CHARGED PARTICLE INTERACTIONS

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REFERENCES:[1]

INTRODUCTION

Gamma-rays, x-rays, neutrons, and neutrinos all have no net charge - they are electrostatically neutral. In order to detect them they must interact with matter and produce an energetic charged particle. In the case of gamma and x-rays, a photo-electron is produced. In the case of neutrons, a proton is given kinetic energy in a billiard ball collision. So we begin our discussion of charged particle interactions by demonstrating that even when detecting neutral particles we must quickly think in terms of the charged particles.

Charged particles can be divided into two families: 1) electrons, including positrons and beta particles from radioactive decay, Auger electrons, and internal conversion electrons, and; 2) heavy charged particles such as the alpha particle, fission fragments, protons, deuterons, tritons, and mu and pi mesons.

The charged particles interact with matter primarily through the Coulomb force. Differences in the energy deposition trails between the two types of charged particles are due to the mass differences, i.e., the more easily altered trajectory of the electron.

The maximum energy that can be transferred from a charged particle to an electron in a single collision is given by 4 T m(e)/m, for about 1/500 of the particle energy per nucleon for heavy particles. Due to this small fraction and the fact that at any time the particle is interacting with more than one electron, the heavy particles appear to continuously slow down, gradually losing their kinetic energy along an unaltered linear path.

TYPES OF INTERACTIONS

When a charged particle gives sufficient energy to an orbital electron such that the freed electron itself can cause secondary ionization, we say that a delta-ray has been produced. The majority of the energy loss of a heavy charged particle such as the alpha occurs via these delta-rays.
We will concern ourselves primarily with two types of interactions: collisional and radiative.

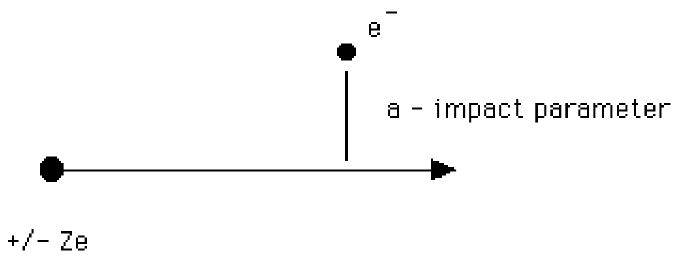
1. TYPE 1, COLLISIONAL. Inelastic collision with atomic electrons. This results in excitation or ionization. These processes ultimately end with the heating of the absorber (through atomic and molecular vibrations) unless the ions and electrons can be separated using an electric field as is done in radiation detectors.
2. TYPE 2, RADIATIVE. Inelastic collision with nucleus. A quantum of electromagnetic radiation is emitted (a photon). Energy loss is experienced by the particle. Important for electrons. Probability of nuclear excitation is negligible. This process is also known as **RADIATIVE** energy loss. The acceleration of electrons near a nucleus is known as beam braking or bremsstrahlung. Bremsstrahlung will be discussed further in the next section.

STOPPING POWER

The stopping of charged particles in matter is by collisional and radiative processes which occur in frequencies dictated by their interaction cross sections (or probabilities). What we observe then is a statistical average of the 2 processes occurring as the particle slows down. Linear Stopping Power is given by
Eqn 1: $S= - dT/dx$,

where T is the charged particle kinetic energy, -dT is the energy increment lost in infinitesimal material thickness of dx. The units of stopping power are keV/micron. The higher the stopping power, the shorter the range into the material the particle can penetrate. The quantity S is also referred to as *specific energy loss* .

The stopping power S increases as the particle velocity is decreased. The classical depiction of the charged particle interaction with an electron is depicted in the following drawing:



Classically, a passing charged particle will impart kinetic energy to the electron given by the following formula:

$$T_e = \frac{Z^2 e^4}{8\pi \varepsilon_o^2 a^2 m_o v^2}$$

The faster the particle is, the less energy is given to the electron (the less time it has to spend imparting energy to the electron).

The classical (i.e, non-Quantum Mechanical) expression that describes the specific energy loss is known as the Bethe-Bloch formula and is written (Eqn 2):

Bethe-Bloch Formula

FOR HEAVY CHARGED PARTICLES:

$$-\frac{dT}{dx} = \frac{4\pi e^4 z^2}{m_o v^2} NB$$

where

$$B \equiv Z\left[\ln\frac{2m_o v^2}{I} - \ln\left(1 - \frac{v^2}{c^2}\right) - \frac{v^2}{c^2}\right]$$

with the following definitions:

- v velocity of the charged particle
- ze charge of the charged particle
- N number density of absorber atoms
- Z atomic number of absorber atoms
- m electron rest mass
- e electron charge
- I A parameter, treated as experimentally determined, representing average excitation and ionization potential
- B is known as the stopping number (atomic number scaled for stoppping)

FOR ELECTRONS:

$$-\frac{dT}{dx} = \frac{2\pi e^4}{m_o v^2} NB$$

where

$$B \equiv Z\left[\ln\frac{m_o v^2 T}{2I^2(1-\beta^2)} - (\ln 2)\left(2\sqrt{1-\beta^2} - 1 + \beta^2\right) + \right.$$

$$\left. + 1 - \beta^2 + \frac{1}{8}\left(1 - \sqrt{1-\beta^2}\right)^2\right]$$

where $\beta = \frac{v}{c}$

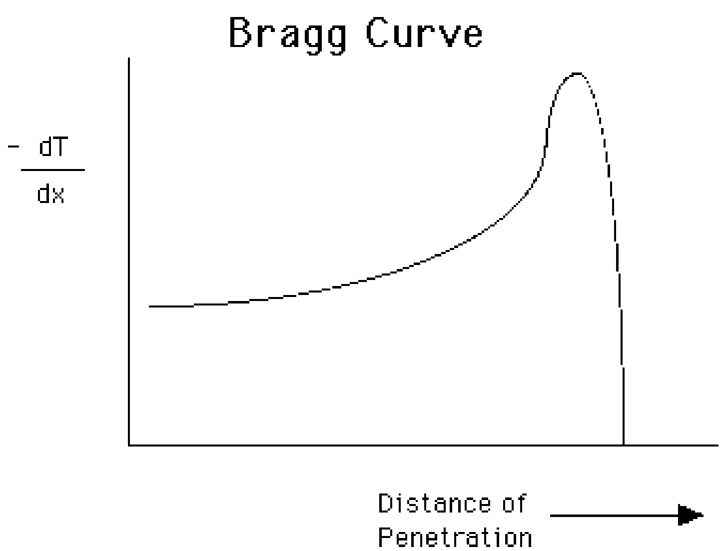
The first term in the stopping number B is sufficient if v< The ionization/excitation parameter I is treated as an experimentally determined quantity. The ratio of I/Z is approximately constant for absorbers with Z greater than 13. For smaller atoms the electrons play a more important role in influencing the value of I.

The Bethe-Bloch formula for electrons contains must take into account two facts: 1) that the electron has a much smaller mass than the heavy particles; and 2) that the electron is identical to the particles with which it is interacting, thus giving the possibility of the exchange of identity, i.e., the incoming particle has a probability of becoming the atomic electron with the atomic electron becoming the outgoing particle. Taking this into account, as well as relativity, gives us the Bethe-Bloch formula for electrons.

THE BRAGG CURVE

Heavy Particles

The plot of specifc energy loss (which can be related to specifc ionization) along the track of a charged particle is called a Bragg Curve. A typical Bragg curve is depicted in the following graphic for an alpha particle of several MeV of initial energy:



As the energy falls, the specific energy loss increases according to the Bethe-Bloch formula. As the energy falls below a threshold, however, an electron will attach to the alpha, dramatically lowering the specific energy loss.

ELECTRONS

The energy deposition of the electron increases more slowly with penetration depth due to the fact that its direction is changed so much more drastically (Johns and Cunningham, p. 213). In fact, these authors state: "There is no increase in energy deposited near the end of the tract and the Bragg peak for electrons is never observed." The explanation of Sorenson and Phelps, however, helps shed light on this topic. They state (p. 170) : "A similar increase in ionization density is seen at the end of an electron track; however, the peak occurs when the electron energy has been reduced to less than about 1 keV, and it accounts for only a small fraction of its total energy." Therefore it seems that for electrons and their tortuous paths the energy deposition is spread in the transverse direction as it progresses forward in the initial direction, until the last 1 keV which is deposited pretty much along a straight line in the forward direction.

Energy Straggling

Energy loss in a material is a statistical or stochastic process. Therefore, a spread of energies always results when an initially monoenergetic beam of particles encounters an absorber. Many particles lose the "average energy," although some will lose not so much and some will lose more than the average. This results in a finite width to the energy distribution curve known as "energy straggling."
The straggling peak is approximately Gaussian shaped, with a width that increases with the ratio Z/A. In other words, at lower atomic numbers (where the Z/A ratio is higher) we have wider widths. This is due to the fact that there is less shielding of inner electrons in the lower Z elements - there is more stopping influence per electron.

Mass Stopping Power

As with the mass attenuation coeficient, which is defined as mu/rho, where mu is the linear attenuation coefficient and rho is the absorber density, we can define a "Mass Stopping Power." The mass stopping power is given by Eqn 3:
(1/rho) * (dT/dx) in units of keV/(gm/cm**2)

Bremsstrahlung and Stopping Power

The total stopping power for electrons can be given as a combination of the collisional and radiative types of interactions:
Eqn 4: [dT/dx] total = [dT/dx]collision + [dT/dx] radiative

(for heavy particles we only consider orbital electron interactions (an atomic process) since the probability of nuclear interaction resulting in energy loss is much smaller.)

The radiative component calculated from electromagnetic and relativistic theory is given by Eqn 5:

$$-\left(\frac{dT}{dx}\right)_r = \frac{NTZ(Z+1)e^4}{137\, m_o^2 c^4}\left(4\ln\frac{2T}{m_o c^2} - \frac{4}{3}\right)$$

which is the basis for the approximation for the percentage of energy lost to radiation for monoenergetic electrons, Eqn. 6:

(dT/dx)r/ (dT/dx)total = EZ/1000 ; where E is in MeV.

, where Z is the atomic number of the absorber and E is the charged particle energy in MeV.

For example, a stream of 100 keV electrons hits a tungsten target (Z=74). Only 0.74% of the energy lost goes to emitted x-rays.

For polyenergetic beta rays, Eqn 6 becomes:

Eqn 7: (dT/dx)r/(dT/dx)total = Z E(max) / 3000

since the average beta energy is 1/3 that of the maximum beta energy.

The fraction of energy lost to radiation is inversely proportional to the charged particle's mass. Bremsstrahlung will therefore be very low for heavy charged particles (compared to electrons). Even at 100 MeV, for example, heavy particles dissipate most of their energy through the collisional process.

For mixtures of elements, one determines the "effective atomic number", Z(eff). This can be found by using the following formula, Eqn 8:

$$Z_{eff} = \frac{\sum_i f_i Z_i^2}{\sum_i f_i Z_i}$$, where f_i is the fraction by weight of the i th element.

RANGE OF CHARGED PARTICLES

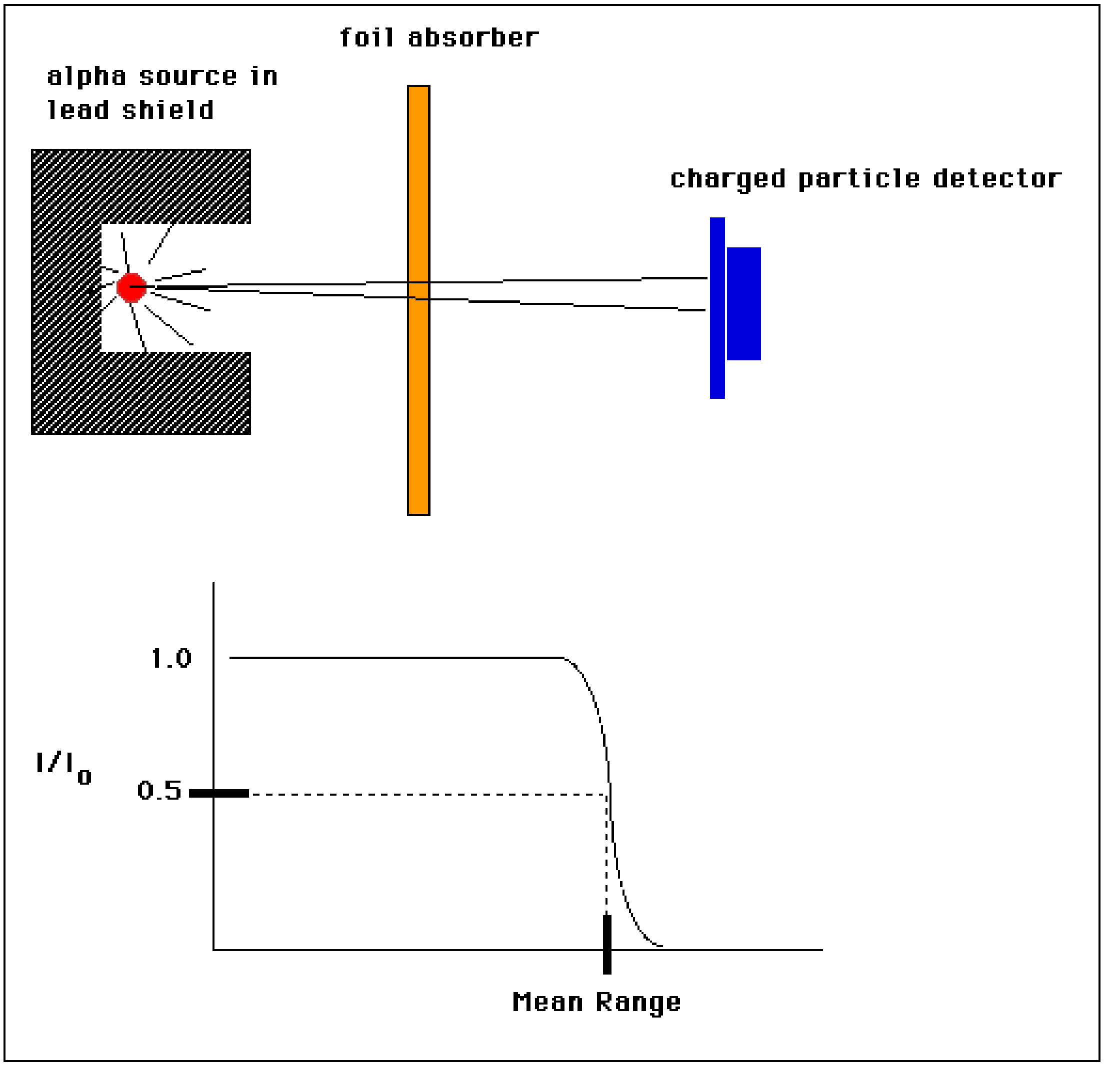
The range of a charged particle can be derived from the stopping power formula by using and gives Eqn 9:

$$R = \int_T^0 dx \ (cm) = \int_T^0 \frac{dT}{dT} dx = -\int_0^T \frac{1}{dT/dx} dT = \int_0^T \frac{dT}{S}$$

The summal distance elements as kinetic energy goes from T down to 0 is the total distance along the incident direction, or the RANGE.

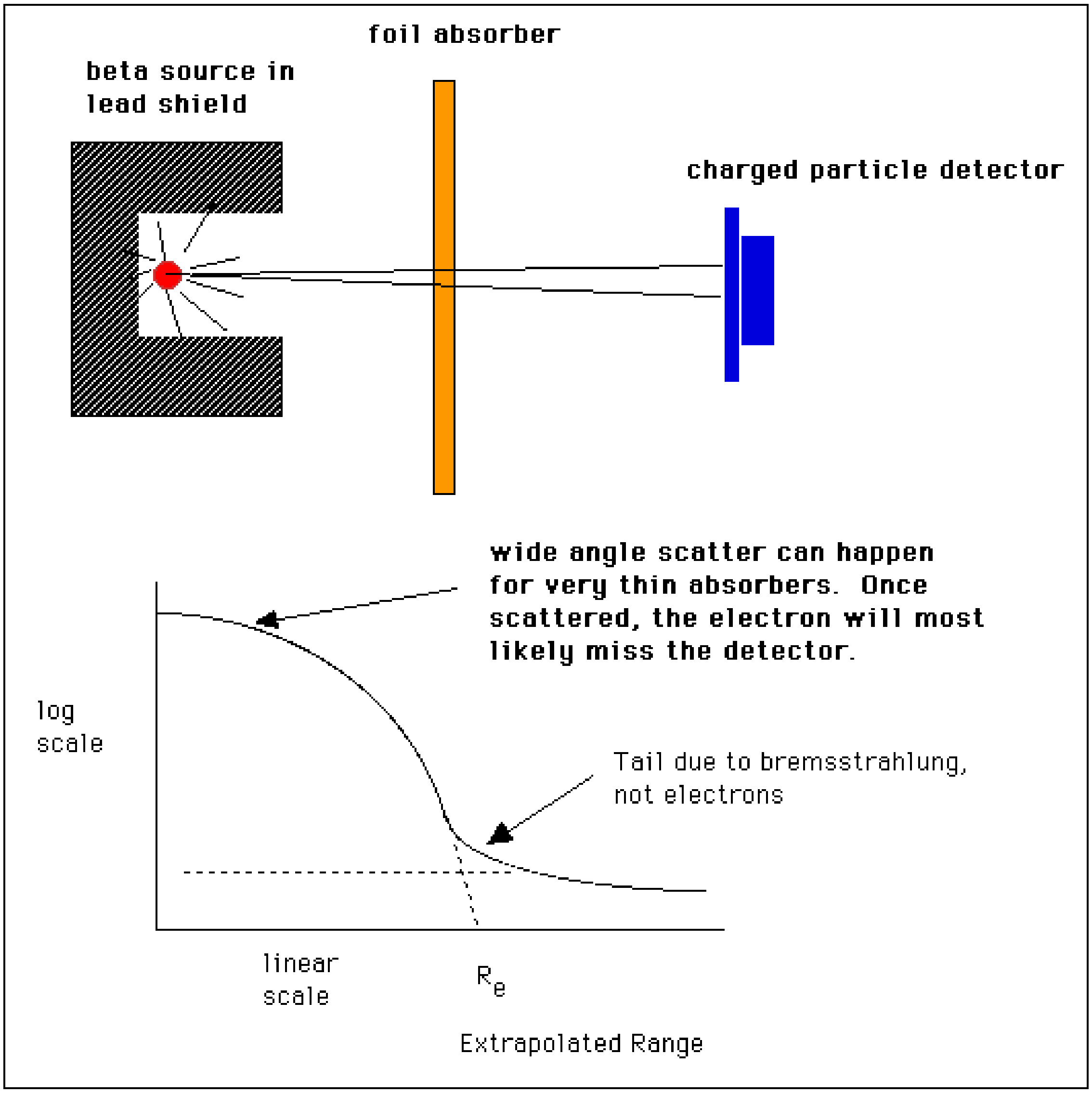
Examine the thought experiment involving alpha particle transmission depicted in the following drawing.

A source of monoenergetic alpha particles is directed at a particle detector through an absorber. We know the initially monoenergetic spectrum will demonstrate energy straggling, but the direction of the particles should not be altered and they should ALL make it to the detector, provided the thickness of the absorber is less than the thickness required to stop the slowest of the distribution. This is shown as the constant horizontal line in the graph at the bottom of the drawing. Once the absorber thickness is such that some of the slower particles are stopped, the width of the energy straggling curve presents itself in the slope of the fall-off from constant particle number detected down to zero.



The mean range is the range at which the number of particles detected is one-half the original value.

The same experiment can be performed with a beam of electrons (or a beta emitter). In this case, however, the electrons are more vulnerable to wide-angle scattering even in the thinnest of foils - so wide an angle that the scattered electrons will not be detected. The result is an immediate falloff in the number of electrons detected as a function of absorber thickness as demonstrated is this drawing:



Factors which affect range R:

Energy: Range is approximately linear with energy since the Bethe-Bloch equation for stopping power is inversely proportional to E.

Mass: for the same kinetic energy, the electron is much faster than the alpha due to its smaller mass, and therefore the electron has less time to spend near orbital electrons. This reduces the effect of Coulomb interactions (hence stopping power) and increases range.

Charge: the more charge, the more stopping power and the lower range. Range is inversely proportional to the square of the charge of the charged particle.

For example, a tritium particle with z=1 will have 1/4 the stopping power of a He-3 particle with z=2.

Density: The stopping power increases with increasing density. The range is inversely proportional to the density of the absorbing medium.

RANGE STRAGGLING

The same statistical factors which account for energy straggling result in a straggling of range - the total path of energy depletion is different for each initially monoenergetic alpha. For protons or alphas, straggling results in a few percent variation about the mean range. The sharper the cut-off in the transmission, the less pronounced the range straggling. Differentiating the cut-off curve often is used to demonstrate a range staggling curve analogous to the energy straggling curve.

SCALING LAWS

It is not possible to fund range experiments for all possible incident particles, all possible absorbing materials at all possible energies. In order to estimate the range or energy loss characteristics of a particular example we must assume the validity of the Bethe-Bloch formula and assume that the stopping power per atom of compounds or mixtures is additive. The latter assumption is known as the Bragg-Kleeman rule, and can be found in Knoll (1989, p. 43).
A formula from the Bragg-Kleeman rule can tell us the range of particles in a material for which we have no energy loss or range information if we are given range information in another material. The approximation becomes less valid as the separation in atomic weights increases. This rule is as follows (Eqn 10):

Bragg-Kleeman Rule:

$$\frac{R_1}{R_o} = \frac{\rho_o}{\rho_1}\frac{\sqrt{A_1}}{\sqrt{A_o}}$$

where r is the density and A is the atomic weight or effective atomic weight) of materials o and 1, respectively.

This rule follows from the fact that the range in cm (linear range) is inversely proportional to the density and we are taking a ratio of ranges - one known, one unknown. The atomic number part is the approximation and the guts of the Bragg-Kleeman rule.

The estimation of a particle's range in a given material is a two-step process:

1) determine the range in air; then

2) determine the range in the material given the Bragg-Kleeman rule .

The range of alpha particles in cm depends upon the material and the energy of the alphas. The first part takes care of the energy in air; the second part takes care of the difference in materials between air and the material of choice. **Determination of the range in air:**

In order to compute the range of alpha particles in any material, we must use an empirical equation for the range of alpha particles in air. The following equation for range in cm is valid for alphas in air in the energy range 4 < E < 15 MeV (Eqn 11):

R = (0.005E + 0.285) E**3/2 ; R in cm

(the above equation is taken from page 202 of Lapp and Andrews textbook.

Sorenson and Phelps (p. 173) use a simpler equation for the range 4 < E < 8 (Eqn. 12):

R = 0.325 E**3/2 ; R also in cm

It is helpful to note that the density of air is 0.001293 g/cm**3 and that the effective atomic number of air is about 14.

EXAMPLE(1):

The alpha particles of Americium-241 have energies of 5.49 MeV and 5.44 MeV. What is the range of these particles in air?

Using the above equation, we find R(cm) = 4.04 cm.

If the material is changed, use the scaling law as follows:

If medium o is air (sqrt A = 3.82), R(1) = 3.4 x 10**-4 R(air) (sqrt(A(1)/ rho(1)) ;

since 0.001293 /3.82 = 3.4 x 10**-4

. R(air) can be calculated using the above equations. Knowing more about the material in question will tell us its density and effective atomic mass number.

EXAMPLE(2):

For Americium-242 what is the range in tissue if the sqrt(A) = 3 and rho = 1.0 g/cm**3?

R(1) = (3.4 x 10**-4)* (3/1)*4.04 cm = 0.0039 cm

Range of Beta Particles

The range of beta particles, when expressed in terms of g/cm**2, is independent of the absorbing material and only depends upon energy. The square root factor of the Bragg-Kleeman rule is not applicable to the electron. This indicates that screening or the extent of the electron orbitals is less important for electrons than for heavy particles. Since the mass electron range depends only on energy, we can define it in terms of semi-empirical equations. Two semi-empirical equations are:

for the range 0.01 < E < 3.0 MeV (Eqn. 13),

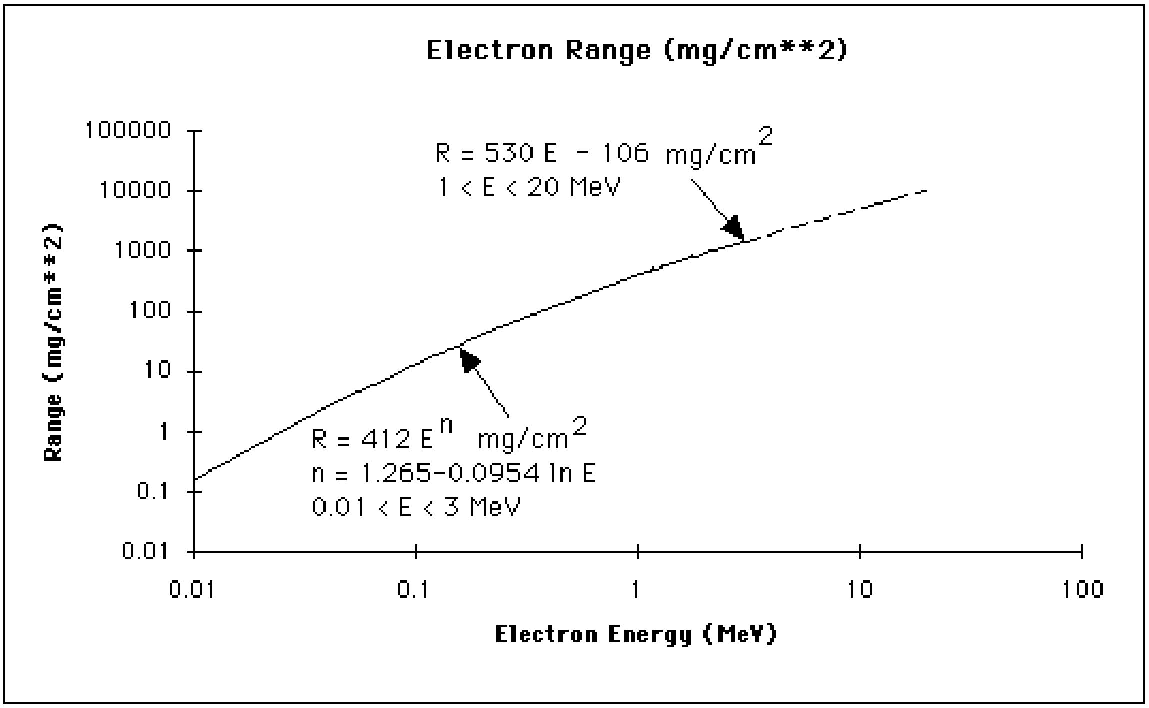
R = 412 E**n ; R in mg/cm**2 ; where

n = 1.265 - 0.0954 ln E

for the range 1 < E < 20 MeV (Eqn. 14),

R = 530 E - 106 ; R in mg/cm**2.

In order to demonstrate that these two empirical equations overlap and represent approximately the same electron range, we show the following graph:



EXAMPLE (1):

What is the maximum range of the principal Beta particle of I-131?

The maximum energy of the principal Beta particle for I-131 is 0.606 MeV. The range is then:

R = 412 (0.606)**n mg/cm**2;

where n = 1.265 - 0.0954 ln (0.606)

plugging in and calculating, we find that R = 0.345 g/cm**2. If the material was water or tissue the range would be 0.345 cm.

EXAMPLE(2):

The following table gives an indication of the order of magnitude of the ranges of monoenergetic alphas and electrons in both soft tissue as well as air:

	Soft Tissue		Air	
	β	α	β	α
100 keV	0.02 cm	1.4 µ	16 cm	0.1 cm
1 MeV	0.4 cm	7.2 µ	330 cm	0.5 cm

LINEAR ENERGY TRANSFER

The Linear Energy Transfer (LET) is similar to the stopping power except that it does not include the effects of radiative energy loss (i.e., Bremsstrahlung) or delta-rays. The difference between LET and S is that LET is local energy deposition only and S is concerned with the total energy lost by the particle. S and LET are nearly equal for heavy charged particles; for betas LET does not include delta-rays nor Bremsstrahlung.
The LET is related to Biologiical Damage. The severity and permenance of biological chages are directly related to the local rate of energy deposition along the particle track. The higher the LET, the higher the Q- quality factor in determining dose equivalent (Severts, where 1 Sv = 100 rem).

POSITRONS

Positrons behave nearly the same as electrons as they lose their energy, they are simply deflected in the opposite direction. As far as imaging is concerned, the 511 keV annihilation photons resulting from positrons come from the point at which the positron as slowed enough to engage in a prolonged encounter with an electron, i.e. it forms the unstable "positronium". This places an isotope-dependent limit on spatial resolution in PET imaging, since each positron emitter has a different E(max) for positrons and hence a different range. The other consideration is the center-of-mass momentum of the positronium, which contributes to non-opposition of the annihilation photons in the laboratory frame of reference.

CHERENKOV EFFECT

When a charged particle is emitted in radioactive decay with velocity greater than c/n (the speed of light in the medium), where n is the index of refraction of the medium, a "shock wave" of photons is generated in the blue light range, analogous to the sonic boom of a jet flying faster than the speed of sound. This is applicable in nuclear medicine in the assaying of P-32, which in a water solution has a detectable (by liquid scintillator apparatus) Cherenkov emission.

ATTENDANCE, October 26, 1994

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1. http://www.med.harvard.edu/JPNM/physics/didactics/physics/charged/refs.html