1 Solution

$$
\begin{aligned}
u_1 &= e^{-xt}\sin(x+t)\\
u_2 &= \cos(x+t)\\
v_1 &= e^{-xt}\cos(x+t)\\
v_2 &= (x-t)^2\\
w_1 &= \tanh(x-t)\\
w_2 &= \sin(x+t)
\end{aligned}
$$

Take $t=0$ for ICs.

2 PDEs

$$
\begin{aligned}
\frac{\partial u_1}{\partial t} &= -xu_1 + v_1 + \frac{\partial^2 u_1}{\partial x^2} - 2tv_1 - \left(t^2-1\right)u_1 + \left(\frac{\partial w_1}{\partial x} + w_1^2 - 1\right) + \cos^{-1}(u_2) - 2t - v_2\\
\frac{\partial u_2}{\partial t} &= \frac{\partial u_2}{\partial x} + \left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial w_2}{\partial x}\right) + \left(\frac{\partial u_1}{\partial x} - tu_1 + v_1\right) + \left(\sqrt{1-u_2^2} - w_2\right)\\
\frac{\partial v_1}{\partial t} &= -xv_1 - u_1 + u_2 - \sqrt{1-w_2^2}\\
\frac{\partial v_2}{\partial t} &= -2\tanh^{-1} w_1 + \sin\left(e^{xt}u_1 - w_2\right) + \left(e^{xt}v_1 - u_2\right) + v_2^{3/2} - (x-t)^3\\
0 &= -\frac{\partial^2 w_1}{\partial x^2} - 2w_1\left(\frac{\partial u_1}{\partial x}\right)^2 + e^{2xt}\left(u_1^2 + v_1^2 - 1\right) + \sin v_2 - \sin\left((x-t)^2\right)\\
0 &= -\frac{\partial^2 w_2}{\partial x^2} + e^{xt}u_1 + \left(u_2^2 + w_2^2\right) - \left(\left(\frac{\partial w_2}{\partial x}\right)^2 + \left(\frac{\partial u_2}{\partial x}\right)^2\right) + \left(\frac{\partial u_1}{\partial x} - tu_1 + v_1\right)
\end{aligned}
$$

3 Boundary conditions

Left boundary:

$$
\begin{aligned}
u_1(1,t) &= e^{-t}\sin(1+t)\\
u_2(1,t) &= \cos(1+t)\\
w_1(1,t) &= \tanh(1-t)\\
w_2(1,t) &= \sin(1+t)
\end{aligned}
$$

Right boundary:

$$
\begin{aligned}
\frac{\partial u_1}{\partial x}(2,t) &= -tu_1(2,t) + v_1(2,t) \\
\frac{\partial u_2}{\partial x}(2,t) &= \sqrt{1 + \left(\frac{\partial w_2}{\partial x}(2,t)\right)^2} \\
\frac{\partial w_1}{\partial x}(2,t) &= 1 - w_1^2(2,t) + v_2(2,t) - (2-t)^2 \\
\frac{\partial w_2}{\partial x}(2,t) &= u_2(2,t) + \frac{\partial u_2}{\partial x}(2,t) + w_2(2,t)
\end{aligned}
$$