

```

In[1]:= S := 16
        S
        Sc := S/2
Out[2]= 16

In[4]:= cr := {{0, 1}, {0, 0}}

In[5]:= an := {{0, 0}, {1, 0}}
        n = .

In[7]:= cr ;

In[8]:= an ;

In[9]:= id2 := {{-1, 0}, {0, 1}}

In[10]:= id := IdentityMatrix[2]

In[11]:= c[n_] :=
        SparseArray[KroneckerProduct @@ (Table[id, {n - 1}] ~Join~ {cr} ~Join~ Table[id2, {Sc - n}])]
        cd[n_] :=
        SparseArray[KroneckerProduct @@ (Table[id, {n - 1}] ~Join~ {an} ~Join~ Table[id2, {Sc - n}])]

In[13]:= c[1].cd[1] + cd[1].c[1] ;

In[14]:= Do[ψ[n] = (1/Sqrt[2]) * (c[n] + cd[n]), {n, Sc}]

In[15]:= Do[ψ[Sc + n] = (1/Sqrt[2]) * (-I * c[n] + I * cd[n]), {n, Sc}]

In[16]:= Do[ψ[i1, i2] = ψ[i1].ψ[i2], {i1, S}, {i2, S}] // Timing
        Do[ψ[i3, i4] = ψ[i3].ψ[i4], {i3, S}, {i4, S}]

Out[16]= {0.053491, Null}

        Hamiltonian q = 2

In[18]:= ψ[1, 1] ;
        q = 2;
        J = 1;

In[21]:= Js = RandomVariate[NormalDistribution[0, Sqrt[(J^2) * ((q - 1)! / (S^(q - 1)) )]], {S, S}]
Out[21]= {{-0.0185682, 0.0813419, -0.128479, 0.239844, 0.22736, -0.350823, 0.142102, -0.308322,
        0.284913, 0.2914, -0.330952, 0.232132, -0.303041, 0.42197, -0.0546247, -0.11685},
        {0.0808133, -0.245592, -0.157307, 0.075899, -0.0238774,
        -0.00834184, 0.222065, 0.02999, 0.168742, -0.217821, 0.247966,
        -0.0850806, 0.0980814, -0.213642, 0.0754028, 0.153616},

```

```

{0.236621, 0.105905, 0.377715, -0.121772, -0.472456, 0.235731,
 -0.0367851, 0.0559142, 0.0905272, -0.261941, 0.163565,
 -0.0805816, -0.170531, -0.316105, -0.191298, 0.0300723},
{0.0162516, -0.00643119, 0.477207, -0.0395425, 0.273054,
 -0.0538415, 0.229286, 0.0272972, 0.246929, -0.16397, 0.242679,
 -0.343837, -0.039801, 0.178668, 0.0659389, -0.0859803},
{-0.181663, 0.411483, -0.38823, 0.128016, 0.354363, 0.0219805, 0.119375,
 -0.203528, -0.430089, 0.258972, -0.199695, -0.289749, -0.109075,
 0.446223, 0.176359, -0.360282}, {-0.164931, 0.381811, 0.141785, 0.310543,
 -0.0933857, -0.0223687, 0.295682, 0.224615, 0.516219, -0.312496,
 -0.0762332, -0.156209, 0.0677654, -0.396108, -0.153188, 0.132668},
{0.35028, -0.229309, 0.00435406, -0.208909, 0.223748, -0.0981613,
 -0.175216, 0.0379475, 0.0104412, -0.0414482, 0.220603,
 -0.304237, -0.397266, -0.483425, 0.035444, 0.15142},
{0.0659013, -0.269399, -0.138294, 0.206026, 0.335549, -0.354037,
 0.151409, -0.0471368, 0.0468704, -0.149032, 0.520608,
 -0.0225646, -0.0697258, 0.293352, 0.278538, -0.0292264},
{0.250068, 0.217015, -0.202745, -0.38172, 0.21253, -0.617142,
 0.0309666, -0.223985, -0.0684681, 0.0976588, 0.508088,
 -0.478943, -0.0631594, -0.403391, -0.452167, -0.0968736},
{-0.625446, 0.0413842, -0.232988, 0.0789123, 0.230316, -0.287653,
 0.129549, -0.402354, 0.0102112, -0.0590402, -0.537105,
 -0.15263, -0.315799, -0.167862, 0.0162314, 0.000863623},
{0.0946308, -0.155643, -0.156213, -0.0555633, -0.106783, 0.302932,
 -0.16324, -0.596382, 0.303915, 0.00876554, -0.0384043, 0.105449, 0.134669,
 0.0995717, 0.570861, -0.29389}, {0.093291, -0.336771, 0.0547772, 0.103397,
 -0.0980638, -0.108337, -0.0590698, -0.101773, 0.444428, 0.0419434,
 -0.120888, -0.285785, -0.0813816, 0.0813147, -0.147761, 0.597231},
{-0.138755, 0.165813, -0.144877, -0.156108, -0.114813, -0.0166544,
 0.104446, -0.215631, -0.064301, -0.0700499, 0.00368724, -0.432806,
 0.285202, 0.0704872, 0.094447, 0.282856}, {0.0877155, -0.1285, 0.0798081,
 -0.406225, -0.24075, -0.107971, 0.176449, 0.170271, -0.267172, -0.13497,
 0.253713, 0.000839945, -0.0834465, -0.222055, 0.0799468, 0.326504},
{0.0338193, -0.21241, 0.259281, 0.189664, -0.276032, -0.317419, -0.214508,
 -0.225475, -0.110681, 0.110473, -0.00300268, 0.0904052, 0.265896,
 -0.152707, 0.126262, 0.242296}, {0.0559251, 0.530681, 0.386412,
 -0.139585, 0.187521, -0.259078, -0.0626649, 0.031829, 0.0489958,
 0.196945, 0.0172631, -0.124216, 0.0685907, 0.319102, -0.21679, 0.321049}}

```

$\ln[22]:= H = \text{Sum}[I * J s[[i1, i2]] * \psi[i1, i2], \{i1, S\}, \{i2, i1 + 1, S\}] // \text{Normal};$

```

In[23]:= iv = H // N // Eigenvalues // Sort
Out[23]=
{-3.05517, -2.9766, -2.93554, -2.85696, -2.59718, -2.5186, -2.48772, -2.47754,
-2.40915, -2.39897, -2.36809, -2.34866, -2.28952, -2.27009, -2.22903, -2.15045,
-2.0956, -2.02973, -2.01703, -1.97597, -1.95115, -1.91009, -1.89739, -1.89067,
-1.83152, -1.81209, -1.78121, -1.77103, -1.71427, -1.70264, -1.69246, -1.66158,
-1.63761, -1.6357, -1.59463, -1.583, -1.55903, -1.52815, -1.51797, -1.51606,
-1.44958, -1.4394, -1.40852, -1.38909, -1.32995, -1.32322, -1.31052, -1.26946,
-1.25627, -1.24464, -1.20358, -1.19088, -1.1777, -1.17547, -1.14682, -1.13664,
-1.12501, -1.09689, -1.07016, -1.06825, -1.05806, -1.05583, -1.02718, -1.00776,
-0.991584, -0.977258, -0.950521, -0.948611, -0.931095, -0.929185, -0.888122,
-0.871948, -0.852522, -0.821644, -0.811459, -0.809549, -0.754699, -0.743071,
-0.732886, -0.71747, -0.702008, -0.688822, -0.676126, -0.638897, -0.635063,
-0.623435, -0.610249, -0.608018, -0.597834, -0.569186, -0.55649, -0.549761,
-0.529445, -0.519261, -0.490613, -0.488382, -0.471188, -0.468957, -0.440309,
-0.430125, -0.409809, -0.390384, -0.363646, -0.361736, -0.351552, -0.349321,
-0.320673, -0.296702, -0.285073, -0.270748, -0.24401, -0.2421, -0.218129,
-0.215898, -0.18725, -0.177065, -0.165437, -0.150021, -0.137325, -0.108677,
-0.0984925, -0.0962614, -0.0714477, -0.0676138, -0.0481884, -0.0303845,
-0.0176885, -0.0109591, 0.0109591, 0.0176885, 0.0303845, 0.0481884, 0.0676138,
0.0714477, 0.0962614, 0.0984925, 0.108677, 0.137325, 0.150021, 0.165437,
0.177065, 0.18725, 0.215898, 0.218129, 0.2421, 0.24401, 0.270748, 0.285073,
0.296702, 0.320673, 0.349321, 0.351552, 0.361736, 0.363646, 0.390384, 0.409809,
0.430125, 0.440309, 0.468957, 0.471188, 0.488382, 0.490613, 0.519261, 0.529445,
0.549761, 0.55649, 0.569186, 0.597834, 0.608018, 0.610249, 0.623435, 0.635063,
0.638897, 0.676126, 0.688822, 0.702008, 0.71747, 0.732886, 0.743071, 0.754699,
0.809549, 0.811459, 0.821644, 0.852522, 0.871948, 0.888122, 0.929185, 0.931095,
0.948611, 0.950521, 0.977258, 0.991584, 1.00776, 1.02718, 1.05583, 1.05806,
1.06825, 1.07016, 1.09689, 1.12501, 1.13664, 1.14682, 1.17547, 1.1777, 1.19088,
1.20358, 1.24464, 1.25627, 1.26946, 1.31052, 1.32322, 1.32995, 1.38909,
1.40852, 1.4394, 1.44958, 1.51606, 1.51797, 1.52815, 1.55903, 1.583, 1.59463,
1.6357, 1.63761, 1.66158, 1.69246, 1.70264, 1.71427, 1.77103, 1.78121, 1.81209,
1.83152, 1.89067, 1.89739, 1.91009, 1.95115, 1.97597, 2.01703, 2.02973,
2.0956, 2.15045, 2.22903, 2.27009, 2.28952, 2.34866, 2.36809, 2.39897,
2.40915, 2.47754, 2.48772, 2.5186, 2.59718, 2.85696, 2.93554, 2.9766, 3.05517}

```

In[24]:= **Histogram[iv]**

Out[24]=



In[25]:=

In[26]:= **q = 4**

Out[26]=

4

In[27]:= **J = 4**

Out[27]=

4

In[28]:= **Jm = RandomVariate[NormalDistribution[0, Sqrt[J^2 * ((q - 1)! / (S^(q - 1))]], {S, S, S, S}]**

Out[28]=

{... 1 ...}

Full expression not available (original memory size: 0.5 MB)



In[29]:= **Dynamic[{i1, i2, i3, i4}]**

Ham =

$$I^{(q/2)} \text{Sum}[\psi[i1, i2].\text{Sum}[Jm[i1, i2, i3, i4] * \psi[i3, i4], \{i3, i2 + 1, S\}, \{i4, i3 + 1, S\}], \{i1, S - 3\}, \{i2, i1 + 1, S - 2\}] // \text{Normal};$$

Out[29]=

{i1, i2, i3, i4}

In[31]:= **Ham**

Out[31]=

$$\left\{ \left\{ -0.00496035 + 0.i, 0., 0., 0.0382303 - 0.019193i, 0., 0.24497 + 0.215397i, 0.0508617 - 0.0798721i, 0., 0., 0.068262 - 0.0775788i, 0.223176 + 0.14239i, 0., 0.0802712 - 0.00212717i, 0., 0., \dots 226 \dots, 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., \dots 254 \dots, \{0., \dots 254 \dots, - \dots 21 \dots + \dots 1 \dots\} \right\} \right\}$$

Full expression not available (original memory size: 2.6 MB)



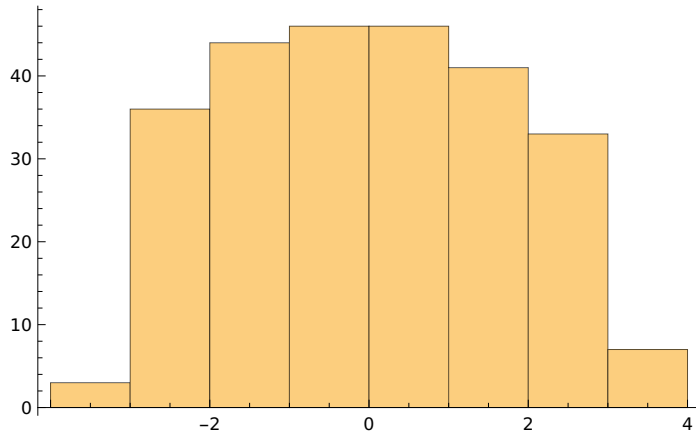
```

In[32]:= ivv = Ham // N // Eigenvalues // Sort
Out[32]=
{-3.15393, -3.10652, -3.0479, -2.99596, -2.9619, -2.90983, -2.88907, -2.85597,
-2.83771, -2.79765, -2.7592, -2.7325, -2.66624, -2.64938, -2.63757, -2.58452,
-2.54662, -2.53809, -2.4864, -2.47722, -2.45806, -2.40174, -2.40091, -2.39696,
-2.3476, -2.33248, -2.30413, -2.2998, -2.237, -2.21424, -2.20566, -2.16525,
-2.16022, -2.15001, -2.13686, -2.10835, -2.07174, -2.07011, -2.02819, -1.99636,
-1.9819, -1.96635, -1.93566, -1.90608, -1.86856, -1.83295, -1.83083, -1.76654,
-1.74765, -1.72318, -1.68069, -1.67145, -1.66507, -1.60885, -1.59112, -1.59036,
-1.58563, -1.54043, -1.5339, -1.5203, -1.46005, -1.44449, -1.43012, -1.40947,
-1.38927, -1.37589, -1.33459, -1.313, -1.30723, -1.27889, -1.27835, -1.21323,
-1.20925, -1.1842, -1.17864, -1.14112, -1.13992, -1.11236, -1.07642, -1.0609,
-1.03818, -1.03186, -1.01814, -0.97022, -0.939554, -0.92913, -0.912727,
-0.892758, -0.880791, -0.845951, -0.829454, -0.785145, -0.772021, -0.73443,
-0.725273, -0.709269, -0.680567, -0.680066, -0.659687, -0.632653, -0.621802,
-0.611461, -0.556774, -0.515769, -0.513991, -0.497525, -0.474301, -0.443479,
-0.432598, -0.425838, -0.387259, -0.349854, -0.343473, -0.335219, -0.331848,
-0.244577, -0.227843, -0.226612, -0.21333, -0.205906, -0.157875, -0.144502,
-0.125099, -0.112903, -0.0921272, -0.0902872, -0.0433065, -0.0365145,
-0.0179059, 0.0083685, 0.030427, 0.0492902, 0.0722906, 0.133836, 0.1387,
0.169526, 0.176701, 0.189919, 0.210435, 0.220032, 0.248892, 0.257996, 0.304276,
0.327352, 0.335847, 0.342508, 0.370833, 0.422849, 0.424439, 0.431497, 0.452227,
0.46055, 0.474033, 0.49051, 0.525461, 0.547257, 0.552287, 0.562874, 0.563939,
0.586495, 0.633622, 0.667828, 0.680381, 0.695521, 0.737431, 0.753339, 0.766574,
0.802023, 0.829939, 0.834095, 0.860952, 0.872262, 0.932428, 0.94378, 0.986731,
1.02014, 1.03677, 1.05433, 1.07567, 1.11305, 1.14929, 1.1532, 1.16792, 1.17628,
1.21135, 1.2427, 1.2557, 1.26103, 1.2792, 1.31786, 1.36856, 1.37047, 1.38766,
1.414, 1.45214, 1.48494, 1.49536, 1.50391, 1.52899, 1.57661, 1.58133, 1.60463,
1.60685, 1.65688, 1.67952, 1.68596, 1.69569, 1.7572, 1.77677, 1.80961, 1.8498,
1.87178, 1.91575, 1.91888, 1.93244, 1.97612, 2.02571, 2.05472, 2.06365, 2.08523,
2.10933, 2.1188, 2.14928, 2.15977, 2.19041, 2.2376, 2.26396, 2.27301, 2.28944,
2.32166, 2.36923, 2.39878, 2.41619, 2.41792, 2.4848, 2.51164, 2.52082, 2.54365,
2.58173, 2.61074, 2.67386, 2.68102, 2.73962, 2.81295, 2.85661, 2.86482, 2.9252,
2.95324, 2.97501, 3.00078, 3.09907, 3.14445, 3.18392, 3.2358, 3.29155, 3.32172}

```

```
In[33]:= ivv // Histogram
```

Out[33]=



Two Point functions

```
In[34]:=  $\beta$  = 5;
```

H = Ham // N

Out[35]=

$$\left\{ \left\{ -0.00496035 + 0.i, 0., 0., 0.0382303 - 0.019193i, 0., 0.24497 + 0.215397i, 0.0508617 - 0.0798721i, 0., 0., \right. \right. \\ \left. \left. 0.068262 - 0.0775788i, 0.223176 + 0.14239i, 0., 0.0802712 - 0.00212717i, 0., 0., \dots 226 \dots, 0., \right. \right. \\ \left. \left. 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0. \right\}, \dots 254 \dots, \left\{ 0., \dots 254 \dots, - \dots 21 \dots + \dots 1 \dots \right\} \right\}$$

Full expression not available (original memory size: 2.6 MB)



```
In[36]:=
```

```
In[37]:= Clear[Gn];
```

$$\text{Gn}[a_ , b_ , \tau_ , \beta_ , \lambda_] := \text{Gn}[a , b , \tau , \beta , \lambda] = \text{Block}[\{\},$$
$$\text{If } [\tau > 0,$$
$$E\tau = \text{MatrixExp}[-\tau H \lambda];$$
$$E\beta\tau = \text{MatrixExp}\left[\left(-\beta + \tau\right) H \lambda\right];$$
$$(\text{Tr}[\mathbf{E}\beta\tau.\psi[a].\mathbf{E}\tau.\psi[b]]) / (\text{Tr}[\mathbf{E}\beta\tau.\mathbf{E}\tau]),$$
$$E\tau = \text{MatrixExp}[+\tau H \lambda];$$
$$E\beta\tau = \text{MatrixExp}[(-\beta - \tau)H\lambda];$$
$$-(\text{Tr}[\mathbf{E}\beta\tau \cdot \psi[b] \cdot \mathbf{E}\tau \cdot \psi[a]]) / (\text{Tr}[\mathbf{E}\beta\tau \cdot \mathbf{E}\tau]),$$

1

1

In[39]:= **Gn[1, 1, 1/4, 1, 1]**

Out[39]=
 $0.409132 - 1.53934 \times 10^{-18} i$

In[40]:= **Dynamic[tt]**

Gn[1, 1, 1, 1, 1]

Out[40]=
tt

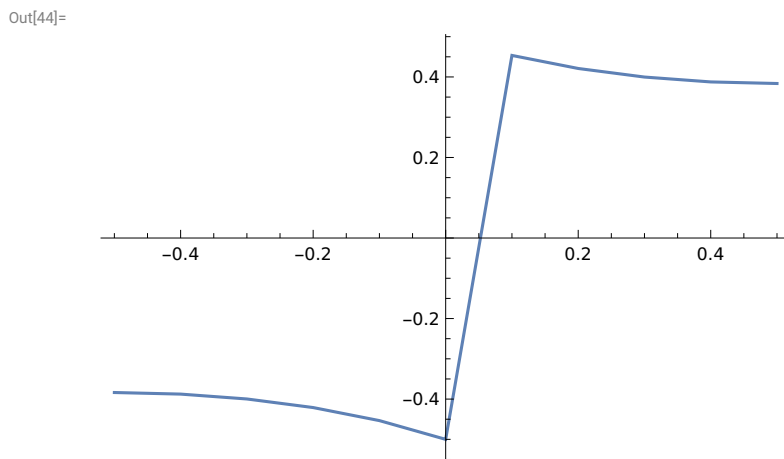
Out[41]=
 $0.5 + 5.77779 \times 10^{-34} i$

In[42]:= **tbGG = Table[{tt, Gn[1, 1, tt, 1, 1]}, {tt, -1/2, 1/2, 1/10}] // Re**

Out[42]=
 $\left\{ \left\{ -\frac{1}{2}, -0.383696 \right\}, \left\{ -\frac{2}{5}, -0.38762 \right\}, \left\{ -\frac{3}{10}, -0.399723 \right\}, \right.$
 $\left. \left\{ -\frac{1}{5}, -0.421034 \right\}, \left\{ -\frac{1}{10}, -0.453445 \right\}, \{0, -0.5\}, \left\{ \frac{1}{10}, 0.453445 \right\}, \right.$
 $\left. \left\{ \frac{1}{5}, 0.421034 \right\}, \left\{ \frac{3}{10}, 0.399723 \right\}, \left\{ \frac{2}{5}, 0.38762 \right\}, \left\{ \frac{1}{2}, 0.383696 \right\} \right\}$

In[43]:=

In[44]:= **plt = ListPlot[tbGG, Joined → True]**



```
In[45]:= Spec[τ_, β_] := Spec[τ, β] = Block[{ },
  If[τ > 0,
    Zβτm = Tr[MatrixExp[(-β * H) - (I * H * τ)]];
    Zβτp = Tr[MatrixExp[(-β * H) + (I * H * τ)]];
    Zβ = Tr[MatrixExp[(-β * H)]];
    Abs[(Zβτm * Zβτp) / (Zβ)^2]
  ]
]
```

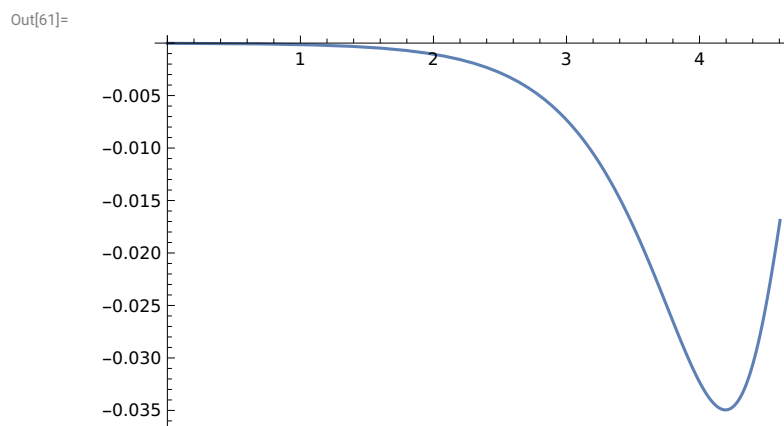
```
In[46]:= Spec[3, 1]
```

```
Out[46]=
0.0591718
```

```
In[59]:= tbSpec = Table[{tt, Spec[tt, 100]}, {tt, 1, 100, 1}];
```

```
In[62]:= tap = Table[{i, Sum[tbSpec[[i]][[2]], {i, 1, i}]}, {i, 1, 100, 1}];
```

```
In[61]:= pltSpec = ListPlot[Log[tbSpec], Joined → True]
```



```
In[63]:= pltSpec2 = ListPlot[tap, Joined → True]
```

