Strategic coalitions in stochastic games

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Abstract

The article compares two different approaches of incorporating probability into coalition logics. One is based on the semantics of games with stochastic transitions and the other on games with the stochastic failures. The work gives an example of a non-trivial property of coalition power for the first approach and a complete axiomatization for the second approach. It turns out that the logical properties of the coalition power modality under the second approach depend on whether the modal language allows the empty coalition. The main technical results for the games with stochastic failures are a strong completeness theorem for the logical system without the empty coalition and an incompleteness theorem which shows that there is no strongly complete logical system in the language with the empty coalition.

Keywords: stochastic game, logic, probability, axiomatization, completeness.

1 Introduction

1.1 Coalition logic

Coalition logics study strategic abilities of coalitions expressible through modality $[C]\varphi$ that stands for 'coalition C has a strategy to achieve φ'. Several such logics for different settings were originally introduced by Pauly [58, 59], who also gave a complete axiomatization of the basic coalition logic to achieve in one step. The key axiom in his system is what we call the cooperation axiom:

$$[C](\varphi \to \psi) \to ([D]\varphi \to [C \cup D]\psi), \tag{1}$$

where coalitions (sets of agents) C and D are disjoint. Informally, this axiom states that if coalition C has a strategy to achieve $\varphi \to \psi$ and coalition D has a strategy to achieve φ, then working together these two coalitions can achieve ψ. The condition that coalitions C and D are disjoint is important because otherwise any agent $a \in C \cap D$ would be facing a dilemma between acting according to the strategy of coalition C and the strategy of coalition D.

Coalition logics have been widely studied in the literature [3–6, 9, 15, 19, 33–36, 61, 62]. Alur *et al.* [11] introduced alternating-time temporal logic (ATL) that combines temporal and coalition modalities. Goranko and van Drimmelen [37] gave a complete axiomatization of ATL. Decidability and model checking problems for ATL-like systems have also been studied [14, 17, 18]. Alternative approaches to expressing the power to achieve a goal in a temporal setting are the STIT logic [16, 39–41, 57] and strategy logic [12, 18, 26, 48]. Broersen *et al.* [20] have shown that coalition logic can be embedded into a variation of STIT logic.

A complete logical system for modality 'coalition C has a strategy that uses a fixed amount of resources' has been proposed by Alechina *et al.* [9] under name resource-bounded coalition logic (RBCL). Model checking for RBCL and resource-bounded ATL has been widely studied [6–8, 21–

Vol. 31, No. 7,
Advance Access Publication on 17 May 2021 https://doi.org/10.1093/logcom/exab032

23, 29]. Various ways to incorporate knowledge into coalition logic have been also proposed [2, 25, 50–52].

1.2 Stochastic games

In Pauly's semantics of coalition logics, the complete action profile of all players uniquely determines the outcome state of the game. The same is true about concurrent game semantics of ATL. It is natural to consider games in which the complete action profile of all players only predetermines a distribution of probabilities on the set of possible outcomes. We refer to such games as *games with stochastic transitions*. Chen and Lu [28] added the probability of achieving a goal to the syntax of ATL and developed model checking algorithms for the proposed system. Another version of ATL with probabilistic success was proposed by Bulling and Jamroga [24]. They considered modality $\langle\langle A\rangle\rangle^{p}_{\omega}\varphi$ that stands for 'coalition C can bring about φ with success level of at least p when the opponents behave according to ω' and investigated its model checking properties. Huang *et al.* [42] combined perfect recall and coalition power to achieve a goal with a certain probability and discussed the model checking properties of their logical system. Coalition power to achieve a goal with a certain probability is also used in PRISM-games, a model checker for stochastic multi-player games [27, 44]. Aminof *et al.* [13] incorporated probability into strategy logic. Nguyen and Rakib [54] added probability to resource-bounded version of ATL. None of these works on probabilistic extensions of ATL contain completeness results.

Novák and Jamroga [55] proposed to distinguish probability of *achieving* a goal from probability of *non-failure*. They define the former as the probability of the chosen action to lead to a state in which a goal φ is satisfied. To capture the latter, they annotate each action a with a condition $\mathfrak{Ann}(a)$ that represents the expected (non-failure) behaviour of the action. Condition $\mathfrak{Ann}(a)$, generally speaking, is different from the goal φ. For example, goal φ might be to drive from point A to point B, while condition $\mathfrak{Ann}(a)$ might represent non-failure of brakes if action a is 'pushing the breaks'. By failure, they mean any execution where an action does not achieve the expected ('annotated') effect. They define the probability of non-failure ('ok') of action a in state s as

$$P_{ok}(s,a) = \sum_{s' \Vdash \mathfrak{Ann}(a)} P(s,a,s'). \tag{2}$$

In this article, we modify Novák and Jamroga's approach in two ways. First, we generalize it from a single-agent to a multi-agent setting. In such a setting, 'probability of non-failure' function $P_{ok}(s,\delta)$ depends on a complete action profile δ of all agents and the state s of the system in which these actions are executed. Second, we simplify their approach by assuming that function $P_{ok}(s,\delta)$ is given as a part of the game specification rather than defined by an equation similar to (2). We refer to such games as *games with stochastic failures*. Unlike Novák and Jamroga, we prove completeness results for our logical system.

1.3 Outline

The rest of the article is organized as follows. In the next section, we discuss the games with stochastic transitions. After defining the formal language of coalition power with probabilities, we introduce the games with stochastic transitions and discuss the properties of these games expressible in our language. We observe that the properties of games with stochastic transitions expressible in this language are very complicated and conclude, based on the existing works on the logics of probabilities, that the complete axiomatization of these properties is problematic. In Section 3, we define semantics of the same language based on the games with stochastic failures and give a sound and strongly complete axiomatization of the properties of the coalition power for this setting. We also show that strong completeness does not hold for a language that allows empty coalition. Section 5 concludes.

2 Games with stochastic transitions

In this section, we define the formal system and semantics of the logic of coalition power in the games with *stochastic transitions* and discuss examples of properties that can be captured in this language. The logical system for games with *stochastic failures* is introduced in Section 3. It uses the same language Φ as the logical system from the current section.

2.1 Syntax and semantics

We assume a fixed finite set of agents A and a fixed set of propositional variables. A *coalition* is any nonempty subset of A. We discuss empty coalitions in Section 3.5.

DEFINITION 2.1
Let Φ be the minimal set of formulae such that

1. $v \in \Phi$ for each propositional variable v,
2. $\neg\varphi, \varphi \to \psi \in \Phi$ for all formulae $\varphi, \psi \in \Phi$,
3. $[C]_p\varphi \in \Phi$ for each coalition C, each real number $p \in [0,1]$ and each formula $\varphi \in \Phi$.

In other words, Φ is the language specified by the following grammar:

$$\varphi := v \mid \neg\varphi \mid \varphi \to \varphi \mid [C]_p\varphi.$$

We assume that conjunction $\wedge$, disjunction $\vee$ and Boolean constants true $\top$ and false $\bot$ are defined in our language in the standard way. Let X^Y be the set of all functions from set Y to set X.

DEFINITION 2.2
A tuple (S, D, P, π) is a game with stochastic transitions, if

1. S is a finite set (of states),
2. D is a nonempty set (domain of actions),
3. P is a function from set $S \times D^A \times S$ into set $[0,1]$ such that
$$\sum_{s' \in S} P(s, \delta, s') = 1$$
for each state $s \in S$ and each function $\delta \in D^A$,
4. π is a function from propositional variables into subsets of S.

Informally, in each state s of the game, each agent chooses an action from the domain of the actions D. These choices are captured by a complete action profile $\delta \in D^A$. After the choices are made, the game transitions from the current state s to a random new state s'. The probability of transition to a specific new state s' is $P(s, \delta, s')$. As usual, valuation function π specifies which propositional variables are true in each state. Note that our games with stochastic transitions are *Markovian* because probability $P(s, \delta, s')$ depends only on the current state s but not on the previous states of the system.

The formal semantics of our logical system for games with stochastic transitions is given in the definition below:

DEFINITION 2.3
For any state $s \in S$ of a game with stochastic transitions (S, D, P, π) and any formula $\varphi \in \Phi$, the satisfiability relation $s \Vdash \varphi$ is defined recursively as follows:

1. $s \Vdash v$ if $s \in \pi(v)$, for any propositional variable v,
2. $s \Vdash \neg\varphi$ if $s \nVdash \varphi$,
3. $s \Vdash \varphi \to \psi$ if $s \nVdash \varphi$ or $s \Vdash \psi$,
4. $s \Vdash [C]_p\varphi$ when there is an action profile $\delta \in D^C$ of coalition C such that for any complete action profile $\delta' \in D^A$ of the set of all agents A, if $\delta \subseteq \delta'$, then $\sum_{t \Vdash \varphi} P(s, \delta', t) \geq p$.

In the above definition, $\delta \subseteq \delta'$ means that, as a set of pairs, function $\delta \in D^C$ is a subset of function $\delta' \in D^A$. In other words, functions δ and δ' are equal on the set C, where both of them are defined.

2.2 Cooperation axiom for games with stochastic transitions

One might suggest that the following form of the cooperation axiom (1) holds for the games with stochastic transitions:

$$[C]_p(\varphi \to \psi) \to ([D]_q\varphi \to [C \cup D]_{pq}\psi). \tag{3}$$

Unfortunately, this is true only if events $t \Vdash \varphi \to \psi$ and $t \Vdash \varphi$ are independent for any given initial state s and complete action profile δ'. Indeed, consider an example of a game in which Alice and Bob choose an action and then, no matter what actions have been chosen, they throw a coin. The coin lands heads up with probability 0.5 and tails up with probability 0.5 as well. Let H and T be statements 'coin landed heads up' and 'coin landed tails up', respectively. Note that no matter what the actions are, statement H will be *false* with probability 0.5. Thus, statement $H \to H \wedge T$ will be *true* with probability 0.5. Hence, Alice has a strategy (any action would do) to achieve $H \to H \wedge T$ with probability 0.5. In our notations, $[\text{Alice}]_{0.5}(H \to H \wedge T)$. Similarly, $[\text{Bob}]_{0.5}H$. At the same time, statement $[\text{Alice,Bob}]_\varepsilon(H \wedge T)$ is false for any positive value ε because statements H and T cannot be both true in the same state. This provides a counterexample for formula (3).

There are versions of the cooperation axiom that hold for games with stochastic transitions. For example, the following principle is universally true:

$$[C]_{0.9}(\varphi \to \psi) \to ([D]_{0.9}\varphi \to [C \cup D]_{0.8}\psi).$$

Indeed, assumption $[C]_{0.9}(\varphi \to \psi)$ means, see Item 4 of Definition 2.3, that once an initial state s and a complete action profile δ' are fixed, statement $t \Vdash \varphi \to \psi$ will be *false* for at most 10% of outcomes t. Similarly, statement $t \Vdash \varphi$ will be *false* for at most 10% of outcomes t. Thus, both of these statements will be *true* for at least 80% of outcomes. Therefore, statement ψ will be true in at least 80% of outcomes. The same argument can justify the following form of the cooperation axiom:

$$[C]_p(\varphi \to \psi) \to ([D]_q\varphi \to [C \cup D]_{\max\{0, p+q-1\}}\psi). \tag{4}$$

Note that if $p + q \leq 1$, then the conclusion $[C \cup D]_0\psi$ is vacuously true. Hence, this formula captures a non-trivial property of strategies only when $p + q > 1$. In the next section, we give another example of a non-trivial property of strategies in games with stochastic transitions. That property, see Theorem 2.4, is non-trivial even for small values of the probabilities.

2.3 An example

In this section, we give a non-trivial example of a property of games with stochastic transitions. It is captured by the following theorem:

THEOREM 2.4
For any state $s \in S$ of a stochastic game (S, D, P, π), any coalition C and any formulae $\varphi, \psi, \sigma \in \Phi$,

$$s \Vdash [A]_{p+q+r}(\varphi \vee \psi \vee \sigma) \to [A]_{2p}(\varphi \vee \psi) \vee [A]_{2q}(\psi \vee \sigma) \vee [A]_{2r}(\varphi \vee \sigma).$$

Recall that A is the set of all agents. The proof of the theorem is based on the two lemmas below. The first lemma follows from Item 4 of Definition 2.3.

LEMMA 2.5
$s \Vdash [A]_p\varphi$ iff there is a complete action profile $\delta \in D^A$ such that $\sum_{t \Vdash \varphi} P(s, \delta, t) \geq p$.

The second lemma can be shown using a much more general principle of Rényi [60, p. 38, Theorem 11]. To keep the article self-contained, we prove this lemma directly.

LEMMA 2.6
For arbitrary events B, C and D in an arbitrary probability space with probability function P,

$$\mathsf{P}(B \cup C) + \mathsf{P}(C \cup D) + \mathsf{P}(B \cup D) \geq 2 \cdot \mathsf{P}(B \cup C \cup D).$$

PROOF. Since probability of the union of any two events is always no more than the sum of the probabilities of these events,

$$\mathsf{P}(C \setminus B) + \mathsf{P}(D \setminus B) \geq \mathsf{P}((C \setminus B) \cup (D \setminus B)).$$

Thus, because $(C \setminus B) \cup (D \setminus B) = (C \cup D) \setminus B)$,

$$\mathsf{P}(C \setminus B) + \mathsf{P}(D \setminus B) \geq \mathsf{P}((C \cup D) \setminus B).$$

Hence, by adding $\mathsf{P}((C \cup D) \setminus B)$ to both sides of the inequality,

$$\mathsf{P}(C \setminus B) + \mathsf{P}(D \setminus B) + \mathsf{P}((C \cup D) \setminus B) \geq 2 \cdot \mathsf{P}((C \cup D) \setminus B).$$

At the same time, because $\mathsf{P}((C \cup D) \cap B) \geq 0$,

$$\mathsf{P}(B) + \mathsf{P}(B) + \mathsf{P}((C \cup D) \cap B) \geq 2 \cdot \mathsf{P}(B).$$

Then, by adding the two inequalities above,

$$[\mathsf{P}(C \setminus B) + \mathsf{P}(B)] + [\mathsf{P}(D \setminus B) + \mathsf{P}(B)] + [\mathsf{P}((C \cup D) \setminus B) + \mathsf{P}((C \cup D) \cap B)] \geq 2 \cdot [\mathsf{P}((C \cup D) \setminus B) + \mathsf{P}(B)].$$

Hence, since the sum of probabilities of two disjoint events is equal to the probability of the union, $\mathsf{P}(B \cup C) + \mathsf{P}(D \cup B) + \mathsf{P}(C \cup D) \geq 2 \cdot \mathsf{P}(C \cup D \cup B)$. □

We are now ready to prove Theorem 2.4.

PROOF. By Lemma 2.5, assumption $s \Vdash [A]_{p+q+r}(\varphi \vee \psi \vee \sigma)$ implies that there is a complete action profile $\delta \in D^A$ such that $\sum_{t \Vdash \varphi\vee\psi\vee\sigma} P(s,\delta,t) \geq p+q+r$. Hence, by Lemma 2.6,

$$\begin{aligned}\sum_{t\Vdash\varphi\vee\psi} P(s,\delta,t)+\sum_{t\Vdash\psi\vee\sigma} P(s,\delta,t)+\sum_{t\Vdash\varphi\vee\sigma} P(s,\delta,t) &\geq 2\sum_{t\Vdash\varphi\vee\psi\vee\sigma} P(s,\delta,t)\\ &\geq 2(p+q+r).\end{aligned}$$

Thus, at least one of the following inequalities holds

$$\begin{aligned}\sum_{t\Vdash\varphi\vee\psi} P(s,\delta,t) &\geq 2p,\\ \sum_{t\Vdash\psi\vee\sigma} P(s,\delta,t) &\geq 2q,\\ \sum_{t\Vdash\varphi\vee\sigma} P(s,\delta,t) &\geq 2r.\end{aligned}$$

Then, by Lemma 2.5, one of the following statements holds,

$$\begin{aligned}s &\Vdash [A]_{2p}(\varphi\vee\psi),\\ s &\Vdash [A]_{2q}(\psi\vee\sigma),\\ s &\Vdash [A]_{2r}(\varphi\vee\sigma).\end{aligned}$$

Therefore, $s \Vdash [A]_{2p}(\varphi\vee\psi)\vee[A]_{2q}(\psi\vee\sigma)\vee[A]_{2r}(\varphi\vee\sigma)$. □

2.4 From stochastic transitions to stochastic failures

Pauly's cooperation axiom (1) captures a property of the interplay among strategic powers of different coalitions. As we have observed, variation (4) of this axiom holds for the games with stochastic transitions. A more complicated example of a property of such games is captured by Theorem 2.4. Note that the statement of Theorem 2.4 is more about properties of *probabilities* than *strategies*. Logical systems for reasoning about just probabilities have been proposed before.

Heifetz and Mongin proposed a sound and complete axiomatization of a logic that uses modalities $M_p\varphi$ and $L_p\varphi$ that stand for 'formula φ is true with probability *at most* p' and 'formula φ is true with probability *at least* p', respectively [38]. Their axiomatization is very complex. For example, for arbitrary formulae $\varphi_1,\dots,\varphi_m,\psi_1,\dots,\psi_n$, their B axiom has the following form:

$$\left(\bigwedge_{k=1}^{max(m,n)}\left(\varphi^{(k)}\leftrightarrow\psi^{(k)}\right)\right)\rightarrow$$
$$\left(\left(\bigwedge_{i=1}^{m} L_{\alpha_i}\varphi_i\right)\wedge\left(\bigwedge_{j=1}^{n-1} M_{\beta_j}\psi_j\right)\rightarrow L_{\sum_{i=1}^{m}\alpha_i-\sum_{j=1}^{n-1}\beta_j}\psi_n\right),$$

where $\varphi^{(k)}$ and $\psi^{(k)}$ are formulae

$$\bigvee_{1\leq\ell_1<\dots<\ell_k\leq m}(\varphi_{\ell_1}\wedge\dots\wedge\varphi_{\ell_k})$$

and

$$\bigvee_{1\leq \ell_1<\cdots<\ell_k\leq n}(\psi_{\ell_1}\wedge\cdots\wedge\psi_{\ell_k}),$$

respectively.

Abadi and Halpern [1] have shown that the first-order probability logic is Π^2_1-complete and, thus, does not have a finitary axiomatization. Ognjanović and Raškovic [56] gave an axiomatization for this logic using an infinitary inference rule.

There have been many attempts in the literature to develop logical systems that circumvent the described-above complexity of probability logics. Fagin *et al.* proposed a sound and complete modal logic-like system without nested modalities that describes properties of inequalities among probabilities of different events [31]. The complexity of their axiomatization is hidden in the *Ineq* axiom that contains 'all instances of valid formulas about linear inequalities'.

Kokkinis *et al.* [43] added probability modality to the logic of justifications. However, their definition of a probability space allows algebras of events that are closed only with respect to finite unions and finite intersections. The standard definition of a probability space requires the set of events to be closed with respect to countable unions and countable intersections [10, p.10].

Others suggested to replace probability with a probability-like uncertainty parameter whose meaning is not defined using a probability space. The existing literature on this approach includes papers on possibilistic logic [30], quantitative modal logic [45] and graded logic of justifications [32, 47]. Another non-probabilistic way to interpret uncertainty is through confidence intervals in a metric space [49, 53].

Given the complexity of probability logics and of the example captured by Theorem 2.4, it is clear that properties of games with stochastic transitions are unlikely to have a simple axiomatization that could be described as a natural extension of Pauly's original logic of coalition power. In the rest of this article, we focus on a different approach to probabilities and coalition power that goes back to the described earlier Novák and Jamroga [55] proposal to distinguish probability of *achieving* a goal from probability of *non-failure*.

Unlike Novák and Jamroga, we treat probability of non-failure P_{ok} as a basic function which is given as a part of the game specification. To do this, we modify Definition 2.2 of games with stochastic transitions into Definition 3.1 of games with stochastic failures, which is given below. We use the latter games to give formal semantics of modality 'coalition C has a strategy that guarantees that system will not fail with probability at least p and, if the system does not fail, statement φ will be true'. Our technical results are soundness and strong completeness of a logical system capturing the properties of this modality. We also show that a strongly complete logical system does not exist if the language of the system allows the empty coalition.

3 Games with stochastic failures

This section contains the main contribution of this article. It proposes a strongly complete logical system for coalition power in games with stochastic failures.

3.1 Semantics

DEFINITION 3.1
A tuple (S, D, P_{ok}, M, π) is game with stochastic failures, if

TABLE 1 A game with stochastic failures.

	d_1		d_2		d_3	
	P_{ok}	Outcome	P_{ok}	Outcome	P_{ok}	Outcome
d_1	0.6	p, q	0.7	p, q	0.8	$p, \neg q$
d_2	0.5	p, q	0.9	p, q	0.1	$p, \neg q$
d_3	0.7	$\neg p, q$	0.1	$\neg p, q$	0.2	$\neg p, \neg q$

1. S is a set (of states);
2. D is a nonempty set (domain of actions);
3. P_{ok} is a function from set $S \times D^A$ into interval $[0, 1]$;
4. $M \subseteq S \times D^A \times S$, where, for each state $s \in S$ and each complete action profile $\delta \in D^A$, there is at least one state $s' \in S$ such that $(s, \delta, s') \in M$;
5. π is a function from propositional variables into subsets of S.

Note that unlike Definition 2.2, we no longer assume that set S is finite. This is because Item 3 of Definition 2.2 includes a summation over the set of all states S and Definition 3.1 does not. In addition, a summation over a subset of S is present in Definition 2.3 and it is not present in the definition of satisfiability for games with stochastic failures that we give in Definition 3.2 below.

In this section, we only discuss global failures of the game. In other words, if the game fails, then it terminates and does not reach any state. Informally, a transition of such a game from a state s could be viewed as three-step process. First, agents choose actions that determine an action profile δ. Second, the game is terminated with probability $1 - P_{ok}(s, \delta)$ due to a failure. Third, if the game survived on step two, then the game nondeterministically transitions into a next state s' such that $(s, \delta, s') \in M$. Note that the third step is nondeterministic, but we do not assign any probability distribution to it. Assigning such a distribution would mean adding stochastic transitions as discussed in Section 2. Item 4 of Definition 3.1 requires that there is at least one state s' such that $(s, \delta, s') \in M$. In other words, the game will have at least one possible next state s' after it survives step two.

As an example, consider the two-agent game with stochastic failures depicted in Table 1. The domain of actions D in this game is $\{d_1, d_2, d_3\}$. Thus, the game has nine complete action profiles that we denote as pairs of actions: $\{(x, y) \mid x, y \in D\}$. The set of all states S of this game consists of the initial state s_0 and nine outcome states. We assume that this game is deterministic. Thus, there is a unique state corresponding to each complete action profile. We use the same pairs $\{(x, y) \mid x, y \in D\}$ to denote the outcomes of the corresponding complete action profiles. That is, from state s_0 under action profile (x, y) the game transitions to outcome state (x, y). More formally, $M = \{(s_0, (x, y), (x, y)) \mid x, y \in D\}$. The values of function P_{ok} are shown in Table 1. For example, $P_{ok}(s_0, (d_2, d_1)) = 0.5$. Valuation function π is also specified in Table 1. For example, the table shows $p, \neg q$ for outcome (d_2, d_3). This means that $(d_2, d_3) \in \pi(p)$ and $(d_2, d_3) \notin \pi(q)$.

Next is the key definition of this section. Its Item 4 formally specifies the semantics of the modality $[C]_p$ for games with stochastic failures.

DEFINITION 3.2
For any game with stochastic failures (S, D, P_{ok}, M, π), any state $s \in S$ and any formula $\varphi \in \Phi$, satisfiability relation $s \Vdash \varphi$ is defined recursively as follows:

1. $s \Vdash p$ if $s \in \pi(p)$,

2. $s \Vdash \neg\varphi$ if $s \nVdash \varphi$,
3. $s \Vdash \varphi \to \psi$ if $s \nVdash \varphi$ or $s \Vdash \psi$,
4. $s \Vdash [C]_p\varphi$ when there is an action profile $\delta \in D^C$ of coalition C such that for any complete action profile $\delta' \in D^A$ if $\delta \subseteq \delta'$, then
   (a) $P_{ok}(s, \delta') \geq p$ and
   (b) for any state $s' \in S$, if $(s, \delta', s') \in M$, then $s' \Vdash \varphi$.

Informally, $s \Vdash [C]_p\varphi$ means that coalition C has a strategy that guarantees condition φ in each outcome and which is guaranteed not to fail with probability at least p no matter what are the actions of the other agents.

For example, in the game depicted in Table 1, the first agent, whom we refer to as agent a, has two different actions to achieve statement p: action d_1 and action d_2. We do not consider the third action, d_3, because it does not guarantee that condition p will be achieved. Action d_1 guarantees that the system will not fail with probability at least 0.6 because 0.6 is the smallest number in the first row of that table. Similarly, action d_2 guarantees that the system will not fail with probability at least 0.5. Thus, agent a has a strategy that achieves p and will not fail with probability at least 0.6 no matter what is the action of the second player. We denote this by $[a]_{0.6}p$. Note that 0.6 is *maxmin* value in the table for all rows (actions of agent a) that guarantee condition p.

In the same game, the second agent, whom we refer to as agent b, has two actions that guarantee condition q: action d_1 and action d_2. The first of these actions guarantees non-failure with probability 0.5 and the second with probability only 0.1. Thus, $[b]_{0.5}q$. Note that 0.5 is *maxmin* value in the table for all columns (actions of agent b) that guarantee condition q.

If agents a and b decide to use their maxmin actions for achieving p and q, respectively, then they will achieve condition $p \wedge q$ with probability of non-failure at least 0.6. We write this as $[a, b]_{0.6}(p \wedge q)$. Observe that in our example agents a and b will do even better if they deviate from their respective maxmin actions. If they both use action d_2, then they are guaranteed to achieve $p \wedge q$ and not to fail with probability 0.9. Hence, $[a, b]_{0.9}(p \wedge q)$.

3.2 Logical systems

In this section, we introduce axioms that describe the properties of coalition power modality $[C]_p\varphi$ in games with stochastic failures. In addition to propositional tautologies in language Φ, each system contains the following axioms:

1. Monotonicity: $[C]_p\varphi \to [C]_q\varphi$, where $q \leq p$,
2. Unachievability of falsehood: $\neg[C]_p\bot$,
3. Cooperation: $[C_1]_p(\varphi \to \psi) \to ([C_2]_q\varphi \to [C_1 \cup C_2]_{\max\{p,q\}}\psi)$,
4. where $C_1 \cap C_2 = \varnothing$.

The monotonicity axiom says that if a coalition C can achieve goal φ with probability at least p, then coalition C can achieve φ with probability q, where $q \leq p$. The unachievability of falsehood axiom says that no coalition can achieve falsehood.

The cooperation axiom generalizes the original Pauly axiom (1). Informally, it says that two coalitions can combine their strategies to achieve a common goal. Recall that the assumption that coalitions C_1 and C_2 are disjoint is important because a hypothetical common agent of these two coalitions might be required to choose different actions under strategies of these two coalitions.

Our version of the cooperation axiom adds probability of non-failure subscript to the original version of this axiom. To understand our form of this axiom, consider again the example of a stochastic game with failures depicted in Table 1. Recall from our earlier discussion that $[a]_{0.6}p$ and $[b]_{0.5}q$. In other words, if agent a is using her *maxmin* action d_1, then she will achieve condition p with probability of non-failure 0.6 no matter what action agent b chooses. Similarly, if agent b is using his *maxmin* action d_1, then he will achieve condition q with probability of non-failure 0.5 no matter what action agent a chooses. If agents a and b use their respective maxmin actions, then they will achieve condition $p \wedge q$ with probability of non-failure of at least $\max\{0.6, 0.5\} = 0.6$. Note that the probability is the *maximum* of the two probabilities, not the minimum, because action of agent a alone guarantees probability of non-failure to be *at least* 0.6. The cooperation axiom above captures this property for an arbitrary two disjoint coalitions C and D. This axiom states that the combined coalition has an action profile to achieve condition ψ with probability *at least* $\max\{p, q\}$. In a specific game, the combined coalition can avoid failure with even higher probability. Indeed, as we discussed earlier, $[a, b]_{0.9}(p \wedge q)$ for the game depicted in Table 1.

In this article, we assume that, generally speaking, stochastic failures are not independent. If they would be independent and the set of all agents A were finite, then function $P_{ok}(s, \delta)$ would be equal to the product of probabilities of individual actions not to fail:

$$P_{ok}(s, \delta) = \prod_{a \in \mathcal{A}} p_a(s, \delta(a)),$$

where $p_a(s, d)$ is the probability of action d not to fail in state s when executed by an agent a. In this case, the cooperation axiom would have form:

$$[C_1]_p(\varphi \to \psi) \to ([C_2]_q\varphi \to [C_1 \cup C_2]_{pq}\psi), \tag{5}$$

where $C_1 \cap C_2 = \varnothing$. Note that in this case one can define a 'resource' parameter r to be the logarithm of the probability not to fail: $r = \log_2 p$. Using this 'resource' instead of original probability as a subscript, formula (5) could be written as follows:

$$[C_1]_{r_1}(\varphi \to \psi) \to ([C_2]_{r_2}\varphi \to [C_1 \cup C_2]_{r_1+r_2}\psi) \tag{6}$$

because $\log(pq) = \log p + \log q$. Formula (6) is the form of the cooperation axiom in the RBCL [9]. In other words, the general properties of coalition strategies in stochastic games are described by our logical system. However, in the case of a special class of games, where stochastic failures of different agents are independent, the properties are described by a variation of the RBCL with logarithm of the probability of non-failure acting as a 'resource'. We say 'a variation' because the original version of RBCL allows only non-negative integer values of the resource parameters. In our case, logarithm of a probability is a negative real number or negative infinity.

We write $\vdash \varphi$ if formula $\varphi \in \Phi$ is provable from the above axioms using the modus ponens, the necessitation and the monotonicity inference rules:

$$\frac{\varphi, \varphi \to \psi}{\psi} \qquad \frac{\varphi}{[C]_0\varphi} \qquad \frac{\varphi \to \psi}{[C]_p\varphi \to [C]_p\psi}.$$

Notice that the necessitation inference rule with positive subscript is not, generally speaking, valid. Indeed, formula $\top$ is universally true but coalition C may not have a strategy that guarantees the survival of the system with a positive probability. Thus, $[C]_p\top$ is not a universally true formula for $p > 0$.

We write $X \vdash \varphi$ if formula $\varphi \in \Phi$ is provable from the theorems of our logical system and a set of additional axioms X using only the modus ponens inference rule.

The next lemma is a well-known 'deduction' lemma. We reproduce its proof here to keep the article self-contained.

LEMMA 3.3
(deduction).
If $X, \varphi \vdash \psi$, then $X \vdash \varphi \to \psi$.

PROOF. Suppose that sequence $\psi_1, \dots, \psi_n$ is a proof from set $X \cup \{\varphi\}$ and the theorems of our logical system that uses the modus ponens inference rule only. In other words, for each $k \leq n$, either

1. $\vdash \psi_k$, or
2. $\psi_k \in X$, or
3. ψ_k is equal to φ, or
4. there are $i, j < k$ such that formula ψ_j is equal to $\psi_i \to \psi_k$.

It suffices to show that $X \vdash \varphi \to \psi_k$ for each $k \leq n$. We prove this by induction on k through considering the four cases above separately.
Case 1: $\vdash \psi_k$. Note that $\psi_k \to (\varphi \to \psi_k)$ is a propositional tautology and, thus, is an axiom of our logical system. Hence, $\vdash \varphi \to \psi_k$ by the modus ponens inference rule. Therefore, $X \vdash \varphi \to \psi_k$.
Case 2: $\psi_k \in X$. Then, $X \vdash \varphi \to \psi_k$ similarly to the previous case.
Case 3: formula ψ_k is equal to φ. Thus, $\varphi \to \psi_k$ is a propositional tautology. Therefore, $X \vdash \varphi \to \psi_k$.
Case 4: formula ψ_j is equal to $\psi_i \to \psi_k$ for some $i, j < k$. Thus, by the induction hypothesis, $X \vdash \varphi \to \psi_i$ and $X \vdash \varphi \to (\psi_i \to \psi_k)$. Note that formula $(\varphi \to \psi_i) \to ((\varphi \to (\psi_i \to \psi_k)) \to (\varphi \to \psi_k))$ is a propositional tautology. Therefore, $X \vdash \varphi \to \psi_k$ by applying the modus ponens inference rule twice. □

LEMMA 3.4
(Lindenbaum).
Any consistent set of formulae can be extended to a maximal consistent set of formulae.

PROOF. The standard proof of Lindenbaum's lemma applies here [46, Proposition 2.14]. However, since the formulae in our logical system use real numbers in subscript, the set of formulae is uncountable. Thus, the proof of Lindenbaum's lemma in our case relies on the axiom of choice. □

We conclude this section by giving an example of a formal derivation in our logical system. This result is used later in the proof of the completeness.

LEMMA 3.5
If $C \subseteq D$, then $\vdash [C]_p\varphi \to [D]_p\varphi$.

PROOF. If $C = D$, then $\vdash [C]_p\varphi \to [D]_p\varphi$ because formula $[C]_p\varphi \to [D]_p\varphi$ is a propositional tautology.

Suppose now that $C \subsetneq D$. Thus, set $D \setminus C$ is not empty. Note that $\varphi \to \varphi$ is a propositional tautology. Thus, $\vdash [D \setminus C]_0(\varphi \to \varphi)$ by the necessitation inference rule. At the same time, because $(D \setminus C) \cap C = \varnothing$, the following formula is an instance of the cooperation axiom:

$$[D \setminus C]_0(\varphi \to \varphi) \to ([C]_p\varphi \to [(D \setminus C) \cup C]_{\max\{0,p\}}\varphi).$$

Hence, by the modus ponens inference rule,

$$\vdash [C]_p\varphi \to [(D \setminus C) \cup C]_{\max\{0,p\}}\varphi.$$

Then, $\vdash [C]_p\varphi \to [D]_p\varphi$, because $C \subseteq D$ and $0 \leq p$. □

3.3 Soundness

In this section, we prove the soundness of each of our axioms as a separate lemma. The soundness of our system is stated in the end of the section as Theorem 3.9.

LEMMA 3.6
For any game with stochastic failures (S, D, P_{ok}, M, π), any state $s \in S$, any coalition C, any formula $\varphi \in \Phi$ and any real numbers p, q such that $0 \leq q \leq p \leq 1$, if $s \Vdash [C]_p\varphi$, then $s \Vdash [C]_q\varphi$.

PROOF. By Definition 3.2, assumption $s \Vdash [C]_p\varphi$ implies that there is an action profile $\delta_1 \in D^C$ such that for any complete action profile $\delta' \in D^A$ if $\delta_1 \subseteq \delta'$, then

1. $P_{ok}(s, \delta') \geq p$ and
2. for any state $s' \in S$ if $(s, \delta', s') \in M$, then $s' \Vdash \varphi$.

Note that $P_{ok}(s, \delta') \geq p \geq q$ by assumption $q \leq p$ of the lemma. Therefore, $s \Vdash [C]_q\varphi$ by Definition 3.2. □

LEMMA 3.7
$s \nVdash [C]_p\bot$ for any state $s \in S$ of a game with stochastic failures (S, D, P_{ok}, M, π), any coalition C and any $p \in [0, 1]$.

PROOF. Suppose that $s \Vdash [C]_p\bot$. Thus, by Definition 3.2, there is an action profile $\delta \in D^C$ such that for any complete action profile $\delta' \in D^A$ if $\delta \subseteq \delta'$, then

1. $P_{ok}(s, \delta') \geq p$ and
2. for any state $s' \in S$ if $(s, \delta', s') \in M$, then $s' \Vdash \bot$.

Recall that set D is not empty by Item 2 of Definition 3.1. Let d_0 be an arbitrary element of this set. Consider the complete action profile

$$\delta'(a) = \begin{cases} \delta(a), & \text{if } a \in C, \\ d_0, & \text{otherwise.} \end{cases}$$

Then, $\delta \subseteq \delta'$. By Item 4 of Definition 3.1, there exists state $s' \in S$ such that $(s, \delta', s') \in M$. Thus, $s' \Vdash \bot$ by Item 2 above, which contradicts the definition of $\bot$ and Definition 3.2. □

LEMMA 3.8
For any game with stochastic failures (S, D, P_{ok}, M, π), any state $s \in S$, any coalitions C_1 and C_2, any formulae $\varphi, \psi \in \Phi$ and any real numbers $p, q \in [0, 1]$, if $s \Vdash [C_1]_p(\varphi \to \psi)$, $s \Vdash [C_2]_q\varphi$ and $C_1 \cap C_2 = \varnothing$, then $s \Vdash [C_1 \cup C_2]_{\max\{p,q\}}\psi$.

PROOF. By Definition 3.2, assumption $s \Vdash [C_1]_p(\varphi \to \psi)$ implies that there is an action profile $\delta_1 \in D^{C_1}$ such that for any complete action profile $\delta' \in D^A$, if $\delta_1 \subseteq \delta'$, then

1. $P_{ok}(s, \delta') \geq p$ and
2. for any state $s' \in S$, if $(s, \delta', s') \in M$, then $s' \Vdash \varphi \to \psi$.

Additionally, by Definition 3.2, assumption $s \Vdash [C_2]_q\varphi$ implies that there is an action profile $\delta_1 \in D^{C_2}$ such that for any complete action profile $\delta' \in D^A$ if $\delta_2 \subseteq \delta'$, then

1. $P_{ok}(s, \delta') \geq q$ and
2. for any state $s' \in S$ if $(s, \delta', s') \in M$, then $s' \Vdash \varphi$.

Let strategy profile δ of coalitions $C_1 \cup C_2$ be defined as

$$\delta(a) = \begin{cases} \delta_1(a), & \text{if } a \in C_1, \\ \delta_2(a), & \text{if } a \in C_2. \end{cases} \tag{7}$$

Strategy profile δ is well defined because coalitions C_1 and C_2 are disjoint by an assumption of the lemma.

Consider an arbitrary complete strategy profile δ' such that $\delta \subseteq \delta'$. Note that

$$\delta_1 \subseteq \delta \subseteq \delta', \tag{8}$$

$$\delta_2 \subseteq \delta \subseteq \delta' \tag{9}$$

by equation (7) and the assumption $\delta \subseteq \delta'$. By Definition 3.2, it suffices to show that

1. $P_{ok}(s,\delta') \geq \max\{p,q\}$ and
2. for any state $s' \in S$ if $(s,\delta',s') \in M$, then $s' \Vdash \psi$.

First we show that $P_{ok}(s,\delta') \geq \max\{p,q\}$. Indeed, $P_{ok}(s,\delta') \geq p$ by the choice of action profile δ_1 and due to equation (8). Similarly, $P_{ok}(s,\delta') \geq q$ by the choice of action profile δ_2 and equation (9). Thus, $P_{ok}(s,\delta') \geq \max\{p,q\}$.

Finally, consider any state s' such that $(s,\delta',s') \in M$. We will show that $s' \Vdash \psi$. Note that $s' \Vdash \varphi \to \psi$ by the choice of action profile δ_1 and equation (8). Similarly, $s' \Vdash \varphi$ by the choice of action profile δ_2 and equation (9). Therefore, $s' \Vdash \psi$ by Definition 3.2. □

The strong soundness theorem for our logical system with respect to the semantics described above follows from Lemma 3.8, Lemma 3.6 and Lemma 3.7.

THEOREM 3.9
For any state $s \in S$ of a game (S,D,P_{ok},M,π) with stochastic failures, if $X \vdash \varphi$, and $s \Vdash \chi$ for each formula $\chi \in X$, then $s \Vdash \varphi$.

3.4 Completeness

In this section, we prove strong completeness of our logical system with respect to the semantics of games with stochastic failures. This result is stated later in this section as Theorem 3.20. We start the proof by defining the canonical game with stochastic failures (S,D,P_{ok},M,π).

DEFINITION 3.10
S is the set of all maximal consistent sets of formulae.

DEFINITION 3.11
D is the set of all pairs (φ,p) where $\varphi \in \Phi$ and p is an arbitrary real number.

Informally, by choosing the action (φ,p), the agent is requesting the system to survive with probability at least p and formula φ to be true at the next state. The canonical action aggregation mechanism, which we define later, might grant or ignore this request. In particular, the mechanism ignores the request if $p \notin [0,1]$.

Next we define the probability $P_{ok}(s,\delta)$ of the system to survive in state s under action profile δ. For each $[C]_p\varphi \in s$, we want the system to survive with probability at least p if all members of coalition C choose action (φ, p). Thus, we define $P_{ok}(s,\delta)$ to be the maximum among such p. In the definition below, we assume that the maximum of the empty set is equal to 0.

Definition 3.12
$P_{ok}(s,\delta) = \max\{p \mid [C]_p\varphi \in s, \forall a \in C(\delta(a) = (\varphi, p))\}$.

Lemma 3.13
For each state $s \in S$ and each profile $\delta \in D^A$, value $P_{ok}(s,\delta)$ is well defined and $P_{ok}(s,\delta) \in [0,1]$.

Proof. Consider set $X = \{p \mid [C]_p\varphi \in s, \forall a \in C(\delta(a) = (\varphi, p))\}$. Note that $X \subseteq [0,1]$ by Definition 2.1. To prove that value $P_{ok}(s,\delta)$ is well defined by Definition 3.12, it suffices to show that set X is finite. Recall that set of all agents A is finite. Thus, set $\{p \mid \exists a \in A\, \exists \varphi \in \Phi\ (\delta(a) = (\varphi, p))\}$ is finite. Therefore, set X is finite because any coalition C in a formula $[C]_p\varphi \in \Phi$ is nonempty. $\square$

The canonical action aggregation mechanism M guarantees that if the system is in state s, $[C]_p\varphi \in s$, and all members of coalition C choose action (φ, p), then $\varphi \in s'$, where s' is the next state of the system.

Definition 3.14
$(s,\delta,s') \in M$ if $\{\varphi \mid [C]_p\varphi \in s, \forall a \in C(\delta(a) = (\varphi, p))\} \subseteq s'$.

According to Definition 3.1, we need to prove that for each state $s \in S$ and each complete action profile $\delta \in D^A$, there is at least one state $s' \in S$ such that $(s,\delta,s') \in M$. We will show this in Lemma 3.17.

Definition 3.15
$\pi(v) = \{s \in S \mid v \in s\}$.

This concludes the definition of the canonical game with stochastic failures (S, D, P_{ok}, M, π).

The next lemma is the key lemma in the proof of the completeness. It shows that if $\neg[C]_p\varphi \in s$, then in state s coalition C has no strategy to achieve φ in the next state while surviving with probability at least p.

Lemma 3.16
For each state $s \in S$, each formula $\neg[C]_p\varphi \in s$, and each action profile $\delta \in D^C$, there is a complete action profile $\delta' \in D^A$ such that $\delta \subseteq \delta'$ and one of the following is true

1. $P_{ok}(s,\delta') < p$ or
2. there is a state $s' \in S$ where $(s,\delta',s') \in M$ and $\neg\varphi \in s'$.

Proof. Consider function $\delta' \in D^A$ such that

$$\delta'(a) = \begin{cases} \delta(a), & \text{if } a \in C, \\ (\top, -1), & \text{otherwise.} \end{cases} \tag{10}$$

Suppose that

$$P_{ok}(s,\delta') \geq p. \tag{11}$$

Consider set

$$X_0 = \{\neg\varphi\} \cup \{\psi \mid [B]_q\psi \in s, \forall a \in B(\delta'(a) = (\psi, q))\}.$$

First, we prove that set X_0 is consistent. Suppose the opposite, thus there must exist formulae $[B_1]_{q_1}\psi_1, \dots, [B_n]_{q_n}\psi_n \in s$ such that

$$\forall i \le n\ \forall a \in B_i\ (\delta'(a) = (\psi_i, q_i)) \tag{12}$$

$$\psi_1, \dots, \psi_n \vdash \varphi. \tag{13}$$

Without loss of generality, we can assume that formulae $\psi_1, \dots, \psi_n$ are distinct. Note that sets $B_1, \dots, B_n$ are pairwise disjoint because of statement (12). Due to Definition 3.12,

$$q_1, \dots, q_n \le P_{ok}(s, \delta'). \tag{14}$$

Case 1: $n = 0$. Thus, $P_{ok}(s, \delta') = 0$ by Definition 3.12. Hence, $p = 0$ by assumption (11) and because $p \ge 0$ by Definition 2.1. Note also that $\vdash \varphi$ by statement (13) because $n = 0$. Thus, $\vdash [C]_p\varphi$ by the necessitation inference rule and equation $p = 0$. Then, $[C]_p\varphi \in s$ by the maximality of set s. Hence, $\neg[C]_p\varphi \notin s$ because set s is consistent. This contradicts the assumption of the lemma.
Case 2: $n > 0$. Then, by Definition 3.12, we can suppose that there is an integer m such that $1 \le m \le n$ and

$$q_m = P_{ok}(s, \delta'). \tag{15}$$

Furthermore, we can assume that there is $n' \le n$ such that $B_i \subseteq C$ for each $i \le n'$ and $B_i \nsubseteq C$ for each $i > n'$.

Let us first show that $m \le n'$. Indeed, suppose that there is an agent $a_0 \in B_m \setminus C$. Thus, $\delta'(a_0) = (\top, -1)$ by equation (10). Hence, $q_m = -1$ due to equation (12). Thus, $P_{ok}(s, \delta') = -1$ by equation (15), which contradicts Lemma 3.13. Therefore, $m \le n'$.

Next, note that for each $i > n'$ we have $\psi_i = \top$ because $B_i \nsubseteq C$ and due to equality (10) and equality (12). Hence, $\psi_1, \dots, \psi_{n'} \vdash \varphi$ by statement (13). By Lemma 3.3 applied n' times,

$$\vdash \psi_1 \to (\psi_2 \to \dots (\psi_{n'} \to \varphi) \dots).$$

Note that $n' \ne 0$ because $1 \le m \le n'$. So, by the monotonicity inference rule,

$$\vdash [B_1]_{q_1}\psi_1 \to [B_1]_{q_1}(\psi_2 \to \dots (\psi_{n'} \to \varphi) \dots)).$$

By the modus ponens inference rule,

$$[B_1]_{q_1}\psi_1 \vdash [B_1]_{q_1}(\psi_2 \to \dots (\psi_{n'} \to \varphi) \dots)).$$

By the cooperation axiom and the modus ponens rule,

$$[B_1]_{q_1}\psi_1, [B_2]_{q_2}\psi_2 \vdash [B_1 \cup B_2]_{\max\{q_1, q_2\}}(\psi_3 \to \dots (\psi_{n'} \to \varphi) \dots)).$$

By repeating the previous step $n - 2$ more times,

$$[B_1]_{q_1}\psi_1, \dots, [B_{n'}]_{q_{n'}}\psi_{n'} \vdash [B_1 \cup \dots \cup B_{n'}]_{\max\{q_1, \dots, q_{n'}\}}\varphi.$$

Thus, by the choice of formulae $[B_1]_{q_1}\psi_1, \dots, [B_{n'}]_{q_{n'}}\psi_{n'}$,

$$s \vdash [B_1 \cup \dots \cup B_{n'}]_{\max\{q_1, \dots, q_{n'}\}}\varphi.$$

Then, by Lemma 3.5 and because $B_1, \dots, B_{n'} \subseteq C$,

$$s \vdash [C]_{\max\{q_1,\dots,q_{n'}\}}\varphi.$$

Recall that $m \leq n'$. Thus, $\max\{q_1, \dots, q_{n'}\} = P_{ok}(s, \delta')$ by inequality (14) and equation (15). Hence, $s \vdash [C]_{P_{ok}(s,\delta')}\varphi$. Thus, $s \vdash [C]_p\varphi$ by the monotonicity axiom and assumption (11). Then, $\neg[C]_p\varphi \notin s$ due to consistency of set s, which contradicts the assumption of the lemma. Therefore, set X_0 is consistent. By Lemma 3.4, there is a maximal consistent extension s' of set X_0. Note that $\neg\varphi \in s'$ by the choice of set X_0.

To show that $(s, \delta', s') \in M$, consider any formula $[B]_q\psi \in s$ such that $\delta'(a) = (\psi, q)$ for each $a \in B$. By Definition 3.14, it suffices to show that $\psi \in s'$, which is true due to the choice of set X_0. □

The next lemma establishes that the defined earlier canonical game with stochastic failures satisfies the property from Item 4 of Definition 3.1.

LEMMA 3.17
For each state $s \in S$ and each complete action profile $\delta \in D^A$, there is a state $s' \in S$ such that $(s, \delta, s') \in M$.

PROOF. Consider an arbitrary state $s \in S$ and an arbitrary complete action profile $\delta \in D^A$. By the unachievability of falsehood axiom, $\vdash \neg[A]_{P_{ok}(s,\delta)/2}\bot$. Thus, $\neg[A]_{P_{ok}(s,\delta)/2}\bot \in s$ due to the maximality of set s. Hence, by Lemma 3.16, there exists $\delta' \in D^A$ such that $\delta =_A \delta'$ and one of the following is true

1. $P_{ok}(s, \delta') < P_{ok}(s, \delta)/2$ or
2. there is $s' \in S$ where $(s, \delta', s') \in M$ and $\neg\bot \in s'$.

Condition $\delta =_A \delta'$ implies that $\delta = \delta'$ because A is the set of all agents. Thus, statement $P_{ok}(s, \delta') < P_{ok}(s, \delta)/2$ is false because, by Definition 3.1, the value $P_{ok}(s, \delta')$ is non-negative. Then, condition 1 above is false. Hence, Condition 2 is true. Therefore, there exists $s' \in S$ such that $(s, \delta, s') \in M$. □

The following lemma shows that if $[C]_p\varphi \in s$, then in state s coalition C has a strategy that guarantees that φ will be achieved in the next state and that the system will survive with probability at least p.

LEMMA 3.18
For any state $s \in S$ and any formula $[C]_p\varphi \in s$, there is an action profile $\delta \in D^C$ of coalition C such that for any complete action profile $\delta' \in D^A$ if $\delta \subseteq \delta'$, then

1. $P_{ok}(s, \delta') \geq p$ and
2. for any state $s' \in S$, if $(s, \delta', s') \in M$, then $\varphi \in s'$.

PROOF. Consider any state $s \in S$ and any formula $[C]_p\varphi$. Let action profile $\delta \in D^C$ be defined as following: $\delta(a) = (\varphi, p)$ for each agent $a \in C$.

Let $s' \in S$ be a state and $\delta' \in D^A$ be a complete action profile such that $\delta \subseteq \delta'$ and $(s, \delta', s') \in M$. Note that $\delta'(a) = \delta(a) = (\varphi, p)$ for each agent $a \in C$ by the choice of action profile δ. Thus, $P_{ok}(s, \delta') \geq p$ by Definition 3.12. Also, $\varphi \in s'$ by Definition 3.14 and due to the assumption $[C]_p\varphi \in s$ of the lemma. □

The next lemma is the standard induction lemma in the proof of completeness. It brings together the results established in Lemma 3.16 and Lemma 3.18.

LEMMA 3.19
$\varphi \in s$ iff $s \Vdash \varphi$ for any formula $\varphi \in \Psi$ and any maximal consistent set $s \in S$.

PROOF. We prove the lemma by structural induction on formula φ. The case when formula φ is a propositional variable follows from Definition 3.15 and Definition 3.2. The case when formula φ is a negation or an implication follows from Definition 3.2 and the maximality and the consistency of set s in the standard way. Let us now suppose that formula φ has the form $[C]_p\psi$.
$(\Rightarrow)$: assume that $[C]_p\psi \in s$. Thus, by Lemma 3.18, there is an action profile $\delta \in D^C$ of coalition C such that for any complete action profile $\delta' \in D^A$ if $\delta \subseteq \delta'$, then

1. $P_{ok}(s,\delta') \geq p$ and
2. for any state $s' \in S$, if $(s,\delta',s') \in M$, then $\varphi \in s'$.

Note that statement $\psi \in s'$ is equivalent to $s' \Vdash \psi$ by the induction hypothesis. Therefore, $s \Vdash [C]_p\psi$ by Definition 3.2.
$(\Leftarrow)$: suppose that $s \Vdash [C]_p\psi$. Thus, by Definition 3.2, there is an action profile $\delta_0 \in D^C$ such that for any complete action profile $\delta' \in D^A$ if $\delta_0 \subseteq \delta'$, then

1. $P_{ok}(s,\delta') \geq p$ and
2. for any state $s' \in S$ if $(s,\delta',s') \in M$, then $s' \Vdash \psi$.

Assume that $[C]_p\psi \notin s$. Thus, $\neg[C]_p\psi \in s$ due to the maximality of set s. Hence, by Lemma 3.16, there is $\delta'_0 \in D^A$ such that $\delta_0 \subseteq \delta'_0$ and one of the following is true:

1. $P_{ok}(s,\delta'_0) < p$ or
2. there is $s' \in S$ such that $(s,\delta'_0,s') \in M$ and $\neg\psi \in s'$.

Note that Statement 3 can not be true due to Statement 1 above. Thus, there is $s' \in S$ where $(s,\delta'_0,s') \in M$ and $\neg\psi \in s'$. Hence, $\psi \notin s$ due to the consistency of set s'. Thus, $s' \nVdash \psi$ by the induction hypothesis, which contradicts to Statement 2 above. □

We are now ready to state and to prove the strong completeness for our logical system.

THEOREM 3.20
If $X \subseteq \Phi$, $\varphi \in \Phi$ and $X \nvdash \varphi$, then there is a state s of a game with stochastic failures such that $s \Vdash \chi$ for each $\chi \in X$ and $s \nVdash \varphi$.

PROOF. Let (S,D,P_{ok},M,π) be the canonical game with stochastic failures. Suppose that $X \nvdash \varphi$. Hence, set $X \cup \{\neg\varphi\}$ is consistent. By Lemma 3.4, there is a maximal consistent extension $s \subseteq \Phi$ of set $X \cup \{\neg\varphi\}$. Then, $s \in S$ by Definition 3.10. Note that $\varphi \notin s$ due to the consistency of set s. Also, $\chi \in s$ for each $\chi \in X$ because $X \subseteq s$. Therefore, $s \Vdash \chi$ for each $\chi \in X$ and $s \nVdash \varphi$ by Lemma 3.19. □

3.5 *Incompleteness in language $\Phi_\varnothing$*

Recall that in any formula $[C]_p\varphi \in \Phi$, set C is a coalition. We have defined coalitions to be nonempty sets of agents. One can consider an extension of our language by allowing coalitions to be empty. We will refer to such an extension as language $\Phi_\varnothing$. The semantics of language $\Phi_\varnothing$ is still specified by Definition 3.2. Informally, $s \Vdash [\varnothing]_p\varphi \in \Phi$ means that under any complete action profile δ', the game will not fail with probability at least p and statement φ is guaranteed to be true in the next state. All axioms and inference rules of our logical system stated in language $\Phi_\varnothing$ are sound with respect to the semantics of games with stochastic failures.

In this section, we prove that no strongly sound logical system in language $\Phi_\varnothing$ is strongly complete with respect to the semantics of stochastic failures. This result is formally stated as Theorem 3.25. We start by reminding the definitions of strong soundness and strong completeness.

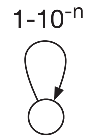

Figure 1 Game with stochastic failure G_1.

DEFINITION 3.21
A logical system $\mathcal{L}$ is strongly sound when for each set of formulae X, each formula φ and each state s of an arbitrary game with stochastic failures, if $s \Vdash \chi$ for each formula $\chi \in X$ and $X \vdash_{\mathcal{L}} \varphi$, then $s \Vdash \varphi$.

DEFINITION 3.22
A logical system $\mathcal{L}$ is strongly complete if for each set of formulae X and each formula φ such that $X \nvdash_{\mathcal{L}} \varphi$, there is a state s of a game with stochastic failures such that $s \Vdash \chi$ for each formula $\chi \in X$ and $s \nVdash \varphi$.

Next we prove our incompleteness result, which is stated below as Theorem 3.25. We start by defining set X as the following infinite subset of $\Phi_{\varnothing}$:

$$X = \{[\varnothing]_{1-10^{-n}}\top \mid n \geq 0\} = \{[\varnothing]_0\top, [\varnothing]_{0.9}\top, [\varnothing]_{0.99}\top, \ldots\}.$$

LEMMA 3.23
For any state $s \in S$ of any game with stochastic failures, if $s \Vdash \chi$ for each $\chi \in X$, then $s \Vdash [\varnothing]_1\top$.

PROOF. Suppose that $s \nVdash [\varnothing]_1\top$. Let $\delta_{\varnothing}$ be the unique strategy profile of the empty coalition. Thus, by Definition 3.2, there exists a complete action profile $\delta' \in D^A$ and such that $\delta \subseteq \delta'$ and either $P_{ok}(s, \delta') < 1$ or there is a state $s' \in S$ such that $(s, \delta', s') \in M$ and $s' \nVdash \top$. Hence, since formula $\top$ is satisfied in each state of each model, $P_{ok}(s, \delta') < 1$. Choose any integer $n > 0$ such that

$$P_{ok}(s, \delta') < 1 - 10^{-n}. \tag{16}$$

By the assumption of the lemma, $s \Vdash [\varnothing]_{1-10^{-n}}\top$. Thus, $P_{ok}(s, \delta') \geq 1 - 10^{-n}$ by Definition 3.2 and because $\delta_{\varnothing} \subseteq \delta'$, which contradicts inequality (16). □

LEMMA 3.24
$X \nvdash_{\mathcal{L}} [\varnothing]_1\top$ for any logical system $\mathcal{L}$ strongly sound with respect to the semantics of the games with stochastic failures.

PROOF. Suppose that $X \vdash_{\mathcal{L}} [\varnothing]_1\top$. Hence, since any proof is using only finitely many assumptions, there must exists an integer $n \geq 0$ such that

$$[\varnothing]_0\top, [\varnothing]_{0.9}\top, [\varnothing]_{0.99}\top, \ldots, [\varnothing]_{1-10^{-n}}\top \vdash_{\mathcal{L}} [\varnothing]_1\top. \tag{17}$$

Consider now game with stochastic failure G_1 depicted in Figure 1. This game has only one state. Under any strategy profile, with probability $1 - 10^{-n}$ the game loops back into the same state and with probability 10^{-n} the game fails. By Item 4 of Definition 3.2, all assumptions in statement (17) hold in the unique state of this game while the conclusion does not. Therefore, logical system $\mathcal{L}$ is not strongly sound. □

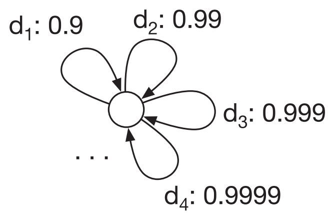

FIGURE 2 Game with stochastic failure G_2 with a single state s.

THEOREM 3.25
Any strongly sound logical system in language $\Phi_\varnothing$ is not strongly complete with respect to the semantics of games with stochastic failures.

PROOF. Consider any strongly complete system $\mathcal{L}$. Thus, $X \vdash_{\mathcal{L}} [\varnothing]_1\top$, by Definition 3.22 and Lemma 3.23. Therefore, system $\mathcal{L}$ is not strongly sound by Definition 3.21 and Lemma 3.24. □

4 Discussion

In Theorem 3.20, we have shown that our logical system is *strongly complete* if the language *does not* include the empty coalition. In Theorem 3.25, we have shown that our logical system is *strongly incomplete* if the language *does* include the empty coalition. It is natural to ask where the proof of Theorem 3.20 fails if we consider the empty coalition and where the proof of Theorem 3.25 fails if instead of the empty coalition we consider a nonempty one.

In case of Theorem 3.20, it is Definition 3.12. Indeed, if coalitions could be empty, then it is possible for the set s to include formulae $[\varnothing]_{0.9}\top$, $[\varnothing]_{0.99}\top$, $[\varnothing]_{0.999}\top$, ... In this case, the set

$$\{p \mid [C]_p\varphi \in s, \forall a \in C(\delta(a) = (\varphi, p))\} \tag{18}$$

has no maximal value and, thus, the value of $P_{ok}(s, \delta)$ is undefined.

In case of Theorem 3.25, it is Lemma 3.23. Namely, this lemma is false if $\varnothing$ in the definition of the set X and in the statement of Lemma 3.23 is replaced with the same nonempty coalition C. Indeed, assume, without loss of generality, that coalition C is a singleton set $\{a\}$ and consider game with stochastic failures G_2 depicted in Figure 2.

Agent a in this game has infinitely many actions $d_1, d_2, d_3, \dots$. Action d_n leads to failure with probability 10^{-n}. In other words, $P_{ok}(s, d_n) = 1 - 10^{-n}$, where s is the only state of this game. Note that $s \Vdash \chi$ for each $\chi \in X$ but $s \nVdash [\varnothing]_1\top$. Therefore, Lemma 3.23 becomes false if $\varnothing$ is replaced with coalition $\{a\}$.

As pointed out above, our canonical game construction fails for the empty coalition because if coalition C is empty, then the set (18) might have no maximal value. In other words, the canonical game construction imposes certain constraints through the actions of the coalition members. If the coalition is empty, then the constraints cannot be easily enforced. It is interesting to note that such a situation happens in at least two other works on coalition logics. Goranko *et al.* [36, p. 11] point out that Pauly's representation theorem fails because 'there is no guarantee that any i [in coalition C] will indeed choose f_i as its strategy ... since the coalition C ... does not include any players'. Similarly, Naumov and Tao [52] developed 'harmony" construction to overcome the fact that they cannot control the actions of the empty coalition.

5 Conclusion

In this article, we explored two different approaches to adding probabilities to coalition logic. The first approach captures the properties of games with stochastic transitions, and the second approach captures the properties of games with stochastic failures. We showed that the logical system under the first approach contains non-trivial properties that go far beyond the axioms of original coalition logic. It also turned out that the situation under the second approach depends on the presence of the empty coalition in the language. We gave a strongly sound and strongly complete logical system for the language without empty coalition and prove that such a system does not exist for the language with the empty coalition.

An important question about decidability of the proposed logical system is left unanswered. The standard technique to prove decidability of a complete logical system is to show completeness with respect to finite models using filtration on subformulae. Unfortunately, this technique fails in the current setting because the proof of Lemma 3.17 forces us to consider formulae $\neg[A]_p\bot$ with arbitrary small values of p. The same issue also prevents us from proving weak completeness for our logical system in language $\Phi_\varnothing$. Both of these questions remain open for future research.

Acknowledgements

The authors are thankful to Italo Siminelli for pointing out that Lemma 2.6 follows from a more general theorem of Alfréd Rényi.

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Received 27 July 2020