

Lecture 10

Classification revisited

Topics

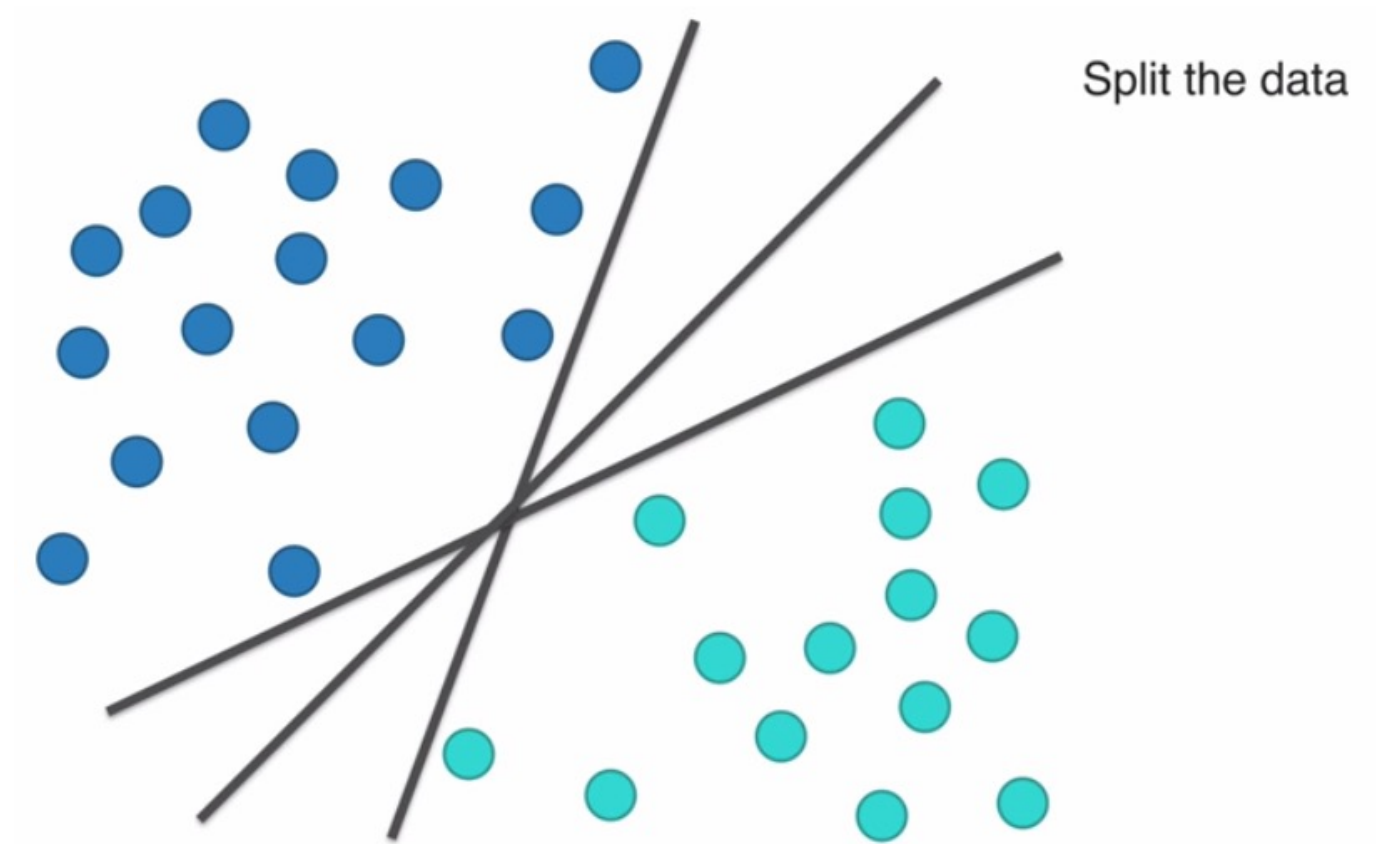
- Support Vector Machines
- Noise
- Kernels
- Decision trees
- Random forests

Support Vector Machines

[Alice Zhao](#)

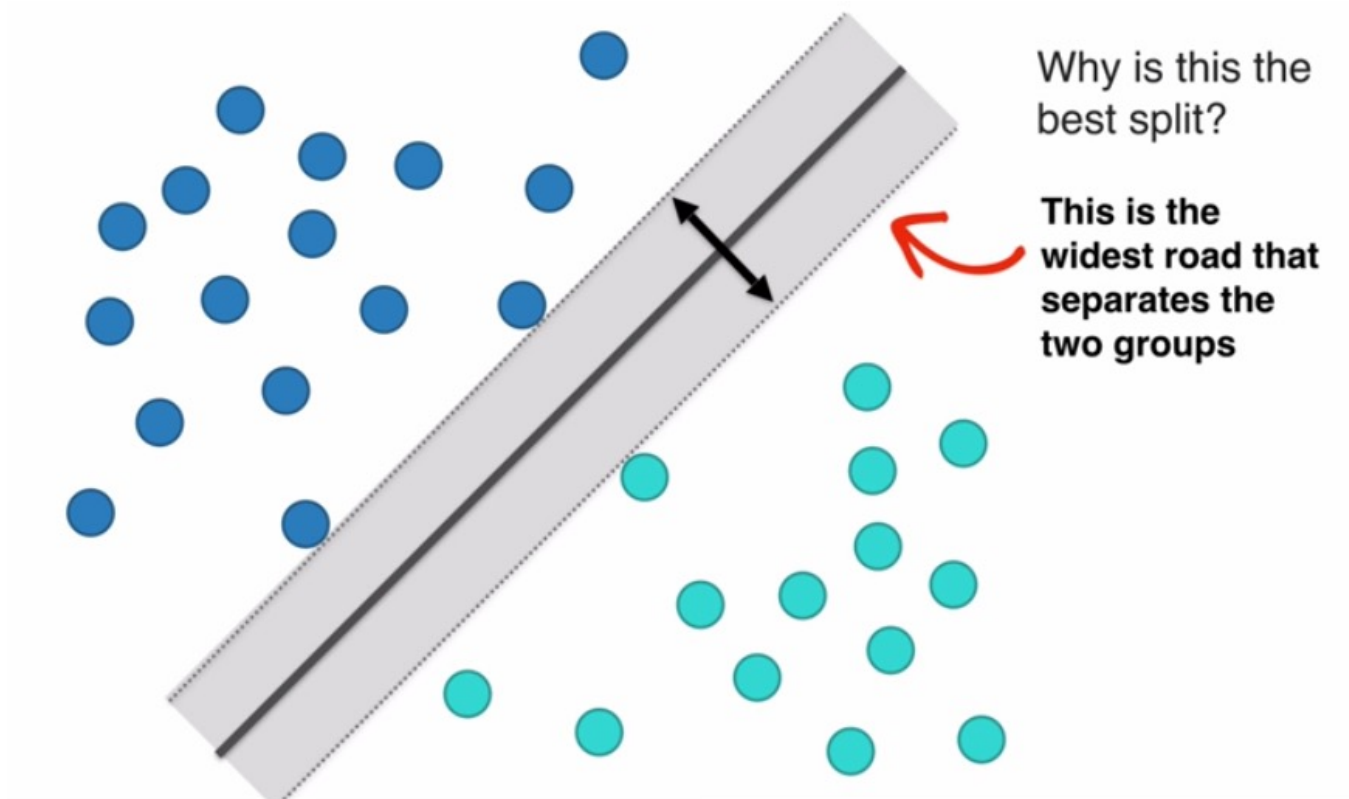
How should we split the data?

In classification
we have this
problem

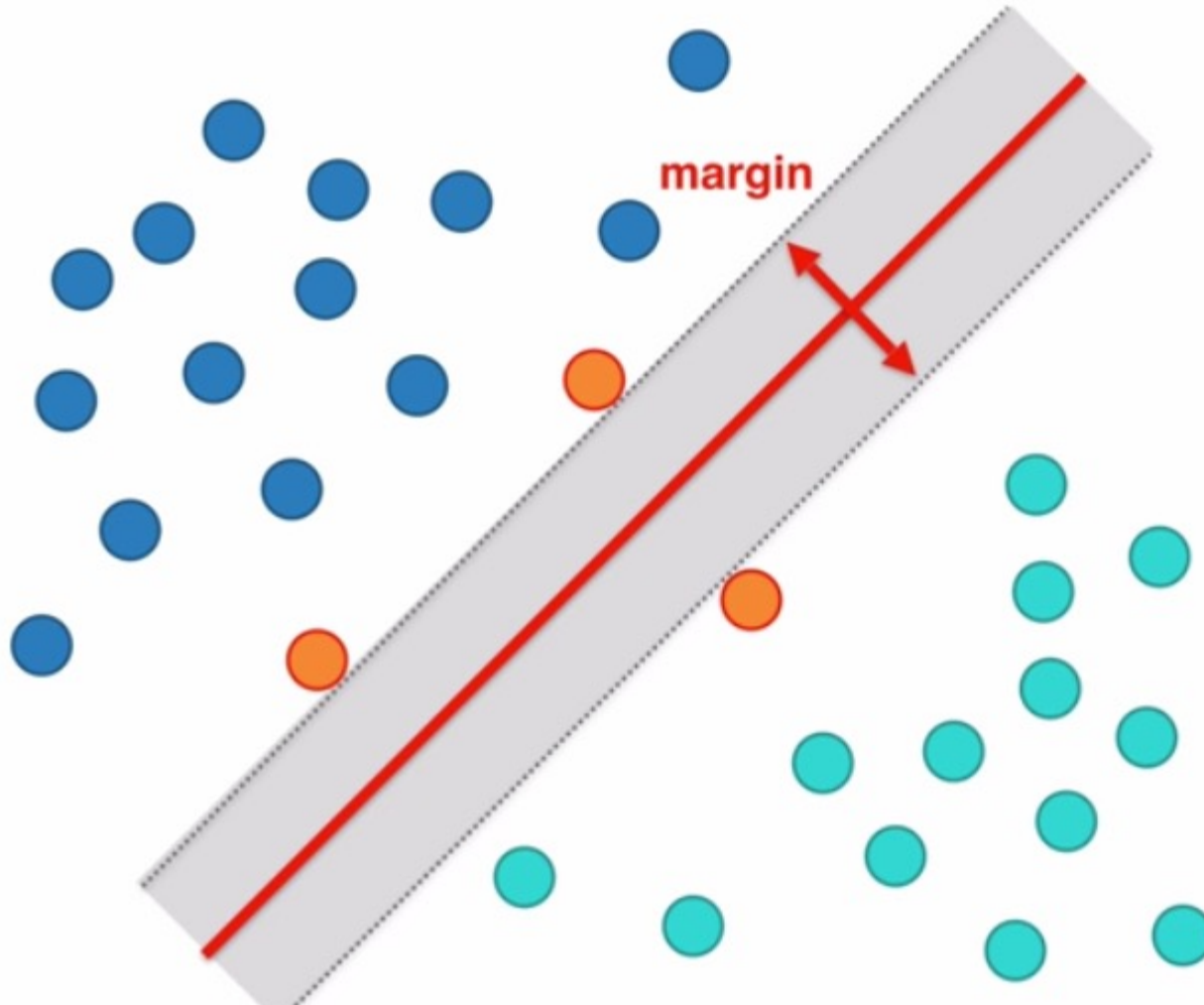


The SVM algorithm will
do this split for us!

Maybe like this?



Motivation



Why is this the best split?

The distance between the **support vectors** and the **hyperplane** are as far as possible

Cupcakes and muffins: What's the difference?

Cupcakes





versus

Muffins



Let's proceed
scientifically!

Collecting the data

Google  

basic **muffin** recipe Remove

basic **cupcake** recipe Remove

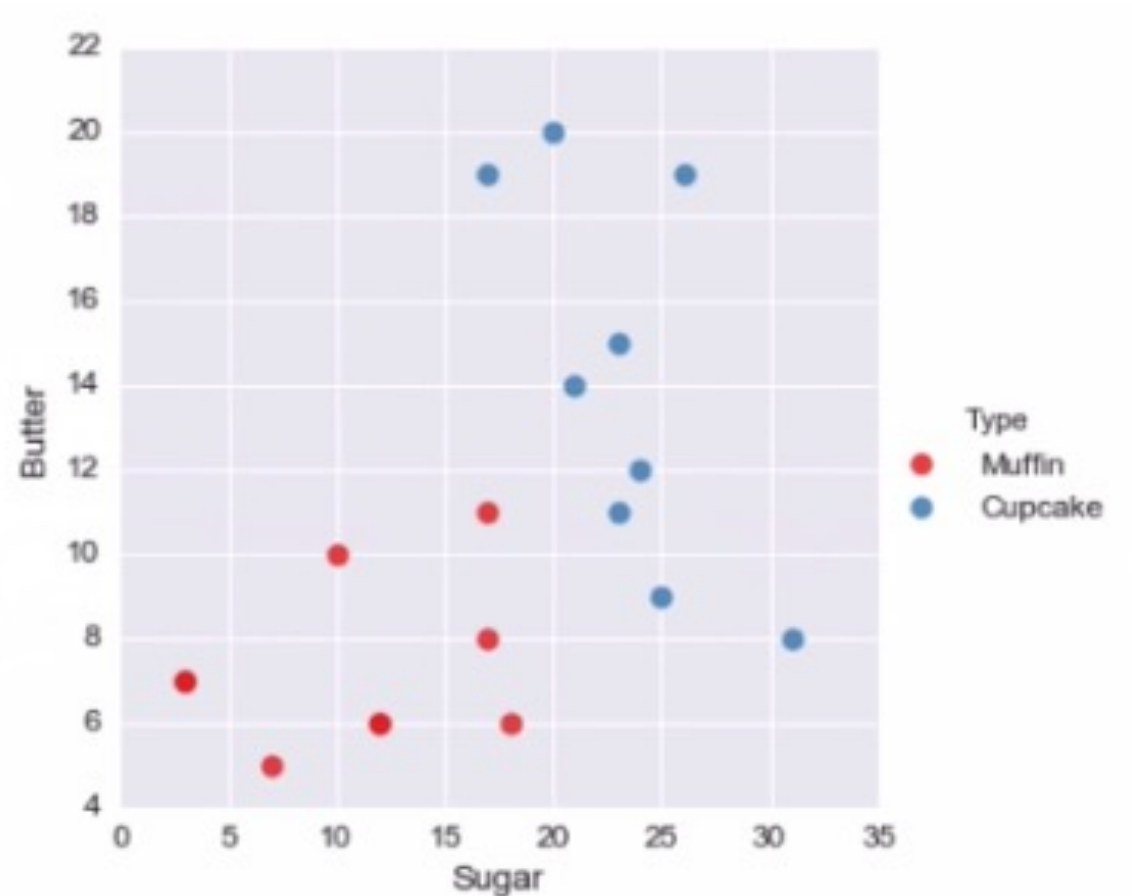
Find 9 recipes of each kind

Type	Flour	Milk	Sugar	Butter	Egg	Baking Powder	Vanilla	Salt
Muffin	55	28	3	7	5	2	0	0
Muffin	47	24	12	6	9	1	0	0
Muffin	47	23	18	6	4	1	0	0
Muffin	50	25	12	6	5	2	1	0
Muffin	55	27	3	7	5	2	1	0
Muffin	54	27	7	5	5	2	0	0
Muffin	47	26	10	10	4	1	0	0
Muffin	50	17	17	8	6	1	0	0
Muffin	50	17	17	11	4	1	0	0
Cupcake	39	0	26	19	14	1	1	0
Cupcake	34	17	20	20	5	2	1	0
Cupcake	39	13	17	19	10	1	1	0
Cupcake	38	15	23	15	8	0	1	0
Cupcake	42	18	25	9	5	1	0	0
Cupcake	36	14	21	14	11	2	1	0
Cupcake	38	15	31	8	6	1	1	0
Cupcake	36	16	24	12	9	1	1	0
Cupcake	34	17	23	11	13	0	1	0

Express the ingredients as weight percentages

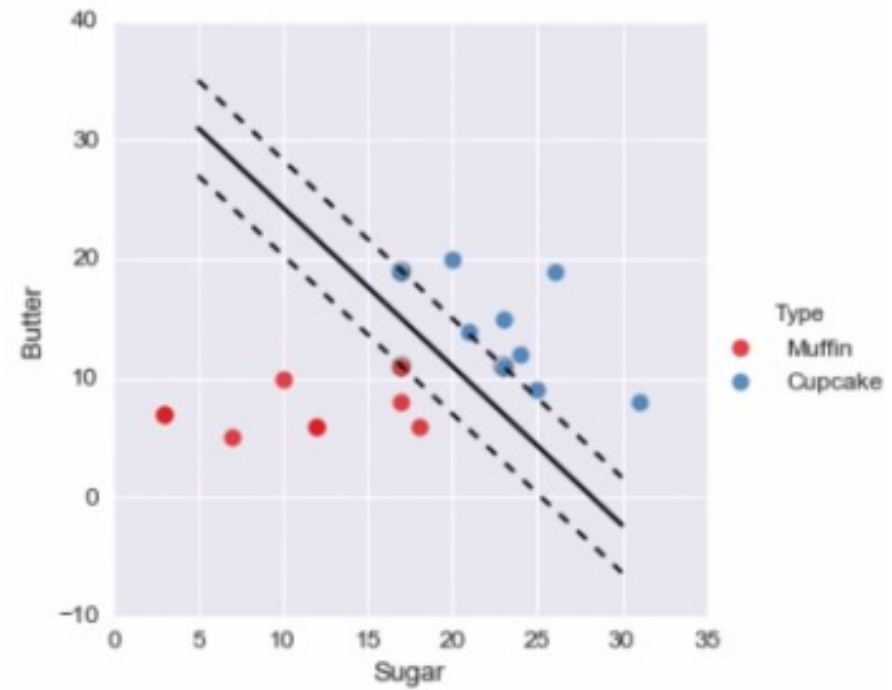
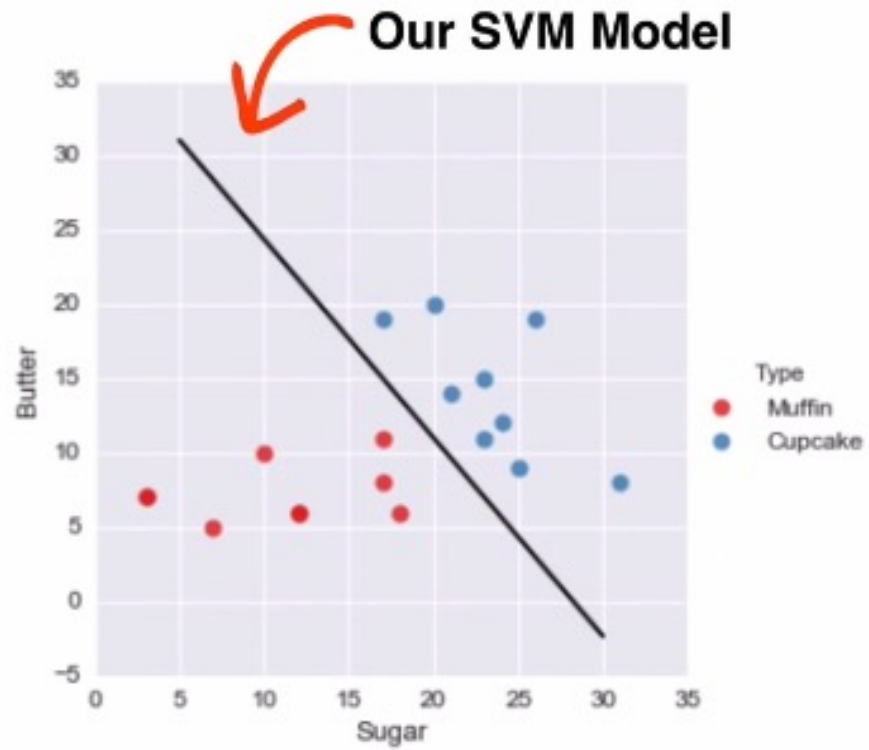
Plotting the data

Focusing on butter
and sugar only



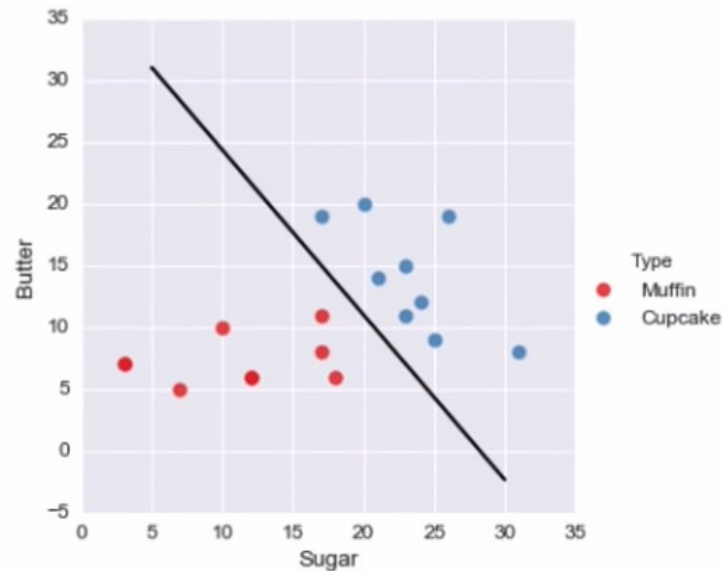
Applying SVM

Finding the SVM
model

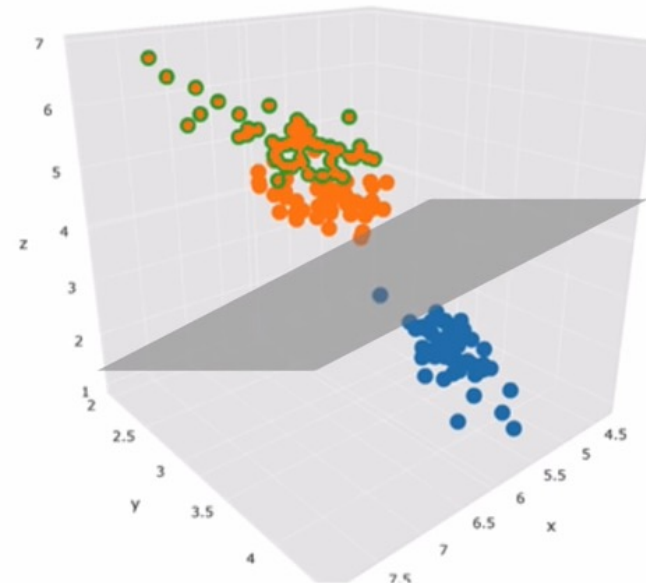


SVMs find separating hyperplanes

2D: Separate with Line



3D: Separate with Plane

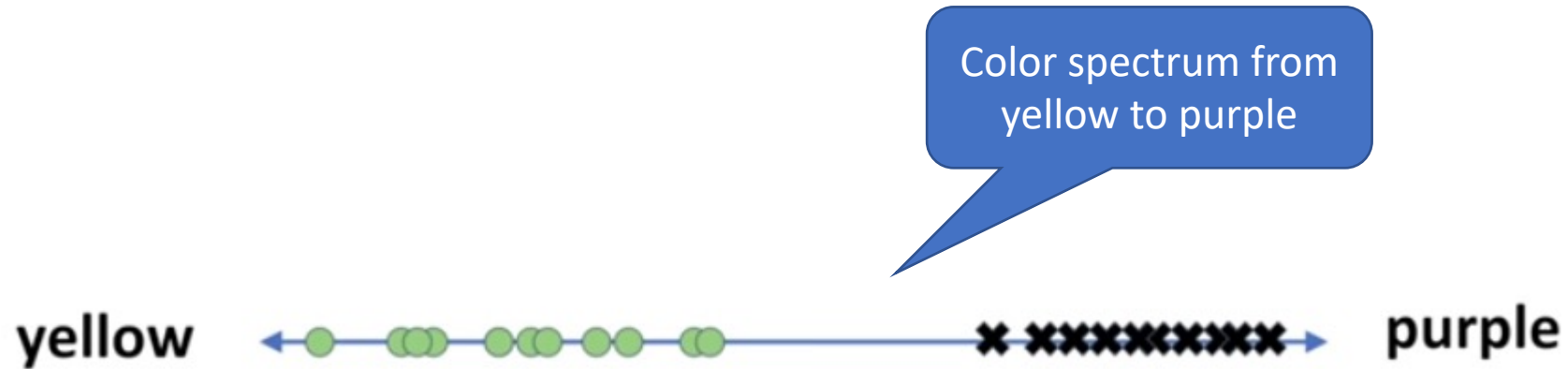


Key point: maximizing the margin can be formulated as a convex optimization problem, and therefore solved efficiently even in high-dimensional spaces.

Noise

[Brandon Rohrer](#)

Peaches



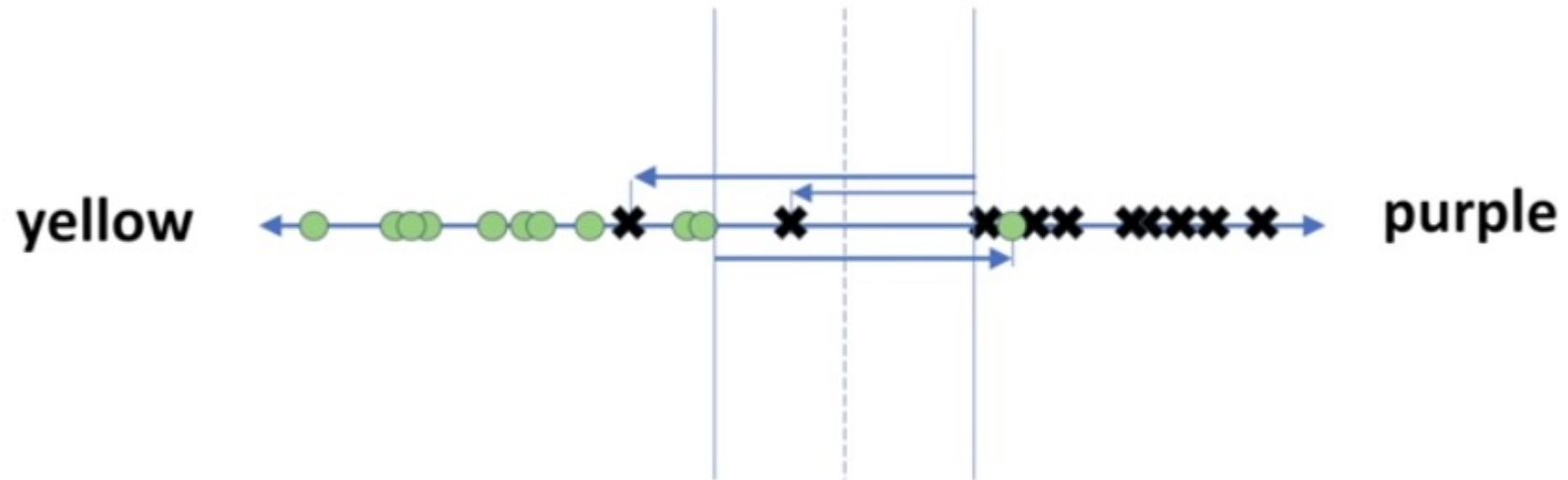
Each green dot represents an edible peach. Each black cross represents a non-edible peach.
Clean situation.

Peaches



Less clean situation.

Peaches

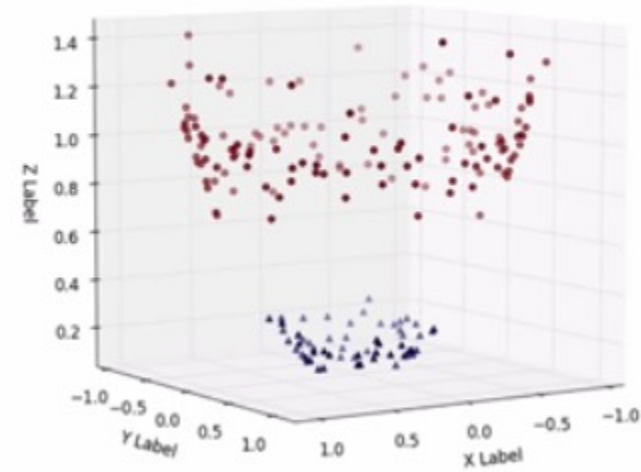
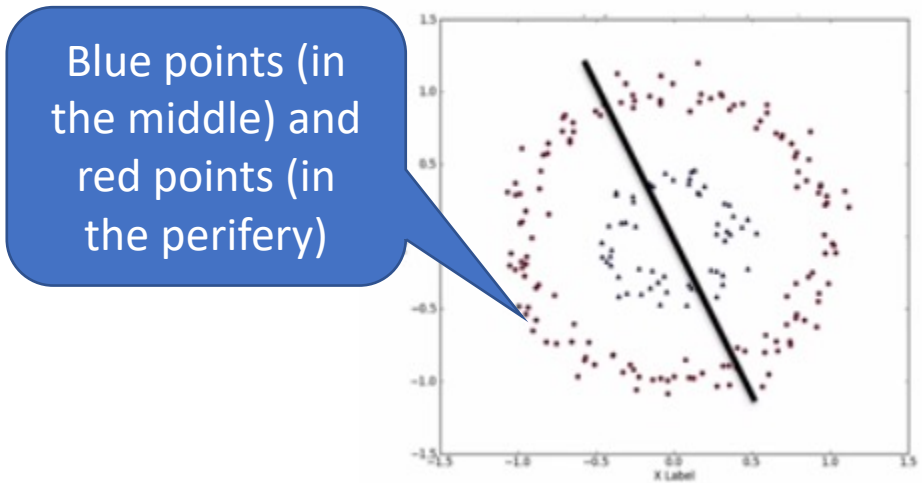


Use penalty for misclassified points. Still possible to solve it.

Kernels

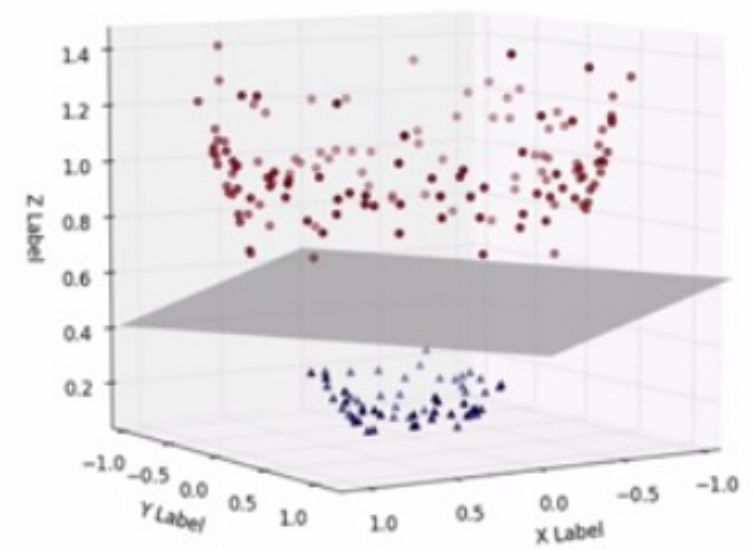
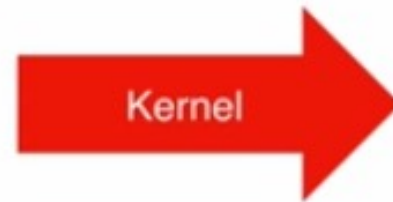
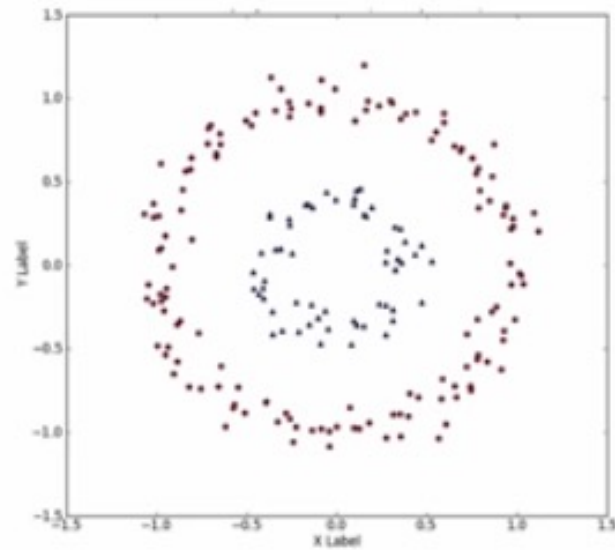
[Brandon Rohrer](#)

Kernels



Here it is impossible to separate the two classes with a line! Idea: add a dimension and a kernel function that will make them separable! In this case: $z = \text{distance of } (x,y) \text{ from the origin}$ works.

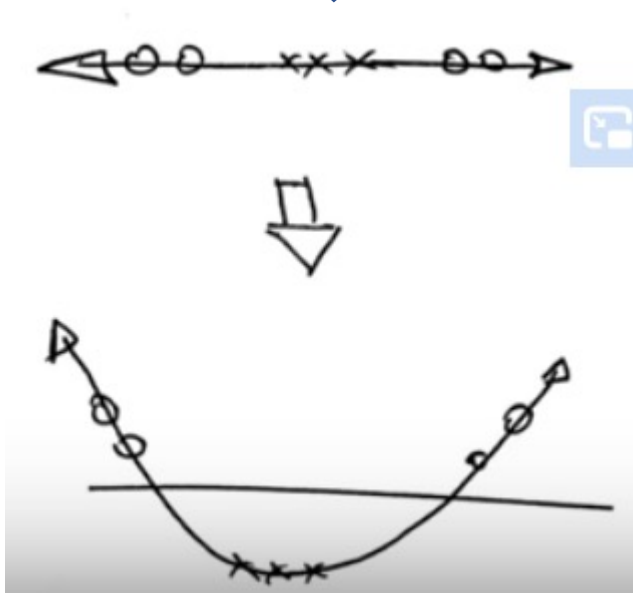
Kernels



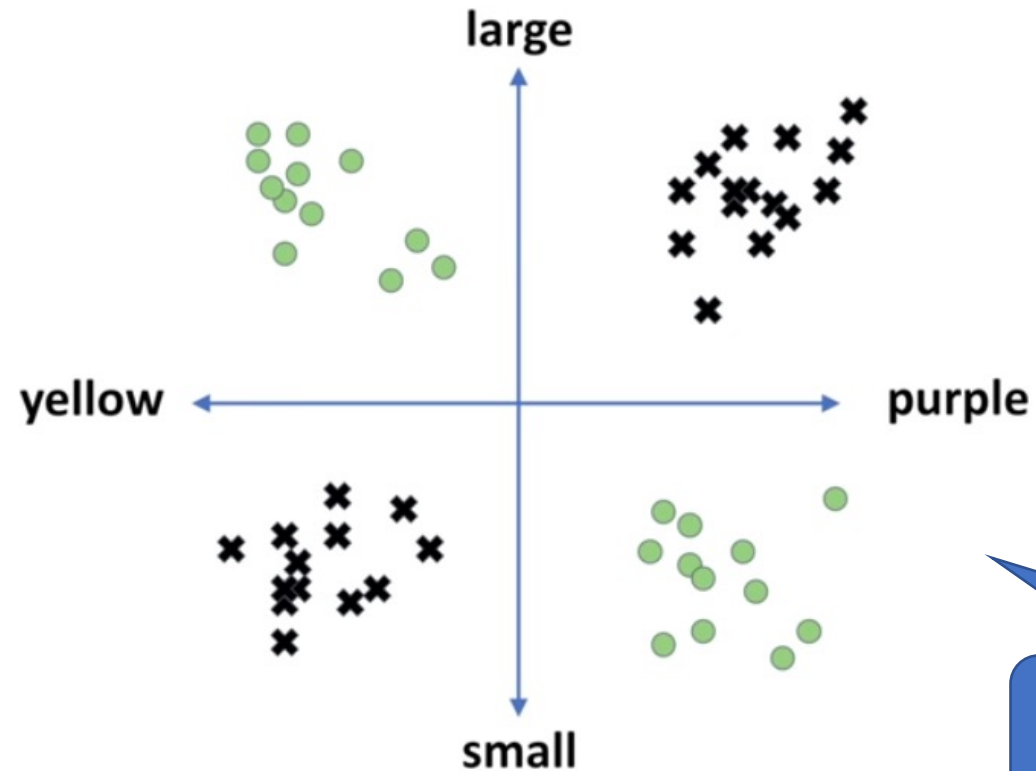
In this new space they are separable! So we can use SVM as a classifier for this 2D set too.

Kernels

Put $x = 0$ here and let $y = x^2$



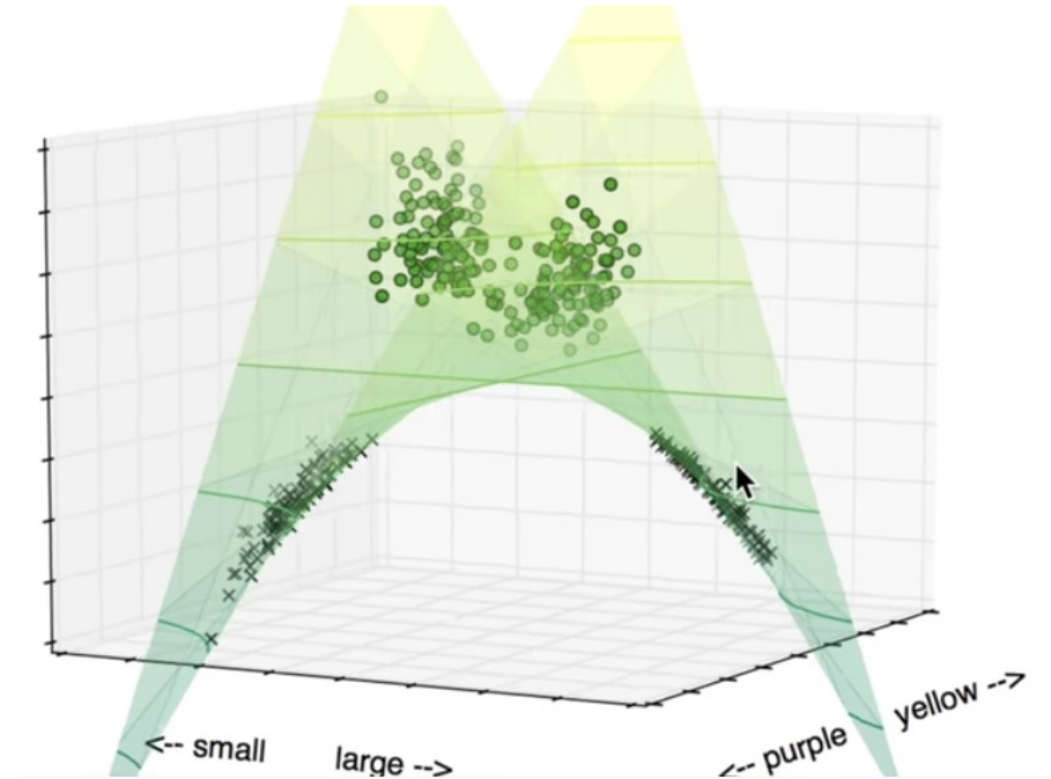
Kernels



Here we have the peaches again but now they have a size as well as a color and we've added plums

The kernel $z = x * y$ will make the two sets linearly separable!

Kernels



Now they are linearly separable. No problem to pay with more dimensions.
Kernel selection is trial and error (an art).

Evaluation of SVM

Advantages

- Works for classification as well as regression
- Works with high-dimensional space
- Works for two or more classes (via one-vs-rest strategy)
- Good accuracy

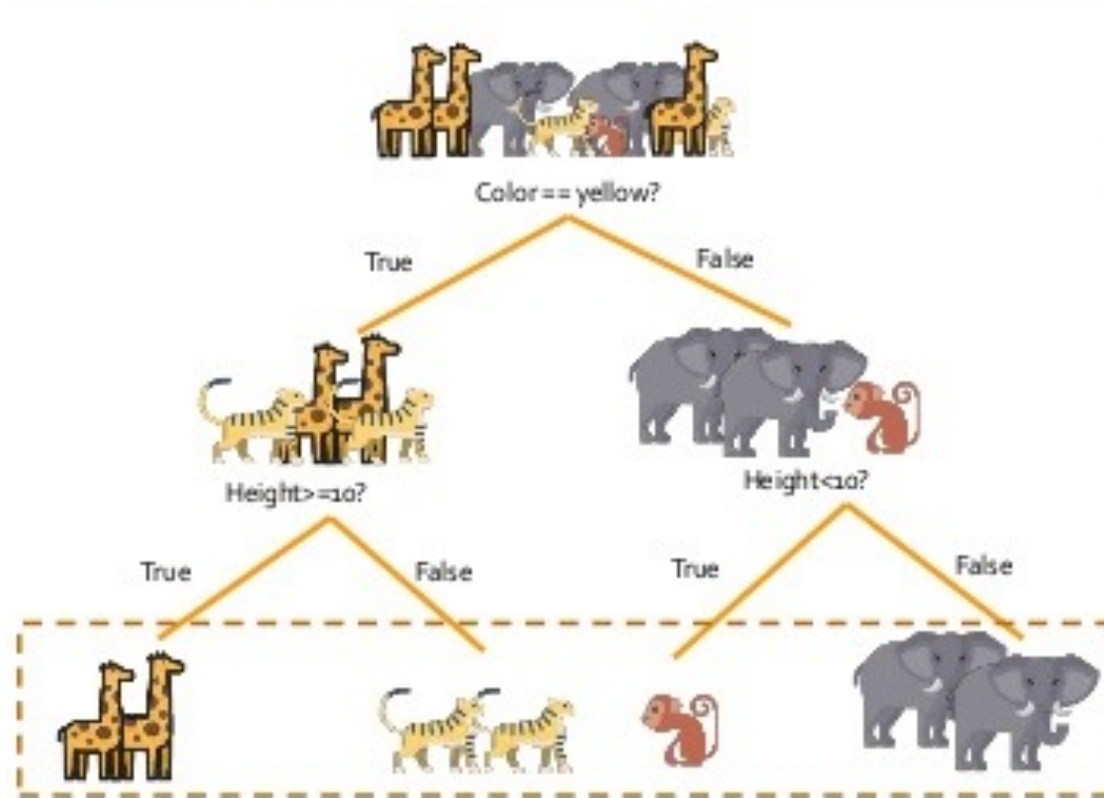
Disadvantages

- Slow on large datasets (compared to Naïve Bayes)
- Works poorly with overlapping classes
- Kernel type must be selected manually.

Decision trees

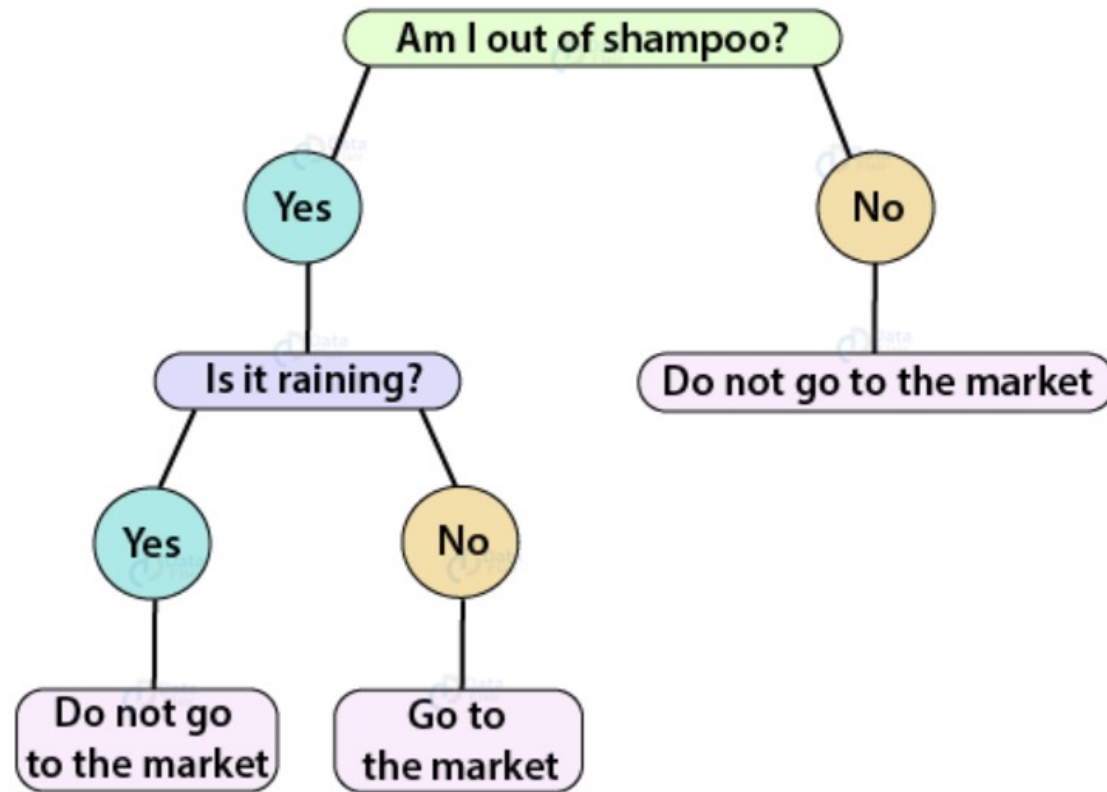
[StatQuest](#)

Example



[Classification of animals](#)

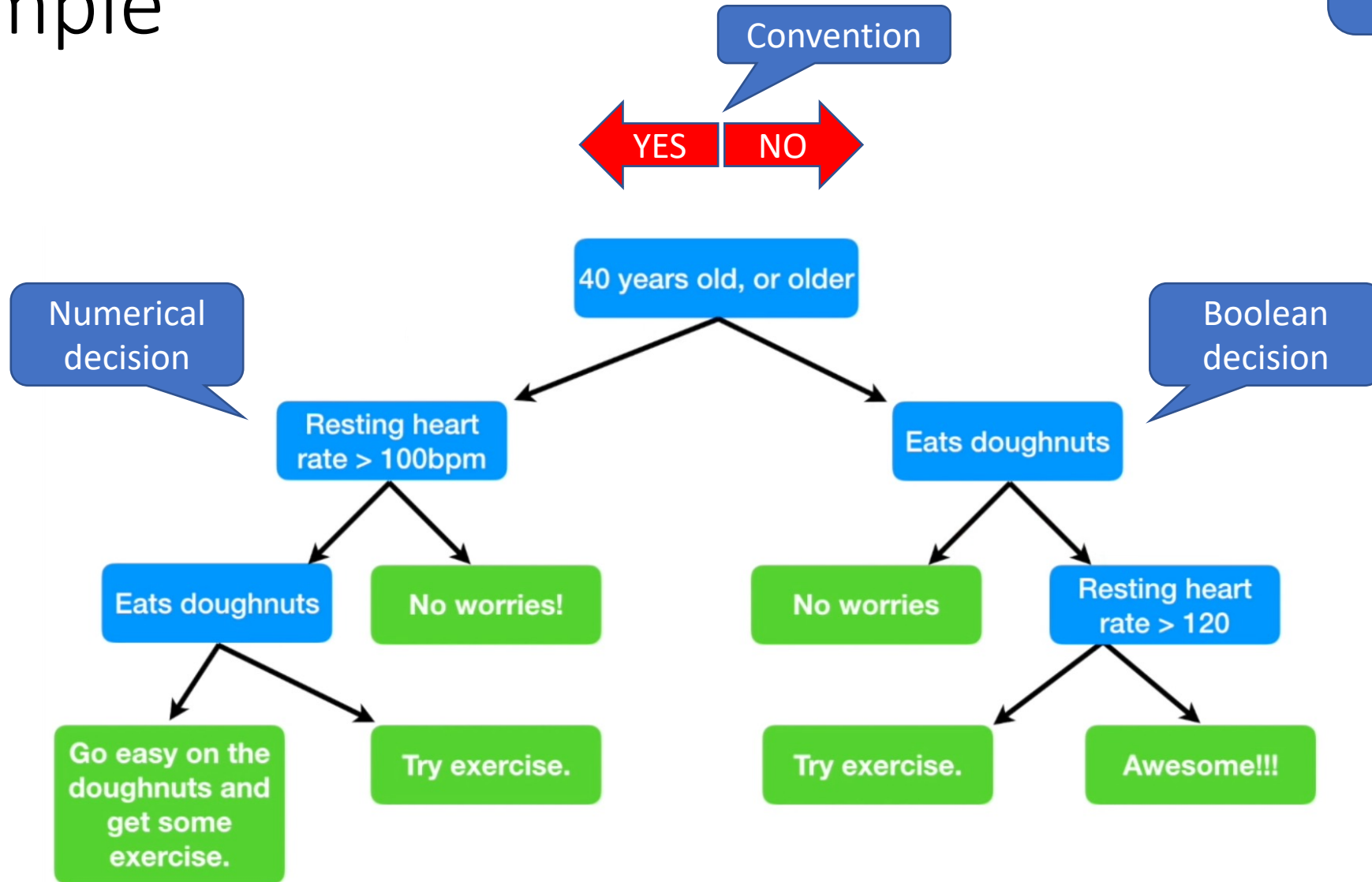
Example



[Classification of activities](#)

Example

A decision-tree for
(questionable)
health advise



From data to decision-tree

	Feature 1	Feature 2	Target	
	Chest Pain	Good Blood Circulation	Blocked Arteries	Heart Disease
Patient 1	No	No	No	No
Patient 2	Yes	Yes	Yes	Yes
	Yes	Yes	No	No
	Yes	No	???	Yes
	etc...	etc...	etc...	etc...

Decision-trees can be made in many ways. One way is to ask Q1 at the root, Q2 everywhere at the next level, and so on, until the labeled leaves. How many leaves will such a tree have?

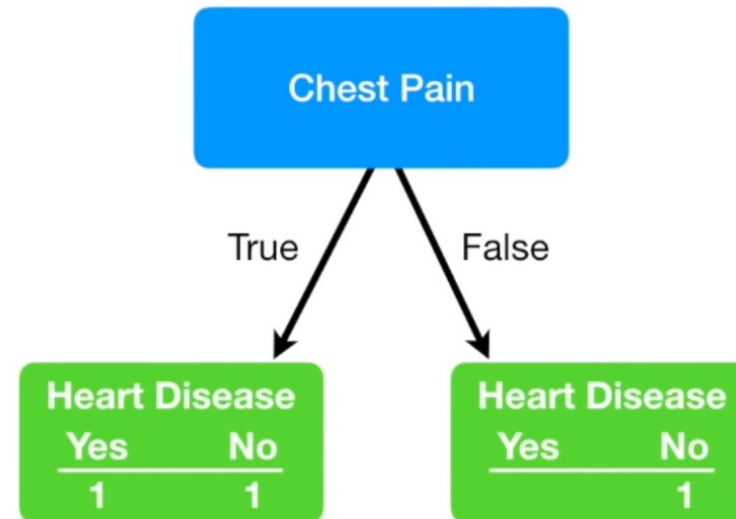
Big trees are bad for generalization + may be expensive to use. (Tests may cause suffering and be expensive.) Making small trees requires a method that finds regularities.

Feature analysis

Which split (or question or feature) should we start with? Let's consider Chest Pain. We calculate the effect of this split in terms of the target variable Heart Disease.

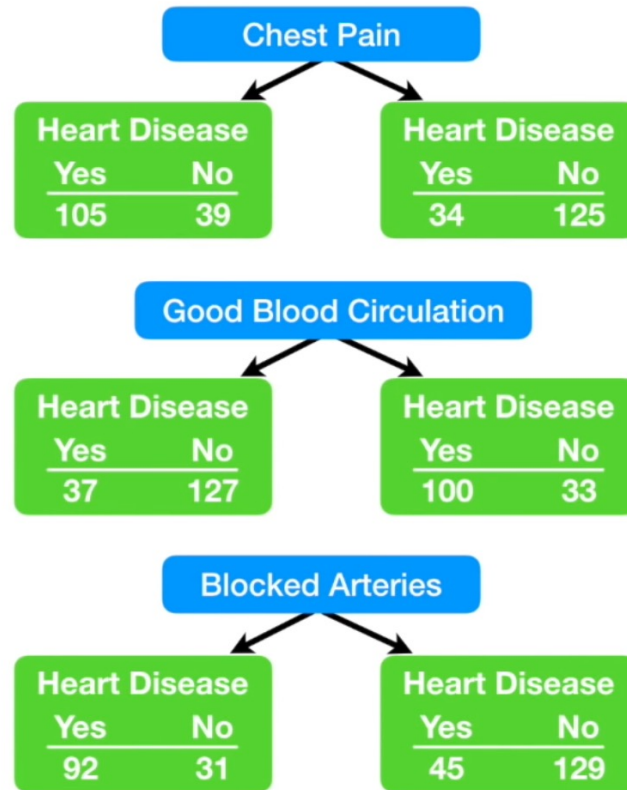
The 4th patient has chest pain and heart disease.

Chest Pain	Good Blood Circulation	Blocked Arteries	Heart Disease
No	No	No	No
Yes	Yes	Yes	Yes
Yes	Yes	No	No
Yes	No	???	Yes
etc...	etc...	etc...	etc...



Go through all the patients (rows) and analyze Chest pain vs Heart Disease

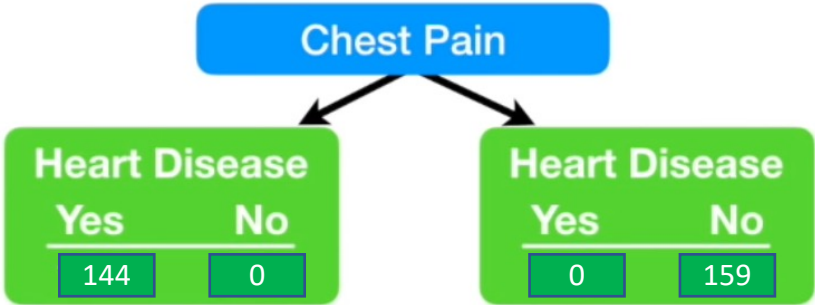
Feature analysis



Result of this analysis for all patients and all three splits (columns). Which split is best to start with?

Wishful thinking...

Gini impurity



Pure sets like this would be ideal, since then we would not need to ask any more questions. The probability of Yes is 0 or 1.

Maybe we could define a purity measure that measures to what extent a question splits the data into pure Yes or pure No sets?

Equivalently we could define a measure of impurity. We will use a common one called the *Gini impurity*. Another common one is Shannon's notion of *entropy*.

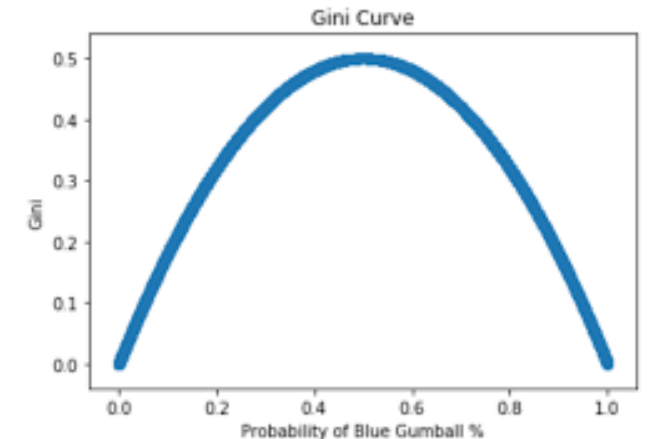
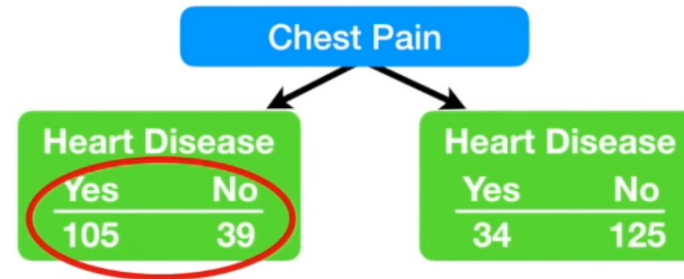
Gini impurity of sets

If the probability of Yes is 1, then the probability of No is 0, so Gini = 0

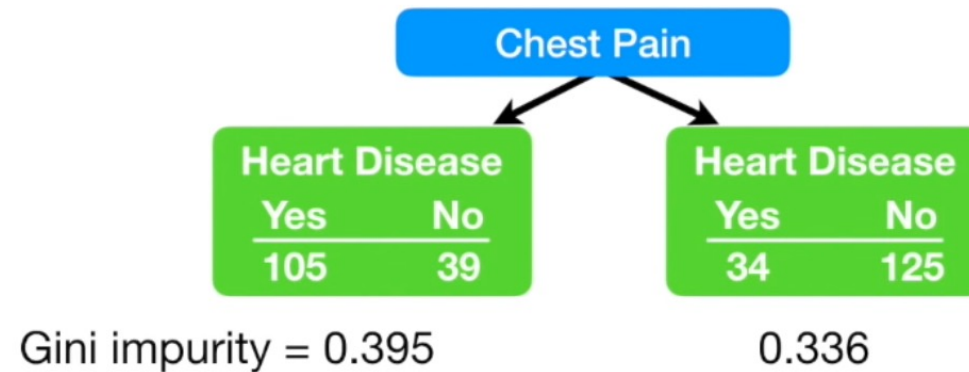
If the probability of Yes is 0.5, then the probability of No is 0.5, so Gini = 0.5

For this leaf, the Gini impurity = $1 - (\text{the probability of "yes"})^2 - (\text{the probability of "no"})^2$

$$= 1 - \left(\frac{105}{105 + 39}\right)^2 - \left(\frac{39}{105 + 39}\right)^2$$



Gini impurity of splits



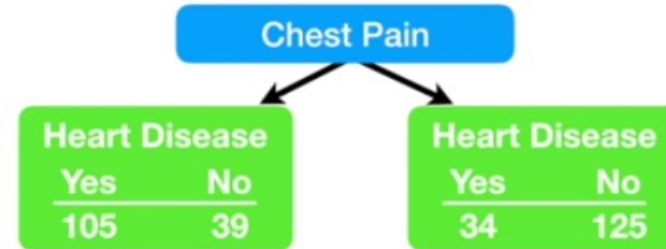
Gini impurity for Chest Pain = weighted average of Gini impurities for the child sets

$$= \left(\frac{144}{144 + 159} \right) 0.395 + \left(\frac{159}{144 + 159} \right) 0.336$$
$$= 0.364$$

With this probability we have that impurity

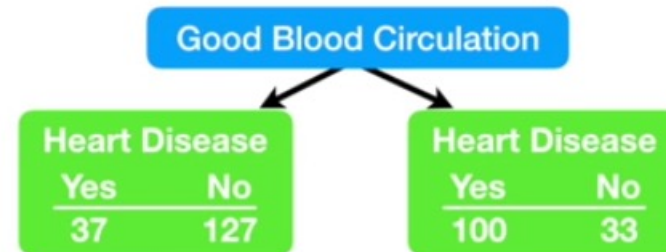
Selecting the best split

Gini impurity for Chest Pain = 0.364

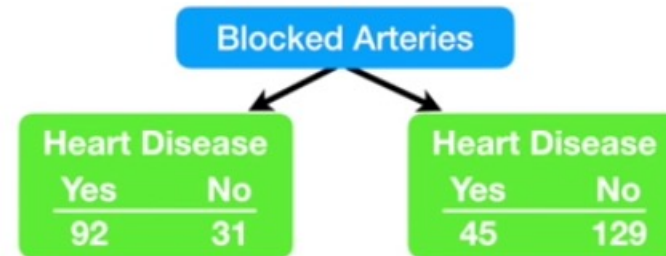


WINNER!

Gini impurity for Good Blood Circulation = 0.360

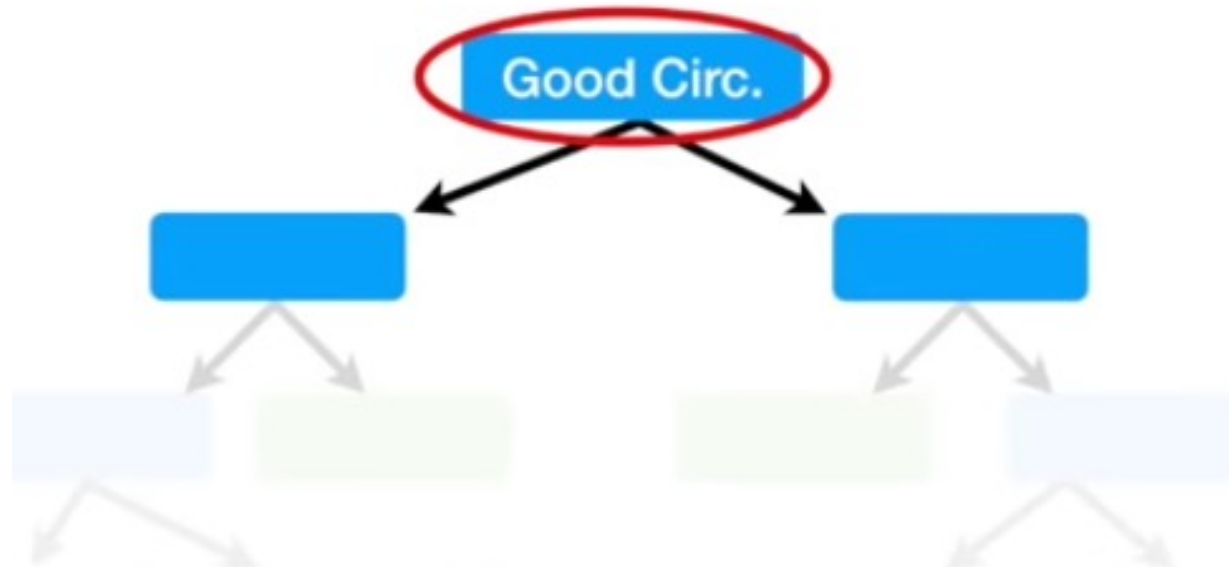


Gini impurity for Blocked Arteries = 0.381

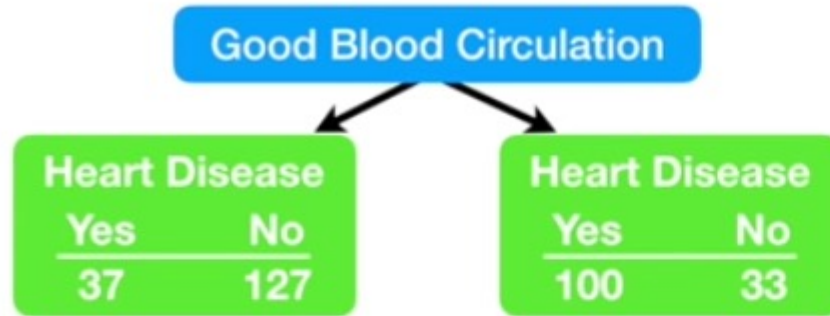


Tree construction

...so we will use it at the root of the tree.

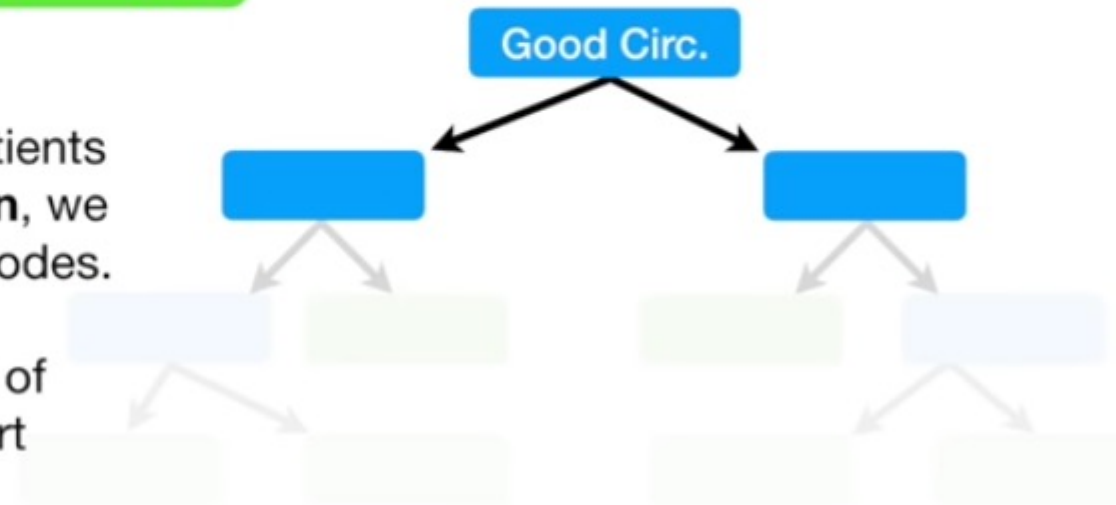


Tree construction



When we divided all of the patients using **Good Blood Circulation**, we ended up with “impure” leaf nodes.

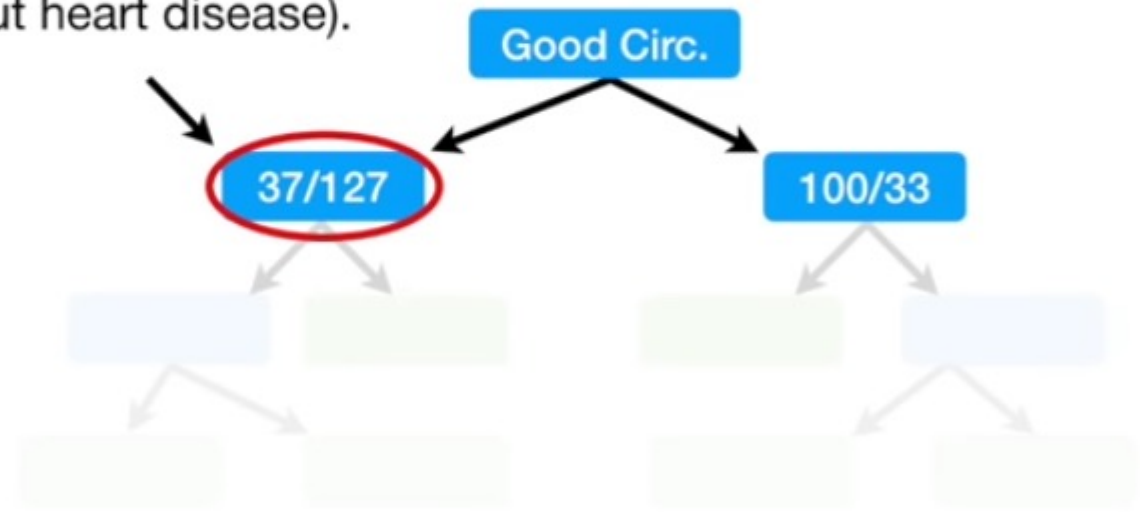
Each leaf contained a mixture of patients with and without Heart Disease.



Let's continue the construction of the tree!

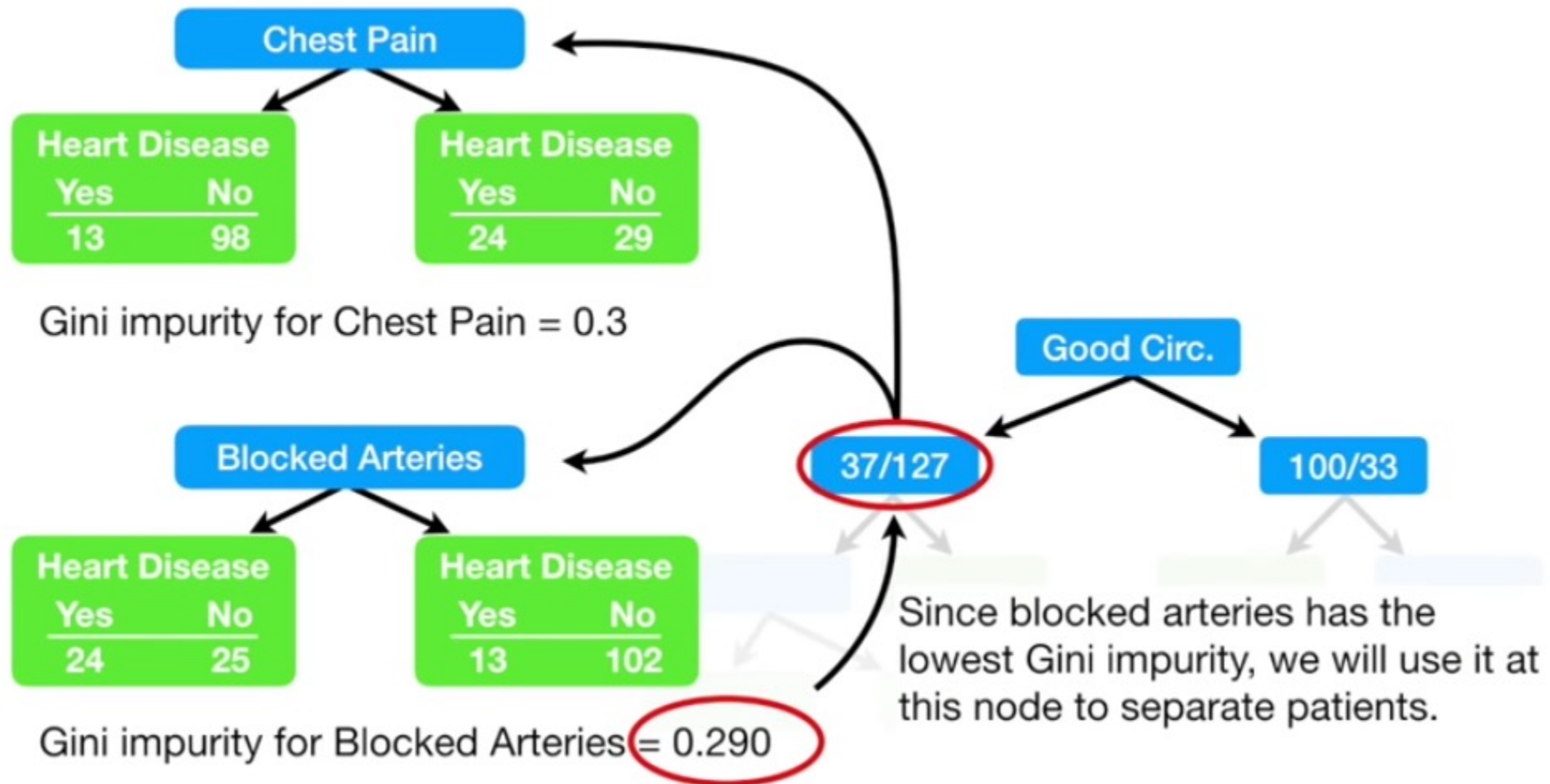
Tree construction

Now we need to figure how well **chest pain** and **blocked arteries** separate these 164 patients (37 with heart disease and 127 without heart disease).

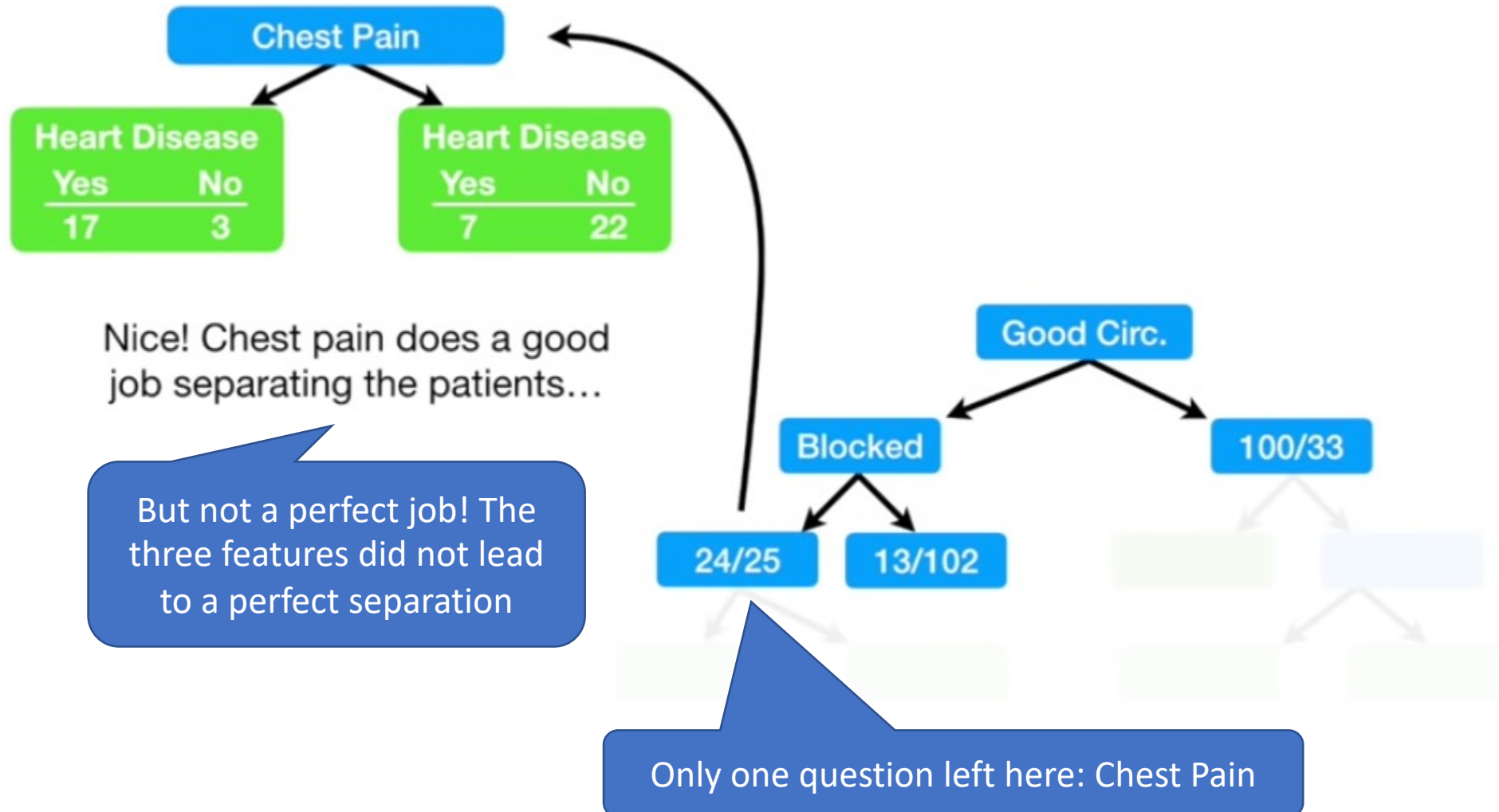


Select the split with the lowest Gini impurity again

Tree construction

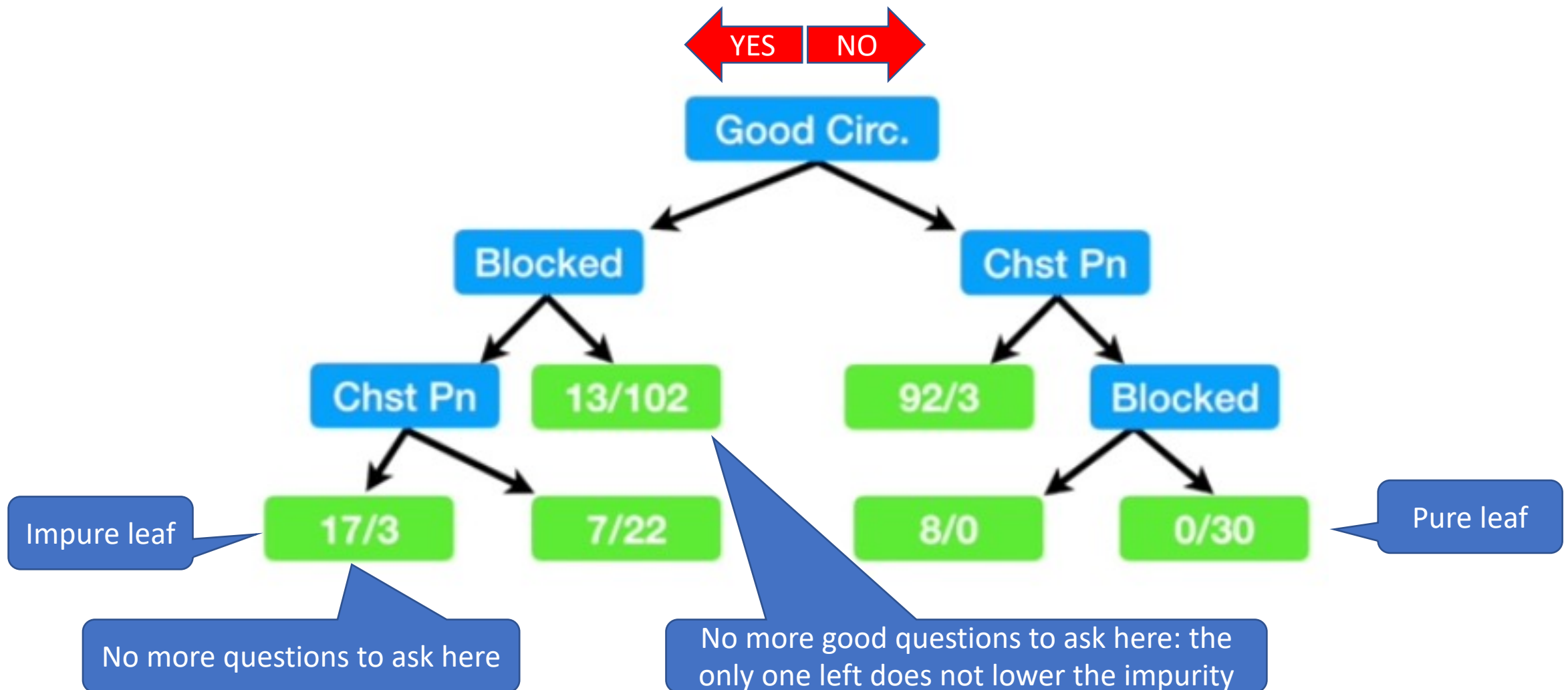


Tree construction



The finished decision-tree!

Tree construction



Numerical decisions

Now suppose we also have column with numerical data, e.g. Weight.

Then we may add decisions of the form $\text{Weight} \geq 180$ or the like

Weight	Heart Disease
220	Yes
180	Yes
225	Yes
190	No
155	No

Numerical decisions

Start by sorting the data by Weight

	Weight	Heart Disease
Lowest	155	No
	180	Yes
	190	No
	220	Yes
Highest	225	Yes

Then compute the Gini impurities of $\text{Weight} \geq x$ for different x (that appear in the column).

Finally select the best split among all splits like before!

Decision tree applications

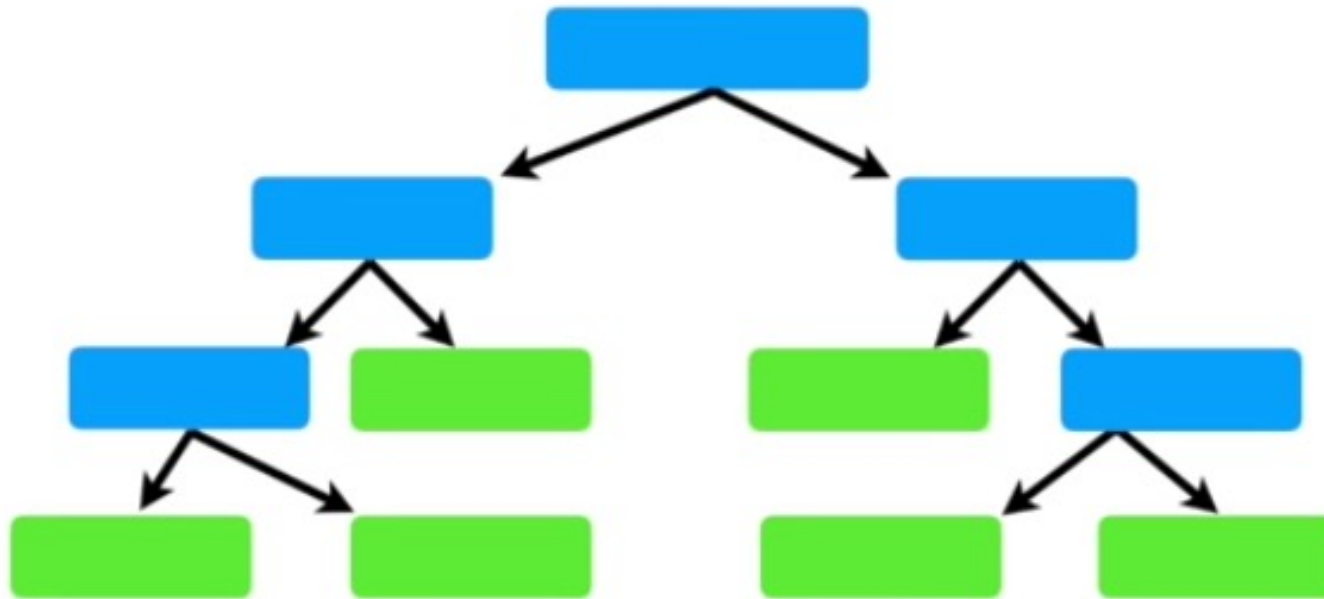
- **Marketing and Sales** – Decision Trees play an important role in a decision-oriented sector like marketing. In order to understand the consequences of marketing activities, organizations make use of Decision Trees to initiate careful measures. This helps in making efficient decisions that help the company to reap profits and minimize losses.
- **Reducing Churn Rate** – Banks make use of Decision Trees to retain their customers. It is always cheaper to keep customers than to gain new ones. Banks are able to analyze which customers are more vulnerable to leaving their business. Based on the output, they are able to *make decisions by providing better services, discounts as well as several other features*.
- **Anomaly & Fraud Detection** – Industries like finance and banking suffer from various cases of fraud. In order to filter out anomalous or fraud loan applications, information and insurance fraud, these companies deploy decision trees to provide them with the necessary information to identify fraudulent customers.
- **Medical Diagnosis** – Classification trees identifies patients who are at risk of suffering from serious diseases such as cancer and diabetes.

Random Forests

[StatQuest](#)

Decision trees

Decision Trees are easy to build, easy to use
and easy to interpret...



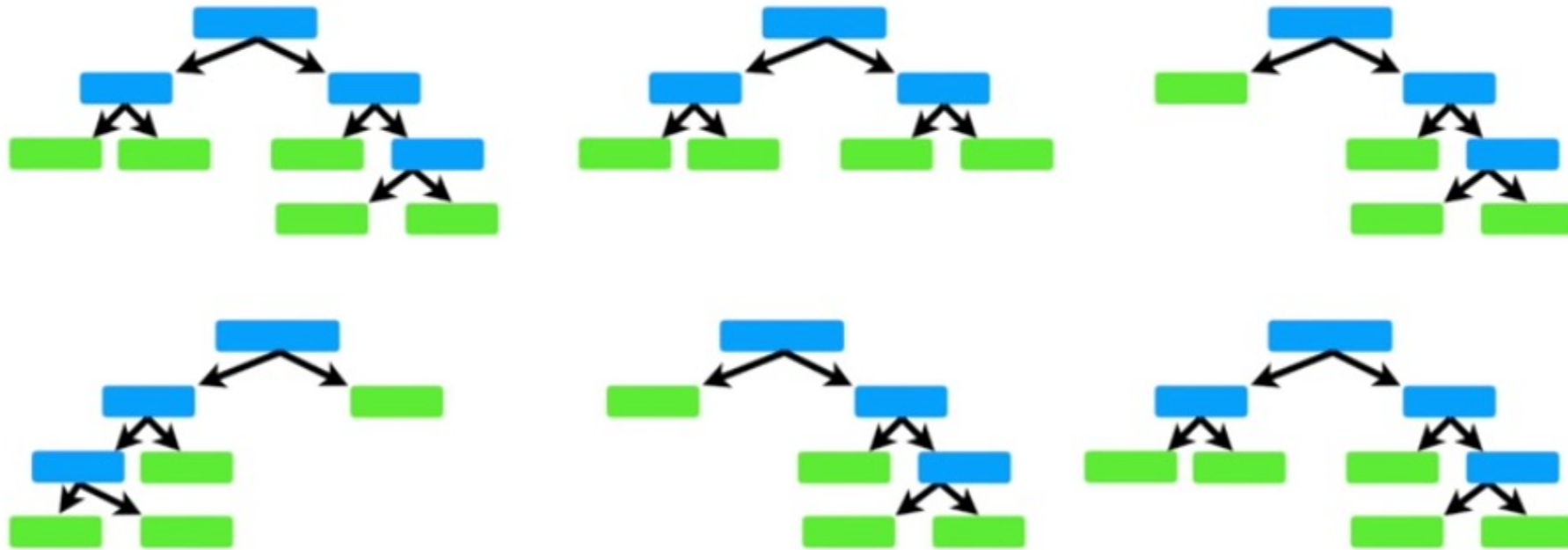
Decision trees

To quote from ***The Elements of Statistical Learning*** (aka The Bible of Machine Learning), “Trees have one aspect that prevents them from being the ideal tool for predictive learning, namely **inaccuracy**.”

In other words, they work great with the data used to create them, but **they are not flexible when it comes to classifying new samples.**

Random forests

The good news is that **Random Forests** combine the simplicity of decision trees with flexibility resulting in a vast improvement in accuracy.



Building random forests

Original Dataset

Chest Pain	Good Blood Circ.	Blocked Arteries	Weight	Heart Disease
No	No	No	125	No
Yes	Yes	Yes	180	Yes
Yes	Yes	No	210	No
Yes	No	Yes	167	Yes

Let us use this dataset again and build a random forest!

Building random forests

Step 1: create a bootstrapped dataset by randomly picking lines from the original dataset

Original Dataset

Chest Pain	Good Blood Circ.	Blocked Arteries	Weight	Heart Disease
No	No	No	125	No
Yes	Yes	Yes	180	Yes
Yes	Yes	No	210	No
Yes	No	Yes	167	Yes

Bootstrapped Dataset

Chest Pain	Good Blood Circ.	Blocked Arteries	Weight	Heart Disease
Yes	Yes	Yes	180	Yes
No	No	No	125	No
Yes	No	Yes	167	Yes
Yes	No	Yes	167	Yes

Same line twice!

Building random forests

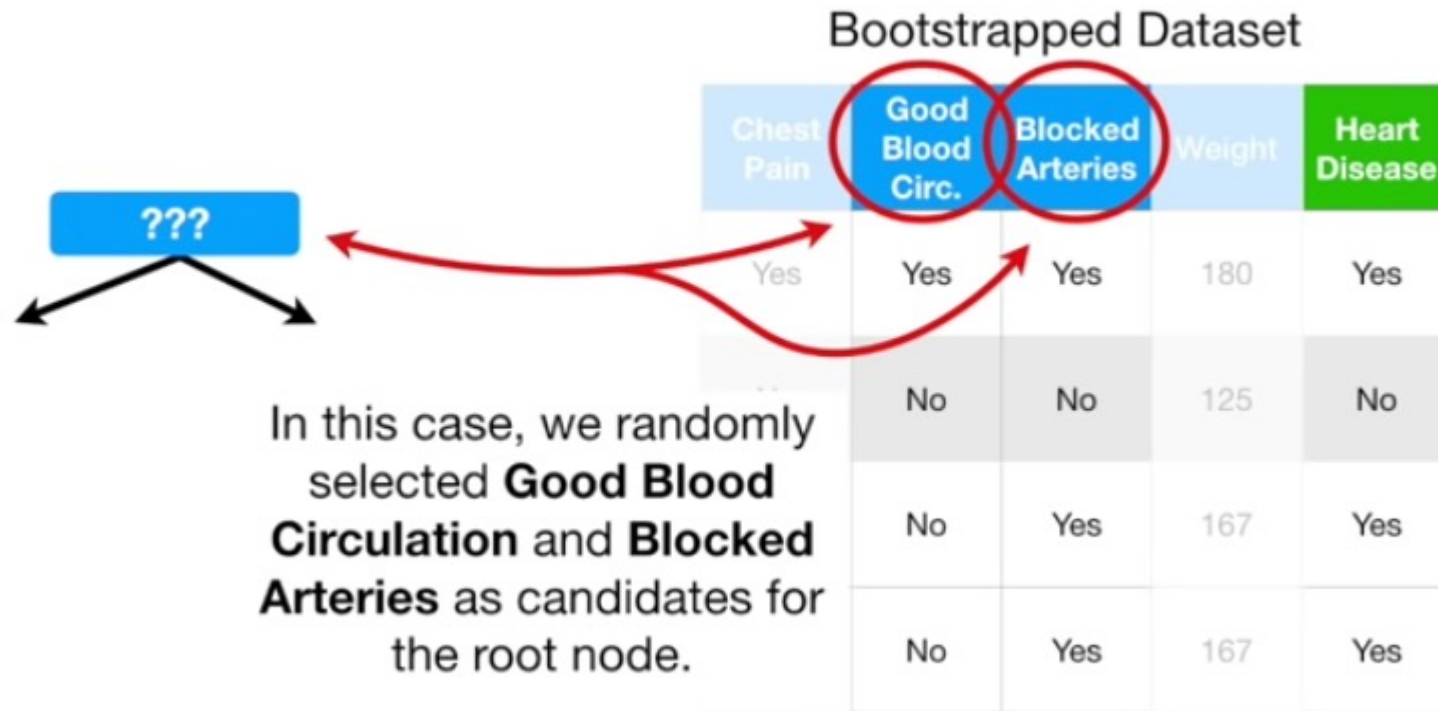
Step 2: Create a decision tree using the bootstrapped dataset, but only use a random subset of variables (or columns) at each step.

In this example, we will only consider 2 variables (columns) at each step.

Bootstrapped Dataset

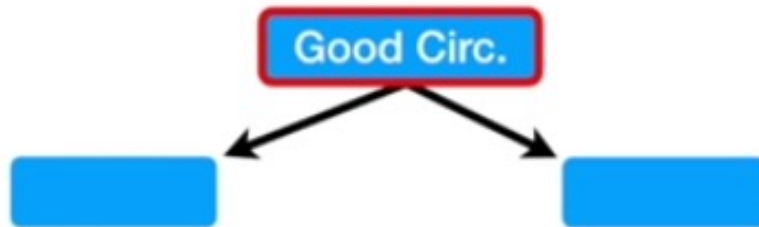
Chest Pain	Good Blood Circ.	Blocked Arteries	Weight	Heart Disease
Yes	Yes	Yes	180	Yes
No	No	No	125	No
Yes	No	Yes	167	Yes
Yes	No	Yes	167	Yes

Building random forests



Building random forests

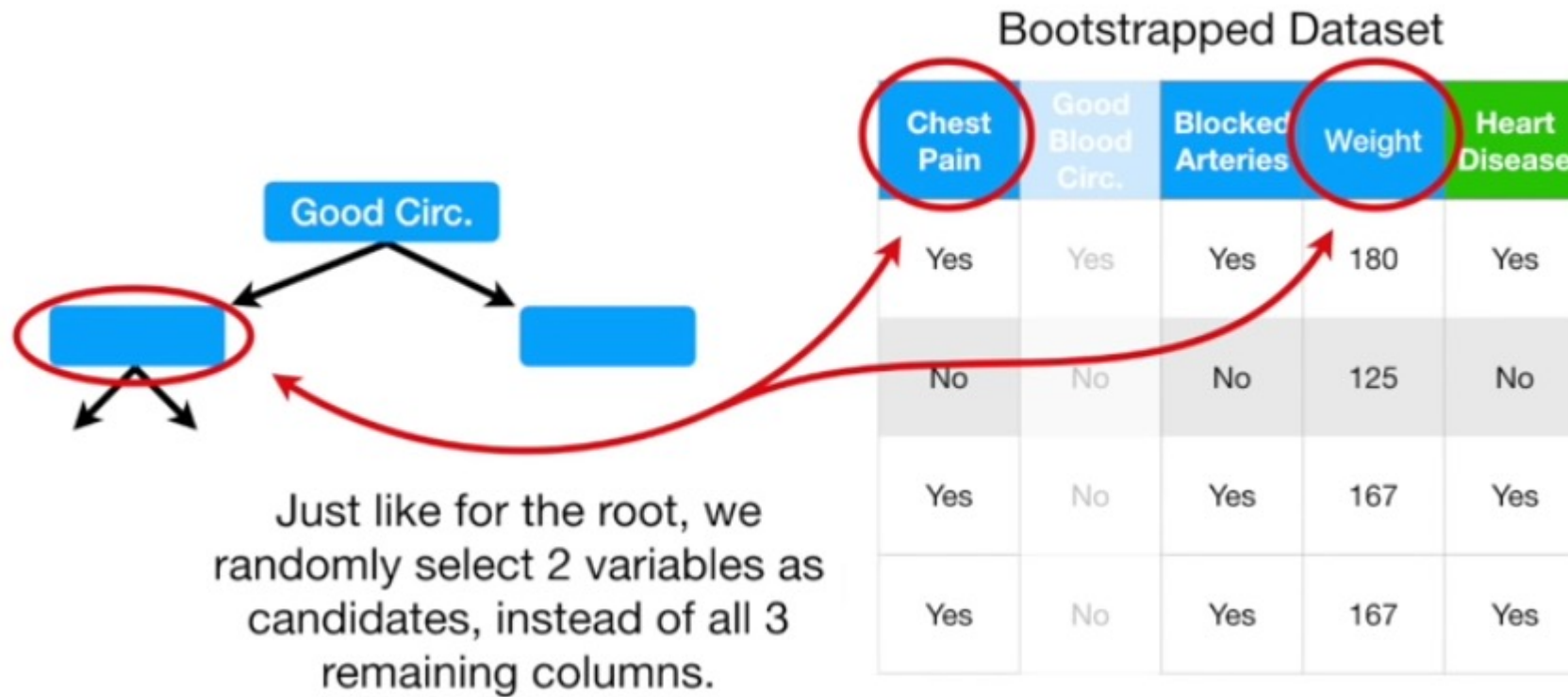
Just for the sake of the example, assume that **Good Blood Circulation** did the best job separating the samples.



Bootstrapped Dataset

Chest Pain	Good Blood Circ.	Blocked Arteries	Weight	Heart Disease
Yes	Yes	Yes	180	Yes
No	No	No	125	No
Yes	No	Yes	167	Yes
Yes	No	Yes	167	Yes

Building random forests

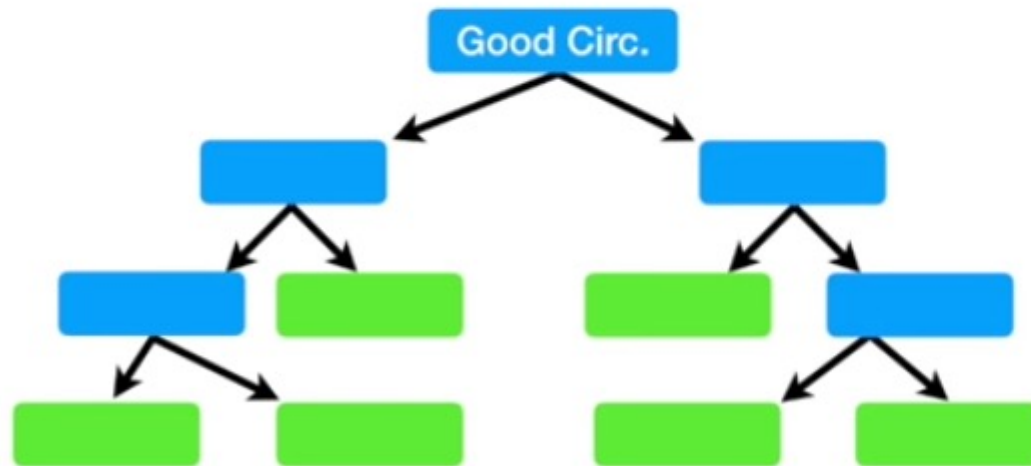


Continue as usual, but select features from a random subset!

Random forests

We built a tree...

- 1) Using a bootstrapped dataset
- 2) Only considering a random subset of variables at each step.

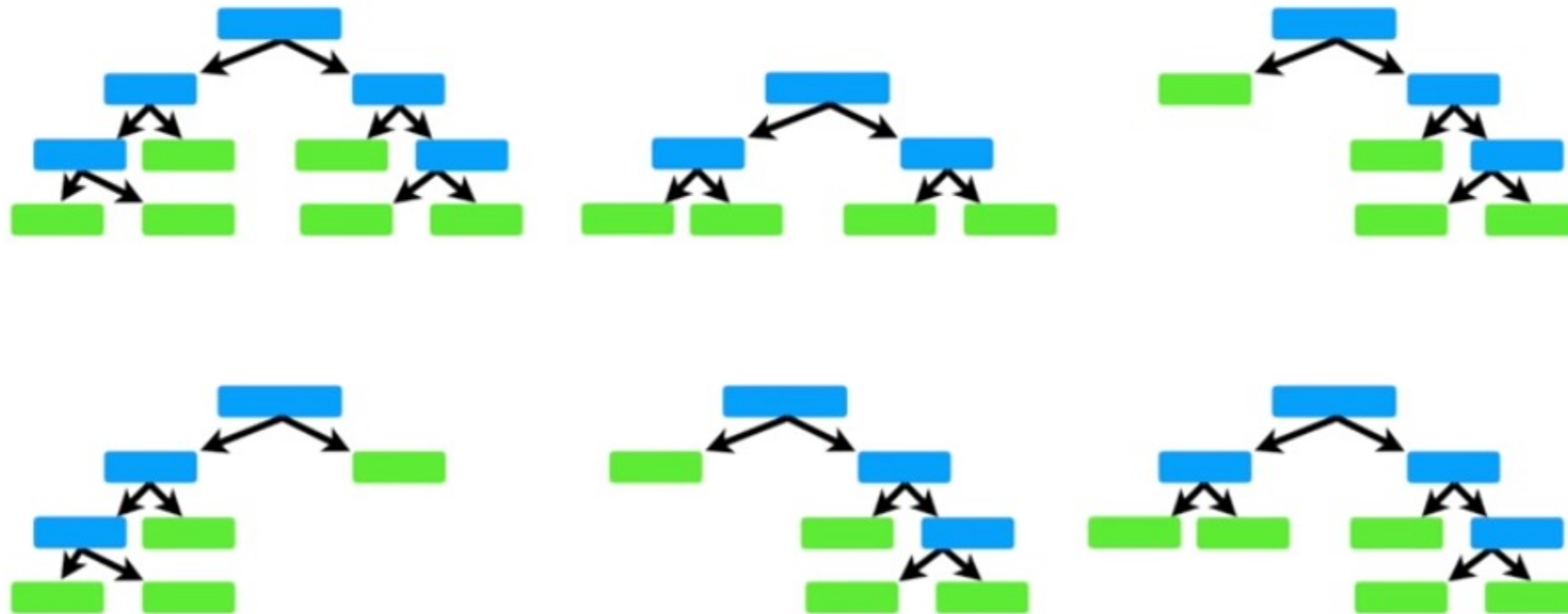


Bootstrapped Dataset

Chest Pain	Good Blood Circ.	Blocked Arteries	Weight	Heart Disease
Yes	Yes	Yes	180	Yes
No	No	No	125	No
Yes	No	Yes	167	Yes
Yes	No	Yes	167	Yes

Building random forests

Now go back to Step 1 and repeat: Make a new bootstrapped dataset and build a tree considering a subset of variables at each step.



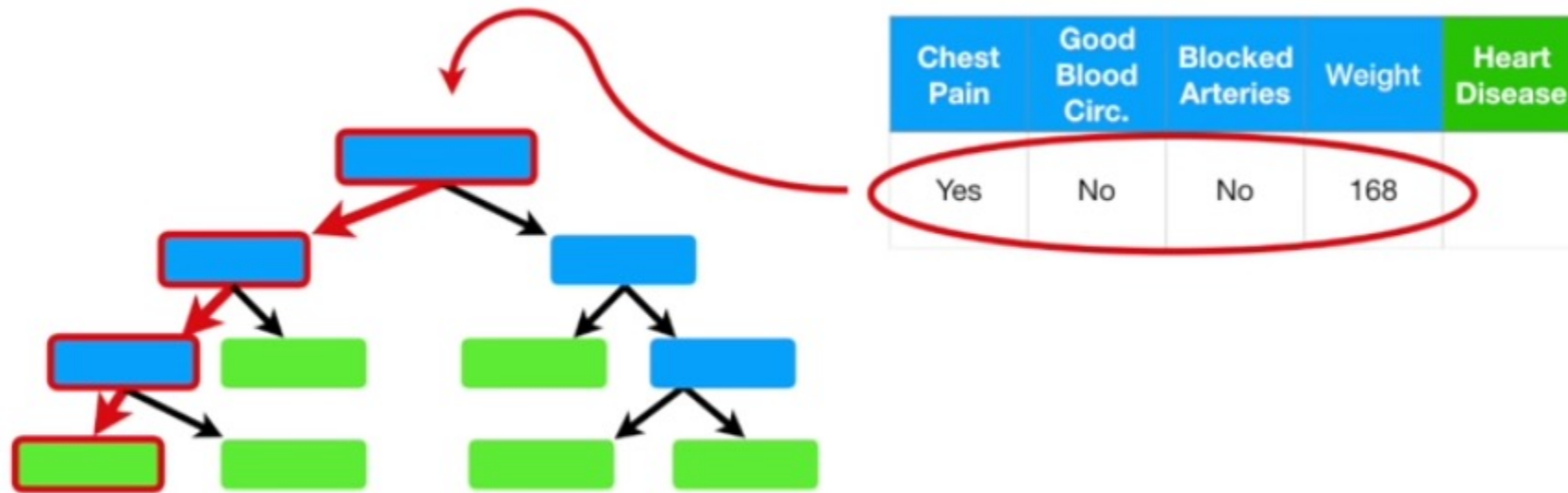
Using random forests

Suppose we have this new patient...

Chest Pain	Good Blood Circ.	Blocked Arteries	Weight	Heart Disease
Yes	No	No	168	

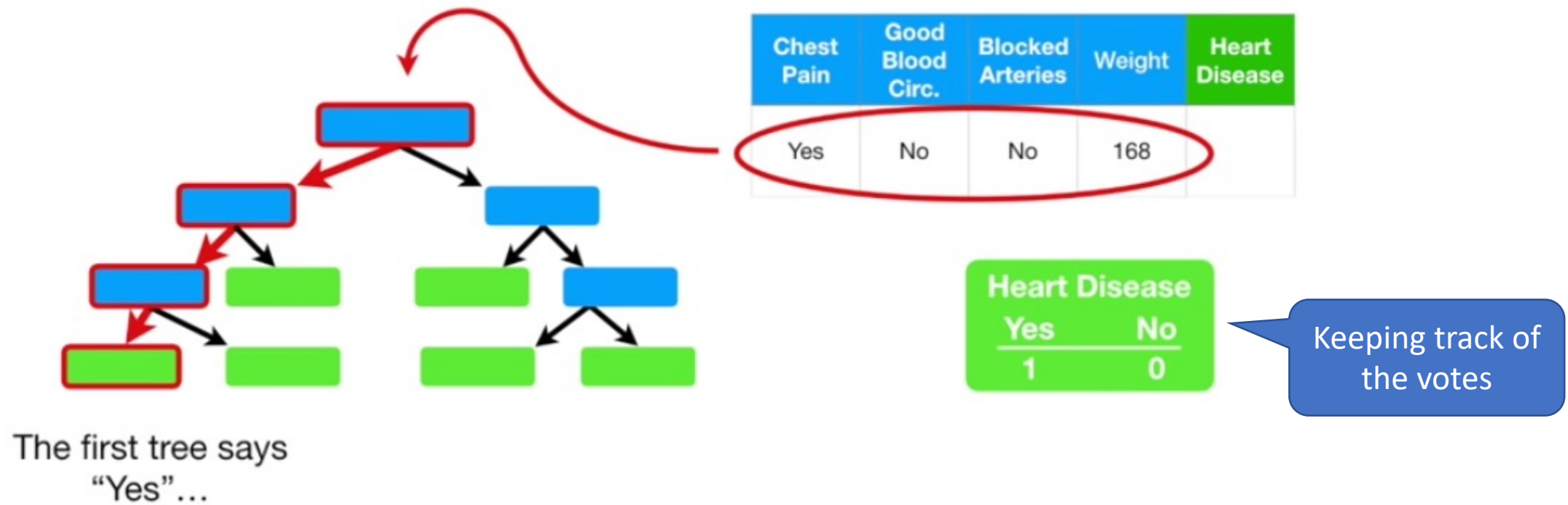
...and now we want to know if they have heart disease or not.

Using random forests



The first tree says
"Yes"...

Using random forests



Using random forests

Chest Pain	Good Blood Circ.	Blocked Arteries	Weight	Heart Disease
Yes	No	No	168	YES

In this case, “**Yes**” received the most votes, so we will conclude that this patient has heart disease.

Heart Disease	
Yes	No
5	1

All 6 trees have voted!
Then we make an aggregated decision.

Using random forests

In other words...

...change the number of
variables used per step...

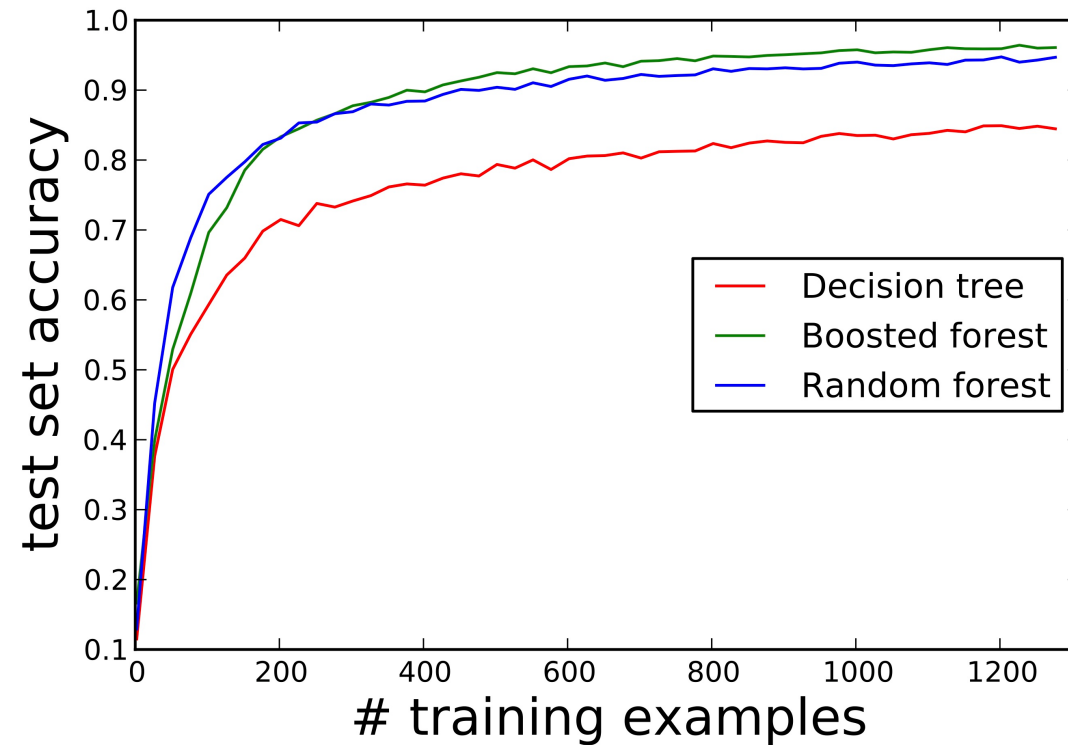
1) Build a Random Forest

2) Estimate the accuracy of a Random Forest.



Typically, start using the square root
of the number of variables and then
try a few more nearby values.

Forests vs. trees



Always evaluate accuracy on datapoints that were not used in the model construction (unseen datapoints). For example separated from the beginning or not included in the Bootstrap sets.

Random forests

Advantages:

- Random forests is considered as a highly accurate and robust method because of the number of decision trees participating in the process.
- It does not suffer from the overfitting problem. The main reason is that it takes the average of all the predictions, which cancels out the biases.

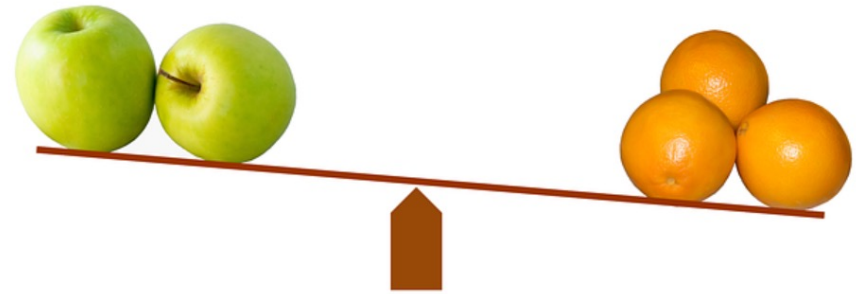
Random forests

Disadvantages:

- Random forests is slow in generating predictions because it has multiple decision trees. Whenever it makes a prediction, all the trees in the forest have to make a prediction for the same given input and then perform voting on it. This whole process is time-consuming.
- The model is difficult to interpret compared to a decision tree, where you can easily make a decision by following the path in the tree.

Comparing classification methods

- Accuracy
- Scalability
- Manual steps
- Multiclass or two-class
- Interpretability/Explainability



Best accuracy: random forest and SVM. Deep learning not tested.

[Comparison of 14 different families of classification algorithms on 115 binary datasets](#)

Comparing classification methods

1	Gender	Age	Salary	Purchased Iphone
2	Male	19	19000	0
3	Male	35	20000	0
4	Female	26	43000	0
5	Female	27	57000	0
6	Male	19	76000	0
7	Male	27	58000	0
8	Female	27	84000	0
9	Female	32	150000	1

Logistic Regression: Mean Accuracy = 82.75% – SD Accuracy = 11.37%

K Nearest Neighbor: Mean Accuracy = 90.50% – SD Accuracy = 7.73%

Kernel SVM: Mean Accuracy = 90.75% – SD Accuracy = 9.15%

Naive Bayes: Mean Accuracy = 85.25% – SD Accuracy = 10.34%

Decision Tree: Mean Accuracy = 84.50% – SD Accuracy = 8.50%

Random Forest: Mean Accuracy = 88.75% – SD Accuracy = 8.46%

Classification Algorithms

Neural networks are not
included in this comparison!