

Index

1 Enviroment Settings

1.1 .vimrc

2 Data Structure

2.1 Fenwick Tree [1, size]

2.2 Fenwick Tree 2D - [1, size][1, size]

2.3 Heap

2.4 Deap

2.5 zkw Segment Tree
(range modify and query)

2.6 劃分樹

2.7 BigInteger

3 Graph

3.1 maximum matching in general graph

4 String

4.1 KMP

4.2 Z Algorithm

4.3 Suffix Array

4.4 Longest Palindromic Substring

5 Math

5.1 Euler's phi function $O(n)$

5.2 Extended Euclid's Algorithm

5.3 Möbius function

5.4 China remainder theorem

6 Others

6.1 8 puzzle - IDA*

1 Enviroment Settings

1.1 .vimrc

```
1 " set encoding
1 set encoding=utf-8
2 set fileencodings=utf-8,big5
2 set showmode
2 syntax on
2 set hlsearch
2 set background=dark
3 set laststatus=2
3 set wildmenu
3 set scrolloff=5 " keep at least 5 lines above/
3 below
4 set ruler
5 set cursorline
5 set ic " ignore case when searching
7 set bs=2 " enable backspace
7 set number
7 set tabstop=4
8 set shiftwidth=4
8 set autoindent
8 set smarttab
8 set smartindent
8 map<F9> :!g++ "%:t" -o "%:r.out" -Wall -Wshadow -
8 O2 -Im && "%:r.out"
9 map<F10> :!g++ "%:t" -o "%:r.out" -Wall -Wshadow
9 -O2 -Im
9
9
9
10
```

2 Data Structure

2.1 Fenwick Tree [1, size]

```
inline int lowbit(int x) { return x&-x; }
template<class T>
class fenwick {
public:
    fenwick(int __size=SIZE) {
        size = __size+10;
        a = new T[size], b=new T[size];
        memset(a, 0, sizeof(T)*size);
        memset(b, 0, sizeof(T)*size);
    }
    ~fenwick() { delete[] a, delete[] b; }
    inline void add(int l, int r, long long n) {
        __add(a, r, r*n), __add(a, l-1, (l-1)*-n);
        __add(b, r, n), __add(b, l-1, -n);
    }
    inline long long sum(int l, int r) { return
        __sum(r)-__sum(l-1); }
private:
    int size;
    T *a, *b;
    inline void __add(T *arr, int x, T n) { for(; x
        &&n&&x<size; x+=lowbit(x)) arr[x]+=n; }
    inline T __sum(T x) { return __sum(a, x)+(__sum
        (b, size)-__sum(b, x))*x; }
    inline T __sum(T *arr, int x) {
        T res=0;
        for(; x; x-=lowbit(x)) res+=arr[x];
        return res;
    }
};
```

2.2 Fenwick Tree 2D - [1, size][1, size]

```
int tree[size+1][size+1]={0};
inline int lowbit(const int &x) {return x&(-x);}
inline void add(int x, int y, int z) {
    for(int i; x<=n; x+=lowbit(x))
        for(i=y; i<=n; i+=lowbit(i)) tree[x][i]+=z;
}
inline int query(short x, short y) {
    int res=0;
    for(int i; x; x-=lowbit(x))
        for(i=y; i; i-=lowbit(i))
            res+=tree[x][i];
    return res;
}
```

2.3 Heap

```
// max heap tree
#define ParentIndex(i) i==0 ? 0 : ((i-1) >> 1)
#define LeftChildIndex(i) ((i)<<1)+1
#define RightChildIndex(i) ((i)<<1)+2
void BuildMaxHeap(int*, const int&);
void MaxHeapBalance(int*, const int&, const int&)
;
void MaxHeapDelete(int*, int&);
inline bool comp(int &a, int &b) {return a>b;}
void BuildMaxHeap(int all[], const int &size) {
    for(int i=(size-1) >> 1; i>=0; i--)
        MaxHeapBalance(all, size, i);
}
void MaxHeapBalance(int all[], const int &size,
    const int &root) {
    int aim = root, aim2;
    while(1) {
        aim2 = aim;
        int L = LeftChildIndex(aim2);
        int R = RightChildIndex(aim2);
        if( L < size && comp( all[aim], all[L] ) )
            aim = L;
        if( R < size && comp( all[aim], all[R] ) )
            aim = R;
        if( aim != aim2 ) swap(all[aim], all[aim2]);
        else return;
    }
}
void MaxHeapAdd(int all[], int &size, const int &
    AddNum) {
    all[size] = AddNum;
    ++size;
    int P, index = size-1;
    while( index != 0 ) {
        P = ParentIndex(index);
        if( comp(all[P], all[index]) ) {
            swap(all[P], all[index]);
            index = P;
        }
        else return;
    }
}
void MaxHeapDelete(int all[], int &size) {
    all[0] = all[size-1], --size;
    MaxHeapBalance(all, size, 0);
}
```

2.4 Deap

```
class deap {
public:
    deap() {size=1;}
    ~deap() {}
    inline void insert(int n) {
        arr[++size]=n;
        int now=size;
        if( (now&1) && arr[now-1]>arr[now] )
            swap(arr[now-1], arr[now]), now--;
        while( now>3 ) {
            if( arr[now>>2<<1]>arr[now] )
                swap(arr[now>>2<<1], arr[now]),
                now=now>>2<<1;
            else if( arr[(now>>2<<1)+1]<arr[now] )
                swap(arr[(now>>2<<1)+1], arr[now]),
                now=(now>>2<<1)+1;
            else break;
        }
    }
    inline int min() {
        int res=arr[2];
        swap(arr[2], arr[size--]), down(2);
        return res;
    }
    inline int max() {
        int res=arr[3];
        swap(arr[3], arr[size--]), down(3);
        return res;
    }
}

private:
    int arr[1000005], size;
    inline void down(int now) {
        while( (now<<1)<=size ) {
            int tmp;
            if( (now&1)==0 ) {
                if( arr[now]>arr[now+1] )
                    {swap(arr[now], arr[now+1]);
                    now++;continue;}
                tmp=now;
            }
            if( arr[tmp]>arr[now<<1] )
                tmp=now<<1;
            if( (now<<1)+2<=size && arr[tmp]>arr[(now<<1)+2] ) tmp=(now<<1)+2;
            if( tmp==now ) break;
            else swap(arr[now], arr[tmp]),
                now=tmp;
        }
        else if( (now&1)==1 ) {
            if( arr[now]<arr[now-1] )
                {swap(arr[now], arr[now-1]);
                now--;continue;}
            tmp=now;
            if( arr[tmp]<arr[(now<<1)-1] )
                tmp=(now<<1)-1;
            if( (now<<1)+1<=size && arr[tmp]<arr[(now<<1)+1] ) tmp=(now<<1)+1;
            if( tmp==now ) break;
            else swap(arr[now], arr[tmp]), now=tmp;
        }
    }
    if( (now&1)==0 && now+1<=size && arr[now]>arr[
        now+1] )
        swap(arr[now], arr[now+1]);
    if( (now&1)==1 && arr[now]<arr[now-1] )
        swap(arr[now], arr[now-1]);
}
};
```

2.5 zkw Segment Tree (range modify and query)

```
class zkw_seg_tree { public:
    struct node {
        node() {add=sum=0, len=1;}
        int len, add, sum;
    };
    zkw_seg_tree(int size) { // [1,size]
        dep=lg2(size-1)+1;
        delta=(1<<dep)-1;
        arr=new node[1<<(dep+1)];
        for(int i=delta; i>0; --i) arr[i].len=arr[i+i]
            .len<<1;
    }
    ~zkw_seg_tree() {delete[] arr;}
    inline void update(int l, int r, int num=1) {
        l+=delta-1, r+=delta+1;
        int l0=l, r0=r;
        while( r-l>1 ) {
            if( (l&1)^1 ) ++l, arr[l].add+=num, arr[l].
                sum+=arr[l].len*num;
            if( (r&1)^0 ) --r, arr[r].add+=num, arr[r].
                sum+=arr[r].len*num;
            l>>=1, r>>=1;
        }
        __update(l0), __update(r0);
    }
    inline int query(int l, int r) {
        __down(l+delta), __down(r+delta);
        l+=delta-1, r+=delta+1;
        int res=0;
        while( r-l>1 ) {
            if( (l&1)^1 ) res+=arr[l+1].sum;
            if( (r&1)^0 ) res+=arr[r-1].sum;
            l>>=1, r>>=1;
        }
        return res;
    }
private:
    node *arr;
    int dep, delta;
    inline int lg2(int x) {int r;for(r=-1; x; x
        >>=1, ++r);return r;}
    inline void __update(int x) {
        while( x>1 ) x>>=1, arr[x].sum=arr[x+x].sum+
            arr[x+x+1].sum+arr[x].len+arr[x].add;
    }
    inline void __down(int x) {
        for(int i=dep, tmp; i>0; --i) {
            tmp=x>>1;
            arr[tmp<<1].add+=arr[tmp].add;
            arr[(tmp<<1)+1].add+=arr[tmp].add;
            arr[tmp<<1].sum+=arr[tmp].add*arr[tmp<<1].
                len;
            arr[(tmp<<1)+1].sum+=arr[tmp].add*arr[tmp
                <<1].len;
            arr[tmp].add=0;
        }
    }
} segtree(N);
```

2.6 劃分樹

```
#include <iostream>
#include <cstdio>
#include <algorithm>
using namespace std;
#define N 100005
int a[N], as[N]; // 原數組，排序後數組
int n, m;
int sum[20][N]; // 紀錄第 i 層的 1~j
               // 劃分到左子樹的元素個數 (包括 j)
int tree[20][N]; // 紀錄第 i 層元素序列
void build(int c, int l, int r) {
    int i, mid=(l+r)>>1, lm=mid-l+1, lp=1, rp=mid
        +1;
    for (i=l; i<=mid; i++)
        if (as[i] < as[mid]) lm--;
        // 先假設左邊的 (mid-l+1) 個數都等於 as[mid],
        // 然後把實際上小於 as[mid] 的減去
    for (i = 1; i <= r; i++){
        if (i == 1) sum[c][i] = 0;
        // sum[i] 表示 [l, i] 內有多少個數分到左邊，用
        // DP 來維護
        else sum[c][i] = sum[c][i-1];
        if (tree[c][i] == as[mid]){
            if (lm){
                lm--;
                sum[c][i]++;
                tree[c+1][lp++] = tree[c][i];
            } else
                tree[c+1][rp++] = tree[c][i];
        } else if (tree[c][i] < as[mid]){
            sum[c][i]++;
            tree[c+1][lp++] = tree[c][i];
        } else
            tree[c+1][rp++] = tree[c][i];
    }
    if (l == r) return;
    build(c+1, l, mid);
    build(c+1, mid+1, r);
}
int query(int c, int l, int r, int ql, int qr,
          int k){
    int s; // [l, ql] 內將被劃分到左子樹的元素數目
    int ss; // [ql, qr] 內將被劃分到左子樹的元素數目
    int mid=(l+r)>>1;
    if (l == r)
        return tree[c][l];
    if (l == ql){ // 這裡要特殊處理！
        s = 0;
        ss = sum[c][qr];
    } else {
        s = sum[c][ql-1];
        ss = sum[c][qr]-s;
    } // 假設要在區間 [l, r] 中查找第 k 大元素，t
    // 為當前節點，lch, rch 為左右孩子，left, mid
    // 為節點 t 左邊界和中間點。
    if (k <= ss) // sum[r]-sum[l-1]>=k, 查找 lch[t],
        // 區間對應為 [left+sum[l-1], left+sum[r]-1]
        return query(c+1, l, mid, l+s, l+s+ss-1, k);
    else
        // sum[r]-sum[l-1]<k, 查找 rch[t], 區間對應為
        // [mid+1+left-sum[l-1], mid+1+r-left-sum[r]]
        return query(c+1, mid+1, r, mid-l+1+ql-s, mid
            -l+1+qr-s-ss, k-ss);
}
int main(){
    int i, j, k;
    while(~scanf("%d%d", &n, &m)){
        for(i=1; i<=n; i++){
            scanf("%d", &a[i]);
            tree[0][i] = as[i] = a[i];
        }
        sort(as+1, as+1+n);
        build(0, 1, n);
        while(m--){
            scanf("%d%d%d", &i, &j, &k);
            // i, j 分別為區間起始點，k 為該區間第 k
            // 大的數。
            printf("%d\n", query(0, 1, n, i, j, k));
        }
    }
    return 0;
}
```

2.7 BigInteger

```
#include <cstdio>
#include <cstring>
#include <iostream>
#include <iomanip>
using namespace std;
template<class T>
T abs(const T& n) {return n>=T(0)?n:-n;}
class BigInteger {
public:
    BigInteger(const int& num=0) : len(0), sign(1)
    {
        int num2=num;
        memset(arr, 0, sizeof(arr));
        if( num2<0 ) sign=-1, num2*=-1;
        while( num2 ) arr[len++]=num2%step, num2/=
            step;
    }
    BigInteger(const char* num0) : len(0), sign(1)
    {
        *this = num0;
    }
    BigInteger(const BigInteger& b) : len(b.len),
        sign(b.sign) {
        memset(arr, 0, sizeof(arr));
        for(int i=0; i<len; ++i) arr[i]=b.arr[i];
    }
    ~BigInteger() {}
    BigInteger & operator = (const BigInteger& b) {
        len=b.len;
        sign=b.sign;
        memset(arr, 0, sizeof(arr));
        for(int i=0; i<len; ++i) arr[i]=b.arr[i];
        return *this;
    }
    BigInteger & operator = (const int& num) {
        int num2=num;
        memset(arr, 0, sizeof(arr));
        len=0, sign=1;
        if( num2<0 ) sign=-1, num2*=-1;
        while( num2 ) arr[len++]=num2%step, num2/=
            step;
        return *this;
    }
    BigInteger & operator = (const char* num0) {
        char num[strlen(num0)];
        int offset = 0;
        len = 0;
        sign = 1;
        if( num0[0] == '-' ) sign = -1, ++offset;
        else if( num0[0] == '+' ) ++offset;
        while( num0[offset]!='0' ) ++offset;
        strcpy(num, num0+offset);
        int tmp = strlen(num);
        for(int i=tmp-digit; i>=0; i-=digit) {
            arr[len] = 0;
            for(int j=0; j<digit; ++j) arr[len] = arr[
                len]*10 + num[i+j]-'0';
            ++len;
        }
        arr[len] = 0;
        for(int j=0; j<tmp%digit; ++j) arr[len] = arr[
            len]*10 + num[j]-'0';
        if( tmp%digit ) ++len;
        return *this;
    }
    BigInteger operator + (const BigInteger& b)
        const {
        if( *this>0 && b<0 ) return *this-(-b);
        if( *this<0 && b>0 ) return -(-*this-b);
        BigInteger res=*this;
        int len2=max(res.len, b.len);
        for(int i=0; i<len2; ++i) {
            res.arr[i]+=b.arr[i];
            if( res.arr[i]>=step ) res.arr[i]-=step,
                res.arr[i+1]++;
        }
        res.len=len2;
        if(res.arr[len2]) ++res.len;
        return res;
    }
    BigInteger operator - (const BigInteger& b)
        const {
        if( *this<b ) return -(b-*this);
        if( *this<0 && b<0 ) return -(-*this+b);
        if( *this>0 && b<0 ) return *this+(-b);
        BigInteger res=*this;
        int len2=max(res.len, b.len);
        for(int i=0; i<len2; ++i) {
            res.arr[i]-=b.arr[i];
            if( res.arr[i]<0 ) res.arr[i]+=step, res.
                arr[i+1]--;
        }
        while( len2>0 && res.arr[len2-1]==0 ) --len2;
        res.len=len2;
        return res;
    }
    BigInteger operator * (const BigInteger& b)
        const {
        if( *this==0 || b==0 ) return BigInteger(0);
        BigInteger res;
        for(int i=0; i<len; ++i) {
            for(int j=0; j<b.len; ++j) {
                res.arr[i+j]+=arr[i]*b.arr[j];
                res.arr[i+j+1]+=res.arr[i+j]/step;
                res.arr[i+j]%=step;
            }
        }
        res.len=len+b.len-1;
        while( res.arr[res.len] ) ++res.len;
        res.sign=sign*b.sign;
        return res;
    }
    BigInteger operator / (const int& b) const {
        if( b==0 ) return 0;
        BigInteger res;
        long long reduce=0;
        int signb=b>0?1:-1, b2=b*signb;
        for(int i=len-1; i>=0; --i) {
            res.arr[i] = (arr[i]+reduce*step)/b2;
            reduce = (arr[i]+reduce*step)%b2;
        }
        res.len = len;
        while( res.len>0 && res.arr[res.len-1]==0 )
            --res.len;
        if( res.len==0 ) res.sign=1;
        else res.sign=sign*signb;
        return res;
    }
    BigInteger operator / (const BigInteger& b)
        const {
        BigInteger abs_this=abs(*this);
        if( b==0 ) return 0;
        BigInteger st=0, ed, md;
        if( b.arr[0]>0 ) ed=abs_this/b.arr[0];
        else if( b.arr[1]*b.step+b.arr[0]>0 ) ed=
            abs_this/b.arr[1]*b.step+b.arr[0];
        else ed=abs_this;
        while( st<ed ) {
            md = (st+ed)/2+1;

```

```

        if( md*b<=abs_this ) st=md;
        else ed=md-1;
    }
    if( st.len==0 ) st.sign=1;
    else st.sign=sign*b.sign;

    return st;
}
BigInteger operator % (const int& b) const {
    if( b<=0 ) return 0;
    BigInteger res;
    long long reduce=0;
    for(int i=len-1; i>=0; --i)
        reduce = (arr[i]+reduce*step)%b;
    return reduce*sign;
}
BigInteger operator % (const BigInteger& b)
    const {
    if( b.isInt() ) return *this%int(b.toInt());
    if( b<=0 ) return 0;
    return *this-*this/b*b;
}
bool operator < (const BigInteger& b) const {
    if( sign!=b.sign ) return sign<b.sign;
    if( len!=b.len ) return len*sign<b.len*b.sign
        ;
    for(int i=len-1; i>=0; --i)
        if( arr[i]!=b.arr[i] ) return arr[i]*sign<b
            .arr[i]*b.sign;
    return false;
}
bool operator == (const BigInteger& b) const {
    if( sign!=b.sign ) return false;
    if( len!=b.len ) return false;
    for(int i=len-1; i>=0; --i)
        if( arr[i]!=b.arr[i] ) return false;
    return true;
}
bool operator <= (const BigInteger& b) const {
    return *this<b || *this==b; }
bool operator > (const BigInteger& b) const {
    return b<*this; }
bool operator >= (const BigInteger& b) const {
    return b<=*this; }
bool operator != (const BigInteger& b) const {
    return !(*this==b); }
BigInteger operator-() const {
    BigInteger res = *this;
    if( res.len>0 ) res.sign*=-1;
    return res;
}
template<class T> BigInteger operator + (const
    T& b) const {return *this+BigInteger(b);}
template<class T> BigInteger operator - (const
    T& b) const {return *this-BigInteger(b);}
template<class T> bool operator == (const T&
    b) const {return *this==BigInteger(b);}
void print(const char *str="") const {
    if( len==0 ) printf("0");
    else {
        printf("%d", arr[len-1]*sign);
        for(int i=len-2; i>=0; --i) printf("%04d",
            arr[i]);
    }
    printf("%s", str);
}
bool isInt() const {
    if( len>2 ) return false;
    if( len<2 ) return true;
    long long res=toInt();
    return res<(111<<31) && res>=-(111<<31);
}
friend ostream& operator << ( ostream& out,
    const BigInteger &rhs ) {
    if( rhs.len==0 ) out << '0';
    else {
        out << rhs.arr[rhs.len-1]*rhs.sign;
        for(int i=rhs.len-2; i>=0; --i) out <<
            setfill('0') << setw(BigInteger::digit)
                << rhs.arr[i];
    }
    return out;
}
long long toInt() const {return sign*(111*arr
    [1]*step+arr[0]);}
private:
    static const int length = 100;
    static const int digit = 4, step = 10000;
    int arr[length];
    int len, sign;
};
istream& operator >> ( istream& in, BigInteger &
    rhs ) {
    char s[1000];
    in >> s;
    rhs = s;
    return in;
}

```

3 Graph

3.1 maximum matching in general graph

```
//Problem:http://acm.timus.ru/problem.aspx?space
=1&num=1099
#include <cstdio>
#include <cstdlib>
#include <cstring>
#include <iostream>
#include <algorithm>
using namespace std;
const int N=250;
int n;
int head;
int tail;
int Start;
int Finish;
int link[N];    //表示哪個點匹配了哪個點
int Father[N];  //這個就是增廣路的Father……
                但是用起來太精髓了
int Base[N];    //該點屬於哪朵花
int Q[N];
bool mark[N];
bool map[N][N];
bool InBlossom[N];
bool in_Queue[N];

void CreateGraph(){
    int x,y;
    scanf("%d",&n);
    while (scanf("%d%d",&x,&y)!=EOF)
        map[x][y]=map[y][x]=1;
}

void BlossomContract(int x,int y){
    fill(mark,mark+n+1,false);
    fill(InBlossom,InBlossom+n+1,false);
    #define pre Father[link[i]]
    int lca,i;
    for (i=x;i;pre) {i=Base[i]; mark[i]=true; }
    for (i=y;i;pre) {i=Base[i]; if (mark[i]) {lca
        =i; break;} } //尋找lca之旅…… 一定要注意i
        =Base[i]
    for (i=x;Base[i]!=lca;i=pre){
        if (Base[pre]!=lca) Father[pre]=link[i]; //
            對於BFS樹中的父邊是匹配邊的點，Father
            向後跳
        InBlossom[Base[i]]=true;
        InBlossom[Base[link[i]]]=true;
    }
    for (i=y;Base[i]!=lca;i=pre){
        if (Base[pre]!=lca) Father[pre]=link[i]; //
            同理
        InBlossom[Base[i]]=true;
        InBlossom[Base[link[i]]]=true;
    }
    #undef pre
    if (Base[x]!=lca) Father[x]=y;    //注意不能從
        lca這個奇環的關鍵點跳回來
    if (Base[y]!=lca) Father[y]=x;
    for (i=1;i<=n;i++){
        if (InBlossom[Base[i]]){
            Base[i]=lca;
            if (!in_Queue[i]){
                Q[++tail]=i;
                in_Queue[i]=true; //要注意如果本來連向
                    BFS樹中父結點的邊是非匹配邊的點，
                    可能是沒有入隊的
            }
        }
    }
}
```

```
void Change(){
    int x,y,z;
    z=Finish;
    while (z){
        y=Father[z];
        x=link[y];
        link[y]=z;
        link[z]=y;
        z=x;
    }
}

void FindAugmentPath(){
    fill(Father,Father+n+1,0);
    fill(in_Queue,in_Queue+n+1,false);
    for (int i=1;i<=n;i++) Base[i]=i;
    head=0; tail=1;
    Q[1]=Start;
    in_Queue[Start]=1;
    while (head!=tail){
        int x=Q[++head];
        for (int y=1;y<=n;y++){
            if (map[x][y] && Base[x]!=Base[y] && link[x]
                !=y) //無意義的邊
                if ( Start==y || link[y] && Father[link[y]
                    ]) //精髓地用Father表示該點是否
                    BlossomContract(x,y);
            else if (!Father[y]){
                Father[y]=x;
                if (link[y]){
                    Q[++tail]=link[y];
                    in_Queue[link[y]]=true;
                }
                else{
                    Finish=y;
                    Change();
                    return;
                }
            }
        }
    }
}

void Edmonds(){
    memset(link,0,sizeof(link));
    for (Start=1;Start<=n;Start++){
        if (link[Start]==0)
            FindAugmentPath();
    }
}

void output(){
    fill(mark,mark+n+1,false);
    int cnt=0;
    for (int i=1;i<=n;i++){
        if (link[i]) cnt++;
    }
    printf("%d\n",cnt);
    for (int i=1;i<=n;i++){
        if (!mark[i] && link[i]){
            mark[i]=true;
            mark[link[i]]=true;
            printf("%d %d\n",i,link[i]);
        }
    }
}

int main(){
    CreateGraph();
    Edmonds();
    output();
    return 0;
}
```

4 String

4.1 KMP

```
int KMP(char ts[5005], char ss[5005]) {
    if( strlen(ts)>strlen(ss) ) return -1;
    int failure[5005];
    int len=strlen(ts);
    for(int i=1, j=failure[0]=-1; i<len; ++i) {
        while( j>=0 && ts[j+1]^ts[i] ) j=failure[j];
        if( ts[j+1]==ts[i] ) ++j;
        failure[i]=j;
    }
    for(int i=0, j=-1; ss[i]; ++i) {
        if( j>=0 && ss[i]^ts[j+1] ) j=failure[j];
        if( ss[i]==ts[j+1] ) ++j;
        if( j==len-1 ) {
            return i-len+1; // rec this!!
            j=failure[j];
        }
    }
    return -1;
}
```

4.2 Z Algorithm

```
void Z(char G[], int z[]){
    int len = strlen(G);
    z[0] = len;
    int L = 0, R = 1;
    for ( int i = 1 ; i < len ; i++ ) {
        if ( i >= R || z[i-L] >= R-i ) {
            int x = (i>=R) ? i : R;
            while ( x < len && G[x] == G[x-i] )
                x++;
            z[i] = x - i;
            if ( x > i ) L = i , R = x;
        }
        else z[i] = z[i-L];
    }
}
```

4.3 Suffix Array

```
int rank[LEN], sa[LEN];
int height[LEN];
int y[LEN], cnt[LEN], rr[2][LEN];
inline bool same(int *rank, int a, int b, int l)
{ return rank[a]==rank[b]&&rank[a+l]==rank[b+l]; }
void sa2(char str[], int n, int m) {
    printf("%s!! %d %d\n", str, n, m);
    int *rank1=rr[0], *rank2=rr[1];
    MSET(rr[1], 0);
    int i, p;
    for(i=0; i<m; ++i) cnt[i]=0;
    for(i=0; i<n; ++i) rank2[i]=str[i], cnt[rank2[i]]++;
    for(i=1; i<m; ++i) cnt[i]+=cnt[i-1];
    for(i=n-1; i>=0; --i) sa[--cnt[rank2[i]]]=i;
    for(int j=1; p<n; j<=1, m=p) {
        // 表示用第二個key(rank2)排序後 從 y[i]
        // 開始的後綴排第i名
        for(p=0, i=n-j; i<n; ++i) y[p++]=i;
        for(i=0; i<n; ++i) if( sa[i]>=j ) y[p++] = sa[i]-j;
        for(i=0; i<m; ++i) cnt[i]=0;
        for(i=0; i<n; ++i) cnt[ rank2[y[i]] ] ++;
        for(i=1; i<m; ++i) cnt[i]+=cnt[i-1];
        for(i=n-1; i>=0; --i) sa[ --cnt[ rank2[y[i]] ] ]=y[i];
        for(p=i=1, rank1[sa[0]]=0; i<n; ++i)
            rank1[sa[i]]=same(rank2, sa[i], sa[i-1], j)
                ? p-1:p++;
        std::swap(rank1, rank2);
    }
    for(int i=0; i<n; ++i) rank[i]=rank2[i];
}
void make_height(char str[]) {
    int len=strlen(str);
    height[0]=0;
    for(int i=0, j=0; i<len; ++i, j=height[rank[i]-1]-1) {
        if( rank[i]==1 ) continue;
        if( j<0 ) j=0;
        while( i+j<len && sa[rank[i]-1]+j<len &&
            str[i+j]==str[sa[rank[i]-1]+j] ) ++j;
        height[rank[i]]=j;
    }
}
int main() {
    char str[LEN];
    scanf("%s", str);
    int len = strlen(str);
    sa2(str, len+1, 256);
    make_height(str);
    for(int i=1; i<=len; ++i) printf("%d %d %s\n",
        i, height[i], str+sa[i]);
}
```


4.4 Longest Palindromic Substring

```
char t[1001]; // 要處理的字串
char s[1001 * 2]; // 中間插入特殊字元的t。
int Z[1001 * 2], L, R; // Gusfield's Algorithm
// 由a往左、由b往右，對稱地作字元比對。
int match(int a, int b) {
    int i = 0;
    while (a-i>=0 && b+i<N && s[a-i] == s[b+i]) i
        ++;
    return i;
}
void longest_palindromic_substring()
{
    int N = strlen(t);
    // 在t中插入特殊字元，存放到s。
    memset(s, '.', N*2+1);
    for (int i=0; i<N; ++i) s[i*2+1] = t[i];
    N = N*2+1;
    // modified Gusfield's lgorithm
    Z[0] = 1;
    L = R = 0;
    for (int i=1; i<N; ++i) {
        int ii = L - (i - L); // i的映射位置
        int n = R + 1 - i;
        if (i > R) {
            Z[i] = match(i, i);
            L = i;
            R = i + Z[i] - 1;
        }
        else if (Z[ii] == n) {
            Z[i] = n + match(i-n, i+n);
            L = i;
            R = i + Z[i] - 1;
        }
        else Z[i] = min(Z[ii], n);
    }
    // 尋找最長迴文子字串的長度。
    int n = 0, p = 0;
    for (int i=0; i<N; ++i)
        if (Z[i] > n) n = Z[p = i];
    // 記得去掉特殊字元。
    cout << "最長迴文子字串的長度是" << (n-1) / 2;
    // 印出最長迴文子字串，記得別印特殊字元。
    for (int i=p-Z[p]+1; i<=p+Z[p]-1; ++i)
        if (i & 1) cout << s[i];
}
```

5 Math

5.1 Euler's phi function $O(n)$

1. $\gcd(x, y) = d \Rightarrow \phi(xy) = \frac{\phi(x)\phi(y)}{\phi(d)}$
2. $p \text{ is prime} \Rightarrow \phi(p^k) = p^{k-1}\phi(p)$
3. $p \text{ is prime} \Rightarrow \phi(p^k) = \phi(p^{k-1}) \times p$
4. $n = p_1^{k_1} p_2^{k_2} \cdots p_m^{k_m}$
 $\Rightarrow \phi(n) = p_1^{k_1-1} \phi(p_1) p_2^{k_2-1} \phi(p_2) \cdots p_m^{k_m-1} \phi(p_m)$

```
const int MAXN = 100000;
int phi[MAXN], prime[MAXN], pn=0;
memset(phi, 0, sizeof(phi));
for(int i=2; i<MAXN; ++i) {
    if( phi[i]==0 ) prime[pn++]=i, phi[i]=i-1;
    for(int j=0; j<pn; ++j) {
        if( i*prime[j]>=MAXN ) break;
        if( i%prime[j]==0 ) {
            phi[i*prime[j]] = phi[i] * prime[j];
            break;
        }
        phi[i*prime[j]] = phi[i] * phi[prime[j]];
    }
}
```

5.2 Extended Euclid's Algorithm

$$ax + by = \gcd(a, b)$$

```
int ext_gcd(int a, int b, int &x, int &y){
    int x2;
    if( b==0 ) {
        x=1, y=0;
        return a;
    }
    int gcdn=ext_gcd(b, a%b, x, y), x2=x;
    x=y, y=x2-a/b*y;
    return gcdn;
}
int ext_gcd(int a, int b, int &x, int &y){
    int t, px=1, py=0, tx, ty;
    x=0, y=1;
    while(a%b!=0) {
        tx=x, ty=y;
        x=x*(-a/b)+px, y=y*(-a/b)+py;
        px=tx, py=ty;
        t=a, a=b, b=t%b;
    }
    return b;
}
```

5.3 Möbius function

```
memset(mobius, 0, sizeof(mobius));
mobius[1]=1;
for(int i=0; i<flag; ++i) mobius[prime[i]]=-1;
for(int i=2, tmp, cntprime; i<MAXN; ++i)
{
    if( !~mobius[i] ) continue;
    tmp=i, cntprime=0;
    for(int j=0; !mobius[tmp]&&prime[j]<=tmp; ++j){
        if( tmp%prime[j]==0 )
            ++cntprime, tmp/=prime[j];
        if( tmp%prime[j]==0 ) {cntprime=0; break;}
    }
    if( cntprime && mobius[tmp] )
        mobius[i]=mobius[tmp]*(cntprime&1?-1:1);
}
```

5.4 China remainder theorem

$ans \equiv a_i \pmod{m_i}$

```
int ans, gcdn, x, y, reduce, tmp;
for(int i=1; i<n; ++i) {
    gcdn=ext_gcd(mi[i-1], mi[i], x, y);
    reduce=ai[i]-ai[i-1];
    if( reduce%gcdn!=0 ) {
        ans=-1;
        break;
    }
    tmp=mi[i]/gcdn;
    x=(reduce/gcdn*x%tmp+tmp)%tmp;
    ai[i] = ai[i-1] + mi[i-1]*x;
    mi[i] = mi[i-1]*tmp;
}
```

6 Others

6.1 8 puzzle - IDA*

```
// 一個盤面。其數值1~8代表方塊號碼，0代表空格。
int board[3][3] = {2, 3, 4, 1, 5, 0, 7, 6, 8};
// 檢查 permutation inversion。檢查不通過，
// 表示盤面不合理。
bool check_permutation_inversion(int board[3][3])
{
    int inversion = 0;
    for (int a=0; a<9; ++a)
        for (int b=0; b<a; ++b) {
            int i = a / 3, j = a % 3;
            int ii = b / 3, jj = b % 3;
            if (board[i][j] && board[ii][jj]
                && board[i][j] < board[ii][jj])
                inversion++;
        }
    int row_number_of_0 = 0;
    for (int i=0; i<3 && !row_number_of_0; ++i)
        for (int j=0; j<3 && !row_number_of_0; ++j)
            if (board[i][j] == 0)
                row_number_of_0 = i+1;
    return (inversion + row_number_of_0) % 2 == 0;
}
//////////
// heuristic function,
// 採用不在正確位置上的方塊個數。
int h(int board[3][3])
{
    int cost = 0;
    for (int i=0; i<3; ++i)
        for (int j=0; j<3; ++j)
            if (board[i][j])
                if (board[i][j] != i*3 + j + 1)
                    cost++;
    return cost;
}
//////////
int taxicab_distance(int x1, int y1, int x2, int
y2)
{return abs(x1 - x2) + abs(y1 - y2);}

// heuristic function, 採用taxicab distance。
int h(int board[3][3]) {
    // 每塊方塊的正確位置。{0,0}
    // 是為了方便編寫程式而多加的。
    static const int right_pos[9][2] = {
        {0,0},
        {0,0}, {0,1}, {0,2},
        {1,0}, {1,1}, {1,2},
        {2,0}, {2,1}
    };
    // 計算每個方塊與其正確位置的 taxicab distance
    // 的總和。
    int cost = 0;
    for (int i=0; i<3; ++i)
        for (int j=0; j<3; ++j)
            if (board[i][j])
                cost += taxicab_distance(
                    i, j,
                    right_pos[board[i][j]][0],
                    right_pos[board[i][j]][1]
                );
    return cost;
}

// 上下左右
const string operator[4] = {"up", "down", "right",
    , "left"};
```

```

const int dx[4] = {-1, 1, 0, 0}, dy[4] = {0, 0, 1, -1};
char solution[30];
// 正確的推動方式，其數值是方向0~3。
const int reverse_dir[4] = {1, 0, 3, 2};
// 用表格紀錄每一個方向的反方向。
// 可用於避免來回推動的判斷。

int board[3][3] = {2, 3, 4, 1, 5, 0, 7, 6, 8};
// 起始狀態。其數值1~8代表方塊號碼，0代表空格。

int sx = 1, sy = 2;
// 空格的位置。可馬上知道推動方塊的目的地。

bool onboard(int x, int y)
{return x>=0 && x<3 && y>=0 && y<3;}

int IDAstar(int x, int y, int gv, int prev_dir,
            int& bound, bool& ans) {
    int hv = h(board);
    if (gv + hv > bound) return gv + hv;
    // 超過，回傳下次的bound
    if (hv == 0) {ans = true; return gv;}
    // 找到最佳解

    int next_bound = 1e9;
    for (int i=0; i<4; ++i) {
        // 四種推動方向
        int nx = x + dx[i], ny = y + dy[i];
        // 空格的新位置
        if (reverse_dir[i] == prev_dir) continue;
        // 避免來回推動
        if (!onboard(nx, ny)) continue;
        // 避免出界
        solution[gv] = oper[i];
        // 紀錄推動方向
        swap(board[x][y], board[nx][ny]);
        // 推動
        int v = IDAstar(nx, ny, gv+1, i, bound, ans);
        if (ans) return v;
        next_bound = min(next_bound, v);
        swap(board[nx][ny], board[x][y]);
        // 回復原狀態
    }
    return next_bound;
}

void eight_puzzle() {
    if (!check_permutation_inversion(board)) {
        cout << "盤面不合理，無法解得答案。" << endl;
        return;
    }
    // IDA*
    bool ans = false;
    int bound = 0;
    while (!ans && bound <= 50)
        bound = IDAstar(sx, sy, 0, -1, bound, ans);
    if (!ans) {
        cout << "50步內無法解得答案。" << endl;
        return;
    }
    // 印出移動方法
    for (int i=0; i<bound; ++i)
        cout << operation[solution[i]] << ' ';
    cout << endl;
}

```

The End