MFE-230M: Homework Set 1

Due: (August 21, 2019 – submitted electronically by midnight)

Instructions:

- submit a MAXIMUM 2-page writeup and working code via bcourses
- your code should be easy to run: the grader should be able to download all your files (could be multiple; including data files) in a SINGLE directory of her choice and run ONE main file to obtain ALL numerical results; do not hardcode your work directory and do not create any subdirectories; try to avoid additional instructions on how to run your code
- include short but informative comments in your code
- use software of your choice (Matlab or Python).
- work in teams of 3 or 4; you are HIGHLY advised not to work alone; contact the instructor if you can not find a team

Problem 1: Hull-White (HW) Model

Calibrate HW term structure model,

$$dr(t) \quad = \quad (\theta(t) - \kappa r(t))dt + \sigma dW(t),$$

using the semiannually-compounded zero rates, the forward Libor rates and the at-the-money (ATM) USD cap data (Black flat volatilities) as of a trade date of 08/30/2004 and a settlement date of 9/1/2004 provided:

(a) (5 points) Calculate discount factors, $Z(D_i)$, from the semiannually-compounded zero rates, $r(D_i)$, for provided maturity dates D_i. Hint:

$$T_i^{30/360} = \frac{Days_{30}(D_0, D_i)}{360},$$

where $D_0 = 9/1/2004$, and following Veronesi Eq. (2.9), the discount factor $Z(D_i)$ implied from $r(D_i)$ is

$$Z(D_i) = \frac{1}{(1 + r_i(D_i)/2)^{2 \cdot T_i^{30/360}}}.$$

(b) (5 points) Calculate quarterly-compounded forward rates between each maturity date D_i using the discount factors, $Z(D_i)$, calculated from (a) and the maturity dates D_i. Hint:

$$f(T_{i-1}, T_i) = \frac{1}{\tau_i} \left(\frac{Z(D_{i-1})}{Z(D_i)} - 1 \right),$$

where

$$\tau_i = T_i - T_{i-1}, \text{ where } T_i = T_i^{ACT/360} = \frac{Days_{ACT}(D_i, D_0)}{360}.$$

(c) (5 points) Calculate the at-the-money (ATM) strike rates for each of the 15 caps. Hint: A n-year USD cap is comprised of $4n-1$ quarterly paying caplets, e.g. a 1-year cap is comprised of 3-caplets. The swap rate for a given n-year ATM USD cap is

$$X_n = \frac{\sum_{i=2}^{4n} \tau_i \cdot f_i \cdot Z_i}{\sum_{i=2}^{4n} \tau_i \cdot Z_i},$$

where τ_i is as in (b) and and $i = 0$ is D_0 and $i = 1$ is D_1 =12/1/2004, and represents the first omitted caplet on the forward Libor rate of 1.79%, thus we calculate the swap rate beginning at $i = 2$ corresponding to the forward Libor rate of 2.18785%.

(d) (25 points) use Black (1976) formula and estimate parameters κ and σ that best fit all of the fifteen ATM caps by minimizing the function

$$\min_{\kappa,\sigma} \sqrt{\sum_{i=1}^{15} \left(Cap_i^{Blk} - Cap_i^{HW}(\kappa,\sigma)\right)^2},$$

where $Cap_i^{HW}(\kappa,\sigma)$ and Cap_i^{Blk} are, correspondingly, modeled and market cap i cash prices.

(e) (5 point) use the technique you learned in the MFE-230I: Fixed Income class and estimate monthly $\theta(t)$'s from today t_0 to 30-years with a time step of 1/12 (i.e. one-month). Graph and report your results in a table.

(f) (5 points Extra Credit) Convert the Hull-White-implied ATM cap prices into Black-implied (flat) volatilities, and compare with market quotes with a plot. Comment on discrepancies.

Problem 2: Pricing REMIC Bonds

The prospectus for Freddie Mac, Multiclass Certificate, Series 2848 is provided. The REMIC issues nine classes of bonds. An Excel spreadsheet with

- characteristics of two Freddie Mac PC pools that constitute the collateral of the REMIC
- amortization schedules for the pools and the bonds (except for the residual class R)
- Conditional Prepayment Rates (CPR) with the corresponding prepayment probabilities for 100% PSA

is also provided. The information in the spreadsheet for the pools and the bonds came from Freddie Mac website. If you see any inconsistencies between the prospectus and the spreadsheet always give priority to the spreadsheet.

According to the prospectus, payments of principal and interest on the bonds are made on the 15th of each month. Assume that the payments are collected at the beginning of each month and the interest earned in the interim is paid to the residual class R.

(a) (45 points) Price all nine bonds using Monte Carlo simulations and antithetic variates (10,000 total paths) based on the HW model you estimated in Problem 1. Assume a 150% PSA speed. Report bond prices and standard errors.

(b) (20 points) Compute effective durations and convexities for all bonds.

(c) (Extra credit: 10 points) If market prices of each of the bonds were par what would be the implied OAS'? How should you interpret the OAS values?