1   Lesson 12 Example 3

The state proposes a lottery in which you select 6 numbers from 1 to 15. When it is time to draw, the lottery selects 8 different numbers, and you win if at least 4 of the 6 numbers you picked are among the 8 numbers that the lottery drew. What is the probability you win the prize?

2   Answer

2.1   Parameters of the Hypergeometric Distribution

The problem fits a hypergeometric distribution with the following parameters:

- $N = 15$ (Total number of possible numbers)

- $N_1 = 6$ (Number of numbers you selected)

- $N_0 = 9$ (Number of numbers you did not select)

- $n = 8$ (Number of numbers drawn by the lottery)

2.2   Winning Condition

You win if at least 4 of your selected numbers match the drawn numbers. Therefore, we need to calculate:

$$P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$$

where X is the number of matches.

2.3   Calculations

- $P(X = 4)$:

$$P(X = 4) = \frac{\binom{6}{4}\binom{9}{4}}{\binom{15}{8}} = \frac{15 \times 126}{6435} = \frac{1890}{6435} \approx 0.2937$$

- $P(X = 5)$:

$$P(X = 5) = \frac{\binom{6}{5}\binom{9}{3}}{\binom{15}{8}} = \frac{6 \times 84}{6435} = \frac{504}{6435} \approx 0.0783$$

- $P(X = 6)$:

$$P(X = 6) = \frac{\binom{6}{6}\binom{9}{2}}{\binom{15}{8}} = \frac{1 \times 36}{6435} = \frac{36}{6435} \approx 0.0056$$

2.4　Final Probability

Summing these probabilities gives:

$$P(X \geq 4) \approx 0.2937 + 0.0783 + 0.0056 = 0.3776$$

Thus, the probability that you win the prize is approximately 0.3776.