ADS - Lab 1 solutions

Part 1

$2/N$, 37, $\sqrt{N}$, N, $N \log \log N$, $N \log N$, $N \log(N^2)$, $N \log^2 N$, $N^{1.5}$, N^2, $N^2 \log N$, N^3, $2^{N/2}$, 2^N.

$N \log N$ and $N \log (N^2)$ grow at the same rate.

Part 2

For all these programs, the following analysis will agree with a simulation:

(I) The running time is $O(N)$.

(II) The running time is $O(N^2)$.

(III) The running time is $O(N^3)$.

(IV) The running time is $O(N^2)$.

(V) j can be as large as i^2, which could be as large as N^2. k can be as large as j, which is N^2. The running time is thus proportional to $N\ N^2\ N^2$, which is $O(N^5)$.

(VI) The *if* statement is executed at most N^3 times, by previous arguments, but it is true only $O(N^2)$ times (because it is true exactly i times for each i). Thus the innermost loop is only executed $O(N^2)$ times. Each time through, it takes $O(j^2) = O(N^2)$ time, for a total of $O(N^4)$. This is an example where multiplying loop sizes can occasionally give an overestimate.

Part 3

(a) Five times as long, or 2.5 ms.

(b) Slightly more than five times as long.

(c) 25 times as long, or 12.5 ms.

(d) 125 times as long, or 62.5 ms.

Part 4

(a) True

(b) True

(c) False. A counter example is $T_1(N) = N^2$, $T_2(N) = N$, and $f(N) = N^{2,}$

$T_1(N) / T_2(N)$ = N and N is not equal to O(1)

(d) False. The same counterexample as in part (c) applies :

N^2 is not equal to O(N)