```mathematica
In[1]:= S := 16
        S
        Sc := S / 2
```

Out[2]= 16

```mathematica
In[4]:= cr := {{0, 1}, {0, 0}}
```

```mathematica
In[5]:= an := {{0, 0}, {1, 0}}
        n =.
```

```mathematica
In[7]:= cr ;
```

```mathematica
In[8]:= an ;
```

```mathematica
In[9]:= id2 := {{-1, 0}, {0, 1}}
```

```mathematica
In[10]:= id := IdentityMatrix[2]
```

```mathematica
In[11]:= c[n_] :=
         SparseArray[KroneckerProduct @@ ( Table[id, (n - 1)] ~Join~ {cr} ~Join~ Table[id2, {Sc - n}])]
         cd[n_] :=
         SparseArray[KroneckerProduct @@ (Table[id, (n - 1)] ~Join~ {an} ~Join~ Table[id2, {Sc - n}])]
```

```mathematica
In[13]:= c[1].cd[1] + cd[1].c[1] ;
```

```mathematica
In[14]:= Do[ψ[n] = (1 / Sqrt[2]) * (c[n] + cd[n]), {n, Sc}]
```

```mathematica
In[15]:= Do[ψ[Sc + n] = ( 1 / Sqrt[2]) * (-I * c[n] + I * cd[n]), {n, Sc}]
```

```mathematica
In[16]:= Do[ψ[i1, i2] = ψ[i1].ψ[i2], {i1, S}, {i2, S}] // Timing
         Do[ψ[i3, i4] = ψ[i3].ψ[i4], {i3, S}, {i4, S}]
```

Out[16]=

{0.048712, Null}

Hamiltonian q = 2

```mathematica
In[18]:= ψ[1, 1] ;
         q = 2;
         J = 1;
```

```mathematica
In[21]:= Js = RandomVariate[NormalDistribution[0, Sqrt[ (J^2) * ( (q - 1)! / (S^(q - 1)) )]], {S, S}]
```

Out[21]=

{{-0.0489194, -0.224879, 0.0187036, -0.371167,
   -0.138508, 0.395436, -0.44221, -0.457569, -0.2664, -0.363714,
   -0.531623, -0.205805, 0.0795659, 0.282232, 0.126332, 0.465541},
  {-0.200289, -0.00435953, -0.431903, 0.0189042, -0.0855657,
   -0.430771, 0.112109, -0.152356, -0.273491, 0.00651877,

```
        -0.085554, -0.192404, -0.096431, 0.15297, 0.0070294, -1.06287},
     {-0.255307, 0.033957, 0.0759824, 0.0320758, 0.0962582, 0.370996,
       0.107507, 0.0687588, -0.0839152, 0.433298, -0.356612,
       0.403281, 0.135204, 0.437046, -0.334126, 0.0829714},
     {0.430137, 0.444869, -0.0744296, -0.402576, 0.0986474, -0.00408623,
       0.0970411, 0.301268, -0.0308262, 0.134501, -0.107024,
       -0.387914, 0.0474704, -0.0317709, 0.00790474, -0.0814116},
     {0.0518719, -0.344281, 0.0845449, 0.230998, 0.262122, 0.370759, -0.025712,
       -0.164649, -0.0452081, -0.130557, 0.210295, 0.355222, -0.147962,
       -0.0257641, 0.18679, 0.0255095}, {-0.0656935, 0.169862, -0.174865,
       0.134809, 0.0998637, 0.235993, 0.203056, 0.150366, -0.275852, 0.632073,
       -0.314667, -0.197691, 0.176283, -0.143823, 0.0197202, 0.157545},
     {0.264084, 0.274682, 0.112717, 0.0977801, -0.0127269, -0.409067, 0.150623,
       -0.0827522, 0.0333648, -0.122857, -0.400112, 0.00978636, 0.0913825,
       0.623874, 0.221513, 0.105811}, {0.0791087, -0.324637, -0.381881,
       0.349991, -0.235657, -0.145257, 0.272524, 0.204774, 0.192138, -0.15918,
       -0.0277359, -0.189252, -0.309301, -0.28628, 0.0754534, 0.209504},
     {-0.440218, -0.05193, -0.016513, -0.356275, 0.292087, -0.222934, 0.134252,
       0.145083, -0.314893, -0.106409, 0.0153258, -0.112823, -0.404042,
       -0.0814831, 0.0955721, 0.109544}, {-0.488301, -0.32397, -0.11593,
       0.130338, -0.0327356, -0.634696, -0.158616, -0.477979, 0.173413,
       -0.356227, -0.127181, 0.102639, -0.278777, 0.277286, -0.409001, -0.20749},
     {0.06026, 0.418725, -0.0835344, 0.160769, -0.169849, 0.0221477,
       0.00228497, -0.235567, -0.0488625, 0.00827652, 0.104685,
       -0.499306, 0.144849, -0.000226318, 0.0268021, -0.0174341},
     {0.363205, -0.125351, 0.0490961, -0.326095, -0.104901, -0.219652,
       0.244861, -0.141203, 0.223259, -0.0666131, 0.0562673,
       0.0776461, -0.47226, 0.126234, 0.0475763, -0.00703472},
     {-0.0535108, -0.108836, -0.251072, -0.18953, 0.199074, -0.411943, -0.45982,
       -0.152635, -0.181599, -0.079928, -0.61038, -0.0708233, 0.254521,
       0.14149, -0.204024, 0.164644}, {-0.176319, -0.0375434, -0.00819487,
       -0.153621, 0.0195032, -0.361097, -0.323413, -0.202394, -0.100857,
       0.113418, -0.242055, -0.481977, 0.21348, -0.323273, -0.405068, -0.275863},
     {0.0208431, -0.0864243, -0.12985, 0.110045, 0.0952625, -0.0558473,
       -0.0892359, -0.414265, -0.0475589, -0.137744, -0.322507,
       0.037335, -0.152465, 0.0953758, -0.417748, -0.518968},
     {-0.164533, -0.140917, -0.393414, 0.0302401, -0.304285, -0.234782,
       -0.0541714, -0.00559783, -0.233323, -0.0828849, -0.424313,
       0.527539, -0.0207522, 0.121031, 0.110824, 0.0455267}}
```

In[22]:= H = Sum[I * Js[[i1, i2]] * ψ[i1, i2], {i1, S}, {i2, i1 + 1, S}] // Normal;

In[23]:= `iv = H // N // Eigenvalues // Sort`

Out[23]=

```
{-3.39148, -3.38928, -3.18396, -3.18177, -2.98707, -2.98487, -2.77956, -2.77736,
 -2.70626, -2.70407, -2.49875, -2.49655, -2.36258, -2.36039, -2.30185, -2.29966,
 -2.15507, -2.15288, -2.09434, -2.09215, -2.09191, -2.08971, -1.95858, -1.95818,
 -1.95638, -1.95598, -1.8844, -1.8822, -1.75107, -1.75066, -1.74887, -1.74847,
 -1.6875, -1.6853, -1.67737, -1.67517, -1.66921, -1.66701, -1.55417, -1.55197,
 -1.47999, -1.47779, -1.46986, -1.46766, -1.4617, -1.4595, -1.40669, -1.4045,
 -1.34666, -1.34446, -1.27337, -1.27296, -1.27117, -1.27077, -1.2648, -1.2626,
 -1.19918, -1.19699, -1.06586, -1.06545, -1.06366, -1.06325, -1.06302, -1.06082,
 -1.05729, -1.05509, -1.00228, -1.00009, -0.983993, -0.981797, -0.929689,
 -0.927493, -0.868957, -0.86676, -0.855507, -0.853311, -0.794774, -0.792578,
 -0.776483, -0.774287, -0.722179, -0.719983, -0.661446, -0.65925, -0.659013,
 -0.658608, -0.656817, -0.656412, -0.640317, -0.638121, -0.579584, -0.577388,
 -0.52528, -0.523084, -0.451502, -0.451098, -0.449306, -0.448902, -0.432806,
 -0.43061, -0.377804, -0.375608, -0.372074, -0.369878, -0.369641, -0.367444,
 -0.31777, -0.315573, -0.254604, -0.252408, -0.244475, -0.242279, -0.236312,
 -0.235908, -0.234116, -0.233711, -0.170293, -0.168097, -0.16213, -0.159934,
 -0.0470932, -0.0448971, -0.0369649, -0.0347688, -0.0288017, -0.0283971,
 -0.0266056, -0.026201, 0.026201, 0.0266056, 0.0283971, 0.0288017, 0.0347688,
 0.0369649, 0.0448971, 0.0470932, 0.159934, 0.16213, 0.168097, 0.170293,
 0.233711, 0.234116, 0.235908, 0.236312, 0.242279, 0.244475, 0.252408, 0.254604,
 0.315573, 0.31777, 0.367444, 0.369641, 0.369878, 0.372074, 0.375608, 0.377804,
 0.43061, 0.432806, 0.448902, 0.449306, 0.451098, 0.451502, 0.523084, 0.52528,
 0.577388, 0.579584, 0.638121, 0.640317, 0.656412, 0.656817, 0.658608, 0.659013,
 0.65925, 0.661446, 0.719983, 0.722179, 0.774287, 0.776483, 0.792578, 0.794774,
 0.853311, 0.855507, 0.86676, 0.868957, 0.927493, 0.929689, 0.981797, 0.983993,
 1.00009, 1.00228, 1.05509, 1.05729, 1.06082, 1.06302, 1.06325, 1.06366,
 1.06545, 1.06586, 1.19699, 1.19918, 1.2626, 1.2648, 1.27077, 1.27117, 1.27296,
 1.27337, 1.34446, 1.34666, 1.4045, 1.40669, 1.4595, 1.4617, 1.46766, 1.46986,
 1.47779, 1.47999, 1.55197, 1.55417, 1.66701, 1.66921, 1.67517, 1.67737,
 1.6853, 1.6875, 1.74847, 1.74887, 1.75066, 1.75107, 1.8822, 1.8844, 1.95598,
 1.95638, 1.95818, 1.95858, 2.08971, 2.09191, 2.09215, 2.09434, 2.15288,
 2.15507, 2.29966, 2.30185, 2.36039, 2.36258, 2.49655, 2.49875, 2.70407,
 2.70626, 2.77736, 2.77956, 2.98487, 2.98707, 3.18177, 3.18396, 3.38928, 3.39148}
```

In[24]:= `Histogram[iv]`

Out[24]=
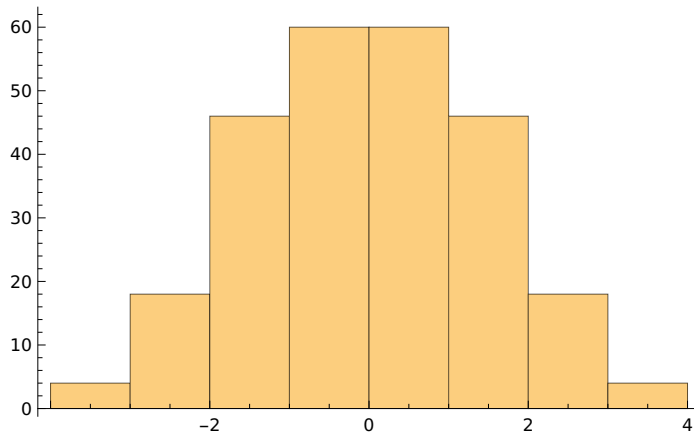
In[25]:=

In[26]:= `q = 4`

Out[26]=

4

In[27]:= `J = 4`

Out[27]=

4

In[28]:= `Jm = RandomVariate[NormalDistribution[0, Sqrt[J^2 * ((q-1)!)/(S^(q-1))]], {S, S, S, S}]`

Out[28]=

```
{{  ··· 1 ···  }, {  ··· 1 ···  },   ··· 12 ···  ,
  {{{0.0830187, -0.0413969, 0.0654496, 0.375797, 0.0362138, 0.151266, 0.146041, 0.0467501,
      -0.03787, 0.0203225, 0.139444, -0.060923, -0.334651, -0.210695, 0.157979, -0.0749213},
     ··· 14 ··· , {  ··· 1 ···  }},   ··· 14 ···  , {  ··· 1 ···  }}, {  ··· 1 ···  }}
```

Full expression not available (original memory size: 0.5 MB)   ⚙

In[29]:= `Dynamic[{i1, i2, i3, i4}]`

`Ham =`

`  I^(q/2) Sum[ψ[i1, i2].Sum[ Jm⟦i1, i2, i3, i4⟧ * ψ[i3, i4], {i3, i2+1, S}, {i4, i3 + 1, S}],`

`    {i1, S-3}, {i2, i1 + 1, S-2}] // Normal;`

Out[29]=

{i1, i2, i3, i4}

In[31]:= **Ham**

Out[31]=

{{0.067201 + 0. *i*, 0., 0., −0.00988416 − 0.0406664 *i*, 0., −0.0114813 + 0.0856677 *i*, −0.387348 + 0.1061 *i*, 0., 0.,
0.00482001 + 0.0257768 *i*, 0.203356 + 0.11987 *i*, 0., −0.291403 + 0.115416 *i*, 0., 0., ⟨ ··· 226 ··· ⟩, 0., 0.,
0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.}, ⟨ ··· 254 ··· ⟩, {0., ⟨ ··· 254 ··· ⟩, ⟨ ··· 19 ··· ⟩ + ⟨ ··· 1 ··· ⟩}}

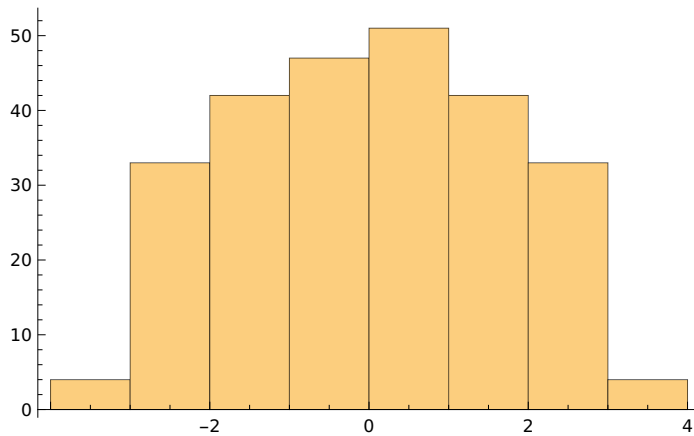Full expression not available (original memory size: 2.6 MB)

In[32]:= **ivv = Ham // N // Eigenvalues // Sort**

Out[32]=

{−3.21233, −3.08829, −3.07375, −3.06042, −2.9987, −2.89827, −2.87913, −2.83403,
−2.80051, −2.78411, −2.74937, −2.71505, −2.67304, −2.61593, −2.58706, −2.58142,
−2.52445, −2.51733, −2.50804, −2.48329, −2.42984, −2.40835, −2.39037, −2.3651,
−2.36134, −2.26887, −2.19966, −2.18963, −2.16794, −2.1514, −2.14808, −2.12688,
−2.07957, −2.07051, −2.05201, −2.04743, −2.01189, −1.95527, −1.95027, −1.93067,
−1.89424, −1.8942, −1.85561, −1.83701, −1.83336, −1.76759, −1.76375, −1.7418,
−1.69618, −1.67686, −1.65411, −1.64343, −1.63567, −1.55338, −1.55257, −1.52002,
−1.51695, −1.4792, −1.45234, −1.45163, −1.40489, −1.40405, −1.37218, −1.36119,
−1.33217, −1.32422, −1.30439, −1.29474, −1.24979, −1.24898, −1.22571, −1.20189,
−1.19345, −1.15205, −1.10406, −1.084, −1.0795, −1.04143, −1.01209, −0.984516,
−0.97921, −0.952279, −0.926971, −0.913277, −0.890661, −0.86754, −0.853716,
−0.809956, −0.787521, −0.769042, −0.761224, −0.716052, −0.708551, −0.70548,
−0.675971, −0.657549, −0.626881, −0.620124, −0.597331, −0.577691, −0.552121,
−0.54648, −0.52139, −0.509553, −0.494865, −0.462701, −0.44388, −0.435027,
−0.400474, −0.371482, −0.358241, −0.357532, −0.327205, −0.313993, −0.299741,
−0.295283, −0.262896, −0.217802, −0.193863, −0.158334, −0.140936, −0.137372,
−0.122543, −0.0956337, −0.0263718, −0.0214707, 0.00813232, 0.0330346,
0.0340849, 0.0430188, 0.0670332, 0.105804, 0.112833, 0.137231, 0.143766,
0.154109, 0.159944, 0.198722, 0.206839, 0.230006, 0.243736, 0.26967, 0.282236,
0.302709, 0.360966, 0.382921, 0.388692, 0.430904, 0.453356, 0.457936, 0.48219,
0.491209, 0.504015, 0.565988, 0.582025, 0.598986, 0.608317, 0.628272, 0.641153,
0.65517, 0.666712, 0.666775, 0.722737, 0.72685, 0.752958, 0.759231, 0.779113,
0.805397, 0.836041, 0.865141, 0.883394, 0.899028, 0.912422, 0.947885, 0.959043,
0.978537, 0.990601, 1.04413, 1.06143, 1.09378, 1.0984, 1.10659, 1.13226, 1.1417,
1.18243, 1.19204, 1.24452, 1.2634, 1.27539, 1.30095, 1.33003, 1.35138, 1.37223,
1.39767, 1.39822, 1.43579, 1.46356, 1.46447, 1.4879, 1.4928, 1.51247, 1.5661,
1.5781, 1.61671, 1.61847, 1.64115, 1.65858, 1.66834, 1.73865, 1.76497, 1.77349,
1.80103, 1.81176, 1.85468, 1.86367, 1.89066, 1.9256, 1.93301, 1.93721, 2.04245,
2.04973, 2.0498, 2.06102, 2.06569, 2.11002, 2.12824, 2.15812, 2.18198, 2.21311,
2.21737, 2.23145, 2.27701, 2.31199, 2.33234, 2.38011, 2.38741, 2.39525, 2.43501,
2.47927, 2.52817, 2.60369, 2.60481, 2.63194, 2.64268, 2.7003, 2.73598, 2.78332,
2.78944, 2.80859, 2.84669, 2.9252, 2.97259, 3.06176, 3.09391, 3.14191, 3.16807}

In[33]:= `ivv // Histogram`

Out[33]=

Two Point functions

In[34]:= β = 5;

`H = Ham // N`

Out[35]=

```
{{0.067201 + 0. i, 0., 0., -0.00988416 - 0.0406664 i, 0., -0.0114813 + 0.0856677 i, -0.387348 + 0.1061 i, 0., 0.,
   0.00482001 + 0.0257768 i, 0.203356 + 0.11987 i, 0., -0.291403 + 0.115416 i, 0., 0., ⋯ 226 ⋯ , 0., 0.,
   0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.}, ⋯ 254 ⋯ , {0., ⋯ 254 ⋯ , ⋯ 19 ⋯ + ⋯ 1 ⋯ }}}
```
Full expression not available (original memory size: 2.6 MB)   ⚙

In[36]:=

In[37]:=
```
Clear[Gn];
Gn[a_, b_, τ_, β_, λ_] := Gn[a, b, τ, β, λ] = Block[{},
If[τ > 0,
Eτ = MatrixExp[-τ H λ];
Eβτ = MatrixExp[(-β + τ) H λ];
(Tr[Eβτ.ψ[a].Eτ.ψ[b]]) / (Tr[Eβτ.Eτ]),

Eτ = MatrixExp[+τ H λ];
Eβτ = MatrixExp[(-β - τ) H λ];
-(Tr[Eβτ.ψ[b].Eτ.ψ[a]]) / (Tr[Eβτ.Eτ]),
]
]
```
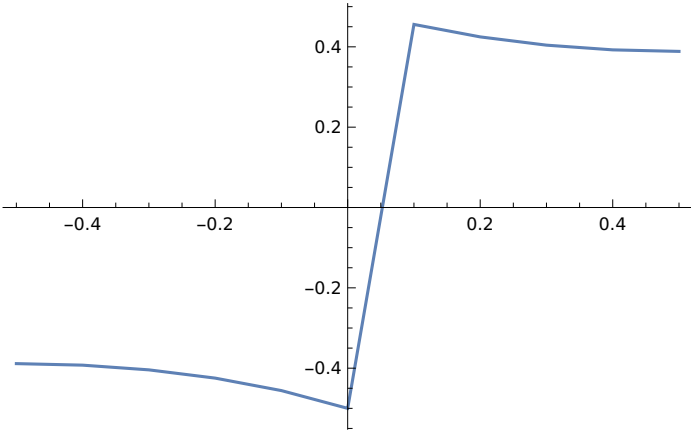
In[39]:= `Gn[1, 1, 1/4, 1, 1]`

Out[39]=

$0.413257 - 2.48759 \times 10^{-19} \, i$

In[40]:= `Dynamic[tt]`
`Gn[1, 1, 1, 1, 1]`

Out[40]=

`tt`

Out[41]=

$0.5 + 9.62965 \times 10^{-34} \, i$

In[42]:= `tbGG = Table[{tt, Gn[1, 1, tt, 1, 1]}, {tt, -1/2, 1/2, 1/10}] // Re`

Out[42]=

$\left\{ \left\{ -\frac{1}{2}, -0.388634 \right\}, \left\{ -\frac{2}{5}, -0.392445 \right\}, \left\{ -\frac{3}{10}, -0.404169 \right\}, \right.$

$\left\{ -\frac{1}{5}, -0.424723 \right\}, \left\{ -\frac{1}{10}, -0.455777 \right\}, \{0, -0.5\}, \left\{ \frac{1}{10}, 0.455777 \right\},$

$\left. \left\{ \frac{1}{5}, 0.424723 \right\}, \left\{ \frac{3}{10}, 0.404169 \right\}, \left\{ \frac{2}{5}, 0.392445 \right\}, \left\{ \frac{1}{2}, 0.388634 \right\} \right\}$

In[43]:=

In[44]:= `plt = ListPlot[tbGG, Joined → True]`

Out[44]=

In[45]:= $\text{Spec}[\tau\_, \beta\_] := \text{Spec}[\tau, \beta] = \text{Block}[\{\},$

$\text{If}[\tau > 0,$

$Z\beta\tau = \text{Tr}[\text{MatrixExp}[(-\beta * H) - (I * H * \tau)]];$

$Z\beta = \text{Tr}[\text{MatrixExp}[(-\beta * H)]];$

$\text{Abs}[(Z\beta\tau / Z\beta)]^{\wedge}2$

]

]

In[46]:= **Spec[3, 1]**

Out[46]=

0.0464621

In[47]:= **tbSpec = Table[{tt, Spec[tt, 1]}, {tt, 1, 100, 1}] // Re**

Out[47]=

{{1, 0.426558}, {2, 0.123482}, {3, 0.0464621}, {4, 0.0205906}, {5, 0.0125959},
 {6, 0.00697112}, {7, 0.00423197}, {8, 0.00266093}, {9, 0.0000779763}, {10, 0.0037585},
 {11, 0.00651388}, {12, 0.00629446}, {13, 0.00206251}, {14, 0.00154685},
 {15, 0.0000798716}, {16, 0.00153077}, {17, 0.00748674}, {18, 0.0107484},
 {19, 0.0107598}, {20, 0.0122338}, {21, 0.01131}, {22, 0.00581723}, {23, 0.00201071},
 {24, 0.00130226}, {25, 0.00363828}, {26, 0.00784598}, {27, 0.0164146}, {28, 0.0175225},
 {29, 0.0110508}, {30, 0.00467426}, {31, 0.00123695}, {32, 0.00282228},
 {33, 0.00174583}, {34, 0.005009}, {35, 0.0112438}, {36, 0.0194597}, {37, 0.0230191},
 {38, 0.0146953}, {39, 0.00734952}, {40, 0.00252354}, {41, 0.0063552}, {42, 0.00302503},
 {43, 0.010962}, {44, 0.0161163}, {45, 0.012504}, {46, 0.00285685}, {47, 0.00353849},
 {48, 0.00565794}, {49, 0.00471523}, {50, 0.000520056}, {51, 0.000596735},
 {52, 0.00966103}, {53, 0.0188496}, {54, 0.0199004}, {55, 0.012993}, {56, 0.00930855},
 {57, 0.00661776}, {58, 0.00335229}, {59, 0.017371}, {60, 0.0387269}, {61, 0.0371406},
 {62, 0.0268538}, {63, 0.0107101}, {64, 0.0069357}, {65, 0.0189361}, {66, 0.0301469},
 {67, 0.0207475}, {68, 0.0101175}, {69, 0.00853557}, {70, 0.00437852}, {71, 0.00506673},
 {72, 0.00301983}, {73, 0.000936984}, {74, 0.00169537}, {75, 0.00272581},
 {76, 0.00797721}, {77, 0.0132834}, {78, 0.0143474}, {79, 0.0260525}, {80, 0.0197781},
 {81, 0.015472}, {82, 0.0189118}, {83, 0.0159613}, {84, 0.00314894}, {85, 0.00589658},
 {86, 0.0105076}, {87, 0.0105795}, {88, 0.0123804}, {89, 0.00929381}, {90, 0.013701},
 {91, 0.012429}, {92, 0.00780954}, {93, 0.0117052}, {94, 0.00465697}, {95, 0.00938819},
 {96, 0.00890247}, {97, 0.00221777}, {98, 0.000795609}, {99, 0.0110453}, {100, 0.0237849}}

In[48]:= `tap = Table[{i, Sum[tbSpec〚i〛〚2〛, {i, 1, i}]}, {i, 1, 100}]`

Out[48]=

{{1, 0.426558}, {2, 0.55004}, {3, 0.596502}, {4, 0.617093}, {5, 0.629689}, {6, 0.63666},
  {7, 0.640892}, {8, 0.643553}, {9, 0.643631}, {10, 0.647389}, {11, 0.653903},
  {12, 0.660198}, {13, 0.66226}, {14, 0.663807}, {15, 0.663887}, {16, 0.665418},
  {17, 0.672904}, {18, 0.683653}, {19, 0.694413}, {20, 0.706646}, {21, 0.717956},
  {22, 0.723774}, {23, 0.725784}, {24, 0.727087}, {25, 0.730725}, {26, 0.738571},
  {27, 0.754986}, {28, 0.772508}, {29, 0.783559}, {30, 0.788233}, {31, 0.78947},
  {32, 0.792292}, {33, 0.794038}, {34, 0.799047}, {35, 0.810291}, {36, 0.829751},
  {37, 0.85277}, {38, 0.867465}, {39, 0.874815}, {40, 0.877338}, {41, 0.883693},
  {42, 0.886718}, {43, 0.89768}, {44, 0.913797}, {45, 0.926301}, {46, 0.929157},
  {47, 0.932696}, {48, 0.938354}, {49, 0.943069}, {50, 0.943589}, {51, 0.944186},
  {52, 0.953847}, {53, 0.972697}, {54, 0.992597}, {55, 1.00559}, {56, 1.0149},
  {57, 1.02152}, {58, 1.02487}, {59, 1.04224}, {60, 1.08097}, {61, 1.11811}, {62, 1.14496},
  {63, 1.15567}, {64, 1.16261}, {65, 1.18154}, {66, 1.21169}, {67, 1.23244},
  {68, 1.24255}, {69, 1.25109}, {70, 1.25547}, {71, 1.26054}, {72, 1.26356},
  {73, 1.26449}, {74, 1.26619}, {75, 1.26891}, {76, 1.27689}, {77, 1.29017},
  {78, 1.30452}, {79, 1.33057}, {80, 1.35035}, {81, 1.36582}, {82, 1.38474}, {83, 1.4007},
  {84, 1.40385}, {85, 1.40974}, {86, 1.42025}, {87, 1.43083}, {88, 1.44321}, {89, 1.4525},
  {90, 1.46621}, {91, 1.47863}, {92, 1.48644}, {93, 1.49815}, {94, 1.50281},
  {95, 1.51219}, {96, 1.5211}, {97, 1.52331}, {98, 1.52411}, {99, 1.53516}, {100, 1.55894}}

In[49]:= `pltSpec = ListPlot[tbSpec, Joined → True]`

Out[49]=
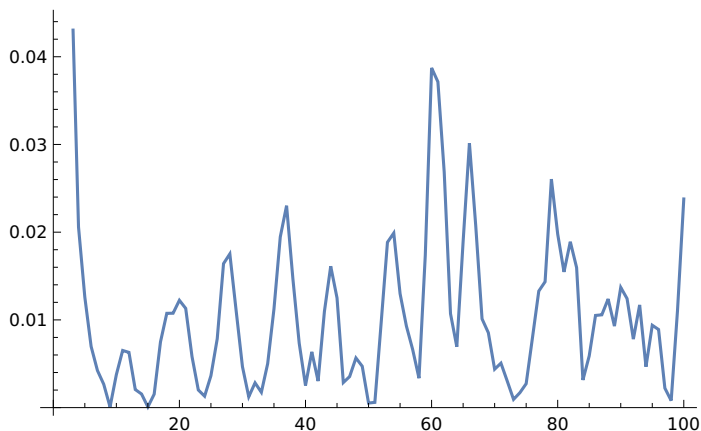
In[50]:= **pltSpec = ListPlot[tap, Joined → True]**

Out[50]=