1 Lesson 6 Example 2

Dr. No has captured James Bond and forces him to play a game of Russian roulette. (Note: Russian roulette is very different from the casino game roulette!) Dr. No shows him a revolver with 6 chambers, all initially empty. He places 2 bullets into adjacent chambers. He makes Bond spin the cylinder, place the muzzle against his head, and pull the trigger. He survives! Luckily for Bond, the cylinder stopped on one of the empty chambers.

Now Dr. No gives Bond two options: he can re-spin the cylinder before firing again or he can fire with the gun in its current state. (Keep in mind that the cylinder rotates to the next chamber each time the gun is fired.) What option should Bond choose to maximize his chance of surviving?

a. Clearly write out the conditional probability of interest using $P(B|A)$ notation.

b. Find the probability. *(Hint: You should not need to do any calculations. You should be able to find the probability just by thinking carefully about the information you have. Make sure you explain your reasoning carefully.)*

2 Answer

Let:

- A be the event that Bond survives the first trigger pull (i.e., the first chamber is empty).
- B be the event that Bond survives the second trigger pull.

The conditional probability of interest, $P(B|A)$, is the probability that Bond survives the second trigger pull given that he has survived the first pull.

Analysis with Conditional Probability

1. Re-spin the Cylinder:

If Bond re-spins the cylinder, the scenario resets:

$$P(B|A)_{\text{re-spin}} = \frac{4}{6} = \frac{2}{3}$$

2. Do Not Re-spin the Cylinder:

There are exactly four possible positions where the first chamber could be empty. Given that the bullets are adjacent:

- **One of these four cases** results in the next chamber being loaded because the bullets are adjacent, and the gun barrel rotates in one direction. If the empty chamber is next to the first bullet, the next chamber will contain a bullet.

- In the other three cases, the next chamber will be empty.

Thus, the probability that the next chamber is loaded is $\frac{1}{4}$. The probability that the next chamber is empty (and thus, Bond survives) is:

$$P(B|A)_{\text{no re-spin}} = 1 - \frac{1}{4} = \frac{3}{4}$$

Since $P(B|A)_{\text{no re-spin}} = \frac{3}{4}$ is greater than $P(B|A)_{\text{re-spin}} = \frac{2}{3}$, Bond should **not re-spin** the cylinder to maximize his chance of survival.