TAOR Assignment 3

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Exercise 2

The random index (RI) values computed using 500 randomly generated pairwise comparison matrices has been presented in figure 1 against the results presented in the lecture. The values we obtain appear to be fairly close to Saaty's values presented in class and generally follows the same trend, while small discrepancies exist. There are two possible reasons for this. Firstly, the number of random matrices we used is 500, which may not be a sufficiently large sample size to provide accurate estimates of the RI values. As we increase the sample size used, the values obtained would be expected to approach Saaty's values. Secondly, the method we used to calculate the consistency index (CI) values is based on the eigenvalue approach, whereas the values presented in class may have been obtained using a different method, such as the geometric mean method presented in class.

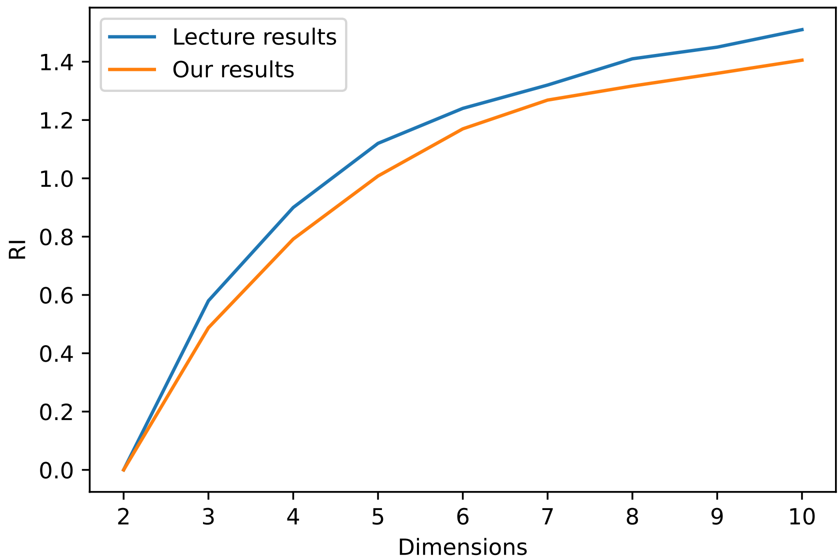

Figure 1: RI computed compared to the results presented in class

Exercise 3

Let us consider the example of choosing between laptops based on 4 different criteria: price, performance, battery life, design. We will now construct 4 different pairwise comparison matrices in terms of the 4 objectives. These matrices have been constructed so that the diagonal entries are 1 and $a_{ij} = 1/a_{ji}$, where a_{ij} is the (i,j)th entry in the pairwise comparison matrix. For price:

$$A_{Pr} = \begin{array}{c} \\ \mathbf{L1} \\ \mathbf{L2} \\ \mathbf{L3} \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{L1} & \mathbf{L2} & \mathbf{L3} \end{array} \\ \begin{bmatrix} 1 & 2 & 6 \\ \frac{1}{2} & 1 & 3 \\ \frac{1}{6} & \frac{1}{3} & 1 \end{bmatrix} \end{array}$$

For performance:

$$A_{Pe} = \begin{array}{c} \\ \mathbf{L1} \\ \mathbf{L2} \\ \mathbf{L3} \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{L1} & \mathbf{L2} & \mathbf{L3} \end{array} \\ \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{4} \\ 2 & 1 & \frac{1}{2} \\ 4 & 2 & 1 \end{bmatrix} \end{array}$$

For battery life:

$$A_{BL} = \begin{array}{c} \\ \mathbf{L1} \\ \mathbf{L2} \\ \mathbf{L3} \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{L1} & \mathbf{L2} & \mathbf{L3} \end{array} \\ \begin{bmatrix} 1 & \frac{1}{3} & \frac{2}{3} \\ 3 & 1 & 2 \\ \frac{3}{2} & \frac{1}{2} & 1 \end{bmatrix} \end{array}$$

For design:

$$A_{De} = \begin{array}{c} \\ \mathbf{L1} \\ \mathbf{L2} \\ \mathbf{L3} \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{L1} & \mathbf{L2} & \mathbf{L3} \end{array} \\ \begin{bmatrix} 1 & \frac{5}{2} & \frac{1}{2} \\ \frac{2}{5} & 1 & \frac{1}{5} \\ 2 & 5 & 1 \end{bmatrix} \end{array}$$

These pairwise comparison matrices indicate how well each laptop performs against others under each criteria, for example laptop 2 only has $\frac{1}{2}$ the performance but twice the battery life of laptop 3. Since we would like to minimise price when selecting a laptop as opposed to the other objectives, a better performance in A_{pr} indicates a lower price. The CI values of all of these 4 matrices are computed to be 0.

The scores of each laptop for each criterion can be computed by computing the weights of each pairwise comparison matrix. The scores obtained are:

$$S = \begin{array}{c} \\ \mathbf{Pr} \\ \mathbf{Pe} \\ \mathbf{BL} \\ \mathbf{De} \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{L1} & \mathbf{L2} & \mathbf{L3} \end{array} \\ \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.143 & 0.286 & 0.571 \\ 0.182 & 0.545 & 0.273 \\ 0.294 & 0.118 & 0.588 \end{bmatrix} \end{array}$$

We will now construct a pairwise comparison matrix for the criteria, make a decisions, and analyse the consistency. Suppose we arbitrarily construct the following pairwise comparison matrix:

$$A = \begin{array}{c} \\ \mathbf{Pr} \\ \mathbf{Pe} \\ \mathbf{BL} \\ \mathbf{De} \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{Pr} & \mathbf{Pe} & \mathbf{BL} & \mathbf{De} \end{array} \\ \begin{bmatrix} 1 & 1 & 2 & 5 \\ 1 & 1 & 3 & 5 \\ \frac{1}{2} & \frac{1}{3} & 1 & \frac{5}{3} \\ \frac{1}{5} & \frac{1}{5} & \frac{3}{5} & 1 \end{bmatrix} \end{array}$$

It can be noted that there are some inconsistencies in A, for example, we feel that price and performance are equally important ($a_{12} = a_{21} = 1$), but we feel that price is 2 times as important as battery life

($a_{13} = 2$) while performance is 3 times as important as battery life ($a_{23} = 3$). If we were consistent, these two entries should be equal. The CI value of A is 6.873×10^{-3}. The weights computed are:

$$w = \begin{bmatrix} 0.366 & 0.403 & 0.150 & 0.081 \end{bmatrix}$$

We can now obtain a weighted score of each of the laptops to make a decision. The weighted scores are calculated as:

$$Z = wS = \begin{bmatrix} 0.3283 & 0.3163 & 0.3554 \end{bmatrix}$$

Since laptop 3 has the highest weighted score, our decision is to purchase laptop 3.

As stated earlier, the CI value for A is 6.87×10^{-3}, and since A has 4 dimensions, we can calculated the RI value at $n = 4$ using our implemented function, which yields a result of 0.8601. The consistency ratio can be calculated as:

$$\text{CR} = \frac{\text{CI}}{\text{RI}} = 8.612 \times 10^{-3}$$

Since $\text{CR} > 0$, we can say that we are inconsistent as a decision maker, but not, and since $\text{CR} < 0.1$, we can conclude that we are only slightly and not seriously inconsistent.