1   Lesson 11 Example 2

Consider a random variable Z with c.d.f. given by the formula:

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - 3^{-\lfloor x \rfloor} & \text{if } x \geq 0 \end{cases}$$

(Note that $\lfloor x \rfloor$ denotes the floor operator, which rounds x down to the nearest integer.)
     Graph the c.d.f. $F(x)$. Then, use it to calculate:

a. $P(Z > 3)$

b. $P(Z = 2)$

2   Answer

2.1   Evaluation of the c.d.f.

We evaluate the c.d.f. $F(x)$ at integer values of x:

- For $x = 0$:
$$F(0) = 1 - 3^{-\lfloor 0 \rfloor} = 1 - 3^0 = 1 - 1 = 0$$

- For $x = 1$:
$$F(1) = 1 - 3^{-\lfloor 1 \rfloor} = 1 - 3^{-1} = 1 - \frac{1}{3} \approx 0.6667$$

- For $x = 2$:
$$F(2) = 1 - 3^{-\lfloor 2 \rfloor} = 1 - 3^{-2} = 1 - \frac{1}{9} \approx 0.8889$$

- For $x = 3$:
$$F(3) = 1 - 3^{-\lfloor 3 \rfloor} = 1 - 3^{-3} = 1 - \frac{1}{27} \approx 0.9630$$
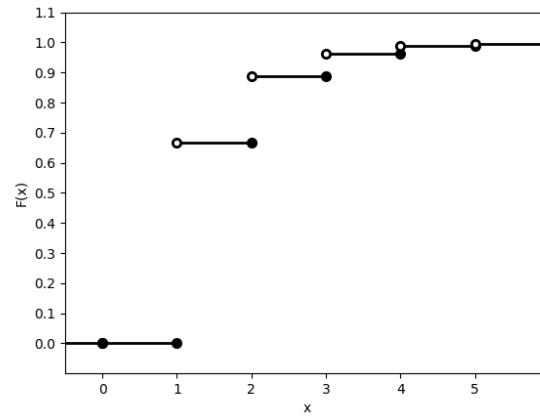
- For $x = 4$:
$$F(4) = 1 - 3^{-\lfloor 4 \rfloor} = 1 - 3^{-4} = 1 - \frac{1}{81} \approx 0.9877$$

- For $x = 5$:
$$F(5) = 1 - 3^{-\lfloor 5 \rfloor} = 1 - 3^{-5} = 1 - \frac{1}{243} \approx 0.9959$$

2.2 Graph of the c.d.f. $F(x)$

2.3 Calculation of Probabilities

a. $P(Z > 3)$:

$$P(Z > 3) = 1 - P(Z \leq 3) = 1 - F(3) = 1 - 0.9630 \approx 0.0370$$

b. $P(Z = 2)$:

$$P(Z = 2) = F(2) - F(1) = 0.8889 - 0.6667 \approx 0.2222$$