

1 Lesson 2 Example 2

A deck of 52 cards is shuffled thoroughly. What is the probability that the four aces are all next to each other? (Hint: First, count the number of positions that the block of four aces can go, then multiply this by the number of ways of ordering the four aces.)

2 Answer

To find the probability that the four aces in a shuffled deck of 52 cards are all next to each other, we need to follow these steps:

1. **Total Number of Possible Outcomes:** The total number of ways to arrange 52 cards is $52!$.

2. **Number of Favorable Outcomes:** - The aces can be arranged in $4!$ ways. - The remaining 48 cards can be arranged in $48!$ ways. - The block of four aces can be placed in one of 49 positions in the sequence of 52 cards.

So, the number of favorable outcomes is:

$$49 \times 4! \times 48!$$

3. **Probability Calculation:** The probability is the ratio of the number of favorable outcomes to the total number of possible outcomes:

$$P(\text{four aces are next to each other}) = \frac{49 \times 4! \times 48!}{52!}$$

4. **Simplifying the Factorials:**

$$\begin{aligned} \frac{49 \times 4! \times 48!}{52!} &= \frac{49 \times 24 \times 48!}{52 \times 51 \times 50 \times 49 \times 48!} \\ &= \frac{49 \times 24}{52 \times 51 \times 50 \times 49} \\ &= \frac{24}{52 \times 51 \times 50} \end{aligned}$$

5. **Final Probability:**

$$P(\text{four aces are next to each other}) = \frac{24}{52 \times 51 \times 50}$$

6. **Conclusion:**

The probability that the four aces are all next to each other in a shuffled deck of 52 cards is

$$\frac{24}{52 \times 51 \times 50} = \frac{1}{5525}$$