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CS-225: Discrete Structures in CS
Homework 8, Part 1
Exercise Set 1.4: (Canvas Problem)
Exercise Set 4.9: Problem #(10, 15, 18, 21.c, 23.f, 24.d)

Exercise Set #1.4 Problem on Canvas

Graph 1)

I. Find all edges incident on v_2
e_2
e_3
e_4
e_5
e_7

II. Find all vertices adjacent to v_4

v_1
v_2
v_4 (adjacent to itself, as an endpoint of a loop)

III. Find all edges adjacent to e_1

e_7
e_2
e_6

IV. Find all loops

e_3
e_6

V. Find all parallel edges

e_4
e_5

VI. Find the degree of V_2

$e_2 + e_3 + e_4 + e_5 + e_7 = 1 + 2 + 1 + 1 + 1 = 6$

Graph 2)

I. Find all edges incident on v_2

e_2
e_3
e_7
e_5

II. Find all vertices adjacent to v_4

v_2
v_3
v_5

III. Find all edges adjacent to e_1

e_2
e_8
e_9
e_{10}

IV. Find all loops

e_{10}

V. Find all parallel edges

There are no parallel edges in Graph 2.

VI. Find the degree of V_2

$e_2 + e_3 + e_7 + e_5 = 4$

<u>Exercise Set #4.9: Problem #(10, 15, 18, 21.c, 23.f, 24.d)</u>

10.

The simple graph as described in this problem has 5 vertices, described as 2, 3, 3, and 5. Since a simple graph can't have loops or parallel edges, the vertex with 5 degrees must be connected to 5 vertices through one edge, which it does cannot. Therefore, a simple graph with the 5 vertices as described in the problem statement cannot exist.

15.
a.

Let the people in the social network be vertices, and let edges between the people indicate that they are friends within the network. Then, we are given:

- 3 people are network friends with 6 other people in the network
- 1 person is network friend with 5 other people in the network
- 5 people are network friends with 4 other people in the network
- remainder (x) are network friends with 3 others in the network

The degree of the graph described in this statement is: $3 \cdot 6 + 1 \cdot 5 + 5 \cdot 4 + (x) \cdot 3 = 3x + 43$.

Using the handshake theorem, we find that the total degree is 41 edges (network connections) $\cdot 2 = 82$, thus

$3x + 43 = 82$

$x = 13$ by algebra

Hence there are 13 people which are friends with 3 other people in the network.

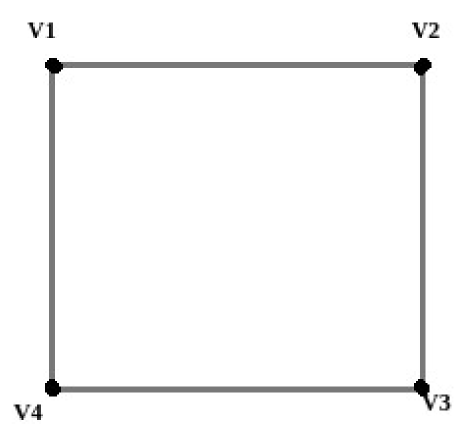
b.

The total number of people in the network =

(# of people friends with 6 others) + (# of people friends with 5 others) + (# of people friends with 4 others) + (# of people friends with 3 others)

$= 3 + 1 + 5 + 13 = 22$.

18. Yes, there exists a simple graph whose vertices each have an even degree, for example, the following graph has vertices with a degree of 2.



21.c

No.

Suppose that the simple graph has vertices of integer $n \geq 5$ of all different degrees, and that a vertex's degree cannot be < 0.

In 21.a, we established that the maximum degree of a vertex is n-1. Since the vertices of all of different degrees, the degrees of the graph must be 0, 1, 2, 3, … n-1. The vertex of degree n-1 must connect to the other vertices through one edge, which is impossible as one vertex has a degree of 0.

Therefore, this graph cannot exist.

23.f

Using the handshake theorem, the total degree of the graph is equal to 2 · the number of edges of $K_{m,\ n}$ so

$2 \cdot (e) =$ total degree of $K_{m,\ n}$

We determined that the total degree of $K_{m,\ n}$ in 23.d and 23.e, which was 2mn, by summing the degrees of $K_{m,\ n}$'s vertices. Thus,

$2(e) = 2mn$

$e = mn$ (number of edges of $K_{m,\ n}$)

24.d

Graph (d.) is not a bipartite graph.

In this graph, let v_2 be in set V_1. Since v_2 and v_5 connect via edge, v_5 must be in set V_2. However, v_2 also connects by edge to v_4, and v_4 connects to v_5 by edge. Then v_4, is not in set V_1 nor is it in set V_2.

Since vertex v_4 cannot be placed in either of the two sets V_1 and V_2, the graph is not bipartite.