1 Lesson 12 Additional Exercise 1

You are enrolled in 3 courses this quarter, and the breakdown of majors by class is as follows:
Class 1: 4 Statistics Majors and 6 Computer Science Majors
Class 2: 17 Statistics Majors and 13 Computer Science Majors
Class 3: 11 Statistics Majors and 9 Computer Science Majors

- If you take a random sample of 20% of the students in Class 1, what is the probability that the number of Statistics majors in your sample is equal to the number of Computer Science majors? (Note: In a simple random sample, each student can be selected at most once.)
- Now, suppose you pick one of your 3 classes at random and then choose a random sample of 20% of students from that class. What is the probability that the number of Statistics majors in your sample is equal the the number of Computer Science majors in your sample?

2 Answer

2.1 Solution to Question 1a

Class 1 Composition:

- 4 Statistics majors
- 6 Computer Science majors
- Total students: $4 + 6 = 10$

Sampling:

- You are taking a simple random sample of 20% of the students in Class 1.
- 20% of 10 students means you are sampling 2 students.

We want to find the probability that the sample contains 1 Statistics major and 1 Computer Science major.

2.2 Hypergeometric Distribution Parameters

This scenario fits a hypergeometric distribution with the following parameters:

- $N = 10$ (Total number of students in Class 1)
- $N_1 = 4$ (Number of Statistics majors)
- $N_0 = 6$ (Number of Computer Science majors)
- $n = 2$ (Number of students sampled)

We are interested in finding the probability that $X = 1$, where X is the number of Statistics majors in the sample.

2.3 Calculations

The hypergeometric probability mass function (PMF) is given by:

$$P(X = x) = \frac{\binom{N_1}{x}\binom{N_0}{n-x}}{\binom{N}{n}}$$

Substituting the values:

$$P(X = 1) = \frac{\binom{4}{1}\binom{6}{1}}{\binom{10}{2}}$$

Calculating the binomial coefficients:

- $\binom{4}{1} = 4$
- $\binom{6}{1} = 6$
- $\binom{10}{2} = \frac{10\times 9}{2} = 45$

Thus:

$$P(X = 1) = \frac{4 \times 6}{45} = \frac{24}{45} = \frac{8}{15} \approx 0.5333$$

2.4 Final Answer to Question 1a

The probability that the number of Statistics majors in your sample is equal to the number of Computer Science majors is $\frac{8}{15}$ or approximately 0.5333.

2.5 Solution to Question 1b

We are to:

1. Randomly pick one of the three classes.
2. Take a simple random sample of 20% of the students from the selected class.
3. Calculate the probability that the number of Statistics majors in the sample equals the number of Computer Science majors.

2.6 Calculate the Sample Size for Each Class

- Class 1: 20% of 10 students = 2 students
- Class 2: 20% of 30 students = 6 students
- Class 3: 20% of 20 students = 4 students

2.7 Use Hypergeometric Distribution

For each class, we'll calculate the probability that the number of Statistics majors equals the number of Computer Science majors in the sample.

2.8 Class 1:

Sample size = 2 students. We need 1 Statistics major and 1 Computer Science major.

$$P(\text{Class 1}) = \frac{\binom{4}{1}\binom{6}{1}}{\binom{10}{2}} = \frac{4 \times 6}{45} = \frac{24}{45} = \frac{8}{15} \approx 0.5333$$

2.9 Class 2:

Sample size = 6 students. We need 3 Statistics majors and 3 Computer Science majors.

$$P(\text{Class 2}) = \frac{\binom{17}{3}\binom{13}{3}}{\binom{30}{6}} = \frac{680 \times 286}{593775} \approx 0.3275$$

2.10 Class 3:

Sample size = 4 students. We need 2 Statistics majors and 2 Computer Science majors.

$$P(\text{Class 3}) = \frac{\binom{11}{2}\binom{9}{2}}{\binom{20}{4}} = \frac{55 \times 36}{4845} = \frac{1980}{4845} \approx 0.4087$$

2.11 Calculate the Final Probability

The probability of picking any of the three classes is $\frac{1}{3}$. Therefore, the final probability is:

$$P(\text{final}) = \frac{1}{3} \times P(\text{Class 1}) + \frac{1}{3} \times P(\text{Class 2}) + \frac{1}{3} \times P(\text{Class 3})$$

Substituting the values:

$$P(\text{final}) = \frac{1}{3} \times 0.5333 + \frac{1}{3} \times 0.3275 + \frac{1}{3} \times 0.4087$$

$$P(\text{final}) \approx \frac{1}{3} \times (0.5333 + 0.3275 + 0.4087) = \frac{1}{3} \times 1.2695 \approx 0.4232$$

2.12 Final Answer

The probability that the number of Statistics majors in your sample equals the number of Computer Science majors in your sample is approximately 0.4232.