EE213M

Digital Circuits

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EE213M Digital Circuits

L1: Background and Number Systems - 1

Benefits of using Digital Representation

✓Reproducibility ✓Ease of design ✓Flexibility & Functionality

✓HDLs – Programmability ✓Speed

- Signals can be processed with minimal degradation (store, transmit, receive, amplify, etc.)
- Transistor circuits need to give an output that is either a high or a low – Easy and Reliable
- Computers can be used to process the data
- Data can be stored in a variety of storage media (ex. CD, DVD, Hard disk, etc.)
- Functionality of the system can be programmed (ex. the behaviour of a digital filter can be changed by simply changing its coefficients)
- Cost-effective

10 ps 2-3 ns

$\frac{1}{2n} s^{-1}$

$\sim 5\times10^{8}\ s^{-1}$

✓Economy

✓Advancement of Technology

Generic Digital Computer

<u>Memory</u>:

- Stores programs and data (input, output, and intermediate)

<u>ALU</u>:

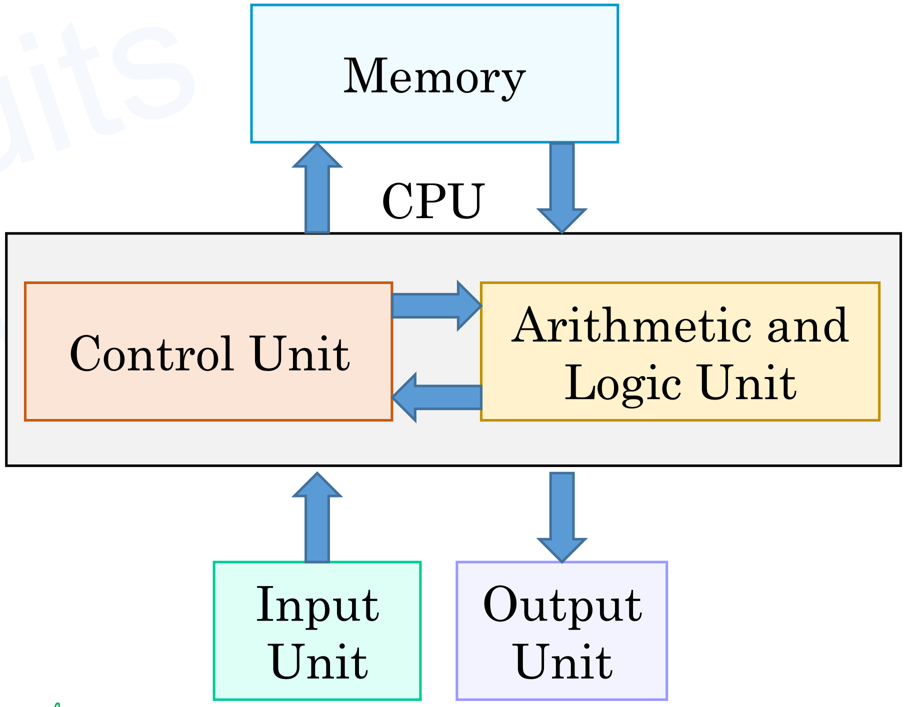
- Arithmetic and other data processing operations as specified by the program

<u>Control Unit</u>:

- Retrieves each instruction from the program stored in the memory
- Manipulates the ALU to execute the operation specified by the instruction
- Supervises the flow of information between various units

<u>Input and Output Units</u>:

- Key board, DVD drives, USB drive, Display, Printer etc.

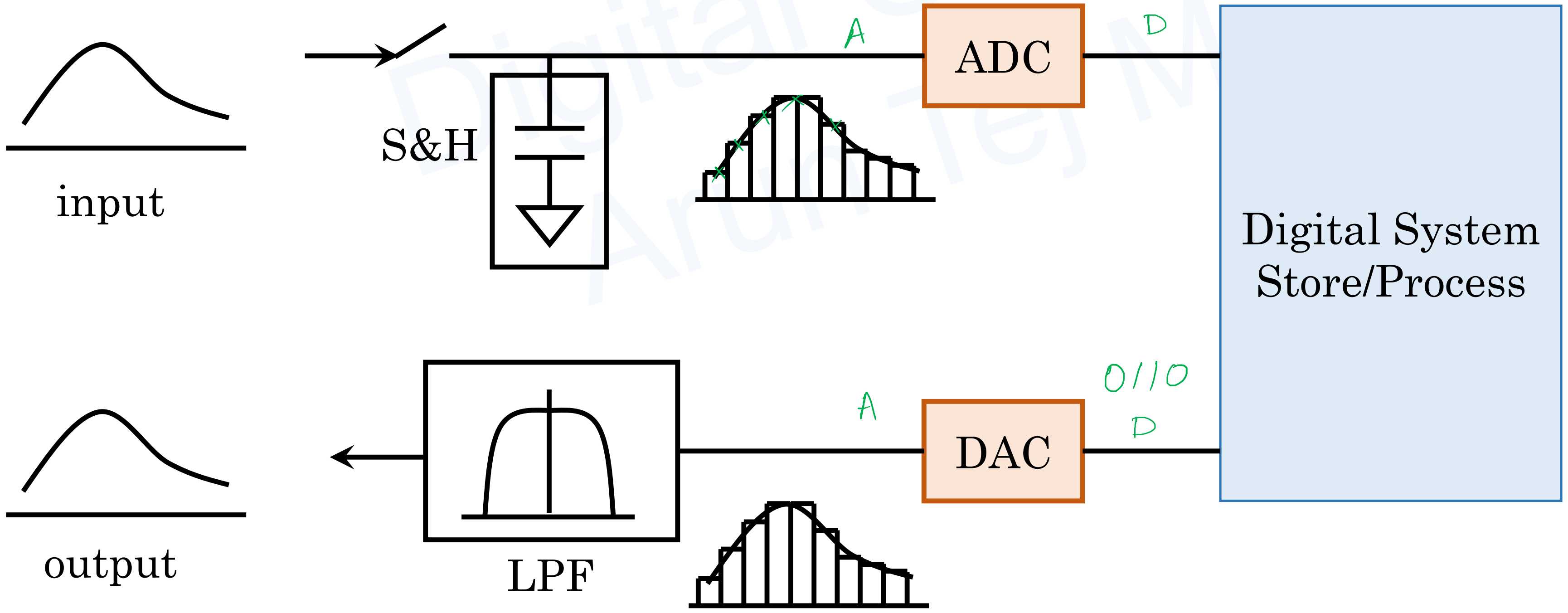


Flexible Displays
Rollable Foldable

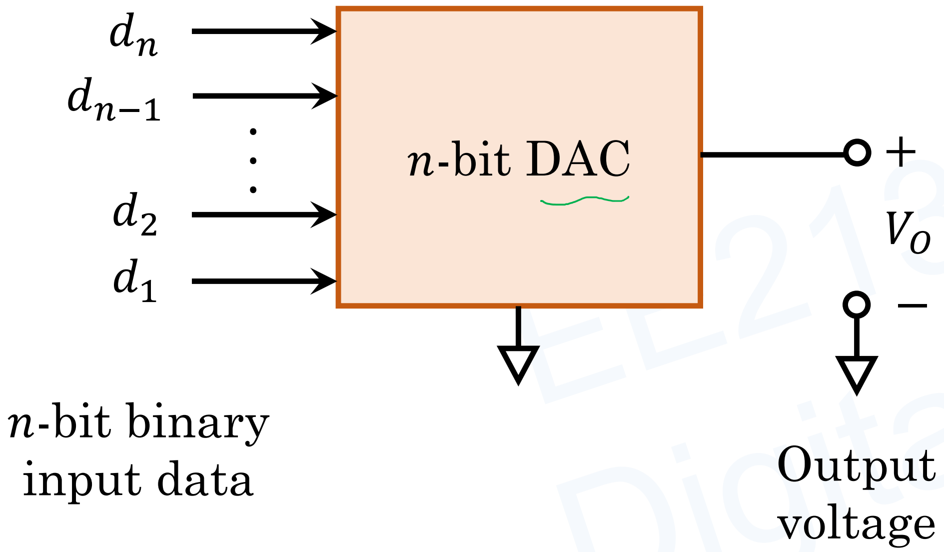
Flexible Electronics

Digital Systems

- Most of the real-life systems - analog in nature ⇒ Any digital system dealing with real-life signals (or data) would inevitably require an analog interface
- Sample and Hold Circuit, Analog-to-Digital Converters (ADCs), Digital-to-Analog Converters (DACs), Low Pass Filter



DAC



$$V_O = (d_1 2^{-1} + d_2 2^{-2} \ldots + d_n 2^{-n})V_{ref}$$

1 0 1 1

0000
0001

$(d_4 2^{-4})V_{ref}$

$(1 \times 2^{-4})\,5\text{ V}$

Smallest possible voltage change or the least significant bit, LSB equals $2^{-n}V_{ref}$

$V_{ref} = 5\text{ V}$

Binary input = 1011

$$V_O = (1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}) \times 5\text{ V}$$

$$= (0.5 + 0 + 0.125 + 0.0625) \times 5\text{ V} = 3.4375\text{ V}$$

$$V_{min} = 2^{-4} \times 5\text{ V}$$

$$= 0.3125\text{ V}$$

Embedded Systems

- Smaller computers called microcomputers/ microcontrollers or special-purpose computers called digital signal processors (DSPs) – *more prevalent, less powerful, as parts of products* we regularly use
- As a consequence of being integral parts of other products and often enclosed within them, they are called *embedded systems*.
- Different from general purpose computers → *software programs are permanently stored* and perform only those functions that are specific to the product → *embedded software*
- Human interface of the microcomputer → very limited or nonexistent
- Digital camera, Microwave oven, Automobiles (many embedded systems such as engine control unit, automatic breaking system, stability control unit etc.) …

Embedded Systems

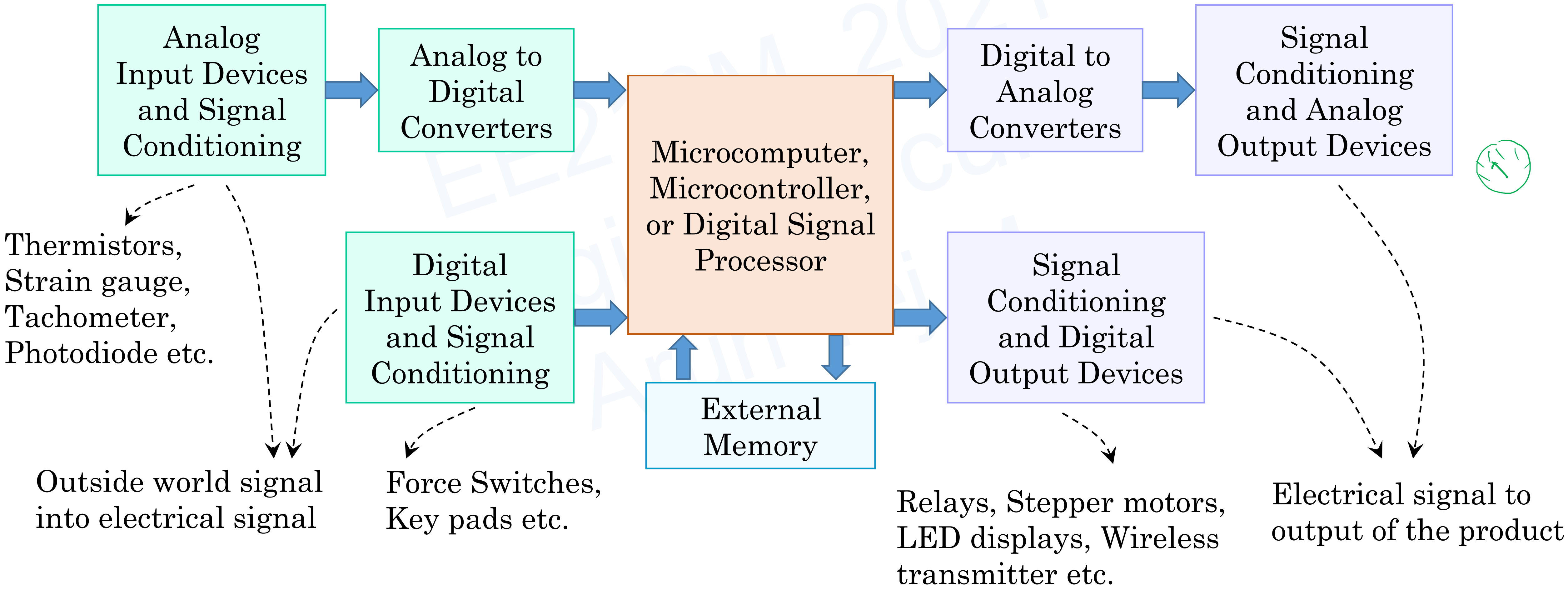
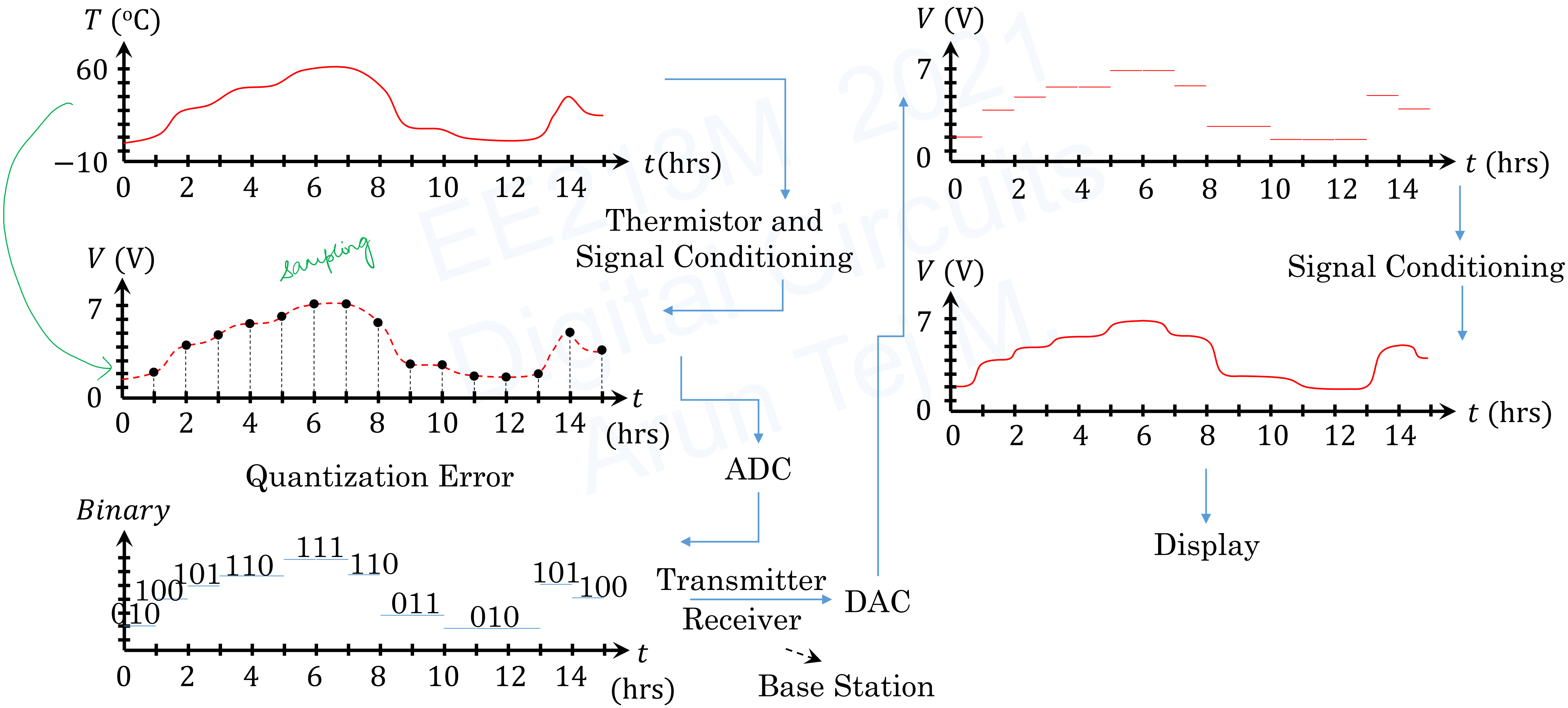


TABLE 1-1
Embedded System Examples

Application Area	Product
Banking, commerce and manufacturing	Copiers, FAX machines, UPC scanners, vending machines, automatic teller machines, automated warehouses, industrial robots, 3D printers
Communication	Wireless access points, network routers, satellites
Games and toys	Video games, handheld games, talking stuffed toys
Home appliances	Digital alarm clocks, conventional and microwave ovens, dishwashers
Media	CD players, DVD players, flat panel TVs, digital cameras, digital video cameras
Medical equipment	Pacemakers, incubators, magnetic resonance imaging
Personal	Digital watches, MP3 players, smart phones, wearable fitness trackers
Transportation and navigation	Electronic engine controls, traffic light controllers, aircraft flight controls, global positioning systems

(M.M. Mano, Ch.1)

Example – Wireless Weather Station (M.M. Mano, Ch.1)



Number Systems

- Digital systems operate only with a set of discrete values
- Discrete values – Represented using numbers
- Number systems:
    - Systematic way to represent and manipulate numbers
    - At the heart of any digital system
- Humans evolved to use *decimal* number system – 0 to 9 (10 digits i.e., *base or radix* is 10)
- Most of the ancient systems use base 10 (Indian, Roman, Chinese, Hebrew …)
- Some exceptions:
    - Mayan, Celtic, Muisca etc. (base 20 – called *vigesimal* systems)
    - Babylonian, Ekari, Sumerian etc. (base 60 – called *sexagesimal* systems)
- However, modern computers and other such digital systems, mostly, use other systems

A decimal number actually represents a polynomial in powers of 10

Ex.: $364.91 = 3 \times 10^2 + 6 \times 10^1 + 4 \times 10^0 + 9 \times 10^{-1} + 1 \times 10^{-2}$

$$A_{n-1}A_{n-2} \dots A_1A_0 \,.\, A_{-1}A_{-2} \dots A_{-m+1}A_{-m} \qquad A_i r^i$$

MSD ← → LSD

$$= A_{n-1}r^{n-1} + A_{n-2}r^{n-2} + \cdots A_1r^1 + A_0r^0 + A_{-1}r^{-1} + A_{-2}r^{-2} \dots A_{-m+1}r^{-m+1} + A_{-m}r^{-m}$$

✓ Positional notation – Only coefficients and radix point are used

Commonly used systems

What about base 1?

	System	Base	Symbols
Computers	Binary	2	0, 1
	Octal	8	0, 1, 2, …, 7
	Hexa-decimal	16	0, 1, 2, …, 9, A, B, …, F
Humans	Decimal	10	0, 1, 2, …, 9

Conversion

$4\times \quad 3+$

$$(312.4)_5 \quad = 3 \times 5^2 + 1 \times 5^1 + 2 \times 5^0 + 4 \times 5^{-1} \quad = (82.8)_{10}$$

Alternative method that reduces the number of operations, based on a factored form of the power series

$$(\ldots((A_{n-1}r + A_{n-2})r + (A_{n-3})r + \ldots + A_1)r + A_0$$
$$+ \ (A_{-1} + (A_{-2} + (A_{-3} + \ldots + (A_{-m+2} + (A_{-m+1} + A_{-m}r^{-1})r^{-1})r^{-1} \ldots)r^{-1})r^{-1})r^{-1}$$

$$(312.4)_5 \quad = ((3 \times 5 + 1) \times 5) + 2 + 4 \times 5^{-1} \quad = (82.8)_{10}$$

$3\times \quad 3+$

Conversion

$$(11010)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = (26)_{10}$$

$$(110101.11)_2 = 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$

$$= 32 + 16 + 0 + 4 + 0 + 1 + 0.5 + 0.25 = (53.75)_{10}$$

Powers of Two

2^{10} – K (kilo)

2^{20} – M (mega)

2^{30} – G (giga)

2^{40} – T (tera)

10^3

10^6

10^9

10^{12}

Powers of Two

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096	20	1,048,576
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

$$16\text{M} = 2^4 \times 2^{20} = 2^{24} = 16{,}777{,}216$$

Conversion

$(625)_{10} = (??)_2$

$$
\begin{aligned}
625 - 512 &= 113 = N_1 & 512 &= 2^9 \\
113 - 64 &= 49 = N_2 & 64 &= 2^6 \\
49 - 32 &= 17 = N_3 & 32 &= 2^5 \\
17 - 16 &= 1 = N_4 & 16 &= 2^4 \\
1 - 1 &= 0 = N_5 & 1 &= 2^0
\end{aligned}
$$

$$(625)_{10} = 2^9 + 2^6 + 2^5 + 2^4 + 2^0 = (1001110001)_2$$

Powers of Two

n	2^n	n	2^n	n	2^n
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Conversion

$10 \quad = (??)_3$ Decimal (N) to any other radix (R)

$$N = (a_n a_{n-1} \cdots a_2 a_1 a_0)_R = a_n R^n + a_{n-1} R^{n-1} + \cdots + a_2 R^2 + a_1 R^1 + a_0$$

$$\frac{N}{R} = a_n R^{n-1} + a_{n-1} R^{n-2} + \cdots + a_2 R^1 + a_1 = Q_1, \text{ remainder } a_0$$

$$\frac{Q_1}{R} = a_n R^{n-2} + a_{n-1} R^{n-3} + \cdots + a_3 R^1 + a_2 = Q_2, \text{ remainder } a_1$$

$$\frac{Q_2}{R} = a_n R^{n-3} + a_{n-1} R^{n-4} + \cdots + a_3 = Q_3, \text{ remainder } a_2$$

This process is continued until we finally obtain a_n.

Note that the remainder obtained at each division step is one of the desired digits and the least significant digit is obtained first.

Example

- ✓ Convert 53 to binary

$$\begin{array}{rll} 2\,\big/\,53 & & \\ 2\,\big/\,26 & \text{rem.} = 1 = a_0 & \\ 2\,\big/\,13 & \text{rem.} = 0 = a_1 & \\ 2\,\big/\,6 & \text{rem.} = 1 = a_2 & \\ 2\,\big/\,3 & \text{rem.} = 0 = a_3 & \\ 2\,\big/\,1 & \text{rem.} = 1 = a_4 & \\ 0 & \text{rem.} = 1 = a_5 & \end{array}$$

$$53_{10} = 110101_2$$

Conversion

Decimal fraction (F) to any other radix (R)

$$F = (.a_{-1}a_{-2}a_{-3}\cdots a_{-m})_R = a_{-1}R^{-1} + a_{-2}R^{-2} + a_{-3}R^{-3} + \cdots + a_{-m}R^{-m}$$

$$FR = a_{-1} + a_{-2}R^{-1} + a_{-3}R^{-2} + \cdots + a_{-m}R^{-m+1} = a_{-1} + F_1$$

Integer part

Fractional part

$$F_1R = a_{-2} + a_{-3}R^{-1} + \cdots + a_{-m}R^{-m+2} = a_{-2} + F_2$$

$$F_2R = a_{-3} + \cdots + a_{-m}R^{-m+3} = a_{-3} + F_3$$

… so on till we obtain a_{-m}

Integer part obtained at each step is one of the desired digits. MSD is obtained first.

Example

✓ Convert 0.625 to binary (2)

$$
\begin{array}{rl}
F = & .625 \\
\times & 2 \\
\hline
& 1.250 \\
& (a_{-1} = 1)
\end{array}
\qquad
\begin{array}{rl}
F_1 = & .250 \\
\times & 2 \\
\hline
& 0.500 \\
& (a_{-2} = 0)
\end{array}
\qquad
\begin{array}{rl}
F_2 = & .500 \\
\times & 2 \\
\hline
& 1.000 \\
& (a_{-3} = 1)
\end{array}
$$

$$.625_{10} = .101_2$$

Example

✓ Convert 0.7 to binary

$$
\begin{array}{r}
.7 \\
2 \\ \hline
(1).4 \\
2 \\ \hline
(0).8 \\
2 \\ \hline
(1).6 \\
2 \\ \hline
(1).2 \\
2 \\ \hline
(0).4 \\
2 \\ \hline
(0).8
\end{array}
$$

$$0.7_{10} = (0.1\,\underline{0110}\;\underline{0110}\;\underline{0110}\;\ldots)_2$$

Repeat

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