1 Lesson 17 Example 1

Packets arrive at a certain node on the university's intranet at 10 packets per minute, on average. Assume packet arrivals meet the assumptions of a Poisson process.

- Calculate the probability that exactly 15 packets arrive in the next 2 minutes.
- Calculate the probability that more than 60 packets arrive in the next 5 minutes.
- Calculate the probability that the next packet will arrive in within 15 seconds.

2 Answer

2.1 Part A

The rate of packets arriving is 10 per minute, and we are interested in a time interval of 2 minutes, so the parameter $\lambda = 10 \times 2 = 20$. We need to calculate the probability that exactly 15 packets arrive, which is given by the Poisson distribution formula:

$$P(X = 15) = e^{-20}\frac{20^{15}}{15!}$$

Evaluating this gives:

$$P(X = 15) \approx 0.0516$$

2.2 Part B

For this problem, the rate is 10 packets per minute and we are considering a time interval of 5 minutes, so $\lambda = 10 \times 5 = 50$. We need to calculate the probability that more than 60 packets arrive, which is:

$$P(X > 60) = 1 - P(X \leq 60)$$

Where $P(X \leq 60)$ is the cumulative distribution function (CDF) of the Poisson distribution. The result is:

$$P(X > 60) \approx 0.0722$$

The code for this calculation is as follows:

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(1 - Poisson(10 * 5).cdf(60))
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2.3 Part C

Given:

- Arrival rate: $\lambda = 10$ packets per minute.
- Time interval: $t = 15$ seconds.

Step 1: Convert Time Units

Convert the time interval from seconds to minutes to match the units of λ:

$$t = \frac{15\,\text{seconds}}{60\,\text{seconds per minute}} = 0.25\,\text{minutes}$$

Step 2: Calculate the Expected Number of Arrivals (μ)

$$\mu = \lambda t = 10 \times 0.25 = 2.5$$

Step 3: Calculate the Probability Using the Poisson Distribution

We are interested in the probability that at least one packet arrives in the next 15 seconds, which is:

$$P(X \geq 1) = 1 - P(X = 0)$$

The probability of observing k events in a Poisson distribution is:

$$P(X = k) = e^{-\mu} \frac{\mu^k}{k!}$$

Calculate $P(X = 0)$:

$$P(X = 0) = e^{-2.5} \frac{(2.5)^0}{0!} = e^{-2.5} \approx 0.0821$$

Step 4: Compute the Desired Probability

$$P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-2.5} \approx 1 - 0.0821 \approx 0.9179$$

Answer:

The probability that the next packet arrives within 15 seconds is approximately 0.9179 or 91.79%.