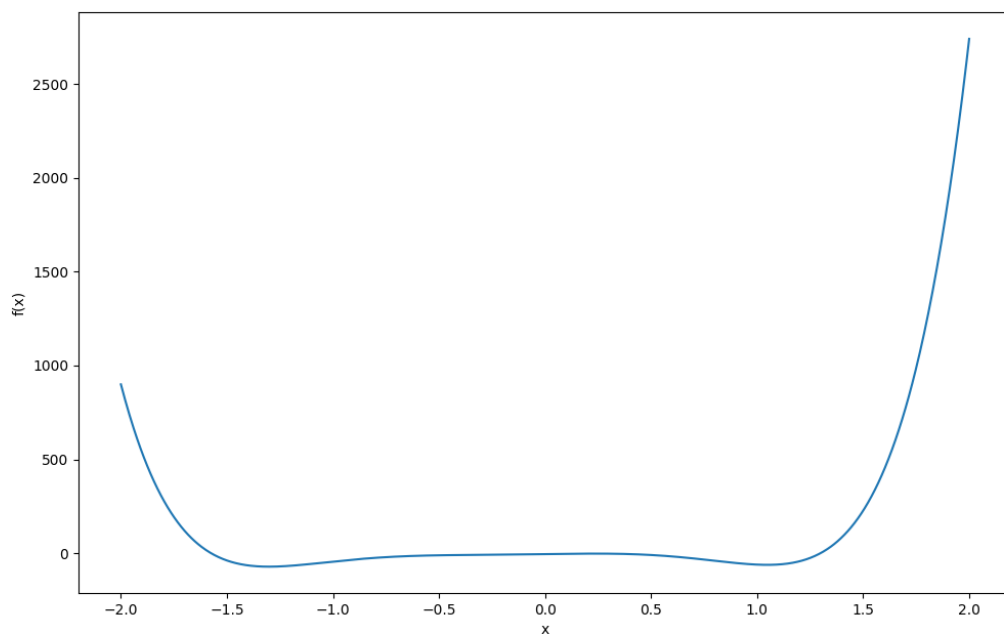


割线法求根作业

函数图像

函数 $f(x) = 54x^2 + 45x^5 - 102x^4 - 69x^3 + 16x - 4$



实现方法

采用编程实现，方法为将 $[-2, 2]$ 区间平均划分为100个小区间，将每个小区间两端点作为初始估计，采用割线法：

$x_0, x_1 = \text{初始估计}$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

1) 求出5个根

根据实验结果，只看到两个不同的根：

$$x_1 = -1.57366$$

$$x_2 = 1.29735$$

```

f(x)=3.4273720206812186e-05, x=-1.5736592318558802
f(x)=3.866733528923305e-07, x=-1.5736591800085173
f(x)=3.624956608661023e-06, x=-1.5736591849631087
f(x)=-0.00010422735420689833, x=-1.573659019948391
f(x)=3.6300631265362426e-06, x=-1.5736591849709218
f(x)=-9.284508927009938e-06, x=-1.5736591652115535
f(x)=-1.283237638105561e-05, x=-1.5736591597832958
f(x)=0.00012832884612379303, x=-1.5736593757606383
f(x)=3.6173394946104054e-07, x=-1.5736591799703599
f(x)=-5.3343271449790564e-05, x=-1.5736590978013807
f(x)=-6.049877612213095e-07, x=1.2973541330044975
f(x)=-5.445784651669783e-06, x=1.2973541250016227
f(x)=9.87769177029918e-06, x=1.297354150334612
f(x)=0.0002788069511758806, x=1.2973545949316014
f(x)=-1.3840000868015068e-05, x=1.2973541111241835
f(x)=-6.48190909657842e-06, x=1.2973541232886872
f(x)=1.2204973316443102e-06, x=1.2973541360224154
f(x)=0.00022738709304093163, x=1.297354509923792
f(x)=1.2946582234718562e-05, x=1.2973541554081447
f(x)=1.2311179382606952e-06, x=1.2973541360399736
f(x)=7.917828683545736e-05, x=1.2973542649032879
f(x)=7.873387808388088e-07, x=1.2973541353063116
f(x)=1.8524952217546797e-05, x=1.2973541646303839
f(x)=0.0002032451730897833, x=1.297354470012119
f(x)=8.718901831628045e-07, x=1.2973541354460931
f(x)=1.0154661595151993e-05, x=1.2973541507925024
f(x)=7.622861319944718e-05, x=1.2973542600268495
f(x)=0.00040911730500070576, x=1.2973548103617338
f(x)=1.3049687446198277e-06, x=1.2973541361620649
f(x)=9.218448528969247e-06, x=1.2973541492447416
f(x)=4.9518868465270316e-05, x=1.297354215869949
f(x)=0.00021248312677002446, x=1.2973544852844012
f(x)=3.5317160751446863e-07, x=1.2973541345885402
f(x)=2.142866751597694e-06, x=1.2973541375472897

```

如果有5个根，那应该是重根。

2) 哪一个根线性收敛？

根 $x_1 = -1.57366$ 是超线性收敛，因为 $\lim_{i \rightarrow \infty} \frac{\|x_{i+1} - x^*\|}{\|x_i - x^*\|} = C = 0$

3) 哪一个根超线性收敛？

根 $x_2 = 1.29735$ 是线性收敛，因为 $\lim_{i \rightarrow \infty} \frac{\|x_{i+1} - x^*\|}{\|x_i - x^*\|} = C$ ，得 $0 < C < 1$

作业代码

```

import numpy as np
import matplotlib.pyplot as plt
import math

def func(x):
    return 54 * np.power(x, 6) + 45 * np.power(x, 5) - 102 * np.power(x, 4) - 69 * np.power(x, 3) + 16 * x - 4

def plot(f):
    """画函数图像"""
    x_list = np.linspace(-2, 2, 1000)
    y_list = [f(x) for x in x_list]
    plt.figure(figsize=(16, 9))
    plt.plot(x_list, y_list)
    plt.xlabel('x')

```

```

plt.ylabel('f(x)')
plt.show()

def find_root(f, x_start, x_end, x_final=None):
    """求区间内的根"""
    x_result = 0
    x1 = x_start
    x2 = x_end
    iter_count = 0
    c_list = []
    while iter_count < 50:
        y1 = f(x1)
        y2 = f(x2)
        x_tmp = x2 - (y2 * (x2 - x1)) / (y2 - y1)
        x1 = x2
        x2 = x_tmp

        if x_final is not None:
            c_list.append(evaluate_convergence_speed(x_result, x_tmp, x_final))

        if math.fabs(x_tmp - x_result) < 1e-6:
            break
        x_result = x_tmp
        iter_count += 1
        # print(x_result)

    print(f'iter count={iter_count}, between=({x_start},{x_end}), x_result={x_result}')
    print(f'c_list = {c_list}')
    return x_result

def evaluate_convergence_speed(x1, x2, x_final):
    """验证收敛速度"""
    c = math.fabs(x2 - x_final) / math.fabs(x1 - x_final)
    return c

if __name__ == '__main__':
    # plot(func)

    x1 = -2
    x2 = x1 + 0.04
    result_list = []
    while len(result_list) < 100 and x2 <= 2:
        x_result = find_root(func, x1, x2, x_final=-1.57366)
        result_list.append(x_result)

        x1 = x2
        x2 = x1 + 0.04

    [print(f'f(x)={func(result)}, x={result}') for result in result_list if math.fabs(func(result)) < 1e-3]

```