CoΛjuΛctioΛ

What do we mean by conjunction

- $P \land Q$ is true if and only if P is true and Q is true
    - In logic, conjunctions use the word “and” to mean that both of the propositions are true
    - In some contexts, a conjunction might take more than two propositions, in which case the conjunction is true if *all* of the propositions are true
- Consider the statement “it is raining and the sprinkler is on”.
    - This proposition is *not* true if:
        - It is raining, and the sprinkler is off
        - It is not raining, but the sprinkler is on
    - This proposition *is* true if:
        - It is raining, and the sprinkler is on

Right associative

- Conjunction is right-associative:
  - $P \land (Q \land R)$ and $P \land Q \land R$ are the same
  - $(P \land Q) \land R$ is syntactically the same
- However, as we will see $P \land (Q \land R)$ and $(P \land Q) \land R$ are semantically the same — conjunction is fully associative
- Syntactically, however, this means that a conjunction of 4 terms is actually a conjunction of two terms, where the second term is a conjunction of two terms, where the second term of the second term is a conjunction of two atomic terms:
  - $P \land Q \land R \land S = P \land (Q \land (R \land S))$

Introduction rule

- A proof of conjunction requires proofs that both propositions are true:

```
{P Q} pfP: P, pfQ: Q
----------------------- (and.intro)
         P ∧ Q
```

```
lemma my_and_intro{P Q: Prop}(pfP: P)(pfQ: Q): P ∧ Q :=
begin
  exact and.intro pfP pfQ
end
```

Elimination Rules

- We have two elimination rules for `and`: `and.elim_left` and `and.elim_right`
- The first, `and.elim_left`, is synonymous with `.1` or `.left` being applied to the proof, so that if pfPandQ is a proof of P and Q, these are the same:
  - `(and.elim_left pfPandQ)`
  - `pfPandQ.1`
  - `pfPandQ.left`
- The second, `and.elim_right`, is synonymous with `.2` or `.right` being applied to the proof

Examples

- See file and_properties.lean in the examples directory

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