1 Lesson 16 Example 1

If you buy a lottery ticket in 50 lotteries, in each of which your chances of winning a prize of 1/100, what is the probability that you will win a prize?

a. at least once?

b. exactly twice?

c. at least twice?

Calculate both the exact probabilities (using the binomial distribution) and the approximate probabilities (using the Poisson distribution).

2 Answer

2.1 Part 1: Exact Probabilities Using the Binomial Distribution

The binomial distribution is defined as:

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where:

- $n = 50$ is the number of lotteries,
- $p = \frac{1}{100} = 0.01$ is the probability of winning in each lottery,
- X is the number of wins (prizes).

Probability of Winning at Least Once

The probability of winning at least once is:

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X = 0) = \binom{50}{0} (0.01)^0 (0.99)^{50} = (0.99)^{50}$$

Thus:

$$P(X \geq 1) = 1 - (0.99)^{50} \approx 0.395$$

Probability of Winning Exactly Twice

The probability of winning exactly twice is:

$$P(X = 2) = \binom{50}{2} (0.01)^2 (0.99)^{48} \approx 0.0756$$

Probability of Winning at Least Twice

The probability of winning at least twice is:

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$$

We already have $P(X = 0)$. Now, calculate:

$$P(X = 1) = \binom{50}{1}(0.01)^1(0.99)^{49} \approx 0.3056$$

$$P(X \geq 2) = 1 - (0.99)^{50} - 0.3056 \approx .0894$$

2.2 Part 2: Approximate Probabilities Using the Poisson Distribution

The Poisson distribution is an approximation of the binomial distribution when n is large and p is small. The Poisson distribution is defined as:

$$f(x) = e^{-\mu}\frac{\mu^x}{x!}$$

Probability of Winning at Least Once

The probability of winning at least once is:

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X = 0) = e^{-0.5}\frac{0.5^0}{0!} = e^{-0.5} \approx 0.60653$$

Thus:

$$P(X \geq 1) = 1 - e^{-0.5} \approx 0.3935$$

Probability of Winning Exactly Twice

The probability of winning exactly twice is:

$$P(X = 2) = e^{-0.5}\frac{0.5^2}{2!} \approx 0.0758$$

Probability of Winning at Least Twice

The probability of winning at least twice is:

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$$

Where:

$$P(X = 1) = e^{-0.5}\frac{0.5^1}{1!} \approx 0.3033$$

$$P(X \geq 2) = 1 - 0.60653 - 0.3033 \approx 0.09017$$