1 Lesson 13 Example 1

Show Mr. Pepys that C (at least 3 sixes in 18 rolls) is actually the least likely of the three options.

2 Answer

We use the binomial distribution to calculate the probabilities of each option.

The binomial probability mass function is given by:

$$P(X = x) = \binom{n}{k} p^x (1-p)^{n-x}$$

2.1 At least 1 six in 6 rolls

We know from the text that the probability of at least 1 six in 6 rolls is

$$P(\text{at least 1 six in 6 rolls}) \approx 0.665$$

2.2 At least 2 sixes in 12 rolls

We also know from the text that the probability of at least 2 sixes in 12 rolls is

$$P(\text{at least 2 sixes in 12 rolls}) \approx 0.6187$$

2.3 At least 3 sixes in 18 rolls

We calculate the probability of fewer than 3 sixes (i.e., 0, 1, or 2 sixes).

$$P(\text{at least 3 sixes in 18 rolls}) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$$

$$P(X = 0) = \binom{18}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{18} = \left(\frac{5}{6}\right)^{18} \approx 0.0376$$

$$P(X = 1) = \binom{18}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{17} = 18 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^{17} \approx 0.1352$$

$$P(X = 2) = \binom{18}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{16} \approx 0.2299$$

$$P(\text{at least 3 sixes}) = 1 - (0.0376 + 0.1352 + 0.2299) \approx 0.5973$$

2.4 Conclusion

The probabilities of each option are:

- $P(\text{at least 1 six in 6 rolls}) \approx 0.665$
- $P(\text{at least 2 sixes in 12 rolls}) \approx 0.6187$
- $P(\text{at least 3 sixes in 18 rolls}) \approx 0.5973$

Thus, getting at least 3 sixes in 18 rolls is the least likely of the three options.