1 Lesson 9 Example 1

The rare mineral unobtanium is present in only 1% of rocks in a mine. You have an unobtanium detector, which never fails to detect unobtanium when it is present. Otherwise, it is still reliable, returning accurate readings 90% of the time when unobtanium is not present.

- What is $P(\text{Unobtanium detected} \mid \text{Unobtanium present})$
- What is $P(\text{Unobtanium present} \mid \text{Unobtanium detected})$

2 Answer

2.1 Given

- $P(\text{Unobtanium present}) = 0.01$ (1% of rocks contain unobtanium)
- $P(\text{Unobtanium detected} \mid \text{Unobtanium present}) = 1$ (the detector never fails to detect unobtanium when it is present)
- $P(\text{Unobtanium not detected} \mid \text{Unobtanium not present}) = 0.90$ (90% accurate when unobtanium is not present)

We need to calculate:

- $P(\text{Unobtanium detected} \mid \text{Unobtanium present})$
- $P(\text{Unobtanium present} \mid \text{Unobtanium detected})$

2.2 Part (a)

$P(\text{Unobtanium detected} \mid \text{Unobtanium present})$ is given directly in the problem statement:

$$P(\text{Unobtanium detected} \mid \text{Unobtanium present}) = 1$$

2.3 Part (b)

To find $P(\text{Unobtanium present} \mid \text{Unobtanium detected})$, we need to use Bayes' Rule.

Bayes' Rule states:

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

In this context:

- A is "Unobtanium present"
- B is "Unobtanium detected"

So,

$$P(\text{Unobtanium present} \mid \text{Unobtanium detected}) = \frac{P(\text{Unobtanium detected} \mid \text{Unobtanium present}) \cdot P(\text{Unobtanium present})}{P(\text{Unobtanium detected})}$$

We already know:

$$P(\text{Unobtanium detected} \mid \text{Unobtanium present}) = 1$$

$$P(\text{Unobtanium present}) = 0.01$$

To find $P(\text{Unobtanium detected})$, we consider two cases:

1. The detector correctly detects unobtanium when it is present.
2. The detector incorrectly detects unobtanium when it is not present.

$$P(\text{Unobtanium detected}) = P(\text{Unobtanium detected} \mid \text{Unobtanium present}) \cdot P(\text{Unobtanium present}) + P(\text{Unobtanium detected} \mid \text{Unobtanium not present}) \cdot P(\text{Unobtanium not present})$$

Substituting the known values:

- $P(\text{Unobtanium not present}) = 1 - 0.01 = 0.99$
- $P(\text{Unobtanium detected} \mid \text{Unobtanium not present}) = 1 - 0.90 = 0.10$ (false positive rate)

$$P(\text{Unobtanium detected}) = (1 \times 0.01) + (0.10 \times 0.99)$$

$$P(\text{Unobtanium detected}) = 0.01 + 0.099 = 0.109$$

Finally, using Bayes' Rule:

$$P(\text{Unobtanium present} \mid \text{Unobtanium detected}) = \frac{1 \times 0.01}{0.109} \approx 0.0917$$

3 Conclusion

1. $P(\text{Unobtanium detected} \mid \text{Unobtanium present}) = 1$
2. $P(\text{Unobtanium present} \mid \text{Unobtanium detected}) \approx 0.0917$ or 9.17%