BRDFs

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BRDFs

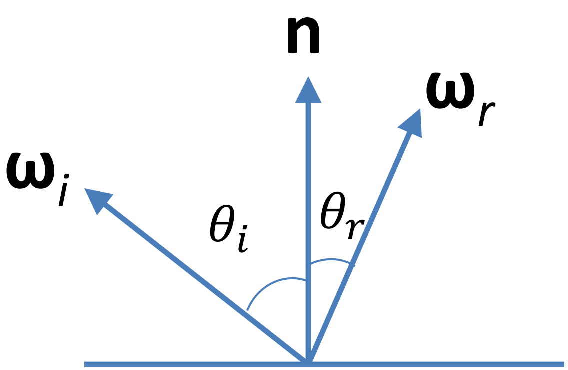
- A *bidirectional reflectance distribution function* is a four dimensional function that describes how light is reflected off a surface
- It relates light coming from the incident direction $\boldsymbol{\omega}_i$ to light reflected in some outward direction $\boldsymbol{\omega}_r$

$$f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_r)$$

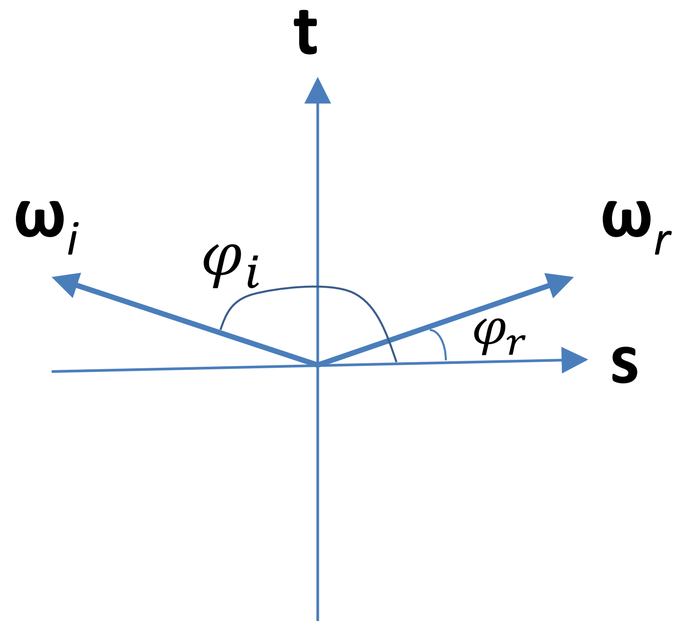
- Note that because the two vectors are unit length direction vectors, they really only have two independent variables each (the two polar angles describing the direction). Sometimes, BRDFs are written like this:

$$f_r(\theta_i, \varphi_i, \theta_r, \varphi_r)$$

BRDFs



Side view, where **n** is surface normal



Top view, where **s** and **t** are surface tangents

BRDFs

- We can think of the BRDF as returning a color, so it is really more of a vector function, but sometimes, it's written as a scalar function of wavelength λ

$$f_r(\lambda, \boldsymbol{\omega}_i, \boldsymbol{\omega}_r)$$

- Also, as surface properties vary based on position **x**, sometimes, it is written as:

$$f_r(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_r)$$

- Also, some authors prefer to use the symbol ρ for the BRDF function instead of f_r

BRDF Properties

- There are two important properties that are required for BRDFs to be physically valid:
  - Conservation of Energy
  - Reciprocity

Conservation of Energy

- Conservation of energy is a fundamental principle in physics
- It says that energy is never created or destroyed- only changed from one form to another
- With respect to light scattering from materials, it tells us that the total light reflected off of the material must be less than the light incident on the material

$$\forall \boldsymbol{\omega}_i, \int_{\Omega} f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_r) \cos\theta_r \, d\,\boldsymbol{\omega}_r \leq 1$$

Reciprocity

- Hermann von Helmholtz (1821-1894) was a German physicist who made many contributions to optics, theory of vision, color, thermodynamics, electrodynamics and more
- In a famous treatise on physiological optics, published in 1856, he formulated the *Helmholtz reciprocity principle*
- The principle states that if we follow a beam of light on any path through an optical system, the loss of intensity is the same as for a beam traveling in the reverse direction

$$f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_r) = f_r(\boldsymbol{\omega}_r, \boldsymbol{\omega}_i)$$

Isotropic & Anisotropic

- A BRDF can be either isotropic or anisotropic
- An *isotropic* BRDF doesn't change with respect to surface rotation about the normal
- Brushed metals and animal fur are good examples of *anisotropic* materials that do change with respect to orientation
- For an isotropic BRDF, there is no fixed frame of reference for the φ angles. The *absolute* angles are irrelevant, but the *difference* between the two is still important. This effectively reduces the isotropic BRDF to a 3 dimensional function:

$$f_r(\theta_i, \theta_r, |\varphi_i - \varphi_r|)$$

Radiance

- *Radiance* is a measure of the quantity of light radiation reflected (and/or emitted) from a surface within a given *solid angle* in a specified direction
- It is typically measured in units of $W/(sr \cdot m^2)$ or Watts per steradian meter squared
- It is the property we usually associate as ‘light intensity’ and is what we measure when we are computing the color to display
- Technically, radiance refers to a single wavelength, and *spectral radiance* refers to the radiance across a spectrum
- Spectral radiance is what we associate as the perceived color (including the intensity), and is what we use to determine the pixel color

Reflected Radiance

- Let's say that $L_i(\boldsymbol{\omega}_i)$ is a function that measures the *incident radiance* from direction $\boldsymbol{\omega}_i$
- If light is shining on the surface from some particular direction $\boldsymbol{\omega}_i$, then the light we see coming off in direction $\boldsymbol{\omega}_r$ is

$$f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_r) L_i(\boldsymbol{\omega}_i) \cos\theta_i$$

- The cosine term is due to the fact that the incident light on the surface is spread out by 1/cosθ, reducing the amount of light that can be reflected
- Note that $\cos\theta_i = \boldsymbol{\omega}_i \cdot \mathbf{n}$

Radiance Equation

- The radiance equation computes the reflected radiance L_r in direction $\boldsymbol{\omega}_r$
- It integrates over all of the possible incident light directions $\boldsymbol{\omega}_i$ in a hemisphere Ω
- The total reflected radiance is the integral (sum) of all of the incident radiances L_i from different directions, times the BRDF function for that direction (and times the cosine of incident angle)

$$L_r(\boldsymbol{\omega}_r) = \int_{\Omega} f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_r) L_i(\boldsymbol{\omega}_i) \cos\theta_i d\boldsymbol{\omega}_i$$

BRDF Variations

- Technically, BRDFs are limited to describing light reflecting off of a surface
- If we want to talk about light transmitted (refracted) through the surface, we need the *bidirectional transmission distribution function* or BTDF
- Some authors combine these into a single *scattering* function or BSDF
- The BSDF is the most general as it includes reflection and scattering
- There is another term called *bidirectional subsurface scattering distribution function* or BSSRDF, which is more complex and is used for modeling translucency

Fundamental BRDFs

Idealized Surfaces

- The most fundamental of the physically based BRDFs are the idealized Fresnel specular and Lambert diffuse models
  - Fresnel metal
  - Fresnel dielectric
  - Lambert diffuse

Fresnel Metal

- The Fresnel metal BRDF is 0 everywhere except in the direction of perfect specular reflection, which happens when $\boldsymbol{\omega}_r$ = reflect($\boldsymbol{\omega}_i$,$\mathbf{n}$)= $(\boldsymbol{\omega}_i \cdot \mathbf{n})\mathbf{n}-2\boldsymbol{\omega}_i$
- In that case, the reflectivity is based on the Fresnel equation for metals
- The reflectivity varies based on the wavelength of the light leading to coloring of the reflection (especially for copper and gold)
- The coloring can be modeled based on actual optical properties of the metals or can be approximated as just constant colors multiplied times the Fresnel equation

Fresnel Dielectric

- Fresnel dielectrics are clear and split an incident ray into a reflected ray and a refracted ray
- The Fresnel dielectric is actually a BSDF, since it contains reflecting and transmitting components
- It is 0 everywhere except where $\boldsymbol{\omega}_r$ is in the direction of perfect reflection or refraction

Lambert Diffuse

- The Lambert diffuse BRDF can be written as:

$$f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_r) = \frac{\mathbf{c}_{diff}}{\pi}$$

where $\mathbf{c}_{diff}$ is the diffuse color

- Note that the BRDF itself is constant. The cosine term usually associated with Lambert reflection comes from the cosine term in the radiance equation itself

Microfacet Models

- The Fresnel metal, dielectric, and Lambert BRDFs are all based on the assumption of a perfectly smooth surface
- These have been extended to roughened surfaces made up of microfacets, where each microfacet is an ideal Fresnel or Lambert reflector
- The roughened models typically account for *shadowing* and *masking* effects between microfacets, as well as *interreflections* between them

Cook-Torrance BRDF

- The Cook-Torrance (1981) model extends the Fresnel metal model to handle rough metal surfaces
- The reflectivity is modeled as the product of three terms: Fresnel F(), Geometry G(), and Distribution D()

$$f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_r) = \frac{F(\boldsymbol{\omega}_i \cdot \mathbf{h}) G(\boldsymbol{\omega}_i, \boldsymbol{\omega}_r) D(\mathbf{h} \cdot \mathbf{n})}{4(\boldsymbol{\omega}_i \cdot \mathbf{n})(\boldsymbol{\omega}_r \cdot \mathbf{n})}$$

where **h** is the halfway vector between $\boldsymbol{\omega}_i$ and $\boldsymbol{\omega}_r$

Oren-Nayar BRDF

- The Oren-Nayar (1994) BRDF takes the ideas from Cook-Torrance and applies them to perfectly diffuse microfacets
- The resulting model is a more realistic diffuse model that accounts for backscattering due to the opposition effect

Oren-Nayar Reflectance Model

$$f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_r) = \frac{\rho}{\pi} \cdot \big(A + (B \cdot \max\big(0, \cos(\varphi_i - \varphi_r)\big)\sin\alpha \cdot \tan\beta)\big)$$

where

$$A = 1 - 0.5\frac{\sigma^2}{\sigma^2 + 0.33}$$

$$B = 0.45\frac{\sigma^2}{\sigma^2 + 0.09}$$

$\alpha = \max(\theta_i, \theta_r)$
$\beta = \min(\theta_i, \theta_r)$
$\sigma =$ roughness (ranges from 0 to around 0.5)
$\varphi_i - \varphi_r =$angle between incident and reflected rays projected onto the plane

Walter BSDF

- The BSDF of Walter, Marschner, Li, and Torrance (2007) applies the same concepts to microfacets made of perfect Fresnel dielectrics
- It is a BSDF (scattering) because it accounts for both reflection and transmission
- They also explore the use of various different expressions for microfacet distributions *D*() and geometric attenuation *G*()

Fundamental BRDFs

	Diffuse	Metal	Dielectric
Ideal (smooth)	Lambert	Fresnel metal	Fresnel dielectric
Microfacet (rough)	Oren-Nayar	Cook-Torrance	Walter

BRDF Sampling

BRDF Evaluation

- In classical ray tracing and rendering, BRDFs are evaluated directly- that is, we have some known view direction and light direction and we want to directly evaluate the reflected intensity
- In these cases, we are just plugging in values of $\boldsymbol{\omega}_i$ and $\boldsymbol{\omega}_r$ to the BRDF and computing the result
- This is called *forward evaluation* of the BRDF

Radiance Estimation

- Recall the radiance equation:

$$L_r(\boldsymbol{\omega}_r) = \int_{\Omega} f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_r) L_i(\boldsymbol{\omega}_i) \cos\theta_i \, d\boldsymbol{\omega}_i$$

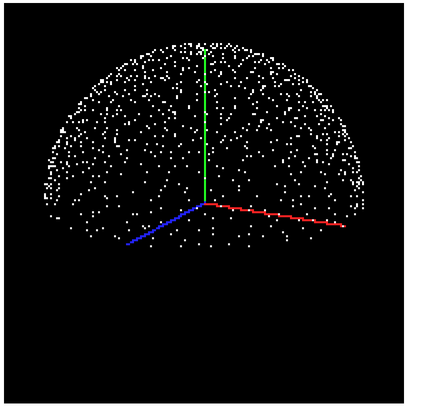
- To determine a pixel color, we have to evaluate (or at least estimate) this integral
- However, it's generally too complex to integrate analytically over all possible directions of incoming light, so we have to approximate it by summing over a subset of the possible directions

Radiance Estimation

$$L_r(\boldsymbol{\omega}_r) = \int_\Omega f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_r) L_i(\boldsymbol{\omega}_i) \cos\theta_i \, d\boldsymbol{\omega}_i$$

- We need to choose a bunch of directions in a hemisphere, so we can use any of the various number generators we've previously discussed (uniform, random, jittered, quasi-random), combined with the hemispherical mapping
- This will give us a set of $\boldsymbol{\omega}_i$ values to plug into a summation to approximate the integral

$$L_r(\boldsymbol{\omega}_r) \approx \frac{2\pi}{N} \sum^{N} f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_r) L_i(\boldsymbol{\omega}_i) \cos\theta_i$$

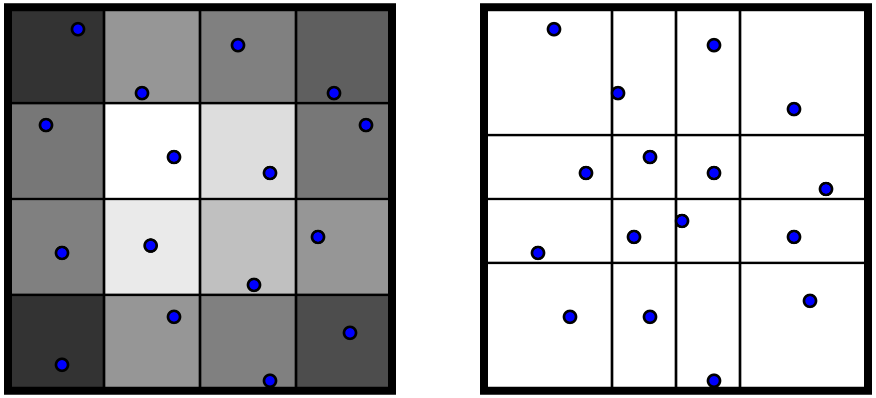


Radiance Estimation

- Picking a bunch of random directions might work fine for diffuse BRDFs that are non-zero over a wide range of angles
- However, for specular BRDFs, picking a random direction and then scaling by the BRDF for that direction will usually lead to 0
- Consider the Fresnel metal BRDF for example. Every random direction chosen would be scaled to 0 except for the impossible chance that it happens to pick the reflected direction
- Therefore, we need a smarter way to choose our directions to estimate the integral

BRDF Sampling

- A good solution is to *sample the BRDF*
- Instead of generating a bunch of evenly distributed samples and then scaling by the BRDF value for the chosen direction, we randomly generate a non-uniform distribution of samples that match the distribution of the BRDF function
- These samples can then be equally weighted
- This is parallel to the discussion we had about pixel sampling for antialiasing



BRDF Sampling

- Instead of passing the BRDF evaluation function a known input and output direction, we wish to *sample* the BRDF by providing an input direction and have it generate a random output direction that is statistically distributed according to the reflected intensity
- In other words, it will generate a random reflected vector, but the randomness will tend to favor the more likely reflected directions
- We can think of it as generating random incident directions that contribute equally to the reflected light towards the viewer
- Or, due to the reciprocity of BRDFs, we can think of it as shining an incident ray and randomly generating a reflected (scattered) ray

Fresnel BRDF Sampling

- The simplest BRDF to sample is the Fresnel metal BRDF: an input vector will always generate the same reflected vector
- A Fresnel dielectric might either reflect or refract the ray. The intensity of each possibility is determined by the Fresnel equations. Instead of generating two rays scaled by the intensity, we generate only one ray at full intensity, but randomly chosen according to the two possible intensities

Lambert BRDF Sampling

- As the Lambert BRDF is constant across the hemisphere, it makes sense to just use the hemispherical mapping to generate the direction uniformly
- However, we are really interested in evaluating the radiance equation, we can compensate for the cosine weighting term in the equation as well by using a cosine weighted hemisphere
- This generates samples that are of equal value

Microfacet BRDF Sampling

- The Cook-Torrance model and its related microfacet models (Oren-Nayar and Walter) were not designed with inverse BRDF sampling in mind
- As a result, there are no perfect ways to inverse sample them
- However, the cosine weighted hemisphere is always a decent fallback option
- It should work well for Oren-Nayar, which is similar to a Lambert surface, but will definitely not be well suited to shiny specular materials such as low-roughness Cook-Torrance materials

Empirical BRDFs

Empirical Models

- The fundamental BRDFs that we looked at so far are all based on real physics to varying degrees of approximation
- There are also several popular BRDFs that are based on empirical formulations that attempt to model the resulting visual phenomena rather than the underlying causes
- Some examples include:
  - Phong
  - Ward
  - Schlick
  - Ashikhman

Phong

- The Phong model (1971) is a classic empirical model in computer graphics
- It is still widely in use due to its simplicity and intuitive control
- When the research focus shifted heavily towards physically based rendering, it was observed that the classic Phong model was not physically valid
- A modern Phong BRDF was developed that is a minor modification of the original in order to enforce reciprocity and conservation of energy

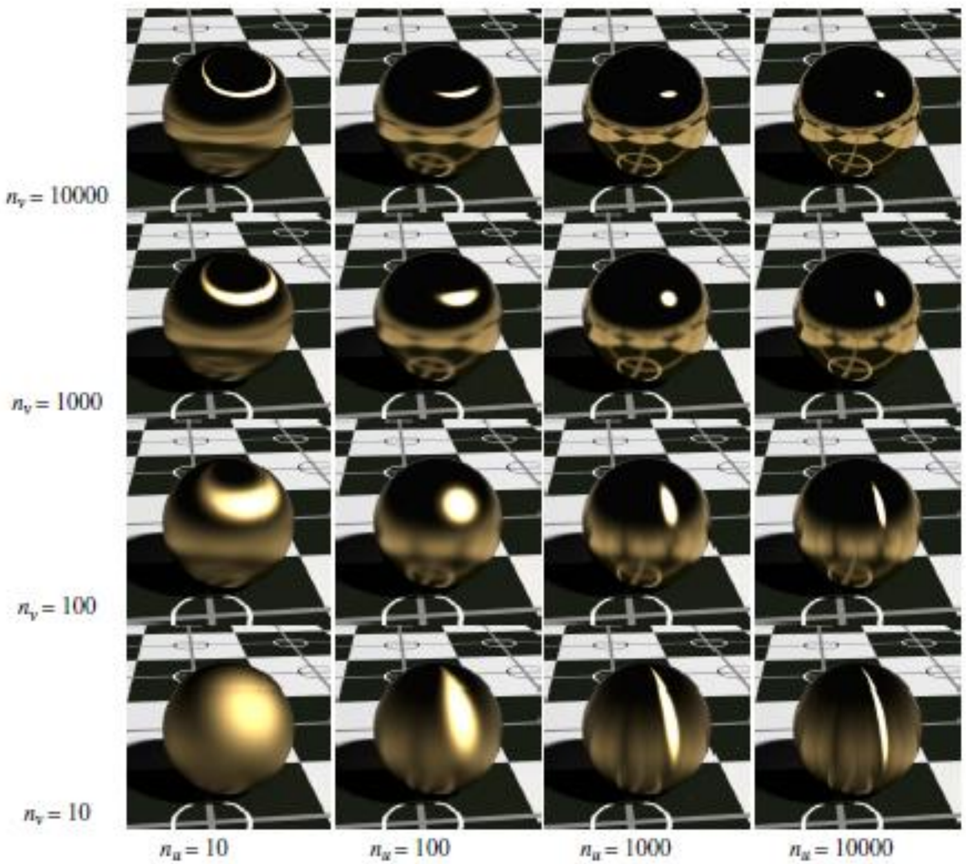
Schlick

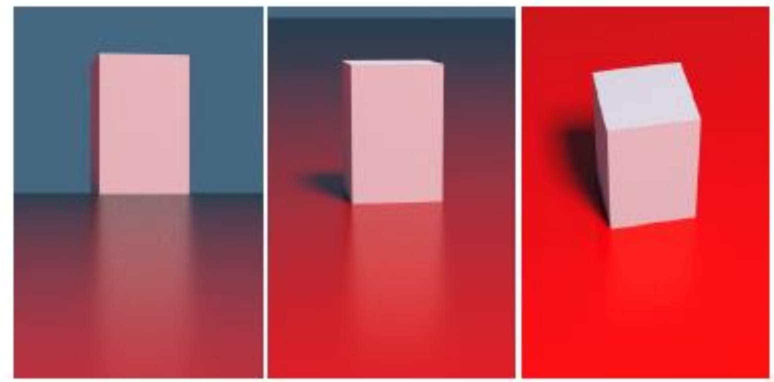
- Christophe Schlick created ‘An Inexpensive BRDF Model for Physically-based Rendering’ in 1994
- It makes some approximations to the Fresnel equation and uses some other fast empirical functions to model diffuse, specular, and a simple layered surface mode
- It was also designed with sampling in mind and has appropriate functions for generating sample ray directions

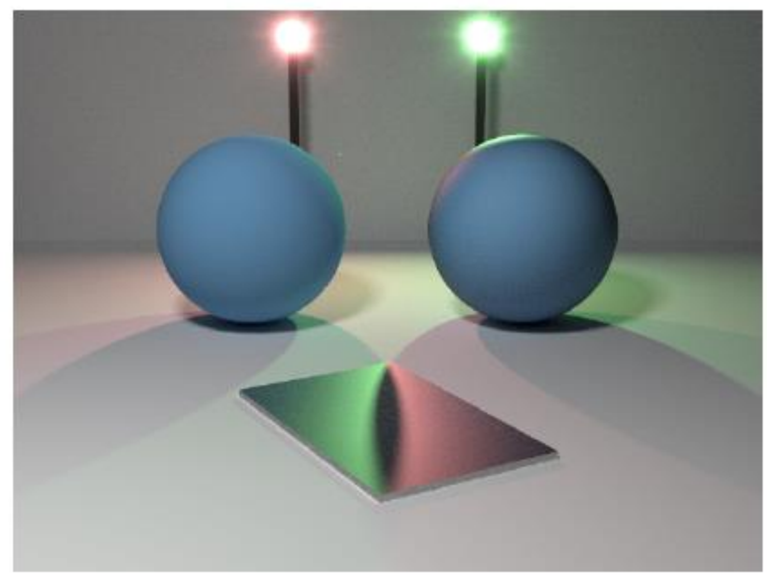
Ashikhmin

- In 2000, Michael Ashikhmin and Peter Shirley published a BRDF that remains popular today
- It is an empirical BRDF intended to model general purpose surfaces from diffuse to plastics to metals
- They listed 6 main goals for the BRDF:
  - Obey conservation of energy and reciprocity
  - Allow *anisotropic* reflection
  - Controlled by intuitive parameters
  - Accounts for Fresnel effect
  - Non-constant diffuse term (diffuse decreases as specularity increases)
  - Well suited to Monte Carlo methods

Ashikhmin







Ashikhmin

- One of the nicest features of the Ashikhmin BRDF is the effectiveness of the sampling functions

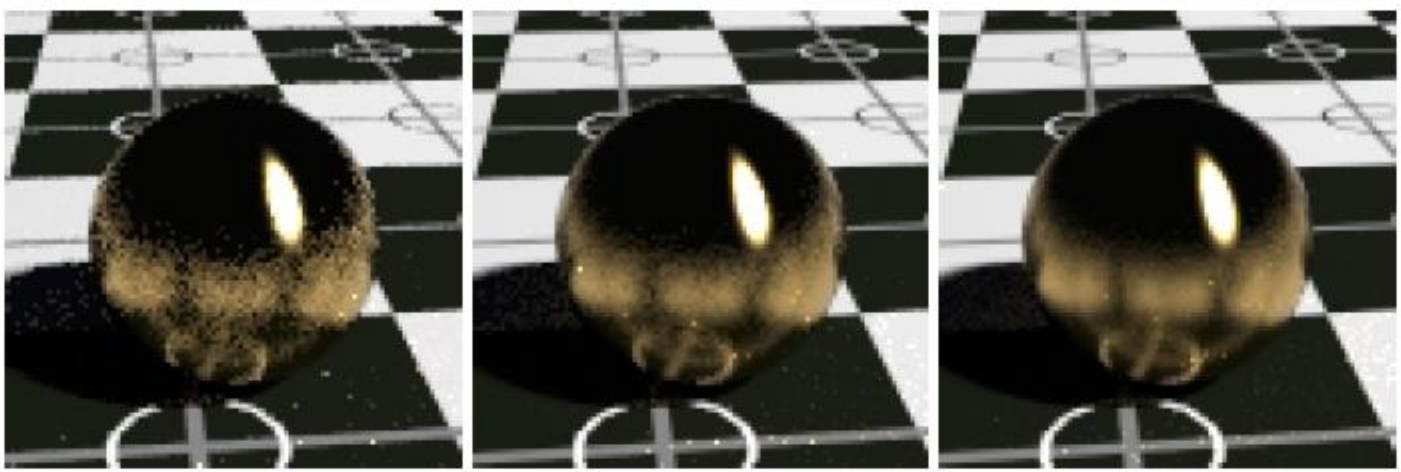


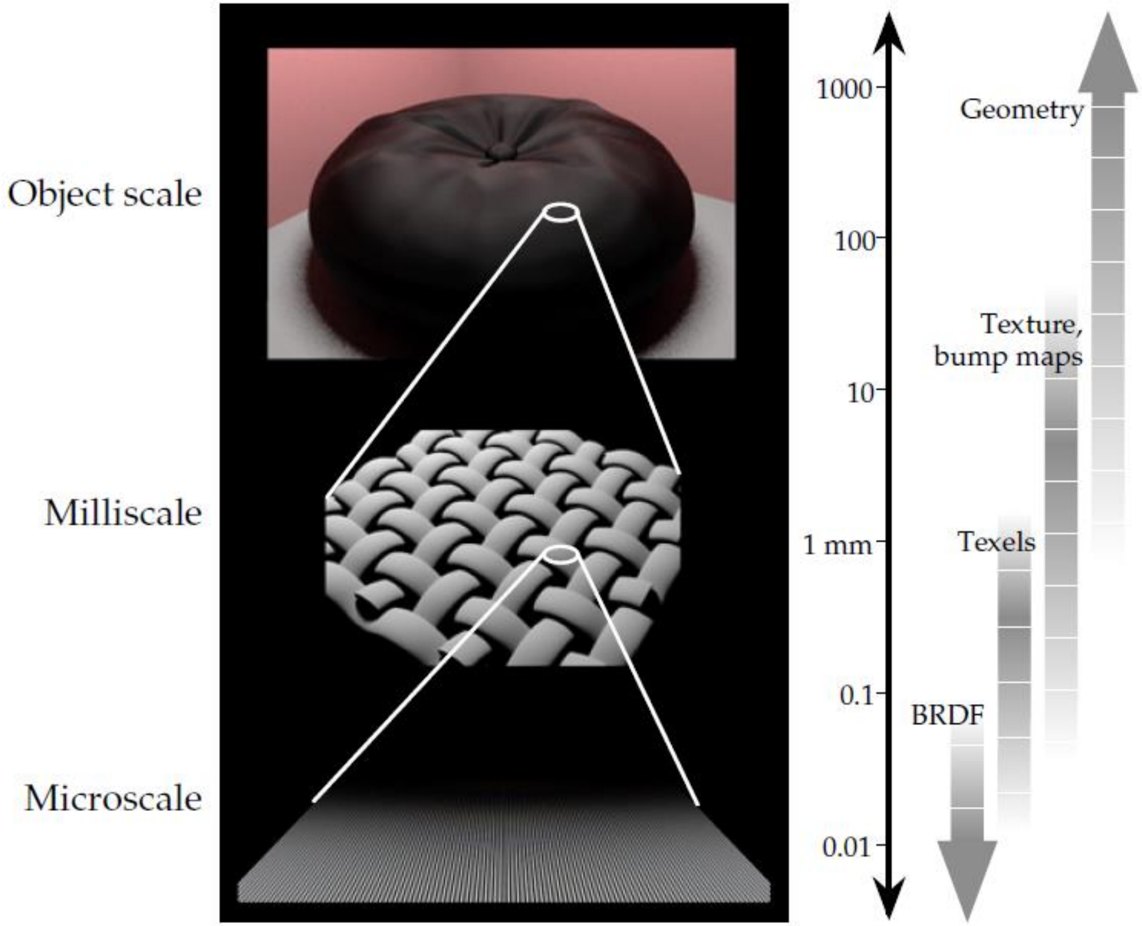
Figure 6: *A closeup of the model implemented in a path tracer with 9, 26, and 100 samples.*

Advanced BRDF Concepts

Layered Models

- Many real surfaces are built up of multiple layers of primers, pigments, scattering particles, and clear coatings
- There have been various attempts to develop BRDFs that account for the interaction of light between these different layers
- One early model was ‘Reflection from Layered Surfaces due to Subsurface Scattering’ by Pat Hanrahan and Wolfgang Krueger

Multiscale BRDFs



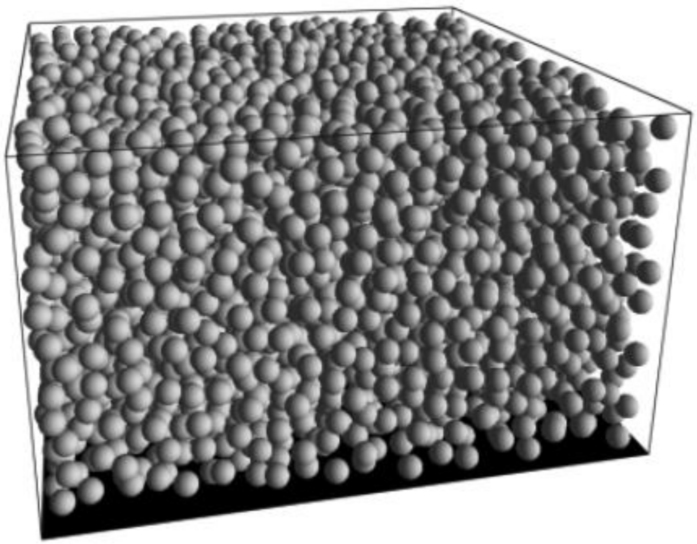
Measured BRDFs

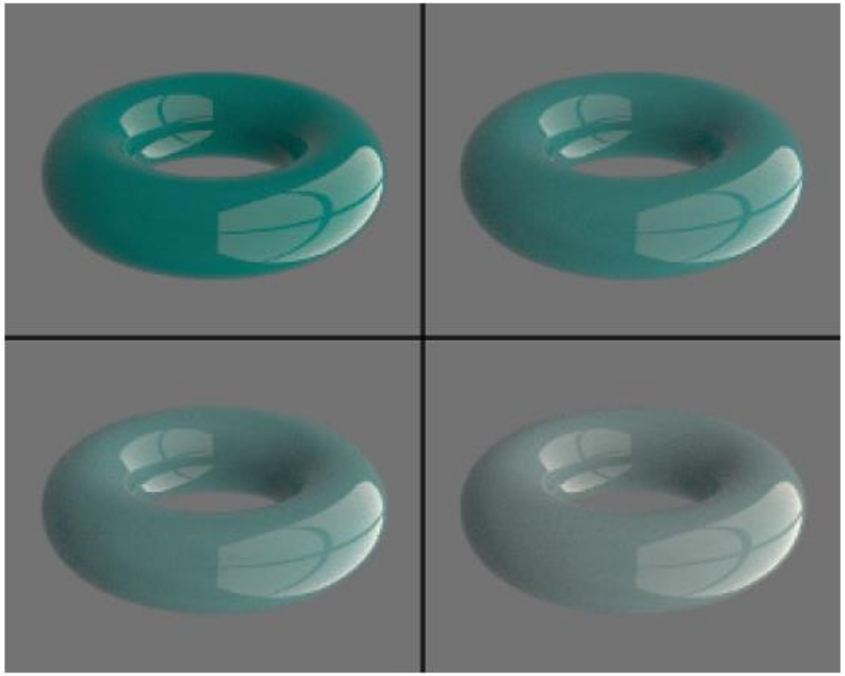
- It is possible to measure a BRDF from a real world material by using a special tool called a *gonioreflectometer*
- It takes a sample of the material and illuminates it with a light source and measures the reflection with a light sensor
- Both the light source and sensor can move to various angles around the sample to measure the BRDF across the whole range of incident and reflected angles

Virtual Gonioreflectometer

- It is also possible to simulate a virtual gonioreflectometer and build BRDF models from actual modeled microgeometry
- A sample of the surface is modeled with actual geometry
- Basic material properties are assigned to the microsurfaces such as Fresnel reflection/refraction, particle scattering, absorption, and diffuse reflection
- Millions of random rays are shot at the material and then bounced around within the material
- The rays that escape off the surface are measured and contribute to the final BRDF

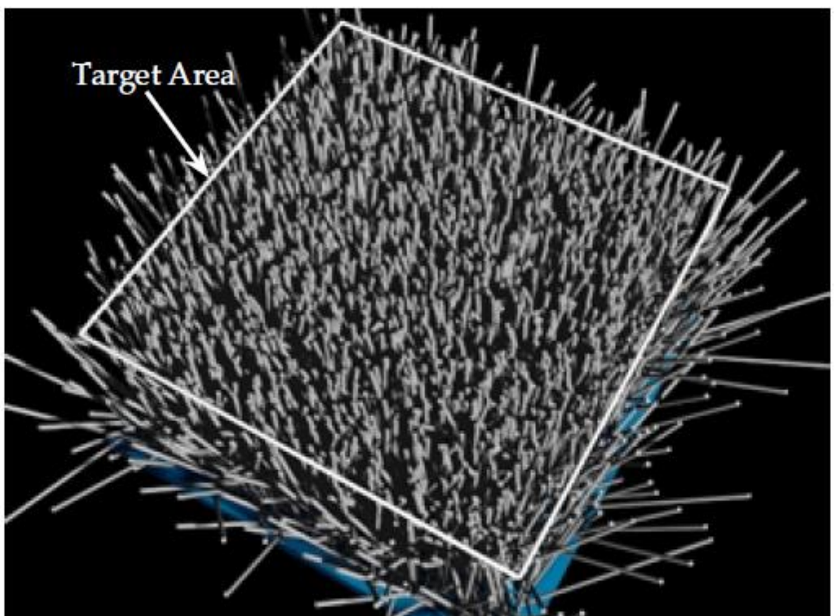
Virtual Gonioreflectometer





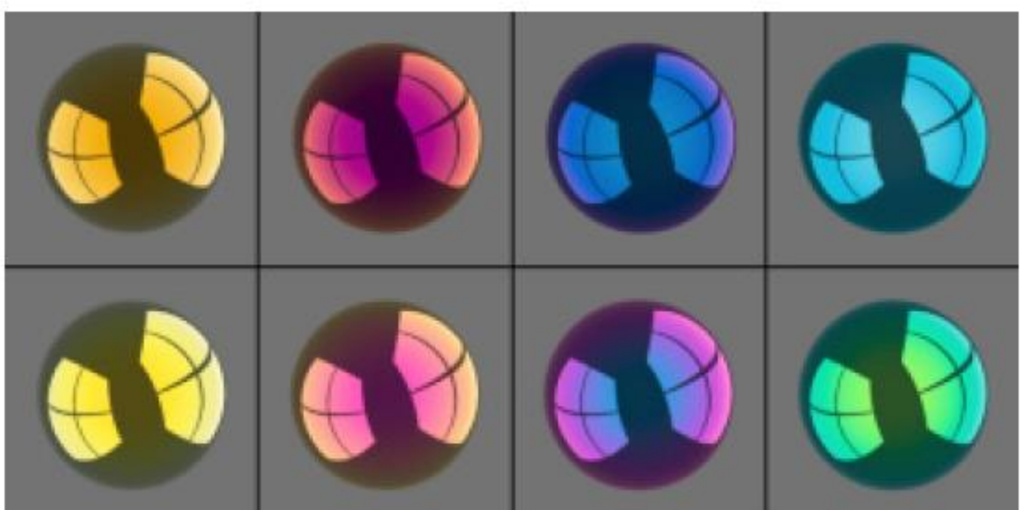
Westin, Arvo, Torrance

- A classic paper on multiscale BRDFs and virtual gonioreflectometers is ‘Predicting Reflectance Functions from Complex Surfaces’ by Stephen Westin, James Arvo, and Kenneth Torrance from 1992



Wavelength Dependence

- Jay Gondek, Gary Meyer, and Jonathan Newman wrote a paper called ‘Wavelength Dependent Reflectance Functions’ that used a virtual gonioreflectometer to model complex materials
- They account for wavelength dependent properties such as interference that lead to iridescent effects

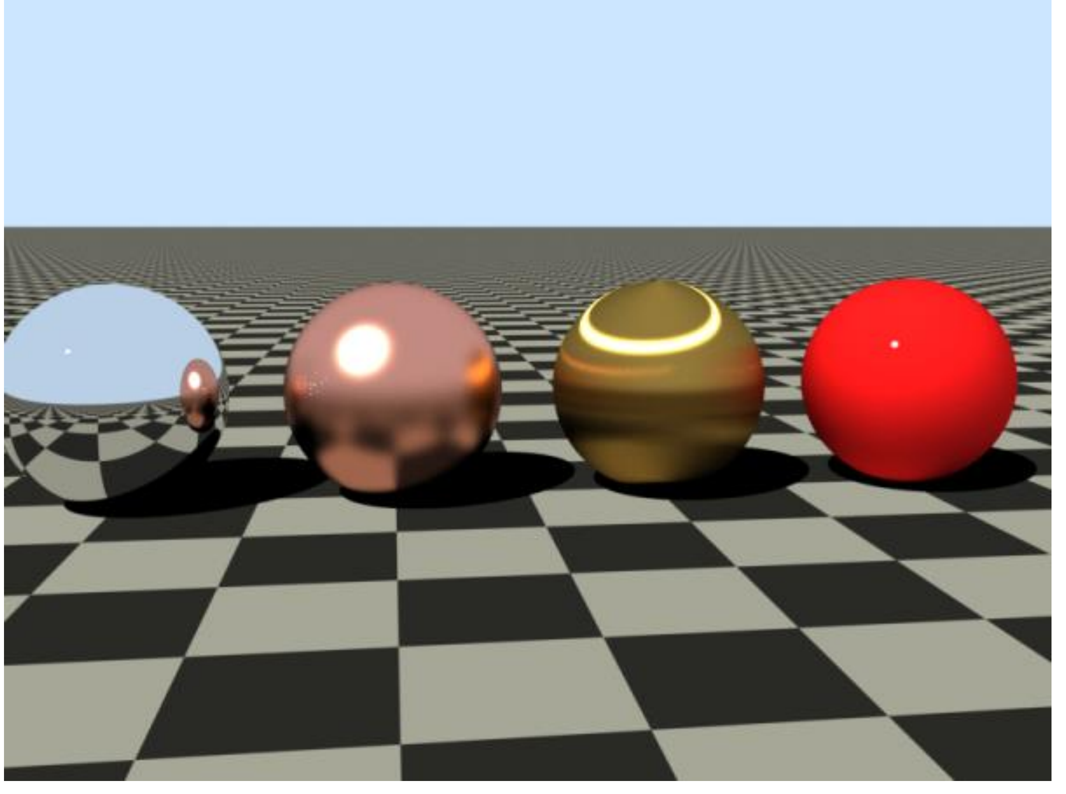


BRDF Representations

- Imagine if you were measuring a real material BRDF or simulating one with a virtual setup
- If you want reasonable resolution, you probably want to sample everything at 1 degree increments or so
- For an isotropic BRDF, this means sampling two variables over 90 degrees and one over 180 degrees, leading to 90x90x180=1,458,000 data points
- An anisotropic BRDF would add another dimension of 180, raising it to 262 million data points
- Various hierarchical, multidimensional data structures have been devised to store and compress BRDF data, and various mathematical tools have been developed to represent and manipulate the data

Project 3

- Project 3 will add the Ashikhmin BRDF and reflections



GenerateSample()

- I suggest adding a new virtual function to the Material base class:

```
virtual void GenerateSample(Color &col,const Vector3
        &in,Vector3 &out,const Intersection &hit);
```

- I also suggest starting with a simpler BRDF like a Fresnel metal or even simpler as a perfect specular material that just reflects a single ray

Recursive Reflections

- I had previously suggested creating a RayTrace class to handle all shading computations
- The camera should only need to call RayTrace::TraceRay(ray,hit) to get a fully shaded result (the hit.Shade contains the color)
- This allows the camera to have a RenderPixel() function that handles antialiasing and everything up to shooting the actual primary rays
- The RayTrace class can handle the ray intersection, all shading & shadow ray testing, and all recursive ray reflections
- Start with a simple perfect mirror BRDF to get reflections working