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CS-225: Discrete Structures in CS
Homework 8, Part 2
Exercise Set #: 9.5: Problem #(7.b.ii.iii., 12, 16.a.b.)
Exercise Set #: 9.6 Problem #(4, 14, 18)

Exercise Set 9.5

7. b. I.

Men | n = 6, r = 3| $\binom{6}{3}$
Women | n = 7, r = 4 | $\binom{7}{4}$

Groups of seven containing 4 women and 3 men = C(6, 3) · C(7, 4) by multiplication rule. Thus

C(n, r) = $\frac{6!}{3!(6-3)!} \cdot \frac{7!}{4!(7-4)!} = 20 \cdot 35 = 700$ combinations containing four women and three men

II.

The number of groups that contain at least one man is equal to the total number of groups minus the group containing 7 women (since that group contains only women, and zero men). Thus

Total number of groups | n = 13, r = 7 | $\binom{13}{7} = \frac{13!}{7!(13-7)!}$
Groups with only women | n = 7, r = 7 | $\binom{7}{7} = \frac{7!}{7!(7-7)!}$
By subtraction rule, the groups of seven containing at least one man = $\frac{13!}{7!(13-7)!} - \frac{7!}{7!(7-7)!} =$
1716 – 1 = $\boxed{1}$

III.

The number of groups of 7 containing at most 3 women = 0 through 3 women with men filling the remaining spots. In other words, the groups of 6 men 1 woman, 5 men 2 women, and 4 men 3 women. Thus, by multiplication rule:

Number of groups so that there are at most 1 woman = $\binom{6}{6} \cdot \binom{7}{1} = \frac{6!}{6!(6-6)!} \cdot \frac{7!}{1!(7-1)!}$
Number of groups so that there are at most 2 women = $\binom{6}{5} \cdot \binom{7}{2} = \frac{6!}{5!(6-5)!} \cdot \frac{7!}{2!(7-2)!}$
Number of groups so that there are at most 3 women = $\binom{6}{4} \cdot \binom{7}{3} = \frac{6!}{4!(6-4)!} \cdot \frac{7!}{3!(7-3)!}$

Finally to find the number of groups containing at most 3 women, we use the addition rule with the above three combinations, adding up the number of ways to select 1, 2, and 3 women:

$$\frac{6!}{6!(6-6)!}\cdot\frac{7!}{1!(7-1)!} \quad + \quad \frac{6!}{5!(6-5)!}\cdot\frac{7!}{2!(7-2)!} \quad + \quad \frac{6!}{4!(6-4)!}\cdot\frac{7!}{3!(7-3)!}$$

= $\boxed{\text{658 ways}}$ to select at most 3 women

12.

Pairs of integers with an even sum = even + even OR odd + odd Out the set of 101, 51 are odd, and 50 are even. Thus:

$\binom{51}{2}+\binom{50}{2}=\frac{51!}{2!(51-2)!}+\frac{50!}{2!(50-2)!}=2500$ pairs of distinct integers of the set {1, 2, 3...100} that have an even sum.

16.a.

n = 40 (production run)
r = 5 (sample)

Number of different samples = $\binom{40}{5}=\frac{40!}{5!(40-5)!}=658{,}008$

b.

Since the number of samples = $\binom{40}{5}$ and 3 in the production run (40) are defective, then the samples that contain at least one defective chip is the total number of samples $\binom{40}{5}$ - $\binom{37}{5}$ = 222,111.

<u>Exercise Set #9.6</u>

4. a.

n = 30 (battery inventory)
8 battery types

= $\binom{n+m=1}{n}=\binom{30+8-1}{30}=\binom{37}{30}=\frac{37!}{30!(37-30)!}=10{,}295{,}472$ total inventory of 30 batteries which can be distributed among the 8 different types

b.

n = 30 (inventory) – 4 (A76 batteries) = 26.

$= \binom{n+m=1}{n} = \binom{26+8-1}{26} = \binom{33}{26} = \frac{33!}{26!(33-26)!} = 4{,}272{,}048$

c.

Total inventory of 30 batteries distributed among 8 different types if the inventory includes AT MOST 3 A76 batteries = part A – part B = 10,295,472 – 4,272,048 = $\boxed{6{,}023{,}424,}$ by the difference rule.

14.

a + b + c d + e = 500, where each integer a, b, c, d, e ≥ 10.

With integer a for example, let a = (some integer) x + 10. This integer must be ≥ 0. Then

$a = x_1 + 10$
$b = x_2 + 10$
$c = x_3 + 10$
$d = x_4 + 10$
$e = x_5 + 10$

where $x_1, x_2, x_3, x_4, x_5 \geq 0$

Then, since a + b + c d + e = 500, then $(x_1 + 10) + (x_2 + 10) + (x_3 + 10) + (x_4 + 10) + (x_5 + 10) = 500$ by substitution

$= x_1 + x_2 + x_3 + x_4 + x_5 + 50 = 500$
$= x_1 + x_2 + x_3 + x_4 + x_5 = 450$ where $x \geq 0$

Let n = 5 integers x_1 to x_5, and let r = 450.

$= \boxed{\binom{n+m=1}{n} = \binom{450+5-1}{450} = \binom{454}{450} = \frac{454!}{450!(454-450)!}}$ solutions to the equation a + b + c + d + e = 500 where a, b, c, d, and e are integers that are ≥ 10.

18. a.

r = 30 (collection of 30 coins)
n = 4 (four types of coin – quarter, dime, nickel, penny)

$= \boxed{\binom{30+4-1}{30} = \binom{33}{30} = \frac{33!}{30!(33-30)!}}$

b.

A set of collection of 30 coins where at most 15 quarters will be chosen = total set of collections of 30 coins – the set where at least 16 quarters will be chosen.

The total collections of 30 coins was calculated in part a, which is

$$\binom{33}{30}=\frac{33!}{30!(33-30)!}$$

And the set of 30 coins with at least 16 quarters is equivalent to

$$\binom{14+4-1}{14}=\binom{17}{14}=\frac{17!}{14!(17-14)!}$$

By the difference rule, the set of only 15 quarters = total set – set of at least 16 quarters. Thus:

$$\boxed{\frac{33!}{30!(33-30)!}-\frac{17!}{14!(17-14)!}}$$

c.

A set of 30 coins where at most 20 dimes will be chosen = Total set of collections of 30 coins – the set where at least 21 dimes will be chosen.

The set of at least 21 dimes chosen = $\binom{9+4-1}{9}=\binom{12}{9}=\frac{12!}{9!(12-9)!}$

The total collections of 30 coins was calculated in part a, which is

$$\binom{33}{30}=\frac{33!}{30!(33-30)!}$$

Then, by the difference rule, the set where at most 20 dimes will be chosen = Total set of collection of 30 coins – the set where at least 21 dimes will be chosen. Thus:

$$\boxed{\frac{33!}{30!(33-30)!}-\frac{12!}{9!(12-9)!}}$$

d.
The set of at most 15 quarters chosen AND at most 20 dimes chosen = Total number of selection of 30 coins – the non-allowed combinations

Incorporating the total set from a, and the set of 30 coins with at least 16 quarters, and the set where at most 20 dimes will be chosen from part b and c respectively, we get

$$\boxed{\frac{33!}{30!(33-30)!}-\frac{12!}{9!(12-9)!}-\frac{17!}{14!(17-14)!}}$$