In[1]:=
```
S := 16
S
Sc := S / 2
```

Out[2]=
```
16
```

In[4]:=
```
cr := {{0, 1}, {0, 0}}
```

In[5]:=
```
an := {{0, 0}, {1, 0}}
n =.
```

In[7]:=
```
cr ;
```

In[8]:=
```
an ;
```

In[9]:=
```
id2 := {{-1, 0}, {0, 1}}
```

In[10]:=
```
id := IdentityMatrix[2]
```

In[11]:=
```
c[n_] :=
 SparseArray[KroneckerProduct @@ ( Table[id, (n - 1)] ~Join~ {cr} ~Join~ Table[id2, {Sc - n}])]
cd[n_] :=
 SparseArray[KroneckerProduct @@ (Table[id, (n - 1)] ~Join~ {an} ~Join~ Table[id2, {Sc - n}])]
```

In[13]:=
```
c[1].cd[1] + cd[1].c[1] ;
```

In[14]:=
```
Do[ψ[n] = (1 / Sqrt[2]) * (c[n] + cd[n]), {n, Sc}]
```

In[15]:=
```
Do[ψ[Sc + n] = ( 1 / Sqrt[2]) * (-I * c[n] + I * cd[n]), {n, Sc}]
```

In[16]:=
```
Do[ψ[i1, i2] = ψ[i1].ψ[i2], {i1, S}, {i2, S}] // Timing
Do[ψ[i3, i4] = ψ[i3].ψ[i4], {i3, S}, {i4, S}]
```

Out[16]=
```
{0.050971, Null}
```

Hamiltonian q = 2

In[18]:=
```
ψ[1, 1] ;
q = 2;
J = 1;
```

In[21]:=
```
Js = RandomVariate[NormalDistribution[0, Sqrt[ (J^2) * ( (q - 1)! / (S^(q - 1)) )]], {S, S}]
```

Out[21]=
```
{{-0.222334, 0.00698013, 0.037111, 0.0103493,
   0.285487, 0.0610865, -0.279779, 0.261791, 0.16915, 0.122718,
   -0.156643, 0.143038, 0.124635, 0.469176, -0.0779159, -0.132833},
 {-0.172104, 0.0800919, -0.273942, -0.236366, 0.377085, -0.668728,
   0.0781306, 0.219698, -0.017242, -0.150584, -0.366617,
```

```
 -0.0571292, 0.0734359, -0.155761, -0.270606, 0.008927},
{0.440368, -0.16876, 0.151811, -0.0734368, -0.025255, 0.541417,
 -0.151943, 0.145076, 0.0488516, -0.268906, -0.00937377,
 -0.660637, -0.137998, -0.0307962, -0.194359, -0.0651111},
{-0.734086, 0.0920415, -0.0197113, 0.159751, 0.346957, -0.112552,
 -0.136032, -0.337754, -0.307429, 0.239606, -0.00490735, -0.236434,
 -0.305224, -0.14914, -0.3738, 0.160444}, {-0.240685, 0.160549, -0.196309,
 -0.313824, -0.196383, 0.381024, -0.0314747, -0.141464, 0.165373,
 -0.3785, 0.188912, 0.362879, -0.740033, 0.398357, -0.153248, -0.200997},
{-0.0442011, -0.121956, -0.176142, 0.083436, -0.143372, 0.00333898,
 0.256324, 0.192612, 0.299317, -0.212919, -0.0395776,
 0.0406036, -0.0998269, 0.0532826, 0.371903, 0.0573962},
{0.0505012, -0.448578, -0.218338, 0.0737837, 0.169146, -0.360805,
 -0.0886327, -0.092244, 0.135164, -0.298589, 0.217463,
 -0.184421, 0.422651, 0.167202, -0.227567, -0.033369},
{-0.186587, 0.0831112, 0.220992, -0.117658, 0.425921, -0.0497812,
 0.150957, -0.50154, -0.597496, -0.152592, -0.0834758, 0.341191,
 0.198529, -0.500235, 0.0340998, -0.0303486}, {-0.0653373, -0.0772164,
 -0.160641, -0.117356, -0.433649, -0.0472011, 0.33038, 0.194103, -0.46468,
 0.236528, 0.14922, 0.0261641, 0.12991, -0.283784, -0.0292464, 0.19512},
{0.274378, -0.273091, 0.509836, -0.204166, -0.186025, -0.179836, 0.288936,
 0.366805, -0.0732883, 0.127112, 0.145367, 0.281279, -0.0771457,
 0.26055, -0.106249, -0.203138}, {0.268255, -0.102787, -0.406927,
 -0.0845377, -0.0497066, 0.205743, -0.0528649, -0.100826, 0.138142,
 0.140444, 0.2167, 0.096786, 0.172829, 0.108477, 0.0255198, 0.0606976},
{-0.0339619, -0.28024, 0.077528, 0.32202, 0.167033, -0.493953, 0.0259582,
 -0.0590814, -0.145771, 0.0563253, 0.0239958, -0.279625, -0.0761722,
 -0.435034, 0.503908, 0.228339}, {0.235138, -0.151171, -0.0184518,
 -0.368813, -0.0118131, 0.322546, -0.00738283, 0.575346, -0.445149,
 0.0808285, -0.12421, 0.0907255, 0.506168, -0.197014, 0.262355, -0.502772},
{0.0572863, -0.432647, -0.264734, -0.274952, -0.0738689, 0.0683015,
 -0.253152, -0.182778, -0.00192728, -0.165572, 0.269514,
 0.219617, 0.0890857, -0.155999, -0.280175, 0.254608},
{-0.0147004, -0.168246, -0.238247, 0.300302, -0.055297, 0.0740459,
 0.116329, -0.126045, 0.274906, 0.065219, -0.183575, -0.0330175, -0.135233,
 -0.273463, -0.329343, -0.490363}, {0.0262038, -0.284585, -0.194328,
 0.205605, 0.0362202, -0.00930332, 0.17124, -0.254221, 0.0196286, 0.171843,
 0.0869533, -0.115743, -0.165056, -0.315712, -0.0878134, -0.170452}}
```

In[22]:=
```
H = Sum[I * Js[[i1, i2]] * ψ[i1, i2], {i1, S}, {i2, i1 + 1, S}] // Normal;
```

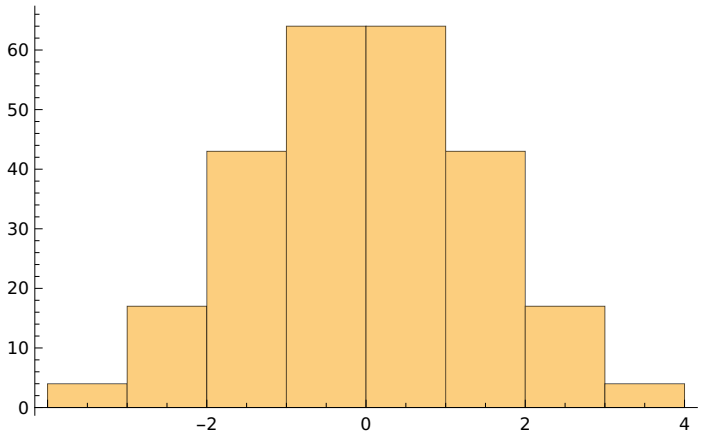
In[23]:=
```
iv = H // N // Eigenvalues // Sort
```

Out[23]=

{-3.49686, -3.34153, -3.16441, -3.00908, -2.84424, -2.72783, -2.68891, -2.5725,
-2.53855, -2.51179, -2.45422, -2.39538, -2.38321, -2.35646, -2.29889, -2.24005,
-2.2061, -2.12177, -2.10397, -2.07521, -2.05076, -1.96644, -1.94864, -1.91988,
-1.88592, -1.80642, -1.8016, -1.77152, -1.76952, -1.74276, -1.73059, -1.68519,
-1.65109, -1.64627, -1.61619, -1.61418, -1.58742, -1.55347, -1.52986, -1.49591,
-1.47397, -1.46915, -1.45135, -1.43706, -1.39814, -1.35274, -1.34057, -1.33494,
-1.31864, -1.31382, -1.29602, -1.28173, -1.19741, -1.17961, -1.16346, -1.1538,
-1.14566, -1.1189, -1.11689, -1.06133, -1.03739, -1.03257, -1.00812, -1.00249,
-0.998466, -0.990323, -0.963565, -0.961559, -0.906, -0.88206, -0.877236,
-0.848106, -0.847159, -0.843282, -0.821347, -0.813205, -0.78444, -0.763782,
-0.728881, -0.726876, -0.704941, -0.700117, -0.692774, -0.68795, -0.682316,
-0.666015, -0.657873, -0.629109, -0.608451, -0.573549, -0.571544, -0.549609,
-0.544785, -0.526985, -0.515655, -0.510831, -0.49303, -0.431332, -0.413531,
-0.408707, -0.394425, -0.384767, -0.376625, -0.360323, -0.3555, -0.349866,
-0.337699, -0.292301, -0.276, -0.258199, -0.253375, -0.239094, -0.229435,
-0.221293, -0.195481, -0.194534, -0.16058, -0.136969, -0.111158, -0.103015,
-0.0810805, -0.0790751, -0.0762566, -0.0742512, -0.0523163, -0.044174,
-0.0401493, -0.00524827, 0.00524827, 0.0401493, 0.044174, 0.0523163, 0.0742512,
0.0762566, 0.0790751, 0.0810805, 0.103015, 0.111158, 0.136969, 0.16058,
0.194534, 0.195481, 0.221293, 0.229435, 0.239094, 0.253375, 0.258199, 0.276,
0.292301, 0.337699, 0.349866, 0.3555, 0.360323, 0.376625, 0.384767, 0.394425,
0.408707, 0.413531, 0.431332, 0.49303, 0.510831, 0.515655, 0.526985, 0.544785,
0.549609, 0.571544, 0.573549, 0.608451, 0.629109, 0.657873, 0.666015, 0.682316,
0.68795, 0.692774, 0.700117, 0.704941, 0.726876, 0.728881, 0.763782, 0.78444,
0.813205, 0.821347, 0.843282, 0.847159, 0.848106, 0.877236, 0.88206, 0.906,
0.961559, 0.963565, 0.990323, 0.998466, 1.00249, 1.00812, 1.03257, 1.03739,
1.06133, 1.11689, 1.1189, 1.14566, 1.1538, 1.16346, 1.17961, 1.19741, 1.28173,
1.29602, 1.31382, 1.31864, 1.33494, 1.34057, 1.35274, 1.39814, 1.43706,
1.45135, 1.46915, 1.47397, 1.49591, 1.52986, 1.55347, 1.58742, 1.61418,
1.61619, 1.64627, 1.65109, 1.68519, 1.73059, 1.74276, 1.76952, 1.77152, 1.8016,
1.80642, 1.88592, 1.91988, 1.94864, 1.96644, 2.05076, 2.07521, 2.10397,
2.12177, 2.2061, 2.24005, 2.29889, 2.35646, 2.38321, 2.39538, 2.45422, 2.51179,
2.53855, 2.5725, 2.68891, 2.72783, 2.84424, 3.00908, 3.16441, 3.34153, 3.49686}

In[24]:=
```
Histogram[iv]
```

Out[24]=



In[25]:=

In[26]:=
```
q = 4
```

Out[26]=
```
4
```

In[27]:=
```
J = 4
```

Out[27]=
```
4
```

In[28]:=
```
Jm = RandomVariate[NormalDistribution[0, Sqrt[J^2 * ((q - 1)!) / (S^(q - 1))]], {S, S, S, S}]
```

Out[28]=

{ ··· 1 ··· }

Full expression not available (original memory size: 0.5 MB)

In[29]:=
```
Dynamic[{i1, i2, i3, i4}]
Ham =
  I^(q / 2) Sum[ψ[i1, i2].Sum[ Jm[[i1, i2, i3, i4]] * ψ[i3, i4], {i3, i2 + 1, S}, {i4, i3 + 1, S}],
    {i1, S - 3}, {i2, i1 + 1, S - 2}] // Normal;
```

Out[29]=
```
{i1, i2, i3, i4}
```

In[31]:=
```
Ham
```

Out[31]=

{{0.127843 + 0. i, 0., 0., −0.00881449 + 0.100599 i, 0., −0.00589959 + 0.0857182 i, 0.0179641 + 0.0990508 i, 0.,
  0., 0.328077 − 0.176273 i, −0.0746503 − 0.0382286 i, 0., 0.0636072 − 0.0903515 i, 0., 0., ··· 226 ···, 0.,
  0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.}, ··· 254 ···, {0., ··· 254 ···, ··· 20 ··· + ··· 1 ···}}

Full expression not available (original memory size: 2.6 MB)

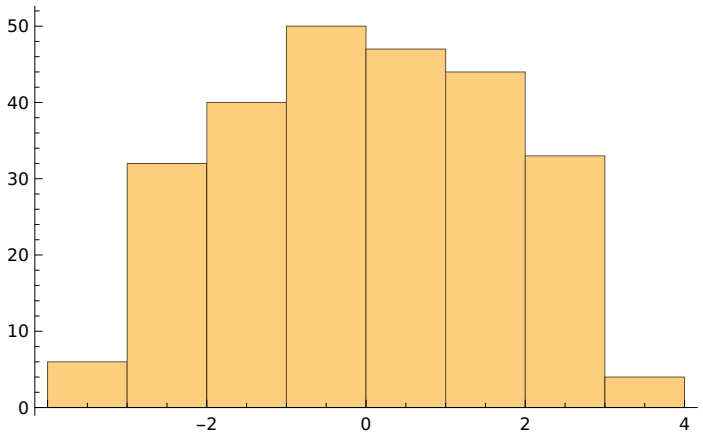
In[32]:=
```
ivv = Ham // N // Eigenvalues // Sort
```

Out[32]=

{-3.33276, -3.23499, -3.15533, -3.10792, -3.05016, -3.04538, -2.9689, -2.90137,
-2.8738, -2.87041, -2.84396, -2.77888, -2.69117, -2.65069, -2.63791, -2.63019,
-2.55147, -2.53328, -2.53068, -2.52705, -2.46628, -2.44616, -2.44529, -2.38079,
-2.37163, -2.31141, -2.29816, -2.26001, -2.22951, -2.22043, -2.19211, -2.12449,
-2.07163, -2.05704, -2.04792, -2.02475, -2.00945, -2.00649, -1.9963, -1.98875,
-1.93858, -1.92729, -1.90926, -1.81507, -1.79116, -1.75904, -1.7431, -1.74018,
-1.73631, -1.71694, -1.66608, -1.65035, -1.63998, -1.62041, -1.59851, -1.54358,
-1.53677, -1.50217, -1.48338, -1.46957, -1.46663, -1.43754, -1.37868, -1.36624,
-1.32142, -1.2996, -1.27843, -1.26248, -1.23366, -1.23049, -1.21048, -1.17875,
-1.15324, -1.14574, -1.12234, -1.10099, -1.07923, -1.05996, -0.982815, -0.943402,
-0.942043, -0.923144, -0.918422, -0.896242, -0.885381, -0.853893, -0.843338,
-0.799364, -0.788159, -0.775598, -0.761739, -0.754585, -0.737783, -0.70494,
-0.655101, -0.620534, -0.619709, -0.606768, -0.586663, -0.582109, -0.535907,
-0.516392, -0.503242, -0.494346, -0.488686, -0.46503, -0.457685, -0.422438,
-0.400702, -0.360453, -0.345464, -0.321141, -0.309486, -0.295546, -0.264774,
-0.251672, -0.222912, -0.201195, -0.193909, -0.169172, -0.158222, -0.107315,
-0.10258, -0.072272, -0.0664912, -0.0504468, -0.0484895, -0.000988025,
0.0169601, 0.0684256, 0.068673, 0.0756012, 0.108658, 0.116786, 0.120678,
0.138239, 0.190596, 0.201141, 0.221143, 0.274652, 0.275144, 0.290589, 0.360889,
0.361468, 0.368439, 0.37546, 0.385965, 0.435415, 0.456338, 0.489348, 0.518695,
0.541665, 0.546461, 0.564923, 0.586721, 0.59525, 0.598756, 0.639223, 0.658068,
0.691891, 0.692301, 0.712851, 0.747832, 0.756745, 0.805041, 0.824166, 0.833017,
0.837327, 0.877708, 0.881413, 0.893244, 0.950015, 0.958542, 0.97163, 0.988473,
1.01862, 1.02326, 1.04089, 1.04564, 1.0682, 1.12411, 1.13147, 1.16224, 1.1681,
1.25166, 1.25246, 1.27605, 1.2803, 1.30357, 1.31092, 1.33797, 1.34476, 1.36949,
1.40628, 1.43226, 1.49325, 1.51066, 1.51893, 1.53172, 1.53535, 1.57485, 1.59024,
1.61706, 1.63252, 1.68677, 1.69959, 1.70346, 1.7103, 1.75038, 1.75709, 1.76902,
1.86266, 1.86595, 1.87137, 1.89228, 1.92196, 1.9501, 1.96127, 1.96694, 2.01841,
2.02669, 2.04291, 2.04802, 2.05546, 2.09887, 2.1294, 2.13222, 2.19941, 2.21868,
2.26349, 2.299, 2.35836, 2.37707, 2.38592, 2.39886, 2.42223, 2.44632, 2.46041,
2.49716, 2.50417, 2.54667, 2.56745, 2.60552, 2.62677, 2.67535, 2.70614, 2.71174,
2.74798, 2.77249, 2.81888, 2.86558, 2.88816, 3.02493, 3.0381, 3.10279, 3.11103}

In[33]:=
```
ivv // Histogram
```

Out[33]=



Two Point functions

In[34]:=
```
β = 5;
H = Ham // N
```

Out[35]=

```
{{0.127843 + 0. i, 0., 0., -0.00881449 + 0.100599 i, 0., -0.00589959 + 0.0857182 i, 0.0179641 + 0.0990508 i, 0.,
  0., 0.328077 - 0.176273 i, -0.0746503 - 0.0382286 i, 0., 0.0636072 - 0.0903515 i, 0., 0., ··· 226 ···, 0.,
  0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.}, ··· 254 ···, {0., ··· 254 ···, ··· 20 ··· + ··· 1 ···}}
```

Full expression not available (original memory size: 2.6 MB)

In[36]:=

In[37]:=
```
Clear[Gn];
Gn[a_, b_, τ_, β_, λ_] := Gn[a, b, τ, β , λ] = Block[{},
If[τ > 0,
Eτ = MatrixExp[-τ H λ];
Eβτ = MatrixExp[(-β + τ) H λ];
(Tr[Eβτ.ψ[a].Eτ.ψ[b]]) / (Tr[Eβτ.Eτ]),

Eτ = MatrixExp[+τ H λ];
Eβτ = MatrixExp[(-β - τ) H λ];
-(Tr[Eβτ.ψ[b].Eτ.ψ[a]]) / (Tr[Eβτ.Eτ]),
]
]
```

In[39]:=
```
Gn[1, 1, 1/4, 1, 1]
```

Out[39]=

$$0.403082 + 8.148 \times 10^{-19}\, i$$

In[40]:=
```
Dynamic[tt]
Gn[1, 1, 1, 1, 1]
```

Out[40]=

tt

Out[41]=

$$0.5 + 1.20371 \times 10^{-35}\, i$$

In[42]:=
```
tbGG = Table[{tt, Gn[1, 1, tt, 1, 1]}, {tt, -1/2, 1/2, 1/10}] // Re
```

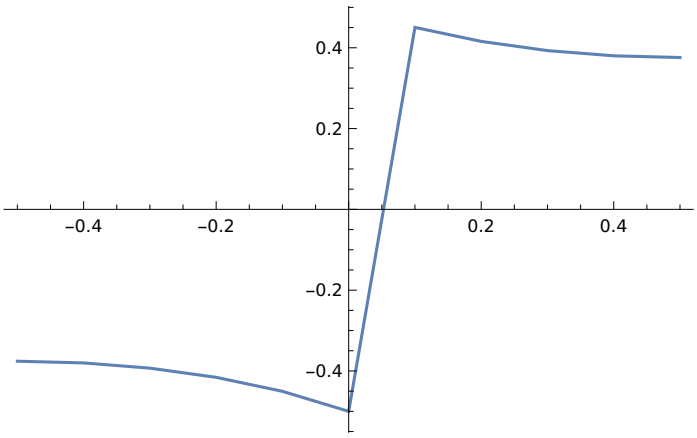
Out[42]=

$$\left\{\left\{-\frac{1}{2}, -0.375894\right\}, \left\{-\frac{2}{5}, -0.38009\right\}, \left\{-\frac{3}{10}, -0.393027\right\},\right.$$
$$\left\{-\frac{1}{5}, -0.415795\right\}, \left\{-\frac{1}{10}, -0.450387\right\}, \{0, -0.5\}, \left\{\frac{1}{10}, 0.450387\right\},$$
$$\left.\left\{\frac{1}{5}, 0.415795\right\}, \left\{\frac{3}{10}, 0.393027\right\}, \left\{\frac{2}{5}, 0.38009\right\}, \left\{\frac{1}{2}, 0.375894\right\}\right\}$$

In[43]:=

In[44]:=
```
plt = ListPlot[tbGG, Joined → True]
```

Out[44]=



In[45]:=
```
Spec[τ_, β_] := Spec[τ, β] = Block[{},
If[τ > 0,
Zβτm = Tr[MatrixExp[(-β * H) - (I * H * τ)]];
Zβτp = Tr[MatrixExp[(-β * H) + (I * H * τ)]];
Zβ = Tr[MatrixExp[(-β * H)]];
Abs[((Zβτm * Zβτp) / (Zβ)^2)]
]
]
```

In[46]:=
```
Spec[3, 1]
```

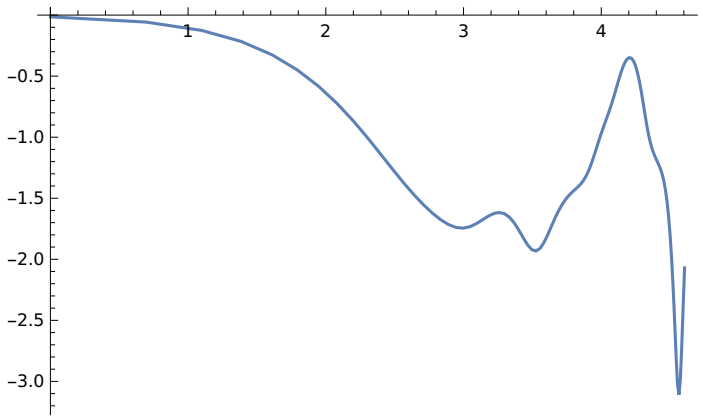
Out[46]=
```
0.0384954
```

In[47]:=
```
tbSpec = Table[{tt, Spec[tt, 10]}, {tt, 1, 100, 1}];
```

In[48]:=
```
tap = Table[{i, Sum[tbSpec[[i]][[2]], {i, 1, i}]}, {i, 1, 100, 1}];
```

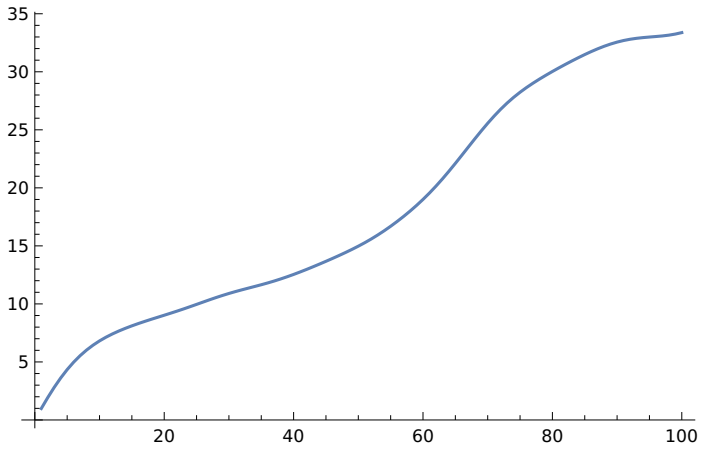
In[49]:=
```
pltSpec = ListPlot[Log[tbSpec], Joined → True]
```

Out[49]=



In[50]:=
```
pltSpec2 = ListPlot[tap, Joined → True]
```

Out[50]=



GRAVITY CALLS

In[51]:=
```
s0 = 1
ρ[M_] := ρ[M] = Exp[s0] * Sinh[(2 * Pi * Sqrt[2 * M]) / (2 * Pi ^ 2)];
```

Out[51]=

1

In[59]:=
```
corrJT[τ_, β_] := corrJT[τ, β] = Block[{},
If[τ > 0,
ZβJ = Integrate[ρ[M] * Exp[-1 * β * M], {M, 0, Infinity}];
ZβαJTp = Integrate[ρ[M] * Exp[(-β * M) + (I * M * τ)], {M, 0, Infinity}];
ZβαJTm = Integrate[ρ[M] * Exp[(-β * M) - (I * M * τ)], {M, 0, Infinity}];
Abs[((ZβαJTp * ZβαJTm) / ((ZβJ)^2))]
]
]
```

In[60]:=
```
ZβJ = Integrate[ρ[M] * Exp[-1 * β * M], {M, 0, Infinity}];
```

In[74]:=
```
JTspectMa = Table[{tt, corrJT[tt, 1]}, {tt, 1, 10, 1}] // N
```

Out[74]=

```
{{1., 0.336088}, {2., 0.0824788}, {3., 0.0288667},
 {4., 0.0129692}, {5., 0.00684273}, {6., 0.00402609}, {7., 0.00256107},
 {8., 0.00172705}, {9., 0.00121846}, {10., 0.000891149}}
```

In[77]:=
```
ListLinePlot[JTspectMa, Mesh → All, AxesOrigin → {0, 0}]
```

Out[77]=