1 Lesson 10 Example 3

A fair coin is tossed 5 times. Let Z be the number of heads. Calculate and graph the p.m.f. of Z. Use the p.m.f. to calculate the probability of getting at least 2 heads.

2 Answer

2.1 Probability Mass Function of Z

Let Z be the number of heads when a fair coin is tossed 5 times. The p.m.f. of Z is given by:

$$P(Z = z) = \binom{5}{z} \times (0.5)^z \times (0.5)^{5-z} = \binom{5}{z} \times (0.5)^5$$

2.2 Calculating the p.m.f.

- $P(Z = 0) = \frac{1}{32} \approx 0.03125$
- $P(Z = 1) = \frac{5}{32} \approx 0.15625$
- $P(Z = 2) = \frac{10}{32} = \frac{5}{16} \approx 0.3125$
- $P(Z = 3) = \frac{10}{32} = \frac{5}{16} \approx 0.3125$
- $P(Z = 4) = \frac{5}{32} \approx 0.15625$
- $P(Z = 5) = \frac{1}{32} \approx 0.03125$

2.3 Probability of Getting at Least 2 Heads

$P(Z \geq 2) = P(Z = 2)+P(Z = 3)+P(Z = 4)+P(Z = 5) = 0.3125+0.3125+0.15625+0.03125 = 0.8125$

2.4 Graph of the p.m.f. of Z