

HPC & Parallel Programming

Start from An Example

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An example of HPC & Parallel Programming: Matrix Multiplication

$$\begin{array}{c} \vec{b}_1 \quad \vec{b}_2 \\ \downarrow \quad \downarrow \\ \vec{a}_1 \rightarrow \begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} \vec{a}_1 \cdot \vec{b}_1 & \vec{a}_1 \cdot \vec{b}_2 \\ \vec{a}_2 \cdot \vec{b}_1 & \vec{a}_2 \cdot \vec{b}_2 \end{bmatrix} \\ \vec{a}_2 \rightarrow \end{array}$$

A B C



Matrix multiply

<https://github.com/kevinsuo/CS7172/blob/master/matrix.c>

```
int main()
{
    initMatrix();

    double time_spent = 0.0;
    clock_t begin = clock();

    matrixMultiply();

    clock_t end = clock();
    time_spent += (double)(end - begin) / CLOCKS_PER_SEC;
    printf("Time elapsed is %f seconds", time_spent);

    return 0;
}
```



Matrix multiply

<https://github.com/kevinsuo/CS7172/blob/master/matrix.c>

```
#include <stdio.h>
#include <time.h>
#include <stdlib.h>
#define N 1000

double A[N][N], B[N][N], C[N][N];

void initMatrix()
{
    int i, j = 0;
    for (i = 0; i < N; i++) {
        for (j = 0; j < N; j++) {
            A[i][j] = rand() % 100 + 1; //generate a number between [1, 100]
            B[i][j] = rand() % 100 + 1; //generate a number between [1, 100]
        }
    }
}
```

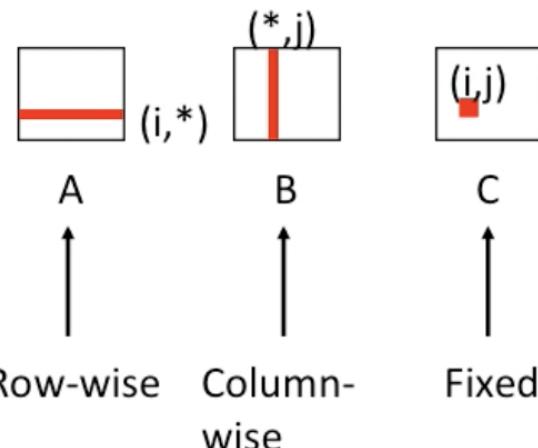


Matrix multiply

<https://github.com/kevinsuo/CS7172/blob/master/matrix.c>

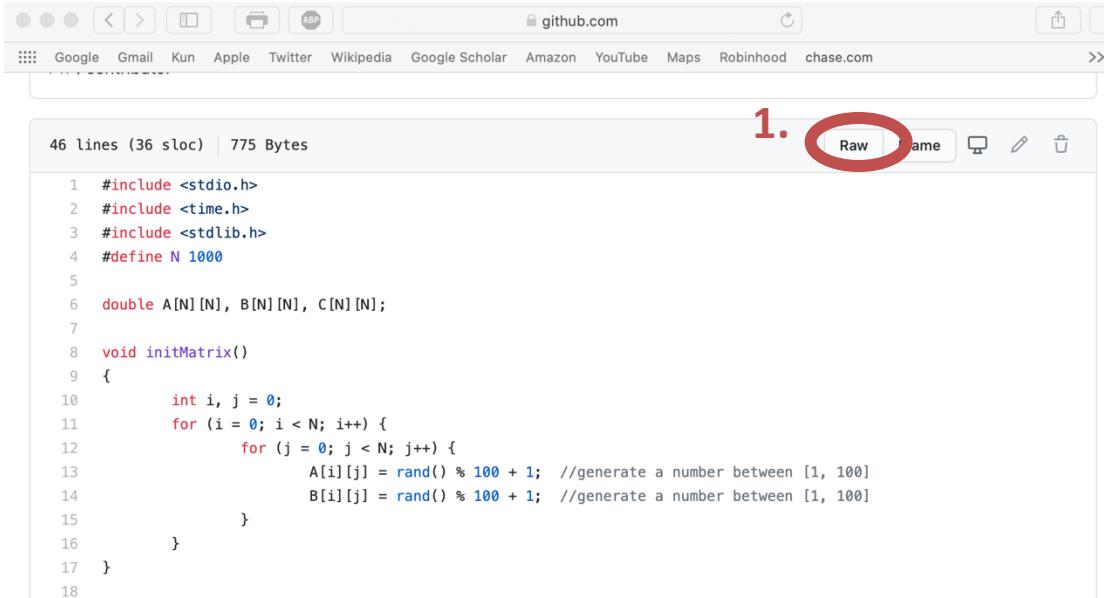
```
void matrixMultiply() {  
    int i, j, k = 0;  
    for (i = 0; i < N; i++) {  
        for (j = 0; j < N; j++) {  
            for (k = 0; k < N; k++) {  
                C[i][j] += A[i][k] * B[k][j];  
            }  
        }  
    }  
}
```

Inner loop:



Matrix multiply

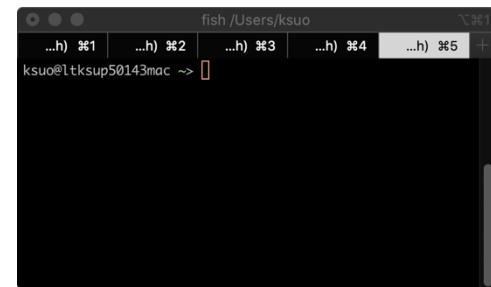
<https://github.com/kevinsuo/CS7172/blob/master/matrix.c>



1. **Raw**

```
46 lines (36 sloc) | 775 Bytes

1 #include <stdio.h>
2 #include <time.h>
3 #include <stdlib.h>
4 #define N 1000
5
6 double A[N][N], B[N][N], C[N][N];
7
8 void initMatrix()
9 {
10     int i, j = 0;
11     for (i = 0; i < N; i++) {
12         for (j = 0; j < N; j++) {
13             A[i][j] = rand() % 100 + 1; //generate a number between [1, 100]
14             B[i][j] = rand() % 100 + 1; //generate a number between [1, 100]
15         }
16     }
17 }
```



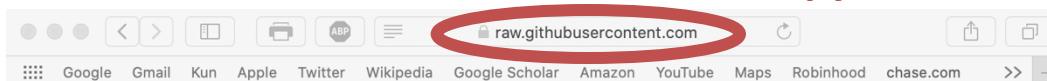
3.

\$ wget URL

\$ gcc filename.c –o filename.o

\$./filename.o

**(if no wget/gcc,
\$ sudo apt install wget, gcc)**



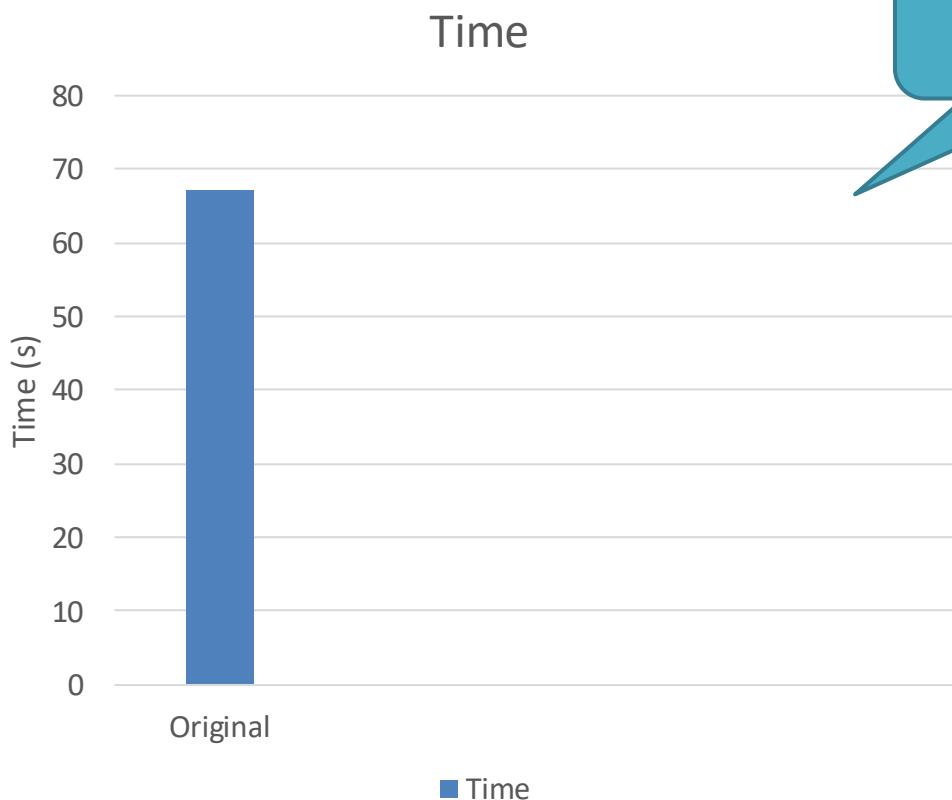
```
#include <stdio.h>
#include <time.h>
#include <stdlib.h>
#define N 1000

double A[N][N], B[N][N], C[N][N];

void initMatrix()
{
    int i, j = 0;
    for (i = 0; i < N; i++) {
```



Matrix multiply

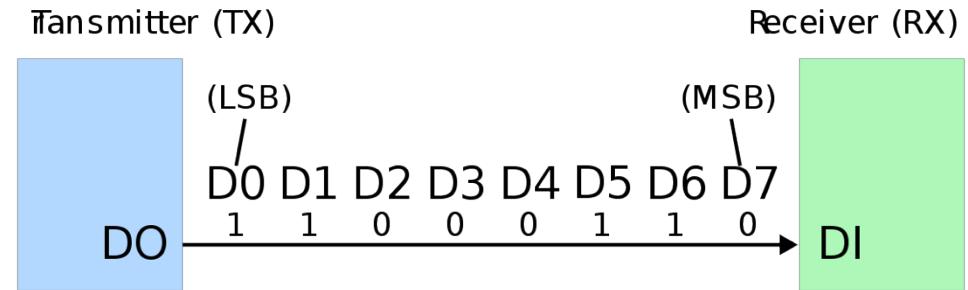


1. Accelerate serial execution
2. Accelerate in parallel

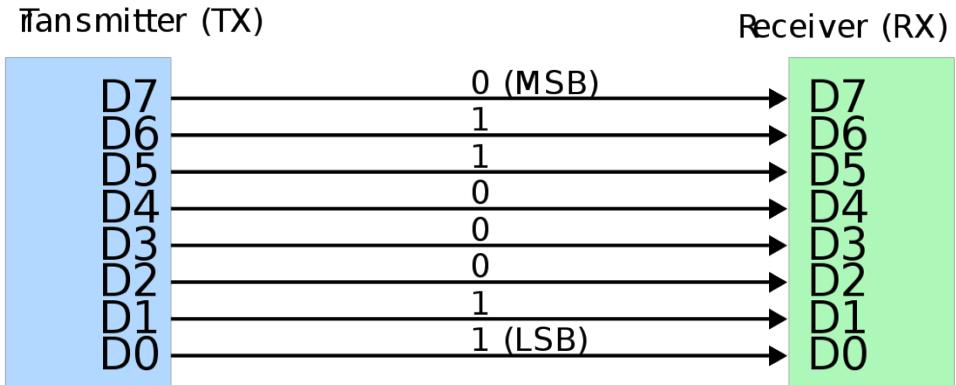


How to run it faster?

1. Accelerate serial execution
Reduce unnecessary steps



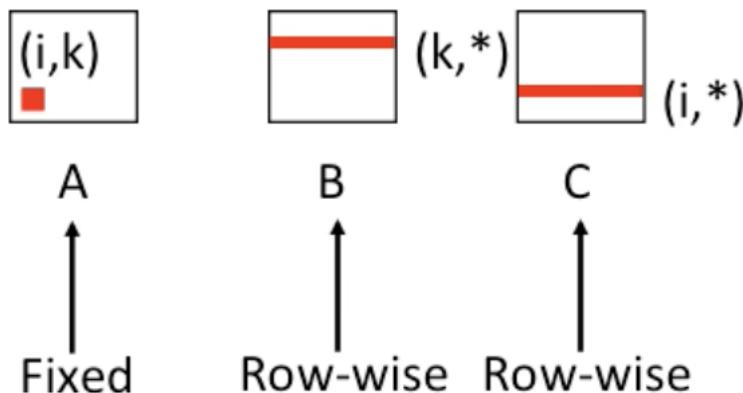
2. Accelerate in parallel



Option 1: Optimization using locality

```
void matrixMultiply() {  
    int i, j, k = 0;  
    for (k = 0; k < N; k++) {  
        for (i = 0; i < N; i++) {  
            for (j = 0; j < N; j++) {  
                C[i][j] += A[i][k] * B[k][j];  
            }  
        }  
    }  
}
```

Inner loop:



[https://github.com/kevinsuo/CS
7172/blob/master/matrix-opt.c](https://github.com/kevinsuo/CS7172/blob/master/matrix-opt.c)

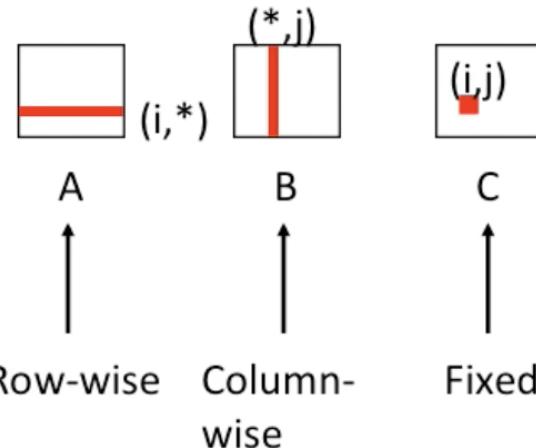
Option 1: Optimization using locality

```
ksuo@ksuo-VirtualBox ~/cs7172> ./a.o  
Time elapsed is 67.452589 seconds  
ksuo@ksuo-VirtualBox ~/cs7172>  
ksuo@ksuo-VirtualBox ~/cs7172>  
ksuo@ksuo-VirtualBox ~/cs7172>  
ksuo@ksuo-VirtualBox ~/cs7172>  
ksuo@ksuo-VirtualBox ~/cs7172>  
ksuo@ksuo-VirtualBox ~/cs7172> ./a2.o  
Time elapsed is 18.149353 seconds
```

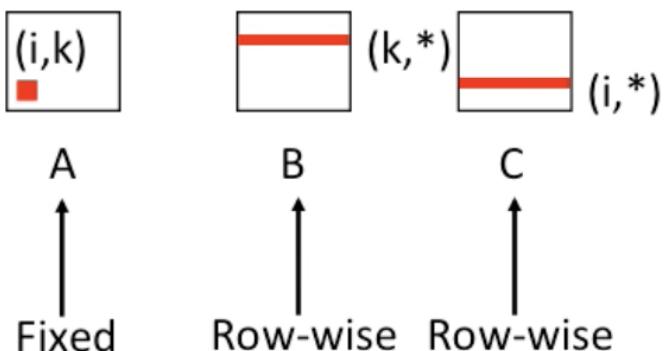
N=2000

3.7x

Inner loop:

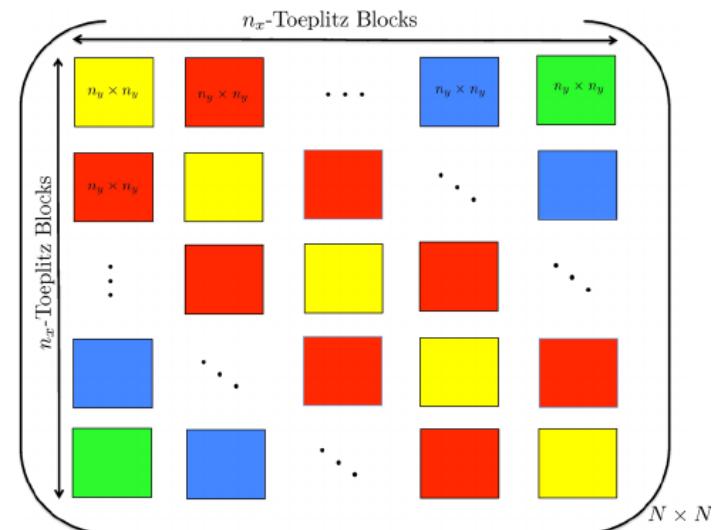
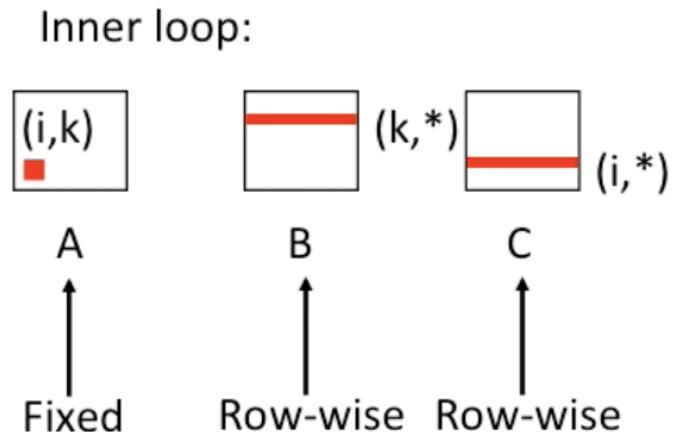


Inner loop:



Option 1: Optimization using locality

- Temporal locality
 - Every inner loop reuse the value of $A[i, k]$
- Spatial locality
 - Divide the large matrix into smaller ones and put it inside the cache during calculation



Option 1: Optimization using locality

```
void matrixMultiply() {  
    int i, j, k = 0;  
    int i2, j2, k2 = 0;  
  
    for (k2 = 0; k2 < N; k2+=BLOCK_SIZE) {  
        for (i2 = 0; i2 < N; i2+=BLOCK_SIZE) {  
            for (j2 = 0; j2 < N; j2+=BLOCK_SIZE) {  
                //inside each block  
                for (k = k2; k < k2+BLOCK_SIZE; k++) {  
                    for (i = i2; i < i2+BLOCK_SIZE; i++) {  
                        for (j = j2; j < j2+BLOCK_SIZE; j++) {  
                            C[i][j] += A[i][k] * B[k][j];  
                        }  
                    }  
                }  
            }  
        }  
    }  
}
```

<https://github.com/kevinsuo/CS7172/blob/master/matrix-opt2.c>

N = 2000

$$\left(\begin{array}{c|cc|cc} J_1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & J_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & J_3 \end{array} \right)$$

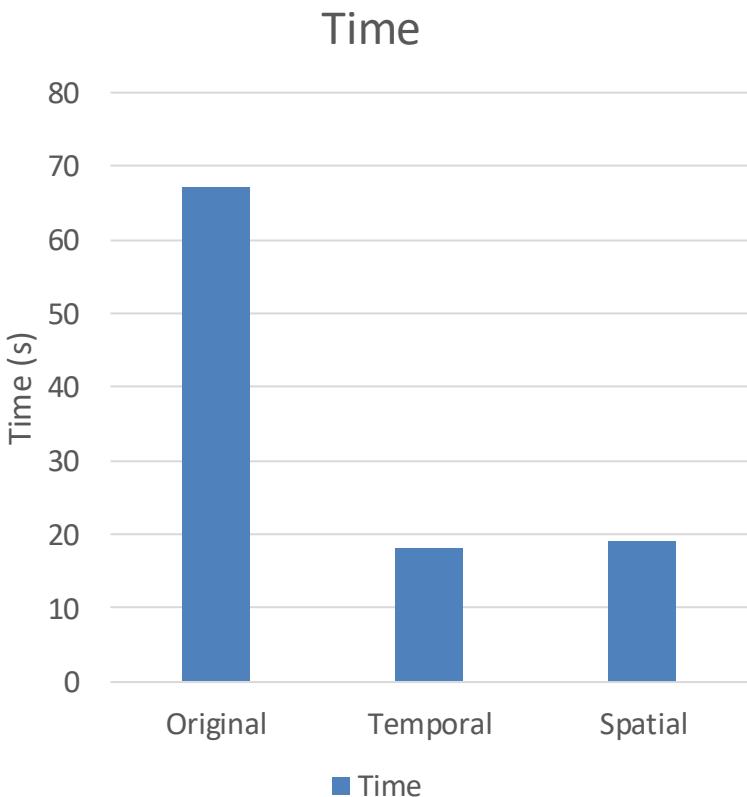


Option 1: Optimization using locality

$$A = \left(\begin{array}{cc|cc} a_{11} & a_{12} & a_{13} & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{array} \right) \implies \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$
$$A_{11} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, A_{12} = \begin{pmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \end{pmatrix}$$
$$A_{21} = \begin{pmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}, A_{22} = \begin{pmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{pmatrix}$$



Option 1: Optimization using locality



```
ksuo@ksuo-VirtualBox ~/cs7172> ./a.o
Time elapsed is 67.845517 seconds↵
ksuo@ksuo-VirtualBox ~/cs7172>
ksuo@ksuo-VirtualBox ~/cs7172>
ksuo@ksuo-VirtualBox ~/cs7172>
ksuo@ksuo-VirtualBox ~/cs7172>
ksuo@ksuo-VirtualBox ~/cs7172> ./a3.o
Time elapsed is 19.115410 seconds↵
```



Optimal 2: Optimization using parallel

- The algorithm uses a divide-and-conquer approach. When the matrix order is very large, a recursive formula is used for calculation. The relevant recursive formula is described as follows:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{24} \end{bmatrix} & \begin{bmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \end{bmatrix} \\ \begin{bmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix} & \begin{bmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{bmatrix} \end{bmatrix}_{A_{21}} \quad \begin{bmatrix} A_{22} \end{bmatrix}$$
$$\times B = \begin{bmatrix} B_{11} & B_{12} \\ \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{24} \end{bmatrix} & \begin{bmatrix} b_{13} & b_{14} \\ b_{23} & b_{24} \end{bmatrix} \\ \begin{bmatrix} b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix} & \begin{bmatrix} b_{33} & b_{34} \\ b_{43} & b_{44} \end{bmatrix} \end{bmatrix}_{B_{21}} \quad \begin{bmatrix} B_{22} \end{bmatrix} \rightarrow \begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix}$$

Strassen algorithm

$$\text{Task 1: } C_{1,1} = A_{1,1}B_{1,1} + A_{1,2}B_{2,1}$$

$$\text{Task 2: } C_{1,2} = A_{1,1}B_{1,2} + A_{1,2}B_{2,2}$$

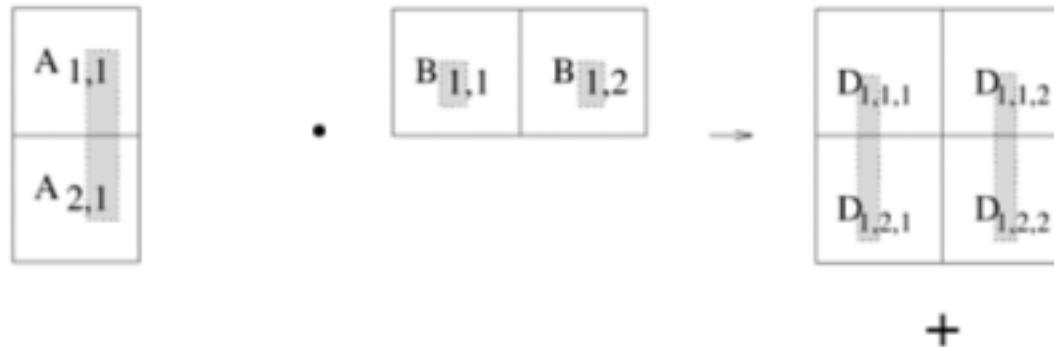
$$\text{Task 3: } C_{2,1} = A_{2,1}B_{1,1} + A_{2,2}B_{2,1}$$

$$\text{Task 4: } C_{2,2} = A_{2,1}B_{1,2} + A_{2,2}B_{2,2}$$

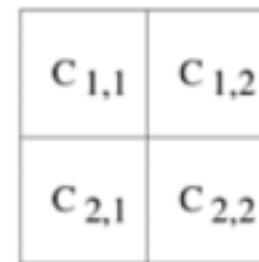
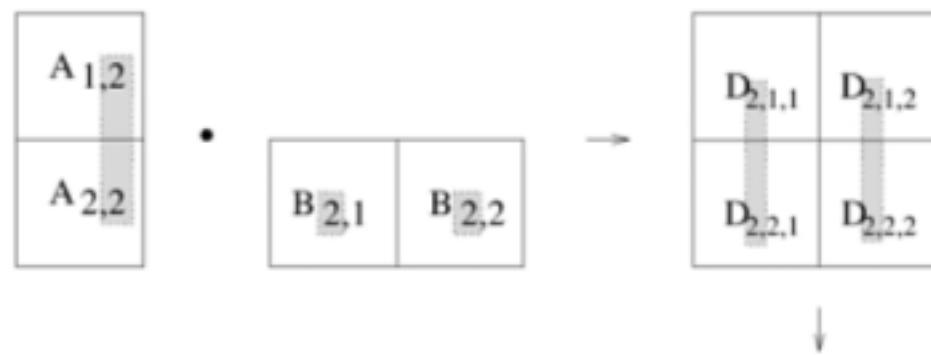


Optimal 2: Optimization using parallel

Thread 1:



Thread 2:



Reading Materials

- Strassen algorithm
 - In 1969, Volker Strassen proposed a matrix multiplication algorithm with a complexity of $O(n^{2.807})$: [Link](#). This was the first time in history that computational complexity of matrix multiplication was reduced below $O(n^3)$.
- Coppersmith–Winograd algorithm
 - In 1990, Don Coppersmith and Shmuel Winograd made a groundbreaking achievement by reducing the complexity to $O(n^{2.3727})$. Paper: [Link](#)



Strassen algorithm

$$\begin{aligned} p_1 &= a(f-h) \\ p_3 &= (c+d)e \\ p_5 &= (a+d)(e+h) \\ p_7 &= (a-c)(e+f) \end{aligned}$$

$$\begin{aligned} p_2 &= (a+b)h \\ p_4 &= d(g-e) \\ p_6 &= (b-d)(g+h) \end{aligned}$$

A

a	b
c	d

X

B

e	f
g	h

=

C

$p_5+p_4-p_2+p_6$	p_1+p_2
p_3+p_4	$p_1*p_5-p_3*p_7$



Strassen algorithm basic idea

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \rightarrow \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$S_1 = B_{12} - B_{22}$$

$$S_2 = A_{11} + A_{12}$$

$$S_3 = A_{21} + A_{22}$$

$$S_4 = B_{21} - B_{11}$$

$$S_5 = A_{11} + A_{22}$$

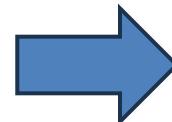
$$S_6 = B_{11} + B_{22}$$

$$S_7 = A_{12} - A_{22}$$

$$S_8 = B_{21} + B_{22}$$

$$S_9 = A_{11} - A_{21}$$

$$S_{10} = B_{11} + B_{12}$$



$$P_1 = A_{11} \times S_1$$

$$P_2 = S_2 \times B_{22}$$

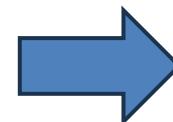
$$P_3 = S_3 \times B_{11}$$

$$P_4 = A_{22} \times S_4$$

$$P_5 = S_5 \times S_6$$

$$P_6 = S_7 \times S_8$$

$$P_7 = S_9 \times S_{10}$$



$$C_{11} = P_5 + P_4 - P_2 + P_6$$

$$C_{12} = P_1 + P_2$$

$$C_{21} = P_3 + P_4$$

$$C_{22} = P_5 + P_1 - P_3 - P_7$$



Coppersmith–Winograd algorithm basic idea

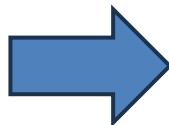
$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \rightarrow \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$S1 = A_{21} + A_{22}$$

$$S2 = S1 - A_{11}$$

$$S3 = A_{11} - A_{21}$$

$$S4 = A_{12} - S2$$



$$T1 = B_{21} - B_{11}$$

$$T2 = B_{22} - T1$$

$$T3 = B_{22} - B_{12}$$

$$T4 = T2 - B_{21}$$

$$M1 = A_{11} * B_{11}$$

$$M2 = A_{12} * B_{21}$$

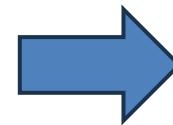
$$M3 = S4 * B_{22}$$

$$M4 = A_{22} * T4$$

$$M5 = S1 * T1$$

$$M6 = S2 * T2$$

$$M7 = S3 * T3$$



$$U1 = M1 + M2$$

$$U2 = M1 + M6$$

$$U3 = U2 + M7$$

$$U4 = U2 + M5$$

$$U5 = U4 + M3$$

$$U6 = U3 - M4$$

$$U7 = U3 + M5$$

$$C_{11} = U1$$

$$C_{12} = U5$$

$$C_{21} = U6$$

$$C_{22} = U7$$



Optimization and Speedup

	N=200	N=400	N=800	N=1600
matrix				
matrix-opt1				
matrix-opt2				

	N=200	N=400	N=800	N=1600
matrix				
matrix-opt1				
matrix-opt2				



Single thread app demo

<https://youtu.be/dlsBhvQ9mA>



The screenshot shows a terminal window with two tabs. The active tab, labeled '#1', displays the fish shell prompt: 'fish /home/administrator'. Below it, the command 'fish' is run, followed by its welcome message: 'Welcome to fish, the friendly interactive shell'. The second tab, labeled '#2', is also titled 'fish /home/administrator (ssh)'. A large blue speech bubble originates from the bottom right of the terminal window, pointing towards the text 'Single thread program'.

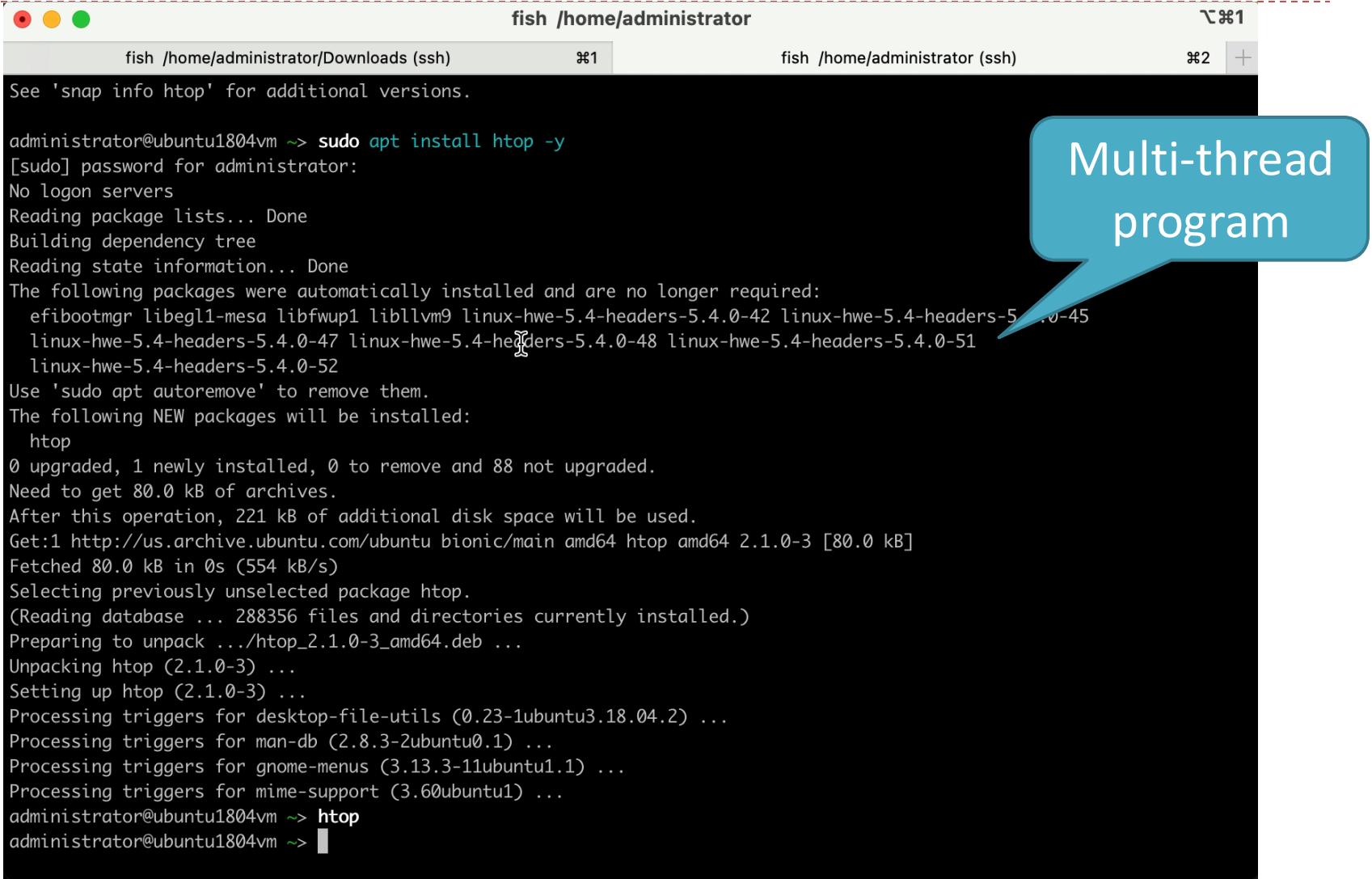
```
fish /home/administrator
fish /home/administrator (ssh)
administrator@ubuntu1804vm:~$ fish
Welcome to fish, the friendly interactive shell
administrator@ubuntu1804vm ~> 
```

Single thread
program



Multi-thread app demo

<https://youtu.be/ubLB2fb8cdc>



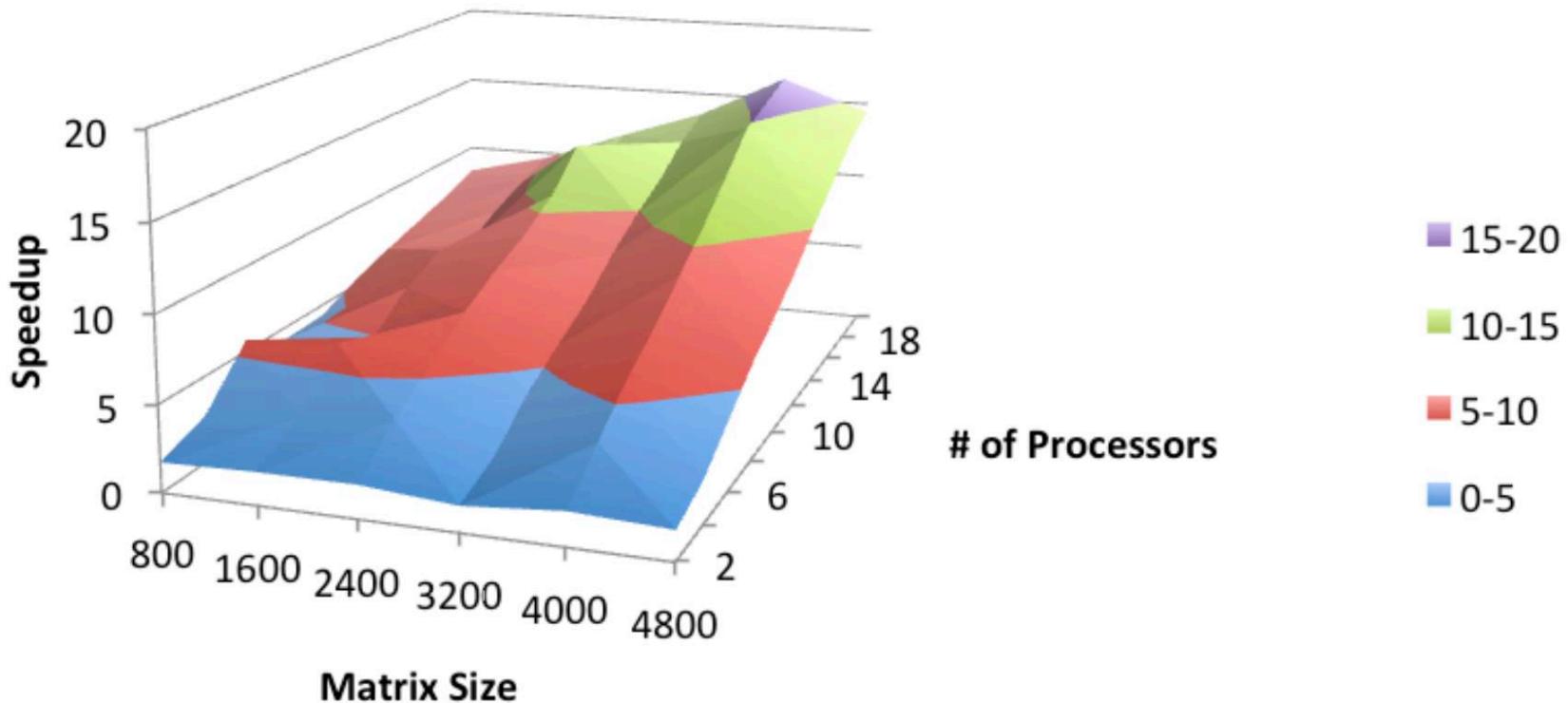
```
fish /home/administrator
fish /home/administrator/Downloads (ssh) #1
See 'snap info htop' for additional versions.

administrator@ubuntu1804vm ~> sudo apt install htop -y
[sudo] password for administrator:
No logon servers
Reading package lists... Done
Building dependency tree
Reading state information... Done
The following packages were automatically installed and are no longer required:
  efibootmgr libegl1-mesa libfwup1 libllvm9 linux-hwe-5.4-headers-5.4.0-42 linux-hwe-5.4-headers-5.4.0-45
    linux-hwe-5.4-headers-5.4.0-47 linux-hwe-5.4-headers-5.4.0-48 linux-hwe-5.4-headers-5.4.0-51
    linux-hwe-5.4-headers-5.4.0-52
Use 'sudo apt autoremove' to remove them.
The following NEW packages will be installed:
  htop
0 upgraded, 1 newly installed, 0 to remove and 88 not upgraded.
Need to get 80.0 kB of archives.
After this operation, 221 kB of additional disk space will be used.
Get:1 http://us.archive.ubuntu.com/ubuntu bionic/main amd64 htop amd64 2.1.0-3 [80.0 kB]
Fetched 80.0 kB in 0s (554 kB/s)
Selecting previously unselected package htop.
(Reading database ... 288356 files and directories currently installed.)
Preparing to unpack .../htop_2.1.0-3_amd64.deb ...
Unpacking htop (2.1.0-3) ...
Setting up htop (2.1.0-3) ...
Processing triggers for desktop-file-utils (0.23-1ubuntu3.18.04.2) ...
Processing triggers for man-db (2.8.3-2ubuntu0.1) ...
Processing triggers for gnome-menus (3.13.3-11ubuntu1.1) ...
Processing triggers for mime-support (3.60ubuntu1) ...
administrator@ubuntu1804vm ~> htop
administrator@ubuntu1804vm ~>
```

Multi-thread
program



Optimal 2: Optimization using parallel



https://www.cse.unr.edu/~fredh/class/415/Nolan/matrix_multiplication/writeup.pdf



Example of distributed system: sorting

- Sorting on a single machine, e.g., Database

```
select field_a from table_b order by field_a limit 100, 10;
```

```
db.collection_b  
.find()  
.sort({"field_a":1})  
.skip(100)  
.limit(10);
```

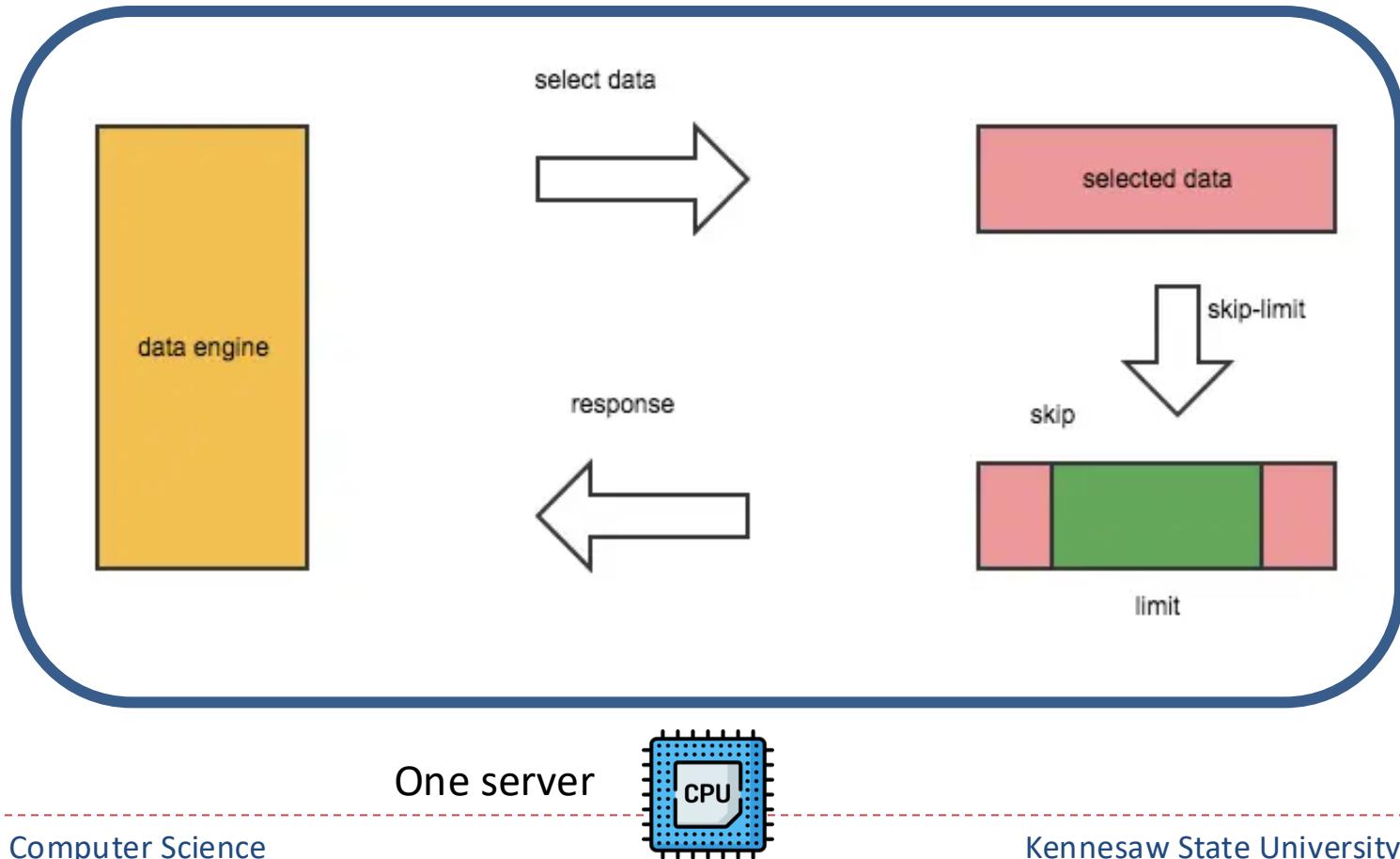
	field_a	field_b	field_c
100			
...			
...			
110			

From line 100
to the next 10
lines of data



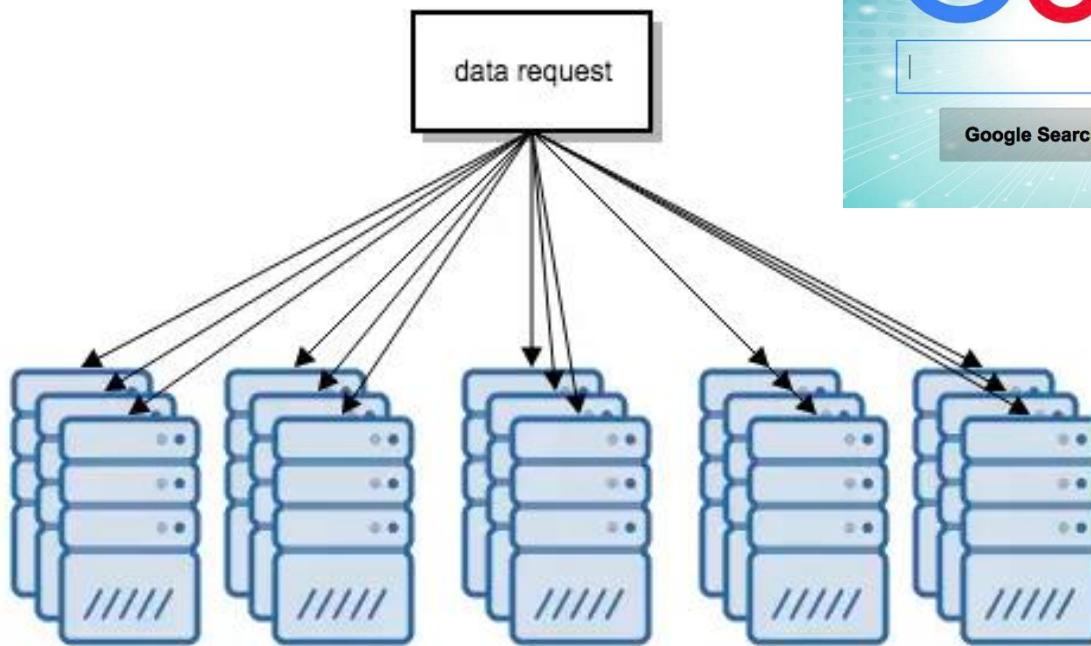
Example of distributed system: sorting

- Workflow on a single node



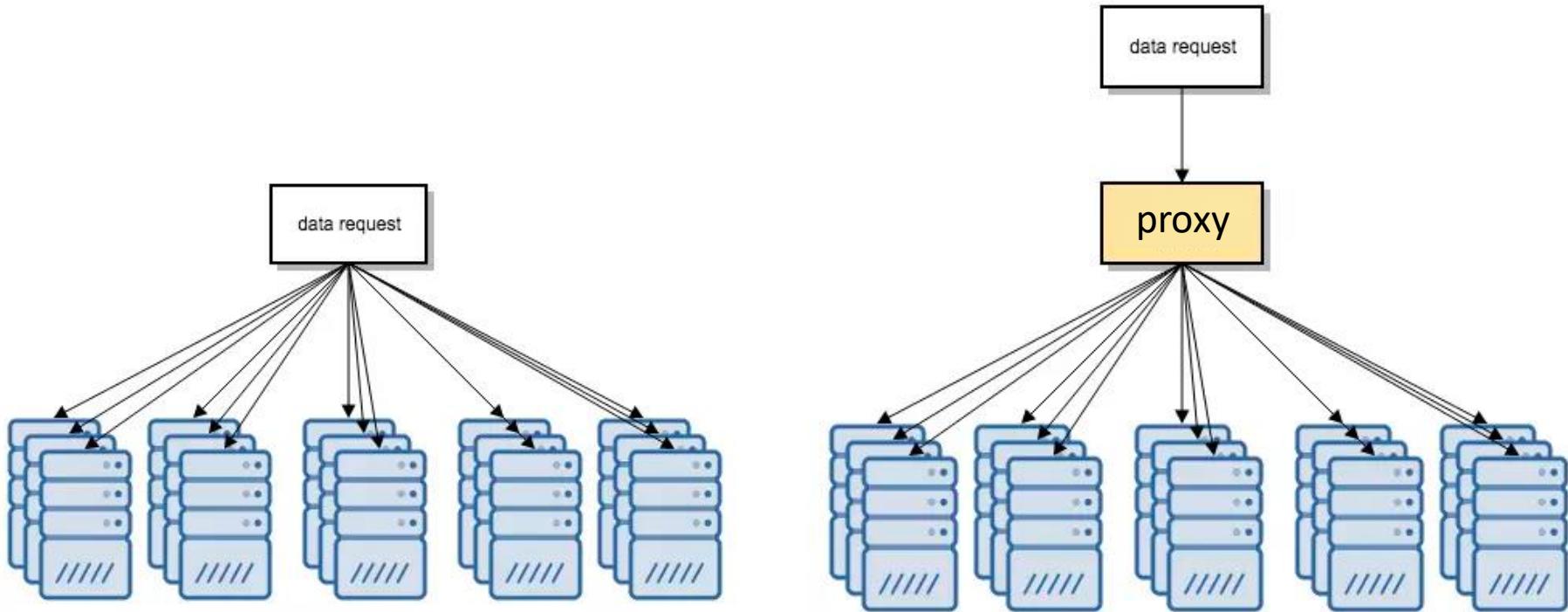
Example of distributed system: sorting

- If the data is too much and single node cannot hold



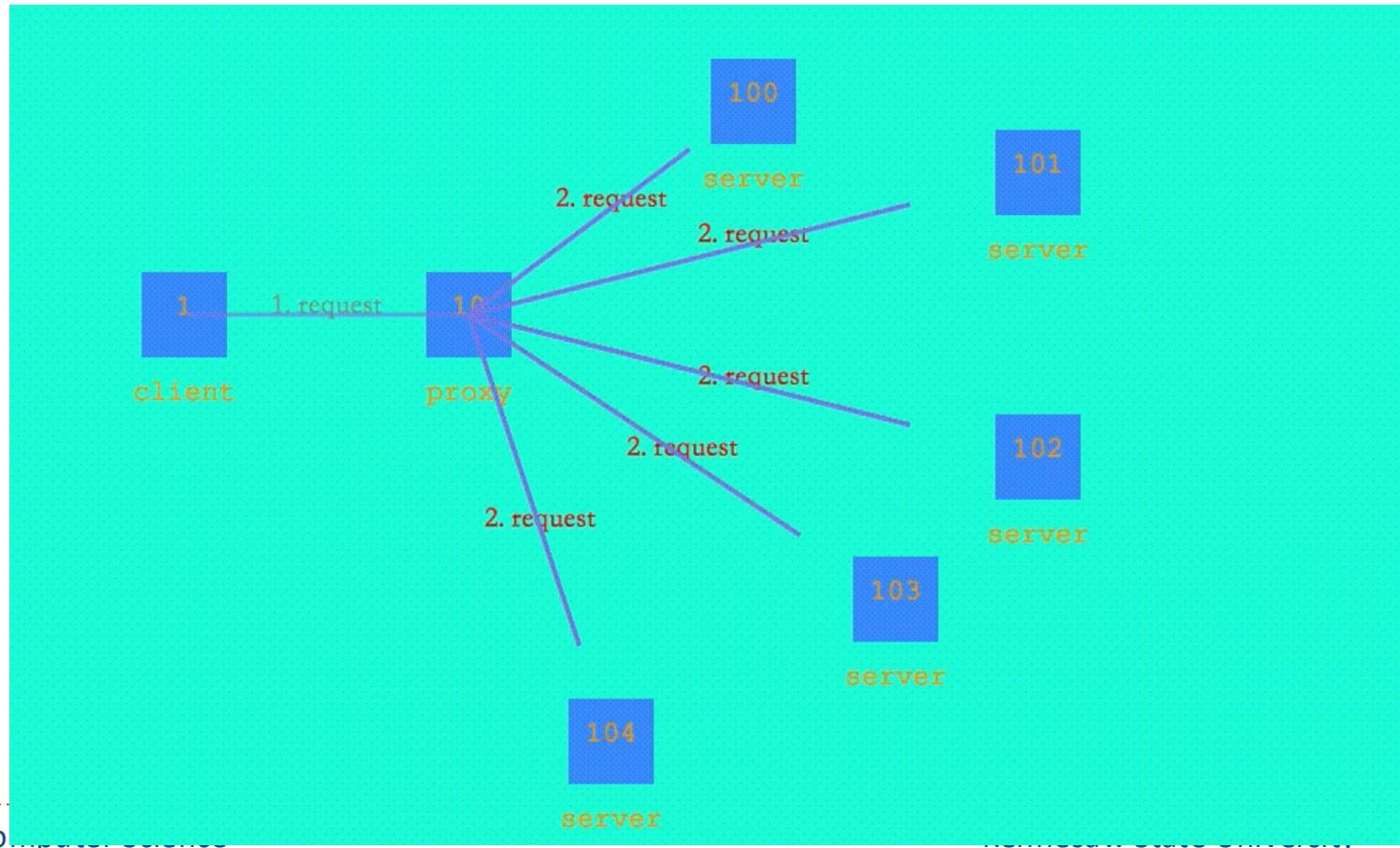
Example of distributed system: sorting

- Choose a node for merge processing



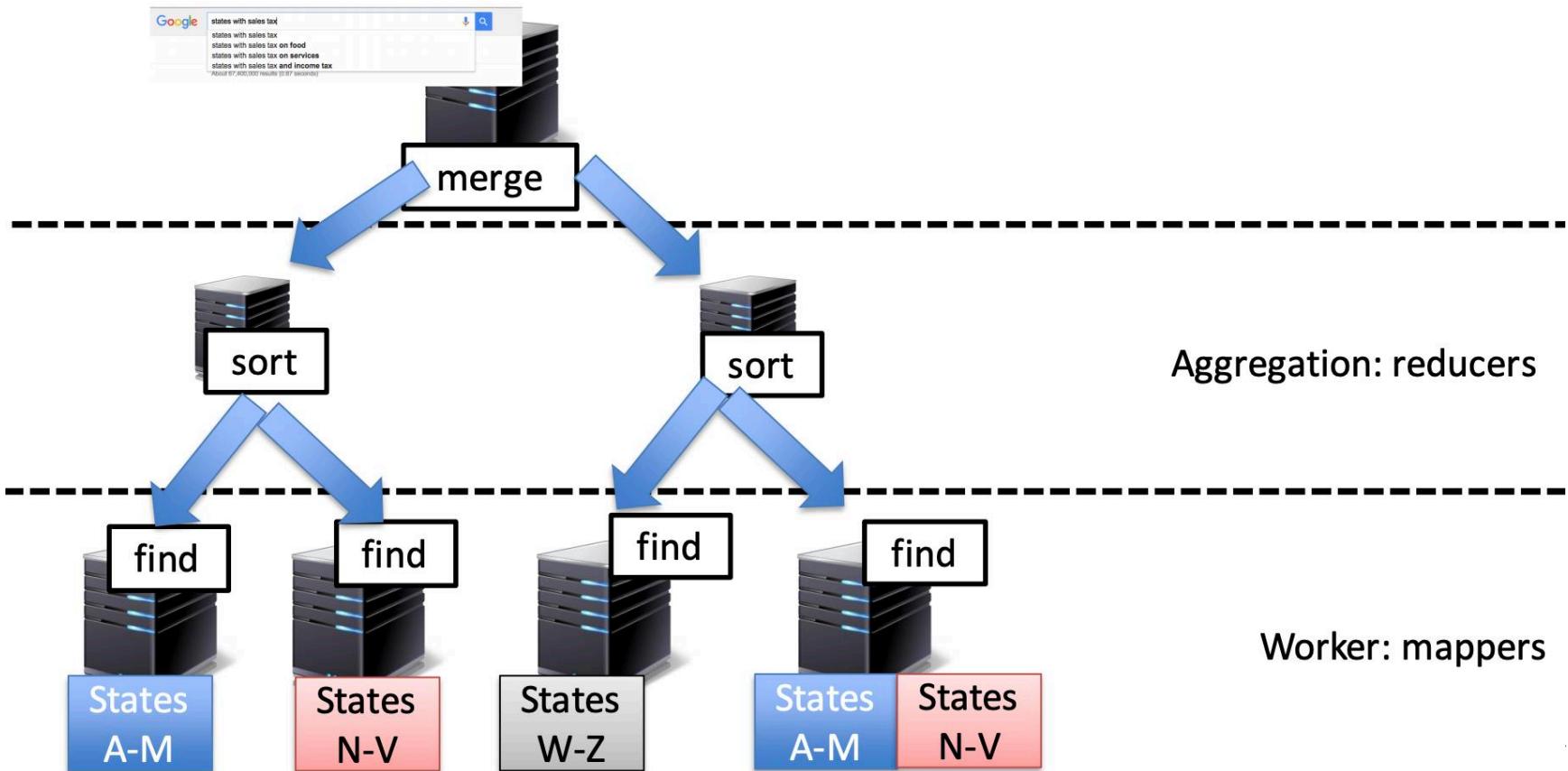
Example of distributed system: sorting

- Workflow



Example of distributed system: sorting

- How Do Request Get Processed in a Data Center



Example of distributed system: sorting

- How Google Search Works
- <https://www.youtube.com/watch?v=0eKVizvYSUQ>



Conclusion

- Why study HPC & parallel programming?
- What to learn?
- Course structure
- Course policy
- An example of HPC & parallel programming

