1 Lesson 5 Example 1

In poker, a "two pair" hand has 2 cards of one rank, 2 cards of another rank, and 1 card of a third rank. For example, the hand 2, 2, Q, Q, J is a "two pair". Your friend calculates the probability of "two pair" as follows:

- There are $\binom{52}{5}$ equally likely hands (where order does not matter).
- We count the number of ways to choose the first pair. There are 13 choices for the rank and $\binom{4}{2}$ choices for the two cards within the rank, so there are $13 \times \binom{4}{2}$ ways.
- Next, we count the ways to choose the second pair. Since one rank has already been chosen, there are 12 choices for the rank and $\binom{4}{2}$ ways to do this.
- Finally, we choose the remaining card. There are 11 ranks left and $\binom{4}{1}$ ways to do this.

Your friend calculates the probability as:

$$\frac{13 \times \binom{4}{2} \times 12 \times \binom{4}{2} \times 11 \times \binom{4}{1}}{\binom{52}{5}} \approx 0.095$$

but then finds online that the actual probability of "two pair" is only 0.0475. This number is exactly half the probability that your friend got, so he suspects that he double-counted. But where?

2 Answer

Identifying the Double-Counting Error

The error is in the calculation of the number of ways to choose the pairs. When calculating the number of ways to choose the first and second pairs, each unique pair of ranks is counted twice, once as (Rank1, Rank2) and once as (Rank2, Rank1). Therefore, we need to divide by 2 to correct for this overcounting.

Detailed Explanation

When our friend calculated the number of ways to choose the first and second pairs, they considered the choices as if the order of selection mattered. Let's break this down:

- Let's say the first pair is chosen to be of Rank1 and the second pair of Rank2.
- There are 13 choices for Rank1 and 12 choices for Rank2, giving us 13×12 combinations.
- Each combination is then multiplied by $\binom{4}{2}$ ways to choose the pairs within the ranks.

However, choosing Rank1 first and Rank2 second is the same as choosing Rank2 first and Rank1 second. Both of these selections result in the same "two pair" hand. Thus, each unique pair of ranks (Rank1, Rank2) has been counted twice:

- Once as (Rank1, Rank2)
- Once as (Rank2, Rank1)

Correct Calculation

1. **Choosing Two Ranks for the Pairs**:

   - Instead of choosing Rank1 and Rank2 in sequence, we should choose 2 ranks out of the 13 available. This is done using the combination formula $\binom{13}{2}$:

$$\binom{13}{2} = \frac{13 \times 12}{2} = 78 \text{ ways}$$

2. **Choosing the Two Pairs and the Fifth Card**:

   - For each chosen rank, there are $\binom{4}{2}$ ways to choose 2 cards from 4:

$$\binom{4}{2} \times \binom{4}{2} = 6 \times 6 = 36$$

   - Number of ways to choose the fifth card:

$$11 \times 4 = 44$$

3. **Total Number of Ways to Get a "Two Pair" Hand**:

$$\binom{13}{2} \times \binom{4}{2} \times \binom{4}{2} \times 11 \times 4 = 78 \times 6 \times 6 \times 11 \times 4 = 123,552$$

4. **Correct Probability Calculation**:

$$\text{Probability} = \frac{123,552}{2,598,960} = 0.0475$$

Conclusion

The friend's error was in not accounting for the double-counting of the two pairs, which led to an overestimation of the number of favorable outcomes. The correct probability of getting a "two pair" hand is:

$$\boxed{0.0475}$$

This is half of the probability that the friend calculated (0.095), which confirms the mistake of double-counting.