

Question 1

a) S^2

$$S = \{\epsilon, ab, abab\}$$

$$S^2 = S \times S.$$

$$S^2 = \{\epsilon, ab, abab\} \times \{\epsilon, ab, abab\}$$

multiplying every element of the first set with all elements of another set

$$S^2 = \{\epsilon, ab, abab, ab, abab, ababab, abab, ababab, abababab\}$$

$$\text{The final set of } S^2 = \{\epsilon, ab, abab, ababab, abababab\}$$

b) S^3

$$S^3 = S^2 \times S$$

$$S^3 = \{\epsilon, ab, abab, ababab, abababab\} \times \{\epsilon, ab, abab\}$$

$$\text{The final set of } S^3 = \{\epsilon, ab, abab, ababab, abababab, ababababab, abababababab\}$$

c) ST

$$S = \{\epsilon, ab, abab\}$$

$$T = \{aa, aba, abba, abbbba, \dots\}$$

The set T is an infinite set. The set ST will have all the elements obtained after multiplying each element of S with the entire set T

$$ST = \{\epsilon, ab, abab\} \times \{aa, aba, abba, abbbba, \dots\}$$

$$\text{The final set of } ST = \{aa, aba, abba, abbbba, \dots, abaa, ababa, ababba, ababbba, \dots, ababaa, abababa, abababba, abababbba, \dots\}$$

d) TS

$$S = \{\epsilon, ab, abab\}$$

$$T = \{aa, aba, abba, abbbba, \dots\}$$

$$TS = \{aa, aba, abba, abbbba, \dots\} \times \{\epsilon, ab, abab\}$$

The final set of TS = { aa, aaab, aaabab, aba, abaab, abaabab, abba, abbaab, abbaabab, abbba, abbbaab, abbbaabab, }

Question 2

a)

- The initial state is the final state. Therefore, The language has epsilon in it.
- The language must start with a. The strings starting with b never make it to the final state.
- The language must end with b. The strings ending with a never make it to the final state.

The language accepts strings starting with a and ending with b. The language also has epsilon in it.

b)

The strings that start with ab are accepted by the DFA.

The strings that start with b or aa are not accepted by the DFA.

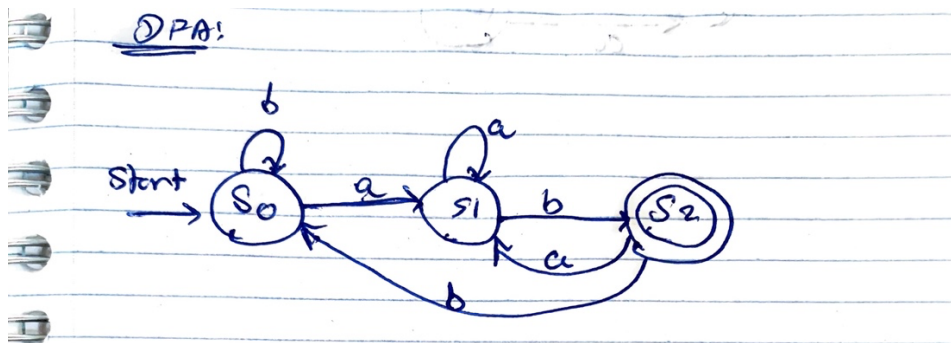
The language accepts strings starting with ab

Question 3.

a) The language of all strings that ending with 'ab'.

For the DFA that accepts all strings ending with 'ab', we need to consider the following points:

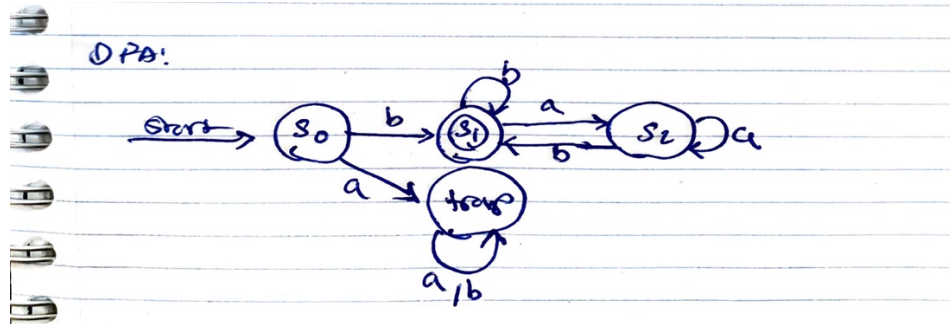
- String could start with either 'a' or 'b'.
- Shortest acceptable string is 'ab'.
- 'a' and 'b' should be the final input before final state.



b) The language of all strings that starting and ending with 'b'.

For DFA that accepts strings starting and ending with 'b', consider the following points:

- If first input is 'a' then DFA goes in a trap state
- Final input should be 'b'
- Smallest acceptable string is 'b'



c) The language of all strings in which the number of both a's and of b's is odd.

Need to consider the following points:

- Possible states according to number of inputs of 'a' and 'b' are:

- | | | | |
|------|----------|----------|------------------|
| i) | a → even | b → even | } Not acceptable |
| ii) | a → even | b → odd | |
| iii) | a → odd | b → even | |
| iv) | a → odd | b → odd | } acceptable |

