1 Lesson 14 Example 1

Carl is a bad driver, so each time he takes the driving test, he only has a 30% chance of passing, independently of all previous attempts. If he takes the driving test over and over until he passes, what is the probability that he passes within his first 5 attempts?

2 Answer

This situation follows a geometric distribution, where Carl keeps taking the test until he passes. The probability of passing the test on any attempt is $p = 0.3$, and the probability of not passing is $1 - p = 0.7$.

The probability mass function (PMF) of a geometric distribution is given by:

$$P(X = x) = (1 - p)^{x-1}p$$

where X is the number of attempts until Carl passes.

We are interested in the probability that Carl passes within his first 5 attempts, which is:

$$P(X \leq 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

Substituting $p = 0.3$ into the formula, we calculate each term:

$$P(X = 1) = (1 - 0.3)^0 \times 0.3 = 0.3$$

$$P(X = 2) = (1 - 0.3)^1 \times 0.3 = 0.7 \times 0.3 = 0.21$$

$$P(X = 3) = (1 - 0.3)^2 \times 0.3 = 0.7^2 \times 0.3 = 0.49 \times 0.3 = 0.147$$

$$P(X = 4) = (1 - 0.3)^3 \times 0.3 = 0.7^3 \times 0.3 = 0.343 \times 0.3 = 0.1029$$

$$P(X = 5) = (1 - 0.3)^4 \times 0.3 = 0.7^4 \times 0.3 = 0.2401 \times 0.3 = 0.07203$$

Summing these probabilities:

$$P(X \leq 5) = 0.3 + 0.21 + 0.147 + 0.1029 + 0.07203 = 0.83193$$

Conclusion

The probability that Carl passes the driving test within his first 5 attempts is approximately 0.8319, or 83.19%.