

## 1 Lesson 12 Example 1

Recall in the example from Lesson 10, there was another player, Bob, who had two Jacks and was looking to get a four-of-a-kind. For Bob, the random variable of interest was the number of Jacks among the community cards. Use a box model to argue that Bob's random variable also has a hypergeometric distribution. What are its parameters?

## 2 Answer

To determine why Bob's random variable (the number of Jacks among the community cards) has a hypergeometric distribution, we analyze the situation using a box model.

### Understanding the Situation

- **Deck Composition:** Initially, there are 52 cards in a deck. Since Bob has 2 Jacks and Alice has two cards, 48 cards remain.
- **Community Cards:** Bob is interested in the number of Jacks among the 5 community cards.
- **Jacks in Deck:** Out of the 48 remaining cards, 2 are Jacks.

### Box Model

We can use a box model to represent this situation:

- **Box Contents:** The box contains 48 tickets, where each ticket represents a card in the remaining deck.
  - 2 tickets are labeled "Jack" (representing the Jacks).
  - 46 tickets are labeled "Not Jack" (representing the non-Jack cards).

### Random Draws

We draw 5 tickets from the box without replacement, corresponding to the 5 community cards. The random variable of interest,  $Y$ , is the number of Jacks among these 5 draws.

### Hypergeometric Distribution

The scenario described matches the conditions for a hypergeometric distribution:

- **Finite Population:** We are drawing from a finite population (48 remaining cards).
- **Two Types of Outcomes:** There are two possible outcomes—drawing a "Jack" (success) or "Not Jack" (failure).
- **Without Replacement:** The cards are drawn without replacement.

- **Fixed Number of Draws:** We draw a fixed number (5) of cards.

Given these conditions, the number of Jacks among the community cards follows a hypergeometric distribution.

## Parameters of the Hypergeometric Distribution

For Bob's random variable  $Y$ , the parameters of the hypergeometric distribution are:

- $N = 48$ : The total number of remaining cards in the deck.
- $N_1 = 2$ : The number of Jacks remaining in the deck (i.e., the number of successes in the population).
- $N_0 = 46$ : The number of Non-Jacks remaining in the deck (i.e., the number of failures in the population).
- $n = 5$ : The number of community cards drawn.

Thus,  $Y$ , the number of Jacks among the community cards, has a hypergeometric distribution with parameters  $N = 48$ ,  $N_1 = 2$ ,  $N_0 = 46$ , and  $n = 5$ .