

MFE 230M

Asset Backed Securities

Assignment 1

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1. (a) In Figure 1 we show the discount factors calculated in our Python program. These stem from six month period zero rates and 30/360 day count convention. Table 2 in Annex shows the exact numbers for the first years. More details are printed in the code.
- (b) Forward rates are shown in Figure 2. These are for 3 month periods, using simple compounding and ACT/360 day count convention. Table 2 in Annex shows the exact numbers for the first years. Complete output is printed in the code.

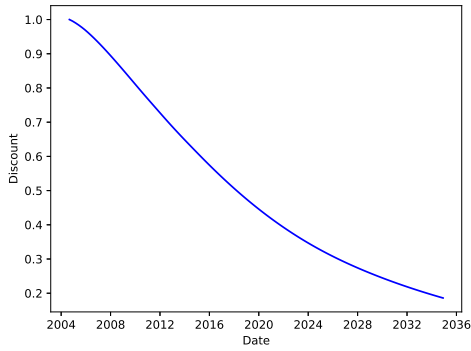


Figure 1: Discount factors

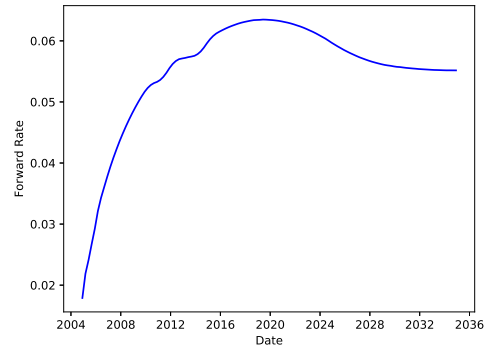


Figure 2: Forward rates

- (c) The ATM strikes for the given 15 caps are shown in Table 3 in the Annex. These are based on caplets on the forward LIBOR.
- (d) The fitted parameters we get after optimization are as follows:

$$\kappa = 0.11469962$$

$$\sigma = 0.01456547$$

- (e) We represent Hull and White θ as a function of time in Figure 3. The fitted discount factor was plotted against the discount factor in Figure 4. To get θ , we used the formula $\theta(t) = \frac{\partial f^M(0,t)}{\partial t} + \kappa f^M(0,t) + \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t})$ with f^M the instantaneous forward rate that we calculated from monthly discount rates, and κ and σ the 2 parameters we just found.

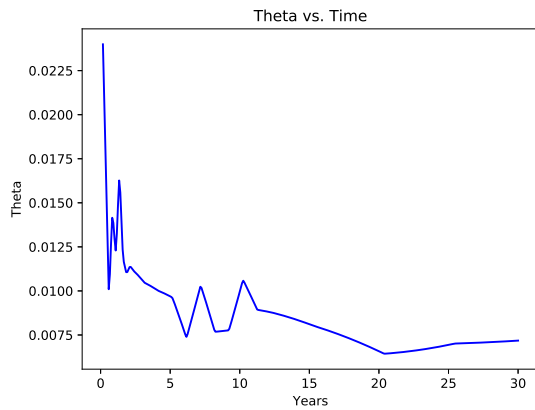


Figure 3: Calibrated θ

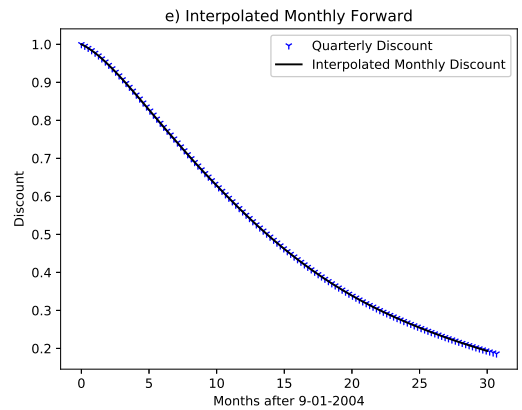


Figure 4: Discount factors vs interpolated discount factors

- (f) Considering the Hull-White ATM cap prices as a Black-implied price, we solve to get the volatilities giving this price with Black formula. We then compare them to initial market volatilities in Figure 5. We see there is a large discrepancy for short maturities, especially for the shortest one. The model seems to require a few values to adjust.

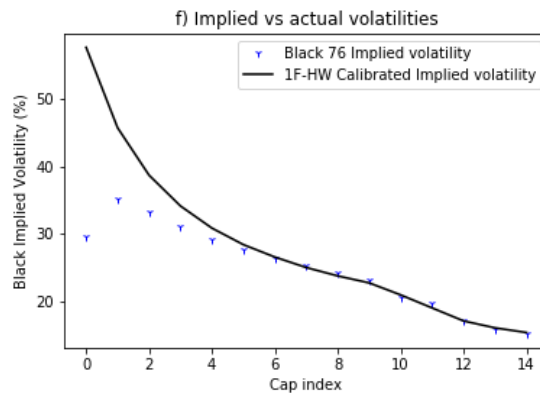


Figure 5: Implied and market volatilities

2. (a) Results of Monte Carlo simulation of interest rates under the Hull and White model with 10,000 total antithetic paths is shown in Table 1.

	Average price	Standard error
CA	33936929.08	0.36
CG	77945715.90	0.24
CM	14715506.37	0.19
CY	13872986.74	1.06
CZ	22902282.08	3.05
GZ	22968168.00	0.68
TC	20575720.77	0.85
VE	5399844.12	0.01
R	0.09	0.00

Table 1: Simulation summary

(b)

(c)

1 Annex

Dates	Zero	Discount	Forward
2004-09-01	-	1	-
2004-12-01	1.81398	0.995496	0.0179
2005-03-01	2.00382	0.99008	0.0218785
2005-06-01	2.16064	0.984011	0.0241341
2005-09-01	2.30631	0.97733	0.0267516
2005-12-01	2.43839	0.970159	0.0292413
2006-03-01	2.56946	0.962428	0.0321317
2006-06-01	2.70501	0.954068	0.0342868
2006-09-01	2.82931	0.94536	0.0360451
2006-12-01	2.94096	0.936421	0.0377613
2007-03-01	3.04225	0.927295	0.0393685
2007-06-01	3.14734	0.917706	0.0408852
2007-09-04	3.2501	0.907567	0.0423361
2007-12-03	3.3401	0.89777	0.043648
2008-03-03	3.42705	0.887702	0.0448709
2008-06-02	3.51291	0.877492	0.0460295
2008-09-02	3.59595	0.867049	0.0471282
2008-12-01	3.6715	0.856737	0.0481452
2009-03-02	3.74254	0.846233	0.0491089
2009-06-01	3.81542	0.835667	0.0500197
2009-09-01	3.88618	0.824939	0.0508835
2009-12-01	3.95131	0.814304	0.0516664

Table 2: Discount factors and forward rates

Maturity	ATM Strike
1	0.024262
2	0.029149
3	0.033024
4	0.036133
5	0.038719
6	0.040851
7	0.042529
8	0.044039
9	0.045260
10	0.046291
12	0.048193
15	0.050302
20	0.052154
25	0.052706
30	0.052900

Table 3: ATM strikes for given caps