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CS-225: Discrete Structures in CS
Homework 7, Part 2
Exercise Set #9.4, Problem #(4, 8, 15, 18, 28, 30)

4.

Initials can be comprised of 26 letters in the alphabet, of which there are two initials for a person's first name and surname. Thus, $26 \cdot 26 = 676$ possible combinations of two initials. By the pigeonhole principle, there are 700 pigeons (700 people) to 676 pigeonholes (combinations of initials), thus there must be two people who share first and last name initials.

8.

T = {1, 2, 3, 4, 5, 6, 7, 8, 9}

Of T, there are 4 pairs of integers which sum to 10, which are: (1, 9), (2, 8), (3, 7), and (4, 6). However, we are choosing 5 integers from T, it is possible that the 5 integers may be {1, 2, 3, 4, 5} of which no two integers in this set have a sum of 10. Thus, the number of pigeons is NOT larger than the number of pigeonholes and no two integers may add up to 10.

15.

In the set of 0 through 2n integers, n+1 must be even and n must be odd.

Odd

Since n+1 even integers can be picked out of this set, we must pick one more in order to select an odd integer, which is $\boxed{n+2.}$

Even

Also, since n odd integers can be picked in the set of 0 through 2n, it means that we must pick $\boxed{n + 1}$ in order to select an even integer.

18.

By definition of divisibility, an if 15 divides an integer k, then $15 \cdot k = n$ with some remainder n where $0 \leq n \leq 15$. By the pigeonhole principle, there are 15 possibilities from 15n + 0, 15n + 1, . . . to 15n + 14.

If we select 16 integers however, at least two integers will have had the same remainder. Thus, by the pigeonhole principle where 16 pigeons > 15 pigeonholes, we would need to select 16 integers.

28.

Let a = number of pigeons and b = number of pigeon holes. Thus, let the lines of code written be $a = 500$ and the total days the code was written be $b = 17$.

Then $a/b = 500/17 = 29.4$, which is less than 30. Then there is at least one day where the programmer wrote >30 liens of code.

30.

We have:

12 pennies from 1967
7 pennies from 1968
11 pennies from 1971

The pigeonholes are the years in the collection, and the pigeons are the total numbers of pennies picked.

If $3 + 1 = 4$ pennies are picked out, then at least two are of the same year. Thus, $2 \times 3 + 1 = 7$ pennies ensures that 3 pennies are of the same year, and $3 \times 3 + 1 = 10$ pennies ensures that four pennies are of the same year.

Then, at least $4 \times 3 + 1$ pennies must be picked from the collection in order to ensure that 5 pennies are from the same year.