1 Lesson 4 Example 2

Complete the solution to Galileo's problem. What is the probability that the sum is 10 when 3 fair dice are rolled? How does this compare with the probability that the sum is 9?

2 Answer

To solve Galileo's problem and determine the probabilities of rolling a sum of 10 or 9 with 3 fair dice, we need to count all possible ordered outcomes and compare the favorable outcomes for each sum.

Total Number of Possible Outcomes

When rolling 3 fair dice, each die has 6 faces, leading to:

$$6 \times 6 \times 6 = 216 \text{ possible outcomes}$$

Counting the Number of Ways to Get Each Sum

Sum of 10

To find all possible ways to get a sum of 10, we list the combinations and count the permutations:

$$\begin{aligned}
&1+3+6 \quad \text{(and its permutations)}\\
&1+4+5 \quad \text{(and its permutations)}\\
&2+2+6 \quad \text{(and its permutations)}\\
&2+3+5 \quad \text{(and its permutations)}\\
&2+4+4 \quad \text{(and its permutations)}\\
&3+3+4 \quad \text{(and its permutations)}
\end{aligned}$$

Now let's calculate the permutations for each combination:

$$\begin{aligned}
&1+3+6 : 3! = 6 \text{ permutations}\\
&1+4+5 : 3! = 6 \text{ permutations}\\
&2+2+6 : \frac{3!}{2!} = 3 \text{ permutations} \quad \text{(since there are two 2's)}\\
&2+3+5 : 3! = 6 \text{ permutations}\\
&2+4+4 : \frac{3!}{2!} = 3 \text{ permutations} \quad \text{(since there are two 4's)}\\
&3+3+4 : \frac{3!}{2!} = 3 \text{ permutations} \quad \text{(since there are two 3's)}
\end{aligned}$$

Total number of ways to get a sum of 10:

$$6+6+3+6+3+3 = 27$$

Sum of 9

To find all possible ways to get a sum of 9, we list the combinations and count the permutations:

$$\begin{aligned}
&1+2+6 \quad \text{(and its permutations)}\\
&1+3+5 \quad \text{(and its permutations)}\\
&1+4+4 \quad \text{(and its permutations)}\\
&2+2+5 \quad \text{(and its permutations)}\\
&2+3+4 \quad \text{(and its permutations)}\\
&3+3+3 \quad \text{(and its permutations)}
\end{aligned}$$

Now let's calculate the permutations for each combination:

$$\begin{aligned}
&1+2+6 : 3! = 6 \text{ permutations}\\
&1+3+5 : 3! = 6 \text{ permutations}\\
&1+4+4 : \frac{3!}{2!} = 3 \text{ permutations} \quad \text{(since there are two 4's)}\\
&2+2+5 : \frac{3!}{2!} = 3 \text{ permutations} \quad \text{(since there are two 2's)}\\
&2+3+4 : 3! = 6 \text{ permutations}\\
&3+3+3 : 1 \text{ permutation} \quad \text{(since all dice are the same)}
\end{aligned}$$

Total number of ways to get a sum of 9:

$$6+6+3+3+6+1=25$$

Probability Calculation

Now, calculate the probabilities by dividing the number of favorable outcomes by the total number of possible outcomes (216).

Probability of Sum 10

$$P(\text{sum}=10)=\frac{27}{216}=\frac{1}{8}=0.125$$

Probability of Sum 9

$$P(\text{sum}=9)=\frac{25}{216}\approx 0.1157$$

Comparison

The probability of rolling a sum of 10 is slightly higher than the probability of rolling a sum of 9.