1 Lesson 16 Example 3

Thelma calculates the exact probability of winning more than 50% of the time when she places 1000 bets on red in roulette. Louise calculates an approximate probability using the Poisson distribution. They get very different answers. What answers did they get, and why did the Poisson approximation fail in this case?

2 Answer

2.1 Thelma's Exact Calculation Using the Binomial Distribution

In roulette, the probability of winning a bet on red is $p \approx \frac{18}{38}$ (for American roulette). Thelma calculates the exact probability using the binomial distribution, where:

- $n = 1000$ is the number of bets,
- $p = \frac{18}{38} \approx 0.4737$ is the probability of winning each bet.

The exact probability of winning more than 50% of the bets (i.e., more than 500 wins) is given by:

$$P(X > 500) = 1 - P(X \leq 500)$$

where $X \sim \text{Binomial}(1000, 0.4737)$. Using the Binomial function in Symbulate, we find:

$$P(X > 500) \approx 0.04489$$

2.2 Louise's Approximate Calculation Using the Poisson Distribution

Louise approximates this probability using the Poisson distribution. The Poisson approximation is typically used when:

- n is large, and
- p is small.

Here, $p = \frac{18}{38} \approx 0.4737$, which is not small. Nevertheless, Louise uses the Poisson distribution with parameter:

$$\mu = n \times p = 1000 \times 0.4737 = 473.7$$

The approximate probability of winning more than 500 bets is (using the Poisson function in Symbulate):

$$P(X > 500) = 1 - P(X \leq 500) \approx 0.1098$$

where $X \sim \text{Poisson}(473.7)$.

2.3 Why the Poisson Approximation Fails

The Poisson approximation fails because the success probability p is not small. The Poisson distribution is derived as an approximation of the binomial distribution when p is small and n is large. In this case, however, p is close to 0.5, meaning the binomial distribution is approximately symmetric. The Poisson distribution, being skewed, does not accurately represent the behavior of the binomial distribution in this scenario.

- Thelma's answer using the binomial distribution is the correct probability, which reflects the symmetry of the binomial distribution with $p \approx 0.4737$.
- Louise's answer using the Poisson approximation is incorrect because the approximation is unsuitable when p is close to 0.5, leading to a significantly different and larger result.