1 Lesson 13 Example 3

In the World Series of baseball, two teams (call them A and B) play a sequence of games against each other, and the first team to win four games wins the series. Suppose team A is slightly better, with a 0.6 probability of winning each game, and assume the games are independent. What is the probability that team A wins the series.

(Hint: After 7 games, one of the teams must have won the Series. Even though the teams only play until one team has won four games, this calculation is easiest if you assume that the teams always play 7 games.)

2 Answer

Let X be the number of games that Team A wins in the 7 games played. Then $X \sim \text{Binomial}(7, 0.6)$, meaning X follows a binomial distribution with $n = 7$ trials and probability $p = 0.6$ of success (i.e., Team A winning a game).

We are interested in calculating $P(X \geq 4)$, the probability that Team A wins at least 4 games in the series. This is given by the sum of the probabilities that Team A wins exactly 4, 5, 6, or 7 games:

$$P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7)$$

The probability mass function of the binomial distribution is:

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

Substituting $n = 7$, $p = 0.6$, and $1 - p = 0.4$, we can compute each term:

$$P(X = 4) = \binom{7}{4} (0.6)^4 (0.4)^3 = 35 \times (0.1296) \times (0.064) = 0.2903$$

$$P(X = 5) = \binom{7}{5} (0.6)^5 (0.4)^2 = 21 \times (0.0778) \times (0.16) = 0.2614$$

$$P(X = 6) = \binom{7}{6} (0.6)^6 (0.4)^1 = 7 \times (0.0467) \times (0.4) = 0.1307$$

$$P(X = 7) = \binom{7}{7} (0.6)^7 = 1 \times (0.02799) = 0.02799$$

Now, summing these probabilities:

$$P(X \geq 4) = 0.2903 + 0.2614 + 0.1307 + 0.02799 = 0.7104$$

Conclusion

The probability that Team A wins the World Series, assuming they play all 7 games, is approximately 0.7104 or 71.04%.