Tessellation

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CSE168: Rendering Algorithms

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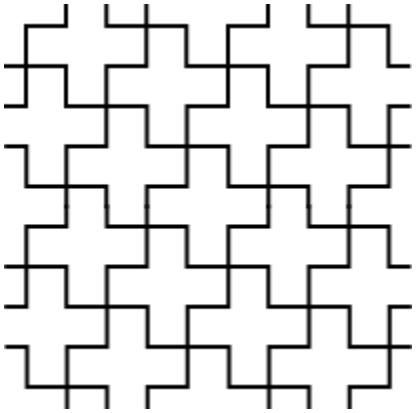
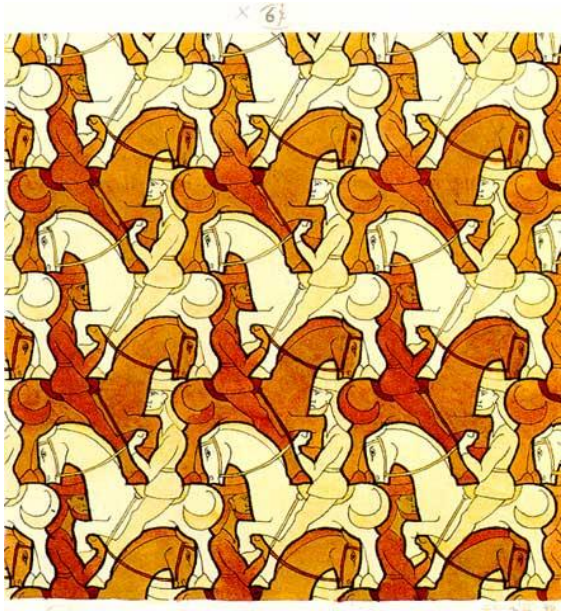
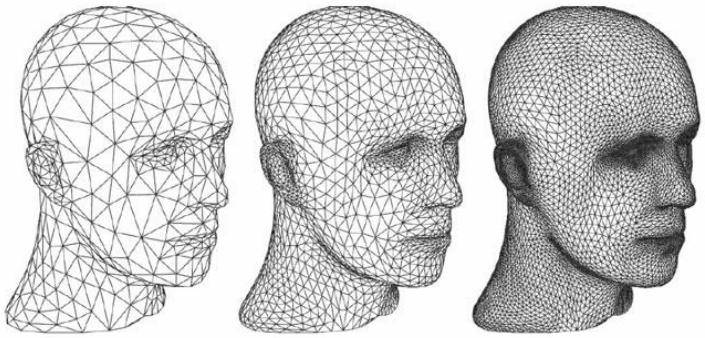
Tessellation

- *Tessellation* refers to the tiling of a surface (or higher dimensional shape) with no overlaps or gaps
- Typically, in computer graphics, it refers more specifically to the triangulation of surfaces







Parametric Surfaces

Parametric Surface

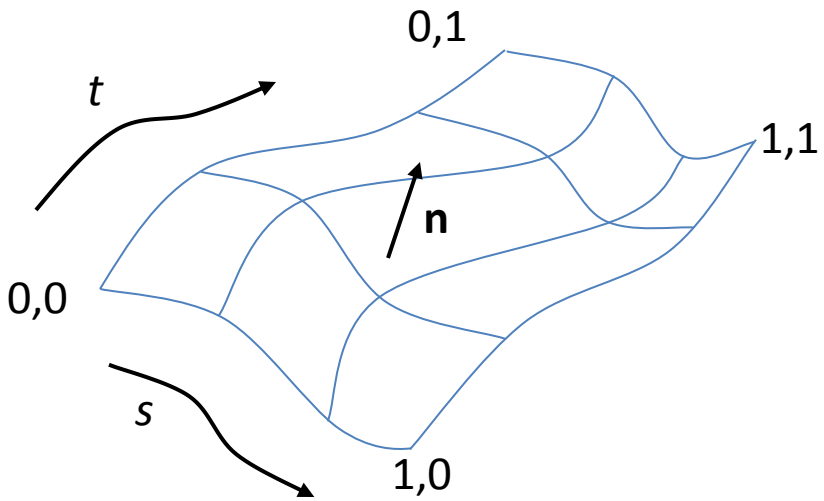
- A *parametric surface* (or *explicit surface*) is a surface that is explicitly defined mathematically as a function of two parameters:

$$\mathbf{x} = \mathbf{f}(s, t)$$

- For some domain of *s* & *t* (typically from 0…1)
- Parametric surfaces have a rectangular topology
- Popular surfaces of this type include:
  - Bezier surfaces
  - B-splines
  - NURBS (non-uniform rational B-splines)

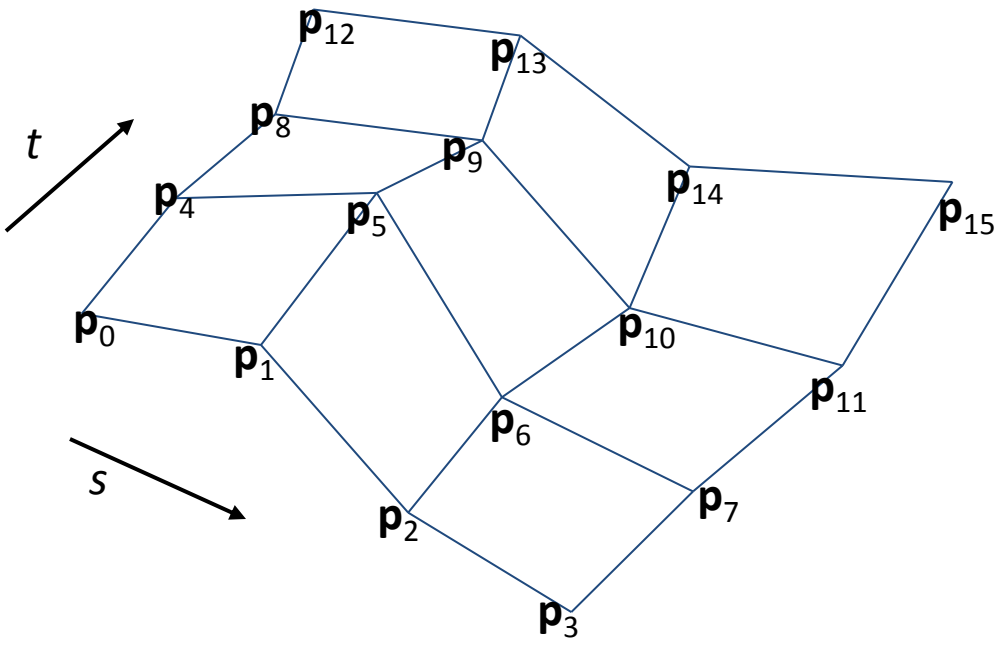
Bezier Surfaces

- Bezier surfaces are a straightforward extension to Bezier curves
- Instead of the curve being parameterized by a single variable *t*, we use two variables, *s* and *t*
- By definition, we choose to have *s* and *t* range from 0 to 1 and we say that an *s*-tangent crossed with a *t*-tangent will represent the normal for the front of the surface



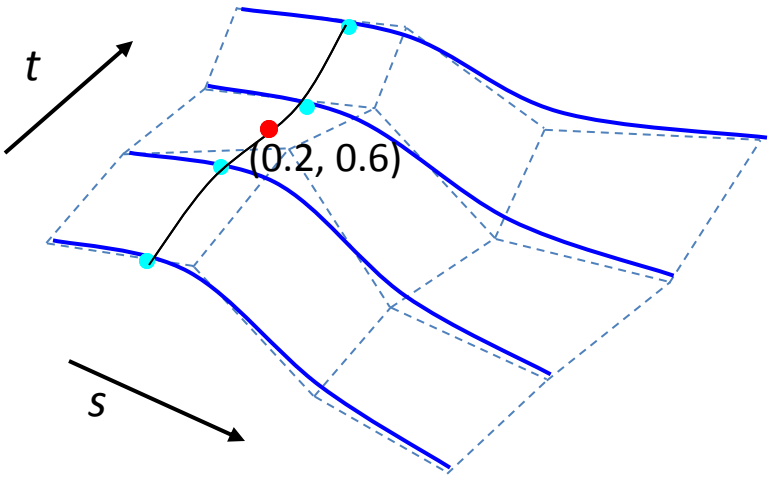
Control Mesh

- Consider a *bicubic* Bezier surface (bicubic means that it is a cubic function in both the *s* and *t* parameters)
- A cubic curve has 4 control points, and a bicubic surface has a grid of 4x4 control points, $\mathbf{p}_0$ through $\mathbf{p}_{15}$



Surface Evaluation

- The bicubic surface can be thought of as 4 curves along the *s* parameter (or alternately as 4 curves along the *t* parameter)
- To compute the location of the surface for some (*s*,*t*) pair, we can first solve each of the 4 *s*-curves for the specified value of *s*
- Those 4 points now make up a new curve which we evaluate at *t*
- Alternately, if we first solve the 4 t-curves and to create a new curve which we then evaluate at *s*, we will get the exact same answer
- This gives a pretty straightforward way to implement smooth surfaces with little more than what is needed to implement curves

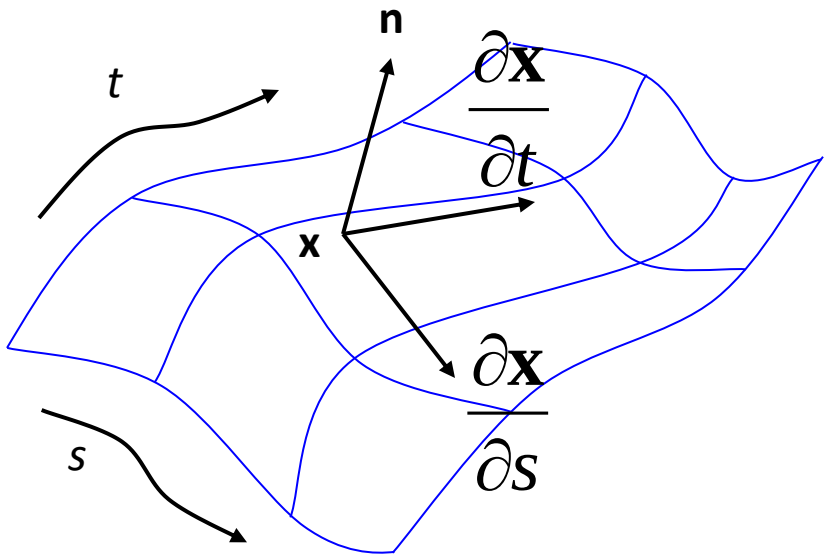


Normals

- To compute the normal of the surface at some location (*s*,*t*), we compute the two tangents at that location and then take their cross product
- Usually, it is normalized as well

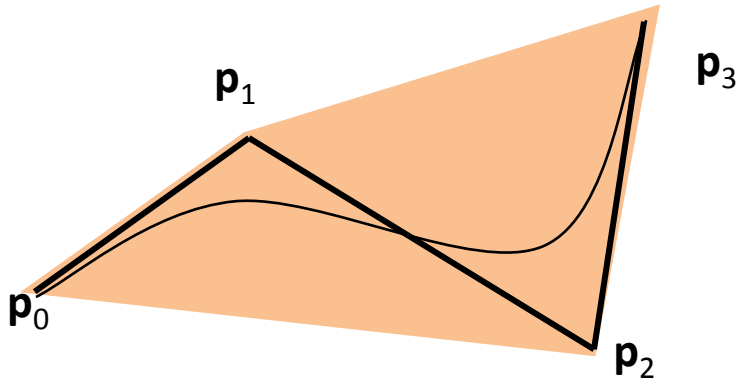
$$\mathbf{n}^* = \frac{\partial \mathbf{x}}{\partial s} \times \frac{\partial \mathbf{x}}{\partial t}$$

$$\mathbf{n} = \frac{\mathbf{n}^*}{\left|\mathbf{n}^*\right|}$$



Convex Hull Property

- If we take all of the control points for a Bezier curve and construct a convex polygon around them, we have the *convex hull* of the curve
- An important property of Bezier curves is that every point on the curve itself will be somewhere within the convex hull of the control points
- Surfaces have this same property



Tessellation

Uniform Tessellation

- The most straightforward way to tessellate a parametric surface is *uniform tessellation*
- With this method, we simply choose some resolution in *s* and *t* and uniformly divide up the surface like a grid
- This method is very efficient to compute, as the cost of evaluating points on the surface is only slightly more costly than evaluating points on a curve
- However, as the generated mesh is uniform, it may have more triangles than it needs in flatter areas and fewer than it needs in highly curved areas

Adaptive Tessellation

- Very often, the goal of a tessellation is to provide the fewest triangles necessary to accurately represent the original surface
- For a curved surface, this means that we want more triangles in areas where the curvature is high, and fewer triangles in areas where the curvature is low
- We may also want more triangles in areas that are closer to the camera, and fewer farther away
- *Adaptive tessellation* schemes are designed to address these requirements

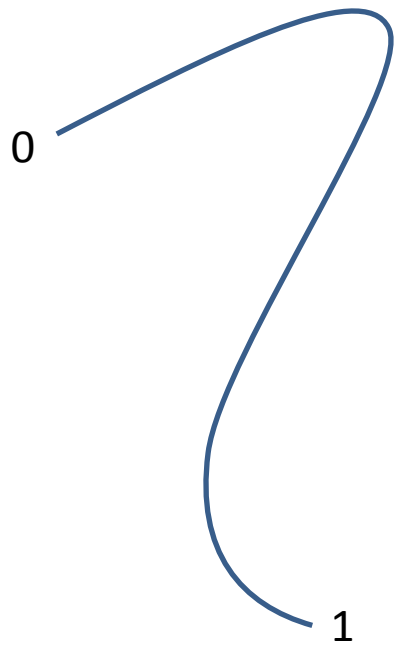
Recursive Tessellation

- Most adaptive tessellation approaches are recursive and use a top-down process
- We will first look at a recursive tessellation of a single curve, and then extend it to surfaces
- We'll assume that the curve is defined over the [0…1] domain in *t*
- We start by examining the entire curve in that domain and consider what would happen if we simply replaced it with a line segment
- We then measure how well the line segment represents the surface, and if we feel it is good enough, then we are done
- Otherwise, the curve is subdivided into 2 curves and we repeat the process recursively on each half

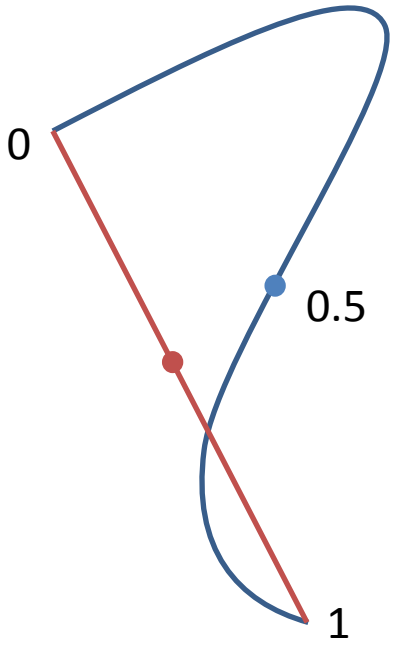
Recursive Tessellation

- How do we determine if a line segment is a good enough approximation to the curve?
- There are many variations on this process, but a simple and common approach is to compare the midpoint of the line segment to the midpoint of the curve
- If they are within some specified distance, then we say it's good enough
- Otherwise, we split

Curve Tessellation



Curve Tessellation



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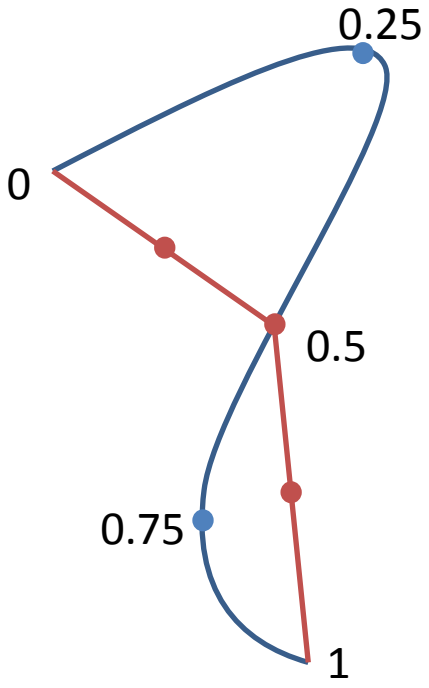
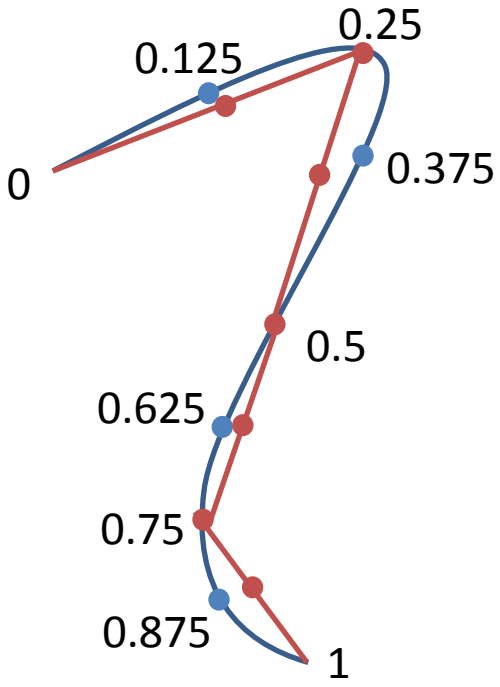
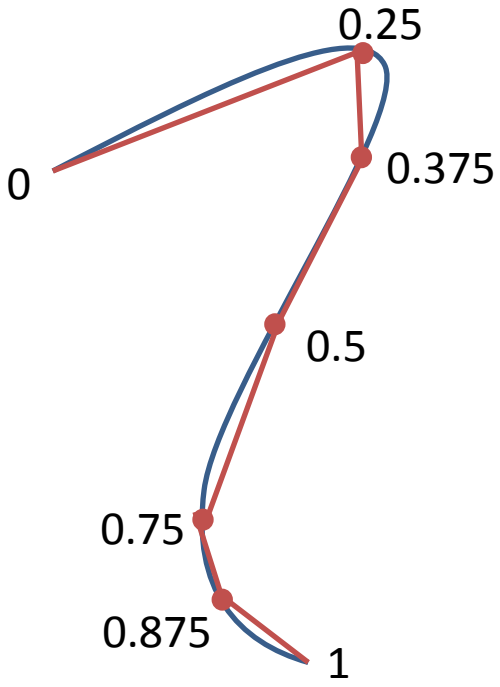


Curve Tessellation



Curve Tessellation

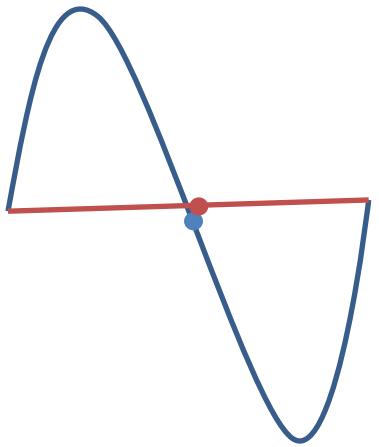


Curve Tessellation Algorithm

```
Curve::Tessellate(float t0, float t1, Vector3 &p0, Vector3 &p1) {
        float t05 = 0.5*(t0+t1);            // mid-parameter
        Vector3 c05 = Evaluate(t05);        // evaluate curve at t05
        Vector3 p05 = 0.5*(p0+p1);          // midpoint of segment
        if(c05.Distance(p05) > TOLERANCE) {
                Tessellate(t0,t05,p0,p05);
                Tessellate(t05,t1,p05,p1);
        }
        else DrawSegment(p0,p1);
}
```

Subdivision Rules

- The previous example was based on comparing the distance of the midpoint of a curve to the midpoint of the approximating segment
- This process is generally pretty good and widely used, however there are a few potential problems and other issues
- For one thing, we could have a situation where the midpoint of the curve happens to hit the midpoint of the segment by chance, even though the two hardly match
- There are some practical ways to handle this



Subdivision Rules

- In addition to the midpoint distance rule, we can add other rules that would trigger a segment to subdivide
- Some possibilities include:
  - Compare distances of other points (0.25 and 0.75 for example)
  - Compare the angle of the tangents at the start and end points
  - Mathematically evaluate the exact curvature at the midpoint (assuming the curve type allows this)
  - Compare the ratio of the midpoint distance to the length of the segment
- All these rules (and others) can be easily combined, since they are just binary decisions and the curve can be subdivided if any one of the rules triggers

Pixel-Based Error

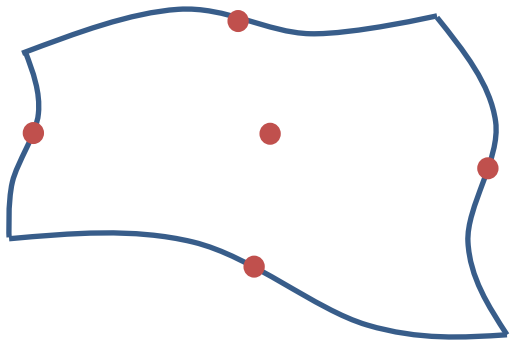
- One nice advantage of the midpoint distance approach is that it can be divided by the distance to the camera to provide a pixel based error measurement
- This provides a scheme for tessellating curves based on the error in pixels rather than the error in 3D distance
- This will cause curves closer to the camera to automatically adapt more than curves far away

Surface Tessellation

- Most adaptive tessellation approaches are recursive and use a top-down process
- We'll assume that the surface is defined over the [0…1] domain in *s* and *t*
- We start by examining the entire surface in that domain and consider what would happen if we simply replaced it with two triangles
- We then measure how well the two triangles represent the surface, and if we feel it is good enough, then we are done
- Otherwise, the surface is subdivided (usually into 2 or 4 sub-surfaces) and we repeat the process recursively on each sub-surface

Surface Tessellation

- As with curves, we can use a rule essentially based on midpoint distances
- One nice approach is to test the midpoints of the 4 edges of the surface as well as the midpoint of the entire surface
- This provides a scheme for splitting the surface only in *s*, only in *t*, or in both *s* and *t*, depending on what is required



Mixed Uniform/Adaptive Tessellation

- Some practical renderers use a mixed tessellation scheme
- First, the original surface patch is adaptively subdivided into several subpatches, each approximately the same size (say around 10 pixels on a side)
- Then, each of the subpatches (which is just a rectangular *s*,*t* range within the larger 0,1 rectangle) is uniformly tessellated to some resolution (say 10 x 10)
- The result is that the curved surface is tessellated into triangles roughly the size of a single pixel
- The bulk of the cost of the algorithm is in the uniform tessellation, which can be implemented in a very efficient way

Ray Tracing Curved Surfaces

Ray Tracing Curved Surfaces

- There are several strategies for including curved surfaces into a ray tracer:
  - Pre-tessellation: before starting to render, we tessellate the surface into a bunch of triangles and add them to a spatial data structure
  - Lazy evaluation: tessellate and build a spatial data structure on the fly, as rays are tested against the surface
  - Direct ray intersection: don’t tessellate at all. Every ray is intersected with the surface through a more direct means

Pre-Tessellation

- The pre-tessellation approach is the most common in practice due to its simplicity and high performance
- Curved surfaces can be pre-tessellated either based on 3D spatial properties and/or properties measured in pixel space
- One catch however, is with instanced objects
- We can’t really use pixel space metrics easily if we are using multiple copies of the geometry, each at different distances to the camera

Lazy Evaluation

- For the lazy evaluation approach, we start by only computing the bounding box of the entire surface
- When the bounding box is intersected by a ray, we then recursively subdivide the surface, computing bounding boxes of the sub-surfaces as we go
- We continue subdividing the boxes that the ray intersects until we generate triangles
- These are then kept in a box-tree-like data structure so they don't need to be recomputed for the next ray that comes by
- This approach does essentially the same work as recursive subdivision combined with box-tree generation, but will have some performance overhead for the additional bookkeeping and complexity
- This approach might be useful if we didn't expect rays to hit all of the surfaces, which might happen for simple renders, but wouldn't be effective in a path tracer that bounces rays all over the place
- This approach can also be useful for extremely large scenes that don't fit into memory. If combined with a system for deleting sections of the box-tree that haven't been intersected for a while (or moving them to a slower, larger memory pool), we can create a system that fits within some fixed memory limit

Direct Intersection

- We can intersect rays directly with the surface if we want
- However, this is guaranteed to be much slower than pre-tessellated triangles, so it is only used in experimental cases or situations where memory is very limited
- Some surfaces can actually be mathematically intersected by solving the roots of a large polynomial
- Other surfaces are too complex and require a recursive subdivision, similar to the lazy evaluation, but without retention of the data structure for later use

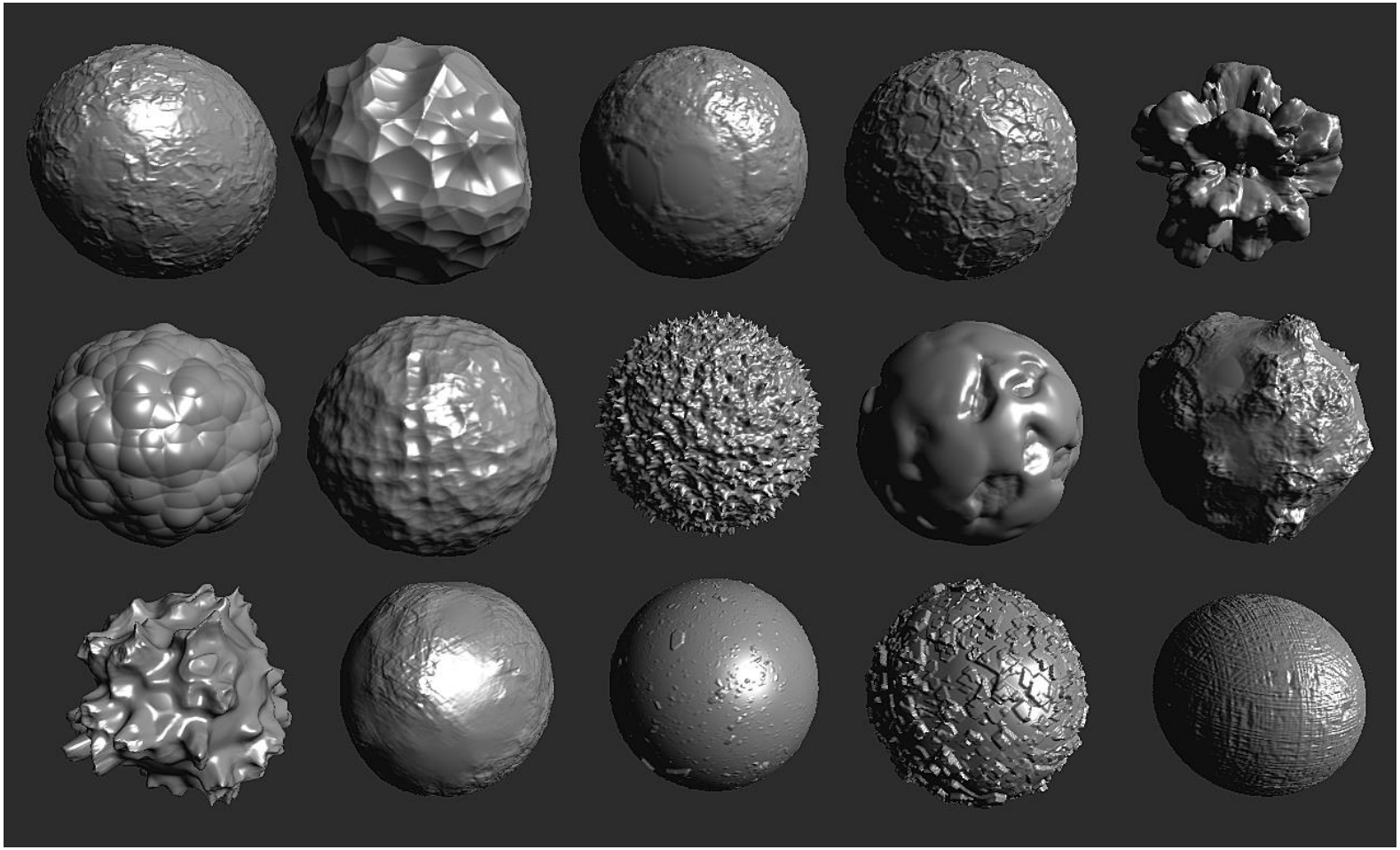
Displacement Mapping

Displacement Mapping

- To add additional geometric detail to a tessellated surface, we can use *displacement mapping*
- We can either store a displacement map as an image or use a procedural displacement function (or some combination)
- Typically, a displacement map is just a height map, representing the displacement along the normal of the surface
- It could however, also include horizontal displacement in the s/t tangent directions

- As we subdivide the surface, we evaluate the curved surface and then displace the position based on the map and the normal (and possibly tangents)
- Once we've displaced our tessellated triangle mesh, we will need to recompute accurate normals, as they will change based on the displacements

Displacement Mapping



Displacement Mapping

- The midpoint distance rule we discussed earlier is far less effective with displacement mapping
- The displacement map might have some localized area of high displacement that would be missed if we just looked at the midpoint
- Therefore, with displacement mapping, one typically has to subdivide the entire surface down to sub-pixel sized triangles to be certain to capture all the detail that might be in the map
- Alternately, one could do a lot of pre-processing on the map itself to identify areas of high displacement ahead of time, and drive the subdivision based on that (a min/max mipmap works well for this)