1 Lesson 8 Example 1

Here are some things we already know about a deck of cards:

- The top card in a shuffled deck of cards has a $\frac{13}{52}$ chance of being a diamond.
- If the top card is a diamond, then the second card has a $\frac{12}{51}$ chance of being a diamond.
- If the top card is not a diamond, then the second card has a $\frac{13}{51}$ chance of being a diamond.

Now, suppose we "burn" (i.e., discard) the top card without looking at it. What is the probability that the second card is a diamond? Use the Law of Total Probability, conditioning on the top card. Does burning cards affect probabilities?

2 Answer

- Let D_1 be the event that the top card is a diamond.
- Let D_2 be the event that the second card is a diamond.

2.1 Known Probabilities

1. **Probability that the top card is a diamond:**
$$P(D_1) = \frac{13}{52} = \frac{1}{4}$$
2. **Probability that the top card is not a diamond:**
$$P(\neg D_1) = \frac{39}{52}$$
3. **Probability that the second card is a diamond given the top card is a diamond:**
$$P(D_2 \mid D_1) = \frac{12}{51}$$
4. **Probability that the second card is a diamond given the top card is not a diamond:**
$$P(D_2 \mid \neg D_1) = \frac{13}{51}$$

2.2 Applying the Law of Total Probability

The overall probability that the second card is a diamond is given by:

$$P(D_2) = P(D_1) \cdot P(D_2 \mid D_1) + P(\neg D_1) \cdot P(D_2 \mid \neg D_1)$$

Substituting the known probabilities:

$$P(D_2) = \left(\frac{13}{52}\right) \cdot \left(\frac{12}{51}\right) + \left(\frac{39}{52}\right) \cdot \left(\frac{13}{51}\right)$$

Simplifying:

$$P(D_2) = \frac{13}{52} \times \frac{12}{51} + \frac{39}{52} \times \frac{13}{51}$$

$$P(D_2) = \frac{156}{2652} + \frac{507}{2652} = \frac{663}{2652} = \frac{13}{52} = \frac{1}{4}$$

Conclusion

The probability that the second card is a diamond, even after burning the top card, is still $\frac{1}{4}$. **Burning the top card does not affect the probability** that the second card is a diamond.