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CS-225: Discrete Structures in CS

Homework 4, Part 2

Exercise Set 6.2: #6 (part 2), #11, #17

Exercise Set 6.3: #13, #40, #42

Exercise Set 6.2

#6. (part 2).

a. or

b. and

c. $x \in A \cap (B \cup C)$

d. $\subseteq$

#11.

Supposition: Suppose A, B, and C are any {*particular but arbitrarily chosen*} sets and $A \cap (B - C)$.

Goal: Prove that for all sets A, B, and C, the relation $A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$ holds.

Deduction:

Let $x \in A \cap (B-C)$. By the definition of intersection, $x \in A$ and $x \in B-C$.

By the definition of set difference,

$X \in B$ and $x \notin C$

Thus, $x \in A$ and $x \in B$ and $x \notin C$.

To show $x \in (A \cap B)-(A \cap C)$, we must show that $x \in A \cap B$ and $x \notin A \cap C$. Since $x \in A$ and $x \in B$, $x \in A \cap B$. Also, since $x \in A$ and $x \notin C$, by the definition of intersection, $x \notin A \cap C$. Thus, according to the previous two conclusions, $x \in (A \cap B)-(A \cap C)$.

Conclusion: Therefore, any $x \in A \cap (B - C)$ is in $(A \cap B) - (A \cap C)$. Thus, by the definition of subsets, $A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$ {*as was to be shown*}.

#17.

Supposition: Suppose A, B, and C are any {*particular but arbitrarily chosen sets*} and $A \subseteq B$.

Goal: Show that for all sets A, B, and C, if $A \subseteq B$ then $A \cup C \subseteq B \cup C$.

Deduction:

Suppose $A \subseteq B$. Let $x \in A \cup C$.

By the definition of union, $x \in A$ or $x \in C$.

1)

In the case $x \in A$,

since $A \subseteq B$, then $x \in B$.

Thus, $x \in B$ or $x \in C$.

Then, by the definition of union, $x \in B \cup C$.

2)

In the case $x \in C$, then

$x \in B$ or $x \in C$. Thus, by the definition of union,

$x \in B \cup C$.

Conclusion: Hence, in both cases, $x \in B \cup C$. Therefore, $A \cup C \subseteq B \cup C$ by the definition of subsets {*as was to be shown*}.

<u>Exercise Set 6.3</u>

#13.

$A \cup (B - C) = (A \cup B) - (A \cup C)$

Let:

$A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

$C = \{2, 4, 6\}$

LHS

<u>$(B - C)$</u>

By the definition of difference:

$B - C = \{3, 4, 5\} - \{2, 4, 6\} = \{3, 5\}$

$\underline{A \cup (B - C)}$

By the definition of union,

$A \cup (B - C) = \{1, 2, 3\} \cup \{3, 5\} = \{1, 2, 3, 5\}$

RHS

$\underline{(A \cup B) - (A \cup C)}$

By the definition of union,

$A \cup B = \{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$

And, by the definition of union

$A \cup C = \{1, 2, 3\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 6\}$

By the definition of difference,

$(A \cup B) - (A \cup C) = \{1, 2, 3, 4, 5\} - \{1, 2, 3, 4, 6\} = \{5\}$.

Thus, every element of each set is not in the other set. In other words, for sets A, B, C,

$A \cup (B - C) \neq (A \cup B) - (A \cup C)$

#40. $(A - B) - (B - C) = A - B$

$(A - B) - (B - C)$

$= (A \cap B^c) - (B \cap C^c)^C = (A \cap B^c) \cap (B \cap C^c)^C$ set difference law

$= (A \cap B^c) \cap (B^c \cup (C^C)^c)$ De Morgan's law

$= (A \cap B^c) \cap (B^c \cup C)$ double complement law

$= ((A \cap B^C) \cap B^C) \cup ((A \cap B^c) \cap C)$ distributive law

$= (A \cap (B^c \cap B^c)) \cup ((A \cap B^c) \cap C)$ associative law ($\cap$)

$= (A \cap B^c) \cup ((A \cap B^c) \cap C)$ idempotent law ($\cap$)

$= A \cap B^c$ absorption law

$= A - B$ set difference law

#42. $(A - (A \cap B)) \cap (B - (A \cap B))$

$= (A \cap (A \cap B^c) \cap (B \cap (A \cap B)^c)$	set difference law
$= A \cap ((A \cap B)^c \cap (B \cap (A \cap B)^c)$	associative law (∩)
$= A \cap (((A \cap B)^c \cap B) \cap (A \cap B)^c)$	associative law (∩)
$= A \cap ((B \cap (A \cap B)^c) \cap (A \cap B)^c)$	commutative law (∩)
$= A \cap (B \cap ((A \cap B)^c \cap (A \cap B)^c))$	associative law (∩)
$= A \cap (B \cap (A \cap B)^c)$	idempotent law (∩)
$= (A \cap B) \cap (A \cap B)^c$	associative law (∩)
$= \emptyset$	complement law (∩)