

Index

1 Enviroment Settings

1.1 .vimrc

2 Computational Geometry

2.1 Geometry on Plane

3 Data Structure

3.1 BigInteger

3.2 Fenwick Tree Range Modify [1, size]

3.3 Fenwick Tree 2D - [1, size][1, size]

3.4 劃分樹

4 Graph

4.1 Dinic

4.2 maximum matching in general graph

5 Math

5.1 China remainder theorem

5.2 Euler's phi function $O(n)$

5.3 Extended Euclid's Algorithm

5.4 Gaussian Elimination

5.5 Miller Rabin

5.6 Möbius function

6 String

6.1 KMP

6.2 Longest Palindromic Substring

6.3 Suffix Array

6.4 Z Algorithm

7 Others

7.1 8 puzzle - IDA*

7.2 recursive to stack

1 Enviroment Settings

1.1 .vimrc

```
1 " set encoding
1 set encoding=utf-8
2 set fileencodings=utf-8, big5
2 set showmode
2 syntax on
2 set hlsearch
2 set background=dark
2 set laststatus=2
4 set wildmenu
4 set scrolloff=5 " keep at least 5 lines above/below
4 set ruler
4 set cursorline
5 set ic " ignore case when searching
5 set bs=2 " enable backspace
6 set number
6 set tabstop=4
7 set shiftwidth=4
7 set autoindent
7 set smarttab
7 set smartindent
7 """""""" abbr
7 syntax on
8 set enc=utf-8 fencs=utf-8, big5
8 set bs=2
8 set smd nu bg=dark hls ls=2 wmnu so=5 ru cul
8 set ts=4 sw=4 ai sta si
9 set list lcs=tab:>\ "# a space after '\ '
9 imap<F9> <ESC>:w<Enter><F9>
9 map<F9> :!g++ "%:t" -o "%:r.out" -Wall -Wshadow -
9 O2 -Im && "./%:r.out"
9 map<F10> :!g++ "%:t" -o "%:r.out" -Wall -Wshadow
9 -O2 -Im
autocmd! BufNewFile * silent! 0r ~/.vim/skel/
Template.:%e
10
10
11
```

2 Computational Geometry

2.1 Geometry on Plane

```
struct node {
    double x,y;
    node() {}
    node(double _x, double _y) : x(_x),y(_y) {}
    node operator+(const node& rhs)const
    { return node(x+rhs.x,y+rhs.y); }
    node operator-(const node& rhs)const
    { return node(x-rhs.x,y-rhs.y); }
    node operator*(const double& rhs)const
    { return node(x*rhs,y*rhs); }
    node operator/(const double& rhs)const
    { return node(x/rhs,y/rhs); }
    double operator*(const node& rhs)const
    { return x*rhs.x+y*rhs.y; }
    double len2()const{ return x*x+y*y; }
    double len()const{ return sqrt(x*x+y*y); }
    node unit()const{ return *this/len(); }
    double operator^(const node& rhs)const{ return
        x*rhs.y-y*rhs.x; }
    node T()const{ return node(-y,x); }
    node rot(double rad)const{ //逆時針旋轉 弧度
        return node(cos(rad)*x-sin(rad)*y, sin(rad)*x
            +cos(rad)*y);
    }
};

node __mirror(node normal, double constant, node
    point){ //2D3D
    double scale=(normal*point+constant)/(normal*
        normal);
    return point-normal*(2*scale);
}

node mirror(node p1, node p2, node p3){ //2D3D
    return __mirror((p2-p1).T(),(p2-p1).T()*p1*(-1)
        ,p3);
}

double ori(const node& p1,const node& p2, const
    node& p3){ //平行四邊形面積(帶正負)
    return (p2-p1)^(p3-p1);
}

bool intersect(const node& p1, const node& p2,
    const node& p3, const node& p4){
    return (ori(p1,p2,p3)*ori(p1,p2,p4)<0 && ori(p3
        ,p4,p1)*ori(p3,p4,p2)<0);
}

pair<node,node> two_circle_intersect(node p1,
    double r1, node p2, double r2){
    double degree=acos(((p2-p1).len2()+r1*r1-r2*r2)
        /(2*r1*(p2-p1).len()));
    return make_pair(p1+(p2-p1).unit().rot(degree)*
        r1, p1+(p2-p1).unit().rot(-degree)*r1);
}

node intersectionPoint(node p1, node p2, node p3,
    node p4){
    double a123 = (p2-p1)^(p3-p1);
    double a124 = (p2-p1)^(p4-p1);
    return (p4*a123-p3*a124)/(a123-a124);
}
```

3 Data Structure

3.1 BigInteger

```
#include <cstdio>
#include <cstring>
#include <iostream>
#include <iomanip>
using namespace std;
template<class T>
T abs(const T& n) {return n>=T(0)?n:-n;}
class BigInteger {
public:
    BigInteger(const int& num=0) : len(0), sign(1)
    {
        int num2=num;
        memset(arr, 0, sizeof(arr));
        if( num2<0 ) sign=-1, num2*=-1;
        while( num2 ) arr[len++]=num2%step, num2/=
            step;
    }
    BigInteger(const char* num0) : len(0), sign(1)
    {
        *this = num0;
    }
    BigInteger(const BigInteger& b) : len(b.len),
        sign(b.sign) {
        memset(arr, 0, sizeof(arr));
        for(int i=0; i<len; ++i) arr[i]=b.arr[i];
    }
    ~BigInteger() {}
    BigInteger & operator = (const BigInteger& b) {
        len=b.len;
        sign=b.sign;
        memset(arr, 0, sizeof(arr));
        for(int i=0; i<len; ++i) arr[i]=b.arr[i];
        return *this;
    }
    BigInteger & operator = (const int& num) {
        int num2=num;
        memset(arr, 0, sizeof(arr));
        len=0, sign=1;
        if( num2<0 ) sign=-1, num2*=-1;
        while( num2 ) arr[len++]=num2%step, num2/=
            step;
        return *this;
    }
    BigInteger & operator = (const char* num0) {
        char num[strlen(num0)];
        int offset = 0;
        len = 0;
        sign = 1;
        if( num0[0] == '-' ) sign = -1, ++offset;
        else if( num0[0] == '+' ) ++offset;
        while( num0[offset]=='0' ) ++offset;
        strcpy(num, num0+offset);
        int tmp = strlen(num);
        for(int i=tmp-digit; i>=0; i-=digit) {
            arr[len] = 0;
            for(int j=0; j<digit; ++j) arr[len] = arr[
                len]*10 + num[i+j]-'0';
            ++len;
        }
        arr[len] = 0;
        for(int j=0; j<tmp%digit; ++j) arr[len] = arr
            [len]*10 + num[j]-'0';
        if( tmp%digit ) ++len;
        return *this;
    }
    BigInteger operator + (const BigInteger& b)
        const {
```

```

    if( *this>0 && b<0 ) return *this-(-b);
    if( *this<0 && b>0 ) return -(*this-b);
    BigInteger res=*this;
    int len2=max(res.len, b.len);
    for(int i=0; i<len2; ++i) {
        res.arr[i]+=b.arr[i];
        if( res.arr[i]>=step ) res.arr[i]-=step,
            res.arr[i+1]++;
    }
    res.len=len2;
    if(res.arr[len2]) ++res.len;
    return res;
}

BigInteger operator - (const BigInteger& b)
    const {
    if( *this<b ) return -(b-*this);
    if( *this<0 && b<0 ) return -(*this+b);
    if( *this>0 && b<0 ) return *this+(-b);
    BigInteger res=*this;
    int len2=max(res.len, b.len);
    for(int i=0; i<len2; ++i) {
        res.arr[i]-=b.arr[i];
        if( res.arr[i]<0 ) res.arr[i]+=step, res.
            arr[i+1]--;
    }
    while( len2>0 && res.arr[len2-1]==0 ) --len2;
    res.len=len2;
    return res;
}

BigInteger operator * (const BigInteger& b)
    const {
    if( *this==0 || b==0 ) return BigInteger(0);
    BigInteger res;
    for(int i=0; i<len; ++i) {
        for(int j=0; j<b.len; ++j) {
            res.arr[i+j]+=arr[i]*b.arr[j];
            res.arr[i+j+1]+=res.arr[i+j]/step;
            res.arr[i+j]%=step;
        }
    }
    res.len=len+b.len-1;
    while( res.arr[res.len] ) ++res.len;
    res.sign=sign*b.sign;
    return res;
}

BigInteger operator / (const int& b) const {
    if( b==0 ) return 0;
    BigInteger res;
    long long reduce=0;
    int signb=b>0?1:-1, b2=b*signb;
    for(int i=len-1; i>=0; --i) {
        res.arr[i] = (arr[i]+reduce*step)/b2;
        reduce = (arr[i]+reduce*step)%b2;
    }
    res.len = len;
    while( res.len>0 && res.arr[res.len-1]==0 )
        --res.len;
    if( res.len==0 ) res.sign=1;
    else res.sign=sign*signb;
    return res;
}

BigInteger operator / (const BigInteger& b)
    const {
    BigInteger abs_this=abs(*this);
    if( b==0 ) return 0;
    BigInteger st=0, ed, md;
    if( b.arr[0]>0 ) ed=abs_this/b.arr[0];
    else if( b.arr[1]*b.step+b.arr[0]>0 ) ed=
        abs_this/b.arr[1]*b.step+b.arr[0];
    else ed=abs_this;

    while( st<ed ) {
        md = (st+ed)/2+1;
        if( md*b<=abs_this ) st=md;
        else ed=md-1;
    }
    if( st.len==0 ) st.sign=1;
    else st.sign=sign*b.sign;

    return st;
}

BigInteger operator % (const int& b) const {
    if( b<=0 ) return 0;
    BigInteger res;
    long long reduce=0;
    for(int i=len-1; i>=0; --i)
        reduce = (arr[i]+reduce*step)%b;
    return reduce*sign;
}

BigInteger operator % (const BigInteger& b)
    const {
    if( b.isInt() ) return *this%int(b.toInt());
    if( b<=0 ) return 0;
    return *this-*this/b*b;
}

bool operator < (const BigInteger& b) const {
    if( sign!=b.sign ) return sign<b.sign;
    if( len!=b.len ) return len*sign<b.len*b.sign;
    ;
    for(int i=len-1; i>=0; --i)
        if( arr[i]!=b.arr[i] ) return arr[i]*sign<b
            .arr[i]*b.sign;
    return false;
}

bool operator == (const BigInteger& b) const {
    if( sign!=b.sign ) return false;
    if( len!=b.len ) return false;
    for(int i=len-1; i>=0; --i)
        if( arr[i]!=b.arr[i] ) return false;
    return true;
}

bool operator <= (const BigInteger& b) const {
    return *this<b || *this==b;
}

bool operator > (const BigInteger& b) const {
    return b<*this;
}

bool operator >= (const BigInteger& b) const {
    return b<=*this;
}

bool operator != (const BigInteger& b) const {
    return !(*this==b);
}

BigInteger operator-() const {
    BigInteger res = *this;
    if( res.len>0 ) res.sign*=-1;
    return res;
}

template<class T> BigInteger operator + (const
    T& b) const {return *this+BigInteger(b);}
template<class T> BigInteger operator - (const
    T& b) const {return *this-BigInteger(b);}
template<class T> bool operator == (const T&
    b) const {return *this==BigInteger(b);}
void print(const char *str="") const {
    if( len==0 ) printf("0");
    else {
        printf("%d", arr[len-1]*sign);
        for(int i=len-2; i>=0; --i) printf("%04d",
            arr[i]);
    }
    printf("%s", str);
}

bool isInt() const {
    if( len>2 ) return false;
}

```

```

    if( len<2 ) return true;
    long long res=toInt();
    return res<((1ll<<31) && res>=-(1ll<<31));
}
friend ostream& operator << ( ostream& out,
    const BigInteger &rhs ) {
    if( rhs.len==0 ) out << '0';
    else {
        out << rhs.arr[rhs.len-1]*rhs.sign;
        for(int i=rhs.len-2; i>=0; --i) out <<
            setfill('0') << setw(BigInteger::digit)
                << rhs.arr[i];
    }
    return out;
}
long long toInt() const {return sign*(1ll*arr
    [1]*step+arr[0]);}
private:
    static const int length = 100;
    static const int digit = 4, step = 10000;
    int arr[length];
    int len, sign;
};
istream& operator >> ( istream& in, BigInteger &
    rhs ) {
    char s[1000];
    in >> s;
    rhs = s;
    return in;
}

```

3.2 Fenwick Tree Range Modify [1, size]

```

inline int lowbit(int x) { return x&-x; }
template<class T>
class fenwick {
public:
    fenwick(int __size=SIZE) {
        size = __size+10;
        a = new T[size], b=new T[size];
        memset(a, 0, sizeof(T)*size);
        memset(b, 0, sizeof(T)*size);
    }
    ~fenwick() { delete[] a, delete[] b; }
    inline void add(int l, int r, long long n) {
        __add(a, r, r*n), __add(a, l-1, (l-1)*-n);
        __add(b, r, n), __add(b, l-1, -n);
    }
    inline long long sum(int l, int r) { return
        __sum(r)-__sum(l-1); }
private:
    int size;
    T *a, *b;
    inline void __add(T *arr, int x, T n) { for(; x
        &&n&&x<size; x+=lowbit(x)) arr[x]+=n; }
    inline T __sum(T x) { return __sum(a, x)+(__sum
        (b, size)-__sum(b, x))*x; }
    inline T __sum(T *arr, int x) {
        T res=0;
        for(; x; x-=lowbit(x)) res+=arr[x];
        return res;
    }
};

```

3.3 Fenwick Tree 2D - [1, size][1, size]

```

int tree[size+1][size+1]={0};
inline int lowbit(const int &x) {return x&(-x);}
inline void add(int x, int y, int z) {
    for(int i; x<=n; x+=lowbit(x))
        for(i=y; i<=n; i+=lowbit(i)) tree[x][i]+=z;
}

```

```

inline int query(short x, short y) {
    int res=0;
    for(int i; x; x-=lowbit(x))
        for(i=y; i; i-=lowbit(i))
            res+=tree[x][i];
    return res;
}

```

3.4 劃分樹

```

#include <iostream>
#include <cstdio>
#include <algorithm>
using namespace std;
#define N 100005
int a[N], as[N]; //原數組，排序後數組
int n, m;
int sum[20][N]; //紀錄第i層の1~j
    劃分到左子樹の元素個數(包括j)
int tree[20][N]; //紀錄第i層元素序列
void build(int c, int l, int r) {
    int i, mid=(l+r)>>1, lm=mid-l+1, lp=l, rp=mid
        +1;
    for (i=l; i<=mid; i++)
        if (as[i] < as[mid]) lm--;
        //先假設左邊の(mid-l+1)個數都等於as[mid],
        然後把實際上小於as[mid]の減去
    for (i = l; i <= r; i++){
        if (i == l) sum[c][i] = 0;
        //sum[i]表示[l, i]內有多少個數分到左邊，用
        DP來維護
        else sum[c][i] = sum[c][i-1];
        if (tree[c][i] == as[mid]){
            if (lm){
                lm--;
                sum[c][i]++;
                tree[c+1][lp++] = tree[c][i];
            }else
                tree[c+1][rp++] = tree[c][i];
        } else if (tree[c][i] < as[mid]){
            sum[c][i]++;
            tree[c+1][lp++] = tree[c][i];
        } else
            tree[c+1][rp++] = tree[c][i];
    }
    if (l == r)return;
    build(c+1, l, mid);
    build(c+1, mid+1, r);
}
int query(int c, int l, int r, int ql, int qr,
    int k){
    int s; // [l, ql]內將被劃分到左子樹の元素數目
    int ss; // [ql, qr]內將被劃分到左子樹の元素數目
    int mid=(l+r)>>1;
    if (l == r)
        return tree[c][l];
    if (l == ql){ //這裡要特殊處理！
        s = 0;
        ss = sum[c][qr];
    }else{
        s = sum[c][ql-1];
        ss = sum[c][qr]-s;
    } //假設要在區間[l, r]中查找第k大元素，t
        為當前節點，lch, rch為左右孩子，left, mid
        為節點t左邊界和中間點。
    if (k <= ss) //sum[r]-sum[l-1]>=k, 查找lch[t],
        區間對應為[ left+sum[l-1], left+sum[r]-1 ]
        return query(c+1, l, mid, l+s, l+s+ss-1, k);
    else
        //sum[r]-sum[l-1]<k, 查找rch[t], 區間對應為
        [mid+1+l-left-sum[l-1], mid+1+r-left-sum[r]]

```

```

        return query(c+1, mid+1, r, mid-l+1+ql-s, mid
        -l+1+qr-s-ss, k-ss);
    }
    int main(){
        int i, j, k;
        while(~scanf("%d%d", &n, &m)){
            for(i=1; i<=n; i++) {
                scanf("%d", &a[i]);
                tree[0][i] = as[i] = a[i];
            }
            sort(as+1, as+1+n);
            build(0, 1, n);
            while(m--){
                scanf("%d%d%d", &i, &j, &k);
                // i,j 分別為區間起始點, k 為該區間第 k
                // 大的數。
                printf("%d\n", query(0, 1, n, i, j, k));
            }
        }
        return 0;
    }
}

```

4 Graph

4.1 Dinic

```

class Flow{
public:
    Flow(int _ncnt) :ncnt(_ncnt), ecnt(1), path(new
        int[_ncnt + 2]), d(new int[_ncnt + 2]),
        visited(new bool[_ncnt + 2]){
        memset(path, 0, sizeof(int)*(_ncnt + 1));
    }
    ~Flow(){
        delete[](path);
        delete[](d);
        delete[](visited);
    }

    void Reset(){
        memset(path, 0, sizeof(int)*(ncnt + 1));
        ecnt = 1;
    }

    void AddEdge(int s, int t, int cap){
        edge[++ecnt].tar = t, edge[ecnt].cap = cap,
        edge[ecnt].next = path[s], path[s] = ecnt
        ;
        edge[++ecnt].tar = s, edge[ecnt].cap = 0,
        edge[ecnt].next = path[t], path[t] = ecnt
        ;
    }

}

int MaxFlow(int s, int t){ // Dinic

    int f = 0, df;
    while (BFS(s, t) < ncnt){
        while (true){
            memset(visited, 0, sizeof(bool)*(ncnt +
                1));
            df = DFS(s, INF, t);
            if (!df) break;
            f += df;
        }
    }
    return f;
}

private:
    static const int eMaxSize = 40002, INF = (int)
        1e9;
    int ecnt, ncnt;
    int *path, *d; // d for Dicic distance
    bool *visited;

    struct Edge{
        int tar, cap, next;
    }edge[eMaxSize];

    int DFS(int a, int df, int t){
        if (a == t) return df;
        if (visited[a]) return 0;
        visited[a] = true;
        for (int i = path[a]; i; i = edge[i].next){
            int b = edge[i].tar;
            if (edge[i].cap > 0 && d[b] == d[a] + 1){
                int f = DFS(b, std::min(df, edge[i].cap),
                    t);
                if (f){
                    edge[i].cap -= f, edge[i ^ 1].cap += f;
                    return f;
                }
            }
        }
    }
}

```

```

    }
    return 0;
}

int BFS(int s, int t){
    memset(d, 0x7f, sizeof(int)*(ncnt + 1));
    memset(visited, 0, sizeof(bool)*(ncnt + 1));
    d[s] = 0; visited[s] = true;
    std::queue<int> Q;
    Q.push(s);
    while (!Q.empty()){
        int a = Q.front(); Q.pop();
        for (int i = path[a]; i; i = edge[i].next){
            int b = edge[i].tar;
            if (visited[b] || edge[i].cap == 0)
                continue;
            visited[b] = true;
            d[b] = d[a] + 1;
            if (b == t) return d[b];
            Q.push(b);
        }
    }
    return d[t];
}

};
Flow flow( 1001 );

```

4.2 maximum matching in general graph

//Problem:<http://acm.timus.ru/problem.aspx?space=1&num=1099>

```

#include <cstdio>
#include <cstdlib>
#include <cstring>
#include <iostream>
#include <algorithm>
using namespace std;
const int N=250;
int n;
int head;
int tail;
int Start;
int Finish;
int link[N];    // 表示哪個點匹配了哪個點
int Father[N];  // 這個就是增廣路的Father……
                // 但是用起來太精髓了
int Base[N];    // 該點屬於哪朵花
int Q[N];
bool mark[N];
bool map[N][N];
bool InBlossom[N];
bool in_Queue[N];

void CreateGraph(){
    int x,y;
    scanf("%d",&n);
    while (scanf("%d%d",&x,&y)!=EOF)
        map[x][y]=map[y][x]=1;
}

void BlossomContract(int x,int y){
    fill(mark,mark+n+1,false);
    fill(InBlossom,InBlossom+n+1,false);
    #define pre Father[link[i]]
    int lca,i;
    for (i=x;i;i=pre) {i=Base[i]; mark[i]=true; }
    for (i=y;i;i=pre) {i=Base[i]; if (mark[i]) {lca=i; break;} } // 尋找lca之旅…… 一定要注意i=Base[i]
    for (i=x;Base[i]!=lca;i=pre){
        if (Base[pre]!=lca) Father[pre]=link[i]; //
        對於BFS樹中的父邊是匹配邊的點, Father

```

```

        向後跳
        InBlossom[Base[i]]=true;
        InBlossom[Base[link[i]]]=true;
    }
    for (i=y;Base[i]!=lca;i=pre){
        if (Base[pre]!=lca) Father[pre]=link[i]; //
        同理
        InBlossom[Base[i]]=true;
        InBlossom[Base[link[i]]]=true;
    }
    #undef pre
    if (Base[x]!=lca) Father[x]=y;    //注意不能從
    lca這個奇環的關鍵點跳回來
    if (Base[y]!=lca) Father[y]=x;
    for (i=1;i<=n;i++){
        if (InBlossom[Base[i]]){
            Base[i]=lca;
            if (!in_Queue[i]){
                Q[++tail]=i;
                in_Queue[i]=true; //要注意如果本來連向
                BFS樹中父結點的邊是非匹配邊的點,
                可能是沒有入隊的
            }
        }
    }
}

```

```

}

void Change(){
    int x,y,z;
    z=Finish;
    while (z){
        y=Father[z];
        x=link[y];
        link[y]=z;
        link[z]=y;
        z=x;
    }
}

void FindAugmentPath(){
    fill(Father,Father+n+1,0);
    fill(in_Queue,in_Queue+n+1,false);
    for (int i=1;i<=n;i++) Base[i]=i;
    head=0; tail=1;
    Q[1]=Start;
    in_Queue[Start]=1;
    while (head!=tail){
        int x=Q[++head];
        for (int y=1;y<=n;y++){
            if (map[x][y] && Base[x]!=Base[y] && link[x]
                !=y) //無意義的邊
                if ( Start==y || link[y] && Father[link[y]]
                    ) //精髓地用Father表示該點是否
                    BlossomContract(x,y);
            else if (!Father[y]){
                Father[y]=x;
                if (link[y]){
                    Q[++tail]=link[y];
                    in_Queue[link[y]]=true;
                }
            }
            else{
                Finish=y;
                Change();
                return;
            }
        }
    }
}

void Edmonds(){
    memset(link,0,sizeof(link));
    for (Start=1;Start<=n;Start++){
        if (link[Start]==0)
            FindAugmentPath();
    }
}

```

```

}
void output(){
    fill(mark,mark+n+1,false);
    int cnt=0;
    for (int i=1;i<=n;i++)
        if (link[i]) cnt++;
    printf("%d\n",cnt);
    for (int i=1;i<=n;i++){
        if (!mark[i] && link[i]){
            mark[i]=true;
            mark[link[i]]=true;
            printf("%d %d\n",i,link[i]);
        }
    }
}
int main(){
    CreateGraph();
    Edmonds();
    output();
    return 0;
}

```

5 Math

5.1 China remainder theorem

$$ans \equiv a_i \pmod{m_i}$$

```

int china_remainder_theorem(int n, int ai[], int
mi[]) {
    int gcdn, x, y, reduce, tmp;
    for(int i=1; i<n; ++i) {
        gcdn=ext_gcd(mi[i-1], mi[i], x, y);
        reduce=ai[i]-ai[i-1];
        if( reduce%gcdn!=0 )
            return -1;
        tmp=mi[i]/gcdn;
        x=(reduce/gcdn*x%tmp+tmp)%tmp;
        ai[i] = ai[i-1] + mi[i-1]*x;
        mi[i] = mi[i-1]*tmp;
    }
    return ai[n-1]%mod;
}

```

5.2 Euler's phi function $O(n)$

1. $\gcd(x, y) = d \Rightarrow \phi(xy) = \frac{\phi(x)\phi(y)}{\phi(d)}$
2. $p \text{ is prime} \Rightarrow \phi(p^k) = p^{k-1}\phi(p)$
3. $p \text{ is prime} \Rightarrow \phi(p^k) = \phi(p^{k-1}) \times p$
4. $n = p_1^{k_1} p_2^{k_2} \dots p_m^{k_m}$
 $\Rightarrow \phi(n) = p_1^{k_1-1}\phi(p_1) p_2^{k_2-1}\phi(p_2) \dots p_m^{k_m-1}\phi(p_m)$

```

const int MAXN = 100000;
int phi[MAXN], prime[MAXN], pn=0;
memset(phi, 0, sizeof(phi));
for(int i=2; i<MAXN; ++i) {
    if( phi[i]==0 ) prime[pn++]=i, phi[i]=i-1;
    for(int j=0; j<pn; ++j) {
        if( i*prime[j]>=MAXN ) break;
        if( i%prime[j]==0 ) {
            phi[i*prime[j]] = phi[i] * prime[j];
            break;
        }
        phi[i*prime[j]] = phi[i] * phi[prime[j]];
    }
}

```

5.3 Extended Euclid's Algorithm

$$ax + by = \gcd(a, b)$$

```

int ext_gcd(int a, int b, int &x, int &y){
    int x2;
    if( b==0 ) {
        x=1, y=0;
        return a;
    }
    int gcdn=ext_gcd(b, a%b, x, y), x2=x;
    x=y, y=x2-a/b*y;
    return gcdn;
}
int ext_gcd(int a, int b, int &x, int &y){
    int t, px=1, py=0, tx,ty;
    x=0, y=1;
    while(a%b!=0) {
        tx=x, ty=y;
        x=x*(-a/b)+px, y=y*(-a/b)+py;
        px=tx, py=ty;
        t=a, a=b, b=t%b;
    }
    return b;
}

```

5.4 Gaussian Elimination

```
// default for module version, comments for
double version
// double mmap[row][column];
const ll modn = 1000000007;
ll mmap[row][column];
ll inv(ll b) {
    return (b==1)?1:inv(modn%b)*(modn-modn/b)%modn;
}
void gauss(int n,int m) {
    int k=0;
    for(int i=0; i<m; i++)
        for(int j=k; j<n; j++)
            if(mmap[j][i]!=0) {
                for(int l=i; l<m; l++)
                    swap(mmap[k][l],mmap[j][l]);
                for(j++; j<n; j++){
                    if(mmap[j][i]==0)
                        continue;
                    //double scale=mmap[j][i]/mmap[k][i];
                    ll scale=mmap[j][i]*inv(mmap[k][i])%
                        modn;
                    for(int p=i+1; p<n; p++)
                        //mmap[j][p]-=mmap[k][p]*scale;
                        mmap[j][p]=(mmap[j][p]-mmap[k][p]*
                            scale%modn+modn)%modn;
                    mmap[j][i]=0;
                }
                k++;
                break;
            }
}
```

5.5 Miller Rabin

```
ll mul(ll a, ll b, ll n) { // a*b%n
    ll r = 0; a %= n, b %= n;
    while(b){
        if(b&1) r = (a+r>=n? a+r-n: a+r);
        a = (a+a>=n? a+a-n: a+a);
        b >>= 1;
    }
    return r;
}
ll powmod(ll a, ll d, ll n) { // a^d%n
    if(d==0) return 1ll;
    if(d==1) return a%n;
    return mul(powmod(mul(a, a, n), d>>1, n), d%2?a
        :1, n);
}
bool miller_rabin(ll a, ll n) {
    if (__gcd(a,n) == n ) return true;
    if (__gcd(a,n) != 1 ) return false;
    ll d = n-1, r = 0, res;
    while(d%2==0) { ++r; d>>=1; }
    res = powmod(a, d, n);
    if( res==1 || res==n-1 ) return true;
    while(r--){
        res = mul(res, res, n);
        if(res==n-1) return true;
    }
    return false;
}
bool isprime(ll n) {
    ll as[7]={2, 325, 9375, 28178, 450775, 9780504,
        1795265022}; // 2, 7, 61
    for(int i=0; i<7; i++)
        if( miller_rabin(n, as[i])==false )
            return false;
    return true;
}
```

}

5.6 Möbius function

```
int* isp;
char fcnt[N+5];
int mobius[N+5];
void make_mobius(int n) {
    isp = mobius;
    memset(mobius, true, sizeof(mobius));
    memset(fcnt, 0, sizeof(fcnt));
    for(int i=2; i<=n; ++i) {
        if( isp[i] ) {
            fcnt[i] = 1;
            for(int j=i+i; j<=n; j+=i) {
                isp[j] = false;
                if( fcnt[j]!=-1 ) fcnt[j]++;
            }
            if( i<=10000 )
                for(int ii=i*i, j=ii; j<=n; j+=ii)
                    fcnt[j] = -1;
        }
    }
    mobius[0] = 0;
    mobius[1] = 1;
    for(int i=2; i<=n; ++i) {
        if( fcnt[i]==-1 ) mobius[i] = 0;
        else if( fcnt[i]&1 ) mobius[i] = -1;
        else mobius[i] = 1;
    }
}
```


6 String

6.1 KMP

```
int KMP(char pat[5005], char str[5005]) {
    if( strlen(pat)>strlen(str) ) return -1;
    int failure[5005];
    int len=strlen(pat);
    for(int i=1, j=failure[0]=-1; i<len; ++i) {
        while( j>=0 && pat[j+1]^pat[i] ) j=failure[j];
        if( pat[j+1]==pat[i] ) ++j;
        failure[i]=j;
    }
    for(int i=0, j=-1; str[i]; ++i) {
        while( j>=0 && str[i]^pat[j+1] ) j=failure[j];
        if( str[i]==pat[j+1] ) ++j;
        if( j==len-1 ) {
            return i-len+1; // rec this!!
            j=failure[j];
        }
    }
    return -1;
}
```

6.2 Longest Palindromic Substring

```
char t[1001]; // 要處理的字串
char s[1001 * 2]; // 中間插入特殊字元的t。
int Z[1001 * 2], L, R; // Gusfield's Algorithm
// 由a往左、由b往右，對稱地作字元比對。
int match(int a, int b) {
    int i = 0;
    while (a-i>=0 && b+i<N && s[a-i] == s[b+i]) i++;
    return i;
}
void longest_palindromic_substring()
{
    int N = strlen(t);
    // 在t中插入特殊字元，存放到s。
    memset(s, '.', N*2+1);
    for (int i=0; i<N; ++i) s[i*2+1] = t[i];
    N = N*2+1;
    // modified Gusfield's Algorithm
    Z[0] = 1;
    L = R = 0;
    for (int i=1; i<N; ++i) {
        int ii = L - (i - L); // i的映射位置
        int n = R + 1 - i;
        if (i > R) {
            Z[i] = match(i, i);
            L = i;
            R = i + Z[i] - 1;
        }
        else if (Z[ii] == n) {
            Z[i] = n + match(i-n, i+n);
            L = i;
            R = i + Z[i] - 1;
        }
        else Z[i] = min(Z[ii], n);
    }
    // 尋找最長迴文子字串的長度。
    int n = 0, p = 0;
    for (int i=0; i<N; ++i)
        if (Z[i] > n) n = Z[p = i];
    // 記得去掉特殊字元。
    cout << "最長迴文子字串的長度是" << (n-1) / 2;
    // 印出最長迴文子字串，記得別印特殊字元。
    for (int i=p-Z[p]+1; i<=p+Z[p]-1; ++i)
```

```
        if (i & 1) cout << s[i];
    }
```

6.3 Suffix Array

```
int rank[LEN], sa[LEN];
int height[LEN];
int y[LEN], cnt[LEN], rr[2][LEN];
inline bool same(int *rank, int a, int b, int l)
{ return rank[a]==rank[b]&&rank[a+l]==rank[b+l]; }
void sa2(char str[], int n, int m) {
    printf("%s!! %d %d\n", str, n, m);
    int *rank1=rr[0], *rank2=rr[1];
    MSET(rr[1], 0);
    int i, p;
    for(i=0; i<m; ++i) cnt[i]=0;
    for(i=0; i<n; ++i) rank2[i]=str[i], cnt[rank2[i]]++;
    for(i=1; i<m; ++i) cnt[i]+=cnt[i-1];
    for(i=n-1; i>=0; --i) sa[--cnt[rank2[i]]]=i;
    for(int j=1; j<n; j<=1, m=p) {
        // 表示用第二個key(rank2)排序後 從 y[i]
        // 開始的後綴排第i名
        for(p=0, i=n-j; i<n; ++i) y[p++]=i;
        for(i=0; i<n; ++i) if( sa[i]>=j ) y[p++]=sa[i]-j;
        for(i=0; i<m; ++i) cnt[i]=0;
        for(i=0; i<n; ++i) cnt[ rank2[y[i]] ] ++;
        for(i=1; i<m; ++i) cnt[i]+=cnt[i-1];
        for(i=n-1; i>=0; --i) sa[ --cnt[ rank2[y[i]] ] ]=y[i];
        for(p=i=1, rank1[sa[0]]=0; i<n; ++i)
            rank1[sa[i]]=same(rank2, sa[i], sa[i-1], j)
                ?p-1:p++;
        std::swap(rank1, rank2);
    }
    for(int i=0; i<n; ++i) rank[i]=rank2[i];
}
void make_height(char str[]) {
    int len=strlen(str);
    height[0]=0;
    for(int i=0, j=0; i<len; ++i, j=height[rank[i]-1]-1) {
        if( rank[i]==1 ) continue;
        if( j<0 ) j=0;
        while( i+j<len && sa[rank[i]-1]+j<len &&
            str[i+j]==str[sa[rank[i]-1]+j] ) ++j;
        height[rank[i]]=j;
    }
}
int main() {
    char str[LEN];
    scanf("%s", str);
    int len = strlen(str);
    sa2(str, len+1, 256);
    make_height(str);
    for(int i=1; i<=len; ++i) printf("%d %d %s\n",
        i, height[i], str+sa[i]);
}
```

6.4 Z Algorithm

```
void Z(char G[], int z[]){
    int len = strlen(G);
    z[0] = len;
    int L = 0, R = 1;
    for ( int i = 1 ; i < len ; i++ ) {
        if ( i >= R || z[i-L] >= R-i ) {
            int x = (i>=R) ? i : R;
            while ( x < len && G[x] == G[x-i] )
                x++;
        }
```

```

        z[i] = x - i;
        if ( x > i ) L = i , R = x;
    }
    else z[i] = z[i-L];
}
}

```

7 Others

7.1 8 puzzle - IDA*

```

// 一個盤面。其數值1~8代表方塊號碼，0代表空格。
int board[3][3] = {2, 3, 4, 1, 5, 0, 7, 6, 8};
// 檢查 permutation inversion。檢查不通過，
// 表示盤面不合理。
bool check_permutation_inversion(int board[3][3])
{
    int inversion = 0;
    for (int a=0; a<9; ++a)
        for (int b=0; b<a; ++b) {
            int i = a / 3, j = a % 3;
            int ii = b / 3, jj = b % 3;
            if (board[i][j] && board[ii][jj]
                && board[i][j] < board[ii][jj])
                inversion++;
        }
    int row_number_of_0 = 0;
    for (int i=0; i<3 && !row_number_of_0; ++i)
        for (int j=0; j<3 && !row_number_of_0; ++j)
            if (board[i][j] == 0)
                row_number_of_0 = i+1;
    return (inversion + row_number_of_0) % 2 == 0;
}
//////////
// heuristic function,
// 採用不在正確位置上的方塊個數。
int h(int board[3][3])
{
    int cost = 0;
    for (int i=0; i<3; ++i)
        for (int j=0; j<3; ++j)
            if (board[i][j])
                if (board[i][j] != i*3 + j + 1)
                    cost++;
    return cost;
}
//////////
int taxicab_distance(int x1, int y1, int x2, int
y2)
{return abs(x1 - x2) + abs(y1 - y2);}

// heuristic function, 採用taxicab distance。
int h(int board[3][3]) {
    // 每塊方塊的正確位置。{0,0}
    // 是為了方便編寫程式而多加的。
    static const int right_pos[9][2] = {
        {0,0},
        {0,0}, {0,1}, {0,2},
        {1,0}, {1,1}, {1,2},
        {2,0}, {2,1}
    };
    // 計算每個方塊與其正確位置的 taxicab distance
    // 的總和。
    int cost = 0;
    for (int i=0; i<3; ++i)
        for (int j=0; j<3; ++j)
            if (board[i][j])
                cost += taxicab_distance(
                    i, j,
                    right_pos[board[i][j]][0],
                    right_pos[board[i][j]][1]
                );
    return cost;
}

// 上下左右
const string operator[4] = {"up", "down", "right",
, "left"};

```

```

const int dx[4] = {-1, 1, 0, 0}, dy[4] = {0, 0, 1, -1};
char solution[30];
// 正確的推動方式，其數值是方向0~3。
const int reverse_dir[4] = {1, 0, 3, 2};
// 用表格紀錄每一個方向的反方向。
// 可用於避免來回推動的判斷。

int board[3][3] = {2, 3, 4, 1, 5, 0, 7, 6, 8};
// 起始狀態。其數值1~8代表方塊號碼，0代表空格。

int sx = 1, sy = 2;
// 空格的位置。可馬上知道推動方塊的目的地。

bool onboard(int x, int y)
{return x>=0 && x<3 && y>=0 && y<3;}

int IDAstar(int x, int y, int gv, int prev_dir,
    int& bound, bool& ans) {
    int hv = h(board);
    if (gv + hv > bound) return gv + hv;
    // 超過，回傳下次的bound
    if (hv == 0) {ans = true; return gv;}
    // 找到最佳解

    int next_bound = 1e9;
    for (int i=0; i<4; ++i) {
        // 四種推動方向
        int nx = x + dx[i], ny = y + dy[i];
        // 空格的新位置
        if (reverse_dir[i] == prev_dir) continue;
        // 避免來回推動
        if (!onboard(nx, ny)) continue;
        // 避免出界
        solution[gv] = oper[i];
        // 紀錄推動方向
        swap(board[x][y], board[nx][ny]);
        // 推動
        int v = IDAstar(nx, ny, gv+1, i, bound, ans);
        if (ans) return v;
        next_bound = min(next_bound, v);
        swap(board[nx][ny], board[x][y]);
        // 回復原狀態
    }
    return next_bound;
}

void eight_puzzle() {
    if (!check_permutation_inversion(board)) {
        cout << "盤面不合理，無法解得答案。" << endl;
        return;
    }
    // IDA*
    bool ans = false;
    int bound = 0;
    while (!ans && bound <= 50)
        bound = IDAstar(sx, sy, 0, -1, bound, ans);
    if (!ans) {
        cout << "50步內無法解得答案。" << endl;
        return;
    }
    // 印出移動方法
    for (int i=0; i<bound; ++i)
        cout << operation[solution[i]] << ' ';
    cout << endl;
}

```

7.2 recursive to stack

replace all variable in data into layer[lay].variable

```
struct data {
```

```

    parameter;
    local variable;
    direction; //new
} layer[10000];
int lay=0; //new
type reval; //new
void go() {
    // at the beginning
start:
    // call recursive function
    direction = 1;
    lay++, parameter = value;
    goto start;
point1:
    variable = reval;
    // return
    reval = value;
    lay--;
    goto trans;
// at the end
trans:
    switch (direction) {
        case 1:
            goto point1;
    }
}

```

The End