

## Question 2

### Part a

- 1) 4.0 GPA - P, the data match the decision tree
- 2) 3.9 GPA - P, the data match the decision tree
- 3) 3.9 GPA - P, the data match the decision tree
- 4) 3.8 GPA - yes publications - P, the data match the decision tree
- 5) 3.6 GPA - no publications - rank 2 university - P, the data match the decision tree
- 6) 3.6 GPA - yes publications - P, the data match the decision tree
- 7) 3.4 GPA - no publications - rank 3 university - N, the data match the decision tree
- 8) GPA 3.4 - No publication - Rank 1 University - N, data match the tree
- 9) GPA 3.2 - N, data match the tree
- 10) GPA 3.1 - N, data match the tree
- 11) GPA 3.1 - N, data match the tree
- 12) GPA 3.0 - N, data match the tree

### Part b

For the entire set, there are 6 positives and 6 negatives. Hence, the entropy of the set is:

$$E(S) = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1 \quad (1)$$

For GPA, the information gained is:

GPA  $\geq 3.9$ : 3 P, 0 N

$$E(GPA \geq 3.9) = -1 \log_2 1 - 0 \log_2 0 = 0 \quad (2)$$

3.2  $\leq$  GPA  $< 3.9$ : 3 P, 2 N

$$E(3.2 \leq GPA < 3.9) = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.9710 \quad (3)$$

GPA  $\leq 3.2$ : 0 P, 4 N

$$E(GPA \leq 3.2) = -0 \log_2 0 - 1 \log_2 1 = 0 \quad (4)$$

$$I(GPA) = \frac{1}{4} * 0 + \frac{5}{12} * 0.9710 + \frac{1}{3} * 0 = 0.4046 \quad (5)$$

$$Gain(GPA) = E(S) - I(GPA) = 1 - 0.4046 = 0.5954 \quad (6)$$

For university rank, the information gained is:

University rank = 1: 3 P, 2 N

$$E(rank = 1) = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.9710 \quad (7)$$

University rank = 2: 2 P, 1 N

$$E(rank = 2) = -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} = 0.9183 \quad (8)$$

University rank = 3: 1 P, 3 N

$$E(rank = 3) = -\frac{1}{4} \log_2 \frac{1}{4} - \frac{3}{4} \log_2 \frac{3}{4} = 0.8113 \quad (9)$$

$$I(rank) = \frac{5}{12} * 0.9710 + \frac{1}{4} * 0.9183 + \frac{1}{3} * 0.8113 = 0.9046 \quad (10)$$

$$Gain(rank) = E(S) - I(rank) = 1 - 0.9046 = 0.0954 \quad (11)$$

For whether the student has publications, the information gained is:

Published = Yes : 3 P, 2 N

$$E(Published = Yes) = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = .9710 \quad (12)$$

Published = No : 3 P, 4 N

$$E(Published = No) = -\frac{3}{7} \log_2 \frac{3}{7} - \frac{4}{7} \log_2 \frac{4}{7} = .9852 \quad (13)$$

$$I(Published) = \frac{5}{12}(.9710) + \frac{7}{12}(.9852) = 0.9792 \quad (14)$$

$$Gain(Published) = E(S) - I(Published) = 1 - 0.9792 = 0.0208 \quad (15)$$

For the quality of the student's recommendations, the information gained is:  
Recommendations = Good : 5 P, 3 N

$$E(\text{Recommendations} = \text{Good}) = -\frac{5}{8} \log_2 \frac{5}{8} - \frac{3}{8} \log_2 \frac{3}{8} = .9544 \quad (16)$$

Recommendations = Normal : 1 P, 3 N

$$E(\text{Recommendations} = \text{Normal}) = -\frac{1}{4} \log_2 \frac{1}{4} - \frac{3}{4} \log_2 \frac{3}{4} = .8113 \quad (17)$$

$$I(\text{Recommendations}) = \frac{8}{12}(.9544) + \frac{4}{12}(.8113) = 0.9067 \quad (18)$$

$$\text{Gain}(\text{Recommendations}) = E(S) - I(\text{Recommendations}) = 1 - 0.9067 = 0.0933 \quad (19)$$

For the root of the decision tree, the best attribute to use is GPA, since it has the highest information gain of all the attributes. Because a GPA of 4.0 always leads to P and a GPA of 3.3 always leads to N, we only need to investigate the next node for when GPA = 3.6.

For the subset, there are 3 positives and 2 negatives. Hence, the entropy of the subset is:

$$E(S) = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.9710 \quad (20)$$

For university rank, the information gained is:

University rank = 1 : 1 P, 1 N

$$E(\text{rank} = 1) = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1 \quad (21)$$

University rank = 2 : 1 P, 0 N

$$E(\text{rank} = 2) = -1 \log_2 1 - 0 \log_2 0 = 0 \quad (22)$$

University rank = 3 : 1 P, 1 N

$$E(\text{rank} = 3) = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1 \quad (23)$$

$$I(rank) = \frac{2}{5} * 1 + \frac{1}{5} * 0 + \frac{2}{5} * 1 = 0.8 \quad (24)$$

$$Gain(rank) = E(S) - I(rank) = 0.9710 - 0.8 = 0.1710 \quad (25)$$

For whether the student has publications, the information gained is:  
Published = Yes : 2 P, 0 N

$$E(Published = Yes) = -1 \log_2 1 - 0 \log_2 0 = 0 \quad (26)$$

Published = No : 2 P, 1 N

$$E(Published = No) = -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} = 0.9183 \quad (27)$$

$$I(Published) = \frac{2}{5}(0) + \frac{3}{5}(.9183) = 0.5510 \quad (28)$$

$$Gain(Published) = E(S) - I(Published) = 0.9710 - 0.5510 = 0.42 \quad (29)$$

For the quality of the student's recommendations, the information gained is:  
Recommendations = Good : 3 P, 2 N

$$E(Recommendations = Good) = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = .9710 \quad (30)$$

Recommendations = Normal : 0 P, 0 N

$$E(Recommendations = Normal) = -0 \log_2 0 - 0 \log_2 0 = 0 \quad (31)$$

$$I(Recommendations) = 1(.9710) + 0(0) = 0.9710 \quad (32)$$

$$Gain(Recommendations) = E(S) - I(Recommendations) = 0.9710 - 0.9710 = 0 \quad (33)$$

The best attribute to use next would be whether the student has publications. Because if the student has publications the answer is always positive, then we only need to examine when the student doesn't have publications to create the next child node in the decision tree.

For the subset, there is 1 positive and 2 negatives. Hence, the entropy of the

subset is:

$$E(S) = -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} = 0.9183 \quad (34)$$

For university rank, the information gained is:

University rank = 1 : 0 P, 1 N

$$E(rank = 1) = -0 \log_2 0 - 1 \log_2 1 = 0 \quad (35)$$

University rank = 2 : 1 P, 0 N

$$E(rank = 2) = -1 \log_2 1 - 0 \log_2 0 = 0 \quad (36)$$

University rank = 3 : 0 P, 1 N

$$E(rank = 3) = -0 \log_2 0 - 1 \log_2 1 = 0 \quad (37)$$

$$I(rank) = \frac{1}{3} * 0 + \frac{1}{3} * 0 + \frac{1}{3} * 0 = 0 \quad (38)$$

$$Gain(rank) = E(S) - I(rank) = 0.9710 - 0 = 0.9710 \quad (39)$$

For the quality of the student's recommendations, the information gained is:

Recommendations = Good : 1 P, 2 N

$$E(Recommendations = Good) = -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} = 0.9183 \quad (40)$$

Recommendations = Normal : 0 P, 0 N

$$E(Recommendations = Normal) = -0 \log_2 0 - 0 \log_2 0 = 0 \quad (41)$$

$$I(Recommendations) = 1(.9183) + 0(0) = 0.9183 \quad (42)$$

$$Gain(Recommendations) = E(S) - I(Recommendations) = 0.9183 - 0.9183 = 0 \quad (43)$$

The best attribute to use next would be the university rank. Because there is no further information necessary to categorize positives and negatives, recommendation quality is not needed to determine the final outcome. Thus, the decision tree is complete.

### Part 3

The decision tree we obtained in part b is identical to the decision tree provided in part a. Because all of the data match the decision tree, this is not surprising.

### Question 4

#### Part a

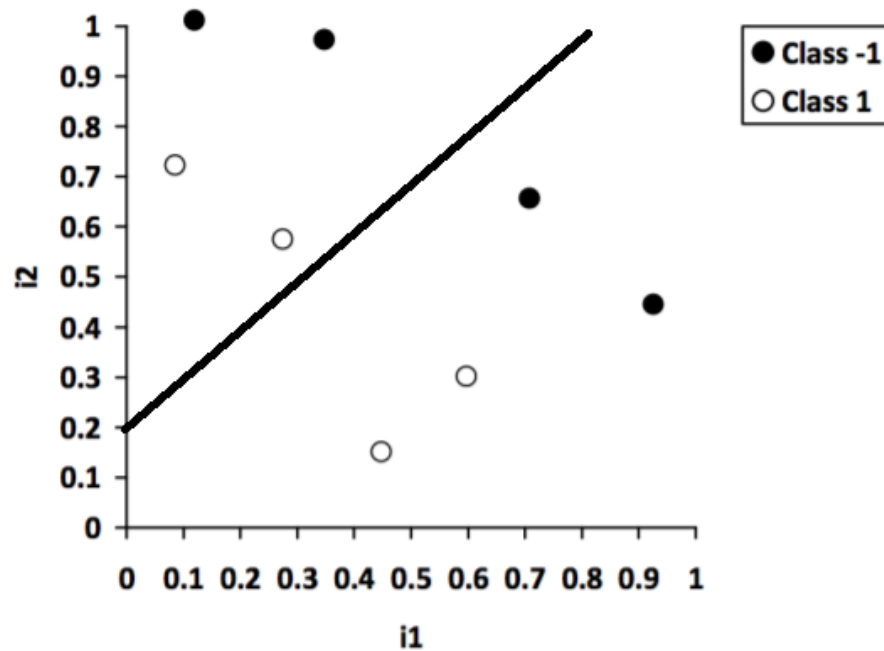


Figure 1: The plot with

## Question 5

Part a

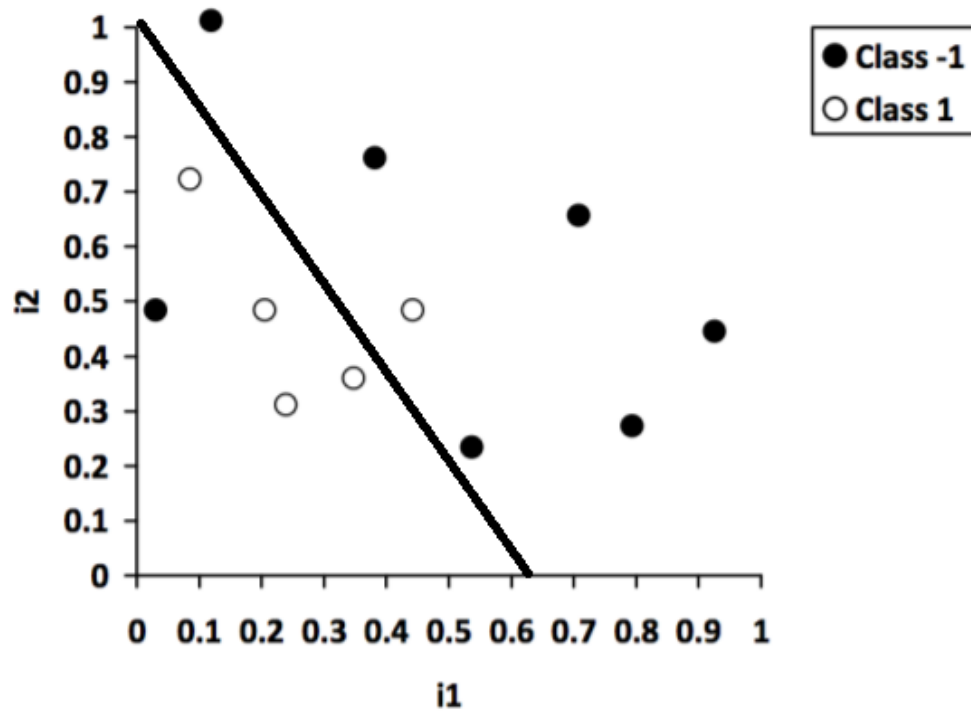


Figure 2: The minimum error possible is 2.

Part b

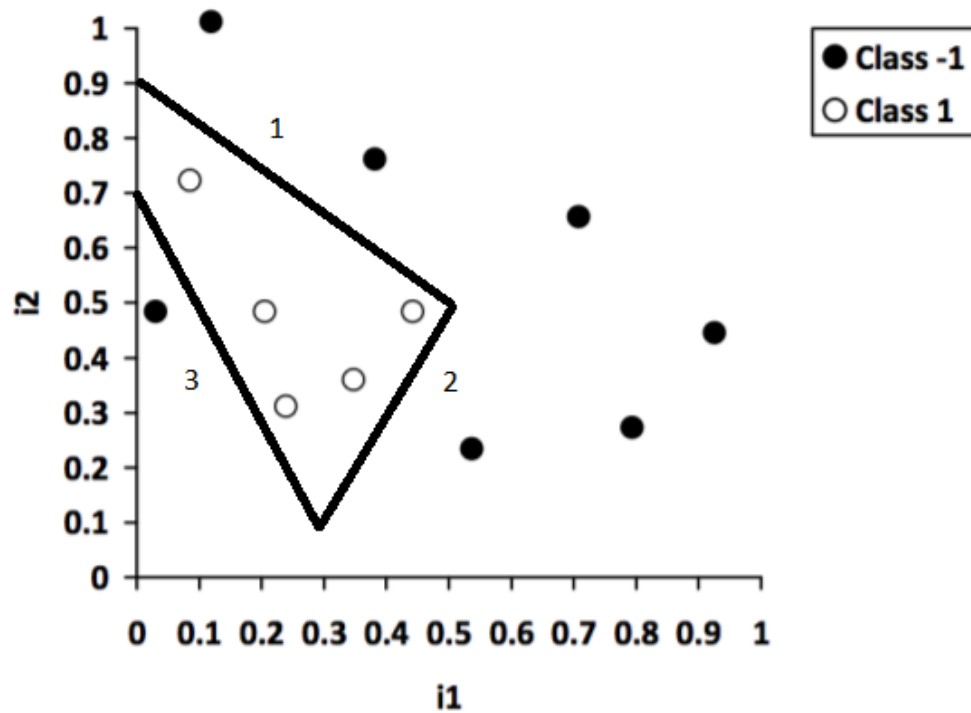


Figure 3: A multilayer perceptron with 3 layers separates the two categories.

The weights for the lines should be  $w_0 = -0.9, w_1 =$