Assignment 1: Exercises for Monte Carlo Methods

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Exercise 1

设圆半径为r，则：

$$\frac{4分圆面积}{正方形面积} = \frac{\frac{1}{4}\pi r^2}{r^2} = \frac{\pi}{4} = \frac{落在4分圆的点数}{落在正方形点数}$$

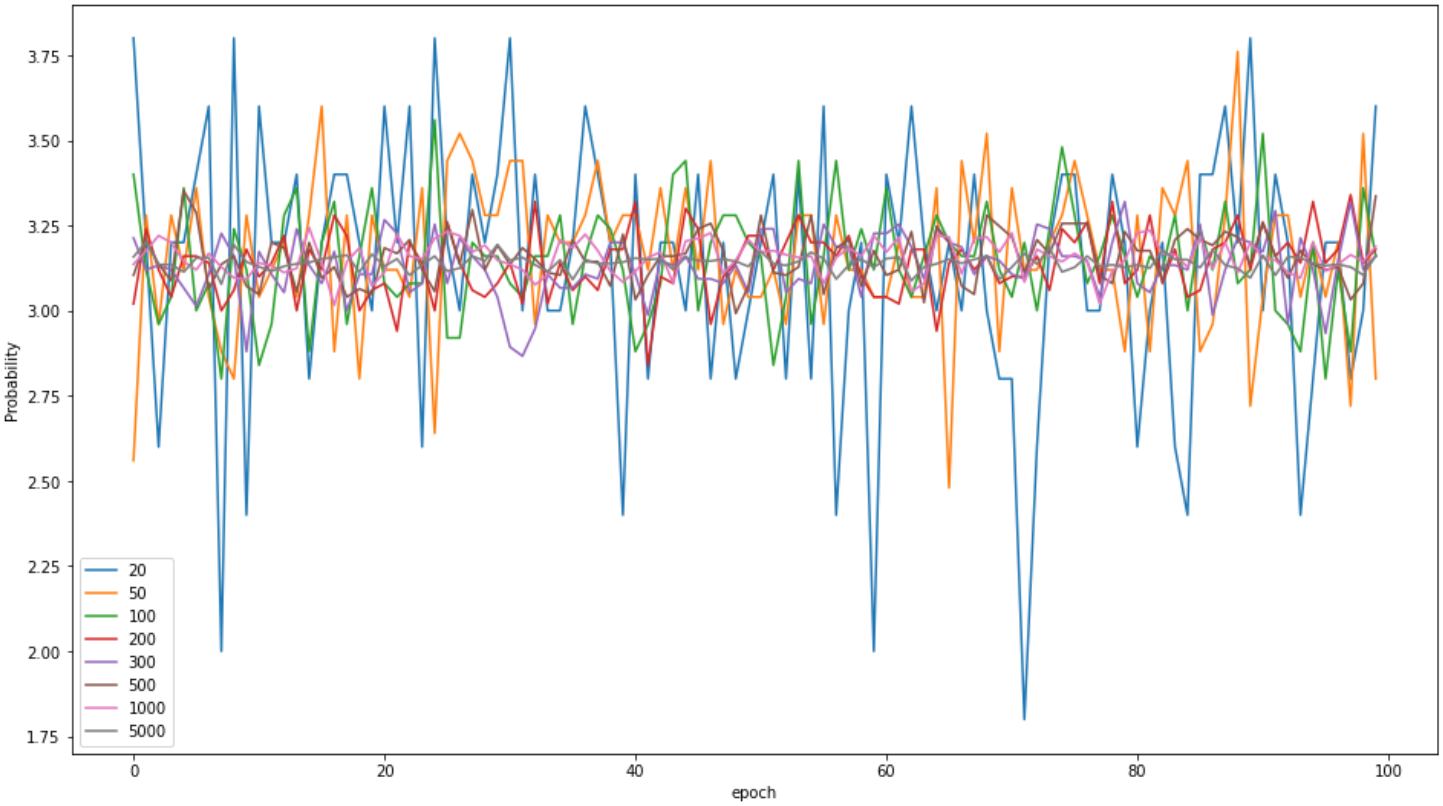
即：

$$\pi = \frac{4 * 落在4分圆的点数}{落在正方形点数}$$

实验结果如下：

	采用数目	均值	方差
0	20	3.126000	0.155724
1	50	3.162400	0.050490
2	100	3.145600	0.026417
3	200	3.138800	0.008959
4	300	3.130933	0.008247
5	500	3.156960	0.005747
6	1000	3.155960	0.002403
7	5000	3.140648	0.000517

以下为个采样100个点的分布情况：


3.75
3.50
3.25
3.00
2.75
2.50
2.25
2.00
1.75
Probability
0
20
40
60
80
100
epoch
20
50
100
200
300
500
1000
5000

代码如下：

```
# ex1

def calculatePi(n: int):
    circle_count = 0

    for i in range(n):
        x = random.random()
        y = random.random()
        len_r = np.sqrt(np.power(x, 2) + np.power(y, 2))
        if len_r <= 1:
            circle_count += 1

    pi = circle_count * 4 / n
    return pi

def ex1():
    samples_list = [20, 50, 100, 200, 300, 500, 1000, 5000]
    categories = [str(x) for x in samples_list]
    epochs = 100
    result_dict = {}
    plt.figure(figsize=(16, 9))
    for index, n in enumerate(samples_list):
        result_list = []
        for c in range(epochs):
            result_list.append(calculatePi(n))
        result_dict[categories[index]] = result_list
        plt.plot(range(epochs), result_list)

    plt.legend(categories)
    plt.ylabel('Probability')
    plt.xlabel('epoch')
    plt.show()

    mean_list = [np.mean(v) for k, v in result_dict.items()]
    std_list = [np.var(v) for k, v in result_dict.items()]
    table = pd.DataFrame({'采用数目': samples_list, '均值': mean_list, '方差': std_list})
    print(table)

ex1()
```

Exercise 2

概率约为25.24%

代码如下：

```
# ex2

def canMoveTo(x: int, y: int, grid: np.array):
    result = True
    mid = (grid.shape[0] // 2, grid.shape[0] // 2)
    if (x, y) == mid:
        if grid[(x, y)] >= 2:
            result = False
    else:
        if grid[(x, y)] >= 1:
            result = False
    return result


def didMoveToDesintation():
    size = 7
    mid = (size // 2, size // 2)

    grid = np.zeros((size, size), dtype=np.int32)
    current = (0, 0)
    grid[current] = 1
    path = [current]

    result = False

    while True:
        p = random.random()

        x = current[0]
        y = current[1]
        if current == (0, 0):
            if not canMoveTo(x + 1, y, grid) and not canMoveTo(x, y + 1, grid):
                # 绝路
                break
            else:
                if p < 0.5:
                    x += 1 # 纵走
                else:
                    y += 1 # 横走
        elif current == (0, size - 1):
            if not canMoveTo(x + 1, y, grid) and not canMoveTo(x, y - 1, grid):
                # 绝路
                break
            else:
                if p < 0.5:
                    x += 1
                else:
                    y -= 1
        elif current == (size - 1, 0):
            if not canMoveTo(x - 1, y, grid) and not canMoveTo(x, y + 1, grid):
```

```
            # 绝路
            break
        else:
            if p < 0.5:
                x -= 1
            else:
                y += 1
    elif current == (size - 1, size - 1):
            # 到达，结束!
            result = True
            break
    else:
        if x == 0:
            if not canMoveTo(x, y - 1, grid) and not canMoveTo(x, y + 1, grid) and not canMoveTo(x
                # 绝路
                break
            else:
                if p < 0.3333:
                    y -=1
                elif 0.3333 <= p < 0.6667:
                    y += 1
                else:
                    x += 1
        elif x == size - 1:
            if not canMoveTo(x, y - 1, grid) and not canMoveTo(x, y + 1, grid) and not canMoveTo(x
                # 绝路
                break
            else:
                if p < 0.3333:
                    y -=1
                elif 0.3333 <= p < 0.6667:
                    y += 1
                else:
                    x -= 1
        elif y == 0:
            if not canMoveTo(x - 1, y, grid) and not canMoveTo(x + 1, y, grid) and not canMoveTo(x,
                # 绝路
                break
            else:
                if p < 0.3333:
                    x -=1
                elif 0.3333 <= p < 0.6667:
                    x += 1
                else:
                    y += 1
        elif y == size - 1:
            if not canMoveTo(x - 1, y, grid) and not canMoveTo(x + 1, y, grid) and not canMoveTo(x,
                # 绝路
                break
            else:
                if p < 0.3333:
```

```
                        x -=1
                    elif 0.3333 <= p < 0.6667:
                        x += 1
                    else:
                        y -= 1
            else:
                if not canMoveTo(x - 1, y, grid) and not canMoveTo(x + 1, y, grid) and not canMoveTo(x,
                    # 绝路
                    break
                else:
                    if p < 0.25:
                        x -= 1
                    elif 0.25 <= p < 0.5:
                        x += 1
                    elif 0.5 <= p < 0.75:
                        y -= 1
                    else:
                        y += 1

        if canMoveTo(x, y, grid):
            path.append((x, y))
            current = (x, y)
            grid[current] += 1

#     print('grid: \n', grid)
#     print('path: ', path)
    return result

def ex2():
    total_count = 20000
    succeed_count = 0
    size = 7
    for i in range(total_count):
        if didMoveToDesintation():
            succeed_count += 1

    print('p = ', succeed_count / total_count)

ex2()
```

Exercise 3

验证得概率约为97.73%，符合理论计算。

代码如下：

```
# ex3

def isASucceed():
    result = False
    p = random.random()
    if p <= 0.85:
        result = True
    return result

def isBCSucceed():
    result = False
    p1 = random.random()
    if p1 <= 0.95:
        p2 = random.random()
        if p2 <= 0.9:
            result = True
    return result


def ex3():
    total_count = 60000
    fail_count = 0
    for i in range(total_count):
        if not isASucceed() and not isBCSucceed():
            fail_count += 1

    print(1 - fail_count / total_count)

ex3()
```