**Stat 155–Probability and Statistics for Science and Engineering**
**Second Mid-Term Examination**
**May 20, 2019**

Multiple Choice Questions (5 points each). Please choose the *SINGLE* best answer and indicate your choice on the scantron.

1. **Exhibit I.** Suppose the random variable $X$ has the following probability mass function (pmf):

| $x$ | $-1$ | $0$ | $2$ |
|---|---|---|---|
| $p(x)$ | 0.2 | 0.6 | 0.2 |

   The expected value of $X$ is

   (a) 0
   (b) 0.2
   (c) 0.4
   (d) 0.6
   (e) none of the above

2. Refer to Exhibit I. The variance of $X$ is

   (a) 0.33
   (b) 0.67
   (c) 0.96
   (d) 1.33
   (e) none of the above

3. Refer to Exhibit I. Let $F(x)$ be the cumulative distribution function (cdf) of $X$. Which of the following statements is **true**?

   (a) $F(0) = 0$
   (b) $F(0) = 0.6$
   (c) $F(1) = 0.6$
   (d) $F(1) = 0.8$
   (e) none of the above

4. A geologist has collected 10 specimens of basaltic rock and 10 specimens of granite. The geologist instructs a laboratory assistant to randomly select 15 (different) specimens for analysis. Let $X$ be the number of granite samples selected for analysis. What is the distribution of $X$?

   (a) Binomial
   (b) Hypergeometric
   (c) Negative Binomial
   (d) Poisson
   (e) none of the above

5. Continuing the last question, which of the following is closest to the variance of $X$?

   (a) 1
   (b) 3.75
   (c) 7.5
   (d) 10
   (e) 15

6. Suppose that in one area in California, 40% of all homeowners are insured against earthquake damage. Four homeowners are to be selected at random; let $X$ denote the number among the four who have earthquake insurance. What is the distribution of $X$?

   (a) Binomial
   (b) Hypergeometric
   (c) Negative Binomial
   (d) Poisson
   (e) none of the above

7. Continuing the last question, what is the probability that $X \geq 2$?

   (a) 0.1296
   (b) 0.3456
   (c) 0.4752
   (d) 0.5248
   (e) none of the above

8. Suppose an operator receives phone calls according to a Poisson process, at a rate of 36 calls per hour. Let $X$ be the number of calls that arrive within a 5-minute period. What is the variance of $X$?

   (a) 3
   (b) 5
   (c) 9
   (d) 25
   (e) none of the above

9. A family decides to have children until it has three girls. Assume that different births are independent with $P(\text{boy}) = P(\text{girl}) = 0.5$. Let $X$ be the **total** number of children the family will have eventually. What is the expected value of $X$?

   (a) 3
   (b) 4.125
   (c) 5
   (d) 6
   (e) none of the above

10. Which of the following parameter configurations would make it appropriate to approximate the $Bin\,(n, p)$ distribution with a normal distribution?

    (a) $n = 10$, $p = 0.7$
    (b) $n = 20$, $p = 0.8$
    (c) $n = 30$, $p = 0.4$
    (d) $n = 40$, $p = 0.1$
    (e) none of the above

**Exhibit II.** Suppose a continuous random variable $X$ has the following probability density function (pdf):

$$f(x) = \begin{cases} kx^2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

11. Refer to Exhibit II. What is the value of $k$? (Recall that the total area under the pdf must be 1.)

    (a) 0.25
    (b) 0.375
    (c) 2.667
    (d) 4
    (e) none of the above

12. Refer to Exhibit II. What is the probability that $X \le 1$?

    (a) 0.125
    (b) 0.25
    (c) 0.375
    (d) 0.5
    (e) none of the above

**Exhibit III.** The checkout duration $X$ (hours) for a book on a 2-hour reserve at a college library has the following cumulative distribution function (cdf):

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2/4 & 0 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

13. Refer to Exhibit III. What is the median checkout duration?

    (a) 0.5
    (b) 1
    (c) $\sqrt{2}$
    (d) 1.5
    (e) none of the above

14. Refer to Exhibit III. What is the probability that $X \le 1$?

(a) 1/5

(b) 1/4

(c) 1/3

(d) 1/2

(e) none of the above

15. The time $X$ (minutes) for a lab assistant to prepare the equipment for a certain experiment is believed to follow a uniform distribution with $A = 20$ and $B = 30$. For $20 < a < a + 2 < 30$, what is the probability that $a < X < a + 2$?

(a) 0.1

(b) 0.2

(c) 0.3

(d) 0.5

(e) none of the above

16. Continuing the last question, what is the variance of $X$?

(a) 25/3

(b) 10

(c) 25

(d) 100

(e) none of the above

17. Let $Z$ be a standard normal random variable, $\Phi$ the standard normal cdf, and $c$ an arbitrary positive number. Which of the following expressions is **false**? (Hint: draw a picture.)

(a) $\Phi(c) + \Phi(-c) = 1$

(b) $P(-c < Z < c) = 2\Phi(c) - 1$

(c) $P(Z > -c) = P(Z < c)$

(d) $P(Z < -c \text{ or } Z > c) = 2\Phi(-c)$

(e) none of the above

18. Let $Z$ be a standard normal random variable, $\Phi$ the standard normal cdf, and $c$ an arbitrary positive number. Which of the following quantities is larger than the others?

(a) $\Phi(c)$

(b) $P(0 \leq Z \leq c)$

(c) $P(Z \geq c)$

(d) $P(-c < Z < c)$

(e) not enough information to tell

19. Let $X$ be a normal random variable with mean 85 and standard deviation 10. Knowing that $\Phi(1.5) = 0.9332$, what is the probability that $X \geq 70$?

    (a) 0.0668

    (b) 0.1336

    (c) 0.8664

    (d) 0.9332

    (e) none of the above

20. Let $X$ be the time (hours) between two successive arrivals at the drive-up window of a local bank. Suppose $X$ follows an exponential distribution with $\lambda = 1$; so the cdf of $X$ is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x} & x \geq 0 \end{cases}$$

    Given that the last arrival occurred half an hour ago, what is the probability that no one else will arrive within the next half hour?

    (a) $e^{-1}$

    (b) $1 - e^{-1}$

    (c) $e^{-1/2}$

    (d) $1 - e^{-1/2}$

    (e) none of the above