1 Lesson 7 Example 2

Two cards are dealt off the top of a well-shuffled deck of cards. Is the event that the first card is a heart independent of the event that the second card is a heart?

2 Answer

1. **Probability of Event A (First card is a heart):**

$$P(A) = \frac{13}{52} = \frac{1}{4}.$$

2. **Probability of Event B given Event A (Second card is a heart given the first card is a heart):**

$$P(B \mid A) = \frac{12}{51}.$$

3. **Joint Probability of Events A and B (Both cards are hearts):**

$$P(A \cap B) = P(A) \times P(B \mid A) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{12}{51} = \frac{12}{204}.$$

4. **Product of Individual Probabilities $P(A) \times P(B)$:** Assuming independence, $P(B)$ (the probability that the second card is a heart) would be $\frac{13}{52}$ (but this assumption doesn't hold true because the first card has been removed from the deck).

   So, if independent:

$$P(A) \times P(B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}.$$

Conclusion

The probability $P(A \cap B) = \frac{12}{204}$ is not equal to $P(A) \times P(B) = \frac{1}{16}$. Therefore, the events are **not independent**.

The event that the first card is a heart affects the probability of the second card being a heart, meaning the two events are dependent on each other.