

1 Lesson 2 Example 3

If a five-letter word is formed at random (meaning that all sequences of five letters are equally likely), what is the probability that no letter occurs more than once?

2 Answer

To find the probability that a randomly formed five-letter word has no repeated letters, we proceed as follows:

1. Total Number of Possible 5-Letter Words:

There are 26 letters in the English alphabet. Since each position in the 5-letter word can be any of the 26 letters, the total number of possible 5-letter words is:

$$26^5$$

2. Number of Favorable Outcomes (No Repeated Letters):

- For the first letter, we can choose any of the 26 letters.
- For the second letter, we can choose any of the remaining 25 letters.
- For the third letter, we can choose any of the remaining 24 letters.
- For the fourth letter, we can choose any of the remaining 23 letters.
- For the fifth letter, we can choose any of the remaining 22 letters.

So, the number of ways to form a 5-letter word with no repeated letters is:

$$26 \times 25 \times 24 \times 23 \times 22$$

3. Probability Calculation:

The probability that no letter occurs more than once in the 5-letter word is the ratio of the number of favorable outcomes to the total number of possible outcomes:

$$P(\text{no repeated letters}) = \frac{26 \times 25 \times 24 \times 23 \times 22}{26^5}$$

Simplifying the expression:

$$P(\text{no repeated letters}) = \frac{26 \times 25 \times 24 \times 23 \times 22}{26 \times 26 \times 26 \times 26 \times 26} = \frac{25 \times 24 \times 23 \times 22}{26^4}$$

Therefore, the probability that a randomly formed five-letter word has no repeated letters is:

$$P(\text{no repeated letters}) = \frac{25 \times 24 \times 23 \times 22}{26^4}$$