

1 Lesson 3 Example 6

You toss a coin 60 times. Each toss results in two equally likely outcomes, heads or tails. What is the probability that you get exactly 30 heads in the 60 tosses?

2 Answer

Steps to Solve

1. **Calculate the Total Number of Combinations:**

$$\binom{60}{30} = \frac{60!}{30! \cdot 30!}$$

2. **Calculate the Probability:** Since each coin toss is independent and the probability of heads or tails in each toss is $\frac{1}{2}$, the probability of any specific sequence of 30 heads and 30 tails is:

$$\left(\frac{1}{2}\right)^{60}$$

Therefore, the probability of getting exactly 30 heads in 60 tosses is:

$$P(30 \text{ heads}) = \binom{60}{30} \left(\frac{1}{2}\right)^{60}$$

Calculation

1. **Calculate $\binom{60}{30}$:**

$$\binom{60}{30} = \frac{60!}{30! \cdot 30!}$$

2. **Calculate $\left(\frac{1}{2}\right)^{60}$:**

$$\left(\frac{1}{2}\right)^{60} = \frac{1}{2^{60}}$$

Numerical Computation

1. **Calculate $\binom{60}{30}$:**

$$\binom{60}{30} \approx 1.182 \times 10^{17}$$

2. **Calculate $\left(\frac{1}{2}\right)^{60}$:**

$$\left(\frac{1}{2}\right)^{60} \approx 8.673 \times 10^{-19}$$

Final Probability

Multiply the two values:

$$P(30 \text{ heads}) = 1.182 \times 10^{17} \times 8.673 \times 10^{-19} \approx 0.102578$$

Thus, the probability of getting exactly 30 heads in 60 coin tosses is approximately:

$$P(30 \text{ heads}) \approx 0.102578$$