1 Lesson 12 Example 2

In order to ensure safety, a random sample of cars on each production line are crash-tested before being released to the public. The process of crash testing destroys the car. Suppose that a production line contains 10 defective and 190 working cars. If 4 of these cars are chosen at random for crash-testing, what is:

- the probability that at least 1 car will be found defective?
- the probability that exactly 2 cars will be found defective?

2 Answer

2.1 Parameters of the Hypergeometric Distribution

The scenario fits a hypergeometric distribution with the following parameters:

- $N = 200$ (Total number of cars)
- $N_1 = 10$ (Number of defective cars)
- $N_0 = 190$ (Number of working cars)
- $n = 4$ (Number of cars chosen for testing)

The probability mass function (PMF) for the hypergeometric distribution is given by:

$$P(X = x) = \frac{\binom{N_1}{x}\binom{N_0}{n-x}}{\binom{N}{n}}$$

2.2 Probability that at least 1 car will be found defective

Using the complement rule:

$$P(\text{at least 1 defective car}) = 1 - P(\text{no defective cars})$$

The probability of finding no defective cars is:

$$P(X = 0) = \frac{\binom{10}{0}\binom{190}{4}}{\binom{200}{4}} = \frac{1 \times 52602165}{64684950} \approx 0.8132$$

So,

$$P(\text{at least 1 defective car}) = 1 - 0.8132 = 0.1868$$

2.3 Probability that exactly 2 cars will be found defective

$$P(X = 2) = \frac{\binom{10}{2}\binom{190}{2}}{\binom{200}{4}} = \frac{45 \times 17955}{64684950} \approx 0.0125$$

2.4 Final Answers

- The probability that at least 1 car will be found defective is ≈ 0.1868.
- The probability that exactly 2 cars will be found defective is ≈ 0.0125.