Kevin Sekuj

CS-225: Discrete Structures in CS

Homework 2, Part 1

Exercise Set 3.1, Problem #18(c, d), #29 (a, b, c)

Exercise Set 3.2, Problem #4(b, d), #12, #14, #23, #29, #40, #44

Extra problem on Canvas

<u>Exercise Set 3.1</u>

#18.

Let:

D = set of all students

M(*s*) = "*s* is a math major"

C(*s*) = "*s* is a computer science student"

E(*s*) = "*s* is an engineering student"

c. $\forall s \in D, C(s) \rightarrow \neg E(s)$

d. $\exists s \in D, M(s) \wedge C(s)$

#29.

Let:

Domain of *x* = set of all geometric figures in the plane

Square(*x*) = "*x* is a square"

Rect(*x*) = "*x* is a rectangle"

a. There exists a geometric figure in the plane that is a rectangle and that is a square.

This statement is **true** because a square is a geometric figure within the set of all geometric figures in the plane and a square is also a rectangle.

b. There exists a geometric figure in the plane that is a rectangle and that is not a square.

This statement is **true** because a rectangle is a geometric figure in the plane and a rectangle is not a square.

c. All squares are also rectangles.

This statement is **true** because all squares are rectangles by definition (geometric shape with four right angles).

<u>Exercise Set 3.2</u>

#4.

b. Some graphs are not connected.

d. All estimates are not accurate (or All estimates are inaccurate).

#12.

The proposed negation is incorrect as the original statement is a universal statement, meaning that "for all" ∀ must be changed to the form "there exists" ∃.

The correct negation is: "The product of some irrational number and some rational number is not irrational."

#14.

The proposed negation is incorrect for the same reason as #12 – a universal statement implies ∀, and its negation must be in the form ∃.

The correct negation is "There exists real numbers x_1 and x_2, such that $x_1^2 = x_2^2$ and $x_1 \neq x_2$.

#23.

There is at least one function that is differential but is not continuous.

#29.

Referenced statement: $\forall n \in Z$, if n is prime then n is odd or n = 2

Contrapositive:

$\forall n \in Z$, if n is not odd and $n \neq 2$ then n is not prime.

This is **true**. Not odd (in other words, even) numbers besides 2 are not prime.

Converse:

$\forall n \in Z$, if n is odd or n = 2 then n is prime.

This is **false.** Let n = 9, for example: 9 is an odd number yet it is not prime.

Inverse:

$\forall n \in Z$, if n is not prime then n is even and $n \neq 2$.

This is **false.** As in the example for the converse, 9 is not prime, yet 9 is odd.

#40. If a number is divisible by 8, then the number is divisible by 4.

#44. If a polygon is a square, then it has four sides.

<u>Extra Problem</u>

Let

B(x): x is a female

W(x): x is a good athlete

S(x): x is young

D: Set of all people

a. "All good athletes are not young"

   $\forall x \in D, (W(x) \rightarrow \neg(S(x)))$

b. "A person is a good athlete only if it is the case that both she is a female and she is young."

   $\forall x \in D, (B(x) \land S(x)) \rightarrow W(x)$

c. “No young people are good athletes.”

$\forall x \in D, [S(x) \rightarrow \neg(W(x))]$

d. All good athletes are neither young nor they are female.

$\forall x \in D, W(x) \rightarrow \neg[(S(x)) \land \neg (B(x))]$

e. Every person is a good athlete unless he/she is not young.

$\forall x \in D, S(x) \rightarrow W(x)$