1 Lesson 3 Example 1

Prove the identity:

$$k\binom{m}{k} = m\binom{m-1}{k-1}$$

By expanding the combinations and simplifying using:

$$\binom{N}{n} = \frac{N!}{n!(N-n)!}$$

2 Answer

1. Expand the left-hand side (LHS):

$$k\binom{m}{k} = k\frac{m!}{k!(m-k)!}$$

2. Simplify the expression:

   The factor k in the numerator and denominator cancels out:

$$k\frac{m!}{k!(m-k)!} = \frac{k \cdot m!}{k!(m-k)!} = \frac{m!}{(k-1)!(m-k)!}$$

3. Recognize that this is the same as the right-hand side (RHS):

$$m\binom{m-1}{k-1} = m\frac{(m-1)!}{(k-1)!((m-1)-(k-1))!} = m\frac{(m-1)!}{(k-1)!(m-k)!}$$

4. Notice that $m \cdot (m-1)!$ simplifies to $m!$:

$$m\frac{(m-1)!}{(k-1)!(m-k)!} = \frac{m!}{(k-1)!(m-k)!}$$

5. Therefore, we have:

$$k\binom{m}{k} = m\binom{m-1}{k-1}$$