Lab 3 - Computation of receiver position from code and phase measurements

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1 Introduction

The task is to compute the coordinates of a GPS receiver at given time using P1-code pseudo ranges. My specific observation epoch is: 2004-02-02, 1 h 14 min 00 sec. The calculations were done with observation files and navigation files in RINEX format. The computation of this position is difficult because the satellite and receiver clocks are not synchronised, the satellite and receiver are both moving with varying velocities, and propagation times. There are many more problems that need to be accounted for like ionospheric and tropospheric corrections, but we have neglected them in this lab because we want to gain a general understanding before we take on the harder tasks

2 Methodology

Matlab was used to solve for the following parameters and all of the following equations were taken from the technical report: GPS single point positioning algorithm.[1]

The steps and equations from this algorithm are documented in the Matlab code, Section ??. The pseudo ranges and navigation messages are used to calculate a more precise position of the receiver. The shift in position is a measure of the time for the signal to travel from satellite to receiver.

This time traveled by the signal multiplied by the speed of light is the pseudo range.

All the satellites were used in this calculation in order to get the best possible approximation of the receiver coordinates. Since we will be using least-squares method to solve for the parameters, the more independent equations we have, the better the approximation will be.

First, the signal propagation time was calculated using,

$$\Delta t_A^s = P_A^s(\tilde{t_A})/c$$

which was then used to calculate the nominal transmission time,

$$\tilde{t}^s = \tilde{t_A} - P_A^s(\tilde{t_A})/c$$

where $\tilde{t_A}$ is the nominal time of signal reception by the receiver. For this lab report, $\tilde{t_A} = 90840$ seconds, or 2 days, 1 hour, and 14 minutes. The weeks in the year was omitted because the difference between epoch times was less than a day and therefore would have cancelled out.

Next the satellite clock correction, δt_{L1}^s, was computed using equations,

$$\begin{aligned} \delta t_{L1}^s &= \Delta t_{SV} - T_{GD} \\ \Delta t_{SV} &= a_{f0} + a_{f1}(\tilde{t}^s - t_{oc}) + a_{f2}(\tilde{t}^s - t_{oc})^2 + \Delta t_r \end{aligned}$$

but the relativistic correction, $\Delta t_r = Fe\sqrt{A}\sin(E_k)$ was neglected until the eccentric anomaly and satellite coordinates were computed. Next, the system time of signal transmission, t^s was calculated using the satellite clock correction. The eccentric anomaly was calculated using Kepler's equation,

$$E_k = M_k + e\sin(E_k)$$

and the solution was iterated until convergence. The relativistic correction is now calculated and t^s is corrected in order to compute the satellite coordinates at that time given by the equations,

$$
\begin{aligned}
X^s &= x'_k \cos\Omega_k - y'_k \cos i_k \sin\Omega_k \\
Y^s &= x'_k \sin\Omega_k + y'_k \cos i_k \cos\Omega_k \\
Z^s &= y'_k \sin i_k
\end{aligned}
$$

Now that the satellite coordinates have been calculated, the satellite clock correction is corrected again to include the relativistic correction, Δt_r. Now the approximate distance of the satellite is calculated using the equation,

$$
\rho^s_{\mathrm{A0}}\left(\tilde{t}_A\right) = \sqrt{\left(X^s - X_{\mathrm{A0}} + \dot{\Omega}_e Y_{\mathrm{A0}} \Delta \mathrm{t}^s\right)^2 + \left(Y^s - Y_{\mathrm{A0}} - \dot{\Omega}_e X_{\mathrm{A0}} \Delta \mathrm{t}^s\right)^2 + \left(Z^s - Z_{\mathrm{A0}}\right)^2}
$$

The calculation of these parameters is done for all satellites we are interested in. Since we are computing four unknown parameters, the satellite lower limit is also four. More than four should only increase our accuracy assuming they aren't outliers. The result will be used to get more accurate coordinates for the receiver.

The least squares method is used to calculate the corrections to the receiver position and system time of signal reception. Using values from different satellites, we solve,

$$
\mathbf{L} - \mathbf{v} = \mathbf{A}\mathbf{X}
$$

where $\mathbf{v}$ is a vector of unknown residuals, and the vector $\mathbf{X}$ is solved for with the LSQ solution,

$$
\mathbf{X} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{L} = [\Delta X \quad \Delta Y \quad \Delta Z \quad \mathrm{c}\delta \mathrm{t_A}]^{\mathrm{T}}
$$

where $\mathbf{A}$ and $\mathbf{L}$ represent

$$
\mathbf{A} = \begin{bmatrix} a_x^{(1)} & a_y^{(1)} & a_z^{(1)} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ a_x^{(n)} & a_y^{(n)} & a_z^{(n)} & 1 \end{bmatrix}
$$

$$
\mathbf{L} = \begin{bmatrix} P_A(\tilde{t}_A) - \rho_{A0}^{(1)} + c\delta t_{L1}^{1} \\ \vdots \\ P_A(\tilde{t}_A) - \rho_{A0}^{(n)} + c\delta t_{L1}^{n} \end{bmatrix}
$$

The new estimated coordinates are computed by adding onto the approximate receiver position given in the observation file.

$$
\begin{aligned}
X_A &= X_{A0} + \Delta X \\
Y_A &= Y_{A0} + \Delta Y \\
Z_A &= Z_{A0} + \Delta Z
\end{aligned}
$$

Now the calculation of $\rho^s_{\mathrm{A0}}\left(\tilde{t}_A\right)$, $\mathbf{L}$, $\mathbf{A}$ is done again using the updated receiver coordinates until the solution has converged. We say that the solution has converged if the following condition is fulfilled: $|(\mathrm{v}^{\mathrm{T}}\mathrm{v})_i - (\mathrm{v}^{\mathrm{T}}\mathrm{v})_{i-1}| < 1e-5$. The results fulfilling the convergence condition are given in Table **??**.

3 Results

The results for the receiver position are given in Table **??**. The results agree completely with the reference material given because both calculations used all of the 11 satellites available. The PDOP value was calculated using,

$$
PDOP = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}
$$

4 Analysis and Discussion

- Are the results reasonable? Compare the results with your expectations.

  Yes the results are reasonable. The final position of the receiver is very close to the approximate position and this is shown in Table **??**. We also get very consistent results if we compute the same calculations for different epochs.

- Can we draw any conclusion/implications from the results?

  We can conclude that the approximate coordinates are very close to the true receiver position, even though we did not factor in ionospheric and tropospheric effects.

- Are results reliable and accurate?

  I believe the results are reliable and accurate because the σ value is very small for all x,y,z and all 11 satellites were taken into account. But the least squares method can give poor results if there are any outliers or the errors aren't Gaussian distributed.[2] Looking at other epochs in the SPP Results, we can see that they also give approximately the same position for the receiver.

- Would it have been more appropriate to use another method? Does the method need to be further developed?

  I think it is only necessary to further develop this method if you needed a higher degree of accuracy. This method could be further developed to take into account the ionospheric and tropospheric effects and factor in all the given epochs for the satellites. I think it could me more appropriate to use the difference method is best used when you have more satellites and tropospheric and ionospheric effects are cancelled out in the equations. This means our final solution should be more accurate using the difference method.

References

[1] M. Horemuž, "Relative positioning with gps," Royal Institute of Technology, Tech. Rep., 2014.

[2] A. L. Garcia, *Numerical Methods for Physics (2Nd Edition)*, 2nd ed., A. Reeves and P. F. Corey, Eds. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1999.