Lab 3 - Computation of receiver position from code and phase measurements

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1 Introduction

The task is to compute the coordinates of receiver ROV at given time using P1-code and L1 phase observations from two GPS receivers: REF and ROV. The receiver REF has known coordinates. The calculations were done with observation files and other values given in the Excel document *DiffAssignmentL1.xls*. Unlike the last lab, where the satellite and receiver clocks were not synchronized, this lab synchronizes clocks of all the satellites to the system time.

2 Methodology

Matlab was used to solve for the following parameters and all of the following equations were taken from the technical report: Relative positioning with GPS.[1]

The steps and equations from this algorithm are documented in the Matlab code, Section 5. The pseudo ranges and phase observations for REF and ROV are given from the beginning in this lab. Two GPS receivers measured the pseudoranges to each satellite at the same time. Of the two points, we used point A as REF and B as ROV. The first step is to synchronize all the clocks for each satellite so we can combine the results. The equation used to synchronize the clocks,

$$\begin{aligned} P_A^s &= P_A^s(t_A) = P_A^s(\tilde{t}_A) + \dot{\rho}_A^s(\tilde{t}_A)\delta\hat{t}_A \\ \lambda\varphi_A^s &= \lambda\varphi_A^s(t_A) = \lambda\varphi_A^s(\tilde{t}_A) + \dot{\rho}_A^s(\tilde{t}_A)\delta\hat{t}_A \end{aligned}$$

where A is REF or ROV. Next, the ionospheric and tropospheric effects are eliminated common to both stations. The single differences are,

$$\begin{aligned} \lambda\varphi_{AB}^s &= \lambda\varphi_B^s - \lambda\varphi_A^s = \rho_{AB}^s + c\delta t_{AB} + \lambda N_{AB}^s \\ P_{AB}^s &= P_B^s - P_A^s = \rho_{AB}^s + c\delta t_{AB} \end{aligned}$$

which are then taken to form double differences and leads to the most important feature of this method: cancellation of the receiver clock errors. The double difference equations are computed by choosing a reference satellite and subtracting the phase and code observations of each satellite from it. Satellite 20 was chosen to be the reference in this lab. Double differences from reference satellite 20 is given from,

$$\lambda\varphi_{AB}^{st} = \lambda\varphi_B^t - \lambda\varphi_A^s = \rho_{AB}^{st} + \lambda N_{AB}^{st} \quad (1)$$

$$P_{AB}^{st} = P_B^t - P_A^s = \rho_{AB}^{st} \quad (2)$$

Linearizing Equation (1) and (2) and converting to matrix form, we get

$$
\begin{aligned}
L &= Ax \\
L &= \begin{bmatrix}
\lambda\varphi_{AB}^{20,4} - \rho_{AB,0}^{20,4} \\
\lambda\varphi_{AB}^{20,5} - \rho_{AB,0}^{20,5} \\
\lambda\varphi_{AB}^{20,6} - \rho_{AB,0}^{20,6} \\
\lambda\varphi_{AB}^{20,24} - \rho_{AB,0}^{20,24} \\
\lambda\varphi_{AB}^{20,25} - \rho_{AB,0}^{20,25} \\
P_{AB,1}^{20,4} - \rho_{AB,0}^{20,4} \\
P_{AB,1}^{20,5} - \rho_{AB,0}^{20,5} \\
P_{AB,1}^{20,6} - \rho_{AB,0}^{20,6} \\
P_{AB,1}^{20,24} - \rho_{AB,0}^{20,24} \\
P_{AB,1}^{20,25} - \rho_{AB,0}^{20,25}
\end{bmatrix} \\
A &= \begin{bmatrix}
a_{XB}^{20,4} & a_{YB}^{20,4} & a_{ZB}^{20,4} & \lambda_1 & 0 & 0 & 0 & 0 \\
a_{XB}^{20,5} & a_{YB}^{20,5} & a_{ZB}^{20,5} & 0 & \lambda_1 & 0 & 0 & 0 \\
a_{XB}^{20,6} & a_{YB}^{20,6} & a_{ZB}^{20,6} & 0 & 0 & \lambda_1 & 0 & 0 \\
a_{XB}^{20,24} & a_{YB}^{20,24} & a_{ZB}^{20,24} & 0 & 0 & 0 & \lambda_1 & 0 \\
a_{XB}^{20,25} & a_{YB}^{20,25} & a_{ZB}^{20,25} & 0 & 0 & 0 & 0 & \lambda_1 \\
a_{XB}^{20,4} & a_{YB}^{20,4} & a_{ZB}^{20,4} & 0 & 0 & 0 & 0 & 0 \\
a_{XB}^{20,5} & a_{YB}^{20,5} & a_{ZB}^{20,5} & 0 & 0 & 0 & 0 & 0 \\
a_{XB}^{20,6} & a_{YB}^{20,6} & a_{ZB}^{20,6} & 0 & 0 & 0 & 0 & 0 \\
a_{XB}^{20,24} & a_{YB}^{20,24} & a_{ZB}^{20,24} & 0 & 0 & 0 & 0 & 0 \\
a_{XB}^{20,25} & a_{YB}^{20,25} & a_{ZB}^{20,25} & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \\
x &= [\Delta X \quad \Delta Y \quad \Delta Z \quad N_{AB}^{20,4} \quad \cdots \quad N_{AB}^{20,4}]^{\mathrm{T}}
\end{aligned}
$$

The LSQ of these equations is given by

$$X = (A^T PA)^{-1} A^T PL \tag{3}$$

The coefficients $a_x,\ a_y,\ a_z,\ \rho_{AB,0}^{pq}$ are from the linearization of the topocentric distance differentiation,

$$
\begin{aligned}
a_x^s &= -\frac{X^s - X_{B0}}{\rho_{BO}^s} \\
a_y^s &= -\frac{Y^s - Y_{B0}}{\rho_{BO}^s} \\
a_z^s &= -\frac{Z^s - Z_{B0}}{\rho_{BO}^s}
\end{aligned}
$$

Similar to the phase and code double differencing, the coefficients are also subtracted differenced with the reference satellites values, in this case, Satellite 20. In Equation (3), the P matrix is the weight matrix and it is given in the excel file. The P matrix contains the standard deviation of the undifferenced phase and code, which for most receivers is 2 mm and 0.3 m respectively.

3 Results

The results for the LSQ of equations is,

$$\mathbf{X}(m) = \begin{bmatrix} \text{-0.03} \\ 0.16 \\ 0.22 \\ 951566.01 \\ 1014451.16 \\ \text{-1138240.42} \\ \text{-9347.42} \\ \text{-1078921.56} \end{bmatrix}$$

with $\mathbf{L}$ and $\mathbf{A}$ equal to,

$$\mathbf{L}(m) = \begin{bmatrix} 181077.04 \\ 193043.66 \\ \text{-216600.08} \\ \text{-1778.67} \\ \text{-205312.07} \\ 0.13 \\ \text{-0.22} \\ \text{-0.09} \end{bmatrix} \quad \mathbf{A}(m) = \begin{bmatrix} \text{-1.13} & 0.25 & \text{-0.08} & 0.19 & 0 & 0 & 0 & 0 \\ \text{-0.199} & 0.58 & \text{-0.31} & 0 & 0.19 & 0 & 0 & 0 \\ 0.306 & \text{-0.30} & \text{-0.34} & 0 & 0 & 0.19 & 0 & 0 \\ \text{-0.605} & 0.52 & \text{-0.10} & 0 & 0 & 0 & 0.19 & 0 \\ \text{-0.338} & \text{-0.84} & \text{-0.049} & 0 & 0 & 0 & 0 & 0.19 \\ \text{-1.13} & 0.25 & \text{-0.08} & 0 & 0 & 0 & 0 & 0 \\ \text{-0.199} & 0.58 & \text{-0.31} & 0 & 0 & 0 & 0 & 0 \\ 0.306 & \text{-0.30} & \text{-0.34} & 0 & 0 & 0 & 0 & 0 \\ \text{-0.605} & 0.52 & \text{-0.10} & 0 & 0 & 0 & 0 & 0 \\ \text{-0.338} & \text{-0.84} & \text{-0.049} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The double differences were computed:

$$\lambda\varphi = \begin{bmatrix} 180848.77 \\ 192567.29 \\ \text{-216415.03} \\ \text{-2197.22} \\ \text{-204701.52} \end{bmatrix} \quad P = \begin{bmatrix} \text{-228.14} \\ \text{-476.60} \\ 184.96 \\ \text{-418.71} \\ 610.13 \end{bmatrix}$$

The results agree completely with the reference material.

4 Analysis and Discussion

- Are the results reasonable? Compare the results with your expectations.

  Yes, the results are reasonable. The double difference observations are composed of the undifferenced phases and code pseudoranges, so we can expect the standard deviations to be greater than or equal to these as well. For most receivers, $\sigma_\Phi = 2$ mm and $\sigma_P = 0.3$ m, so the double differences should give standard deviations greater than or equal to 0.3 m.

- Can we draw any conclusion/implications from the results?

  We can conclude that the approximate coordinates are very close to the true receiver position, even though we did not factor in ionospheric and tropospheric effects.

- Are results reliable and accurate?

  I believe the results are reliable and accurate because the σ values is slightly greater than the change in X,Y, and Z. We should also take into account the calculations that can be done with λ_2 values.

- Would it have been more appropriate to use another method? Does the method need to be further developed?

  I think this method could be further developed by analyzing the double differences using different reference satellites. For example, satellite 20 was used in this report, but we should also analyze using satellites 4,5,6,24, and 25 as reference satellites and compare the differences between each of their results.

References

[1] M. Horemuž, "Relative positioning with gps," Royal Institute of Technology, Tech. Rep., 2014.

5 Main Code

Contents

Lab 3

Difference Equations for GPS

```
clear all; clc;
```

Import file

```
addpath(['/Users/kevin/SkyDrive/KTH Work/'...
    'Period 4 2014/GNSS/Labs/'...
    'L3 - Computation of receiver position '...
    'from code and phase measurements'])
excfile = xlsread('DiffAssignmentL1');
excfile = padarray(excfile,5,'pre');
cellDiff = num2cell(excfile); % same as a cell
```

Constants

```
c = 299792458; % speed of light (m/s)
[lambda1,lambda2] = cellDiff{78:79,2};
mu = 3.986005e14; % universal gravitational parameter (m/s)^3
omega_e_dot = 7.2921151467e-5; % earth rotation rate (rad/s)
F = -4.442807633e-10; % s/m^1/2
nRows = 5;
```

Variables

```
dtA = excfile(24,3);
dtB = excfile(25,3);
sNum = excfile((28:33),1);
refSatNum = 20;
for i = 1:length(sNum) % vector of ones at satellite numbers
    vectOnes(sNum(i),1) = 1;
end
[Xref,Xrov,Yref,Yrov,Zref,Zrov] = cellDiff{(37:38),3:5};
for i = 43:48 %
    rhoDot(excfile(i-15,1),1) = excfile(i-15,2);
    satCoordinates(excfile(i,1),:) = excfile(i,3:5);
    rhoAtoP(excfile(i+23,1),:) = excfile(i+23,2);
    rhoB0toP(excfile(i+23,1),:) = excfile(i+23,3);
end
count = 1;
for i = 6:3:21
   P1ref(sNum(count),1) = excfile(i,3);
   P1rov(sNum(count),1) = excfile(i+1,3);
   phi1ref(sNum(count),1) = excfile(i,4);
   phi1rov(sNum(count),1) = excfile(i+1,4);
   count = count + 1;
end
weightMatrix = excfile(52:61,1:10);
```

Main steps of Calculation

1. Synchronize observables from both receivers using equations (14)

```
P1refeq14 = P1ref + rhoDot*dtA;
P1roveq14 = P1rov + rhoDot*dtB;
phi1refeq14 = lambda1*phi1ref + rhoDot * dtA;
phi1roveq14 = lambda1*phi1rov + rhoDot * dtB;
```

2. Compute single and double differences - equations (15) and (18). Use satellite 20 as a reference satellite for double differencing.

```
P1eq15 = P1refeq14 - P1roveq14;
phi1eq15 = (phi1refeq14 - phi1roveq14);
dtAB = dtB - dtA;
% Double differences
```

```
phi1eq18 = phi1eq15 - phi1eq15(refSatNum)*vectOnes;
P1eq18 = P1eq15 - P1eq15(refSatNum)*vectOnes;
```

3. Compute coefficients a_X , a_Y , a_Z and $rho^{pq}_{AB,0}$ - equations (12).

```
ax_S_rov = -1 * (satCoordinates(:,1)-Xrov*vectOnes)./(rhoB0toP);
ay_S_rov = -1 * (satCoordinates(:,2)-Yrov*vectOnes)./(rhoB0toP);
az_S_rov = -1 * (satCoordinates(:,3)-Zrov*vectOnes)./(rhoB0toP);
aXB_st = -ax_S_rov + ax_S_rov(20);
aYB_st = -ay_S_rov + ay_S_rov(20);
aZB_st = -az_S_rov + az_S_rov(20);
rhoAB_0 = rhoAtoP - rhoB0toP;
```

4. Fill in matrixes A, L and compute least square solution of

equations (21).

```
p_AB_s = rhoB0toP-rhoAtoP;
nv = [4,5,6,24,25];
A = [aXB_st(nv,1), aYB_st(nv,1), aZB_st(nv,1)];
A = [ A , eye(length(nv)) * lambda1 ; A , zeros(length(nv)) ];
% L
L = [phi1eq18(nv) - rhoAB_0(nv) + ones(5,1)*rhoAB_0(20);...
    P1eq18(nv) - rhoAB_0(nv) + ones(5,1)*rhoAB_0(20)];
%
X = inv(A'*weightMatrix*A)*A'*weightMatrix*L;
```