1 Lesson 13 Example 2

About 10% of passengers who are scheduled to take a particular flight fail to show up. For this reason, airlines overbook flights, selling more tickets than they have seats, with the expectation that they will have some no shows. An airline with seating for 100 passengers sells 110 tickets for the flight. What is the probability that they will have enough seats for all the passengers for all of the passengers who show up for the flight? (Assume that passengers independently show up for the flight. Can you think of a situation where this would not be a reasonable assumption?)

2 Answer

The number of passengers who show up follows a binomial distribution with parameters $n = 110$ (the number of passengers) and $p = 0.9$ (the probability a passenger shows up). We need to calculate the probability that 100 or fewer passengers show up:

$$P(X \leq 100) = \sum_{k=0}^{100} \binom{110}{k} (0.9)^k (0.1)^{110-k}$$

Instead of computing this by hand, we can use Python and the `Symbulate` library to calculate the cumulative probability. Here is the Python code:

```
from symbulate import *
probs = Binomial(n=110, p=0.90).pmf(range(0, 101))
sum(probs)
```

In this code:

- `Binomial(n=110, p=0.90)` creates a binomial distribution with 110 trials and a success probability of 0.9.
- `pmf(range(0, 101))` calculates the probability mass function for k ranging from 0 to 100 (the number of passengers showing up).
- `sum(probs)` calculates the cumulative probability that 100 or fewer passengers show up.

Conclusion

Running the code provides the following result:

$$P(X \leq 100) = 0.67101$$

Thus, the probability that the airline will have enough seats for all passengers who show up is approximately 0.67101. A situation where the statistical independence of passengers would not be a reasonable assumption is if a major bus line or highway that connects to the airport is shut down due to an accident.