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CS-225: Discrete Structures in CS

Homework 4, Part 1

Exercise Set 6.1, Question #7, #12, #25(a,b,c,), #27(c,e)

#33(a,c), #34(b), #35(d)

#7.

a. $A \not\subseteq B$ (False)

Let x = 10 and a = 1, then $x \in A$, because x = 6(1) + 4 = 10, but $x \notin B$ because there is no integer b such that 10 = 18b – 2. For if there were an integer, then

$18b - 2 = 10$

$18b = 12$ by algebra

$b = 12/18 = 2/3$

But 2/3 is not an integer. Thus $10 \in A$ but $10 \notin B$, therefore $A \not\subseteq B$.

b. $B \subseteq A$ (True)

Supposition: Suppose y is a particular but arbitrary chosen element of B. Suppose $B \subseteq A$, then any element in B is an element in A.

Goal: We must show that $y \in A$, which means we must show that y = 6 · (some integer) + 4.

Deduction:

Let $y \in B$, then by the definition of B there is an integer b such that y = 18b – 2. Since y is an element in A, then by the definition of subsets there is an integer a so that y = 18b – 2 = 6a + 4.

Then,

$6a + 4 = 18b - 2$

$6a = 18b - 6$ by algebra

$a = 3b - 1$ by algebra

a = 3b – 1 is an integer as it is the difference of two integers 2b and 1.

Then, by substituting in the value of a into A,

$y = 6(3b - 1) + 4 = 18b - 2$. Thus, any element from B is also in A.

Conclusion: Therefore, by the definition of the subset, $B \subseteq A$ is true.

c. $B = C$ (True)

By the definition of equality, $B = C \leftrightarrow (B \subseteq C) \wedge (C \subseteq B)$.

1. $B \subseteq C$

Supposition: There is a particular but arbitrarily chosen element y in B. Suppose $B \subseteq C$, then any element in B is in C.

Goal: We must show that $y \in C$, which means we must show that $y = 18 \cdot$ (some integer) + 16.

Deduction:

Let $y \in B$, then by the definition of B there is an integer b such that $y = 18b - 2$. Since y is an element in C, then by the definition of subsets there is an integer c so that $y = 18b - 2 = 18c + 16$.

Then,

$18b - 2 = 18c + 16$

$18b - 18 = 18c$	by algebra
$b - 1 = c$	by algebra

Since c is the difference of integers b and 1, c is also an integer.

Conclusion: Thus, the element $y \in B$ is an element in C by the definition of subsets.

2. $C \subseteq B$

Supposition: There is a particular but arbitrarily chosen element z in C. Suppose $C \subseteq B$, then any element in C is in B.

Goal: We must show that $z \in B$, which means we must show that $z = 18 \cdot$ (some integer) $- 2$.

Deduction:

Let $z \in C$, then by the definition of C there is an integer c such that $z = 18c + 16$. Since z is an element in B, then by the definition of subsets there is an integer b so that $z = 18c + 16 = 18b - 2$.

Then,

$18b - 2 = 18c + 16$

$18b = 18c + 18$ by algebra

$b = c + 1$ by algebra

Since b is the sum of integers c and 1, b is also an integer.

Conclusion: Thus, the element $z \in C$ is an element in B by the definition of subsets.

Therefore, we've proven that $B \subseteq C$ and $C \subseteq B$, so $B = C$ by the definition of set equality.

#12.

a. [-3, 2)

$A \cup B = \{x \in \mathbf{R} | -3 \leq x \leq 0\} \cup \{x \in \mathbf{R} | -1 < x < 2\}$

$= \{x \in \mathbf{R} | -3 \leq x < 2\} =$

b. (-1, 0]

$A \cap B = \{x \in \mathbf{R} | -3\ x \leq 0\} \cap \{x \in \mathbf{R} | -1 < x < 2\}$

$= \{x \in \mathbf{R} | -1 < x \leq 0\} =$

c. $(-\infty, 3) \cup (0, \infty)$

$A^C = \{x \in \mathbf{R} |$ it is not the case that $x \in [-3, 0]\}$

$= \{x \in \mathbf{R} |$ it is not the case that $x \geq -3$ and $x \leq 0\}$

$= \{x \in \mathbf{R} | x < -3$ or $x > 0\} =$

d. $[-3, 0] \cup (6, 8]$

$A \cup C = \{x \in \mathbf{R} | -3 \leq x \leq 0) \cup \{x \in \mathbf{R} | 6 < x \leq 8\}$

$= \{x \in \mathbf{R} | -3 \leq x \leq 0$ or $6 < x \leq 8\} =$

e. $\emptyset$

$A \cap C = \{x \in \mathbf{R} | -3 \leq x \leq 0\} \cap \{x \in \mathbf{R} | 6 < x \leq 8\}$

$= \{x \in \mathbf{R} | \ -3 \leq x \leq 0 \text{ and } 6 < x \leq 8\} =$

f. $(-\infty, -1] \cup [2, \infty)$

$B^c = \{x \in \mathbf{R} | \text{ it is not the case that } x \in (-1, 2)\}$

$= \{x \in \mathbf{R} | \text{ it is not the case that } x > -1 \text{ and } x < 2\}$

$= \{x \in \mathbf{R} | \ x \leq -1 \text{ or } x \geq 2\} =$

g. $(-\infty, -3) \cup [2, \infty)$

$A^c \cup B^c = [(-\infty, -3) \cup (0, \infty)] \cap [(-\infty, -1] \cup [2, \infty)] =$

h. $(-\infty, -1] \cup (0, \infty)$

$A^c \cup B^c = [(-\infty, -3) \cup (0, \infty)] \cup [(-\infty, -1] \cup [2, \infty)] =$

i. $(-\infty, -1] \cup (0, \infty)$

$(A \cap B)^c = \{ x \in \mathbf{R} | \text{it is not the case that } x \in (-1, 0]\}$

$= \{ x \in \mathbf{R} | \text{it is not the case that } x > -1 \text{ and } x \leq 0\}$

$= \{ x \in \mathbf{R} | \ x \leq -1 \text{ or } x < 0\} =$

j. $(-\infty, -3) \cup [2, \infty)$

$(A \cup B)^c = \{ x \in \mathbf{R} | \text{it is not the case that } x \in [-3, 2)\}$

$= \{ x \in \mathbf{R} | \text{it is not the case that } x \geq -3 \text{ and } x \leq 2\}$

$= \{ x \in \mathbf{R} | \ x < -3 \text{ or } x \geq 2\} =$

#25 (a, b, c).

a.

$$\bigcup_{i=1}^{4} R_1$$

$R_1 = \{x \in \mathbf{R} \mid 1 \leq x \leq 1 + \frac{1}{1}\} = [1,2]$

$R_2 = \{x \in \mathbf{R} \mid 1 \leq x \leq 1 + \frac{1}{2}\} = [1,1\frac{1}{2}]$

$R_3 = \{x \in \mathbf{R} \mid 1 \leq x \leq 1 + \frac{1}{3}\} = [1, 1\frac{1}{3}]$

$R_4 = \{x \in \mathbf{R} \mid 1 \leq x \leq 1 + \frac{1}{4}\} = [1, 1\frac{1}{4}]$

$[1,2] \cup [1,1\frac{1}{2}] \cup [1, 1\frac{1}{3}] \cup [1, 1\frac{1}{4}] = \mathbf{[1,2]}$

b.

$$\bigcap_{i=1}^{4} R_1$$

$[1,2] \cap [1,1\frac{1}{2}] \cap [1, 1\frac{1}{3}] \cap [1, 1\frac{1}{4}] = \mathbf{[1, \frac{5}{4}]}$

c. No, R_1, R_2, and R_3 are not mutually disjoint. Both R_1 and R_2 contain the element 1, for example, which means the three sets cannot be mutually disjoint.

#27 (c, e).

c. No, {{5, 4}, {7, 2}, {1, 3, 4,}, {6, 8}} is not a partition of {1, 2, 3, 4, 5, 6, 7, 8}. This is because {5,4} and {1, 3, 4} are not mutually disjoint as both contain the element 4, as shown below.

$\{5,4\} \cap \{1, 3, 4\} = 4$

e. Yes, {{1, 5}, {4, 7}, {2, 8, 6, 3}} is a partition of {1, 2, 3, 4, 5, 6, 7, 8}.

Let $A_1 = \{1, 5\}$, Let $A_2 = \{4, 7\}$, and let $A_3 = \{2, 8, 6, 3\}$. All three sets are mutually disjoint as none have elements in common, and the union of them $A_1 \cup A_2 \cup A_3 = A = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

#33 (a, c).

a. $\mathcal{P}(\emptyset) = \{\emptyset\}$

c. $(\mathcal{P}\ (\mathcal{P}\ (\mathcal{P}(\emptyset))) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \emptyset\}\}\}$.

$\mathcal{P}(\emptyset) = \{\emptyset\}$

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$(\mathcal{P}\ (\mathcal{P}\ (\mathcal{P}(\emptyset))) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \emptyset\}\}\}$.

#34 (b). $\{(1, m), (1, n), (u, m), (u, n), (v, m), (v, n)\}$

$A_1 \cup A_2 = \{1\} \cup \{u, v\} = \{1, u, v\}$

$(A_1 \cup A_2) \times A_3 = \{1, u, v\} \times \{m, n\}$

$= \{(1, m), (1, n), (u, m), (u, n), (v, m), (v, n)\}$

#35 (d). $\{(a, 2), (b, 2)\}$

Let $A = \{a, b\}$, $B = \{1, 2\}$, and $C = \{2, 3\}$

$A \times B = \{a, b\} \times \{1, 2\} = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

$A \times C = \{a, b\} \times \{2, 3\} = \{(a, 2), (a, 3), (b, 2), (b, 3)\}$

$(A \times B) \cap (A \times C) = \{(a, 1), (a, 2), (b, 1), (b, 2)\} \cap \{(a, 2), (a, 3), (b, 2), (b, 3)\}$

$= \{(a, 2), (b, 2)\}$