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CS-225: Discrete Structures in CS
Homework 7, Part 1
Exercise Set #9.2, Problem #(12.b, 14.e, 15.b, 17.d, 26)
Exercise Set #9.3: Problem #(8.b.c., 24.c, 29.f.g., 33.e, 34.c)

Exercise Set #9.2

12.b.

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F} = 16

1) 4 through D = 10 hexadecimal digits

2) 0 through F = 16 hexadecimal digits

3) 0 through F = 16 hexadecimal digits

4) 0 through F = 16 hexadecimal digits

5) 0 through F = 16 hexadecimal digits

6) 2 through E = 13 hexadecimal digits

By rule of multiplication = $10 \cdot 16 \cdot 16 \cdot 16 \cdot 16 \cdot 13$ = **8,519,680** possible hexadecimals

14.e

The license plate has four uppercase letters followed by 3 digits, but the first two letters are AB and the remaining letters and digits are all distinct. So,

26 letters total – AB = 24 letters AND 10 digits total

A B _ _ _ _ _ = A B letters-2 letters-3 digits digits-1 digits-2

By rule of multiplication = $24 \cdot 23 \cdot 10 \cdot 9 \cdot 8$ = **397,440** possible plates

15.b

Combination lock = 3 selections of 30 numbers (1-30), where no numbers may be used twice

by rule of multiplication = $30 \cdot 29 \cdot 28 = \boxed{\textbf{24,360} \text{ combinations}}$

17.d

10 possible digits = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

5 possible odd digits = {1, 3, 5, 7, 9}

Odd integer from 1000 – 9999 with distinct integers:

Digit 1 __ 8 possibilities (can't be 0 and can't be the last digit)
Digit 2 __ 8 possibilities (can't be digit 1 or last digit)
Digit 3 __ 7 possibilities (can't be digit 1, 2, or last digit)
Digit 4 __ 5 possibilities (must be odd)

By rule of multiplication = $8 \cdot 8 \cdot 7 \cdot 5 = \boxed{\textbf{2240} \text{ odd integers from 1000-9999 with distinct digits}}$

26.

outer loop = 1 to m
middle loop = 1 to n
inner loop = 1 to p

By rule of multiplication = $m \cdot n \cdot p = \boxed{\textbf{mnp} \text{ iterations of innermost loop}}$

<u>Exercise Set #9.3:</u>

8.
b. License plate with no repeated letters

Case 1: 0 digits and four uppercase letters

$26 \cdot 25 \cdot 24 \cdot 23 = 358{,}800$

Case 2: 0 digit and five uppercase letters

$26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 = 7{,}893{,}600$

Case 3: 1 digit and four uppercase letters

$10 \cdot 26 \cdot 25 \cdot 24 \cdot 23 = 3{,}588{,}000$

Case 4: 1 digit and five uppercase letters

$10 \cdot 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 = 78{,}936{,}000$

By addition rule = 358,800 + 7,893,600 + 3,588,000 + 78,936,000= $\boxed{\textbf{90,776,400} \text{ possibilities}}$

c.

License plates with at least one repeated letter = Total number of possible plates – number of plates without repetition

Total plates:

Case 1: 0 digits and four uppercase letters

$26 \cdot 26 \cdot 26 \cdot 26 = 456{,}976$

Case 2: 0 digits and five uppercase letters

$26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 11{,}881{,}376$

Case 3: 1 digit and four uppercase letters

$10 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 4{,}569{,}760$

Case 4: 1 digit and five uppercase letters

$10 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 118{,}813{,}760$

= 456,976 + 11,881,376 + 4,569,760 + 118,813,760 = 135,721,872 total plates

135,721,872 (total plates) - 90,776,400 (plates without repetition) = $\boxed{\textbf{44,945,472}}$ plates with at least one repeated number

24. c.

S = set of all integers from 1-1000
A = set of integers from 1-1000 which are multiplies of 2
B = set of integers from 1-1000 which are multiples of 9

N(A) = 500 as there are 500 multiples of 2 in set S
N(B) = 111 as there are 111 multiples of 9 in set S

Multiples of 2 and 9 = multiples of 18 of which there are 55 in set S, so $N(A \cap B) = 55$.By the inclusion/exclusion rule, $N(A \cup B) = N(A) + N(B) - N(A \cap B) = 500 + 111 - 55 = 556$

Hence the number of integers from 1-1000 that are multiples of 2 or 9 is 556, so the number of integers from 1-1000 that are **neither** multiples of 2 nor multiples of 9 are 1000-556 = $\boxed{444}$.

29.

f.

110_ _ _ _ _ | _ _ _ _ _ _ _ _ | _ _ _ _ _ _ _ _ | _ _ _ _ _ _ _ _

[---------------------Network ID------------------] [---Host ID---]

Block 1 can be 11000000 to 11011111 =

11000000 = $(2^7 + 2^6)$ = **192**

11011111 = $(1 \cdot 1 + 1 \cdot 2 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 1 \cdot 2^6 + 1 \cdot 2^7)$ = **223**

Remaining blocks can be 00000000 to 11111111 =

00000000 = **0**

11111111 = $(1 \cdot 1 + 1 \cdot 2 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 1 \cdot 2^5 + 1 \cdot 2^6 + 1 \cdot 2^7)$ = **255**

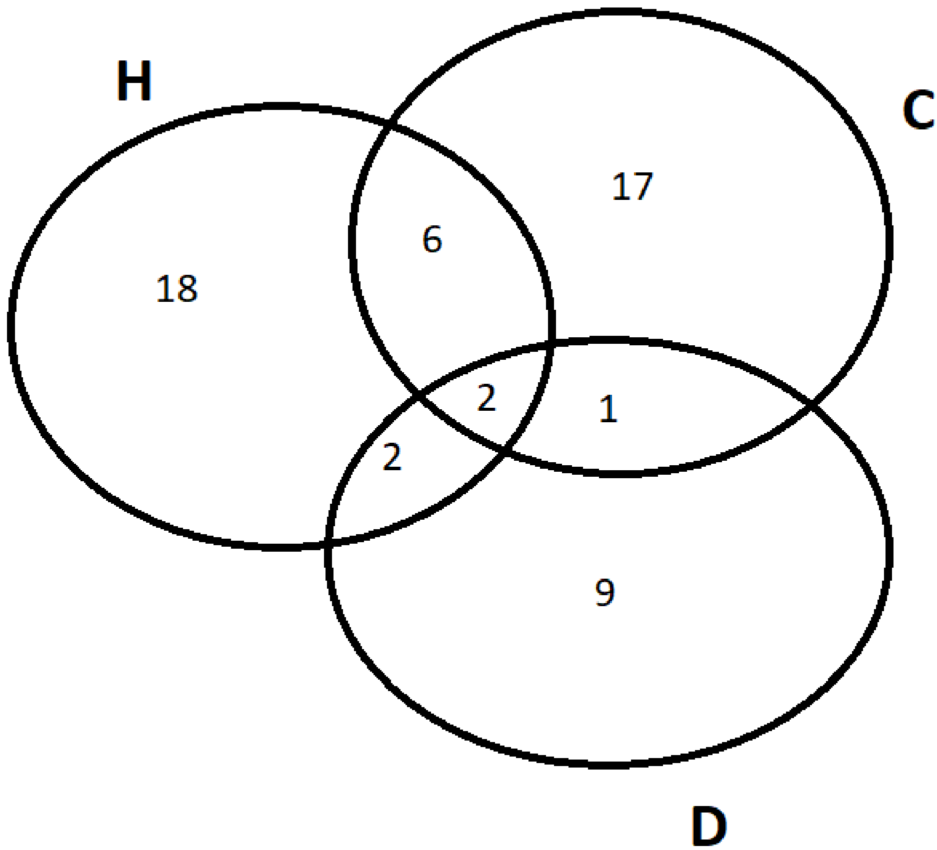
A class C network IP takes the form a.b.c.d, where a is an integer from 192 to 223, and b, c, and d are integers from 0 to 225.

g.

Class C host IDs are 8 bit blocks, where each of the 8 blocks has two possibilities. Thus, $2^8 = 256$, but as all 0 and all 1's are excluded, the total number of host IDs is 256 – 2 = $\boxed{254}$.

33. e.

One student checked 2 and 3 but not 1, as seen from the diagram below:



34.c