1 Lesson 1 Example 1

In the casino game craps, two dice are rolled by a player, called the "shooter". The first roll is called the "come-out roll". The shooter wins on the come-out roll if they roll a 7 or an 11. On the other hand, they lose if the come-out roll is a 2, 3, or 12. Otherwise, the game continues.

- What is the probability that the shooter wins on the come-out roll?
- What is the probability that the shooter loses on the come-out roll?
- What is the probability that the game continues?
- If you add up the three probabilities you just calculated, what do you get? Why does this make sense?

2 Answer

2.1 Possible Outcomes of Rolling Two Dice

Each die has 6 faces, so there are $6 \times 6 = 36$ possible outcomes when rolling two dice.

2.2 Winning on the Come-Out Roll

The shooter wins if they roll a 7 or an 11.

- **Rolling a 7**: The possible combinations are:

$$(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$$

There are 6 ways to roll a 7.

- **Rolling an 11**: The possible combinations are:

$$(5,6), (6,5)$$

There are 2 ways to roll an 11.

Therefore, the total number of winning combinations is $6 + 2 = 8$.

$$P(\text{win}) = \frac{8}{36} = \frac{2}{9} \approx 0.2222$$

2.3 Losing on the Come-Out Roll

The shooter loses if they roll a 2, 3, or 12.

- **Rolling a 2**: The possible combination is:

$$(1, 1)$$

There is 1 way to roll a 2.

- **Rolling a 3**: The possible combinations are:

$$(1, 2), (2, 1)$$

There are 2 ways to roll a 3.

- **Rolling a 12**: The possible combination is:

$$(6, 6)$$

There is 1 way to roll a 12.

Therefore, the total number of losing combinations is $1 + 2 + 1 = 4$.

$$P(\text{lose}) = \frac{4}{36} = \frac{1}{9} \approx 0.1111$$

2.4 Game Continues

The game continues if the shooter rolls any number other than 7, 11, 2, 3, or 12. These numbers are 4, 5, 6, 8, 9, and 10.

- **Rolling a 4**: The possible combinations are:

$$(1, 3), (2, 2), (3, 1)$$

There are 3 ways to roll a 4.

- **Rolling a 5**: The possible combinations are:

$$(1, 4), (2, 3), (3, 2), (4, 1)$$

There are 4 ways to roll a 5.

- **Rolling a 6**: The possible combinations are:

$$(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)$$

There are 5 ways to roll a 6.

- **Rolling an 8**: The possible combinations are:

$$(2,6),(3,5),(4,4),(5,3),(6,2)$$

There are 5 ways to roll an 8.

- **Rolling a 9**: The possible combinations are:

$$(3,6),(4,5),(5,4),(6,3)$$

There are 4 ways to roll a 9.

- **Rolling a 10**: The possible combinations are:

$$(4,6),(5,5),(6,4)$$

There are 3 ways to roll a 10.

Therefore, the total number of combinations that continue the game is:

$$3+4+5+5+4+3=24$$

$$P(\text{continue})=\frac{24}{36}=\frac{2}{3}\approx 0.6667$$

2.5 Sum of Probabilities

The sum of the probabilities is:

$$P(\text{win})+P(\text{lose})+P(\text{continue})=\frac{2}{9}+\frac{1}{9}+\frac{2}{3}$$

Convert $\frac{2}{3}$ to a common denominator of 9:

$$\frac{2}{3}=\frac{6}{9}$$

Now sum them up:

$$\frac{2}{9}+\frac{1}{9}+\frac{6}{9}=\frac{9}{9}=1$$

2.6 Interpretation

The sum of the probabilities equals 1, which makes sense because these three outcomes (winning, losing, or the game continuing) are exhaustive and mutually exclusive. Thus, they account for all possible outcomes of the come-out roll in craps.