Statistical Computing and Simulation(0625)

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Poisson(λ) is Poisson distribution with mean λ. It has the probability mass function :

$$\mathrm{P(x)} = \frac{\lambda^x}{x!} e^{-\lambda}$$

Generate samples from Poisson(5) by Metropolis -Hasting algorithm. Use the following proposal distribution:

$$\mathrm{Q} = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

That is, $\mathrm{q_{00}} = \frac{1}{2}$, $\mathrm{q_{ij}} = \frac{1}{2}$, for $\mathrm{j = i + 1}$ or $\mathrm{j = i - 1}$.

<u>(a) Write down the Metropolis -Hasting algorithm.</u>

< Sol. >

Step1.

Q 為一個轉移矩陣，此矩陣的橫軸與縱軸皆由 0 起始，轉移的方法為以下

現值為 0, 下一次數值為 $\mathrm{y} = \begin{cases} 0, with\ probability\ \frac{1}{2} \\ 1, with\ probability\ \frac{1}{2} \end{cases}$,

現值為$\mathrm{x_t}$，$\mathrm{x_t} \neq 0$, 下一次數值為 $\mathrm{y} = \begin{cases} x_t - 1, with\ probability\ \frac{1}{2} \\ x_t + 1, with\ probability\ \frac{1}{2} \end{cases}$,

Step2.

接受率$\alpha = \min\{\frac{\pi_y}{\pi_x}\frac{q(x_t|y)}{q(y|x_t)}, 1\}$, $\pi \sim \mathrm{poi}(5), \mathrm{q} \sim \text{proposal distribution}$,

而$\mathrm{q}(\mathrm{x_t}|y) = \frac{1}{2} = q(y|x_t)$，$\frac{\pi_y}{\pi_x} = \frac{x_t!}{y!}\lambda^{y-x_t}$，因此$\alpha = \min\{\frac{x_t!}{y!}\lambda^{y-x_t},\ 1\}$，來確認是否接受下一個值。

Step3.

$$\mathrm{x_{t+1}} = \begin{cases} y, with\ probability\ \alpha \\ x_t, with\ probability\ (1-\alpha) \end{cases}$$

(b) Generate an MCMC sequence of length 5000.

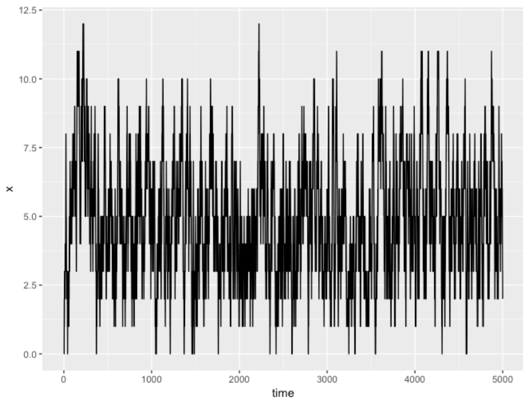
< Sol. >

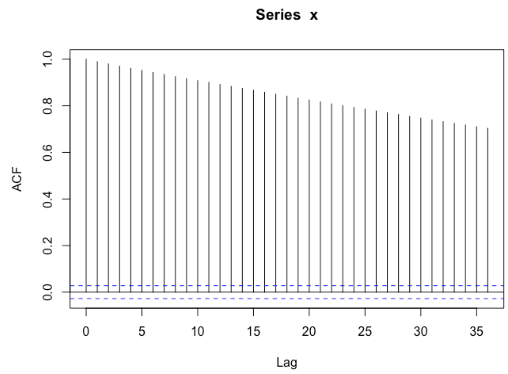
給定起始值$x_0 = 0$

(1) Compute the acceptance rate.

Acceptance rate: 0.818

(2) Draw the time series plot





(3) Discard the first 2000 values, and then compute the mean and variance based on the remaining 3000 value.

Mean: 4.802667; Variance: 5.157445

(4) Discard the first 2000 values, and then compute $P(X = x)$ where x = 0,1,2, …,10 base on the remaining 3000 values.

X	0	1	2	3	4	5
Rate(%)	0.733	4.367	10.533	16.067	17.000	15.700

X	6	7	8	9	10	-
Rate(%)	12.833	9.767	6.467	3.467	1.967	-

Appendix:

Code is in my github:
https://github.com/kevinpiger/Exam-problem-number-1-Metropolis-algorithm-for-Poisson-5-