# A report on a basic group that was implemented and studied


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**Abstract**

A topological group, $G$ was implemented and its representations studied. It was found to have a representation into $GL_5(R)$. This representation was realized. This Abelian Lie was also found to have an 8-dimensional faithful unitary representation.


## 1 Basic group implementation

The construction started by considering two tuples of four numbers henceforth referred to as quartets and a multiplication law. Let us take a quartet to be (s,p,m,n). Let s,p,n $\in R \setminus \{0\}$ , and and m $\in R$. We now take $R \setminus \{0\}$ to be $R^*$ moving forward. A law is now imposed: Given two quartets $(s,p,m,n)$ and $(q,r,t,u)$ then $(s,p,m,n).(q,r,t,u) := (\frac{2}{3}sq, pr, m + (1+t), nu)$. To see that this construction was indeed a group, closure, inverse and identity were checked. The group is found in its most basic form to be composed of three different groups. Let's call them $G_1$,$G_2$,$G_3$. $G_1$ has underling set $R^*$ and has the operation $s.q := \frac{2}{3}sq$. $G_2$ is the multiplicative group over the real numbers excluding 0. This takes care of the second and fourth elements of the quartet, and $G_3$ is has underlying set $R$ with $m.t = m+(1+t)$. These three groups are can be easily shown to be indeed groups [httc]. The group is then $G_1 \times G_2 \times G_3 \times G_2$ It has identity $(e_1, e_2, e_3, e_2) = (\frac{3}{2}, 1, -1, 1)$ and inverse $(\frac{9}{4s}, \frac{1}{p}, -m-2, \frac{1}{n})$. This group is an Abelian Lie group as seen directly from its construction.

## 2 Group Representations

It was suggested that a left regular representation exists for this group. This followed from the fact that locally compact Hausdorff topological groups have finite-dimensional representations [httb]. A representation into $GL_5(R)$ was suggested to exist [httb] and found.

$$\begin{pmatrix} \frac{2}{3}q & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1+t \\ 0 & 0 & 0 & u & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} s \\ p \\ m \\ n \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{3}qs \\ pr \\ 1+m+t \\ nu \\ 1 \end{pmatrix}$$

It was also suggested [htta] that this group would have a faithful unitary representation on the sequence space $l^P(G)$. This follows from the fact that $G_1$ and $G_2$ have finite dimensional unitary representations and thus so does their product. $\rho_1 \times \rho_2 : G_1 \times G_2 \to U(n_1) \times U(n_2) \hookrightarrow U(n_1 + n_2)$ . $G_3$ is isomorphic to $R$ and has a faithful unitary representation[htta] too.

## 3 Algebra

This group is an abelian Lie algebra following from the fact that $G_1$, $G_2$, $G_3$ are abelian groups. The product of the groups $G_i$ is an abelian Lie group. This means the Lie brackets are trivial and structure constants must be trivial.

## 4 Conclusion

The abstract group $R^4 \times C_2^3$ was realized and studied. A representation into $GL_5$ was found, and it was also shown that this group has an 8-dimensional faithful unitary representation.


## Acknowledgements

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## References


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