1 Lesson 12 Additional Exercise 2

In Texas Hold'em, each player has 2 cards of their own, and all players share 5 cards in the center of the table. A player has a flush when there are at least 5 cards of the same suit out of the 7 total cards. The deck is shuffled between hands, so that the probability you obtain a flush is independent from hand to hand. What is the probability that you get a flush at least once in 10 hands of Texas Hold'em? (Hint: First, calculate the probability of a flush of spades. Then, repeat for the other suits, and add the probabilities together to obtain the overall probability of a flush.

2 Answer

In Texas Hold'em, each player has 2 cards, and there are 5 community cards. A flush occurs when at least 5 out of the 7 cards are of the same suit. We calculate the probability of getting a flush at least once in 10 hands using the hypergeometric distribution.

2.1 Probability of a Flush of Spades Using the Hypergeometric Distribution

The probability mass function of the hypergeometric distribution is given by:

$$P(X = x) = \frac{\binom{N_1}{x}\binom{N_0}{n-x}}{\binom{N}{n}}$$

Where:

- $N = 52$ (Total number of cards in the deck)
- $N_1 = 13$ (Number of spades in the deck)
- $N_0 = 39$ (Number of non-spades in the deck)
- $n = 7$ (Number of cards drawn)
- x (Number of spades in the 7 cards)

We need to calculate the probability for at least 5 spades:

- $P(X = 5) = \frac{\binom{13}{5}\binom{39}{2}}{\binom{52}{7}} \approx 0.00713$
- $P(X = 6) = \frac{\binom{13}{6}\binom{39}{1}}{\binom{52}{7}} \approx 0.0005$
- $P(X = 7) = \frac{\binom{13}{7}\binom{39}{0}}{\binom{52}{7}} \approx 0.0000128$

2.2 Sum the Probabilities for a Flush of Spades

$$P(\text{Flush of Spades}) = 0.00713 + 0.0005 + 0.0000128 \approx 0.00764$$

2.3 Extend to All Suits

$$P(\text{Flush}) = 4 \times P(\text{Flush of Spades}) = 4 \times 0.00764 \approx 0.03057$$

2.4 Probability of Getting a Flush at Least Once in 10 Hands

$$P(\text{No Flush}) = 1 - P(\text{Flush}) \approx 1 - 0.03057 \approx 0.96943$$

$$P(\text{No Flush in 10 Hands}) = (P(\text{No Flush}))^{10} \approx 0.96943^{10} \approx 0.7331$$

$$P(\text{At Least One Flush in 10 Hands}) = 1 - P(\text{No Flush in 10 Hands}) \approx 1 - 0.7331 \approx 0.2669$$

2.5 Final Answer

The probability that you get a flush at least once in 10 hands of Texas Hold'em is approximately 0.2669.