1 Lesson 3 Example 2

Prove the identity:

$$\binom{m}{k} = \binom{m}{m-k}$$

(Note I am only giving the algebraic proof here. The story proof can vary).

2 Answer

1. Start with the definition of the binomial coefficient on the left-hand side (LHS):

$$\binom{m}{k} = \frac{m!}{k!(m-k)!}$$

2. Consider the right-hand side (RHS) of the identity:

$$\binom{m}{m-k} = \frac{m!}{(m-k)!(m-(m-k))!}$$

3. Simplify the denominator of the RHS:

$$m - (m-k) = k$$

4. Substitute this into the RHS expression:

$$\binom{m}{m-k} = \frac{m!}{(m-k)!k!}$$

5. Compare the LHS and the RHS:

$$\binom{m}{k} = \frac{m!}{k!(m-k)!} \quad \text{and} \quad \binom{m}{m-k} = \frac{m!}{(m-k)!k!}$$

6. Observe that both expressions are identical:

$$\frac{m!}{k!(m-k)!} = \frac{m!}{(m-k)!k!}$$