1 Lesson 6 Example 1

You and your friend Amy are each dealt two cards: hers face up and yours face down. In which of the following scenarios are you more likely to have a pair:

- When she has a pair of queens?
- When she has a queen and a 5?

2 Answer

Credit to this StackOverflow post: StackOverflow Post

- Let S_{Q5} denote the event that Amy has a queen and a five.
- Let S_{QQ} denote the event that Amy has a queen and a queen.
- Let S_{Pair} denote the event that you have a pair (of any rank).

We will compare the probability of these events using the conditional probability formula:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Specifically, we are looking for:

- $P(S_{Pair}|S_{Q5})$
- $P(S_{Pair}|S_{QQ})$

2.1 Calculation for $P(S_{Pair}|S_{Q5})$

1. Calculate $P(S_{Q5})$:
$$P(S_{Q5}) = \frac{\binom{4}{1} \times \binom{4}{1}}{\binom{52}{2}}$$
2. Calculate $P(S_{Q5} \text{ and } S_{Pair})$:
With a queen and a five drawn by Amy, there are:
$$\left[11 \times \binom{4}{2}\right] + \left[2 \times \binom{3}{2}\right] = 72$$
ways that your cards may be paired up. The 11 refers to the ranks other than queen and five, and the 2 represents the queen and five ranks.
So,
$$P(S_{Q5} \text{ and } S_{Pair}) = \frac{\binom{4}{1} \times \binom{4}{1} \times 72}{\binom{52}{2} \times \binom{50}{2}}$$

3. Calculate $P(S_{Pair}|S_{Q5})$:

$$P(S_{Pair}|S_{Q5}) = \frac{\binom{4}{1} \times \binom{4}{1} \times 72}{\binom{52}{2} \times \binom{50}{2}} \div P(S_{Q5}) = \frac{72}{\binom{50}{2}}$$

2.2 Calculation for $P(S_{Pair}|S_{QQ})$

1. Calculate $P(S_{QQ})$:

$$P(S_{QQ}) = \frac{\binom{4}{2}}{\binom{52}{2}}$$

2. Calculate $P(S_{QQ} \text{ and } S_{Pair})$:

   With two queens drawn by Amy, there are:

$$\left[12 \times \binom{4}{2}\right] + \left[1 \times \binom{2}{2}\right] = 73$$

   ways that your cards may be paired up. The 12 refers to the ranks other than Q, and the 1 represents the Q rank.

   So,

$$P(S_{QQ} \text{ and } S_{Pair}) = \frac{\binom{4}{2} \times 73}{\binom{52}{2} \times \binom{50}{2}}$$

3. Calculate $P(S_{Pair}|S_{QQ})$:

$$P(S_{Pair}|S_{QQ}) = \frac{\binom{4}{2} \times 73}{\binom{52}{2} \times \binom{50}{2}} \div P(S_{QQ}) = \frac{73}{\binom{50}{2}}$$

Conclusion

So, as calculated:

$$P(S_{Pair}|S_{Q5}) = \frac{72}{\binom{50}{2}} \quad \text{and} \quad P(S_{Pair}|S_{QQ}) = \frac{73}{\binom{50}{2}}$$

And therefore:

$$P(S_{Pair}|S_{QQ}) > P(S_{Pair}|S_{Q5})$$

indicating that you are more likely to have a pair when Amy has QQ compared to when she has $Q5$.