

## 1 Lesson 16 Example 2

2. The number of organisms in  $V$  cubic meters of ballast water discharged from a ship follows a  $\text{Poisson}(\mu = 10V)$  distribution. (See "Counting at Low Concentrations: The Statistical Challenges of Verifying Ballast Water Discharge Standards", *Ecological Applications*, 2013:339–351.)

- (a) What is the probability that there are at least 12 organisms in 1.5 cubic meters of discharge?
- (b) For what amount of discharge would the probability of containing at least one organism be 0.999?

## 2 Answer

### 2.1 Probability of at Least 12 Organisms in 1.5 Cubic Meters

For  $V = 1.5$  cubic meters, the Poisson parameter is given by:

$$\mu = 10 \times 1.5 = 15$$

We are asked to find the probability that there are at least 12 organisms, which is:

$$P(X \geq 12) = 1 - P(X \leq 11)$$

Where  $X$  is a Poisson random variable with  $\mu = 15$ . Using the Symbulate code below, we find:

```
from symbulate import *

# Poisson distribution with parameter mu = 15
mu = 15
P_geq_12 = 1 - Poisson(mu).cdf(11)

# Output the result
print(P_geq_12)
```

$$P(X \geq 12) = 1 - P(X \leq 11) \approx 1 - 0.1848 = 0.8152$$

Thus, the probability that there are at least 12 organisms in 1.5 cubic meters of discharge is approximately 0.8152, or 81.52%.

### 2.2 Volume of Discharge for a Probability of 0.999

We are asked to find the volume  $V$  such that the probability of observing at least one organism is 0.999. The number of organisms follows a Poisson distribution with  $\mu = 10V$ , and the probability of at least one organism is:

$$P(X \geq 1) = 1 - P(X = 0)$$

where:

$$P(X = 0) = e^{-10V}$$

Thus, we need to solve:

$$1 - e^{-10V} = 0.999$$

This simplifies to:

$$e^{-10V} = 0.001$$

Taking the natural logarithm of both sides:

$$-10V = \ln(0.001)$$

$$V = -\frac{\ln(0.001)}{10} = \frac{6.9078}{10} = 0.69078$$

Thus, the required volume  $V$  is approximately 0.691 cubic meters.

## 2.3 Conclusion

- The probability of observing at least 12 organisms in 1.5 cubic meters of discharge is approximately 0.8152.
- For the probability of observing at least one organism to be 0.999, the required volume of discharge is approximately 0.691 cubic meters.