1 Lesson 7 Example 1

One card is dealt off the top of a well-shuffled deck of cards. Is the event that the card is a heart independent of the event that the card is an ace?

2 Answer

To determine whether the event that the card is a heart is independent of the event that the card is an ace, we need to assess the probability of these events and their intersection.

Definitions

- **Event A**: The card is a heart.
- **Event B**: The card is an ace.

Two events A and B are independent if:

$$P(A \cap B) = P(A) \times P(B)$$

Probabilities

- $P(A)$: The probability that the card is a heart.
  - There are 13 hearts in a deck of 52 cards.
  - $P(A) = \frac{13}{52} = \frac{1}{4}$.
- $P(B)$: The probability that the card is an ace.
  - There are 4 aces in a deck of 52 cards.
  - $P(B) = \frac{4}{52} = \frac{1}{13}$.
- $P(A \cap B)$: The probability that the card is both a heart and an ace.
  - There is exactly 1 card that is both a heart and an ace (the Ace of Hearts).
  - $P(A \cap B) = \frac{1}{52}$.

Check for Independence

Now, let's check if $P(A \cap B) = P(A) \times P(B)$.

$$P(A) \times P(B) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

$$P(A \cap B) = \frac{1}{52}$$

Since $P(A \cap B) = P(A) \times P(B)$, the events A and B are **independent**.

Conclusion

Yes, the event that the card is a heart is independent of the event that the card is an ace.