac{4}{5}$; a decimal, 0.8; and a percent, 80%.
Convert from decimal to percent format. [6.1]	Move the decimal place two places to the right and then attach the % symbol.	0.125 becomes 12.5%. 0.04 becomes 4%. 2.50 becomes 250%.
Convert from percent to decimal format. [6.1]	Drop the percent symbol and move the decimal point two places to the left.	35% becomes 0.35. 6% becomes 0.06. 200% becomes 2.00.
Proportion [6.2]	A mathematical statement that two ratios are equivalent. In fraction form, a proportion is written as $\frac{a}{b} = \frac{c}{d}$.	$\frac{2}{3} = \frac{50}{75}$ is a proportion because $\frac{50}{75}$ in reduced form equals $\frac{2}{3}$.
Solve a proportion [6.2]	Rewrite $\frac{a}{b} = \frac{c}{d}$ as $a \cdot d = b \cdot c$ and solve for the unknown quantity.	$\frac{7}{30} = \frac{x}{4140}$ $30 \cdot x = 7 \cdot 4140$ $x = \frac{7 \cdot 4140}{30} = 966$
Apply a known ratio to a given piece of information. [6.3]	total · known ratio = unknown part part ÷ known ratio = unknown total	40% of the 350 children play an instrument. 40% is the known ratio; 350 is the total. 350 · 0.40 = 140 children play an instrument. 24 children, comprising 30% of the marching band, play the saxophone. 30% is the known ratio, 24 is the part. 24 ÷ 0.30 = 80, the total number of children in the marching band.
Relative change [6.4]	$\text{relative change} = \frac{\text{actual change}}{\text{original value}}$	A quantity changes from 25 to 35. The actual change is 10; the relative change is $\frac{10}{25} = \frac{2}{5}$, or 40%.

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Growth factor [6.5]	When a quantity increases by a specified percent, the ratio of its new value to its original value is called the growth factor associated with the specified percent increase. The growth factor also is formed by adding the specified percent increase to 100%. The growth factor may also be written in decimal form.	To determine the growth factor associated with a 25% increase, add 25% to 100% to obtain 125% or 1.25.
Apply a growth factor to an original value. [6.5]	original value · growth factor = new value	A \$120 item increases by 25%. Its new value is $\$120 \cdot 1.25 = \$150$.
Apply a growth factor to a new value. [6.5]	new value ÷ growth factor = original value	An item has already increased by 25% and is now worth \$200. Its original value was $\$200 \div 1.25 = \$160$.
Decay factor [6.6]	When a quantity decreases by a specified percent, the ratio of its new value to the original value is called the decay factor associated with the specified percent decrease. The decay factor is also formed by *subtracting* the specified percent decrease from 100%. The decay factor may also be written in decimal form.	To determine the decay factor associated with a 25% decrease, subtract 25% from 100%, $100\% - 25\%$, to obtain 75% or 0.75.
Apply a decay factor to an original value. [6.6]	original value · decay factor = new value	A \$120 item decreases by 25%. Its new value is $\$120 \cdot 0.75 = \$90$.
Apply a decay factor to a new value. [6.6]	new value ÷ decay factor = original value	An item has already decreased by 25% and is now worth \$180. Its original value was $\$180 \div 0.75 = \$240$.
Apply a sequence of percent changes. [6.7]	The cumulative effect of a sequence of percent changes is the *product* of the associated growth or decay factors.	The effect of a 25% increase followed by a 25% decrease is $1.25 \cdot 0.75 = 0.9375$. That is, the item is now worth only 93.75% of its original value. The item's value has decreased by 6.25%.
Use unit or dimensional analysis to solve conversion problems. [6.8]	1. Identify the measurement unit of the result. 2. Set up the sequence of multiplications so the appropriate measurement units cancel, leaving the measurement unit of the result. 3. Multiply the numbers as usual to obtain the numerical part of the result. 4. Check that the appropriate measurement units divide out, leaving the expected measurement unit of the result.	To convert your height of 70 inches to centimeters, $70\ \cancel{\text{in.}} \cdot \frac{2.54\text{ cm}}{1\ \cancel{\text{in.}}}$ $= 177.8$ cm Reading a 512-page book at the rate of 16 pages per hour will take how many 8-hour days? $512\ \cancel{\text{pages}} \cdot \frac{1\ \cancel{\text{hr.}}}{16\ \cancel{\text{pages}}} \cdot \frac{1\text{ day}}{8\ \cancel{\text{hr}}}$ $= 4$ days

Chapter 6 Gateway Review

1. Determine the value of each expression.

a. $20 \cdot \frac{7}{4}$

b. $\frac{4}{5} \div 2$

c. 27% of 44

d. $1300 \div 20\%$

e. $45.7 \div 0.0012$

f. $\frac{1}{6} \cdot 37.34$

2. Solve the proportion for x.

a. $\frac{4}{9} = \frac{x}{45}$

b. $\frac{x}{4} = \frac{5}{4}$

c. $\frac{1}{x} = \frac{2}{3}$

d. $\frac{2.3}{1.7} = \frac{x}{4}$

e. $\frac{\frac{1}{2}}{7} = \frac{x}{6}$

f. $\frac{x}{3} = 4$

3. In a recent survey, 70% of the 1400 female students and 30% of the 1000 male students on campus indicated that shopping was their favorite leisure activity. How many students placed shopping at the top of their list? What percent of the entire student body does this represent?

4. The least favorite responsibility in your summer job is to stuff and seal preaddressed envelopes. After sealing 600 envelopes you discover that you have completed only two-thirds of the job. How large is the entire mailing list?

Answers to all Gateway exercises are included in the Selected Answers appendix.

5. In December 2008, Alcatel-Lucent, the world's leading maker of telecommunications equipment, announced that it was reducing its worldwide workforce from 77,000 to 71,000. What percent of its workforce was being laid off?

6. In 2000, a total of 531,285 live births were recorded in the state of California. In 2006, the total number of live births rose to 562,157. What is the growth factor for live births for the 6-year period from 2000 to 2006, to the nearest thousandth? What is the percent increase (also called the growth rate)?

7. Aspirin is typically absorbed into the bloodstream from the duodenum. In general, 75% of a dose of aspirin is eliminated from the bloodstream in an hour. A patient is given a dose of 650 mg. How much aspirin is expected to remain in the bloodstream after 1 hour?

8. In 2007, your company's sales revenues were $400,000. A vigorous advertising campaign increased sales by 20% in 2008, and then by an additional 30% in 2009. What were your sales revenues in 2009?

9. The Ford Escape Hybrid made its debut in 2004 as the first SUV hybrid. During 2007, 21,386 of these SUVs were sold, an increase of approximately 11% over the previous year. Approximately how many Ford Escape Hybrids were sold in 2006?

10. The recent economic slowdown and consequent decline in state aid has forced public colleges in several states to cut costs and raise tuition. Determine the new tuition (per semester) at the following institutions.

    a. University of Minnesota, which raised its $3975 semester tuition for 2007–2008 by 7%

    b. Clemson University, which raised its 2007–2008 semester tuition of $4943 by 12%

    c. Clemson's trustees voted to raise the 2008–2009 tuition by an additional 10%. What was the new tuition amount?

11. The size of an optical telescope is given by the diameter of its mirror, which is circular in shape. Currently, the world's largest optical telescope is the 10.4-meter Gran Telescopio Canarias, located in the Canary Islands of Spain. What is the diameter of the mirror in feet?

12. In the middle of a playing season, NBA players Kevin Garnet (Minn), Kobe Bryant (LA), and Shaquille O'Neal (MIA) had these respective field goal statistics:

Garnet 87 field goals out of 167 attempts

Bryant 170 field goals out of 342 attempts

O'Neal 229 field goals out of 408 attempts

Rank them according to their relative (%) performance.

13. Hybrid electric/gas cars seem to be friendlier to the environment than all-gas vehicles, and more models are being introduced each year in the United States. One model reports 56 miles per gallon for highway driving. How far can you travel on one tank of gas if the tank holds 10.6 gallons?

14. You run at the rate of 6 miles per hour. Convert your rate to feet per second.

15. Solve each proportion equation for x.

a. $\frac{x}{5} = \frac{120}{300}$ **b.** $\frac{8}{15} = \frac{48}{x}$ **c.** $\frac{18}{x} = \frac{3}{2}$

16. An 8-pound bag of flour costs \$5.60. At that rate, what would a 100-pound bag of flour cost?

Chapter 7

Problem Solving with Geometry

Things around us come in different shapes and sizes and can be measured in a number of ways. A garden can be circular, a room is often rectangular, and a pool of water may be one of many interesting shapes. The study of these shapes and their properties is called geometry. In this chapter you will determine the perimeters and areas of rectangles, triangles, circles, and figures composed of these shapes. You will also calculate some surface areas and volumes.

The Geometry of Two-Dimensional Plane Figures

Activity 7.1

Walking around Bases, Gardens, and Other Figures

Objectives

1. Recognize perimeter as a geometric property of plane figures.
2. Write formulas for, and calculate perimeters of, squares, rectangles, triangles, parallelograms, trapezoids, and polygons.
3. Use unit analysis to solve problems that involve perimeters.

1. Look around your classroom. You probably are surrounded by a variety of geometric shapes. List the ones you see and compare your list with those of your classmates.

2. a. Glance down at the cover of your textbook. What shape is it?

Every rectangle has four sides. Opposite sides must have the same length. Every rectangle also has four corners, of equal size, called *angles*. Angles of this size are called **right angles**, and they measure 90°.

b. Look at the doorway, the walls, or a sheet of paper. Which of these have the shape of a rectangle?

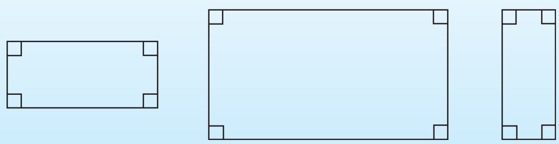
A rectangle has two dimensions, without any thickness. That is, it exists in a **plane**, which is a flat surface like the floor at your feet, except that a plane extends indefinitely.

Rectangles

Definition

A **rectangle** is a closed plane figure whose four sides are at right angles to each other.

Examples:



c. Name two other familiar objects in the shape of a rectangle.

3. a. Using a straightedge, draw your own rectangle here. Label all the right angles with little squares, as shown in the rectangle definition box.

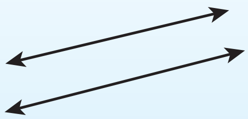
b. On your rectangle in part a, choose any side, and draw a line through it extending past each end of the rectangle. Now find the side opposite and draw another line, again extending beyond the ends of the rectangle. To indicate that those lines could go on forever, put arrowheads on the ends. Do the lines ever intersect?

The lines you have just drawn are called **parallel lines**.

Definition

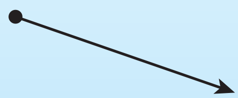
Parallel lines are lines in a plane that never intersect. No matter how far you extend the lines, in either direction, they will never meet.

Example:



A **ray** is a portion of a line that starts from a point and continues indefinitely in one direction, much like a ray of light coming from the Sun.

Example:



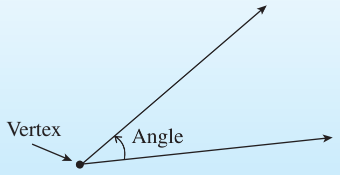
4. Draw a ray; then draw a second ray beginning at the same point. What is the name of the space between the rays?

When two lines of a plane figure meet at one point, they form an **angle**. The point where the two lines meet is called the **vertex** of the angle. Label your vertex in Problem 4.

Definition

An **angle** is formed by two rays that have a common starting point. The common starting point is called the **vertex** of the angle.

Example:

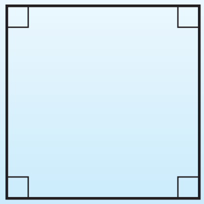


5. a. Draw a new rectangle here, but this time make all the sides equal in length.

b. What is the name of this kind of rectangle?

Definition

Any rectangle with four sides of *equal length* is a **square**.



c. What kind of angles does a square have?

6. a. Draw a new rectangle here, and choose a corner; draw a line segment from the corner you chose to the opposite corner. What new shape do you see?

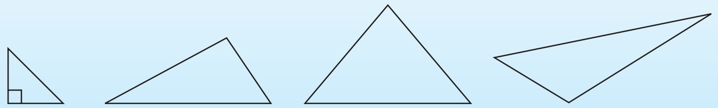
Actually, there are two of these shapes in the diagram, called **triangles**. Like the rectangle, a triangle has two dimensions. Also like a rectangle, adjacent sides meet at a common point forming an angle. Such shapes are called **closed**. Unlike the rectangle, however, a triangle has only three sides, and a triangle might not have a right angle.

Triangles

Definition

A **triangle** is a closed plane figure with three sides.

Examples:



b. Erase or cross out one of your triangles in part a. On the remaining triangle, label the right angle.

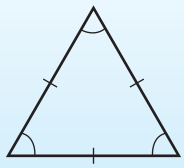
Definition

Any triangle containing a right angle is called a **right triangle**.

c. Draw a triangle with three equal sides. Are there any right angles?

Definition

Any triangle with three sides of equal length is called an **equilateral triangle**. In an equilateral triangle, the angles are also of equal size.

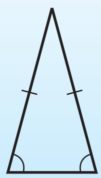


In plane figures, tic marks are customarily used to indicate that side lengths are equal. Angles are marked with curves to show they are equal in size.

d. Draw a triangle with *only two* equal sides. Mark the two equal sides.

Definition

Any triangle containing two equal sides is called an **isosceles triangle**. In an isosceles triangle, the angles opposite the equal sides are equal in size.



7. You are at bat in the middle of an exciting baseball game. You are a fairly good hitter and can run the bases at a speed of 15 feet per second. You know that the baseball diamond has the shape of a square, measuring 90 feet on each side.

a. Draw a square to represent the baseball diamond, labeling each side. What is the total distance in feet you must run from home plate through the bases and back to home?

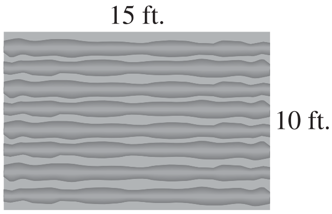
Definition

The total distance around all the edges or sides of any closed two-dimensional (plane) figure is called the **perimeter**.

b. Draw a square and label the length of each side s. Write an expression for the perimeter of your square, and simplify it if possible.

8. You are interested in planting a rectangular garden, 10 feet long by 15 feet wide. To protect your plants, you decide to purchase a fence to enclose your entire garden.

a. How many feet of fencing must you buy to enclose your garden?



b. Draw a rectangle and label any side w, then the side opposite also will be of length w. Label the remaining two sides length l. Write an expression for the perimeter of your rectangle, and simplify, if possible.

9. **a.** The dimensions of the three sides of a triangular garden are 13 feet, 13 feet, and 20 feet. What is the perimeter of the garden? Draw a diagram to represent the garden and label the sides.

b. Draw a triangle with all sides unequal in length and label the sides a, b, and c. Write a formula for the perimeter, P, of your triangle.

Definition

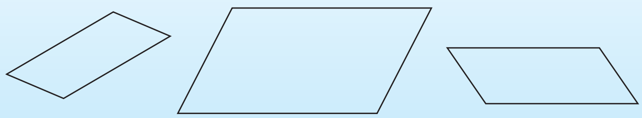
A triangle with all sides unequal in length is called a **scalene triangle.**

Parallelograms

Definition

A **parallelogram** is a closed four-sided plane figure whose opposite sides are parallel and the same length.

Examples:



10. a. You decide to install a walkway diagonally from the street to your front steps. The width of your steps is 3 feet, and you determine that the length of the walkway is 23 feet. What is the perimeter of your walkway?

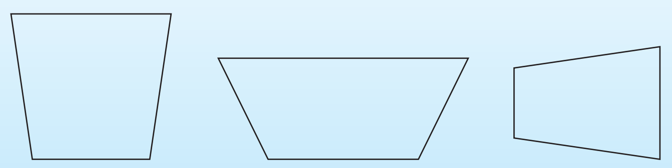
b. Draw a parallelogram and label each side using a or b. Write a formula for the perimeter of your parallelogram, and simplify it if possible.

Trapezoids

Definition

A **trapezoid** is a closed four-sided plane figure with two opposite sides that are parallel and two opposite sides that are not parallel.

Examples:



11. a. You buy a piece of land in the shape of a trapezoid with sides measuring 100 feet, 40 feet, 130 feet, and 50 feet. The parallel sides are 100 feet and 130 feet. Sketch the trapezoid, label the sides, and calculate its perimeter.

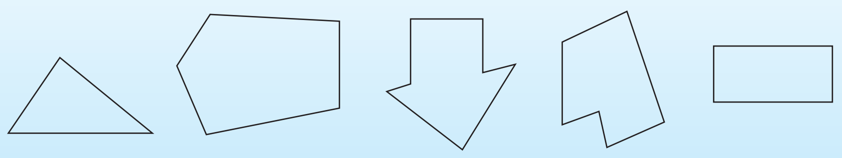
b. The sides of a trapezoid measure a, b, c, and d. Write a formula for the perimeter, P, of a trapezoid.

Polygons

Definition

A **polygon** is a closed plane figure with three or more sides.

Examples:

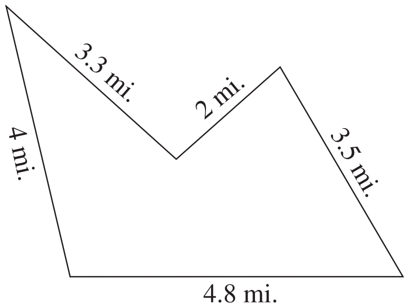


Procedure

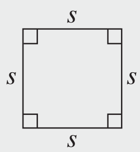
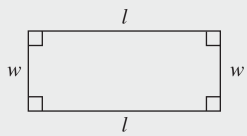
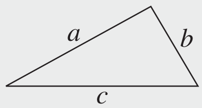
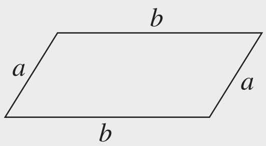
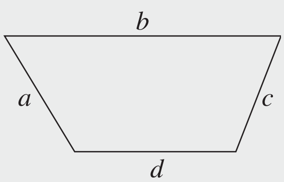
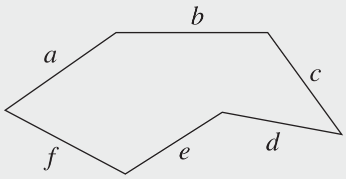
Calculating the Perimeter of a Polygon

The perimeter of a polygon is calculated by adding the lengths of all the sides that make up the figure.

11. Calculate the perimeter of the following polygon.



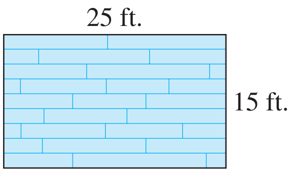
SUMMARY: ACTIVITY 7.1

Two-Dimensional Figure	Labeled Sketch	Perimeter Formula
Square		$P = 4s$
Rectangle		$P = 2l + 2w$
Triangle		$P = a + b + c$
Parallelogram		$P = 2a + 2b$
Trapezoid		$P = a + b + c + d$
Polygon	(A many-sided figure) 	For this example, $P = a + b + c + d + e + f$

EXERCISES: ACTIVITY 7.1

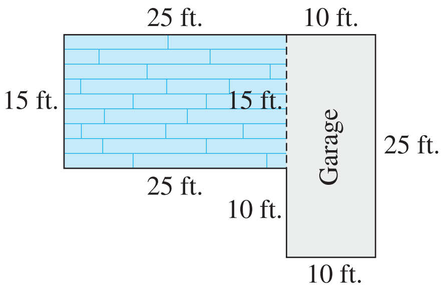
1. You are interested in buying an older home. The first thing you learn about older homes is that they frequently have had additions over the years, and the floor plans often have the shape of a polygon.

   a. When the house you want to buy was built 100 years ago, it had a simple rectangular floor plan.



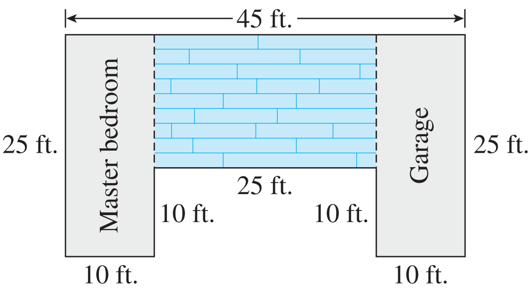
   Calculate the perimeter of the floor plan.

   b. Sixty years ago, a rectangular 10-foot by 25-foot garage was added to the original structure.



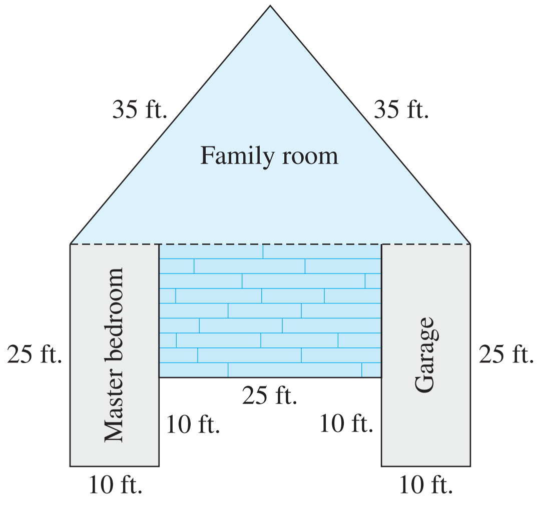
   Calculate the perimeter of the floor plan.

   c. Twenty-five years ago, a new master bedroom, with the same size and shape as the garage, was added onto the other side of the original floor plan.



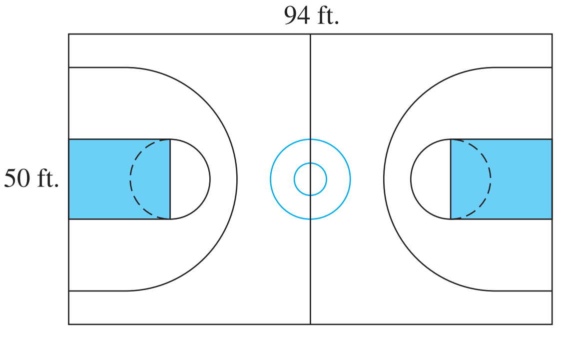
   Calculate the perimeter of the remodeled floor plan.

d. You plan to add a family room with a triangular floor plan, as shown in the following diagram.



Calculate the perimeter of the floor plan of the house after the family room is added.

2. A standard basketball court has the following dimensions:



a. Calculate the perimeter of the court.

b. If you play a half-court game, calculate the perimeter of the half-court.

3. The Bermuda Triangle is an imaginary triangular area in the Atlantic Ocean in which there have been many unexplained disappearances of boats and planes. Public interest was aroused by the publication of a popular and controversial book, *The Bermuda Triangle*, by Charles Berlitz in 1974. The triangle starts at Miami, Florida, goes to San Juan, Puerto Rico (1038 miles), then to Bermuda (965 miles), and back to Miami (1042 miles).

a. What is the perimeter of this triangle?

b. If you were on a plane that was averaging 600 miles per hour, how long would it take you to fly the perimeter of the Bermuda Triangle?

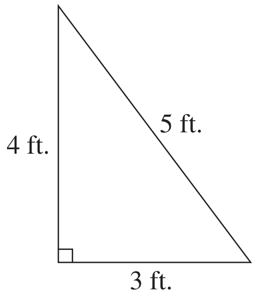
4. Leonardo da Vinci's painting of the *Last Supper* is a 460-cm by 880-cm rectangle.

a. Calculate its perimeter.

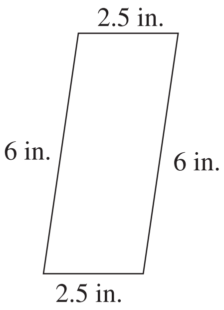
b. Would the painting fit in your living room? Explain.

5. Calculate the perimeters of the following polygons.

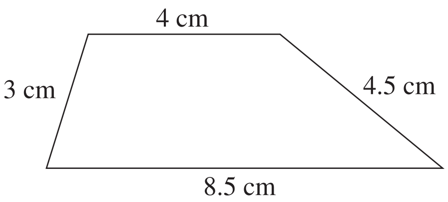
a.



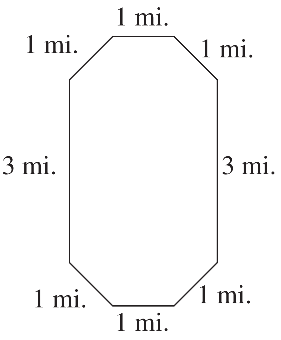
b.



c.



d.



6. A square has a perimeter of 64 feet. Calculate the length of each side of the square.

7. A rectangle has a perimeter of 75 meters and a length of 10 meters. Calculate its width.

Activity 7.2

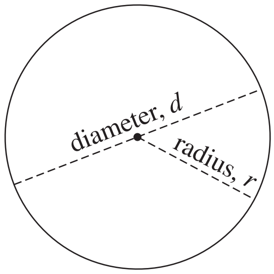
Circles Are Everywhere

Objective

1. Develop and use formulas for calculating circumferences of circles.

Objects in the shape of circles of varying sizes are found in abundance in everyday life. Coins, dartboards, and ripples made by a raindrop in a pond are just a few examples.

The size of a circle is customarily described by the length of a line segment that starts and ends on the circle's edge and passes through its center. This line segment is called the **diameter** of the circle. The **radius** of a circle is a line segment that starts at its center and ends on its edge. Therefore, the length of the radius is one-half the length of the diameter. The distance around the edge of the circle is the perimeter, more commonly called the **circumference**.



$d = 2r$

$r = \frac{1}{2}d$

1. The perimeter and diameter of four circular objects were measured and recorded in the following table. Some of these measurements are given in the metric system and others in the English system. From these measurements, you will develop formulas that relate the circumference of a circle to its diameter or radius.

COLUMN 1 CIRCULAR OBJECTS	COLUMN 2 MEASURED CIRCUMFERENCE, *C*	COLUMN 3 MEASURED DIAMETER, *d*	COLUMN 4 $\frac{C}{d}$, EXPRESSED AS A DECIMAL
Coin	7.62 cm	2.426 cm	
Salt Box	10.56 in.	3.37 in.	
Plate	69 cm	22 cm	
Can	12.38 in.	3.94 in.	

a. Use the data from columns 2 and 3 to determine the ratio $\frac{C}{d}$. Express each ratio as a decimal equivalent to the nearest thousandth and record it in column 4.

b. Are the four values in column 4 the same? How much do they vary?

c. Calculate the average of the four values in column 4, rounding to the hundredths place. What do you conclude?

The remarkable fact is that in every circle, however large or small, the ratio of the circumference, C, to the diameter, d, is always the same $\left(\text{approximately } 3.14, \text{ or } \frac{22}{7}\right)$. This ratio is represented by the Greek letter pi, or π. Most calculators have π keys. Solving $\pi = \frac{C}{d}$ for C results in the formula for the circumference of a circle, $C = \pi d$.

Procedure

Calculating the Circumference of a Circle

The formula for the circumference, C, of a circle with diameter d is

$$C = \pi d.$$

Since $d = 2r$, where r is the radius of the circle, the circumference formula can also be written as

$$C = 2\pi r.$$

2. For each of the following problems, draw and label a circle representing the given description. Then, use the circumference formulas to calculate each circumference. Use π on your calculator and round your answers to the nearest hundredth.

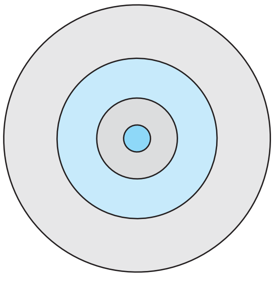
a. The radius of the circle is 8 miles.

b. The diameter of the circle is 10 meters.

c. The radius of the circle is 3 inches.

3. You enjoy playing darts. You decide to make your own dartboard consisting of four concentric circles (that is, four circles with the same center). The smallest circle, C_1 (the "bull's-eye"), has radius 1 cm, the next largest circle, C_2, has radius 3 cm, the third circle, C_3, has radius 6 cm, and the largest circle, C_4, has radius 10 cm. You decide to compare the circumferences of the circles.

a. What are these circumferences?

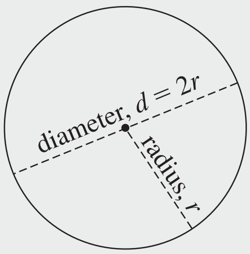


b. Use a ratio to compare the radius of C_1 with the radius of C_2. Compare their circumferences. Is there a connection between the relationship of the radii and the relationship of the circumferences?

c. Use a ratio to compare the radius of C_3 with the radius of C_2. Compare their circumferences. Is there a connection between the relationship of the radii and the relationship of the circumferences?

d. Use a ratio to compare the radius of C_1 with the radius of C_4. Based on your answers to parts b and c, what would you predict the circumference of C_4 to be? Compare their circumferences. Was your prediction correct?

SUMMARY: ACTIVITY 7.2

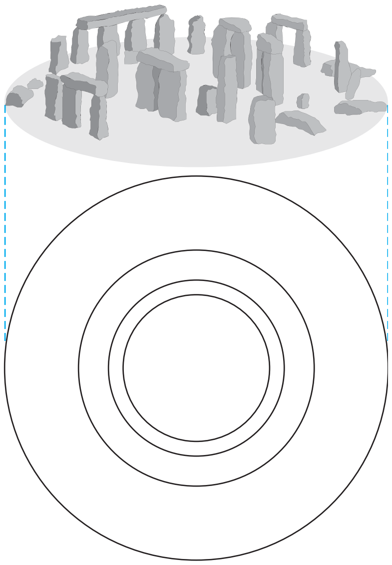
Two-Dimensional Figure	**Labeled Sketch**	**Circumference (Perimeter) Formula**
Circle		$C = \pi d$ or $C = 2\pi r$

EXERCISES: ACTIVITY 7.2

For calculations in Exercises 1–4, use π on your calculator and round your answers to the nearest hundredth unless otherwise indicated.

1. You order a pizza in the shape of a circle with diameter 14 inches. Calculate the "length" of the crust (that is, find the circumference of the pizza).

2. Stonehenge is an ancient site on the plains of southern England consisting of a collection of concentric circles (that is, circles with the same center) outlined with large sandstone blocks. Carbon dating has determined the age of the stones to be approximately 5000 years. Much curiosity and mystery has surrounded this site over the years. One theory about Stonehenge is that it was a ritualistic prayer site. However, even today, there is still controversy over what went on there. One thing everyone interested in Stonehenge can agree on is the mathematical description of the circles.



The diameters of the four circles are 288 feet for the largest circle, 177 feet for the next, 132 feet for the third, and 110 feet for the innermost circle.

a. For each time you walked around the outermost circle, how many times could you walk around the innermost circle? Round to the nearest hundredth.

b. What is the ratio of the diameter of the outermost circle to that of the innermost circle?

c. How does this ratio compare to the ratio of their circumferences from part a?

d. What is the ratio of the diameter of the larger intermediate circle to that of the smaller intermediate circle?

e. Predict how much longer the trip around the larger intermediate circle would be than around the smaller intermediate circle?

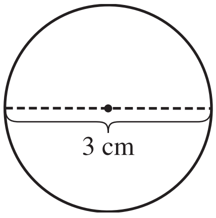
f. Calculate the circumference of each intermediate circle.

g. Are the circumferences you calculated in part f related as you predicted in part e?

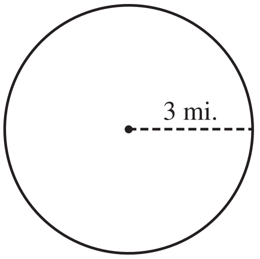
h. How much longer is the trip around one of the two intermediate circles than around the other? Round to the nearest hundredth.

3. Use the appropriate geometric formulas to calculate the circumference, or fraction thereof, for each of the following figures. Round to the nearest tenth.

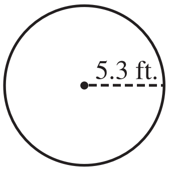
a.



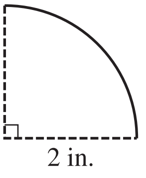
b.



c.



d.



4. If a circle has circumference 63 inches, approximate its radius to the nearest hundredth.

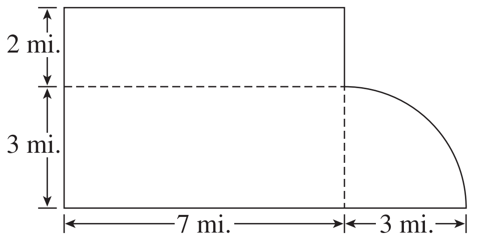
Activity 7.3

Lance Armstrong and You

Objectives

1. Calculate perimeters of many-sided plane figures using formulas and combinations of formulas.
2. Use unit analysis to solve problems that involve perimeters.

The Tour de France is the oldest and most prestigious annual bicycle race. It is approximately 3500 kilometers long, meandering throughout France and a bordering country. In 2005, Lance Armstrong, an American cyclist, won the Tour de France for an unprecedented seventh consecutive time. Inspired by Lance Armstrong's remarkable performance, you decide to experiment with long-distance biking. You choose the route shown by the solid line path in the following figure, made up of rectangles and a quarter circle.



1. Calculate the total length of your bike trip in miles (that is, determine the perimeter of the figure). Use π on your calculator and round to the nearest thousandth.

2. If you can average 9 miles per hour on your bike, how long will it take you to complete the trip?

3. If your bike tires have a diameter of 2 feet, calculate the circumference of the tires to the nearest thousandth.

4. To analyze the wear on your tires, calculate how many rotations of the tires are needed to complete your trip. (*Note*: 1 mile = 5280 feet.)

5. The 2005 Tour de France covered 3607 kilometers. How many hours would it take you to complete the race if you average 9 miles per hour? (*Note*: 1 mi. = 1.609 km.)

6. Lance Armstrong completed the 2005 Tour de France in 86 hours, 15 minutes, and 2 seconds, or approximately 86.25 hours. Compare your time from Problem 5 to his 2005 time.

7. Calculate Lance Armstrong's average speed in miles per hour.

SUMMARY: ACTIVITY 7.3

1. A many-sided closed plane figure may be viewed as a combination of basic plane figures: squares, rectangles, parallelograms, triangles, trapezoids, and circles.

2. To calculate the perimeter of a many-sided plane figure:

  i. Determine the length of each part that contributes to the perimeter.

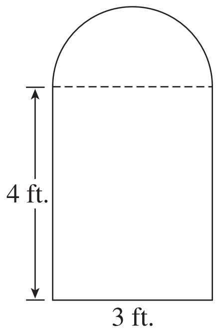
  ii. Add the lengths to obtain the figure's total perimeter.

EXERCISES: ACTIVITY 7.3

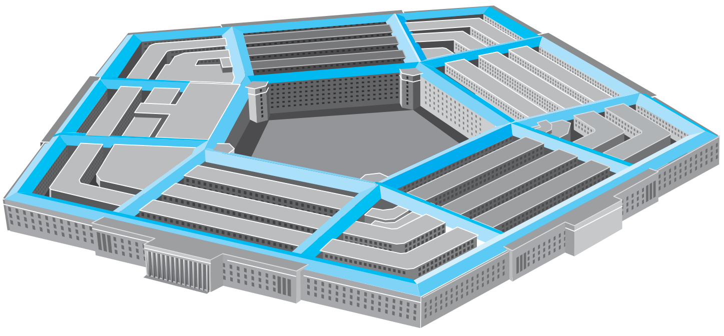
1. You plan to fly from New York City to Los Angeles via Atlanta and return from Los Angeles to New York City via Chicago.

Use the data in the diagram to determine the total distance of your trip.

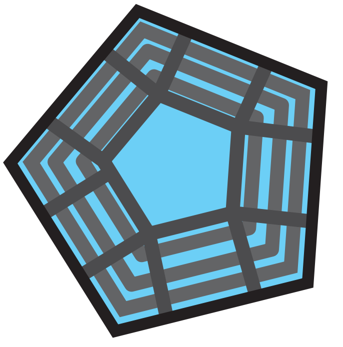
2. A Norman window is a rectangle with a semicircle, or half circle, on top. You decide to install a Norman window in your family room with the dimensions indicated in the diagram. What is its perimeter? Round to the nearest hundredth.



3. The Pentagon, headquarters of the U.S. Department of Defense located in Arlington, VA, is the world's largest office building in terms of the amount of floor space. The Pentagon received its name from the shape of its base, a five-sided polygon. Each floor of this government building consists of five corridors in the shape of concentric (having the same center) pentagons.



Pentagon



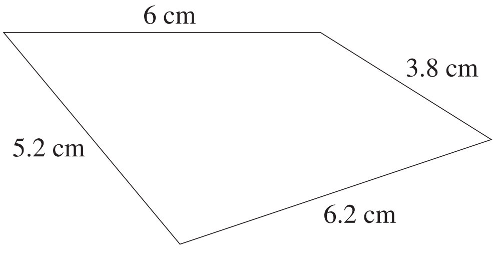
Floor plan

a. Each outside wall of the Pentagon measures 921 feet in length. Your cousin works at the Pentagon and walks 7 miles every day at lunchtime. When weather permits, she prefers to walk outside. Approximately how many times would your cousin need to walk the perimeter of the Pentagon to get in her daily 7 miles?

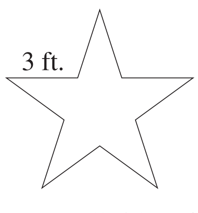
b. The Pentagon contains 17.5 miles of corridors. Your cousin covers 30 inches for every step that she walks. How many steps would your cousin walk if she walked all the corridors of the Pentagon, if that were permissible?

4. Calculate the perimeters of each of the following figures:

a.

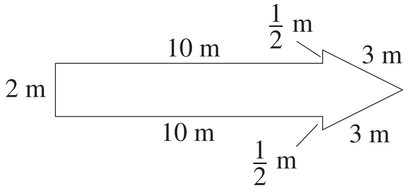


b.

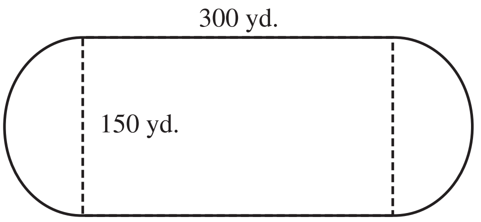


Note: All sides of the star are of equal length.

c.

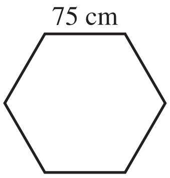


d.

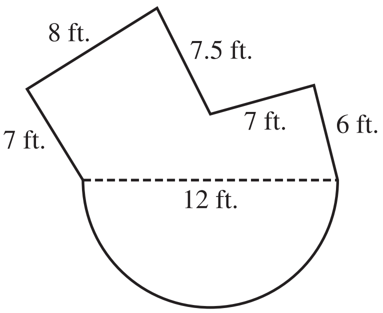


Note: The ends are semicircles.

e.



f.

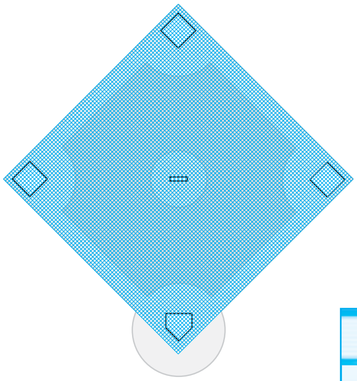


Activity 7.4

Baseball Diamonds, Gardens, and Other Figures Revisited

Objectives

1. Write formulas for areas of squares, rectangles, parallelograms, triangles, trapezoids, and polygons.
2. Calculate areas of polygons using appropriate formulas.



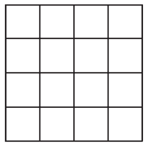
In previous activities, you ran the bases of a baseball diamond, fenced in a rectangular garden, and explored the perimeters of parallelograms, trapezoids, and circles. Now, you will look at those same geometric figures, but from a different perspective.

Squares

As groundskeeper for the local baseball team, you need to guarantee good-quality turf for the infield, that is, the space enclosed by the baselines. In order to plant and maintain this turf, you need to measure this space.

Recall from Chapter 2 that the area of a square is measured by determining the number of unit squares (squares that measure 1 unit in length on each side) that are needed to completely fill the inside of the square.

For example, the square pictured below has sides that are 4 units in length. As illustrated, it takes four rows of four unit squares to fill the inside of the square. So the area, A, of the square is 16 square units.



$A = 16$ square units

1. A regulation baseball diamond is a square with 90-foot sides. How many square feet are needed to cover this baseball diamond? Your answer in square feet is called the area of the square.

Procedure

Calculating the Area of a Square

The formula for the area, A, of a square with sides of length s is

$$A = s \cdot s \text{ or } A = s^2.$$

2. The Little League baseball diamond has sides measuring 60 feet. Calculate the area of the Little League diamond.

Definition

The **area** of a square, or any polygon, is the measure of the region enclosed by the sides of the polygon. Area is measured in square units.

Rectangles

Planting your rectangular 5-foot by 4-foot garden requires that you know how much space it contains.

3. How many square feet are required to cover your garden? Include a sketch to explain your answer.

Procedure

Calculating the Area of a Rectangle

The formula for the area, A, of a rectangle with length l and width w is

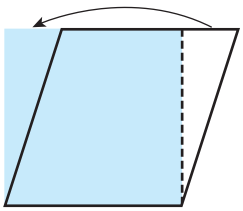
$$A = lw.$$

4. Will doubling the length and width of your garden in Problem 3 double its area? Explain.

Parallelograms

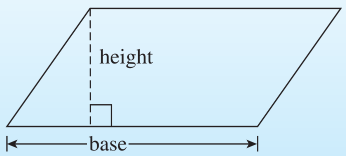
Once you know the formula for the area of a rectangle, then you have the key for determining the formula for the area of a parallelogram. Recall that a parallelogram is formed by two intersecting pairs of parallel sides.

5. If you "cut off" a triangle and move it to the other side of the parallelogram forming the shaded rectangle, is the area different? Explain.

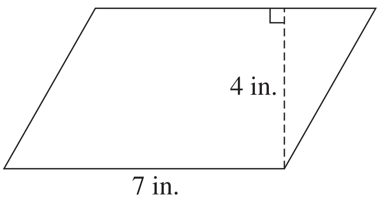


Definition

The **height** of a parallelogram is the distance from one side (the base) to the opposite side, measured along a perpendicular line (see illustration).



6. Use your observations from Problem 5 to calculate the area of the parallelogram with height 4 inches and base 7 inches.



Procedure

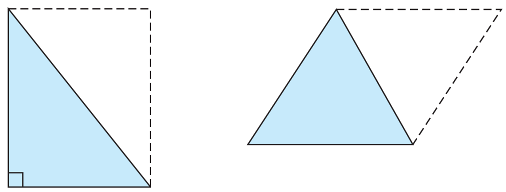
Calculating the Area of a Parallelogram

The formula for the area, A, of a parallelogram with base b (the length of one side) and height h (the perpendicular distance from the base b to its parallel side) is

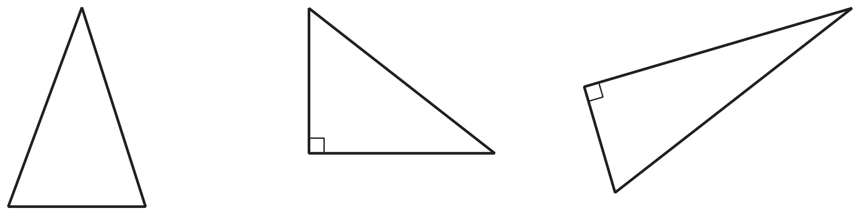
$$A = bh.$$

Triangles

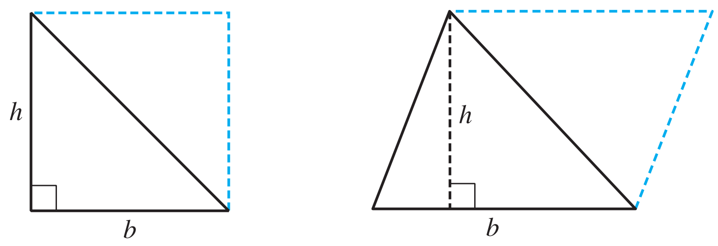
Every triangle can be pictured as one-half of a rectangle or parallelogram.



7. For each of the following triangles, attach an identical triangle to create a rectangle or parallelogram.



8. Use the idea that a triangle is one-half of a rectangle or parallelogram to write a formula for the area of a triangle in terms of its base b and height h. Explain how you obtained the formula.



Procedure

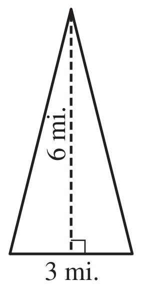
Calculating the Area of a Triangle

The formula for the area, A, of a triangle with base b (the length of one side) and height h (the perpendicular distance from the base b to the vertex opposite it) is

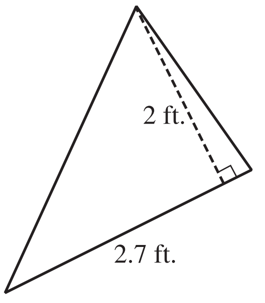
$$A = \frac{1}{2}bh.$$

9. Use the area formula to calculate the areas of the following triangles.

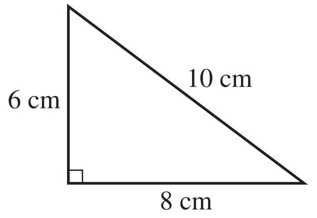
a.



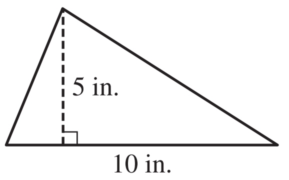
b.



c.

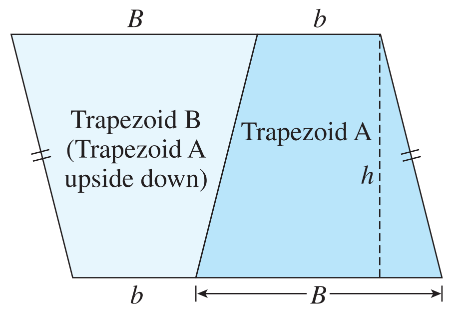


d.



Trapezoids

Trapezoids may be viewed as one-half of the parallelogram that is formed by making a copy of the trapezoid, flipping it vertically, and joining it to the original trapezoid, as shown.



10. a. The parallelogram shown above has base $b + B$ and height h. Determine the area of this parallelogram.

b. Use the idea that a trapezoid can be viewed as one-half of a related parallelogram to write a formula for the area A of a trapezoid in terms of its bases b and B and its height h.

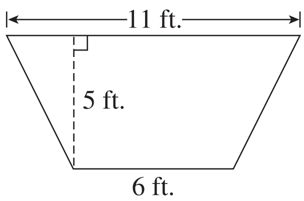
Procedure

Calculating the Area of a Trapezoid

The formula for the area, A, of a trapezoid with bases b and B and height h is

$$A = \frac{1}{2}h(b + B).$$

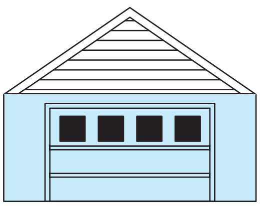
11. Calculate the area of the trapezoid with height 5 feet and bases 6 feet and 11 feet.



12. If the lower base of the trapezoid in Problem 11 is increased by 2 feet and the upper base is decreased by 2 feet, draw the new trapezoid and compute its area.

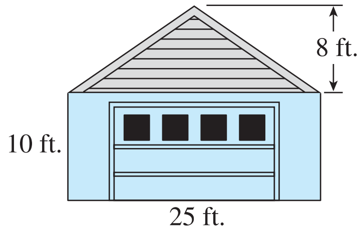
Polygons

You can determine the area of a polygon by seeing that polygons can be broken up into other, more basic figures, such as rectangles and triangles. For example, the front of a garage shown here is a polygon that can also be viewed as a triangle sitting on top of a rectangle.

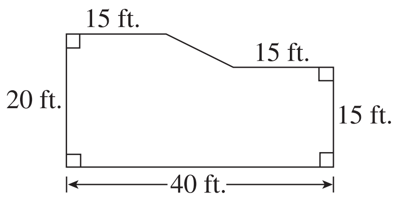


To determine the area of this polygon, use the appropriate formulas to determine the area of each part, and then sum your answers to obtain the area of the polygon.

13. Calculate the area of the front of the garage:



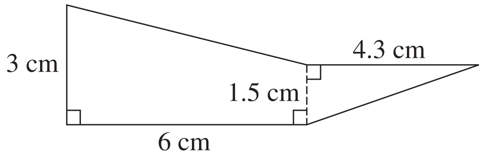
14. You are planning to build a new home with the following floor plan.



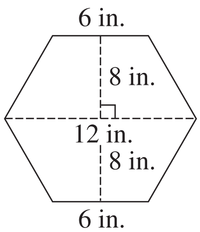
Calculate the total floor area.

15. Calculate the areas of each of the following polygons.

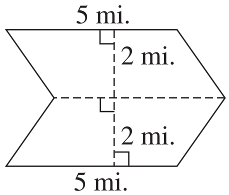
a.



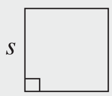
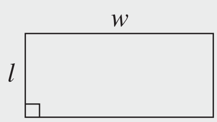
b.

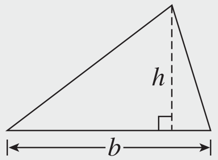
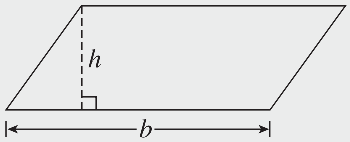
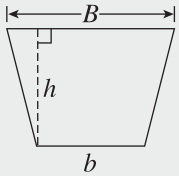
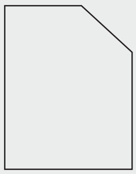
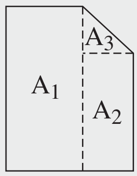


c.



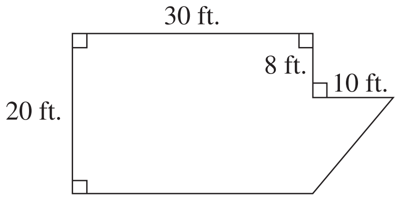
SUMMARY: ACTIVITY 7.4

Two-Dimensional Figure	Labeled Sketch	Area Formula
Square		$A = s^2$
Rectangle		$A = lw$

Two-Dimensional Figure	Labeled Sketch	Area Formula
Triangle		$A = \frac{1}{2}bh$
Parallelogram		$A = bh$
Trapezoid		$A = \frac{1}{2}h(b + B)$
Polygon	A many-sided figure that is subdivided into familiar figures whose areas are given by formulas in this summary. 	$A =$ sum of the areas of each figure $= A_1 + A_2 + A_3$ 

EXERCISES: ACTIVITY 7.4

1. You are carpeting your living room and sketch the following floor plan.



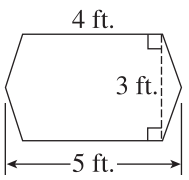
a. Calculate the area that you need to carpet.

b. Carpeting is sold in 10-foot-wide rolls. Calculate how much you need to buy. Explain.

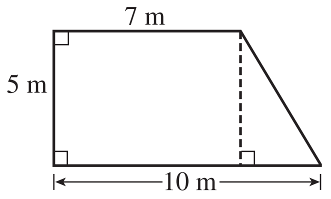
2. You need to buy a solar cover for your 36-foot by 18-foot rectangular pool. A pool company advertises that solar covers are on sale for $0.28 per square foot. Determine the cost of the pool cover before sales tax.

3. Calculate the areas of each of the following figures.

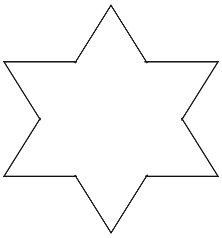
a.



b.



4. How would you break up the following star to determine what dimensions you need to know in order to calculate its area? Explain.



5. A standard basketball court is a rectangle with length 94 feet and width 50 feet. How many square feet of flooring would you need to purchase in order to replace the court?

6. You are building a new double garage and use the length and width of your two cars to estimate the area needed. Your car measurements are

Car 1: 14 ft. 2 in. by 5 ft. 7 in.

Car 2: 14 ft. 6 in. by 5 ft. 9 in.

a. Based on these measurements, what would be a reasonable floor plan for your garage? Explain.

b. What is the area of the floor plan you designed in part a?

Activity 7.5

How Big Is That Circle?

Objectives

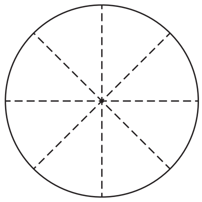
1. Develop formulas for the area of a circle.
2. Use the formulas to determine areas of circles.

Determining the area of a circle becomes a challenge because there are no straight sides. No matter how hard you try, you cannot neatly pack unit squares inside a circle to completely cover the area of a circle. The best you can do in this way is to get an approximation of the area. In this activity you will explore methods for estimating the area of a circle and develop formulas to calculate the exact area.

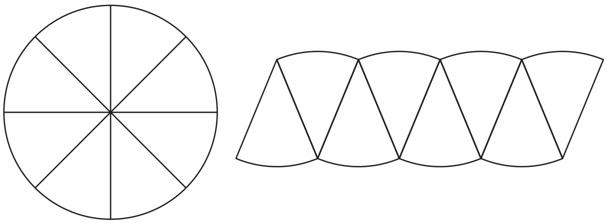
Definition

The **area of a circle** is the measure of the region enclosed by the circumference of the circle.

1. **a.** To help understand the formula for a circle's area, start by folding a paper circle in half, then in quarters, and finally halving it one more time into eighths.



b. Cut the circle along the folds into eight equal pie-shaped pieces (called sectors) and rearrange these sectors into an approximate parallelogram (see accompanying figures).



c. What measurement on the circle approximates the height of the parallelogram? Explain.

d. What measurement on the circle approximates the base of the parallelogram?

e. Recall that the area of a parallelogram is given by the product of its base and its height. Use the approximations from parts c and d to determine the approximate area of the parallelogram and the approximate area of the circle.

Imagine cutting a circle into more than eight equal sectors. Each sector would be thinner. When reassembled, as in Problem 1, the resulting figure will more closely approximate a parallelogram. Hence, the formula for the area of a circle, $A = \pi r^2$, is even more reasonable and accurate.

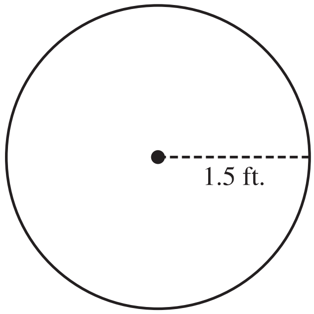
Procedure

Calculating the Area of a Circle

The formula for the area, A, of a circle with radius r is

$$A = \pi r^2.$$

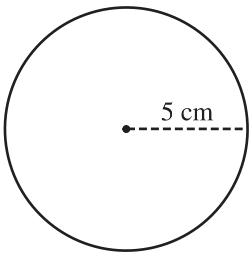
2. You own a circular dartboard of radius 1.5 feet. To figure how much space you have as a target, calculate the area of the dartboard. Be sure to include the units of measurement in your answer.



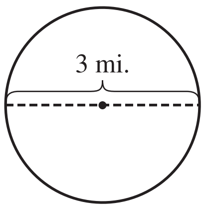
3. The diameter of a circle is twice its radius. Use this fact to rewrite the formula for the area of a circle using its diameter instead of the radius. Show the steps you took.

4. Calculate the areas of the following circles. Round to the nearest hundredth.

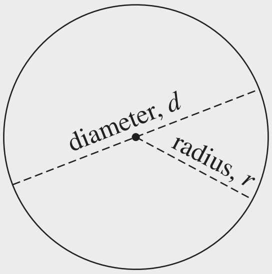
a.



b.



SUMMARY: ACTIVITY 7.5

Two-Dimensional Figure	Labeled Sketch	Area Formula
Circle		$A = \pi r^2$ (involving the radius) $A = \dfrac{\pi d^2}{4}$ (involving the diameter)

EXERCISES: ACTIVITY 7.5

1. You order a pizza with diameter 14 inches. Your friend orders a pizza with diameter 10 inches. Use a ratio to compare the areas of the two pizzas to estimate approximately how many of the smaller pizzas are equivalent to one larger pizza.

2. United States coins are circles of varying sizes. Choose a quarter, dime, nickel, and penny.

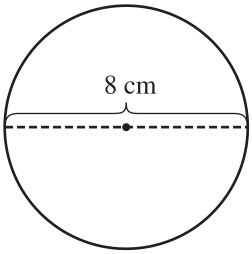
   a. Use a tape measure (or ruler) to estimate the diameter of each coin in centimeters.

   b. Calculate the area of each coin to the nearest hundredth.

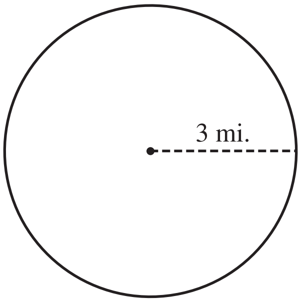
Exercise numbers appearing in color are answered in the Selected Answers appendix.

3. Use an appropriate formula to calculate the area for each of the following figures. Round to the nearest hundredth.

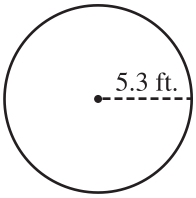
a.



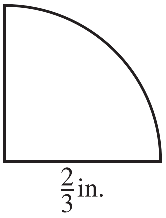
b.



c.



d.



4. You need to polish your circular dining room table, which has a diameter of 7 feet. The label on the polish can claims coverage for 100 square feet, but you notice that the can is only about one-half full. Will you have enough polish to finish your table? Explain.

Activity 7.6

A New Pool and Other Home Improvements

Objectives

1. Solve problems in context using geometric formulas.
2. Distinguish between problems that require area formulas and those that require perimeter formulas.

You are the proud owner of a new circular swimming pool with a diameter of 25 feet and are eager to dive in. However, you quickly discover that having a new pool requires making many decisions about other purchases.

1. Your first concern is a solar pool cover. You do some research and find that circular pool covers come in the following sizes: 400, 500, and 600 square feet. Friends recommend that you buy a pool cover with very little overhang. Which size is best for your needs? Explain.

2. You decide to build a concrete patio around the circumference of your pool. It will be 6 feet wide all the way around. Provide a diagram of your pool with the patio. Show all the distances you know on the diagram. What is the area of the patio alone? Explain how you determined this area.

3. State law requires that all pools be enclosed by a fence to prevent accidents. You decide to completely enclose your pool and patio with a stockade fence. How many feet of fencing do you need? Explain.

4. Next, you decide to stain the new fence. The paint store recommends a stain that covers 500 square feet per gallon. If the stockade fence has a height of 5 feet, how many gallons of stain should you buy? Explain.

5. Lastly, you decide to plant a circular flower garden near the pool and patio but outside the fence. You determine that a circle of circumference 30 feet would fit. What is the length of the corresponding diameter of the flower garden?

Other Home Improvements

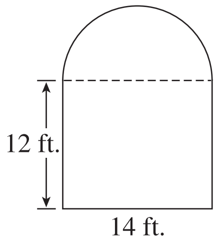
Now that you have a new pool, patio, and flower garden, you want to do some other home improvements in anticipation of enjoying your new pool with family and friends.

You have $1000 budgeted for this purpose and the list looks like this:

- Replace the kitchen floor.
- Add a wallpaper border to the third bedroom.
- Paint the walls in the family room.

To stay within your budget, you need to determine the cost of each of these projects. You expect to do this work yourself, so the only monetary cost will be for materials.

6. The kitchen floor is divided into two parts. The first section is rectangular and measures 12 by 14 feet. The second section is a semicircular breakfast area that extends off the 14-foot side. The cost of vinyl flooring is $21 per square yard plus 6% sales tax. The vinyl is sold in 12-foot widths.



a. How long a piece of vinyl flooring will you need to purchase if you want only one seam, where the breakfast area meets the main kitchen, as shown? Remember that the vinyl is 12 feet wide.

b. How many square feet of flooring must you purchase? How many square yards is that (9 square feet = 1 square yard)?

c. How much will the vinyl flooring cost, including tax?

d. How many square feet of flooring will be left over after you're done? Explain.

7. The third bedroom is rectangular in shape and has dimensions of $8\frac{1}{2}$ by 13 feet. On each 13-foot side, there is a window that measures 3 feet 8 inches wide. The door is located on an $8\frac{1}{2}$-foot side and measures 3 feet wide from edge to edge. You are planning to put up a decorative horizontal wallpaper stripe around the room about halfway up the wall.

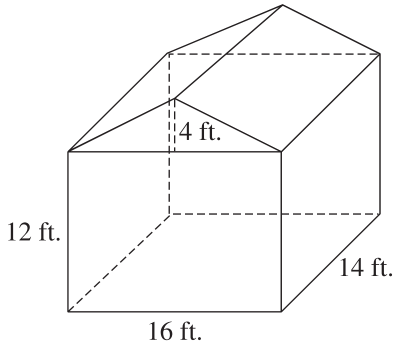
a. How many feet of wallpaper stripe will you need to purchase?

b. The border comes in rolls 5 yards in length. How many rolls will you need to purchase?

c. The wallpaper border costs \$10.56 per roll plus 6% sales tax. Determine the cost of the border.

d. How many feet of wallpaper will be left over after you're done?

8. Your family room needs to be painted. It has a cathedral ceiling with front and back walls that measure the same, as shown in the diagram. The two side walls are 14 feet long and 12 feet high. Each of the walls will need two coats of paint. The ceiling will not be painted. For simplicity, ignore the fact that there are windows.



a. How many square feet of wall surface will you be painting? (Remember, all walls will need two coats.)

b. Each gallon of paint covers approximately 400 square feet. How many gallons of paint will you need to purchase?

c. The paint costs \$19.81 per gallon plus 6% sales tax. What is the total cost of the paint you need for the family room?

9. What is the cost for all your home-improvement projects (not including the new pool, patio, and flower garden)?

10. Additional costs for items such as paint rollers and wallpaper paste amount to approximately \$30. Can you afford to do all the projects? Explain.

SUMMARY: ACTIVITY 7.6

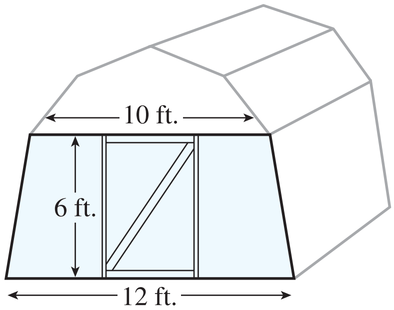
1. Area formulas are used to measure the amount of space *inside* a plane figure.
2. Perimeter or circumference formulas are used to measure the length *around* a plane figure.

EXERCISES: ACTIVITY 7.6

1. Your driveway is rectangular in shape and measures 15 feet wide and 25 feet long. Calculate the area of your driveway.

2. A flower bed in the corner of your yard is in the shape of a right triangle. The perpendicular sides of the bed measure 6 feet 8 inches and 8 feet 4 inches. Calculate the area of the flower bed.

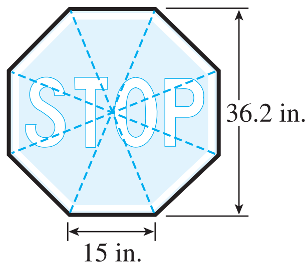
3. The front wall of a storage shed is in the shape of a trapezoid. The bottom measures 12 feet, the top measures 10 feet, and the height is 6 feet. Calculate the area of the front wall (shaded) of the shed.



4. You live in a small, one-bedroom apartment. The bedroom is 10 by 12 feet, the living room is 12 by 14 feet, the kitchen is 8 by 6 feet, and the bathroom is 5 by 9 feet. Calculate the total floor space (area) of your apartment.

5. In your living room, there is a large rectangular window with dimensions of 10 feet by 6 feet. You love the sunlight but would like to redesign the window so that it admits the same amount of light but is only 8 feet wide. How tall should your redesigned window be?

6. A stop sign is in the shape of a regular octagon (an eight-sided polygon with equal sides and angles). A regular octagon can be created using eight triangles of equal area. One triangle that makes up a stop sign has a base of 15 inches and a height of approximately 18.1 inches. Calculate the area of the stop sign.



7. How does the area of the stop sign in Exercise 6 compare with the area of a circle of radius 18.1 inches? Explain.

8. You are buying plywood to board up your windows in preparation for Hurricane Euclid. In the master bedroom you have a Norman window (in the shape of a rectangle with a semicircular top). You need to calculate the area of the window. If the rectangular part of the window is 4 feet wide and 5 feet tall, what is the area of the entire window? Explain.

9. The diameter of Earth is 12,742 kilometers; the diameter of the Moon is 3476 kilometers.

a. If you flew around Earth by following the equator at a height of 10 kilometers, how many trips around the Moon could you take in the same amount of time, at the same height from the Moon, and at the same speed? Explain.

b. The circle whose circumference is the equator is sometimes called the "great circle" of Earth or of the Moon. Use a ratio to compare the areas of the great circles of Earth and the Moon.

Laboratory Activity 7.7

How About Pythagoras?

Objectives

1. Verify and use the Pythagorean Theorem for right triangles.
2. Calculate the square root of numbers other than perfect squares.
3. Use the Pythagorean Theorem to solve problems.
4. Determine the distance between two points using the distance formula.

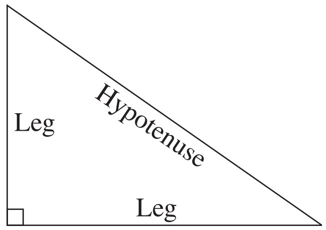
Equipment

In this laboratory activity, you will need the following equipment:

1. a 12-inch ruler
2. a protractor
3. a 12-inch piece of string

In this activity you will experimentally verify a very important formula of geometry. The formula is used by surveyors, architects, and builders to check whether or not two lines are perpendicular, or if a corner truly forms a right angle.

A right triangle is a triangle that has a right angle. In other words, one of the angles formed by the sides of the triangle measures 90°. The two sides that are perpendicular and form the right angle are called the **legs** of the right triangle. The third side, opposite the right angle, is called the **hypotenuse**.



1. Use a protractor to construct three right triangles, one with legs of length 1 inch and 5 inches, a second with legs of length 3 inches and 4 inches, and a third with legs of length 2 inches each. Label the lengths of each leg.

Triangle 1

Triangle 2

Triangle 3

Pythagorean Theorem

2. For each triangle in Problem 1, complete the following table. The first triangle has been done for you as an example. Note that the lengths of the legs of the triangles are represented by a and b. The letter c represents the length of the hypotenuse.

a. Use a ruler to measure the length of each hypotenuse, in inches. Record the lengths in column c.

b. Square each length a, b, and c and record in the table.

	a	b	c	a^2	b^2	c^2
Triangle 1	1	5	5.1	1	25	26.0
Triangle 2	3	4				
Triangle 3	2	2				

3. There does not appear to be a relationship between a, b, and c, but investigate further. What is the relationship between a^2, b^2, and c^2?

The relationship demonstrated in Problem 2 has been known since antiquity. It was known by many cultures but has been attributed to the Greek mathematician Pythagoras, who lived in the sixth century B.C.

The **Pythagorean Theorem** states that, in a right triangle, the sum of the squares of the leg lengths is equal to the square of the hypotenuse length.

Symbolically, the Pythagorean Theorem is written as

$$c^2 = a^2 + b^2 \quad \text{or} \quad c = \sqrt{a^2 + b^2},$$

where a and b are leg lengths and c is the hypotenuse length.

Note that this relationship is true for any right triangle. Also, if this relationship is true for a triangle, then the triangle is a right triangle.

Example 1 ***If a right triangle has legs a = 4 centimeters and b = 7 centimeters, calculate the length of the hypotenuse.***

$$c = \sqrt{a^2 + b^2} = \sqrt{(4\text{ cm})^2 + (7\text{ cm})^2}$$
$$= \sqrt{16\text{ cm}^2 + 49\text{ cm}^2} = \sqrt{65\text{ cm}^2} \approx 8.06\text{ cm}$$

Example 2 ***Estimate $\sqrt{10}$ by guess and check.***

Determining the square root of 10 involves finding a number whose square is 10. Note that 3 is too small, since $3^2 = 9$, and 4 is too large, since $4^2 = 16$. So, $3 < \sqrt{10} < 4$, and $\sqrt{10}$ is closer to 3. Try $3.1^2 = 9.61$ and $3.2^2 = 10.24$. Therefore, $3.1 < \sqrt{10} < 3.2$. Continuing in this manner, one can estimate that $\sqrt{10} \approx 3.16$, easily confirmed on a calculator.

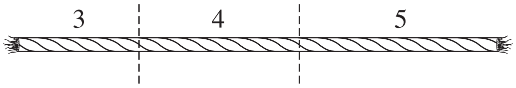
4. Estimate the square root of 53, written $\sqrt{53}$. Then use your calculator to check the answer.

5. A right triangle has legs measuring 5 centimeters and 16 centimeters. Use the Pythagorean Theorem to calculate the length of the hypotenuse to the nearest hundredth.

6. A popular triangle with builders and carpenters has dimensions 3 units by 4 units by 5 units.

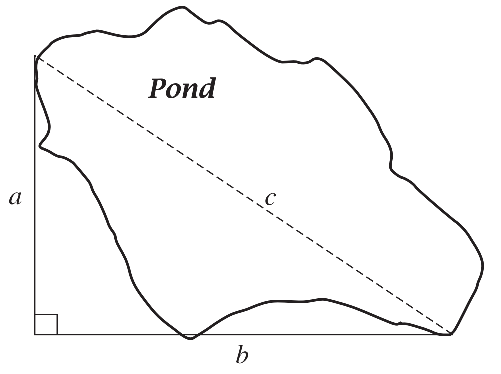
a. Use the Pythagorean Theorem to show that this triangle is a right triangle.

b. Builders use this triangle by taking a 12-unit-long rope and marking it in lengths of 3 units, 4 units, and 5 units (see graphic).



Then, by fitting this rope to a corner, they can quickly tell if the corner is a true right angle. Another special right triangle has legs of length 5 units and 12 units. Determine the perimeter of this right triangle. Explain how a carpenter can use this triangle to check for a right angle.

7. Suppose you wish to measure the distance across your pond but don't wish to get your feet wet! By being clever, and knowing the Pythagorean Theorem, you can estimate the distance by taking two measurements on dry land, as long as your two distances lie along perpendicular lines. (See the illustration.)



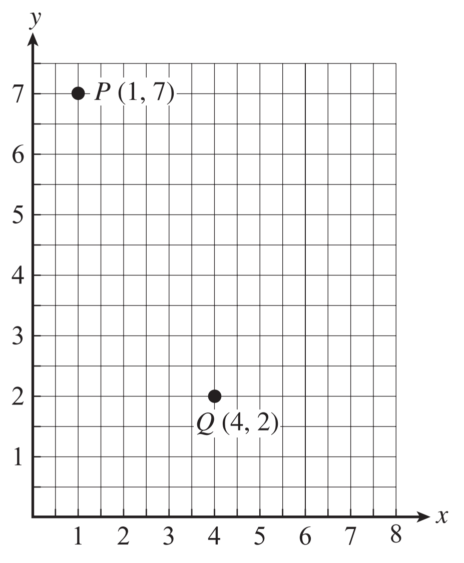
If your measurements for legs a and b are 260 feet and 310 feet, respectively, what is the distance, c, across the pond?

Distance Formula

Archaeologists need to keep precise records of locations of their findings. To accomplish this, some landmark point at the archaeological site is selected. From this key point, a grid of squares, usually made of rope, is laid out over the site to form a coordinate system. This system provides a method in which the location of artifacts, bones, and other items can be located and distances between points can be calculated.

By using the coordinates of the points in the coordinate system and the Pythagorean Theorem, you can determine the distance between any two points in a plane.

8. The following graph shows the points $P(1, 7)$ and $Q(4, 2)$.



Complete a right triangle as follows:

a. Draw a line segment with P and Q as endpoints.

b. Using P as an endpoint, draw a vertical segment downward that intersects with a horizontal segment drawn to the left from endpoint Q. Label this point C. The angle formed by the horizontal and vertical segments is a right angle.

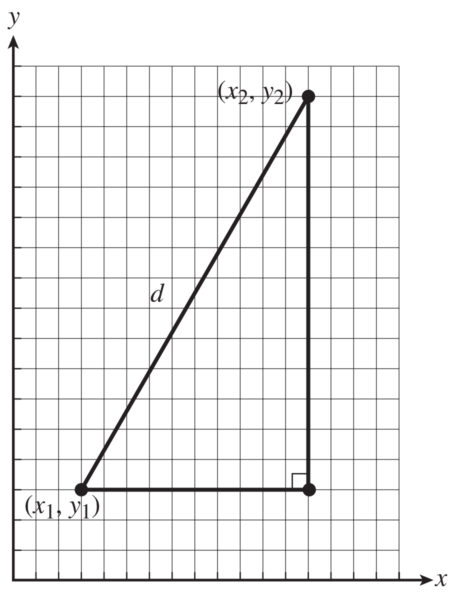
c. What are the coordinates of the point of intersection in part b?

d. Determine the length of the vertical segment PC.

e. Determine the length of the horizontal segment CQ.

f. The distance between the points $P(1, 7)$ and $Q(4, 2)$ is the length of the hypotenuse of the right triangle PCQ. Use the Pythagorean Theorem to determine this distance.

To develop a general formula for the distance, d, between two points, let (x_1, y_1) and (x_2, y_2) represent two points in the plane that do not lie on the same horizontal or vertical line. With these two points, a right triangle can be formed, as shown in the following diagram.



9. a. What are the coordinates of the vertex of the right angle?

b. Write an expression that represents the length of the horizontal side of the right triangle.

c. Write an expression that represents the length of the vertical side of the right triangle.

Applying the Pythagorean Theorem to determine the length, d, of the hypotenuse, you have

$$\begin{array}{ccccc} \text{square of the length} & = & \text{square of the length} & + & \text{square of the length} \\ \text{of the hypotenuse} & & \text{of horizontal side} & & \text{of vertical side} \\ d^2 & = & (x_2 - x_1)^2 & + & (y_2 - y_1)^2 \end{array}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance Formula

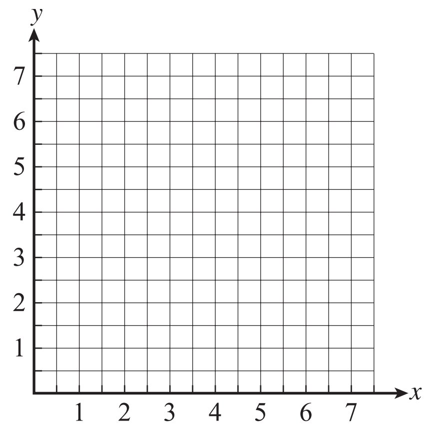
Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ represent two points in the plane. The distance, d, between P and Q is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

10. **a.** Use the distance formula to determine the distance between $P(-8, 4)$ and $Q(3, -2)$.

b. Use the distance formula to determine the distance between the 2 points in Problem 8 and then compare your result to the result from Problem 8f.

11. **a.** Plot the points $P(2, 1)$, $Q(4, 0)$, and $R(5, 7)$ on the following grid. Connect the points to form a triangle.



b. Use the distance formula to determine the length of each of the following.

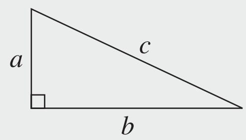
i. side PQ

ii. side PR

iii. side QR

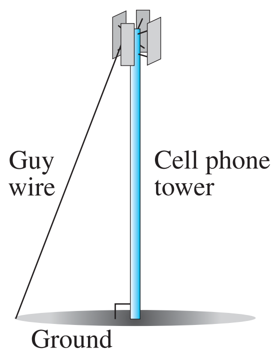
c. Use the Pythagorean Theorem to verify that the points *P*, *Q*, and *R* are vertices of a right triangle. Be sure to identify which vertex represents the right angle and which side represents the hypotenuse.

SUMMARY: ACTIVITY 7.7

Concept/Skill	Description	Example
1. Pythagorean Theorem	In a right triangle, $c^2 = a^2 + b^2$.	
2. Distance formula	Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ represent two points in the plane. The distance, *d*, between *P* and *Q* is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.	 $d = \sqrt{(5-1)^2 + (4-6)^2}$ $= \sqrt{4^2 + (-2)^2}$ $= \sqrt{16 + 4} = \sqrt{20} \approx 4.47$

EXERCISES: ACTIVITY 7.7

1. New cell phone towers are being constructed on a daily basis throughout the country. Typically, they consist of a tall, thin tower supported by several guy wires. Assume the ground is level in the following. Round answers to the nearest foot.

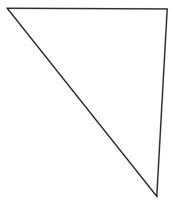


a. The guy wire is attached on the ground at a distance of 100 feet from the base of the tower. The guy wire is also attached to the phone tower 300 feet above the ground. What is the length of the guy wire?

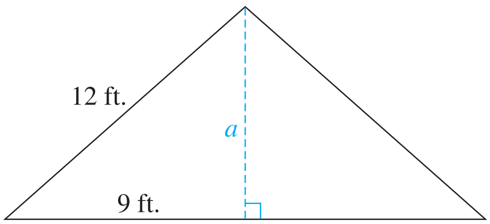
b. You move the base of the guy wire so that it is attached 120 feet from the base of the tower. Now how much wire do you need for the one guy wire?

c. Another option is to attach the base of the guy wire 100 feet from the base of the tower and to the phone tower 350 feet above the ground. How much wire do you need for this option?

2. A building inspector needs to determine if two walls in a new house are built at right angles, as the building code requires. He measures and finds the following information. Wall 1 measures 12 feet, wall 2 measures 14 feet, and the distance from the end of wall 1 to the end of wall 2 measures 18 feet. Do the walls meet at right angles? Explain.



3. Trusses used to support the roofs of many structures can be thought of as two right triangles placed side by side.

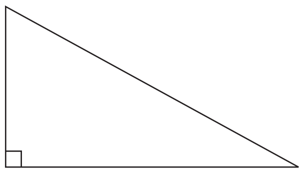


a. If the hypotenuse in one of the right triangles of a truss measures 12 feet and the horizontal leg in the same right triangle measures 9 feet, how high is the vertical leg of the truss?

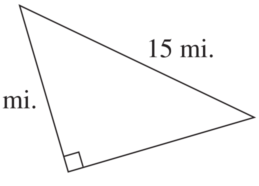
b. To make a steeper roof, you may increase the height of the vertical leg to 10 feet. Keeping the 9-foot horizontal leg, how long will the hypotenuse of the truss be now?

4. For the following right triangles, use the Pythagorean Theorem to compute the length of the third side of the triangle:

a.



b.



5. You are buying a ladder for your 30-foot-tall house. For safety, you would always like to ensure that the base of the ladder be placed at least 8 feet from the base of the house. What is the shortest ladder you can buy in order to be able to reach the top of your house?

6. Pythagorean triples are three positive integers that could be the lengths of three sides of a right triangle. For example, 3, 4, 5 is a Pythagorean triple since $5^2 = 3^2 + 4^2$.

a. Is 5, 12, 13 a Pythagorean triple? Why or why not?

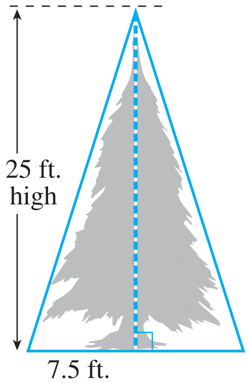
b. Is 5, 10, 15 a Pythagorean triple? Why or why not?

c. Is 1, 1, 2 a Pythagorean triple? Why or why not?

d. Name another Pythagorean triple. Explain.

7. You own a summer home on the east side of Lake George in New York's Adirondack Mountains. To drive to your favorite restaurant on the west side of the lake, you must go directly south for 7 miles and then directly west for 3 miles. If you could go directly to the restaurant in your boat, how long would the boat trip be?

8. You are decorating a large evergreen tree in your yard for the holidays. The tree stands 25 feet tall and 15 feet wide. You want to hang strings of lights from top to bottom draped on the outside of the tree. How long should each string of lights be? Explain.



9. Is it possible to have a right triangle with sides measuring 7 inches, 10 inches, and 15 inches?

10. Determine the distance between the given points.

a. $(3, -11), (-12, -3)$

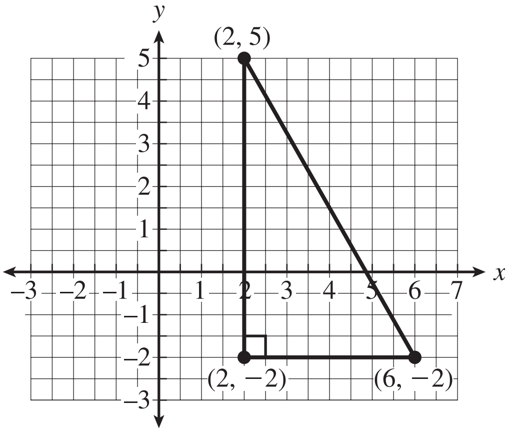
b. $(-7, 3), (2, -9)$

11. Determine the length of the hypotenuse in the following two ways:

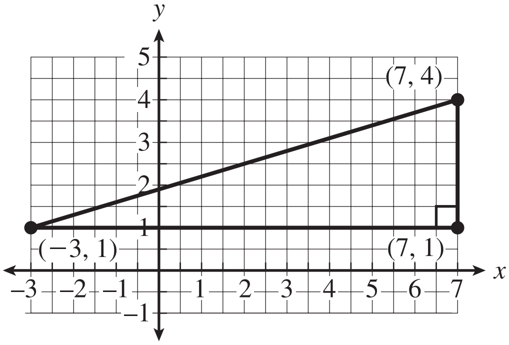
i. use the Pythagorean Theorem

ii. use the distance formula

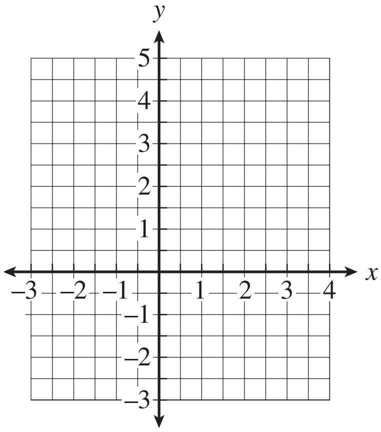
a.



b.



12. **a.** Plot the points $A(1, -3)$, $B(3, 2)$, and $C(-2, 4)$, then connect the points to form a triangle.



b. Use the distance formula to verify that those points are vertices of an isosceles triangle.

Activity 7.8

Painting Your Way through Summer

Objectives

1. Recognize geometric properties of three-dimensional figures.

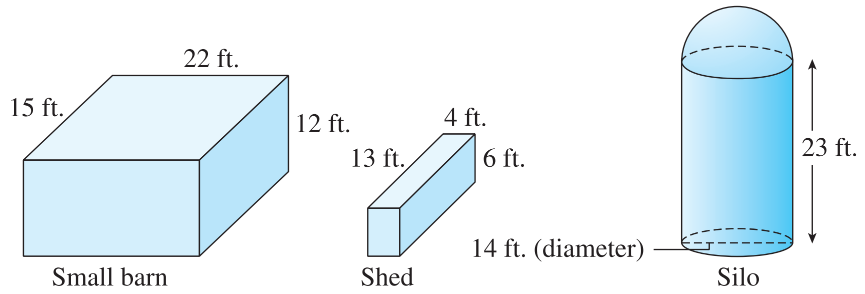
2. Write formulas for and calculate surface areas of rectangular prisms (boxes), right circular cylinders (cans), and spheres (balls).

The Geometry of Three-Dimensional Space Figures

You decide to paint houses for summer employment. To determine how much to charge, you do some experimenting to discover that you can paint approximately 100 square feet per hour. To pay college expenses, you need to make at least \$900 per week during the summer to cover the cost of supplies and to make a profit. You figure that it is reasonable to paint for approximately 40 hours per week.

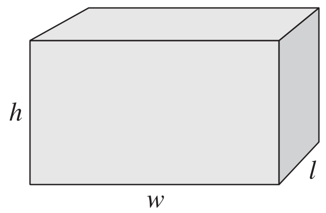
1. Use the appropriate information from above to determine your hourly fee.

Your first job is to paint the exteriors of a three-building farm complex (a small barn, a storage shed, and a silo) with the following dimensions.



In order to determine your fee to the farmer, you need to estimate how many square feet of surface must be painted. Since there are few windows in the buildings, they may be ignored.

2. The small barn and shed are in the shape of rectangular boxes. Each surface is the shape of a rectangle. Such a three-dimensional figure is formally called a **rectangular prism**.



   a. To determine the total surface area, S, of a rectangular prism, you need to add up the areas of the six rectangular surfaces. Using l for length, w for width, and h for height, express each area with a formula.

   Area of front = ________. Area of back = ________.

   Area of one side = ________. Area of other side = ________.

   Area of top = ________. Area of bottom = ________.

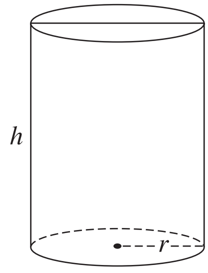
   b. Add the six areas from part a, combining like terms, to obtain the formula for the surface area, S, of a rectangular prism.

3. a. Write a formula for the surface area, S, of a rectangular prism if the bottom area is *not* included.

b. Assume that you will not paint the floors of the small barn and storage shed (but you *will* paint the special flat roofs). Compute the total surface area for the exteriors of these two buildings.

4. The silo consists of a can-shaped base, with half a ball shape for the roof. The three-dimensional can shape is formally called a **right circular cylinder**—circular because the base is a circle, right because the smooth rounded surface is perpendicular to the base.

a. To determine the total surface area of a right circular cylinder, you need to add up the areas of the circular top and bottom and the area of curved side. Using r for the radius and h for the height, express each area with a formula.

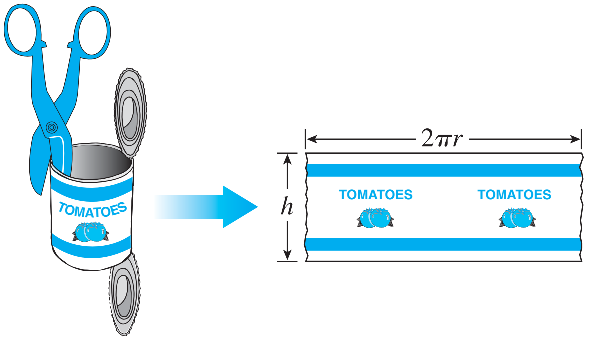


Area of circular top $=$ ____

Area of circular bottom $=$ ____

Area of side $=$ ______

(*Hint*: To determine the area of the side, think about cutting off both circular ends and cutting the side of the can perpendicular to the bottom. Then, uncoil the side of the can into a big rectangle with height h and width the circumference of the circle.)



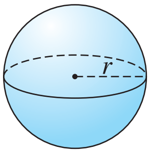
b. Add the three areas from part a to obtain the formula for the surface area, S, of a right circular cylinder.

The roof of the silo is half of a sphere with radius r. The surface area, S, for a ball, or sphere, with radius r is a little more difficult to derive. You may study the formula in future courses. It is provided here.

$$S = 4\pi r^2$$

The formula states that the surface area of a sphere is equal to the sum of the areas of four circles of radius r.

5. Carefully use the surface area formulas for a sphere and a cylinder to compute the total exterior surface area for the silo. Again, assume you will not paint the floor but will paint the hemispherical roof.

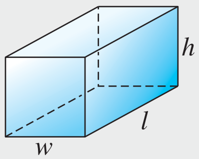
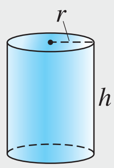
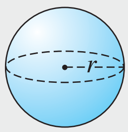


6. What is the total surface area for all three buildings that you must paint?

7. How long will it take you to paint all three buildings?

8. What will you charge for the entire job?

SUMMARY: ACTIVITY 7.8

Figure	Labeled Diagram	Surface Area Formula
Rectangular prism (box)		$S = 2wl + 2hl + 2wh$
Right circular cylinder (can)		$S = 2\pi r^2 + 2\pi rh$
Sphere (ball)		$S = 4\pi r^2$

EXERCISES: ACTIVITY 7.8

1. A basketball has a radius of approximately 4.75 inches.

a. Compute the basketball's surface area.

b. Why would someone want to know this surface area?

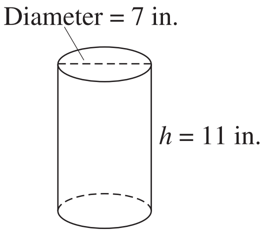
2. Hot air balloons require large amounts of both hot air and fabric material. A hot air balloon is spherical with a diameter of 25 feet.

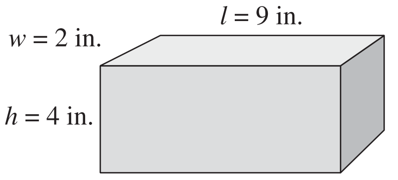
a. Does finding the surface area help you determine the amount of the hot air or the fabric material? Explain.

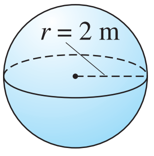
Exercise numbers appearing in color are answered in the Selected Answers appendix.

b. Compute the surface area of the balloon. What are the units?

3. Compute the surface areas of each of the following figures.

a. 

b. 

c. 

4. You need to wrap a rectangular box with dimensions 2 feet by 3.5 feet by 4.2 feet. What is the least amount of wrapping paper you must buy in order to complete the job?

5. A can of soup is 3 inches in diameter and 5 inches in height. How much paper is needed to make a label for the soup can?

6. Calculate the height of a right circular cylinder with a surface area of 300 square inches and a radius of 5 inches to the nearest hundredth.

Activity 7.9

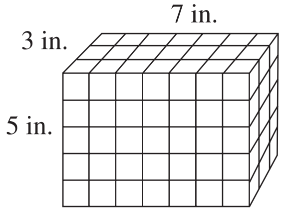
Truth in Labeling

Objectives

1. Write formulas for and calculate volumes of rectangular prisms (boxes) and right circular cylinders (cans).
2. Recognize geometric properties of three-dimensional figures.

When buying a half-gallon of ice cream or a 12-ounce can of soda, have you ever wondered if the containers actually hold the amounts advertised? In this activity, you will learn how to answer this question. The space inside a three-dimensional figure such as a rectangular prism (box) or right circular cylinder (can) is called its **volume** and is measured in cubic units (or unit cubes).

1. A half-gallon of ice cream has estimated dimensions as shown in the figure. In order to determine the number of unit cubes (1 inch by 1 inch by 1 inch) in this box, explain why it is reasonable to multiply the area of the top or bottom by the height.



Definition

The **volume** of a rectangular prism (box) is the measure, in cubic units, of the space enclosed by the sides, top, and bottom of a three-dimensional figure.

Procedure

Calculating the Volume of a Rectangular Prism (Box)

The formula for the volume, V, of a rectangular prism with length l, width w, and height h is

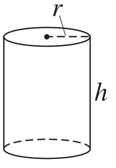
$$V = lwh.$$

2. Use the formula to calculate the volume of the half-gallon of ice cream.

3. One cubic inch contains 0.554 fluid ounces (fl. oz.) of ice cream. Estimate the amount of fluid ounces of ice cream in the carton.

4. There are 64 fluid ounces in a half-gallon. How close is your estimate to the half-gallon?

5. In a way similar to the volume of a box, it is reasonable to define the volume of a right circular cylinder (can) as the product of the area of its circular bottom and its height. If a can has height h and radius r, write a formula for its volume. Explain.



Procedure

Calculating the Volume of a Right Circular Cylinder (Can)

The formula for the volume, V, of a right circular cylinder with height h and radius r is

$$V = \pi r^2 h.$$

6. A can of soda is estimated to be $2\frac{1}{2}$ inches in diameter and $4\frac{3}{4}$ inches high. Use the volume formula to calculate the volume of a can of soda.

7. One cubic inch contains 0.554 fluid ounces (fl. oz.) of soda. Estimate the amount of soda in a can. How close to 12 ounces is your estimate?

8. Assume that you need a long, slim box that can hold up to 100 cubic inches.

a. You want the box to have a base 2 inches long and 4 inches wide. How high would the container be?

b. You need another box to hold 100 cubic inches but with base 3 inches by 4 inches. How high would it stand?

c. Design a cube (length $=$ width $=$ height) to store 100 cubic inches.

i. Let x represent the length $=$ width $=$ height. Write an equation for a volume of 100 cubic inches.

ii. To solve the equation in part i, you need to "uncube," that is, take the cube root of, both sides. You are looking for a number whose cube is 100. You note that 4 is too small, since $4^3 = 64$. Is 5 too large or too small?

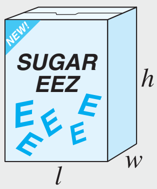
iii. Estimate the cube root of 100.

iv. Most calculators have a cube root key or menu to calculate cube roots. The cube root of 100 is denoted as $\sqrt[3]{100}$. Use your calculator to compute the cube root of 100.

d. Design a cylindrical can that holds 100 cubic inches. How tall will it be if the diameter of the base is 4 inches?

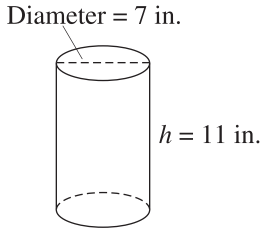
e. Design a "short" can (like a can of tuna fish) with a volume of 100 cubic inches that is 1 inch high. What size must the radius be if the can is 1 inch high?

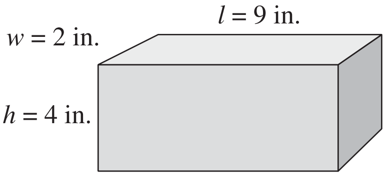
SUMMARY: ACTIVITY 7.9

Three-Dimensional Figure	Labeled Sketch	Volume Formula
Rectangular prism (box)		$V = lwh$
Right circular cylinder (can)		$V = \pi r^2 h$

EXERCISES: ACTIVITY 7.9

1. Compute the volumes in each of the following figures.

   a. 

   b. 

2. You want to buy a pickup truck and are interested in one with a large carrying capacity. One model features a rectangular prism-shaped cargo space, measuring 6 feet by 10 feet by 2 feet; another has a space with dimensions 5 feet by 11 feet by 3 feet. Which truck provides you with the most carrying capacity (that is, the most volume)? Explain.

3. A can of soup is 3 inches in diameter and 5 inches high. How much soup can fit into the can?

4. If the volume of a cube is given as 42 cubic feet, estimate its dimensions.

5. If the volume of a right circular cylinder is given as 50 cubic centimeters and if its radius measures 2 centimeters, calculate its height.

What Have I Learned?

1. What are the differences and similarities between the area and perimeter of a plane figure? Explain.

2. If the perimeter of a plane figure is measured in feet, then what are the usual units of the area of that figure? Explain.

3. If the area of a circular figure is measured in square centimeters, then what are the usual units of the diameter of that circle? Explain.

4. A racetrack can be described as the perimeter of a long rectangle with semicircles on the ends.

   a. At a racetrack, who would be interested in its perimeter? Why?

   b. At a racetrack, who would be interested in knowing the area inside the track? Why?

5. If different figures have the same perimeter, must their areas be the same? Explain.

Exercise numbers appearing in color are answered in the Selected Answers appendix.

6. If different figures have the same area, must their perimeters be the same? Explain.

7. Construct a nonright triangle, measure the sides, and show that the square of the length of the longest side is not equal to the sum of the squares of the lengths of the shorter sides.

8. True or false: If you double the diameter of a circle, then the area of the circle will also double. Give a reason for your answer.

9. The number π has an extraordinary place in the history of mathematics. Many books and articles have been written about this curious number. Research π and report on your findings.

10. What are the differences and similarities between the *surface area* and *volume* of a figure? Explain.

11. If the surface area of a figure is measured in square feet, then what are the units of the volume of that figure? Explain.

12. If different figures have the same surface areas, must their volumes be the same? Explain using an example.

13. If different figures have the same volume, must their surface areas be the same? Explain, using an example.

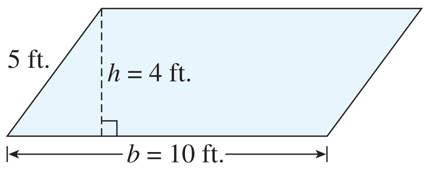
14. If you double the height of a can and keep all other dimensions the same, does the volume double? Explain, using an example.

15. If you double the diameter of a can and keep all other dimensions the same, does the volume double? Explain, using an example.

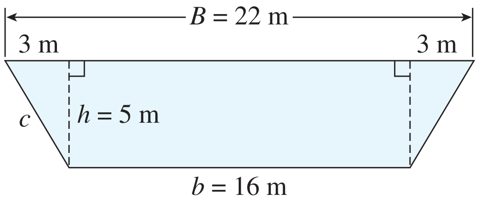
How Can I Practice?

1. Calculate the area and the perimeter for each of the following figures.

a.

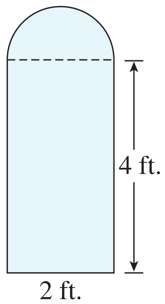


b.

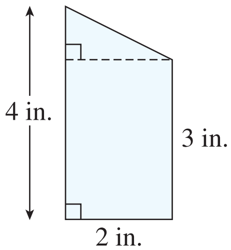


2. Calculate the area and the perimeter for each of the following figures.

a. Rectangle topped by a semicircle.

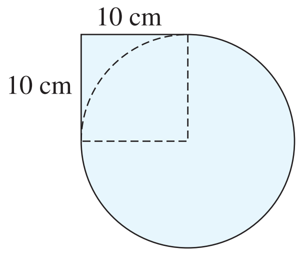


b. Rectangle topped by a right triangle.



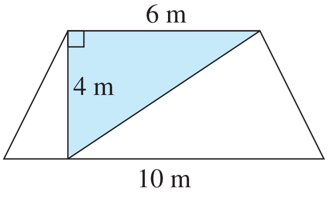
Exercise numbers appearing in color are answered in the Selected Answers appendix.

c. Three-quarters of a circle with a square "corner."

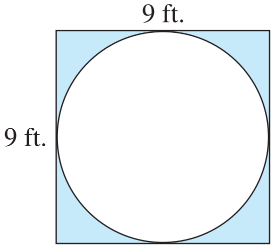


3. Determine the area of the shaded region in each of the following:

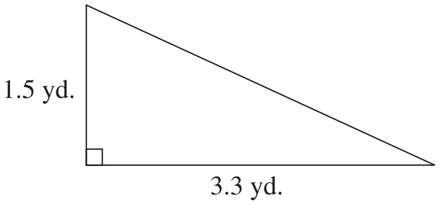
a.



b.



4. Consider the right triangle with dimensions as shown. Round all answers to the nearest tenth.

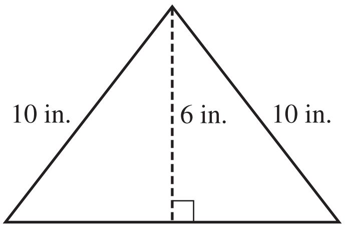


a. Determine the length of the third side of the triangle.

b. Determine the perimeter of the triangle.

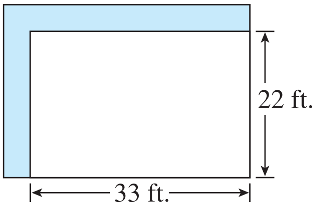
c. Determine the area of the triangle.

5. a. Determine the length of the base of the isosceles triangle.



b. Calculate the perimeter and area of the triangle.

6. You intend to put in a 4-foot-wide concrete walkway along two sides of your house, as shown.



a. Determine the area covered by the walkway only.

b. If you decide to place a narrow flower bed along the outside of the walkway, how many feet of flowers should you plan for?

7. Triangle *A* has sides measuring 3 feet, 5 feet, and 7 feet; triangle *B* has sides measuring 4 feet, 4 feet, and 5 feet; triangle *C* has sides measuring 5 inches, 12 inches, and 13 inches. Which of the three triangles is a right triangle? Explain.

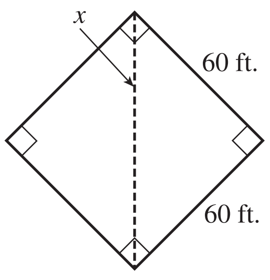
8. If you measure the circumference of a circle to be 20 inches, estimate the length of its radius.

9. You want to know how much space is available between a basketball and the rim of the basket. The distance you want to determine is the difference between the diameter of the rim and the diameter of the ball. Note that if you cannot measure the diameter of an object directly, you can measure its circumference and use the circumference formula to determine the corresponding diameter. Try it!

10. Determine the distance between the points $(-2, -3)$ and (4, 2).

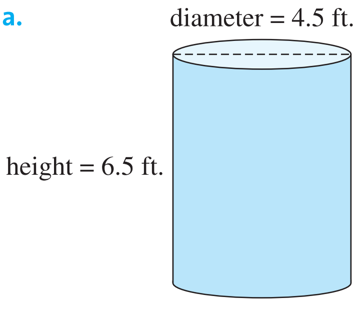
11. Verify that the points, $A(-4, 3)$, $B(3, 5)$, and $C(5, -2)$ are vertices of a right triangle.

12. A softball diamond is in the shape of a square. The bases are 60 feet apart. What is the distance a catcher would have to throw a ball from home plate to second base?

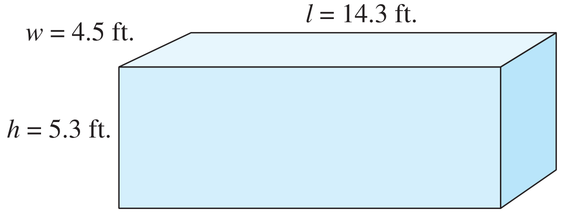


13. Compute the volume and surface area for each of the following geometric figures. Round your answers to the nearest hundredth.

a.



b.



14. Sketch each of the following figures with the desired property or properties. Use the appropriate geometric formula to determine the missing dimension (radius, length, or height). Round to the nearest hundredth.

a. a sphere with a surface area of 53 square centimeters

b. a can with a surface area of 20 square feet and radius of 1 foot

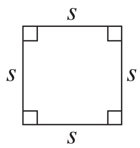
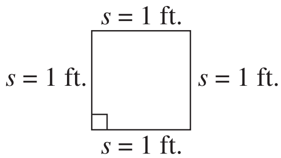
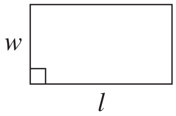
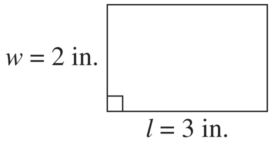
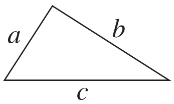
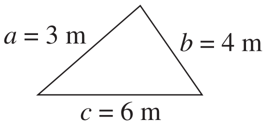
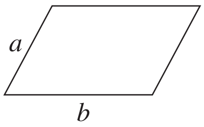
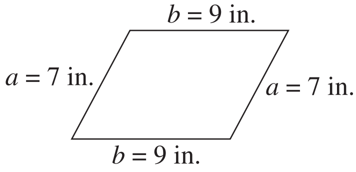
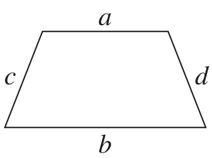
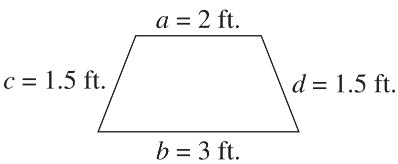
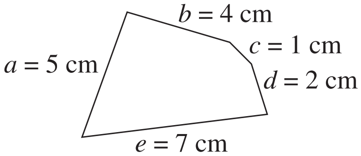
c. a can with a volume of 20 cubic feet and radius of 1 foot

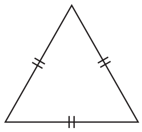
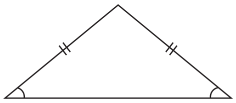
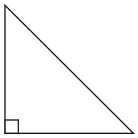
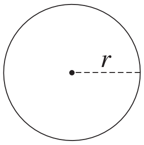
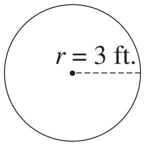
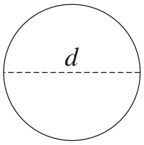
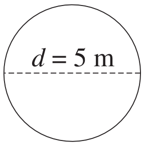
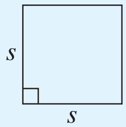
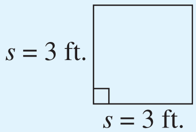
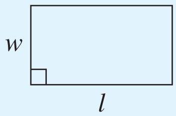
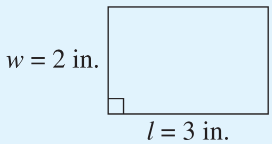
d. a box with a surface area of 40 square feet with width 2 feet and length 4 feet

e. a cubic box with a volume of 27 cubic feet

Chapter 7 Summary

The bracketed numbers following each concept indicate the activity in which the concept is discussed.

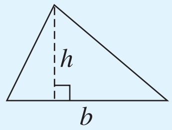
CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Perimeter formulas [7.1]	Perimeter measures the length around the edge of a plane figure, in units of distance.	
Square [7.1]	$P = 4s$ 	$P = 4 \cdot 1 = 4$ ft. 
Rectangle [7.1]	$P = 2l + 2w$ 	$P = 2 \cdot 2 + 2 \cdot 3 = 10$ in. 
Triangle [7.1]	$P = a + b + c$ 	$P = 3 + 4 + 6 = 13$ m 
Parallelogram [7.1]	$P = 2a + 2b$ 	$P = 2 \cdot 7 + 2 \cdot 9 = 32$ in. 
Trapezoid [7.1]	$P = a + b + c + d$ 	$P = 2 + 3 + 1.5 + 1.5 = 8$ ft. 
Polygon [7.1]	$P =$ sum of the lengths of all the sides	$P = 1 + 4 + 5 + 7 + 2 = 19$ cm 

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Equilateral triangle [7.1]	All three sides have the same length. The angles are also of equal measure.	
Isosceles triangle [7.1]	Two sides have the same length. (The two angles opposite the equal sides are also the same size.)	
Right triangle [7.1]	One angle measures 90°.	
Circle [7.2]	$C = 2\pi r$ For circles, perimeter is usually called *circumference.* 	$C = 2\pi \cdot 3 = 6\pi \approx 18.85$ ft. 
	$C = \pi d$ 	$C = \pi \cdot 5 \approx 15.71$ m 
Area formulas [7.4]	Area is the measure in square units of the space inside a plane figure.	
Square [7.4]	$A = s \cdot s$ or $A = s^2$ 	$A = 3 \cdot 3 = 9$ sq. ft. 
Rectangle [7.4]	$A = lw$ 	$A = 2 \cdot 3 = 6$ sq. in. 

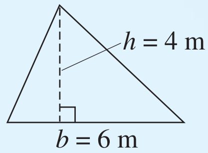
CONCEPT/SKILL	DESCRIPTION	EXAMPLE

Triangle [7.4]

$A = \frac{1}{2}bh$

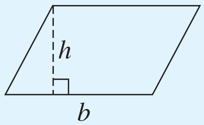


$A = \frac{1}{2} \cdot 4 \cdot 6 = 12$ sq. m

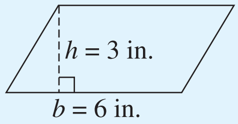


Parallelogram [7.4]

$A = bh$

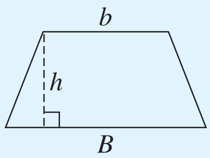


$A = 6 \cdot 3 = 18$ sq. in.

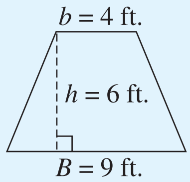


Trapezoid [7.4]

$A = \frac{1}{2}h(b + B)$



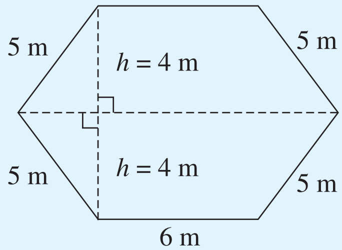
$A = \frac{1}{2} \cdot 6 \cdot (4 + 9) = 39$ sq. ft.



Polygon [7.4]

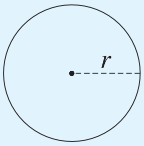
A = sum of the areas of all the parts of the plane figure.

$A = 2 \cdot \frac{1}{2} \cdot 4 \cdot (6 + 12) = 72$ sq. m

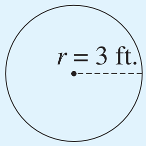


Circle [7.5]

$A = \pi r^2$

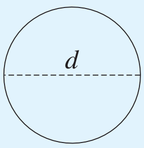


$A = \pi \cdot 3^2 = 9\pi \approx 28.27$ sq. ft.

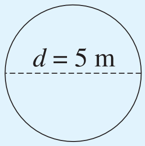


Circle [7.5]

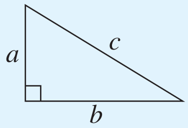
$A = \pi\left(\frac{d}{2}\right)^2$ or $A = \frac{\pi d^2}{4}$

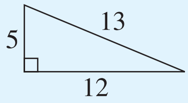


$A = \pi\left(\frac{5}{2}\right)^2 = \frac{\pi \cdot 5^2}{4} \approx 19.63$ sq. m



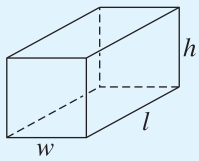
CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Pythagorean Theorem [7.7]	$c^2 = a^2 + b^2$ This important formula for right triangles is useful for indirect measurement.	$13^2 = 5^2 + 12^2$, since $169 = 169$

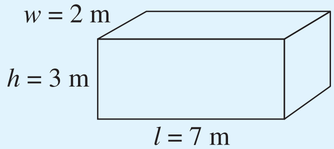




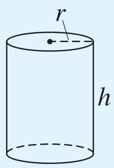
CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Distance formula [7.7]	Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ represent two points in the plane. The distance, d, between P and Q is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.	$d = \sqrt{(5-1)^2 + (4-6)^2}$ $= \sqrt{4^2 + (-2)^2}$ $= \sqrt{16 + 4} = \sqrt{20} \approx 4.47$

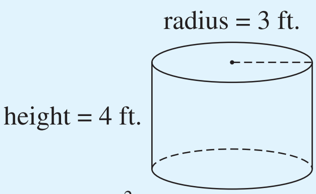
CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Surface area formulas [7.8]	Surface area is the measure in square units of the exterior of the three-dimensional figure.	
Rectangular prism (box) [7.8]	$S = 2lw + 2lh + 2wh$	$S = 2 \cdot 7 \cdot 2 + 2 \cdot 7 \cdot 3 + 2 \cdot 2 \cdot 3$ $S = 28 + 42 + 12 = 82$ sq. m



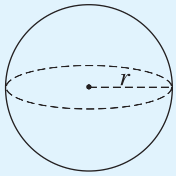


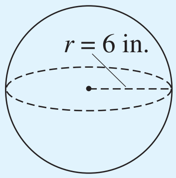
CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Right circular cylinder (can) [7.8]	$S = 2\pi r^2 + 2\pi rh$	$S = 2\pi \cdot 3^2 + 2\pi \cdot 3 \cdot 4 = 42\pi$ $S \approx 131.95$ sq. ft.

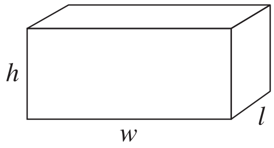
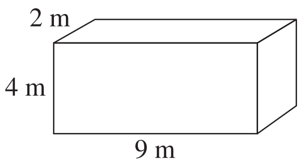
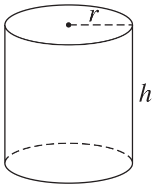
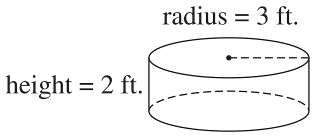




CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Sphere (ball) [7.8]	$S = 4\pi r^2$	$S = 4\pi \cdot 6^2 = 144\pi \approx 452.39$ sq. in.





CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Volume formulas [7.9]	Volume is the measure in cubic units of the space inside a three-dimensional figure.	
Rectangular prism (box) [7.9]	$V = lwh$ 	$V = 2 \cdot 4 \cdot 9 = 72$ cu. m 
Right circular cylinder (can) [7.9]	$V = \pi r^2 h$ 	 $V = \pi \cdot 3^2 \cdot 2 = 18\pi \approx 56.55$ cu. ft.
Taking cube roots [7.9]	$\sqrt[3]{n} = m$ providing $m^3 = n$. Calculators can be very useful in evaluating cube roots. Use $\sqrt[3]{n}$.	$\sqrt[3]{64} = 4$ $\sqrt[3]{100} \approx 4.64$

Chapter 7 Gateway Review

1. Consider the following two-dimensional figures.

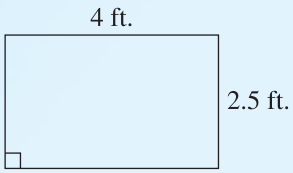


Figure A

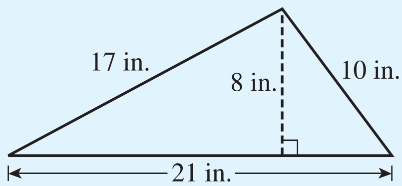


Figure B

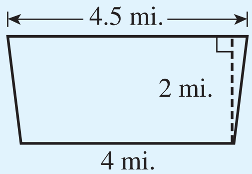


Figure C

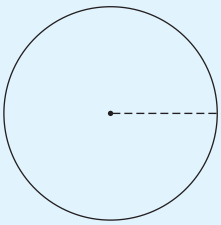
a. The area of Figure A = ______________

b. The perimeter of Figure B = __________

c. The area of Figure B = ______________________________

d. The area of Figure C = __________________________________

2. If a circle has area 23 square inches, what is the length of its radius?



Answers to all Gateway exercises are included in the Selected Answers appendix.

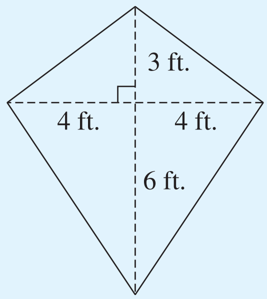
3. You buy a candy dish with a 2-inch by 2-inch square center surrounded on each side by attached semicircles.

a. Sketch a diagram of the candy dish described above and label its dimensions, including the radius of the semicircles.

b. Determine its perimeter and area.

c. If you place the dish on a 1-foot by 1-foot square table, how much space is left on the table for other items? Explain.

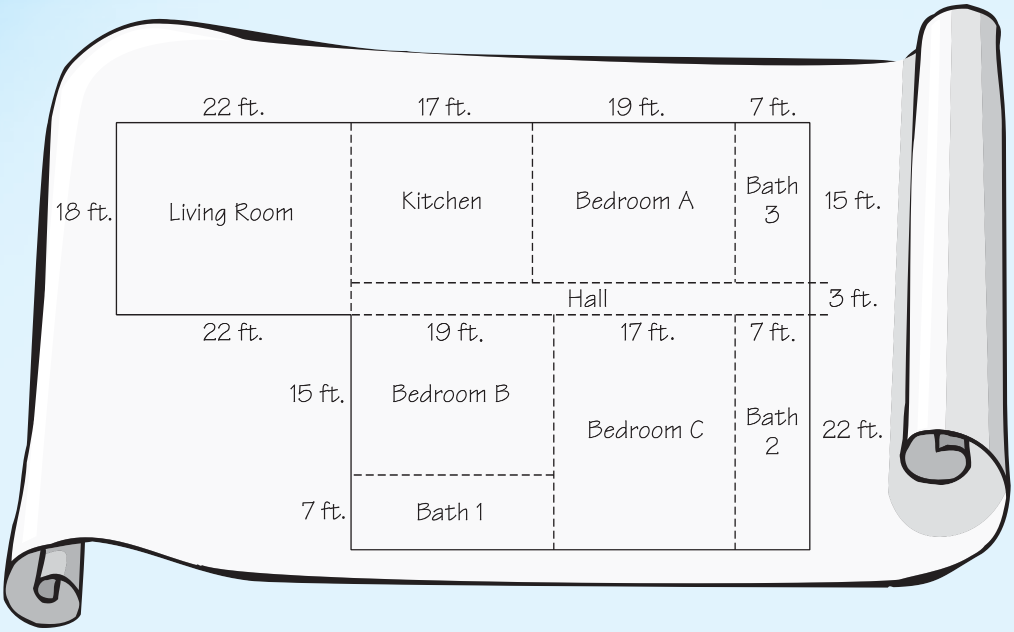
4. You buy a kite in the shape shown. When you open the box, you discover a tear in the kite's fabric. You decide to buy new fabric to place over the entire frame.



a. How much fabric do you need to buy?

b. You also decide to buy gold ribbon to line the perimeter of the kite. How much ribbon must you buy?

5. You are in the process of building a new home and the architect sends you the following floor plan for your approval.



a. What is the perimeter of this floor plan?

b. What is the floor space, in square feet, of the floor plan?

c. Which bedroom has the largest area?

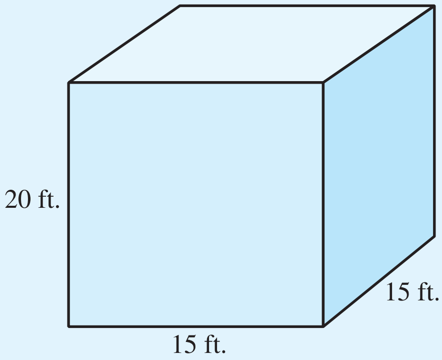
d. If you decide to double the area of the living room, what change in dimensions should you mark on the floor plan that you send back to the architect?

6. Your home is located in the center of a 300-foot by 200-foot rectangular plot of land. You are interested in measuring the diagonal length of that plot. Use the Pythagorean Theorem to make that indirect measurement.

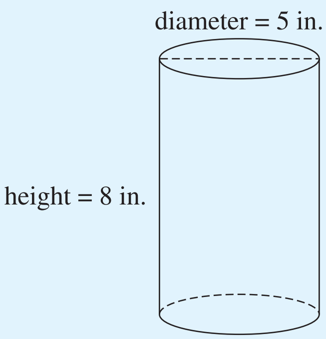
7. Use the distance formula to show that the following points are the vertices of a right triangle: $P(-2, 5)$, $Q(12, 3)$, and $R(10, -11)$.

8. Calculate the surface area and volumes of the following three-dimensional figures.

a.



b.



9. Biologists studying a lake determine that the lake's shape is approximately circular with a diameter of 1 mile and an average depth of 187 feet. Estimate how much water is in the lake. Note that 1 mile $=$ 5280 feet.

10. The Alaska pipeline is a cylindrical pipe with diameter 48 inches that carries oil for 800 miles through Alaska. It is an engineering marvel, with many safety and business concerns.

a. What is the total capacity of the pipeline for oil at any given time, assuming that it could be filled to capacity? Explain.

b. Estimate the amount of material needed to construct the pipeline. Explain.

11. Determine the distance between the points $(6, -3)$ and $(-3, -1)$.

12. Determine whether the points $A(5, 2)$, $B(1, -1)$, and $C(-4, 5)$ are vertices of a right triangle.

Chapter 8

Problem Solving with Mathematical Models

Throughout this book you have solved problems from everyday life—science, business, sports, social issues, space design, and so on—that require mathematics. In the past, a search for solutions to everyday problems resulted in the development of some of the mathematics we use today. For example, the early Egyptians developed concepts of geometry to recalculate the boundaries of the lands owned by farmers due to the seasonal flooding of the Nile River.

When mathematics is applied to solve a problem or to describe a situation, it is known as the **mathematical model** for the problem or situation. In this chapter you will use mathematical models to develop your problem-solving skills.

Activity 8.1

A Model of Fitness

Objectives

1. Describe a mathematical situation as a set of verbal statements.
2. Translate verbal rules into symbolic equations.
3. Solve problems that involve equations of the form $y = ax + b$.
4. Solve equations of the form $y = ax + b$ for the input x.
5. Evaluate the expression $ax + b$ in an equation of the form $y = ax + b$ to obtain the output y.

You have decided to get serious about your health. Along with a gym membership, you sign up for a personal trainer to help you become … a model of fitness! Are you going to be able to afford this? Your gym membership is \$35 per month. The personal trainer costs \$23.50 per half-hour session.

1. a. The total monthly cost for the gym depends upon the number of personal trainer sessions. Identify the input variable and the output variable.

   b. Note that the monthly cost is expressed by the sum of two parts, a fixed cost of \$35 and a cost of \$23.50 per personal trainer session.

   Write a verbal rule to determine the output (monthly cost) in terms of the input (number of sessions).

   c. Translate the verbal rule obtained in part b into an equation. Use n for the input variable and c for the output variable.

2. a. Use the equation obtained in Problem 1c to determine the monthly cost if you have 12 personal trainer sessions each month.

   b. Calculate the monthly cost if you have 4 personal trainer sessions each month.

In Problem 1, the equation, $c = 23.50n + 35$ was developed to represent your costs at the gym. In Problem 2, you determined the gym fees by evaluating the expression $23.50n + 35$ for a known number of sessions. The following example shows an algebraic approach to determine the number of sessions that can be scheduled for a known monthly cost.

Example 1 ***Suppose the monthly budget for your get-serious fitness plan is $200. How many sessions can you afford with the personal trainer for that budgeted amount?***

SOLUTION

Step 1. Recognize that the budgeted $200 is the output value and the unknown is the corresponding input value (number of sessions with the personal trainer).

Step 2. Replace c with 200 in the equation $c = 23.50n + 35$ to obtain the equation

$$200 = 23.50n + 35.$$

Step 3. Solve the equation to determine the input value.

a.
$$\begin{array}{rcl} 200 &=& 23.50n + 35 \\ -35 &=& \quad\quad - 35 \\ \hline 165 &=& 23.50n \end{array}$$

Subtract 35 from each side of the equation to obtain $23.50n$ as a single term on the right side of the equation.

b.
$$\frac{165}{23.50} = \frac{23.50}{23.50}n$$

Divide each term in the equation by 23.50.

$$7.02 \approx n$$

Seven personal trainer sessions are possible.

Step 4. Check the solution:

$$c = 23.50(7) + 35$$
$$c = 164.50 + 35$$
$$c = 199.50$$

Step 5. Interpret the solution of the equation by stating: I can have 7 sessions per month on a budget of $200.

3. How many personal trainer sessions can you have on a fitness budget of $100 per month?

4. One month you have an injury and you cannot train, but you still go to the gym for therapeutic massage. Massage is $30 per half-hour session.

a. Write an equation to represent the cost of the gym that month, where x represents the number of massage therapy visits.

b. For your $200 budget, solve the equation in part a to determine the number of therapeutic massage sessions you can afford.

5. a. The gym billed you \$125 for the month you were injured, but you think they made a mistake because you went for two massages. Use the equation from Problem 4a to determine how many therapeutic massages you were billed for.

   b. You are back in training. The gym billed you \$152.50. Use the equation from Problem 1c to determine how many personal trainer sessions your bill represents.

6. In each of the following, an equation is given along with a specific output value. Replace the output variable y in the given equation with the specified value and solve the resulting equation for the unknown input x. Check each answer.

   a. $y = 0.75x - 21$ and $y = -9$

   b. $y = -2x + 15$ and $y = \frac{1}{3}$

Verbal rules and equations can also be used to make lists of paired input/output values. For example, since you have to pay sales tax on most nonfood items, you could use an equation to determine the final cost after taxes. Or, for quick reference, you can make a table listing the most common costs before and after taxes.

7. You are going shopping for some running shoes and maybe even a treadmill to go with them. You know that in Monroe County, New York, you will have to pay 8% sales tax for these items. You will determine the final cost of your fitness gear by multiplying the price of the item by the growth factor, 108%, or 1.08, corresponding to 8% sales tax.

a. Use y to represent the final cost of a purchase and x to represent the cost of the items before tax. Write an equation to calculate the final cost of a purchase.

b. Use the equation from part a to complete the following table.

COST BEFORE TAX, x ($)	FINAL COST AFTER TAX, y ($)
100	
	260
375	
	621

c. For your birthday, you received a $200 Dick's Sporting Goods gift card to buy some new workout clothes. How much can you spend before tax, if you want to stay within the $200 at checkout?

SUMMARY: ACTIVITY 8.1

1. To determine the output y for a specified input x in an equation of the form $y = ax + b$:

Step 1. Replace the input variable x in the expression $ax + b$ with the specified numerical value.

Step 2. Evaluate the numerical expression to obtain the numerical value for y.

Example: Solve $y = 2x + 1$ for y, given $x = 3$.

$$y = 2 \cdot 3 + 1$$
$$y = 7$$

2. To solve an equation of the form $y = ax + b$ for the unknown value of the input x, given a specified numerical value of the output y:

Use inverse operations to isolate the unknown x. Apply the inverse operations in the following order.

Step 1. Add the opposite of b to each side of the equation.

Step 2. Divide each term in the resulting equation by a to isolate x and then read the value for x.

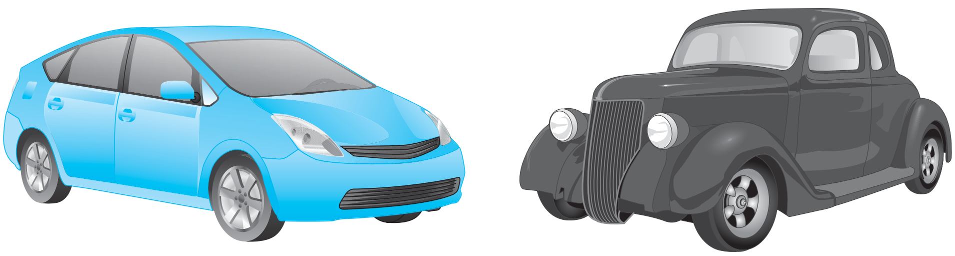
Example: Solve $y = 2x - 1$ for x, given $y = 5$.

Solution: Solve $5 = 2x - 1$ for x.

$$\begin{aligned} 5 &= 2x - 1 \\ +1 &\qquad\; + 1 \\ \hline 6 &= 2x \\ \frac{6}{2} &= \frac{\cancel{2}x}{\cancel{2}} \\ x &= 3 \end{aligned}$$

EXERCISES: ACTIVITY 8.1

1. In the summer of 2009, during a period of economic woes for the United States, President Barack Obama's "Cash for Clunkers" program was wildly popular. The program was intended to aid the auto industry, thereby boosting the economy. It was also a "green" initiative, because consumers replaced their cars with cars having better fuel economy. Your 1995 Jeep Grand Cherokee 4WD had a combined city/highway fuel economy of 15 miles per gallon. In return for it, you would receive a \$4500 credit toward the purchase of a new car provided that you chose one with fuel economy at least 10 miles per gallon more than your Jeep. This incentive, in addition to the 0% interest loans that were being offered by the troubled auto industry, was too much for you to resist. Your car payment is \$420 per month.



a. Write an equation to determine the total amount, p, paid on the car after n months, including the \$4500 credit.

b. Use the equation you wrote in part a to complete the following table.

Number of Months, n	1	2	3	4	5	6
Total Amount Paid, p (\$)						

c. Determine the amount, p, paid on the car after 9 months.

d. The car you chose was a 2009 Ford Escape Hybrid, with combined fuel economy of 30 miles per gallon, and the price of your car after negotiation, including taxes and all fees, was \$29,700. How many years will it take you to pay off your loan?

2. The value of many things we own depreciates (decreases) over time. When an asset's value decreases by a fixed amount each year, the depreciation is called straight-line depreciation. Suppose your Ford Escape from Exercise 1 depreciates \$1950 per year.

a. Let v represent the value of your Ford Escape after t years. Write a symbolic rule that expresses v in terms of t.

b. What is the value of the car after 4 years?

c. How long will it take for the value of the car to decrease below \$5000?

3. Use an algebraic approach to solve each of the following equations for x. Check your answers by hand or with a calculator.

a. $10 = 2x + 12$

b. $-27 = -5x - 7$

c. $-\frac{1}{6} = 1 - \frac{1}{3}x$

d. $0.24 - 2x = 0.38$

e. $0.5x - 0.15 = 0.15$

f. $-4x + \frac{1}{8} = \frac{1}{8}$

g. $12 + \frac{1}{5}x = 9$

h. $\frac{2}{3}x - 12 = 0$

i. $0.25x - 14.5 = 10$

j. $5 = 2.5x - 20$

4. Your Sprint Basic cell phone rate plan costs \$29.99 a month plus 45 cents per minute for every minute over 200 minutes during the month.

   a. Write a verbal rule to determine the total monthly cost for your cell phone, assuming you go over 200 minutes.

   b. Translate the verbal rule in part a into an equation using c to represent the total monthly cost and n to represent the overage minutes for the month.

   c. Determine the monthly cost for 250 overage minutes.

   d. If your total bill for the month was \$61.49, how many overage minutes did you have?

5. Archaeologists and forensic scientists use the length of human bones to estimate the height of individuals. A person's height, h, in centimeters can be determined from the length of his/her femur, f (the bone from the knee to the hip socket), in centimeters using the following formulas:

   Male: $h = 69.089 + 2.238f$ Female: $h = 61.412 + 2.317f$

   a. A partial skeleton of a male is found. The femur measures 50 centimeters. How tall was the man?

   b. What is the length of the femur for a female who is 150 centimeters tall?

6. The recommended weight for an adult male is given by the formula $w = \frac{11}{2}h - 220$, where w represents the recommended weight in pounds and h represents the height of the person in inches. Determine the height of an adult male whose recommended weight is 165 pounds.

7. Complete the following tables using algebraic methods.

a. $y = 2x - 10$

x	y
4	
	14

b. $y = 20 + 0.5x$

x	y
3.5	
	−10

c. $y = -3x + 15$

x	y
$\frac{2}{3}$	
	−3

d. $y = 12 - \frac{3}{4}x$

x	y
−8	
	−6

Activity 8.2

Comparing Energy Costs

Objectives

1. Write symbolic equations from information organized in a table.
2. Produce tables and graphs to compare outputs from two different mathematical models.
3. Solve equations of the form $ax + b = cx + d$.

You obtained the following information regarding the installation and operating costs for two types of heating systems: solar and electric. You will use the information to compare the costs for each heating system over a period of years. The following questions will guide you in making the comparisons.

Some Like It Hot

TYPE OF SYSTEM	OPERATING COST PER YEAR	INSTALLATION COST
Solar	$200	$19,000
Electric	$1400	$7000

1. a. Use the information in the table to write an equation to represent the total cost of using solar heat in terms of the number of years it is in use. Let x represent the number of years of use and let S represent the total cost.

   b. Write an equation to represent the total cost of using electric heat in terms of the number of years it is in use. Let x represent the number of years of use and let E represent the total cost.

You can use the equations you wrote in Problem 1 to compare costs for the two systems in several ways. One way is to produce a table of costs for specific periods of use.

2. a. Use the equations from Problem 1 to complete the following table.

Number of Years in Use, x	0	5	10	15
Total Cost for Solar Heat, S ($)				
Total Cost for Electric Heat, E ($)				

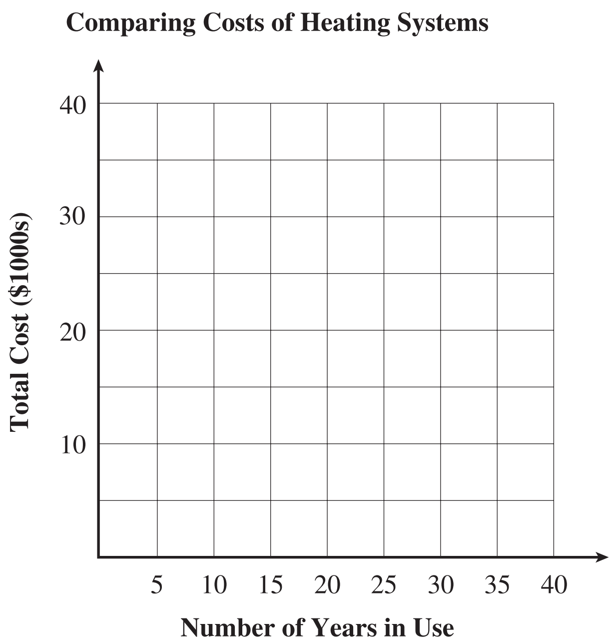
   b. Use the information in the table to compare the total costs of each system after each 5-year period.

   c. From comparing the total costs in the table, which system do you think is the better one for the house you are building?

You can also do comparisons by graphing the information from the table.

3. a. On the following grid, plot the data points for the solar heating system that you determined in Problem 2. Then connect the points by drawing a line through them.

b. On the same grid, also plot the data points for the electric heating system and connect the points by drawing a line through them.



c. From the graph, when are the total costs for the heating systems the same? How much is the cost?

d. Which heating system costs less at 15 years? At 20? Explain.

e. From comparing the total costs on the graph, which system do you think is the better one for the house you are building?

In Problems 2 and 3 you observed that the table and the graph led to the same result: that the costs are the same at 10 years of use and that the cost of the solar system was less after 10 years. The point at which both systems cost the same is useful in making decisions about which heating method is better in terms of cost. When used to compare costs, this point is often referred to as the **break-even point**. You can calculate the break-even point using algebra, as the following example shows.

Example 1 ***The comparison model for the two heating systems in the preceding problems includes two equations.***

$$S = 200x + 19{,}000$$
$$E = 1400x + 7000$$

The break-even point occurs when the outputs from the solar and electric heating systems are the same at the same time (for the same input value). This means $S = E$, or equivalently,

$$200x + 19{,}000 = 1400x + 7000.$$

This single equation can be solved for x to determine the number of years of use when the costs will be the same for both systems.

SOLUTION

First, add and/or subtract terms appropriately so that all terms involving the variable are on one side of the equals sign and all other terms are on the other side.

$$\begin{array}{rcl}
200x + 19{,}000 &=& 1400x + 7000 \\
-\,200x \qquad\quad &=& -\,200x \\ \hline
19{,}000 &=& 1200x + 7000 \\
-\;\; 7{,}000 &=& \qquad\; -\,7000 \\ \hline
12{,}000 &=& 1200x \\
x &=& 10 \text{ yr.}
\end{array}$$

Subtract 200*x* from both sides and combine like terms.

Subtract 7000 from each side.

Divide each term by 1200.

In checking the solution to the equation, you will also determine the total cost at 10 years.

$$200(10) + 19{,}000 = 21{,}000$$
$$1400(10) + 7000 = 21{,}000$$

The check shows that 10 is the correct solution and that the total cost after 10 years is $21,000. Therefore, the break-even point occurs at 10 years.

4. You are interested in purchasing a new car. You have narrowed the choice to a Honda Accord LX (4 cylinder) and a Volkswagen Passat GLS (4 cylinder). You are concerned about the depreciation of the cars' values over time and want to make some comparisons. You search the Internet and obtain the following information:

Driven Down. . .

MODEL OF CAR	MSRP (Manufacturer's Suggested Retail Price)	STRAIGHT LINE DEPRECIATION EACH YEAR
Accord LX	$21,060	$2240
Passat GLS	$27,000	$3230

a. Let A represent the value, in dollars, of the Accord LX after x years of ownership. Use the information in the table to write an equation to determine A in terms of x.

b. Write an equation to determine the value, P, in dollars, of the Passat GLS after x years of ownership.

c. Use the equations from parts a and b to write a single equation to determine when the value of the Accord LX will equal the value of the Passat GLS.

d. Solve the equation in part c to determine the year in which the cars will have the same value.

e. Complete the following table to compare the values of the cars over several years.

NUMBER OF YEARS YOU OWN CAR	VALUE OF ACCORD LX ($)	VALUE OF PASSAT GLS ($)
1		
4		
8		

5. Solve each of the following equations for x. Check your answers by hand or with a calculator.

a. $2x + 9 = 5x - 12$

b. $21 - x = -3 - 5x$

c. $2x - 6 = -8$

d. $2x - 8 + 6 = 4x - 7$

SUMMARY: ACTIVITY 8.2

General Strategy for Solving Equations for an Unknown, *x*

1. Rewrite the equation to obtain a single term containing x on one side of the equation. Do this by adding and/or subtracting terms to move the x terms to one side of the equation and all the other terms to the other side. Then combine like terms that appear on the same side of the equation.
2. Solve for the unknown x by dividing each side of the equation by the coefficient of x.
3. Check the result in the original equation to be sure that the value of the unknown produces a true statement.

EXERCISES: ACTIVITY 8.2

1. Finals are over and you are moving back home for the summer. You need to rent a truck to move your possessions from the college residence hall. You contact two local rental companies and get the following information for the 1-day cost of renting a truck.

   Company 1: \$39.95 per day plus \$0.19 per mile

   Company 2: \$19.95 per day plus \$0.49 per mile

   Let x represent the number of miles driven in one day.

   a. Write an equation that represents the total cost in dollars of renting a truck for 1 day from Company 1.

   b. Write an equation that represents the total cost in dollars of renting a truck for 1 day from Company 2.

   c. Using the equations in parts a and b, write a single equation to determine the mileage for which the cost would be the same from both companies.

   d. Solve the equation in part c and interpret the result.

Exercise numbers appearing in color are answered in the Selected Answers appendix.

e. You actually live 90 miles from the campus. Which rental company would be the better deal?

2. Two companies sell software products. In 2010, Company 1 had total sales of $17.2 million. Its marketing department projects that sales will increase by $1.5 million per year for the next several years. Company 2 had total sales of $9.6 million for software products in 2010 and predicts that its sales will increase $2.3 million each year on average. Let x represent the number of years since 2010.

   a. Write an equation that represents the total sales, in millions of dollars, of Company 1 since 2010. Let S represent the total sales in millions of dollars.

   b. Write an equation that represents the total sales, in millions of dollars, of Company 2 since 2010. Let S represent the total sales in millions of dollars.

   c. Write a single equation to determine when the total sales of the two companies will be the same.

   d. Solve the equation in part c and interpret the result.

3. You are considering installing a security system in your new house. You get the following information from two local home security dealers for similar security systems.

   Dealer 1: $3560 to install and $15 per month monitoring fee.

   Dealer 2: $2850 to install and $28 per month for monitoring.

   Note that the initial cost of the security system from Dealer 1 is much higher than from Dealer 2, but the monitoring fee is lower.

   Let x represent the number of months that you have the security system.

   a. Write an equation that represents the total cost of the system with Dealer 1.

   b. Write an equation that represents the total cost of the system with Dealer 2.

   c. Write a single equation to determine when the total cost of the systems will be equal.

d. Solve the equation in part c and interpret the result.

e. If you plan to live in the house and use the system for 10 years, which system would be less expensive?

4. The life expectancies for men and women in the United States can be approximated by the following formulas:

$$\text{Women: } E = 0.126t + 76.74$$

$$\text{Men: } E = 0.169t + 69.11$$

where E represents the length of life in years and t represents the year of birth, measured as the number of years since 1975.

a. Write a single equation that can be used to determine in what year of birth the life expectancy of men and women would be the same.

b. Solve the equation in part a.

c. What is the life expectancy for the year of birth determined in part b?

In Exercises 5–10, solve each of the given equations for x. Check your answers by hand or with a calculator.

5. $5x - 4 = 3x - 6$

6. $3x - 14 = 6x + 4$

7. $0.5x + 9 = 4.5x + 17$

8. $4x - 10 = -2x + 8$

9. $0.3x - 5.5 = 0.2x + 2.6$

10. $4 - 0.025x = 0.1 - 0.05x$

Activity 8.3

Mathematical Modeling

Objectives

1. Develop an equation to model and solve a problem.
2. Solve problems using formulas as models.
3. Recognize patterns and trends between two variables using a table as a model.
4. Recognize patterns and trends between two variables using a graph as a model.

A model of an object is usually an alternative version of the actual object, produced in another medium. You are familiar with model airplanes and model cars. Architects and engineers build models of buildings and bridges. Computer models are used to simulate complicated phenomena like the weather and global economies. To be useful, a model must accurately describe an object or situation, in order to provide realistic results.

In this course, you have been using **mathematical models**. A mathematical model uses equations, formulas, tables, or graphs to describe the important features of an object or situation. Such models can then be used to solve problems, make predictions, and draw conclusions about the given situation. For example, in Activity 8.1, you developed the equation $c = 23.50n + 35$ to use as a mathematical model for determining the total monthly charges for gym membership and fees.

The process of developing a mathematical model is called **mathematical modeling**. The development of models to solve simple to extremely complicated problems is one of the most important uses of mathematics.

Equations as Mathematical Models

1. As part of a community service project at your college, you are organizing a fund-raiser at the neighborhood roller rink. Money raised will benefit a summer camp for children with special needs. The admission charge is \$4.50 per person, \$2.00 of which will be used to pay the rink's rental fee. The remainder will be donated to the summer camp fund.

   a. The first step in developing a mathematical model is to identify the problem and develop a well-defined question about what you want to know. State a question that you want answered in this situation.

   b. The next step is to identify the variables involved. What two variables are involved in this problem?

   c. Which variable can best be designated as the input variable? As the output variable?

   d. Next, look for relationships and connections between the variables involved in the situation. State in words the relationship between the input and output variables.

   e. Now, translate the features and relationships you have identified into an equation. Use appropriate letters to represent the variables, stating what each represents.

   f. If 91 people attended, use the equation model developed in part e to determine the amount donated to the summer camp fund.

g. If the maximum capacity of the rink is 200 people, what is the maximum amount that can be donated?

Formulas as Mathematical Models

In previous chapters, you used formulas that model situations in geometry, business, and science. In almost every field of study, you are likely to encounter formulas that relate two or more variables represented by letters. Problems 2 and 3 feature formulas used in the health field.

2. In order for exercise to be beneficial, medical researchers have determined that the desirable heart rate, R, in beats per minute, can be approximated by the formulas

$$R = 143 - 0.65a \text{ for women}$$
$$R = 165 - 0.75a \text{ for men,}$$

where a represents the person's age in years.

a. If the desirable heart rate for a woman is 130 beats per minute, how old is she?

b. If the desirable heart rate for a man is 135 beats per minute, how old is he?

3. The basal energy rate is the daily amount of energy (measured in calories) needed by the body at rest to maintain the basic life functions. The basal energy rate differs for individuals, depending on their gender, age, height, and weight. The formula for the basal energy rate for males is

$$B = 655.096 + 9.563W + 1.85H - 4.676A,$$

where: B is the basal energy rate (in calories)

W is the weight (in kilograms)

H is the height (in centimeters)

A is the age (in years).

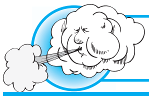
a. A male patient is 70 years old, weighs 55 kilograms, and is 172 centimeters tall. He is prescribed a total daily caloric intake of 1000 calories. Determine if the patient is being properly fed.

b. A male is 178 centimeters tall and weighs 84 kilograms. If his basal energy rate is 1500 calories, how old is the male?

Tables as Mathematical Models

You have encountered tables of ordered pairs of values throughout this course. Those tables can be viewed as a type of mathematical model that represents relationships between variables.

4. The following table gives the windchill temperature (how cold your skin feels) for various air temperatures when there is a 20-mile-per-hour wind.



Blowin' in the Wind

Air Temperature, °F	40	30	20	10	0	−10	−20
Windchill Temperature (20-mph wind), °F	30	17	4	−9	−22	−35	−48

a. What relationship do you observe between the windchill temperature and the air temperature?

b. How does the windchill temperature change as the air temperature drops by 10°F?

c. Estimate the windchill temperature for an air temperature of −30°F.

d. If the wind speed is 30 miles per hour, how would you expect the windchill temperatures in the table to change?

The following Windchill Chart, published by the National Weather Service, displays windchill temperature for many different wind speeds and air temperatures.

WINDCHILL CHART

Wind (mph) \ Air Temperature (°F)	40	35	30	25	20	15	10	5	0	−5	−10	−15	−20	−25	−30	−35	−40	−45
5	36	31	25	19	13	7	1	−5	−11	−16	−22	−28	−34	−40	−46	−52	−57	−63
10	34	27	21	15	9	3	−4	−10	−16	−22	−28	−35	−41	−47	−53	−59	−66	−72
15	32	25	19	13	6	0	−7	−13	−19	−26	−32	−39	−45	−51	−58	−64	−71	−77
20	30	24	17	11	4	−2	−9	−15	−22	−29	−35	−42	−48	−55	−61	−68	−74	−81
25	29	23	16	9	3	−4	−11	−17	−24	−31	−37	−44	−51	−58	−64	−71	−78	−84
30	28	22	15	8	1	−5	−12	−19	−26	−33	−39	−46	−53	−60	−67	−73	−80	−87
35	28	21	14	7	0	−7	−14	−21	−27	−34	−41	−48	−55	−62	−69	−76	−82	−89
40	27	20	13	6	−1	−8	−15	−22	−29	−36	−43	−50	−57	−64	−71	−78	−84	−91
45	26	29	12	5	−2	−9	−16	−23	−30	−37	−44	−51	−58	−65	−72	−79	−86	−93
50	26	19	12	4	−3	−10	−17	−24	−31	−38	−45	−52	−60	−67	−74	−81	−88	−95
55	25	18	11	4	−3	−11	−18	−25	−32	−39	−46	−54	−61	−68	−75	−82	−89	−97
60	25	17	10	3	−4	−11	−19	−26	−33	−40	−48	−55	−62	−69	−76	−84	−91	−98

Frostbite Times: 30 minutes, 10 minutes, 5 minutes

5. a. In the Windchill Chart on the previous page, locate the row that lists windchill temperatures for a 30-mile-per-hour wind and use it to complete the following table.

Air Temperature, °F	40	30	20	10	0	−10	−20
Windchill Temperature (30-mph wind), °F							

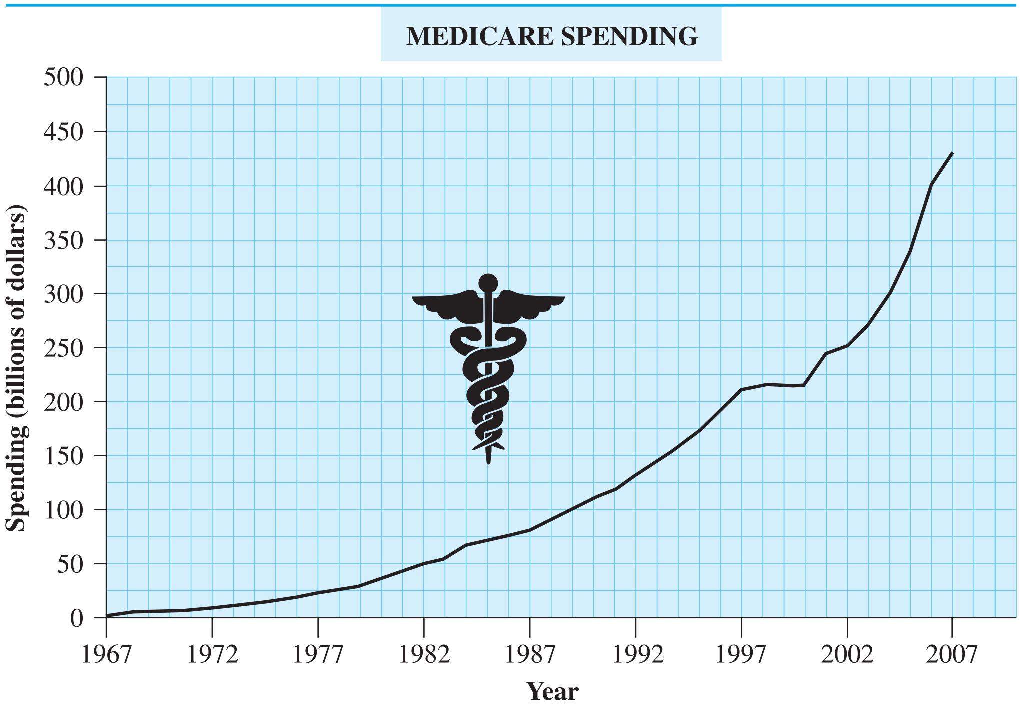
b. If the wind speed is 40 miles per hour, what is the windchill temperature if the air temperature is −20°F?

c. Approximately how long could you be exposed to a 20-mile-per-hour wind when the air temperature is 0°F before being in danger of frostbite? (That would be the frostbite time in the chart.)

Graphs as Mathematical Models

Graphs are very effective mathematical models that can be used to visualize patterns and trends between two variables.

6. Medicare is a government program that helps senior citizens pay for medical expenses. As the U.S. population ages and greater numbers of senior citizens join the Medicare rolls each year, the expense and quality of health service becomes an increasing concern. The following graph presents Medicare expenditures from 1967 through 2007. Use the graph to answer the following questions.



Source: Centers for Medicare and Medicaid Services

a. Identify the input variable and the output variable on the graph.

b. Use the graph to estimate the Medicare expenditures for the years in the following table.

The Cost of Care

YEAR, *y*	MEDICARE EXPENDITURES, *e* (billions of dollars)
1967	
1972	
1977	
1984	
1989	
1994	
1999	
2001	
2003	
2005	
2007	

c. What letters in the table in part b are used to represent the input variable and the output variable?

d. Estimate the year in which the expenditures reached $100 billion.

e. Estimate the year in which the expenditures reached $25 billion.

f. During which 10-year period did Medicare expenditures change the least?

g. During which 10-year period was the change in Medicare expenditures the greatest?

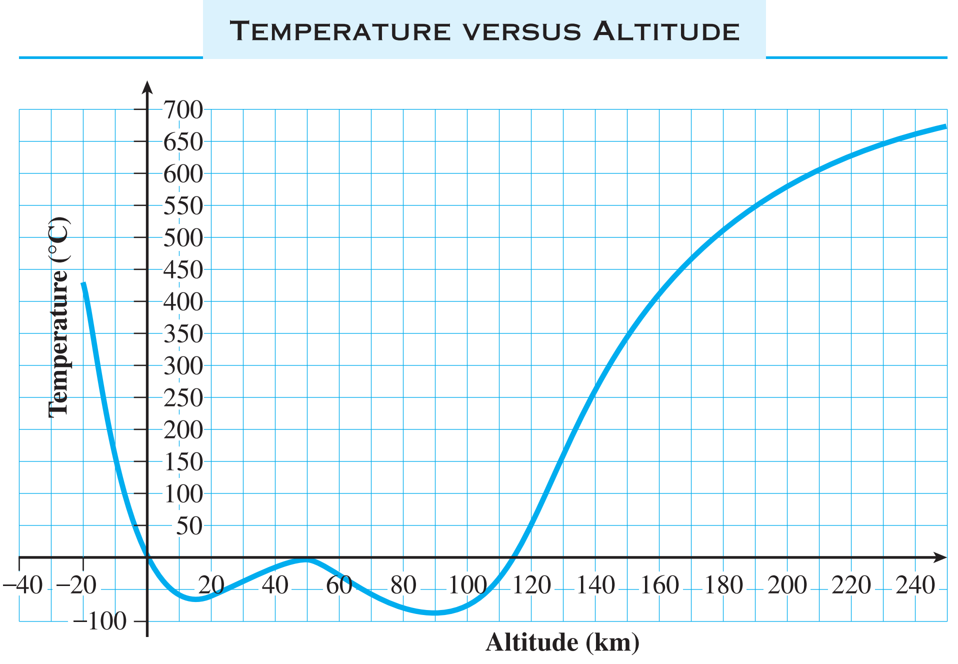
h. For which 1-year period does the graph indicate the most rapid increase in Medicare expenditures?

i. In what period does the graph indicate there was almost no change in Medicare expenditures?

j. From the graph, predict the Medicare expenditures in 2010.

k. What assumptions are you making about the change in Medicare expenditures from 2007 to 2010?

7. Living on Earth's surface, you experience a relatively narrow range of temperatures. But if you could visit below Earth's surface or high up above the surface, even above the atmosphere, you would experience a wider range of temperatures. The graph represents a model that predicts the temperature for a given altitude. Assume the altitude is 0 kilometers at Earth's surface.



a. What is the temperature of Earth 10 kilometers below the surface?

b. The ozone layer is approximately 50 kilometers above Earth's surface. What is the approximate temperature in the ozone layer?

c. Is it warmer or cooler in the nearby altitudes above and below the ozone layer?

d. Describe how the temperature changes as one moves up through the atmosphere.

e. At what depth under Earth's surface is the temperature 400°C?

f. At what height above Earth's surface is the temperature 400°C?

SUMMARY: ACTIVITY 8.3

1. A **mathematical model** uses equations, formulas, tables, or graphs to describe the important features of an object or situation. Such models are used to solve problems, make predictions, and draw conclusions about the given situation.

2. **Mathematical modeling** is the process of developing a mathematical model for a given situation.

EXERCISES: ACTIVITY 8.3

1. The value of almost everything you own, such as a car, computer, or appliance, depreciates over time. When the value decreases by a fixed amount each year, the depreciation is called straight-line depreciation.

   Suppose your car has an initial value of $16,750 and depreciates $1030 per year.

   a. State a question that you might want answered in this situation.

   b. What two variables are involved in this problem?

   c. Which variable do you think should be designated as the input variable?

   d. Complete the following table.

Years Car is Owned	1	2	3	4	5
Value of Car ($)					

Exercise numbers appearing in color are answered in the Selected Answers appendix.

e. State in words the relationship between the value of the car and the number of years the car is owned.

f. Use appropriate letters to represent the variables involved and translate the written statement in part e to an equation.

g. If you plan to keep the car for 7 years, determine the value of the car at the end of this period. Explain the process you used.

2. The formula used to convert a temperature in degrees Celsius to a temperature in degrees Fahrenheit is $F = \frac{9}{5}C + 32$.

a. Use the formula to determine the Fahrenheit temperature when the Celsius temperature is 25°.

b. Use the formula to determine the Celsius temperature when the Fahrenheit temperature is 59°.

3. In 1966, the U.S. Surgeon General's health warnings began appearing on cigarette packages. Data gathered after 1965 indicate that public awareness of the health hazards of smoking has had some effect on consumption of cigarettes.

The percentage, p, of the total population (18 and older) who smoked t years after 1965 can be approximated by the model $p = -0.555t + 41.67$, where t is the number of years after 1965. Note that $t = 0$ corresponds to the year 1965, $t = 9$ corresponds to 1974, etc.

a. Determine the percentage of the population who smoked in 1981 ($t = 16$).

b. Use the formula to determine the year when the percentage of smokers will be 15% $(p = 15)$?

4. As you might guess, medical-related expenses tend to increase with age. The average annual per capita (per person) expense for health care in America can be modeled by the formula

$$E = 136A - 1116,$$

where E is the average per capita expense (dollars) associated with a person of age A (years).

a. Determine the average per capita health care expense for a 30-year-old person.

b. At approximately what age will a person typically incur annual health care expenses of \$8000?

5. The following formula is used by the National Football League (NFL) to calculate quarterback ratings.

$$R = \frac{6.25A + 250C + 12.5Y + 1000T - 1250I}{3A}$$

where:
R = quarterback rating
A = passes attempted
C = passes completed
Y = passing yardage
T = touchdown passes
I = number of interceptions

In the 2008 regular season, quarterbacks Kurt Warner of the Arizona Cardinals and Donovan McNabb of the Philadelphia Eagles had the following player statistics.

Take a Pass

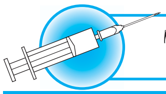
PLAYER	PASSES ATTEMPTED (A)	PASSES COMPLETED (C)	PASSING YARDAGE (Y)	NUMBER OF TOUCHDOWN PASSES (T)	NUMBER OF INTERCEPTIONS (I)
Kurt Warner	598	401	4583	30	14
Donovan McNabb	571	345	3916	23	11

a. Determine the quarterback rating for Kurt Warner for the 2008 NFL football season.

b. Determine the quarterback rating for Donovan McNabb.

c. Visit www.nfl.com and select stats to obtain the rating of your favorite quarterback.

6. The following table gives the number of people infected by the flu over a given number of months.

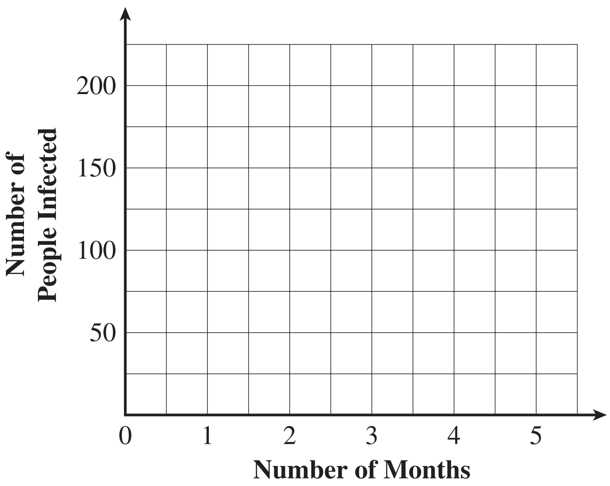


The Spread of the Flu

Number of Months	0	1	2	3	4	5
Number of People Infected	1	5	13	33	78	180

a. Describe any trends or patterns that you observe in the table.

b. Graph the ordered pairs in the table on the following grid. Connect successive points with line segments.



c. Describe any trends or patterns that you observe on the graph. Compare with those you made based on the table.

What Have I Learned?

1. The equation $120 = 3x + 90$ is solved for x as follows.

$$\begin{array}{lrcl} & 120 & = & 3x + 90 \\ \textbf{Step 1.} & -90 & & -90 \\ \hline & 30 & = & 3x \end{array}$$

Step 2. $$\frac{30}{3} = \frac{3x}{3}$$

$$10 = x \quad \text{or} \quad x = 10$$

a. In step 1, what operation is used to remove 90 from each side of the equation and why?

b. In step 2, what operation is used to remove 3 from the term $3x$ and why?

2. Solve the equation $12 = 5x - 8$ for x. At each step, state the operation you used and state why you used it.

3. During President Obama's "Cash for Clunkers" program in the summer of 2009, one of your instructors traded his 1994 Ford Pickup truck for a 2009 Honda Civic LX five-speed coupe with a MSRP sticker price of $18,965. Since the pickup truck had a combined city/highway fuel rating of less than 15 miles per gallon and the Honda Civic had a combined rating of 29 miles per gallon, your instructor qualified for the $4500 "Cash for Clunkers" credit. The dealership's offer of 0% financing was an added incentive.

a. Your instructor had budgeted $220 per month for a car payment. Write an equation to determine the desired purchase price, p. Use n as your input variable for the number of months of the loan and include the $4500 since the credit is given after a price is negotiated.

b. Car loans usually have terms of 36, 48, or 60 months. Determine the loan term that comes closest to the sticker price.

c. What purchase price should your instructor actually negotiate with the car dealer to stay within his budget?

d. What was the final cost of the new Honda Civic once your instructor successfully negotiated his desired price and the credit had been applied? Did your instructor get a good deal?

How Can I Practice?

In Exercises 1–14, solve each equation for x. Check your results by hand or with a calculator.

1. $2x - 5 = 4x + 7$
2. $6x + 3 = 9$
3. $2x + 2 = 5x - 3$
4. $14 - 6x = -2x + 3$
5. $2.1x + 15 = 3.5 - 1.9x$
6. $8x + 4 = 14x + 14$
7. $3x - 6 + 10 = 5x$
8. $0.25x - 0.5 = 0.2x + 2$
9. $\frac{1}{2}x - 6 = 3x + 4$
10. $\frac{1}{3}x - \frac{1}{3} = 6x - 6$
11. $21 + 3x - 12 = 24$
12. $8x - 6 = 6x + 18$
13. $2 - 5x - 25 = 3x - 6 - 1$
14. $0.16x + 2.4 - 0.24x = 1.8$

Answers to all How Can I Practice exercises are included in the Selected Answers appendix.

15. Since 1960, the median age of men at their first marriage has steadily increased. If a represents the median age of men at their first marriage, then a can be approximated by the formula

$$a = 0.11t + 22.5,$$

where t represents the number of years since 1960.

a. What is the median age of men who are married for the first time in 2010?

b. According to the formula, in what year will the median age of men at first marriage be 30?

c. Solve the formula $a = 0.11t + 22.5$ for t.

d. Answer part b using the new formula in part c.

16. The cost of printing a brochure to advertise your lawn-care business is a flat fee of \$10 plus \$0.03 per copy. Let c represent the total cost of printing and x represent the number of copies.

a. Write an equation that will relate c and x.

b. Use the equation from part a to complete the following table. Begin with 1000 copies, increase by increments of 1000, and end with 5000 copies.

Duplicated Effort

NUMBER OF COPIES, x	TOTAL COST, c (\$)
1000	

c. What is the total cost of printing 8000 copies?

d. You have $300 to spend on printing. How many copies can you have printed for that amount?

17. You contact the local print shop to produce a commemorative booklet for your college theater group's twenty-fifth anniversary. The print shop has quoted you a price of $750 to typeset the booklet and 25 cents for each copy produced.

a. Write an equation that gives the total cost, C, in terms of the number, x, of booklets produced.

b. Determine the total cost of producing 500 booklets.

c. How many booklets can be produced for $1000?

d. Suppose the booklets are sold for 75 cents each. Write an equation for the total revenue R from the sale of x booklets.

e. How many booklets must be sold to break even? That is, for what value of x is the total cost of production, C, equal to the total amount of revenue, R?

f. Recall that profit is revenue minus cost $(P = R - C)$. How much profit will be made from the sale of 2500 books?

18. In each of the following problems, use the equation to complete the table.

a. $y = 4x - 11$

x	y
6	
	53

b. $y = -5x - 80$

x	y
12	
	35

19. You are planning on taking some courses at your local community college on a part-time basis for the upcoming semester. For a student who carries fewer than 12 credits (the full-time minimum), the tuition is $155 for each credit hour taken. All part-time and full-time students must pay a $20 parking fee for the semester. This fixed fee is added directly to your tuition bill.

a. Write an equation to determine the total tuition bill, t, for a student carrying fewer than 12 credit hours. Use n for the number of credit hours.

b. Use the equation you wrote in part a to complete the following table.

$$$ DIPLOMA Paying for Credit

Number of Credit Hours, n	1	2	3	4	5	6
Total Tuition, t						

c. Determine the tuition bill if you take 9 credit hours.

d. You have $1000 to spend on tuition. How many credit hours can you carry for the semester?

Chapter 8 Summary

The bracketed numbers following each concept indicate the activity in which the concept is discussed.

CONCEPT/SKILL	DESCRIPTION	EXAMPLE
Evaluate $y = ax + b$ for y, given x. [8.1]	1. Replace the input variable x in the expression $ax + b$ with the specified numerical value. 2. Evaluate the numerical expression to obtain the numerical value for y.	Evaluate $y = 2x + 10$ for y, given $x = -2$. Substitute -2 for x: $y = 2(-2) + 10$ $y = -4 + 10$ $y = 6$
Solve $y = ax + b$ for x, given y. [8.1]	1. Add the opposite of b to each side of the equation. 2. Divide each term in the resulting equation by a to isolate x and then read the value for x.	Solve $y = 2x - 5$ for x, given $y = 11$. $2x - 5 = 11$ $\underline{\quad +5 \quad +5}$ $2x = 16$ $\frac{2x}{2} = \frac{16}{2}$; $x = 8$
General strategy for solving an equation for an unknown x [8.2]	1. Combine like terms on the same side of the equation. 2. Isolate the variable term. 3. Divide out the nonzero coefficient of the variable. 4. Check.	$2x + 8 + 3x = 2x - 1$ $5x + 8 = 2x - 1$ $\underline{-2x - 8 \quad -2x - 8}$ $3x = -9$ $\frac{3x}{3} = \frac{-9}{3}$ $x = -3$ *Check:* $2(-3) + 8 + 3(-3) = 2(-3) - 1$ $2 - 9 = -6 - 1$ $-7 = -7$
Mathematical model [8.3]	Description of mathematical features of an object or situation that uses equations, formulas, tables, and/or graphs.	See Activity 8.3.
Mathematical modeling [8.3]	The process of developing a mathematical model for a given situation.	See Activity 8.3.

Chapter 8 Gateway Review

In Exercises 1–8, solve the equation for x. Check your answers by hand or with a calculator.

1. $-18 = 2x + 8$

2. $25 + 0.15x = 70.90$

3. $1.6x - 49 = -10$

4. $3x + 10 = 6x - 11$

5. $4x + 20 - x = 80$

6. $38 = 57 - x - 32$

7. $5x + 6x - 24 = 2x + 12$

8. $2.5x + 10 = 5.8x - 11.6$

9. Evaluate the expression for the given value(s).

a. $2x^2 - 3x + 5$ for $x = -3$

b. $2a + b$ for $a = 17.3$ and $b = 11.8$

c. $a - (b + c)$ for $a = 10$, $b = -6$, and $c = -8$

10. When applying for a job at a clothing store, you are offered two options for your salary. The following table shows these options.

Option 1	\$100 per wk	Plus 30% of all sales
Option 2	\$150 per wk	Plus 15% of all sales

a. Write an equation to represent the total salary S for option 1 if the total sales are x dollars per week.

b. Write an equation to represent the total salary S for option 2 if the total sales are x dollars per week.

c. Write an equation that you could use to determine how much you would have to sell weekly to have the same weekly salary under both plans.

Answers to all Gateway exercises are included in the Selected Answers appendix.

d. Solve the equation in part c. Interpret your result.

e. What is the salary for the amount of sales found in part d?

11. A triathlon includes swimming, long-distance running, and cycling.

a. Let x represent the number of miles the competitors swim. If the long-distance run is 10 miles longer than the distance swum, write an expression that represents the distance the competitor runs in the event.

b. The distance the athletes cycle is 55 miles longer than they run. Use the result in part a to write an equation in terms of x that represents the cycling distance, C, of the race.

c. Write a formula that represents the total distance, d, of all three phases of the triathlon in terms of x, the number of miles that the competitors swim.

d. If the total distance of the triathlon is 120 miles, use the formula in part c to write and solve an equation to determine the swimming distance.

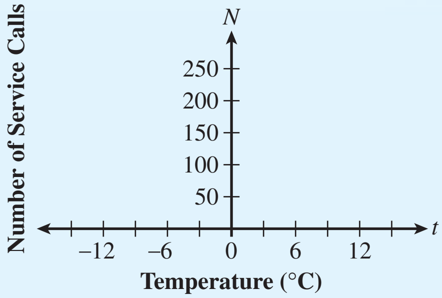
e. What are the lengths of the running and the cycling portions of the race?

12. On an average winter day, a certain office of the Automobile Association of America (AAA) receives 125 calls from persons who need help starting their cars. The number varies depending on the temperature. Here are some data giving the number of calls as a function of the temperature (in degrees Celsius).

AAA on the Way!

TEMPERATURE (°C)	NUMBER OF AUTO CLUB SERVICE CALLS
−12	250
−6	190
0	140
4	125
9	100

a. Plot the data from the table.



The data can be approximated by the equation

$$C = -7.11t + 153.9$$

where C is the total number of calls and t is the temperature $(°C)$.

b. Determine how many service calls AAA can expect if the temperature drops to $-20°C$.

c. Use the given equation to predict the temperature for which AAA should expect 50 calls.

13. ABC Rent-a-Car offers cars at \$50 a day and 20 cents a mile. Its competition rents cars for \$60 a day and 15 cents a mile.

a. For each company, write an equation that determines C, the cost of renting a car for a day in terms of the distance, x, traveled.

b. Write a single equation to determine the number of miles driven in which the cost of renting is the same for each company.

c. Solve the equation in part b and interpret the result.

d. If you were planning a round trip from Washington, D.C. to New York City, a distance of about 465 miles, which company would be cheaper to rent from? Explain.

14. You have an opportunity to be the manager of a day camp for the summer. You know that your fixed costs for operating the camp are \$600 per week, even if there are no campers. Each camper costs the management \$10 per week. The camp charges each camper \$40 per week. Let x represent the number of campers.

a. Write an equation in terms of x that represents the total cost, C, of running the camp per week.

b. Write an equation in terms of x that represents the total income (revenue), R, from the campers per week.

c. Write an equation in terms of x that represents the total profit, P, from the campers per week.

d. How many campers must attend the camp to break even with revenue and costs?

e. The camp would like to make a profit of \$600. How many campers need to enroll to make that profit?

f. How much money would the camp lose if only 10 campers attend?

This page intentionally left blank

Appendix

A

Learning Math Opens Doors: Twelve Keys to Success

1. Are You in the Right Math Course for Your Skill Level?

Students are sometimes placed in the wrong math course for a variety of reasons. Your answers to the following questions will help you and your instructor determine if you are in the correct math course for your skill level. This information will also help your instructor understand your background more quickly, thus providing you with a better learning experience.

a. Did you register for this math course because you took a placement exam and, as a result of your exam score, you selected or were placed in this course?

b. If you were not placed in this class because of a placement exam, please list reasons you are taking this course.

c. Name ______________________________

d. Phone and/or e-mail address ______________________________

e. List the mathematics courses you have taken at the *college level*.

Course Title	Grade
1. ______________________	________
2. ______________________	________

f. List the mathematics courses you have taken at the *high school level*.

Course Title	Grade
1. ______________________	________
2. ______________________	________
3. ______________________	________

g. Have you taken this course before? Yes __________ No __________

h. When did you take your last mathematics course? ______________

i. Are you a full-time or part-time student? ______________

j. Do you have a job? ______________ If yes, how many hours per week do you work? ______________

k. Do you take care of children or relatives at home? ______________

You should now share your information with your instructor to make sure you are in the correct class. It is a waste of a semester of time and money if the course is too easy for you or too difficult for you. Take control of and responsibility for your learning!

2. What Is Your Attitude About Mathematics?

a. Write one word that describes how you feel about learning mathematics.

____

b. Was the word that you wrote a positive word, a negative word, or a neutral word?

____

If your word was a positive word, you are on your way to success in this course. If your word was negative, then before progressing any further, you may want to determine why you have negative feelings toward mathematics.

c. Write about a positive or negative experience that you have had related to mathematics.

____

____

____

____

If you have not had success in the past, try something different. Here are a few suggestions:

d. Make a list of all the materials (pencils, notebook, calculator) that you might need for the course. Make sure you have all the materials required for the course.

____

____

e. Find a new location to study mathematics. List two or three good places you can study.

1. ______________________ 2. ______________________ 3. ______________________

f. Study with a classmate. Help each other organize the material. Ask each other questions about assignments. Write down names, phone numbers/e-mail addresses of two or three other students in your class to study with.

1. ____

2. ____

3. ____

g. How did you study mathematics in the past? Write a few sentences about what you did outside of class to learn mathematics.

____

____

____

h. What will you change from your past to help you become more successful? Write down your strategy for success.

____

____

____

3. Do You Attend All Classes on Time and Are You Organized?

Class work is vital to your success. Make it a priority to attend every class. Arrive in sufficient time and be ready to start when class begins. Start this exercise by recording the months and dates for the entire semester in each box. Then write in each box when and where your class meets corresponding to your schedule. Keep track of exam and quiz dates, deadlines, study sessions, homework, and anything else that can help you succeed in the course.

Schedule routine medical and other appointments so you don't miss class. Allow time for traffic jams, finding a parking space, bus delays, and other emergencies that may occur. Arriving late interferes with your learning and the learning of your fellow students. Make sure assignments are completed and handed in on time. Attending class and being on time and organized is in your control.

✓ Place a check in each day that you are in class. Circle the check to indicate that you were on time. Place an *a* in each day you miss class.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday

4. When Is That Assignment Due?

In mathematics, assignments are usually given each class time. Assignments are meant to reinforce what you learned in class. If you have trouble completing your assignment, take charge and get help immediately. The following table will help you organize your assignments and the dates they are due. Keeping a record of your assignments in one location will help you know at a glance what and when your assignment is due.

Date Assigned	Assignment	Specific Instructions	Dates	
			Due	Completed

Date Assigned	Assignment	Specific Instructions	Dates	
			Due	Completed

5. Do You Keep Track of Your Progress?

Keeping track of your own progress is a great way to monitor the steps you are taking toward success in this course. Different instructors may use different methods to determine your grade.

a. Explain how your instructor will determine your final grade for this course.

____

____

____

____

b. Use the following table to keep track of your grades in this course. (You may want to change the headings to reflect your instructor's grading system; an Excel spreadsheet may also be helpful for this exercise.)

Date	Type of Assessment	Topic(s) and/ or Chapters	Number of Points Correct	Total Number of Points	Comments

c. Determine your final average using your instructor's grading system.

6. How Well Do You Know Your Textbook?

Knowing the structure of your textbook helps you use the book more effectively to reach your learning goals. The Preface and "To the Student" sections at the beginning of the book provide guidance and list other valuable resources. These include CDs/DVDs, Web sites, and computer software with tutorials and/or skill practice for added help in the course. Use the following guidelines to learn the structure and goals of the textbook.

a. According to your syllabus, which chapters will you be studying this semester?

b. Find and underline the titles of the chapters in the table of contents.

c. Read the Preface and summarize the key points.

d. In the Preface, you should have found the student supplements for the course. List them.

e. What are the key points the authors make to you in the "To the Student" section?

f. Each chapter is made up of smaller sections called activities. What is the title of Activity 1 in Chapter 3?

g. Within an activity you will see information that has a box around it. Why is this information important? Look at Chapter 2, Activity 1 before you respond.

h. Look at the end of each activity. There you will find the Exercises section. Doing these exercises should help you better understand the concepts and skills in the activity. Why are some of the numbers of the exercises in color?

i. Chapter 5 is divided into two parts called clusters. Name the clusters in Chapter 5.

j. Look at the end of Cluster 1 in Chapter 5. You will see a section entitled "What Have I Learned?" This section will help you review the key concepts in the cluster. What are the concepts taught in this cluster?

k. At the end of each chapter you will see a Gateway and a Summary. Look at the end of Chapter 7 and write how each of these sections will help you in this course.

l. Locate and briefly review the Glossary. Select a mathematical term from the Glossary and write the term and its definition here.

7. Where Does Your Time Go?

Managing your time is often difficult. A good rule to follow when studying is to allow 2 hours of study time for each hour of class time. It is best not to have marathon study sessions, but, rather, to break up your study time into small intervals. Studying a little every day rather than "cramming" once a week helps to put the information into your long-term memory. Ask yourself this question: Is going to school a top priority? If you answered yes, then your study time must be a priority if you are going to be successful.

a. In the past, approximately how many hours per week did you study mathematics? Reflect on whether it was enough time to be successful.

b. The commitments in your life can be categorized. Estimate the number of hours that you spend each week on each category. (There are 168 hours in a week.)

Class Hours (Class) __________

Study Hours (ST) __________

Work Hours (W) __________

Commute Hours (T) __________

Chore Hours (C) __________

Sleep Hours (Sl) __________

Personal Hours (P) __________

Leisure Hours (L) __________

Other (O) __________

c. Use the information from part b to fill in the grid with a workable schedule using the appropriate letter to represent each category.

Time	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
12 midnight							
1:00 A.M.							
2:00							
3:00							
4:00							
5:00							
6:00							
7:00							
8:00							
9:00							
10:00							
11:00							
12:00 noon							
1:00 P.M.							
2:00							
3:00							
4:00							
5:00							
6:00							
7:00							
8:00							
9:00							
10:00							
11:00							

d. Is your schedule realistic? Does it represent a typical week in *your* life? If not, make adjustments to it now.

e. Circle the hours in your schedule that you will devote to studying.

f. Shade or highlight the hours that you will use for studying mathematics.

g. Do you have some times scheduled each day to study mathematics? How much time is devoted to mathematics each day?

h. Studying math soon after class time will help you review what you learned in class. Did you place any hours to study math close to your actual class time?

i. Scheduling time before class can help you prepare for the class. Did you schedule any study time just before class time?

j. Review your schedule once more and make any changes needed.

k. Periodically review your schedule to see if it is working and whether you are following it. How are you doing?

8. Where Will You Use Mathematics?

Students often ask math instructors, "Where will I ever use this?" Here is an opportunity to show where you have used mathematics, including what you have learned from this course.

a. Describe one way in which you use math in your everyday life.

____

b. Find an article in a newspaper that contains a graph or some quantitative information.

c. Describe ways in which you use math in your job.

____

____

d. Describe a problem in a course you are taking or took (not a mathematics course) that involved a math concept or skill. Include a description of the math concept or skill that is involved.

____

____

After you have been in this course for a few weeks, answer parts e, f, and g.

e. Write a problem from one of your courses (not a mathematics course) or from your work experience where you used concepts or skills that you have learned in *this* mathematics class.

____

____

f. Show how you solved the problem.

____

____

g. Write the math skill(s) or concept(s) that you learned in this math course that helped you solve this problem.

____

____

9. Do You Need Extra Help?

Receiving extra help when you need it may mean the difference between success and failure in your math course. If you are struggling, don't wait. Seek help immediately. When you go for extra help, identify specific areas in which you need help. Don't just say "I'm lost."

a. Your instructor usually will note office hours right on or near the office door or they may be listed in the syllabus. Take a few extra minutes and locate the office. List your instructor's office hours below. Write the office room number next to the hours. Circle the office hours that work with your schedule.

b. Does your college have a center where you can go for tutoring or extra help? If there is one, what is the room number? Take a minute and locate the center. When is the center open? Determine what hours would be good for you if you should need help.

c. Students in your class may also help. Write the phone numbers or e-mail addresses of two or three students in your class whom you could call for help. These students may form a study group with you. When would it be a good time for all of you to meet? Set up the first meeting.

d. You can also help yourself by helping others learn the information. By working a problem with someone else, it will help you to reinforce the concepts and skills. Set aside some time to help someone else. What hours are you available? Tell someone else you are available to help. Write the person's name here.

Extra help is also there for you during class time. Remember to ask questions in class when you do not understand. Don't be afraid to raise your hand. There are probably other students who have the same question. If you work in groups, your group can also help. Use the following chart to record the times you have needed help.

Date	Topic you needed help with	Where did you find help?	Who helped you?	Were your questions answered?	Do you still have questions?

10. Have You Tried Creating Study Cards to Prepare for Exams?

a. Find out from your instructor what topics will be on your exam. Ask also about the format of the exam. Will it be multiple choice, true/false, problem solving, or some of each?

b. Organize your quizzes, projects, homework, and book notes from the sections to be tested.

c. Read these quizzes, projects, homework, and book notes carefully and pick out those concepts and skills that seem to be the most important. Write this information on your study card. Try to summarize in your own words and include an example of each important concept.

d. Make sure you include:

Vocabulary Words, with definitions:

Key Concepts, explained *in your own words*:

Skills, illustrated with an example or two:

e. Reviewing this study card with your instructor may be a good exam preparation activity.

11. How Can You Develop Effective Test-taking Strategies?

a. A little worry before a test is good for you. Your study card will help you feel confident about what you know. Wear comfortable clothing and shoes to your exam and make yourself relax in your chair. A few deep breaths can help! Don't take stimulants such as caffeine; they only increase your anxiety!

b. As soon as you receive your test, write down on your test paper any formulas, concepts, or other information that you might forget during the exam. This is information that you would recall from your study card.

c. You may want to skim the test first to see what kind of test it is and which parts are worth the most points. This will also help you to allocate your time.

d. Read all directions and questions carefully.

e. You may want to do some easy questions first to boost your confidence.

f. Try to reason through tough problems. You may want to use a diagram or graph. Look for clues in the question. Try to estimate the answer before doing the problem. If you begin to spend too much time on one problem, you may want to mark it to come back to later.

g. Write about your test-taking strategies.

	What test-taking strategies will you use?	***What test-taking strategies did you use?***
Test 1		
Test 2		
Test 3		
Test 4		

12. How Can You Learn From Your Exams?

a. Errors on exams can be divided into several categories. Review your exam, identify your mistakes, and determine the category for each of your errors.

Type of Error	Meaning of Error	Question Number	Points Deducted
A. Concept	You don't understand the properties or principles required to answer the question.		
B. Application	You know the concept but cannot apply it to the question.		
C. Skill	You know the concept and can apply it, but your skill process is incorrect.		
D. Test-taking	These errors apply to the specific way you take tests. **1.** Did you change correct answers to incorrect answers? **2.** Did you miscopy an answer from scrap paper? **3.** Did you leave an answer blank? **4.** Did you miss more questions at the beginning or in the middle or at the end of your test? **5.** Did you misread the directions?		
E. Careless	Mistakes you made that you could have corrected had you reviewed your answers to the test.		
	TOTAL:		

b. You should now correct your exam and keep it for future reference.

i. What questions do you still have after making your corrections?

ii. Where will you go for help to get answers to these questions?

iii. Write some strategies that you will use the next time you take a test that will help you reduce your errors.

This page intentionally left blank

Selected Answers

Chapter 1

Activity 1.1 Exercises: 1. \$53,472; **3.** 1,325,082,380; **4.** \$70,000, \$70,200; **8. a.** odd; 22,225 ends with an odd digit; **9. c.** 51 is composite since 51 has factors other than 1 and 51, namely, 3 and 17. **11. a.** \$31,000; \$29,000

Activity 1.2 Exercises: 1. a. 5, **c.** 50; **4. a.** 2, **b.** 5; **6. a.** 9, **g.** $80 + 290 + 220 + 110 + 170 + 50 + 60 + 70 + 160 = 1210$ waste sites total

Activity 1.3 Exercises: 1. a. 291, **b.** 766, **c.** 510; **4. b.** $49 + 71 = 120$; **5. a.** $200 + 100 + 200 = 500$; **7. a.** 32, **d.** 752, **f.** 312; **10. b.** 825; **11. a.** 0, **b.** 30; **12. f.** $28 - 13$, **g.** $85 + 8$

Activity 1.4 Exercises: 1. b. 4232, **h.** 206,535; **3. a.** $12(30 + 6) = 12 \cdot 30 + 12 \cdot 6 = 360 + 72 = 432$; **6. a.** 1656, **b.** 1656, **c.** yes, **d.** The commutative property is demonstrated. **9. a.** $20 \cdot 20 = 400$ hot dog rolls, **b.** The actual number of rolls is 384. **c.** The estimate is slightly higher.

Activity 1.5 Exercises: 1. a. quotient 8, remainder 0, **f.** quotient 25, remainder 12, **h.** undefined; **3. b.** Each student will get $1500 \div 4 = 375$ index cards. **4. a.** 3, **b.** no, **c.** no; **6. a.** $4000 \div 50 = 80$, **b.** 74R2, **c.** My estimate is higher.

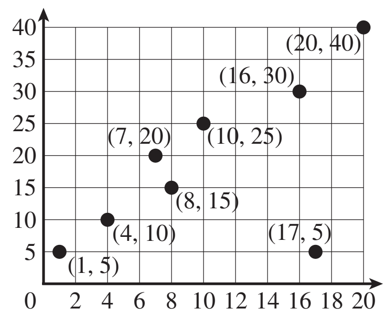
Activity 1.6 Exercises: 1. 10,000,000,000,000,000; **3.** 53, 59; **5. d.** $22 \cdot 1 = 11 \cdot 2 = 22$; **6. a.** $30 \cdot 1 = 15 \cdot 2 = 10 \cdot 3 = 6 \cdot 5 = 2 \cdot 3 \cdot 5 = 30$; $105 \cdot 1 = 35 \cdot 3 = 21 \cdot 5 = 15 \cdot 7 = 3 \cdot 5 \cdot 7 = 105$; **8. d.** $2^5 \cdot 3$; **10. d.** 32, **e.** 144; **11. a.** 5^{11}; **12. e.** 15; **13. d.** yes, 16 ft. by 16 ft.

Activity 1.7 Exercises: 1. b. 145, **d.** 55; **2. d.** 60; **4. a.** $884 - 34 = 850$; **5. c.** $12 + 45 - 4 = 53$; **7. e.** $100 \div 10 = 10$, **f.** $8 \cdot 20 - 4 = 160 - 4 = 156$; **8. b.** $80 - 27 = 53$; **9. e.** $56 - 18 + 5 = 43$, **j.** $5^2 = 25$

What Have I Learned? 4. a. addition and multiplication: $3 + 5 = 8 = 5 + 3$; $3 \cdot 5 = 15 = 5 \cdot 3$, **b.** subtraction and division: $5 - 1 = 4$ and $1 - 5 \neq 4$; $8 \div 2 = 4$ and $2 \div 8 \neq 4$; **5. a.** yes, $90 + 17 = 107$ and $69 + 38 = 107$, **c.** no, $48 - 17 = 31$ and $69 - 4 = 65$

How Can I Practice? 3. $3{,}240{,}000{,}000 \div 395{,}000{,}000 \approx 8.2 \approx 8$ times as much; **5. a.** 92,956,000; **6. c.** 4660; **8. c.** $98 - 15 = 83$, **i.** $95 - 25 = 70$; **9. a.** 225; **13.** 31, 37, 41, 43, 47; **14. d.** 1, 2, 3, 6, 9, 18; **15. d.** $3 \cdot 3 \cdot 7 = 3^2 \cdot 7$; **18. d.** 9^6; **20. b.** approximately 5, **c.** 15; **21. d.** $(36 - 18)/6 = 18/6 = 3$, **e.** $45 - 5 \cdot 8 + 5 = 45 - 40 + 5 = 10$, **h.** $49 + 49 = 98$

Gateway Review 1. two hundred forty-three dollars; **2.** twenty-two million, five hundred twenty-eight thousand, seven hundred thirty-seven; **3.** 108,091; 108,901; 108,910; 109,801; 180,901; **4.** 0 is thousands and 9 is tens; **5. a.** odd; ends with odd digit, **b.** even; ends with even digit, **c.** odd; ends with odd digit; **6. a.** composite, since $145 = 5 \cdot 29$, **b.** prime, since $61 = 1 \cdot 61$, the only factors, **c.** prime, the first one, **d.** composite, since $121 = 11 \cdot 11$; **7. a.** 1,253,000, **b.** 900; **8. a.** Emily, **b.** Daniel, **c.** Andrew, **d.** Sophia; **9. a.** 1973; 53 in., **b.** 1954 and again in 2008; 14 in., **c.** between 20 and 30 in.: 28 years; between 30 and 40 in.: 24 years. So, the number of years of 20 to 30 in. of rain per year is more than from 30 to 40 in. **10. a.** 2, **b.** See grid, **c.** 5, **d.** See grid. **e.**



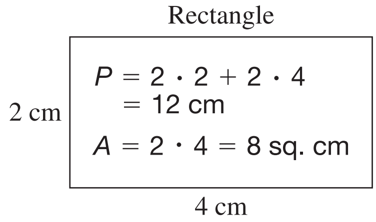
11. a. 8951, **b.** 896, **c.** 177, **d.** 662, **e.** 475; **12.** $113 = 72 + 41$. **13. a.** associative property, **b.** commutative property, **c.** The two expressions are equal because adding 0 does not change the value of the sum. **14.** They are not equal; subtraction is not commutative. **15. a.** $510 + 90 + 120 + 350 = 1070$ (estimate), **b.** 1066, **c.** Estimate was a little higher than actual since 1070 is larger than 1066. **16. a.** $30 + 22$, **b.** $67 - 15$, **c.** $125 - 44$, **d.** $250 - 175$, **e.** $25 \cdot 36$, **f.** $55 \div 11$, **g.** 13^2, **h.** 2^5, **i.** $\sqrt{49}$, **j.** $27(50 + 17)$;

17. a. $61 + 61 + 61 + 61 + 61 = 305$, **b.** It is the same: 305. **c.** $5(61) = 5(60 + 1) = 300 + 5 = 305$; **18. a.** $4(29) = 4(30 - 1) = 120 - 4 = 116$, **b.** It is faster to mentally multiply $4 \cdot 30 - 4 \cdot 1$ to obtain 116. **c.** $6 \cdot 98 = 6(100 - 2) = 600 - 12 = 588$; **19. a.** associative property, **b.** commutative property, **c.** Because multiplying any number by 1 does not change the value of the number. **20. a.** $72 - 12 = 60$; $60 - 12 = 48$; $48 - 12 = 36$; $36 - 12 = 24$; $24 - 12 = 12$; $12 - 12 = 0$; So, the quotient is 6 since there were six subtractions of 12 with a remainder of 0. **b.** $86 - 16 = 70$; $70 - 16 = 54$; $54 - 16 = 38$; $38 - 16 = 22$; $22 - 16 = 6$; So, the quotient is 5 since there were five subtractions of 16 with a remainder of 6. **21.** $12 \div 6 = 2$ but $6 \div 12$ is not a whole number. So, since the answers are different, division is not commutative. **22. a.** 182,352, **b.** Quotient is 15 and remainder is 6. **c.** 861, **d.** 861, **e.** 1, **f.** 0, **g.** undefined, **h.** Quotient is 273 and remainder is 20. **23. a.** $300 \cdot 80 = 24{,}000$ (estimate), **b.** 24,675, **c.** Estimate is lower. **24. a.** $2000 \div 40 = 50$ (estimate), **b.** Quotient is 44 and remainder is 2. **c.** Estimate is higher. **25.** No, I made a mistake since $3 \cdot 20 = 60$. The correct answer is 21. **26.** $1 \div 0$ is undefined, whereas $0 \div 1 = 0$. So, the answers are different. **27. a.** The answer is that whole number; for example, $15 \div 1 = 15$. **b.** The result is 1. It applies to any number except 0, since you cannot divide by 0. **28. a.** 1, 2, 5, 7, 10, 14, 25, 35, 50, 70, 175, 350, **b.** 1, 3, 9, 27, 81, **c.** 1, 2, 3, 4, 6, 9, 12, 18, 36; **29. a.** $2 \cdot 5^2 \cdot 7$, **b.** 3^4, **c.** $2^2 \cdot 3^2$; **30. a.** 1, **b.** 49, **c.** 32, **d.** 1; **31. a.** 9^2, **b.** 3^4, **c.** 25^2, **d.** 5^4; **32. a.** 3^{10}, **b.** 5^{45}, **c.** 21^{19}; **33. a.** a perfect square since $5^2 = 25$, **b.** not a perfect square since $125 = 5 \cdot 5 \cdot 5$, **c.** a perfect square since $11^2 = 121$, **d.** a perfect square since $10^2 = 100$, **e.** not a perfect square, since $200 = 2 \cdot 10 \cdot 10$; **34. a.** 4, **b.** 6, **c.** 17; **35. a.** neither, **b.** perfect square: $20^2 = 400$, **c.** neither, **d.** perfect square: $100^2 = 10{,}000$, **e.** perfect cube: $10^3 = 1000$; **f.** both: $8^2 = 64$; $4^3 = 64$; **36. a.** $48 - 3(4) + 9 = 48 - 12 + 9 = 45$; **b.** $16 + 4 \cdot 4 = 16 + 16 = 32$; **c.** $243/(35 - 8) = 243/27 = 9$; **d.** $(160 - 5)/10 = 110/10 = 11$; **e.** $7 \cdot 8 - 9 \cdot 2 + 5 = 56 - 18 + 5 = 43$; **f.** $8 \cdot 9 = 72$; **g.** $36 + 64 = 100$; **h.** $(9 - 8)^2 = 1^2 = 1$. **37.** They do not, since $6 + 10 \div 2 = 6 + 5 = 11$ and $(6 + 10) \div 2 = 16 \div 2 = 8$.

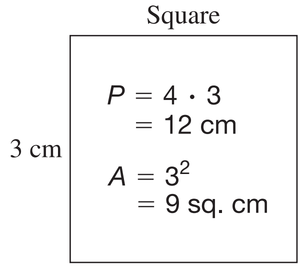
Chapter 2

Activity 2.1 Exercises: 1. a. $P = 4s = 20$ cm, **b.** $A = lw = 805$ sq. in., **c.** $P = 2l + 2w = 28$ cm, **d.** $A = s^2 = 225$ sq. in.; **3. a.** $P = a + b + c$, **b. i.** $P = 21$ ft., **ii.** $P = 102$ cm, **iii.** $P = 41$ in.;

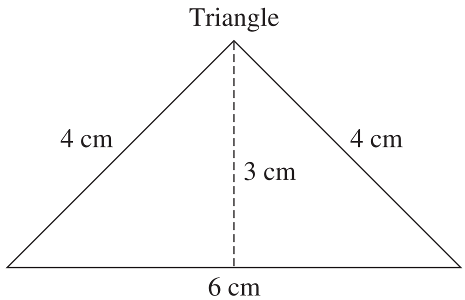
5. a.



b.



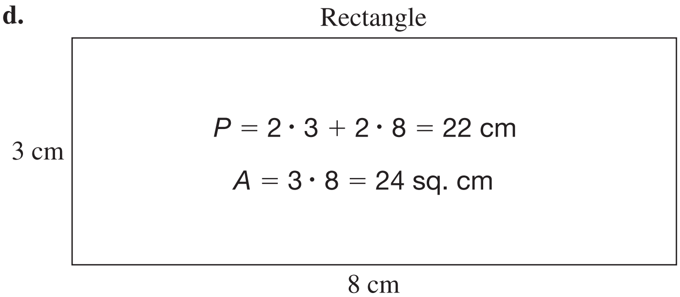
c.



$P = 4 + 4 + 6 = 14$ cm

$A = 6 \cdot 3 \div 2 = 9$ sq. cm

d.



7. Revenue and cost represent input and profit represents output.

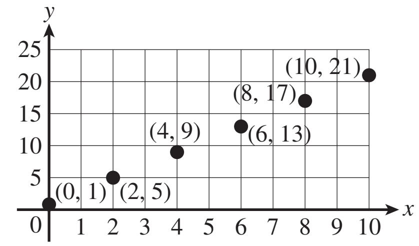
P = profit	$P = R - C$
R = revenue	$P = 400{,}000 - 156{,}800$
C = cost	$P = \$243{,}200$ is the profit.

9. Original cost, remaining value, and estimated life in years represent input and annual depreciation represents output.

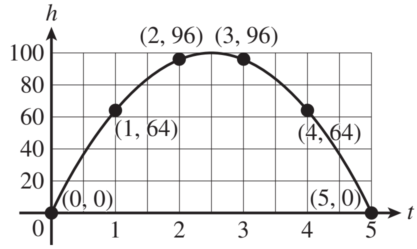
D = annual depreciation	$D = (c - v) \div y$
c = original cost	$D = (25{,}000 - 2000) \div 10$
v = remaining value	$D = \$2300$ is the annual
y = estimated life in years	depreciation of the car.

11. a. $C = 30°$, **b.** $C = 5°$; **13. a.** $y = x + 10$, **c.** $y = x - 5$, **e.** $y = 6x - 3$, **g.** $y = x^2 - 25$

Activity 2.2 Exercises: 2. a. 1, 5, 9, 13, 17, 21;
b. The x variable is the input.
c.

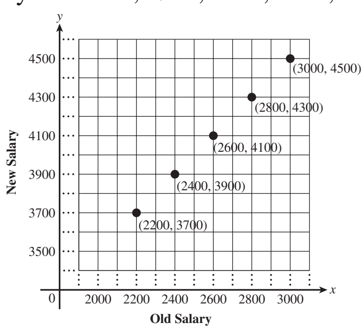


5. a. 0, 64, 96, 96, 64, 0
b.



c. I think it will reach 100 feet. **d.** At 2.5 seconds the ball will be 100 feet above the ground. **e.** The ball will hit the ground at 5 seconds, since that is when the height is again 0. **7. a.** x, T; x, F; F, C; s, A

Activity 2.3 Exercises: 1. a. My bid is equal to my opponent's bid plus \$20. **b.** The input variable is my opponent's bid, which I will call x. The output variable is my bid, which I will call y. **c.** $y = x + 20$, **d.** $y = 575 + 20 = 595$. I should bid \$595. **e.** $995 = x + 20$. My opponent bid \$975. **3. a.** $x = 148$, **c.** $w = 139$, **e.** $t = 0$, **g.** $x = 289$, **i.** $W = 5845$, **k.** $x = 1950$, **m.** $y = 0$, **o.** $x = 648$; **5. a.** Add \$1500 to the manager's old salary to obtain the new salary. **b.** $y = x + 1500$, x represents the old salary, and y represents the new salary after the increase. **c.** The input variable is x, the old salary. The output variable is y, the new salary. **d.** 3700, 3900, 4100, 4300, 4500; **e.** $x = 2600$,
f.



g. The points would fall on a straight line.

Activity 2.4 Exercises: 1. a. $x = 9$, **c.** $80 = y$, **e.** $13 = z$, **g.** $w = 2$, **i.** $t = 0$, **k.** $y = 42$;
3. a. 2000 sq. ft., **b.** 6 gal., **c.** *Wall Areas:* 448, 736, 528, 448, 416; Total: 3200

$$400g = 3200$$
$$g = 8 \text{ gal.}$$

4. a. m = number of miles $\quad 5280m = f$
f = number of feet $\quad f = 21{,}120$ ft

c. m = number of meters
c = number of centimeters

$$100m = c$$
$$c = 4900 \text{ cm}$$

5. a. $x = 132$, **c.** $w = 128$, **e.** $s = 25$, **g.** $g = 2$, **k.** $53 = y$, **m.** $w = 12$, **o.** $6413 = t$

Activity 2.5 Exercises: 1. a. Three terms are in the expression. **b.** The coefficients are 5, 3, and 2. **c.** The like terms are $5t$ and $2t$. **d.** $7t + 3s$; **2. a.** $6x + 12y$, **c.** $12x + 2y$, **e.** $39p + 46n$; **3. a.** $x = 40$, **c.** $40 = s$, **e.** $x = 15$, **g.** $y = 89$, **i.** $592 = t$, **k.** $28 = x$;
5. a. $12p + 15p + 21p = 9{,}360$, **b.** The retail price is \$195. **c.** \$1950

Activity 2.6 Exercises:

1. $x + 425 = 981$, $x = 556$ **3.** $x - 541 = 198$, $x = 739$

5. $x - 15 = 45$, $x = 60$ **7.** $13 + x = 51$, $x = 38$

9. $642 = 6x$
$107 = x$

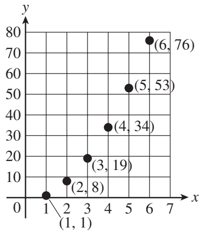
11. x = number of days driving; rental cost per day = \$75; total budget = \$600. The cost of the rental per day times the number of days driving is equal to the total cost of the rental. $75x = 600$, $x = 8$ days; Check: $75(8) = 600$; $600 = 600$. I can drive for 8 days.
13. x = amount to save each month; t = 5 months; total cost of books and fees = \$1200. The amount that I will save each month times the number of months that I will save is equal to the total cost of books. $5x = 1200$, $x = \$240$, Check: $5(240) = 1200$; $1200 = 1200$. I must save \$240 per month. **15.** better set: \$35; cheaper set: \$20; total receipts: \$525. x represents the number of better sets sold; $2x$ represents the number of cheaper sets sold. The number of sets sold times the cost per set equals the total receipts for that set. $35x + 20(2x) = 525$, $35x + 40x = 525$, $75x = 525$, $x = 7$; Check: $35(7) + 20(2)(7) = 525$;
$245 + 280 = 525$;
$525 = 525$.
I sold seven of the better sets.

17. $P = 2l + 2w$ $\quad 540 = 10w + 2w$
$P = 2(5w) + 2w$ $\quad 540 = 12w$
$P = 540$ $\quad 45 = w$

The dimensions of the field are 45 by 5(45), or 45 ft by 225 feet.
Check: $540 = 2(5)(45) + 2(45)$
$540 = 450 + 90$
$540 = 540$

How Can I Practice? 1. $P = 76$ cm; $A = 325$ sq. cm;
3. $A = (b \cdot h) \div 2$
$A = 16$ sq. in.
5. a. $t = 12h$; where h represents the number of hours worked and t represents the total weekly earnings.
c. $D = 5280m$; where D represents the distance measured in feet and m represents the distance measured in miles.

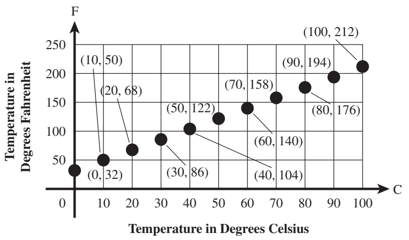
6. a. I would earn \$420 in a week. **c.** I traveled for 12 hours. **7.** Substitute 40° for C in the formula $F = (9C \div 5) + 32$. $F = (9(40) \div 5) + 32$. Multiply 9 times 40, divide by 5, and add 32. $F = 104°$. The temperature will be 104 degrees Fahrenheit when it is 40 degrees Celsius.
9. a. $y = x + 5$, **c.** $y = 3x^2$; **10. a.** 1, 8, 19, 34, 53, 76; **b.** y is the output variable.
c.



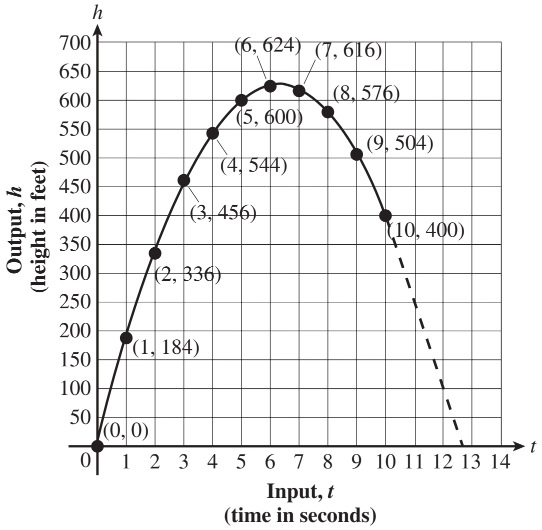
11. a. $13a + 4w$; **c.** $8x + 3y + 9$; **12. a.** $x = 11$, **c.** $37 = s$,
e. $t - 14 = 90$, $t = 104$ **g.** $15x = 765$, $x = 51$;
13. a. $30x = 1350$, $x = 45$ **c.** $2x + 5x = 56$, $x = 8$

Gateway Review

1. a. $P = 4s$, $P = 96$ in. **b.** $A = lw$, $A = 1148$ sq. cm
c. $P = 2l + 2w$, $P = 28$ ft. **d.** $A = s^2$, $A = 1089$ sq. cm
2. a. C is the input variable. **b.** (0, 32), (10, 50), (20, 68), (30, 86), (40, 104), (50, 122), (60, 140), (70, 158), (80, 176), (90, 194), (100, 212),
c.



3. a. (0, 0), (1, 184), (2, 336), (3, 456), (4, 544), (5, 600), (6, 624), (7, 616), (8, 576), (9, 504), (10, 400),
b.



c. The ball will be 400 feet above the ground at about 2.5 seconds and again at 10 seconds. **d.** The ball will go up to approximately 625 feet in the air. **e.** The ball will hit the ground at approximately 12.5 seconds.
4. a. $x = 86$, **b.** $y = 55$,
c. $12 = t$, **d.** $x = 54$,
e. $x = 5$, **f.** $168 = w$,
g. $13 = y$, **h.** $w = 47$;
5. x = amount you need to save
$240 + 420 + 370 + x = 2500$
$x = \$1470$ is the amount left to be saved.
6. $r \cdot t = A$
r = rate (20 pages of text per hour)
A = amount (260 pages)
$20t = 260$
$t = 13$ hr. It will take 13 hours to review 260 pages.
7. x = cost of each pizza

$3x + 6x + 5x = 126$
$x = \$9$ (cost of each pizza)
8. a. $351 + x = 718$, $x = 367$ **b.** $624 = 48x$, $x = 13$
c. $27x = 405$, $x = 15$ **d.** $x - 38 = 205$, $x = 243$
e. $x + 2x = 273$
$3x = 273$
$x = 91$
9. 160 words per minute is a rate. 800 words on each page and 50 pages of text means 800 times 50 pages is the total number of words that will be read (40,000).
I am asked to find the time.
The formula rate · time = amount will apply.
$r \cdot t = A$
$160t = 40{,}000$ Divide both sides of the equation by 160.
$t = 250$ min.
Check: $160(250) = 40{,}000$
It will take approximately 4 hours to read 50 pages.

Chapter 3

Activity 3.1 Exercises: 2. a. −120 points, **b.** −145 ft., **c.** \$50, **d.** −15 yd., **e.** −\$75; **3. b.** <, **d.** >, **e.** <; **4. c.** 32, **d.** 7; **5. a.** 6 is the opposite of −6. **6. b.** −100 feet is farther below sea level.

Activity 3.2 Exercises: 1. a. −2, **i.** −6, **o.** −3; **2. c.** −12, **i.** −5; **4.** The nighttime temperature is −12°F. **6.** The noon temperature was 11°F. **8.** My new elevation is −1100 feet; **11.** The total profit is \$30 million.

Activity 3.3 Exercises: 3. a. $139 - I = -7$, **b.** $I = 146$, I weighed 146 pounds at the beginning of the month.
5. a. $S = P - D$, **b.** $35 = P - 8$
$P = \$43$, the regular price;
7. b. $D = P - S$;

8. b. $x - 9 = 16$
$x = 25,$
d. $10 - x = 6$
$x = 4.$

Activity 3.4 Exercises: 1. b. $T = M + 2,$ **c.** $T = 38,$ The total cost is \$38. **3. a.** $x - y = 5.$

Activity 3.5 Exercises: 1. a. $-42,$ **c.** $-160,$ **e.** $0,$ **j.** $240,$ **l.** $5,$ **o.** $0;$ **2. b.** $192,$ **d.** $-55,$ **e.** $-36,$ **f.** $36;$ **4.** The daily drilling rate is -25 feet per day. $5(-25) = -125$. The approximate depth of the water supply is 125 feet. **8.** 96 was the score; **11.** $-\$220$, a debit.

Activity 3.6 Exercises: 1. Tiger's score was 71. **3. b.** $11,$ **d.** $-14,$ **j.** 7 **l.** $-8,$ **o.** $-225;$ **4. a.** $5,$ **c.** $-32;$ **5. a.** $3 \cdot (-5)^2 + 4(-3) = 63,$ **c.** $-5(3)[14 \cdot 3 \cdot (-2) - 3(-2)^2] = 1440,$ **d.** $\dfrac{3(-2)(-8) - 4(4)}{4(-2)(4)} = -1;$ **7. b.** $(-30)x = -150; x = 5.$ **d.** $2x = -28; x = -14;$ **8. b.** $s = -9,$ **e.** $x = 3,$ **f.** $x = 5;$ **10.** $19d = -57;$ $d = -3$, so the average drop in temperature is 3°F; **12. a.** $-3y + 2x,$ **c.** $7x + 8y;$ **13. a.** $x = 5,$ **c.** $x = -6.$

What Have I Learned? 4. b. negative, **e.** negative.

How Can I Practice? 1. a. $-3;$ **2. a.** $13,$ **b.** $15;$ **3. a.** $51,$ **c.** $-17,$ **e.** $-12,$ **h.** $5,$ **i.** $-14;$ **4. c.** $14,$ **d.** $-12,$ **g.** $-7,$ **i.** $-17;$ **5. c.** $160,$ **i.** $9,$ **k.** $-6,$ **n.** $-30;$ **6. b.** $15,$ **d.** $39,$ **f.** $9;$ **7. d.** $36,$ **f.** $-45,$ **g.** $18,$ **i.** $40;$ **8. a.** $-11,$ **d.** $-12;$ **9. c.** $85;$ **10. b.** $60,$ **c.** $= 22;$ **11. a.** $x = -84$; Check: $-84 + 21 = -63,$ **e.** $x = 36;$ Check: $36 - 17 = 19,$ **f.** $x = 77,$ Check: $77 - 35 = 42,$ **h.** $x = 39$; Check: $-28 + 39 = 11,$ **i.** $x = 13;$ Check: $10 - 13 = -3,$ **l.** $y = -39$; Check: $2(-39) = -78;$

12. c. $x - 8 = 6$
$x = 14,$
d. $x - 3 = 12$
$x = 15,$
h. $3x = -15$
$x = -5,$
k. $90 = (-3)(-6)x$
$x = 5;$

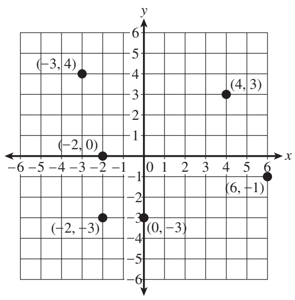
13. a. $2x + 10y,$ **c.** $10x + 7y;$ **19.** $7 - (-6) = 13;$ The change in temperature was 13°F. **22.** Her score became \$600. **24. b.** $-4;$ **26. b.** -3°F.

Gateway Review 1. a. $15,$ **b.** $23,$ **c.** $0,$ **d.** $42,$ **e.** $67;$ **2. a.** $-9,$ **b.** $-6,$ **c.** $16,$ **d.** $-14,$ **e.** $12,$ **f.** $-8,$ **g.** $-53,$ **h.** $-6,$ **i.** $-152,$ **j.** $27,$ **k.** $7,$ **l.** $54,$ **m.** $-26,$ **n.** $-24,$ **o.** $9,$ **p.** $-16,$ **q.** $-49;$ **3.** $(-1)^{13} = -1;$ **4.** $(-5)^2 = (-5)(-5) = 25, -5^2 = -(5)(5) = -25;$ **5. a.** $6,$ **b.** $-7,$ **c.** $10,$ **d.** $-7,$ **e.** $10,$ **f.** $-15,$ **g.** $-4,$ **h.** $2,$ **i.** $-40;$ **6. a.** $-21,$ **b.** $-23;$ **7. a.** $4y + 7x,$ **b.** $-2x + 5y;$ **8. a.** $x = -2,$ **b.** $x = -19,$ **c.** $x = -15,$ **d.** $x = 43,$ **e.** $x = -38,$ **f.** $6,$ **g.** $-5,$ **h.** $6,$ **i.** $5,$ **j.** $6;$

9. a. $x + 18 = -7$
$x = -25,$
b. $x + 11 = 29$
$x = 18,$
c. $15 + x = -28$
$x = -43,$
d. $x - 20 = 39$
$x = 59,$
e. $8 - x = 12$
$x = -4,$
f. $x - 17 = 42$
$x = 59,$
g. $x - 13 = -34$
$x = -21,$
h. $x \cdot (-15) = 30$
$x = -2,$
i. $3x = -15$
$x = -5,$
j. $18 = x(2)$
$x = 9;$

10. a. The noon temperature was -6°F. **b.** The evening temperature was -2°F. **c.** The change was 5°F, **d.** The change was -5°F. **11.** $27 \div 3 = 9$. Each child can have 9 Gummi Bears; **12. a.** $5(12) = 60$. I will have saved \$60. **b.** $240 \div 12 = 20$. I am saving \$20 per week now;

13.



Chapter 4

Activity 4.1 Exercises: 1. $8;$ **4.** $\dfrac{2}{7};$ **6.** $\dfrac{3}{7} > \dfrac{2}{5};$

8. We painted the same amount. **10.** $\frac{1}{20}$ of the applicants are accepted.

Activity 4.2 Exercises: 2. a. $\dfrac{6}{35},$ **d.** $-\dfrac{35}{44},$ **e.** $\dfrac{-4}{9},$ **h.** $\dfrac{16}{21};$ **3. a.** $2,$ **d.** $-\dfrac{45}{52},$ **e.** $\dfrac{27}{50},$ **f.** $-\dfrac{1}{38};$ **4. a.** $\dfrac{9}{10}$ **b.** He can expect 2250 plants to sprout; **7.** \$3245 is allocated for tuition; **10. b.** $\dfrac{2}{3}x = -4, x = -6,$ **c.** $\dfrac{x}{7} = 9, x = 63;$ **11. b.** $x = \dfrac{-3}{8},$ **d.** $y = -\dfrac{4}{9},$ **f.** $x = \dfrac{12}{13};$

Activity 4.3 Exercises: 1. $\dfrac{3}{4};$ **2.** $\dfrac{3}{5};$ **5.** $0;$ **8.** $-\dfrac{5}{7};$ **11.** $\dfrac{2}{3};$ **13.** $\dfrac{1}{2};$ **14.** $-\dfrac{4}{5};$ **17.** $-\dfrac{1}{8};$ **20.** $-\dfrac{1}{3};$ **23.** $\dfrac{7}{12};$ **27. a.** Two out of three slices, or $\frac{2}{3}$ (two-thirds) of the turkey is left for me, **b.** Four out of six ounces remain: $\frac{4}{6} = \frac{2}{3}$. **29. a.** About an additional $\frac{1}{6}$ quart will bring it up to the fill line. **b.** $\frac{1}{3}$ quart would spill out. **31. a.** $x + \dfrac{1}{5} + \dfrac{1}{3} + \dfrac{1}{4} = 1,$ **b.** $x = 1 - \dfrac{47}{60} = \dfrac{13}{60}$ So, $\frac{13}{60}$ of friend's your final grade is determined by class participation. **32.** $c = \frac{1}{2};$ **33.** $-\frac{1}{2};$ **34.** $\dfrac{11}{15};$

35. $b = \frac{1}{6}$; **36.** $-\frac{17}{21}$; **37.** $-\frac{7}{18}$;

Activity 4.4 Exercises: 1. c. $\frac{16}{49}$, **d.** $\frac{32}{243}$, **h.** 0;

2. a. $2\left(\frac{1}{2}\right) - 4\left(\frac{1}{2} - 3\right) = 11$,

b. $\frac{1}{3}\left(\frac{1}{3} - 2\right) + \frac{1}{3}\left[3\left(\frac{1}{3}\right) - 2\right] = -\frac{8}{9}$,

c. $2\left(\frac{3}{4}\right) + 6\left(\frac{3}{4} + 2\right) = 18$,

d. $6\left(\frac{5}{16}\right)\left(\frac{1}{8}\right) - \left(\frac{1}{8}\right)^2 = \frac{14}{64} = \frac{7}{32}$; **3. a.** $\frac{9}{16}$ sq. in.,

b. 9 sq. in. **c.** 9 sq. in.; **4. a.** $\frac{7}{12}$, **b.** $-\frac{11}{13}$, **c.** $\frac{4}{7}$,

d. $-\frac{6}{7}$; **6. a.** $d = 16\left(\frac{1}{2}\right)^2 = 4$ ft; **7. a.** $\frac{1}{2}$ sec.,

b. $\frac{1}{4}$ sec., **c.** $\frac{1}{3}$ sec.; **8. a.** $\frac{8}{13}$, **b.** $-\frac{8}{11}$, **c.** $-\frac{3}{8}$, **d.** $\frac{1}{6}$

How Can I Practice? 1. a. $\frac{9}{33}$, **d.** $\frac{21}{36}$; **2. c.** $\frac{12}{13}$,

e. $\frac{27}{41}$; **4. a.** JFK $\frac{1}{16}$; LaGuardia $\frac{1}{8} = \frac{2}{16}$; Newark $\frac{1}{2} = \frac{8}{16}$. Newark Airport had the best visibility. **5. b.** $\frac{8}{17}$,

e. $\frac{29}{36}$, **h.** $-\frac{19}{63}$, **i.** $-\frac{74}{91}$; **6. b.** $\frac{17}{54}$, **e.** $-\frac{3}{56}$,

f. $-\frac{17}{72}$, **l.** $\frac{23}{33}$, **n.** $-\frac{1}{60}$; **7. b.** $\frac{30}{143}$, **d.** $\frac{12}{35}$,

e. $\frac{2}{15}$, **h.** $-\frac{1}{8}$; **8. a.** $-\frac{1}{3}$, **d.** $\frac{16}{25}$, **f.** $\frac{25}{54}$; **9. b.** $\frac{9}{11}$,

c. $-\frac{27}{125}$; **11. f.** $-\frac{5}{42}$, **i.** $-\frac{1}{20}$, **j.** $\frac{3}{4}$, **k.** $-\frac{6}{7}$,

l. 100; **12. b.** $\frac{11}{15} - x = \frac{3}{5}$, $x = \frac{2}{15}$, **c.** $\frac{4}{13}x = -20$, $x = -65$, **e.** $\frac{x}{-7} = -25$, $x = 175$

Gateway Review 1. a. $>$, **b.** $>$, **c.** $<$, **d.** $>$, **e.** $>$, **f.** $<$; **2. a.** $\frac{6}{7}$, **b.** $-\frac{2}{5}$, **c.** $\frac{7}{11}$, **d.** $\frac{37}{56}$, **e.** $\frac{2}{3}$, **f.** $-\frac{29}{36}$,

g. $\frac{1}{15}$, **h.** $-\frac{2}{33}$, **i.** $\frac{2}{3}$, **j.** $-\frac{4}{5}$, **k.** $-\frac{3}{4}$, **l.** $-\frac{5}{6}$, **m.** $-\frac{3}{4}$,

n. $-\frac{1}{36}$, **o.** $-\frac{1}{10}$, **p.** $-\frac{19}{24}$, **q.** $\frac{37}{45}$, **r.** $\frac{41}{54}$, **s.** $-\frac{1}{2}$,

t. $\frac{1}{28}$, **u.** $-\frac{10}{21}$, **v.** $\frac{4}{9}$, **w.** $-\frac{3}{25}$, **x.** $-\frac{7}{9}$; **3. a.** $\frac{64}{169}$,

b. $-\frac{9}{16}$, **c.** $\frac{11}{12}$; **4. a.** $-\frac{13}{24}$, **b.** $-\frac{2}{23}$; **5. a.** $-\frac{3}{4}$,

b. $-\frac{3}{4}$; **6. a.** $x = \frac{19}{24}$, **b.** $x = -\frac{22}{63}$, **c.** $x = -\frac{1}{2}$,

d. $x = -\frac{1}{12}$, **e.** $x = -\frac{11}{14}$, **f.** $x = \frac{2}{45}$, **g.** $x = 6$,

h. $x = -84$, **i.** $x = \frac{2}{81}$, **j.** $s = -\frac{11}{12}$, **k.** $x = -9$,

l. $x = \frac{10}{29}$; **7. a.** $x + \frac{1}{2} = \frac{5}{6}, x = \frac{1}{3}$,

b. $x + \frac{2}{3} = \frac{1}{4}, x = -\frac{5}{12}$, **c.** $\frac{3}{7} - x = \frac{2}{5}, x = \frac{1}{35}$,

d. $\frac{x}{-15} = -7, x = 105$, **e.** $-\frac{3}{11}x = 18, x = -66$,

f. $\frac{2}{3} = x \cdot \frac{8}{9}, x = \frac{3}{4}$; **8. a.** $\frac{21}{50}$; 21 of every 50 tiles are vowels. **b.** Regular Scrabble has a greater proportion of vowels. **9. a.** I should only drink $\frac{8}{20}$, or $\frac{2}{5}$, of the bottle for one serving. **b.** I would consume about $\frac{11}{125}$ of the daily carbohydrates. **10. a.** $\frac{1}{29}$ of the bricks were used for one figure.

b. $\frac{1}{29}(1{,}000{,}000) = \frac{1{,}000{,}000}{29} \approx 34{,}483$ bricks,

c. I need $\frac{26}{100}(34{,}483) \approx \$8966$ to build the figure, so I have $\frac{3000}{8966}$ of the money I need. **d.** I have only $\frac{3000}{9000}$, or $\frac{1}{3}$, of the money I need. **11.** Three-eighths of my paycheck may be used for other purposes.

12. a. An estimated $\frac{10{,}000}{50{,}000{,}000}$, or $\frac{1}{5000}$, of those infected with H1N1 died as a result.

b. $\frac{1}{20}$ of those hospitalized due to H1N1 died from it.

Chapter 5

Activity 5.1 Exercises: 1. $1\frac{2}{3}$; **4.** $-\frac{23}{4}$; **8.** $1\frac{1}{3}$;

9. $7\frac{3}{5}$; **12.** $2\frac{3}{5}$; **13.** $-2\frac{2}{9}$; **17.** $-21\frac{1}{13}$;

18. I will need 45 feet of wallpaper border.

Activity 5.2 Exercises: 1. $8\frac{2}{3}$; **3.** $1\frac{1}{2}$; **6.** $19\frac{3}{20}$;

9. $8\frac{1}{3}$; **10.** $4\frac{4}{7}$; **14.** $-5\frac{11}{24}$; **16.** $5\frac{37}{56}$; **17.** $-11\frac{38}{45}$;

19. The recipe makes $4\frac{19}{24}$ cups of batter. **21.** $6\frac{3}{4}$ in. Yes, it will fit. **23.** $x = 10\frac{1}{4}$; **26.** $x = -12\frac{7}{12}$

Activity 5.3 Exercises: 1. a. $\frac{33}{5} = 6\frac{3}{5}$, **b.** $-\frac{48}{5} = -9\frac{3}{5}$,

d. 129; **2. b.** $-2\frac{1}{2}$, **d.** 4; **3. c.** $\frac{16}{49}$, **d.** $\frac{32}{243}$,

h. $\sqrt{25} = 5$; **4. b.** $8\frac{4}{9}$, **d.** $\frac{31}{32}$; **5. a.** $A = 2\frac{1}{4}$ sq. ft.,

b. I'd need at least 40 tiles to cover the floor.

7. b. $w = -4$, **c.** $t = 5\frac{1}{2}$; **9. a.** 36 ft.

What Have I Learned? 3. a. The statement is true by definition of improper fractions. **c.** The statement is false because the negative sign of the mixed number applies to both the integer part and the fractional part.

How Can I Practice? 1. a. $\frac{34}{9}$, **d.** $-\frac{138}{13}$, **e.** $-\frac{58}{31}$;
2. b. $2\frac{2}{13}$, **c.** $7\frac{5}{12}$; **3. b.** $13\frac{26}{63}$, **e.** $11\frac{17}{54}$, **f.** $4\frac{17}{72}$,
h. $13\frac{3}{8}$, **i.** $-33\frac{23}{24}$, **l.** $-20\frac{27}{70}$, **o.** $4\frac{1}{3}$, **p.** $7\frac{3}{4}$, **r.** $\frac{14}{15}$,
s. $-8\frac{11}{42}$; **4. b.** $\frac{225}{7}$ or $32\frac{1}{7}$, **c.** $\frac{6}{5}$ or $1\frac{1}{5}$; **5. a.** $\frac{3}{2}$ or $1\frac{1}{2}$,
d. $\frac{7}{6}$ or $1\frac{1}{6}$; **6. b.** $-6\frac{11}{18}$, **d.** 3, **f.** $5\frac{1}{2}$; **7. a.** $-\frac{64}{49}$,
d. $\frac{10}{9}$; **8. b.** $\frac{19}{48}$; **9. c.** $x = 31\frac{17}{42}$, **d.** $x = 11\frac{2}{9}$;
12. I will need $21\frac{1}{2}$ ounces of split peas.
14. a. $V = 2475$ cu. ft., **b.** Weight is $2475 \cdot 62\frac{2}{5} =$ 154,440 lb.

Activity 5.4 Exercises: 3. a. \$0.95, **b.** \$1.00; **5. a.** fifty-two thousandths, **d.** 0.0064, **f.** 2041.0673; **7. a.** False, since $2 > 0$ in the hundred-thousandths place.

Activity 5.5 Exercises: 1. a. 31.79, **e.** 1.227, **g.** -20.404, **i.** 15.216, **k.** -0.00428, **l.** -0.2019;
5. b.

COUNTRY	TOTAL
Australia	176.525
Brazil	174.875
China	188.900
France	175.275
Japan	176.700
Romania	181.525
Russian Federation	180.625
United States	186.525

Gold: China; Silver: USA; Bronze: Romania

6. b. $x = 0.00093$, **d.** $b = 12.877$, **f.** $z = 0.0037$;
7. $x = 91.4 - 11$
$x = 80.4$
The difference between the two kinds is 80.4 meters.

Activity 5.6 Exercises: 2. a. 102.746, **c.** -10.965, **f.** 70,250, **g.** 5000; **4.** \$1380.73 paid in taxes; **6. a.** \$2 million per show, **b.** \$2.02 million per show.

Activity 5.7 Exercises: 1. 42.82; **3.** 13.5236;
6. 0.19625 sq. in.; **10. a.** $\frac{x}{5.3} = -6.7, x = -35.51$,
c. $42.75 = x \cdot (-7.5), x = -5.7$, **e.** $(-9.4)x = 47$, $x = -5$; **11. a.** $x = 5.1$, **c.** $a = 8.5$, **f.** $x \approx -1.9$

Activity 5.8 Exercises: 1. 360 cm.; **4.** 4.705 m.; **7.** 4255 cg.; **10.** 0.742 L; **12.** 124 milligrams is smaller, since 0.15 gram = 150 milligrams. **15. a.** 56 grams of protein are needed. **b.** A 100-kilogram person would need 80 g of daily protein. **c.** Two eggs would be enough daily protein for the child.

How Can I Practice? 1. c. thirteen thousand, fifty-three and three thousand eight hundred ninety-one ten-thousandths;
2. b. 611,712.00068, **e.** 9,000,000,005.28;
3. b. $0.0983 < 0.0987 < 0.384 < 0.392 < 0.561$;
4. c. 182.1000, **d.** 60.0; **5. a.** -160, **d.** 500;
6. b. 968.366, **d.** 245.212, **f.** -138.709, **i.** 378.2877, **l.** 245.1309, **o.** -80.488, **p.** 55.708, **q.** -30.763; **7. a.** 0.0972, **d.** 1.1345, **f.** 504.6, **g.** 340.8; **8. a.** 3.5; **9. b.** -11.147, **d.** 15.188, **e.** 2.9575, **g.** 19.2; **10. b.** $x = 82.31$, **d.** $x = -22.98$, **f.** $x = -25.5$; **11. a.** $x + 4.3 = 6.28; x = 1.98$, **c.** $x - 18 = -14.76; x = 3.24$,
f. $\frac{x}{-9.5} = 78.3; x = -743.85$, **g.** $x \cdot (-9.76) = 678.32, x = -69.5$; **13.** I spent \$116.82. **14.** My take-home pay is \$1568.20. **16. a.** In 1921, Babe Ruth's batting average was 0.378. **b.** Barry Bonds's batting average was 0.328 in 2001. **c.** Babe Ruth's average is higher by 0.050. **18.** 5.01, 12.00, 6.99, 14.68, 6.66, 45.34, 2.83. **20. a.** 3.872 g, **c.** 0.392 L, **f.** 14 mL; **22.** 1580 g or 1.58 kg. **25. a.** $\frac{72}{125}$, **c.** $-7\frac{14}{25}$;
26. c. $0.31818\ldots \approx 0.318$, **d.** 0.45;

Gateway Review Exercises: 1. a. $12\frac{3}{5}$, **b.** 9, **c.** $5\frac{2}{15}$,
d. $5\frac{12}{13}$; **2. a.** $\frac{23}{5}$, **b.** $\frac{19}{7}$, **c.** $\frac{57}{11}$, **d.** $\frac{83}{8}$;
3. a. eight hundred forty-nine million, eighty-three thousand, six hundred fifty-nine and seven hundred twenty-five ten-thousandths, **b.** thirty-two billion, four million, three hundred eighty-nine thousand, four hundred twelve and twenty-three thousand, four hundred eighteen hundred-thousandths, **c.** two hundred thirty-five million, eight hundred sixty-four and five hundred eighty-seven thousand two hundred thirty-four millionths, **d.** seven hundred eighty-four million, six hundred thirty-two thousand, five hundred forty-one and eight hundred nineteen hundred-thousandths;
4. a. 65,073,412.0682, **b.** 89,000,549,613.048,
c. 7,612,011.05601; **5. a.** $\frac{85}{10{,}000} = \frac{17}{2000}$,
b. $\frac{3834}{1000} = \frac{1917}{500} = 3\frac{417}{500}$, **c.** $\frac{425}{100} = \frac{17}{4} = 4\frac{1}{4}$,
d. $\frac{150{,}125}{10{,}000} = \frac{1201}{80} = 15\frac{1}{80}$, **e.** $\frac{712}{100} = \frac{178}{25} = 7\frac{3}{25}$;
6. a. 0.63, **b.** 0.05, **c.** 0.61, **d.** -0.76;
7. a. 692,895.10, **b.** 692,895.098, **c.** 692,895.1, **d.** 692,895, **e.** 692,900, **f.** 690,000;
8. a. $4.00078 < 4.0078 < 4.07008 < 4.0708 < 4.078 < 4.78$, **b.** $3.00085 < 3.00805 < 3.0085 < 3.0805 < 3.085 < 3.85$, **c.** $8.00046 < 8.00406 < 8.0046 < 8.0406 < 8.046 < 8.46$;
9. a. 25, **b.** $19\frac{2}{3}$, **c.** $24\frac{17}{24}$, **d.** $19\frac{17}{18}$, **e.** $11\frac{3}{8}$,

f. $11\frac{2}{13}$, **g.** $2\frac{1}{6}$, **h.** $5\frac{9}{10}$, **i.** $4\frac{1}{3}$, **j.** $3\frac{3}{7}$, **k.** $4\frac{1}{2}$, **l.** $-14\frac{7}{18}$, **m.** $-1\frac{19}{24}$, **n.** $15\frac{7}{45}$, **o.** $-13\frac{5}{36}$, **p.** $-12\frac{5}{18}$, **q.** $-7\frac{4}{5}$, **r.** $-\frac{10}{21}$, **s.** $1\frac{1}{3}$, **t.** $7\frac{1}{7}$, **u.** -4, **v.** $1\frac{1}{4}$; **10. a.** -17.62, **b.** 29.07, **c.** -32.12, **d.** -23.37, **e.** -40.97, **f.** -152, **g.** 7, **h.** 15,400, **i.** 0.003912; **11. a.** -1.332, **b.** $-\frac{1}{4}$; **12. a.** $12.68 + 7.05 = 19.73$, **b.** $98.99 - 14.85 = 84.14$, **c.** $-17.32 + (-4.099) = -21.419$, **d.** $3.98 - 0.125 = 3.855$, **e.** 24, **f.** ≈ 16.28; **13. a.** $x = 9\frac{19}{24}$, **b.** $x = -7\frac{22}{63}$, **c.** $x = -2\frac{7}{12}$, **d.** $x = -33\frac{1}{2}$, **e.** $x = -24\frac{1}{14}$, **f.** $x = 8\frac{31}{36}$, **g.** $x = 25.55$, **h.** $x = -46.05$, **i.** $x = 48.5$, **j.** $x = -18.97$, **k.** $x = 7.47$, **l.** $x = 40\frac{1}{2}$, **m.** $x = -60$, **n.** $w = -1\frac{1}{11}$, **o.** $n = 9.23$, **p.** $x = -20$; **14. a.** $\frac{x}{-15} = -0.7$; $x = 10.5$, **b.** $\left(\frac{-11}{12}\right)x = 2\frac{1}{16}$, $x = -2\frac{1}{4}$, **c.** $1.08 = x(0.2)$; $x = 5.4$; **15.** I would use $41\frac{1}{3}$ ft. of border. **16.** I ran $1\frac{14}{15}$ mi. each way. **17.** $10\frac{7}{12}$ ft. taken up by the couch and end tables; $3\frac{5}{12}$ ft. remaining. The bookcase will fit. **18.** The price of the TV was $194.84. **19.** I would receive $1.65 in change. **20.** The sale price of the dress is $57.76. **21.** The cook's take-home pay is $1187.20. **22.** I will have $219.59 in my bank account. **23.** 202 servings, plus a little left over; **24.** The average population decrease per year was approximately 552 persons per year. **25.** $\approx$ 0.002 sq. mi. per person; **26.** 2.03 million people had farm-related occupations in 1820. **27.** $48.60; **28.** Estimating, $\frac{7000}{10} = 700$ mph. $7318 = r \cdot 11\frac{3}{4}$. So, solving for r, $r \approx 623$ mph, close to the estimate. **29. a.** revenue $= \$15.85b + \$9.95h$, **b.** revenue $= (15.85)(26) + (9.95)(13) = \$541.45$. **30. a.** $87.00, $130.50, $174.00, $217.50, **b.** I multiplied the number of hours by $7.25 per hour. **c.** $200 = 7.25x$, $x \approx 27.59$ hr. I would need to work 28 hr. **31. a.** $x + 2$, **b.** $x + 2 + 2x$, **c.** $x + 2 + 2x + x - 4 + \frac{x}{4} = 4\frac{1}{4}x - 2$, **d.** $d = (52)\left(4\frac{1}{4}x - 2\right)$, **e.** $d = 1443$ mi. I would have traveled 1443 miles. **32. a.** 0.795 g, **b.** 2750 m, **c.** 0.025 L, **d.** 1050 g, **e.** 2350 m = 2,350,000 mm, **f.** 85 mL; **33.** 2.479 km; **34.** 965 mg or 0.965 g; **35.** There are 1495 mL left, or 1.495 L.

Chapter 6

Activity 6.1 Exercises: 2. a. See answers to part b for the matching. **b.** $\frac{12}{27} = \frac{20}{45} \approx 0.444 = 44.4\%$, $\frac{28}{36} = \frac{21}{27} \approx 0.778 = 77.8\%$, $\frac{45}{75} = \frac{42}{70} = 0.6 = 60\%$, $\frac{64}{80} = \frac{60}{75} = 0.80 = 80\%$, $\frac{35}{56} = \frac{25}{40} = 0.625 = 62.5\%$; **4. a.** 0.296; **5. a.** $\frac{1720}{3200}$, **b.** $\frac{43}{80}$, **c.** 0.5375, **d.** 53.75%;

7.

	NUMBER OF GAMES	RATIO OF WINS TO GAMES PLAYED
Regular season:	$92 + 70 = 162$	$\frac{92}{162} \approx 0.568 = 56.8\%$
Playoff season:	$11 + 3 = 14$	$\frac{11}{14} \approx 0.786 = 78.6\%$

The Phillies played better in the playoffs, relatively speaking.

9. Brand A: $\frac{2940}{13{,}350} \approx 0.22 = 22\%$

Brand B: $\frac{730}{1860} \approx 0.39 = 39\%$

Brand A dishwasher has a better repair record.

Activity 6.2 Exercises: 2. a. $x = 24$; **5.** My friend earned $175 last year. **7.** 1809 consumers in the sample reported problems with transactions online.

Activity 6.3 Exercises: 1. a. 8, **b.** 27, **c.** 15, **e.** 18; **l.** 60,000; **3.** About 32.8 million people worldwide are infected with AIDS. **5.** The state sales tax is $1800. **7.** There are approximately 49,000 registered voters. **10.** I need to make about 1000 phone calls. **12.** Bookstore makes $47 on the resale.

Activity 6.4 Exercises: 1. a. 360 students, **b.** The full-time enrollment increased by 11.25% from last year. **3.** Actual increase: $0.50; percent increase: 4%; **5.** Amount of decrease: 600 calories; percent decrease: 25%; **7. b.** Actual increase: 25; percent increase: 100%; **8.** For a quantity that triples in size, the percent increase is 200%.

Activity 6.5 Exercises: 2. a. growth factor: 1.08375, **b.** total cost: $19,446.81; **5.** The 2000 population was approximately 5,130,371. **7.** She must earn $71,400. **8.** The investment will be worth $3072. **10. a.** growth factor: 1.73, **b.** 843 billion gigabytes.

Activity 6.6 Exercises: 2. The length of the article must be reduced by 25%. **3.** decay factor: $\approx$ 0.914; percent decrease: 8.6%; **6.** decay factor: 0.70; sale price: $90.97;

8. a. 100,000 tigers is the lower estimate for the tiger population. **b.** 140,000 tigers is the upper estimate for the tiger population. **9.** 0.95 is the decay factor. \$530.67 is the discounted online fare for the fully refundable ticket. \$259.92 is the discounted online fare for the restricted ticket.

Activity 6.7 Exercises: 1. a. 30% off means 0.70 is the decay factor. 20% off means 0.80 is the decay factor, **b.** Sale price of the suit is \$168; **3.** The remaining inventory is 500 toys. **5.** The current budget is \$514,425. **7. a.** The effective decay factor is $(0.60)(0.75) = 0.45$. So the final cost is \$180. **b.** The effective percent discount is 55%. **9.** Her starting salary for the new job was \$29,925, which was less than her starting salary for her previous job.

Activity 6.8 Exercises: 1. 300 ft. **3.** It will take 15 min. **5.** \$119,600 is the total gross salary for the next 5 years. **7.** ≈ 5.5 mi., ≈ 8.848 km, ≈ 8848 m; **9.** 4.8 qt., 9.5 pt.; **12.** 4.8 g, 0.168 oz. ≈0.2 oz. **15.** A 150-lb. woman on Earth will weigh about 53.1 lb. on Mars. **17.** 4.4 million births were expected in 2007, based on this data.

What Have I Learned? Exercises: 1. I answered more questions correct on the practice exam (32) than on the actual exam (16). I scored $\frac{32}{40} = \frac{4}{5} = 0.8 = 80\%$ on the practice exam. I scored $\frac{16}{20} = \frac{4}{5} = 0.8 = 80\%$ on the actual exam. No, the relative scores were the same. So, I did the same on the actual exams as I did on the practice exam; **3.** Approximately 3,183,116 Florida residents were 65 years or older in 2007; **6. a.** Decay factor: $100\% - 10\% = 90\% = 0.90$, $199 \cdot 0.90 = 179.1$ lb. My relative will weigh 179.1 lb. **b.** $179.1 \cdot 0.90 \approx 161.2$ lb., $161.2 \cdot 0.90 \approx 145$ lb. He must lose 10% of his body weight 3 times to reach 145 lb.

How Can I Practice? Exercises: 1. a. 0.25, **e.** 2.50, **f.** 0.003; **5.** current staff: 3750 employees; **7. a.** final cost: \$224, **b.** decay factor: 0.56. effective percent discount: 44%; **9.** former rent: \$750. **11.** decay factor: 0.77, Number of new Ford Explorers sold in 2006 was 178,983. **13.** I would earn \$93,600 over the next 2 years.

Gateway Review Exercises: 1. a. 35, **b.** $\frac{2}{5}$, **c.** 11.88, **d.** 6500, **e.** ≈ 38,083.33, **f.** ≈ 6.22; **2. a.** $20 = x$, **b.** $x = 5$, **c.** $\frac{3}{2} = x$, **d.** $5.41 \approx x$, **e.** $\frac{3}{7} = x$, **f.** $x = 12$; **3.** 1280 students placed shopping at the top of their list. This represents approximately 53.3% of the student body. **4.** total mailing list: 900 envelopes; **5.** number of layoffs: ≈ 6000, percent of workforce: ≈ 8%; **6.** growth factor: 1.058, percent increase: ≈ 5.8%; **7.** decay factor: 0.25, amount remaining in the bloodstream: 162.5 mg; **8.** sales revenue: \$624,000; **9.** growth factor: 1.11, number of new Escape Hybrids sold in 2006 was 19,267. **10. a.** \$4253, **b.** \$5536, **c.** \$6090; **11.** ≈ 34.12 ft.; **12.** 1. O'Neal: 56.1%, 2. Garnet: 52.1%, 3. Bryant: 49.7%; **13.** I can travel almost 594 mi. **14.** I run at 8.8 ft/sec. **15. a.** $x = 2$,**b.** $x = 90$, **c.** $x = 12$; **16.** 100-pound bag of flour will cost \$70.

Chapter 7

Activity 7.1 Exercises: 1. a. $P = 80$ ft., **b.** $P = 120$ ft., **c.** $P = 160$ ft.; **3. a.** $P = 3045$ mi., **b.** 5.075 hr.; **5. b.** $P = 17$ in., **d.** $P = 12$ mi.; **7.** Since $P = 2l + 2w$, then $75 = 2(10) + 2w$. Solving this equation, $w = \frac{55}{2} = 27.5$ m.

Activity 7.2 Exercises: 2. a. For the outer circle, the circumference is $C_1 \approx 904.78$ ft. For the inner circle, the circumference is $C_2 \approx 345.58$ ft. I could walk around the innermost circle ≈ 2.62 times.

3. a. $C = \pi \cdot 3 \approx 9.4$ cm, **d.** $\frac{1}{4}C = \frac{1}{4} \cdot 2 \cdot \pi \cdot 2 = \pi \approx 3.1$ in.; **4.** Since $C = 2\pi r$, then $63 = 2\pi r$. Solving for r, $r = \frac{63}{2\pi} \approx 10.03$ inches.

Activity 7.3 Exercises: 2. $P \approx 15.71$ ft.; **4. a.** $P = 21.2$ cm, **c.** $P = 29$ m.

Activity 7.4 Exercises: 1. a. $A = 660$ sq. ft., **b.** 72 feet of 10-foot widths are needed. **3. a.** $A = 13.5$ sq. ft.; **5.** $A = 4700$ sq. ft.

Activity 7.5 Exercises: 1. The larger pizza has an area of approximately 153.94 sq. in.; the smaller pizza has an area of approximately 78.54 sq. in. So two smaller pizzas are about the same size as one larger pizza. **2. a.** The diameters for a quarter, nickel, penny, and dime are: 2.4 cm, 2.1 cm, 1.9 cm, and 1.8 cm. **b.** The areas for a quarter, nickel, penny, and dime are 4.52 sq. cm., 3.46 sq. cm., 2.84 sq. cm., and 2.54 sq. cm. **3. b.** $A \approx 28.27$ sq. mi., **d.** $A \approx 0.35$ sq. in.

Activity 7.6 Exercises: 1. $A = 15 \cdot 25 = 375$ sq. ft.; **3.** $A = 66$ sq. ft.; **6.** $A \approx 1086$ sq. in.

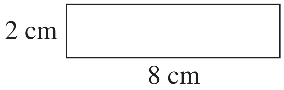
Activity 7.7 Exercises: 2. If they did meet at right angles, then the Pythagorean Theorem would apply, and $18^2 = 12^2 + 14^2$. But $18^2 = 324$ and $12^2 + 14^2 = 144 + 196 = 340$. And $324 \neq 340$. So, the walls do not meet at right angles. **3. a.** $12^2 = a^2 + 9^2$, $a \approx 7.94$ ft., **b.** $c^2 = 10^2 + 9^2 = 181$, $c = \sqrt{181} \approx 13.45$ ft. The hypotenuse of the cross becames 13.45 feet; **4. a.** $c^2 = 6^2 + 11^2 = 157$, so $c \approx 12.53$ cm; **6. a.** Yes, because $13^2 = 169$ and $5^2 + 12^2 = 169$. **b.** No, beacuse $15^2 = 325$ and $5^2 + 10^2 = 125$. **7.** $c \approx 7.62$ mi.; **9.** no, since $15^2 \neq 7^2 + 10^2$

Activity 7.8 Exercises: 1. a. $S \approx 284$ sq. in.; **3. a.** $S \approx 319$ sq. in.; **4.** $S = 60.2$ sq. ft.;

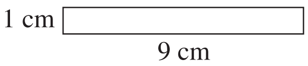
6. $S = 2\pi \cdot r^2 + 2\pi \cdot rh$
$300 = 2\pi \cdot 5^2 + 2\pi \cdot 5h$
$300 \approx 157.08 + 31.42h$
$h \approx 4.55$ in.

Activity 7.9 Exercises: 1. a. $V \approx 423$ cu. in., **b.** $V = 72$ cu. in.; **3.** $V \approx 35$ cu. in.; **4.** $V = x^3 = 42$
$x = \sqrt[3]{42} \approx 3.48$ So the dimensions are 3.48 feet by 3.48 feet by 3.48 feet.

What Have I Learned? Exercises: 5. No, for example:

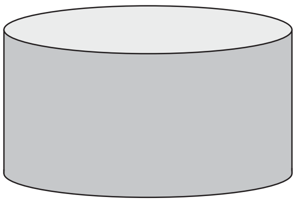


has perimeter 20 cm and area 16 sq. cm.



has perimeter 20 cm and area 9 sq. cm.

So two rectangles can have the same perimeter and different areas.
14. Yes; for example, consider a can of height 1 foot and radius 1 foot.



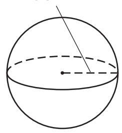
$V = \pi \cdot 1^2 \cdot 1 = \pi$ cu. ft. Next, if you double the height to 2 ft, then $V = \pi \cdot 1^2 \cdot 2 = 2\pi$ cu. ft., which *is* double the original volume.

Because of the placement of h as a factor of $\pi \cdot r^2 \cdot h$ in the formula for volume of a cylinder, replacing the height with its double will always just double the volume.

15. $V = \pi \cdot r^2 \cdot h = \pi \cdot \left(\frac{d}{2}\right)^2 \cdot h = \frac{\pi \cdot d^2 \cdot h}{4}$, the formula for volume of a cylinder in terms of its diameter and height. So doubling d means replacing d by $2d$ in the formula. The new formula becomes $V = \frac{\pi \cdot (2d)^2 \cdot h}{4} = \pi \cdot d^2 \cdot h$. So, comparing volumes, the new volume is four times larger than the initial volume. Therefore, the volume does not double; it quadruples.

How Can I Practice? Exercises: 1. a. Perimeter = 30 ft., Area = 40 sq. ft.; **2. a.** Perimeter ≈ 13.14 ft., Area ≈ 9.57 sq. ft.; **3. a.** Area = 12 sq. m;
7. Triangle C is a right triangle because $a^2 + b^2 = c^2$: $13^2 = 169 = 12^2 + 5^2$.
13. a. $V = \pi \cdot \left(\frac{4.5}{2}\right)^2 \cdot 6.5 \approx 103.38$ cu. ft.
$S = 2\pi \cdot \left(\frac{4.5}{2}\right)^2 + 2\pi \cdot \left(\frac{4.5}{2}\right) \cdot 6.5 \approx 31.81 + 91.89$
≈ 123.70 sq. ft.,
b. $V \approx 341.06$ cu. ft.,
$S = 128.7 + 151.58 + 47.7 = 327.98$ sq. ft.;

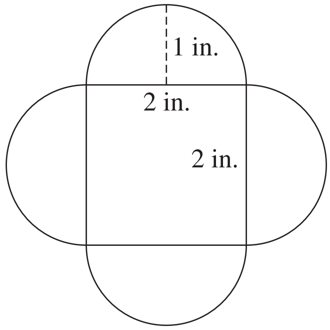
14. a. 2.05 cm.



$53 = 4\pi \cdot r^2$. Solving, $r^2 \approx 4.22$. So, $r \approx \sqrt{4.22} \approx 2.05$ cm.

Gateway Review 1. a. 10 sq. ft., **b.** 48 in., **c.** 84 sq. in., **d.** 8.5 sq. mi.; **2.** $23 = \pi \cdot r^2$. Solving, $r^2 \approx 7.32$. Therefore, $r \approx \sqrt{7.32} \approx 2.71$ in.

3. a.



b. $P \approx 12.57$ in., $A \approx 10.28$ sq. in.,
c. $A_{\text{table}} = 12 \cdot 12 = 144$ sq. in. The remaining space is $144 - 10.28 = 133.72$ sq. in. **4. a.** $A = 36$ sq. ft. I need to buy at least 36 sq. ft. of material. **b.** I need to determine the hypotenuses of the four right triangles:
$c_1 = \sqrt{3^2 + 4^2} = 5$ ft.
$c_2 = \sqrt{6^2 + 4^2} = \sqrt{52} \approx 7.21$ ft.
So, $P \approx 2 \cdot 5 + 2 \cdot 7.21 = 24.42$ ft.
I need to buy at least 25 ft. of ribbon.
5. a. $P = 210$ ft., **b.** $A = 2116$ sq. ft., **c.** A: 285 sq. ft., B: 285 sq. ft., and C: 374 sq. ft., so, bedroom C is the largest. **d.** (Answers will vary.) The area of the living room is $18 \cdot 22 = 396$ sq. ft. Doubling this area would require 792 sq. ft. So, perhaps the easiest way to do this is to increase the length of the living room to 36 ft. and keep the width the same, 22 ft. This produces an area of 792 sq. ft.
6. $c = \sqrt{300^2 + 200^2} = \sqrt{130{,}000} \approx 360.56$ ft. The diagonal length of the plot is approximately 361 feet;
7. $PQ = \sqrt{200}$, $QR = \sqrt{200}$, $PR = \sqrt{400}$, so $(PR)^2 = (PQ)^2 + (QR)^2$; **8. a.** $S = 1650$ sq. ft., $V = 4500$ cu. ft.; **b.** $S \approx 164.93$ sq. ft., $V = \approx 157.08$ cu. in.; **9.** $V \approx 4{,}094{,}485{,}458$ cu. ft.,
10. a. $800 \text{ mi.} \cdot \frac{5280 \text{ ft.}}{1 \text{ mi.}} = 4{,}224{,}000$ ft.
So, $V = \pi \cdot 2^2 \cdot 4{,}224{,}000 \approx 53{,}080{,}350$ cu. ft.
b. $S \approx 53{,}080{,}350$ sq. ft. of material.;
11. $\sqrt{85} \approx 9.22$ **12.** The distance from A to $B = 5$. The distance from B to $C = \sqrt{61}$. The distance from A to $C = \sqrt{90}$. No, the three points do not form a right triangle because the sum of the squares of the lengths of the two shorter sides, $25 + 61 = 86$, does not equal the square of the length of the longest side, 90. The lengths of the sides do not satisfy the Pythagorean Theorem.

Chapter 8

Activity 8.1 **Exercises: 1. a.** $p = 420n + 4500$;
b.

Number of Months, n	1	2	3	4	5	6
Total Amount Paid, p ($)	4920	5340	5760	6180	6600	7020

c. $p = 8280$, After 9 months, I had paid $8280 toward the price of the car. **d.** $n = 60$, It will take $60/12 = 5$ years to pay off the loan. **3. a.** $x = -1$; **c.** $x = \frac{7}{2} = 3\frac{1}{2}$, **e.** $x = 0.6$, **g.** $x = -15$, **i.** $x = 98$;
4. a. Multiply the number of minutes over 200 by 0.45 and then add 29.99 to determine the total monthly cost. **b.** $c = 0.45n + 29.99$, **c.** $c = 0.45 \cdot 250 + 29.99 = \$142.49$, The total bill would be $142.49 **d.** $n = 70$, I went over by 70 minutes.
6. $h = 70$ in.;
7. a.

x	y
4	−2
12	14

c.

x	y
$\frac{2}{3}$	13
6	−3

Activity 8.2 **Exercises: 1. a.** cost $= 0.19x + 39.95$, **b.** cost $= 0.49x + 19.95$, **c.** $0.19x + 39.95 = 0.49x + 19.95$, **d.** $x = 66.\overline{6} \approx 67$ mi., **e.** Company 1 would be the better deal. **3. a.** cost $= 3560 + 15x$, **b.** cost $= 2580 + 28x$, **c.** $3560 + 15x = 2850 + 28x$, **d.** $x \approx 55$ months, The total costs would be the same after 55 months, or about $4\frac{1}{2}$ years. **e.** 10 years is equal to 120 months. Dealer 1 has the better deal for 10 years of use. **6.** $x = -6$; **8.** $x = 3$; **10.** $x = -156$

Activity 8.3 **Exercises: 2. a.** $F = 77$, When the Celsius temperature is 25°, the Fahrenheit temperature is 77°. **b.** $C = 15$, When the Fahrenheit temperature is 59°, the Celsius temperature is 15°. **4. a.** $E = 2964$, The average per capita health care expenses for a 30-year-old person are about $2964. **b.** $A \approx 67$, According to the model, expenses for health care reach $8000 at approximately age 67.

How Can I Practice? 1. $x = -6$; **2.** $x = 1$; **3.** $x = \frac{5}{3} = 1\frac{2}{3}$; **4.** $x = \frac{11}{4} = 2\frac{3}{4}$; **5.** $x = \frac{-11.5}{4} = -2.875$; **6.** $x = \frac{-10}{6} = -1\frac{2}{3}$; **7.** $x = 2$; **8.** $x = 50$; **9.** $-4 = x$; **10.** $1 = x$; **11.** $x = 5$; **12.** $x = 12$; **13.** $x = -2$; **14.** $x = \frac{-0.6}{-0.08} = 7.5$;
15. a. $t = 2010 - 1960 = 50$
$a = 0.11(50) + 22.5 = 28$ years,
b. $t = \frac{7.5}{0.11} \approx 68$
$1960 + 68 = 2028$,
c. $t = \frac{a - 22.5}{0.11}$, **d.** $t = \frac{30 - 22.5}{0.11} \approx 68$; $1960 + 68 = 2028$; **16. a.** $c = 10 + 0.03x$,
b.

NUMBER OF COPIES, x	TOTAL COST, c ($)
1000	40
2000	70
3000	100
4000	130
5000	160

c. $c = \$250$, **d.** $x = \frac{290}{0.03} = 9666.\overline{6}$. I can have almost 9700 copies printed for $300. **17. a.** $C = 750 + 0.25x$, **b.** $C = 875$. The total cost is $875. **c.** $x = \frac{250}{0.25} = 1000$, 1000 booklets can be produced. **d.** $R = 0.75x$,
e. $R = C$
$0.75x = 750 + 0.25x$
$x = 1500$
1500 booklets must be sold to break even. The cost and revenue would both be $1125. **f.** $P = 500$, The sale of 2500 books will result in a profit of $500.
18. a.

x	y
6	13
16	53

b.

x	y
12	−140
−23	35

19. a. $t = 155n + 20$,
b.

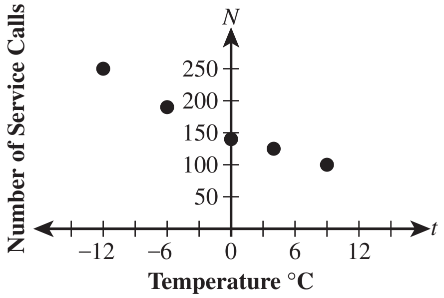
Number of Credit Hours, n	1	2	3	4	5	6
Total Tuition, t	175	330	485	640	795	950

c. $t = \$1415$, **d.** $n \approx 6.32$, I can take 6 credits.

Gateway Review 1. $x = -13$; **2.** $x = 306$; **3.** $x = 24.375$; **4.** $x = 7$; **5.** $x = 20$; **6.** $x = -13$; **7.** $x = 4$; **8.** $x = 6.\overline{54}$; **9. a.** 32, **b.** 46.4, **c.** 24; **10. a.** $S = 100 + 0.30x$, **b.** $S = 150 + 0.15x$, **c.** $100 + 0.30x = 150 + 0.15x$, **d.** $x \approx 333.33$ For $333.33 in sales per week, the salaries for both options would be the same. **e.** The salary is approximately $100 + 0.30(333.33) = 199.999 \approx \$200$. **11. a.** $x + 10$,

b. $C = x + 10 + 55 = x + 65$, **c.** $x + x + 10 + x + 65 = 3x + 75$, **d.** $x = 15$ mi. is the swimming distance, **e.** Running: $15 + 10 = 25$ mi. Cycling: $25 + 55 = 80$ mi. or $15 + 65 = 80$ mi.,

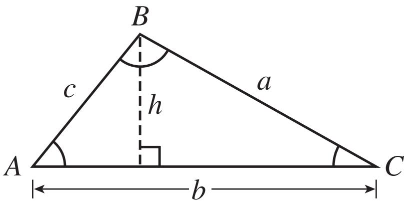
12. a.



b. $C = 296.1 \approx 296$ calls, **c.** $t \approx 14.6$, about 15°C, **13. a.** ABC: $C = 50 + 0.20x$ Competition: $C = 60 + 0.15x$, **b.** $50 + 0.20x = 60 + 0.15x$, **c.** $x = 200$, If I drive the car 200 miles, the cost would be the same. **d.** It would be cheaper to rent from the competition. **14. a.** $C = 600 + 10x$, **b.** $R = 40x$, **c.** $P = 40x - (600 + 10x) = 30x - 600$, **d.** $x = 20$ campers, **e.** $x = 40$ campers, **f.** $30(10) - 600 = 300 - 600 = -300$. The camp would lose \$300.

Geometric Formulas

Perimeter and Area of a Triangle, and Sum of the Measures of the Angles

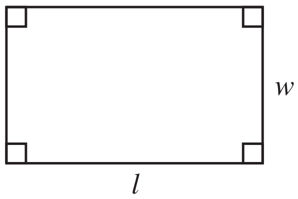


$P = a + b + c$

$A = \frac{1}{2}bh$

$A + B + C = 180°$

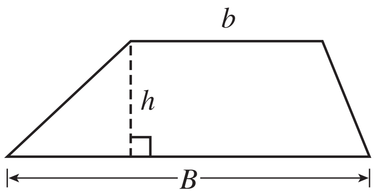
Perimeter and Area of a Rectangle



$P = 2l + 2w$

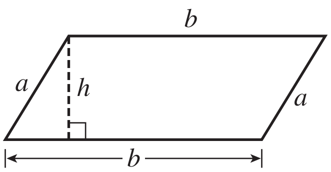
$A = lw$

Area of a Trapezoid



$A = \dfrac{1}{2}h(b + B)$

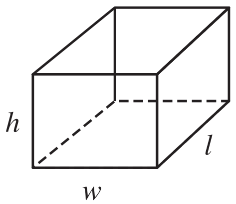
Perimeter and Area of a Parallelogram



$P = 2a + 2b$

$A = hb$

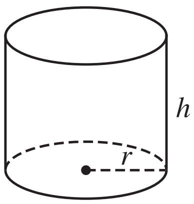
Volume and Surface Area of a Rectangular Solid



$V = lwh$

$SA = 2lw + 2lh + 2wh$

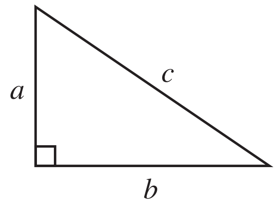
Volume and Surface Area of a Right Circular Cylinder



$V = \pi r^2 h$

$SA = 2\pi r^2 + 2\pi rh$

Pythagorean Theorem

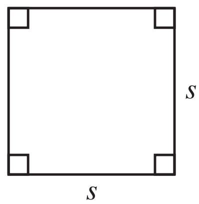


$a^2 + b^2 = c^2$

Distance Formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, where (x_1, y_1) and (x_2, y_2) are two points in the plane.

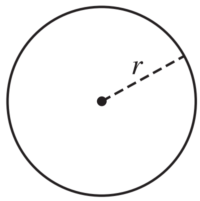
Perimeter and Area of a Square



$P = 4s$

$A = s^2$

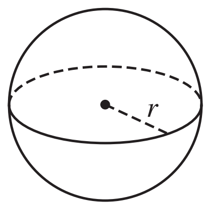
Circumference and Area of a Circle



$C = 2\pi r$

$A = \pi r^2$

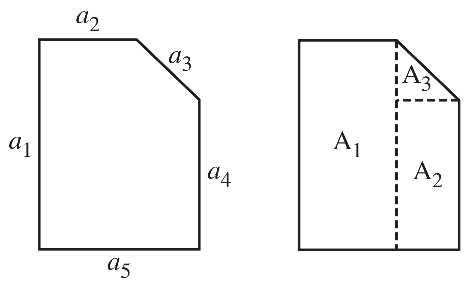
Volume and Surface Area of a Sphere



$V = \dfrac{4}{3}\pi r^3$

$SA = 4\pi r^2$

Perimeter and Area of a Polygon



$P =$ sum of the lengths of the sides

$= a_1 + a_2 + a_3 + a_4 + a_5$

$A =$ sum of the areas of each figure

$= A_1 + A_2 + A_3$

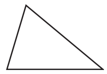
Glossary

absolute value of a number The size or magnitude of a number. It is represented by the distance of the number from zero on a number line. The absolute value is always nonnegative.

actual change The difference between a new value and the original value of a quantity.

acute angle An angle that is smaller than a right angle (measures less than 90°).

acute triangle All angles in the triangle measure less than 90°.



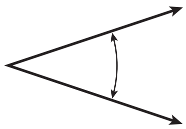
addends The numbers that are combined in addition. For example, in $1 + 2 + 7 = 10$, the numbers 1, 2, and 7 are the addends.

addition The arithmetic operation that determines the total of two or more numbers.

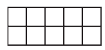
addition property of zero The sum of any number and zero is the same number.

algebraic expression A symbolic code of instructions for performing arithmetic operations with variables and numbers.

angle The figure formed by two rays meeting at the same point. The size of the angle is the amount of turn needed to move one ray to coincide with the other ray.



area A measure of the size of a region that is entirely enclosed by the boundary of a plane figure, expressed in square units.



associative property of addition For all numbers a, b, and c, $(a + b) + c = a + (b + c)$. For example, $(2 + 3) + 4 = 5 + 4 = 9$ and $2 + (3 + 4) = 2 + 7 = 9$.

associative property of multiplication For all numbers a, b, and c, $ab(c) = a(bc)$. For example, $(2 \cdot 3) \cdot 4 = 6 \cdot 4 = 24$ and $2 \cdot (3 \cdot 4) = 2 \cdot 12 = 24$.

average (arithmetic mean) A number that represents the center of a collection of data values. The arithmetic mean is determined by adding up the data values and dividing the sum by the number of data values in the collection.

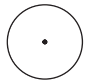
bar graph A diagram that shows quantitative information by the lengths of a set of parallel bars of equal width. The bars can be drawn horizontally or vertically.

base number The number that is raised to a power. In N^P, N is the base number and P is the exponent.

break-even point The point at which two mathematical models for determining cost or value will result in the same quantity.

Cartesian (rectangular) coordinate system in the plane A two-dimensional scaled grid of equally spaced horizontal lines and equally spaced vertical lines that is used to locate the positions of points on a plane.

circle A collection of points that are the same distance from a given point, called the center of the circle.



circumference of a circle The distance around a circle. If a circle has radius r, diameter d, and circumference C, then $C = 2\pi r$ and $C = \pi d$.

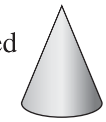
coefficient (numerical) A number written next to a variable that multiplies the variable. For example, $2x$ means that 2 multiplies the variable x.

commutative property of addition For all numbers a and b, $a + b = b + a$, which means that changing the order of the addends does not change the result.

commutative property of multiplication For all numbers a and b, $ab = ba$, which means that changing the order of the factors does not change the result.

composite number A whole number greater than 1 that is not prime.

cone A three-dimensional figure consisting of a curved surface that is joined to a circular base at the bottom and a fixed point at the top.



constant A term in an expression that consists of only a number.

cumulative effect of consecutive sequence of percent changes The product of the associated growth or decay factors.

coordinates of a point An (input, output) label to locate a point in a rectangular coordinate system.

counting numbers The set of positive whole numbers $\{1, 2, 3, 4, \ldots\}$.

cube root The cube root of a number N is a number M whose cube is N. The symbol for the cube root is $\sqrt[3]{}$. For example, the cube root of 8 is 2 $\left(\sqrt[3]{8} = 2\right)$ since $2^3 = 8$.

cylinder *See* **right circular cylinder.**

decay factor When a quantity decreases by a specified percent, the ratio of its new value to its original value is called the decay factor associated with the specified percent decrease.

$$\text{decay factor} = \frac{\text{new value}}{\text{original value}}$$

The decay factor is also formed by subtracting the specified percent decrease from 100% and then changing the percent into decimal form.

decimal number A number with a whole part, to the left of the decimal point, and a fractional part, to the right of the decimal point.

denominator The number written below the line of a fraction; the denominator represents the number of equal parts into which a whole unit is divided.

diameter of a circle A line segment through the center of a circle that connects two points on the circle; the measure of the diameter is twice that of the radius.

difference The result of subtracting the subtrahend from the minuend. For example, in $10 = 45 - 35$, the number 10 is the difference, 45 is the minuend, and 35 is the subtrahend.

digit Any whole number from 0 to 9.

dimensional analysis *See* **unit analysis.**

distributive properties For all numbers a, b, and c, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.

dividend In a division problem, the number that is divided into parts. For example, in $27 \div 9 = 3$, the number 27 is the dividend, 9 is the divisor, and 3 is the quotient.

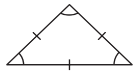
division by 0 Division by 0 is undefined.

division by 1 Any number divided by 1 is the number itself.

divisor In a division problem, the number that divides the dividend. For example, in $27 \div 9 = 3$, the number 27 is the dividend, 9 is the divisor, and 3 is the quotient.

equation A statement that says two expressions are equal, or represent the same number.

equiangular triangle All three angles in the triangle have equal measure, which is 60°.



equilateral triangle All three sides of the triangle have equal length.

equivalent expressions involving a single variable If, for every possible value of the variable, the values of the expressions are equal, the expressions are said to be equivalent.

equivalent fractions Fractions that represent the same ratio. For example, $\frac{4}{6}$ and $\frac{10}{15}$ both represent the ratio $\frac{2}{3}$. Divide both the numerator and denominator of $\frac{4}{6}$ by 2 to obtain $\frac{2}{3}$. Similarly, divide both the numerator and denominator of $\frac{10}{15}$ by 5 to obtain $\frac{2}{3}$.

estimate of a numerical calculation An estimate is the result of rounding numbers in a calculation so that the numbers are easier to combine. The estimate is close to the actual result.

evaluate an algebraic expression Substitute a number for a variable and simplify the resulting numerical expression by applying the order of operations rules.

even number A whole number that is divisible by 2, leaving no remainder.

exponent If a number N is raised to a power P, meaning

$$N^P = \underbrace{N \cdot N \cdot \ldots \cdot N}_{(P \text{ factors})}$$

the power P is also called an *exponent.*

exponential form A number written in the form b^x, where b is called the base and x is the power or exponent. For example, 2^5.

factor A number a that divides another number b and leaves no remainder is called a factor of b. For example, 4 is a factor of 12 since $\frac{12}{4} = 3$.

factor, to To write a number as a product of its factors.

factorization The process of writing a number as a product of factors.

formula An equation, or a symbolic rule, consisting of an output variable, usually on the left-hand side of the equal sign, and an algebraic expression of input variables, usually on the right-hand side of the equal sign.

fundamental principle of algebra Performing the same operation on both sides of the equal sign in a true equation results in a true equation.

graph A collection of points that are plotted on a grid, the coordinates of which are determined by an equation, formula, or table of values.

greatest common divisor (GCD) The greatest common divisor of two numbers is the largest number that divides into each of the numbers, leaving no remainder.

growth factor When a quantity increases by a specified percent, the ratio of its new value to its original value is called the growth factor associated with the specified percent increase.

$$\text{growth factor} = \frac{\text{new value}}{\text{original value}}$$

The growth factor is also formed by adding the specified percent increase to 100% and then changing the percent into decimal form.

histogram A bar graph that uses adjacent bars to represent frequencies for a set of data.

horizontal (x-) axis The horizontal number line in the Cartesian coordinate system that is used to represent input values.

hypotenuse of a right triangle The longest side, opposite the right angle, in a right triangle.

improper fraction A fraction in which the absolute value of the numerator is greater than or equal to the absolute value of the denominator.

input values Replacement values for the variable(s) in the algebraic expression. For example, in the formula $P = 2l + 2w$, l and w may be replaced by the input values 5 and 10, respectively. Then, $P = 2(5) + 2(10) = 30$.

integers The collection of all of the positive counting numbers $\{1, 2, 3, 4, \ldots\}$, zero $\{0\}$, and the negatives of the counting numbers $\{-1, -2, -3, -4, \ldots\}$.

inverse operation An operation that "undoes" another operation. For example, subtraction is the inverse of addition, so if 4 is added to 7 to obtain 11, then 4 would be subtracted from 11 to get back to 7.

isosceles triangle A triangle that has two sides of equal length. The third side is called the base. The two base angles formed on the base are the same size.

least common denominator (LCD) The smallest number that is a multiple of each denominator in two or more fractions.

leg of a right triangle One of the two sides that form the right angle in a right triangle.

like terms Terms that contain identical variable factors, including exponents. For example, $3x^2$ and $4x^2$ are like terms but $3x^2$ and $3x^4$ are not like terms.

lowest terms Phrase describing a fraction whose numerator and denominator have no factors in common other than 1.

mathematical model Description of the important features of an object or situation that uses equations, formulas, tables, or graphs to solve problems, make predictions, and draw conclusions about the given object or situation.

mean The arithmetic average of a set of data. See *average*.

median The middle value in an ordered list containing an odd number of values. In a list containing an even number of values, the median is the arithmetic average of the two middle values.

minuend In subtraction, the number being subtracted from. For example, in $10 = 45 - 35$, the number 45 is the minuend, 35 is the subtrahend, and 10 is the difference.

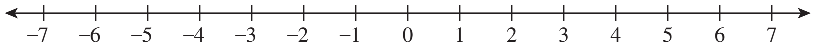
mixed number The sum of an integer and a fraction, written in the form $a\frac{b}{c}$, where a is the integer, and $\frac{b}{c}$ is the fraction.

multiplication by 0 Any number multiplied by 0 is 0.

multiplication by 1 Any number multiplied by 1 is the number itself.

negative numbers Numbers that are less than zero.

number line A straight line that extends indefinitely in opposite directions on which points represent numbers by their distance from a fixed origin or starting point.



numeral A symbol or sequence of symbols called digits that represent a number.

numerator The number written above the line in a fraction; the numerator specifies the number of equal parts of a whole that is under consideration. For example, $\frac{3}{4}$ means there are three parts under consideration and each part is a quarter of the whole.

numerical expression A symbolic representation of two or more numbers combined by arithmetic operations.

obtuse angle An angle that is larger than a right angle (measures more than 90°).

obtuse triangle A triangle where one angle measures greater than 90°.

odd number Any whole number that is not even, that is, if the number is divided by 2, the remainder is 1.

operation, mathematical A procedure that generates a number from one or more other numbers. The basic mathematical operations are negation, addition, subtraction, multiplication, and division. For example, -3 is the result of negating the number 3. The number 8 is the result of adding 1, 2, and 5.

opposite numbers Two numbers with the same absolute value but different signs.

order of operations An agreement for evaluating an expression with multiple operations. Reading left to right, do operations within parentheses first; then exponents, followed by multiplications or divisions as they occur, left to right; and lastly, additions or subtractions, left to right.

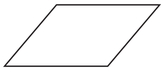
ordered pair Pair of input/output values, separated by a comma, and enclosed in a set of parentheses. The input is given first and the corresponding output is listed second. An ordered pair serves to locate a point in the coordinate plane with respect to the origin.

origin The point at which the horizontal and vertical axes of a rectangular coordinate system intersect.

output Values produced by evaluating the algebraic expression in a symbolic rule or formula.

parallel lines Lines in a plane that never intersect.

parallelogram A four-sided plane figure whose opposite sides are parallel.



percent(age) A fraction expressed as a number of parts out of 100. For example, 25 percent, or 25%, means 25 parts out of a hundred.

percent change *See* **relative change.**

perimeter A measure of the distance around the edge of a plane geometric figure.

perpendicular lines Two intersecting lines that form four angles of equal measure (four right angles).

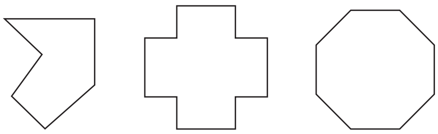
pi (π) The ratio of the circumference of a circle to its diameter. Pi has an approximate value of 3.14, or $\frac{22}{7}$.

plane A plane is a flat surface where a straight line joining any two points on the plane will also lie entirely in the plane.

plotting points Placing points on a grid as determined by the point's coordinates.

polygon A closed plane figure composed of three or more sides that are straight line segments.

For example,



prime factorization A way of writing an integer as a product of its prime factors and their powers. For example, the prime factorization of 18 is $2 \cdot 3^2$. For each integer there is only one prime factorization, except for the order of the factors.

prime factor A factor of a number that is a prime number.

prime number A whole number greater than 1 whose only whole-number factors are itself and 1. For example, 2 and 5 are prime numbers.

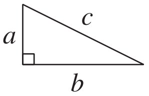
product The resulting number when two or more numbers are multiplied.

proportion An equation stating that two ratios are equal.

proportional reasoning The thought process by which a known ratio is applied to one piece of information to determine a related, but yet unknown, second piece of information.

protractor A device for measuring the size of angles in degrees.

Pythagorean Theorem The relationship between the sides of a right triangle: the sum of the squares of the lengths of the two perpendicular sides (legs) is equal to the square of the length of the side opposite the right angle (hypotenuse).



$a^2 + b^2 = c^2$

quadrant One of four regions that result when a plane is divided by the two coordinate axes in a rectangular coordinate system.

quotient The result of dividing one number by another.

radius of a circle The distance from the center point of a circle to the outer edge of the circle.

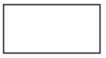
rate The comparison of an actual amount one quantity changes in relation to another quantity in different units; for example, 30 miles per hour.

ratio A quotient that represents the relative measure of similar quantities. Ratios can be expressed in several forms—verbal, fraction, decimal, or percent. For example, 4 out of 5 or 80% of all dentists recommend toothpaste X.

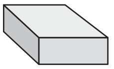
rational number A number that can be written in the form $\frac{a}{b}$, where a and b are integers and b is not zero.

ray Part of a line that has one end point but extends indefinitely in the other direction. Sometimes a ray is called a *half-line*.

reciprocal Two numbers are reciprocals of each other if their product is 1. Obtain the reciprocal of a fraction by switching the numerator and denominator. For example, the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

rectangle A four-sided plane figure with four right angles. 

rectangular prism A three-dimensional figure with three pairs of identical parallel rectangular sides that meet at right angles; a box.



reduced fraction A fraction whose numerator and denominator have no factors in common other than 1.

relative change The comparison of the actual change to the original value. Relative change is measured by the ratio

$$\text{relative change} = \frac{\text{actual change}}{\text{original value}}.$$

Since relative change is frequently reported as a percent, it is often called *percent change*.

remainder In division of integers, the whole number that remains after the divisor has been subtracted from the dividend as many times as possible.

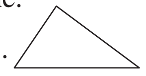
right angle One of four equal-size angles that is formed by two perpendicular lines, measuring 90°.

right circular cylinder A three-dimensional figure with two identical circular bases connected by sides perpendicular to the bases; a can.

right triangle A triangle in which one angle measures 90° (a right angle).

rounding The process of approximating a number to a specified place value.

scale The spacing of numbers on a coordinate axis, or number line.

scalene triangle A triangle whose sides all have different lengths. 

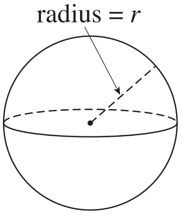
sign of a fraction The sign of a fraction may be placed in one of three positions, preceding the numerator, preceding the denominator, or preceding the fraction itself. For example,

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}.$$

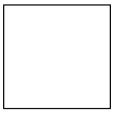
similar triangles Triangles whose corresponding angles are equal. The ratios of the lengths of the corresponding sides (sides opposite equal angles) are equal and are said to be proportional.

solution of an equation The value of a variable that makes the equation a true statement. For example, in the equation $x - 7 = 3$, the value 10 for the variable x is a solution, since $10 - 7 = 3$.

sphere A three-dimensional figure consisting of all points that are the same distance (measured by the radius) from a given point called its center; a ball.



square A closed four-sided plane figure with sides of equal length and with four right angles.



square number A number that can be written as the product of two equal integer factors. For example, 9 is a square number because $3 \cdot 3 = 9$.

square root The square root of a nonnegative number N is a number M whose square is N. The symbol for square root is $\sqrt{}$. For example, $\sqrt{9} = 3$ because $3^2 = 9$.

subtraction The operation that determines the difference between two numbers.

subtrahend In subtraction, the number that is subtracted. For example, in $10 = 45 - 35$, the number 45 is the minuend, 35 is the subtrahend, and 10 is the difference.

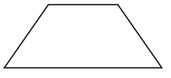
sum In addition, the total of the addends. For example, in $1 + 2 + 7 = 10$, the number 10 is the sum.

surface area The total area of all the surfaces of a three-dimensional figure.

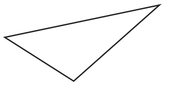
table of input/output values A listing of input values with their corresponding output values.

terms of an expression Parts of an algebraic expression separated by addition or subtraction.

trapezoid A closed four-sided plane figure with two opposite sides parallel and the other two opposite sides not parallel.



triangle A closed three-sided plane figure.



unit analysis Also called *dimensional analysis.* A process using measurement units in a rate problem as a guide to obtain the desired unit for the result. The process involves a sequence of multiplications and/or divisions that cancel units, leaving the desired unit.

variable A quantity, usually represented by a letter, that varies from one situation to another.

variable expression A symbolic code giving instructions for performing arithmetic operations with variables and numbers.

verbal rule A description of a rule using words.

vertex of an angle The point where the two rays of an angle intersect.

vertical (*y*-) axis The vertical number line in the Cartesian coordinate system that is used to represent the output values.

volume The measure, in cubic units, of the space enclosed by the surfaces of a three-dimensional figure.

whole numbers The collection of numbers that includes the counting numbers, 1, 2, 3, . . ., and the number 0.

zero power Any nonzero number raised to the 0 power is 1. For example, $10^0 = 1$.

This page intentionally left blank

Index

I

J

L

M

N

O

P