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CS-225: Discrete Structures in CS

Homework 3. Part 1

Exercise Set 4.2: Problem #8, #28, #40

Exercise Set 4.3: Problem #26, #39

Exercise Set 4.4: Problem #17, #27

Exercise Set 4.2

#8.

Supposition: Suppose m is any [*particular but arbitrarily chosen*] even integer and that n is any [particular but arbitrarily chosen] odd integer.

Goal: [*We must show that 5m + 3n is odd*].

Deduction: By the definition of even and odd,

m = 2r and n = 2s + 1

Then

5m + 3n	= 5(2r) + 3(2s + 1)	by substitution
	= 10r + 6s + 3	by multiplying out
	= 2(5r + 3s + 1) + 1	by factoring out 2

Let t = 5r + 3s + 1. Then t is an integer, because r, s, 5, 3, and 1 are integers, and because the products and sums of integers are integers.

Conclusion: Hence 5m + 3n = 2t + 1, where t is an integer, and so by the definition of odd integers, 5m + 3n is odd [*as was to be shown*].

#28.

Supposition: Suppose that n and m are [*particular but arbitrarily chosen*] integers such that if n – m is even.

Goal: [*We must show that $n^3 – m^3$ is even*].

Deduction: By the definition of even

n – m = 2k for some integer k

Then

$n^3 - m^3 = (n - m)(m^2 + nm + n^2)$ factoring out n-m and multiplying out

$= (2k)(m^2 + nm + n^2)$ by substitution

$= (2(k((m^2 + nm + n^2)))$ by associative property of multiplication

Let $t = k(m^2 + nm + n^2)$, such that t is an integer because the integers$(m^2 + nm + n^2)$ are closed under a multiplication operation, and because the products of integers are integers.

Conclusion: Hence, $n^3 - m^3 = 2t$ is even by the definition of an even integer [*as was to be shown*].

#40.

The statement is **false**.

Counterexample:

Let p = 11

Then 2^p-1 = 2^{11}- 1 = 2048 – 1 = 2047 = 23 x 89.

Because neither 23 or 89 are equal to 1, by the definition of a prime, 2^p-1 is not a prime.

<u>Exercise Set 4.3</u>

#26. (Derived using corollaries of theorems 4.3.1, 4.3.2, and exercises 12-17 as stated in text).

Suppose that s is a rational number.

Since every integer is a rational number,

5, 8, and 7 are rational numbers. Theorem 4.3.1

Since the product of two rational numbers is rational, the products

s^2, s^3, $5s^3$, and $8s^2$ are rational. Exercise 15

Since the sum of two rational numbers is a rational number,

$5s^3+8^2$ is rational Theorem 4.3.2

Since the difference of two rational numbers is a rational number,

$5s^3+8s^2- 7$ is rational Exercise 17

Hence, for any rational number s, $5s^3 + 8s^2 - 7$ is rational.

#39.

The proof is **false** due to the writer assuming that "r + s is rational" is true, when that is what is trying to be proved, which is circular reasoning. The correct proof should be:

Suppose r and s are rational numbers where r = a/b and s = c/d for some integers a, b, c, and d, where b ≠ 0 and d ≠ 0. [*We must show that r + is rational*).

Thus,

$$r + s = \frac{a}{b} + \frac{c}{d} \qquad \text{by substitution}$$

$$= \frac{ad+bc}{bd} \qquad \text{by algebra}$$

Let i = ad + bc and j = bd. Then i and j are integers because products and sums of integers are integers and because a, b, c, and d are all integers, and j ≠ 0 by the zero-product property.

Thus,

$$r + s = \frac{i}{j} \text{ where i and j are integers and j} \neq 0$$

Therefore, r + s is rational by definition of a rational number such that rational numbers are quotients of integers [*as was to be shown*].

Exercise Set 4.4

#17.

Supposition: Suppose that a, b, c, and d are [*particular but arbitrarily chosen*] positive integers such that a|c and b|d.

Goal: [*We must show that ab|cd*].

Deduction: By the definition of divisibility,

$c = ar \quad d = bs$ for some integers r and s

Then,

$cd = (ar)(bs)$ by substitution

$cd = arbs$ by algebra

$cd = abrs$ by associative property of multiplication

Let $k = rs$. Then k is an integer because the products of integers are integers.

Thus,

$$cd = abk \qquad \text{where } k \text{ is an integer}$$

Conclusion: Therefore, ab divides cd by the definition of divisibility where $cd = ab \cdot$ (*some integer*).

#27.

This statement is **false**.

Statement: For all integers a, b, and c, if $a|(b + c)$ then $a|b$ or $a|c$.

Counterexample: Let $a = 2$, $b = 3$, and $c = 5$.

Then

$$a|(b + c) = 2|(3 + 5)$$

But $2 \nmid 3$ or $2 \nmid 5$

Hence the statement is false.