

**Formal Verification of Distributed Leader Election Algorithms with  
Model Checking**

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# Chapter 1

## Introduction

TODO: Confidence 1/10

Distributed Systems are very important...

A problem in Distributed Systems is the problem of leader election...

# Chapter 2

## Overview

TODO: Confidence 1/10

This is the main body section.

### 2.1 Terminology

TODO: Confidence 1/10

Before explaining much of the implementation as well as how the algorithms works, it is important to show some of the terminology.

Failure models... Overlay... Logical... Classifications...

### 2.2 Leader Election

TODO: Confidence 3/10

Leader Election is a type of algorithm in Distributed Systems, whose aim is to elect a leader from processes in a distributed systems. This is crucial for when a leader dies. The main challenge in leader election is that any process can die at any time, including the leader. Thus, if a leader dies, a new leader must be agreed upon by the other processes. Leader election assumptions:

- All nodes
- Fail-stop model

### 2.3 Bully Algorithm

TODO: Confidence 3/10

The Bully Algorithm is an algorithm that solves the Leader Election problem. In the bully algorithm, we assume a complete undirected graph.

When a process P recovers from failure, or the failure detector indicates that the current leader has failed, P performs the following actions:

1. If P has the highest process ID, it sends a Victory message to all other processes and becomes the new leader. Otherwise, P broadcasts an Election message to all other processes with higher process IDs than itself.
2. If P receives no alive after sending an Election message, then it broadcasts a Victory message to all other processes and becomes the leader.
3. If P receives an alive from a process with a higher ID, it sends no further messages for this election and waits for a Victory message. (If there is no Victory message after a period of time, it restarts the process at the beginning.)
4. If P receives an Election message from another process with a lower ID it sends an alive message back and if it has not already started an election, it starts the election process at the beginning, by sending an Election message to higher-numbered processes.
5. If P receives a victory message, it treats the sender as the leader.

## 2.4 Ring Algorithm

TODO: Confidence 3/10

The Ring Algorithm is an algorithm that solves the Leader Election problem. In the bully algorithm, we assume a complete directed ring.

## 2.5 TLA+

TODO: Confidence 5/10

TLA+ (Temporal Language of Actions Plus) is a language for modeling specifications, and widely used for proving algorithms. It has proven to be a very useful tool for proving algorithms, because of the way the language works, by forcing you to write mathematically correct implementations, that can be tested in every possible state.

## Chapter 3

# Bully Algorithm

### 3.1 Converting to TLA+

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### 3.2 Setup

TODO: Confidence 6/10: Formatting, proof-reading, “we”, articulation, maybe some of this is not necessary? too much explanation?

The “setup” of the implementation consists of setting up the variables and constants we use, along with setting the initial values of the code.

```

CONSTANT  $N$ 

ASSUME  $N \in \text{Nat}$ 

 $\text{ProcessID} \triangleq 1 \dots N$ 

VARIABLES  $\text{State}, \text{MessageBox}$ 

 $\text{Message} \triangleq \text{ProcessID} \times \{\text{"ELECTION"}, \text{"ALIVE"}, \text{"VICTORY"}\}$ 

 $\text{Init} \triangleq \wedge \text{State} = [p \in \text{ProcessID} \mapsto$ 
     $[ID \mapsto p,$ 
     $\text{Condition} \mapsto \text{"Active"},$ 
     $\text{Leader} \mapsto N,$ 
     $\text{Participating} \mapsto \text{FALSE}]]$ 
 $\wedge \text{MessageBox} = [p \in \text{ProcessID} \mapsto \langle \rangle]$ 

```

This code segment first defines a natural number constant,  $N$ , which chooses the number of processes in the system. The specific number  $N$  is first chosen in the modeling stage, where you check the model. We then model ID's of the processes by simply making a range from 1 to and including  $N$ , meaning that a process can have any natural number ID up including  $N$ .

The variables *State* and *MessageBox* are then declared, so that we can implement their initial values in the *init* function.

Before initializing, we declare that a message is the cross product between a process ID and its message "ELECTION", "ALIVE" or "VICTORY". This is used in the TypeOK invariant, to make sure that a message is a correct message.

In the *init* function we initialize the *State* and *MessageBox* variables. We set *State* to be a tuple of processes which map to a record of the meta-information of the process. The record which each process maps to has the fields ID, Condition, Leader and Participating. The ID is set to be the process ID,  $p$ . We set the condition to be "Active", because we want each process to be active in the initial state. The "condition" field could be set to be "Dead", in which case a leader election would immediately start. We set the Leader field to be the highest process,  $N$ . We also set the *Participating* field to be FALSE, as there is no election ongoing at the initial state. We initialize the *MessageBox* simply by mapping each process id to an empty tuple. Thus, we end with a tuple of tuples.

With this format, we can very simply access information from any process at the current state. The condition for example, can be accessed simply by evaluating  $\text{State}[p].\text{Condition}$ , where  $p$  is the process ID. Similarly, we can access messages from a process' message box easily with  $(\text{MessageBox}[p])[1]$  for the process ID of the sender, and  $(\text{MessageBox}[p])[2]$  for the contents of the message.

The ease of access here makes the rest of the implementation very readable.

### 3.3 Sending Messages

TODO: Confidence 4/10: General formatting, bad formulation etc.

$$\begin{aligned}
\text{SendAlive}(p, q) &\triangleq \\
&\text{MessageBox}' = [\text{MessageBox} \text{ EXCEPT } ![q] = \text{Append}(\text{MessageBox}[q], \langle p, \text{"ALIVE"} \rangle)] \\
\\
\text{SendVictory}(p) &\triangleq \\
&\wedge \text{MessageBox}' = [q \in \text{ProcessID} \mapsto \\
&\quad \text{IF } q \neq p \wedge \text{State}[q].\text{Condition} = \text{"Active"} \\
&\quad \text{THEN } \text{Append}(\text{MessageBox}[q], \langle p, \text{"VICTORY"} \rangle) \\
&\quad \text{ELSE } \text{MessageBox}[q]] \\
&\wedge \text{State}' = [\text{State} \text{ EXCEPT } ![p].\text{Leader} = p] \\
\\
\text{ProcessHasTheHighestIDAlive}(p) &\triangleq \forall q \in \text{HigherIDs}(p) : \text{State}[q].\text{Condition} = \text{"Dead"} \\
\\
\text{SendElection}(p) &\triangleq \forall q \in \text{HigherIDs}(p) : \\
&\quad \wedge \text{State}[q].\text{Condition} = \text{"Active"} \\
&\quad \wedge \text{MessageBox}' = \\
&\quad [\text{MessageBox} \text{ EXCEPT } ![q] = \text{Append}(\text{MessageBox}[q], \langle p, \text{"ELECTION"} \rangle)]
\end{aligned}$$

In general, with the way messages are modeled, we can send a message as simply as this:

$$\begin{aligned}
&\text{MessageBox}' \\
&= [\text{MessageBox} \text{ EXCEPT } ![q] = \\
&\quad \text{Append}(\text{MessageBox}[q], \langle p, \text{"MESSAGE"} \rangle)]
\end{aligned}$$

This simply updates the MessageBox tuple for the receiving process,  $q$  with a message sent from  $p$  with the contents "MESASAGE".

#### SendAlive

SendAlive sends a message from process  $p$  to process  $q$  with the contents "ALIVE". The primary reason for this function is to help readability.

#### SendVictory

SendVictory sends a "VICTORY" message to every process with the condition "Active" which is not the process itself. The part that specifies that the receiving process should be active is not a necessity and could be left out without changing the functionality of the specification. The reason this is not done, is because it helps with a clean MessageBox. We can do this, because we assume **det der hvor processerne ved hvad der sker**. Furthermore, the process  $p$  which sends the message also sets its own leader to be itself, and its participation status to FALSE. Thus concluding it's own election, by assigning a new leader and not participating anymore.

**SendElection** SendElection sends the "ELECTION" message to all processes with a higher id than the sender. To do this, it uses a helper function HigherIDs which is defined as such:

$$\text{HigherIDs}(p) = \{q \in \text{ProcessID} : q > p\}$$

#### Differences in the send functions

As one may think, these functions are quite different. The SendAlive function just sends a message without doing anything else. SendVictory sends a victory

to every active process that is not itself, and SendElection sends only to higher process IDs. **TODO: Skriv hvorfor**

### 3.4 Receiving Messages

TODO: Confidence 5/10: Måske fungerer referencer til algoritme ikke helt?

$$\begin{aligned}
\text{ReceiveAlive}(p) &\triangleq \wedge \text{State}[p].\text{Participating} = \text{TRUE} \\
&\wedge \exists q \in \text{HigherIDs}(p) : \langle q, \text{"ALIVE"} \rangle \in \text{MessageBox}[p] \\
&\wedge \text{State}' = [\text{State} \text{ EXCEPT } ![p].\text{Participating} = \text{FALSE}] \\
\\
\text{ReceiveElection}(p) &\triangleq \wedge \text{State}[p].\text{Participating} = \text{FALSE} \\
&\wedge (\exists q \in \text{LowerIDs}(p) : \langle q, \text{"ELECTION"} \rangle \in \text{MessageBox}[p] \\
&\quad \wedge \text{State}[q].\text{Condition} = \text{"Active"} \\
&\quad \wedge \text{SendAlive}(p, q)) \\
\\
&\wedge \text{State}' = [\text{State} \text{ EXCEPT } ![p].\text{Participating} = \text{TRUE}] \\
&\wedge \text{SendElection}(p) \\
\\
\text{ReceiveVictory}(p, q) &\triangleq \text{State}' = [\text{State} \text{ EXCEPT } ![p].\text{Leader} = q]
\end{aligned}$$

These functions take care of handling a message that has been received.

#### ReceiveAlive

*If P receives an Alive from a process with a higher ID, it sends no further messages for this election and waits for a Victory message. (If there is no Victory message after a period of time, it restarts the process at the beginning.)*

This function makes sure that the process that has received the ALIVE message is currently participating in the election, and that the ALIVE message comes from a process with a higher ID. If both of these are true, the process is no longer participating in the election.

#### ReceiveElection

*If P receives an Election message from another process with a lower ID it sends an ALIVE message back and if it has not already started an election, it starts the election process at the beginning, by sending an Election message to higher-numbered processes. TODO: Måske skal HigherID og LowerID bare forklares for sig selv?*

This function first checks to make sure that the process is not already participating in the election. Then, it sees if the ELECTION message has come from a Lower ID, if this is the case it sends an "ALIVE" message to the process, if it is active. Again, making sure it is active is not a necessity, and only here to help with a clean MessageBox. TODO: Hvorfor virker den her funktion? Burde den ikke blive FALSE hvis det ikke er fra en election?

#### ReceiveVictory

*If P receives a Victory message, it treats the sender as the coordinator.*

ReceiveVictory changes the process' leader to be q.



### 3.4.1 Handling Messages

TODO: Confidence 1/10: Dårlige forklaringer, dårlig grammatik etc.

$$\begin{aligned}
\text{HandleMessages}(p) \triangleq & \wedge \text{State}[p].\text{Condition} = \text{"Active"} \\
& \wedge \text{MessageBox}[p] \neq \langle \rangle \\
& \wedge \vee \wedge \text{State}[\text{Head}(\text{MessageBox}[p])[1]].\text{Condition} = \text{"Dead"} \\
& \quad \wedge \text{MessageBox}' = [\text{MessageBox} \text{ EXCEPT } ![p] = \text{Tail}(\text{MessageBox}[p])] \\
& \quad \wedge \text{UNCHANGED State} \\
& \vee \wedge \text{Head}(\text{MessageBox}[p])[2] = \text{"VICTORY"} \\
& \quad \wedge \text{ReceiveVictory}(p, \text{Head}(\text{MessageBox}[p])[1]) \\
& \quad \wedge \text{MessageBox}' = [\text{MessageBox} \text{ EXCEPT } ![p] = \text{Tail}(\text{MessageBox}[p])] \\
& \vee \wedge \text{Head}(\text{MessageBox}[p])[2] = \text{"ELECTION"} \\
& \quad \wedge \text{ReceiveElection}(p) \\
& \quad \wedge \text{MessageBox}' = [\text{MessageBox} \text{ EXCEPT } ![p] = \text{Tail}(\text{MessageBox}[p])] \\
& \vee \wedge \text{Head}(\text{MessageBox}[p])[2] = \text{"ALIVE"} \\
& \quad \wedge \text{ReceiveAlive}(p) \\
& \quad \wedge \text{MessageBox}' = [\text{MessageBox} \text{ EXCEPT } ![p] = \text{Tail}(\text{MessageBox}[p])]
\end{aligned}$$

This function checks for new messages in a process' message box. It does this by looking at the first entry (the head) of the message box of process  $p$ . The first case is to clean up in case the process is dead. The only thing that is done, is to remove the message from the dead process. However, in the case where the process is not dead, the function looks at the contents of the message. It then calls the appropriate function depending the contents. After calling the function it removes the latest message by modifying the messagebox to be the tail.

### 3.5 Checking the leader condition

TODO: Confidence 2/10: Dårlig grammatik, dårlig formulering etc.

$$\begin{aligned}
\text{CheckLeader}(p) \triangleq & \wedge \text{State}[p].\text{Condition} = \text{"Active"} \\
& \wedge \text{State}[\text{State}[p].\text{Leader}].\text{Condition} = \text{"Dead"} \\
& \wedge \text{State}[p].\text{Participating} = \text{FALSE} \\
& \wedge \text{IF } \text{ProcessHasTheHighestIDAlive}(p) = \text{TRUE} \\
& \quad \text{THEN } \text{SendVictory}(p) \\
& \quad \text{ELSE } \text{SendElection}(p) \wedge \text{State}' = [\text{State} \text{ EXCEPT } ![p].\text{Participating} = \text{TRUE}]
\end{aligned}$$

This function checks whether or not the current leader is dead. It only does this, if there is the process checking is not participating in an election (meaning that we assume there is no election). We then use the helper function defined up in the Sends, to see if the process has the highest ID. If this is the case, we just send a victory message, otherwise we send an election message out, and change the process to be participating in the election. We can "just" say if the process has the highest ID, because we assume **TODO: hvad er det nu vi assumer siden vi bare kan sige det her? noget med at vi ved hvis processer er døde, vi ved bare ikke hvordan** This is also the reason this function is used, rather than not getting a response and then assuming the leader process is dead.

### 3.6 Next predicate and dying leader

TODO: Confidence 2/10: For kort, dårligt beskrevet etc.

$$\begin{aligned}
KillLeader(p) &\triangleq \wedge State[p].Condition = \text{"Active"} \\
&\quad \wedge State[State[p].Leader].Condition = \text{"Active"} \\
&\quad \wedge State' = [State \text{ EXCEPT } ![State[p].Leader].Condition = \text{"Dead"}] \\
&\quad \wedge \text{UNCHANGED } MessageBox \\
\\
Next &\triangleq \exists p \in ProcessID : \\
&\quad \vee CheckLeader(p) \\
&\quad \vee HandleMessages(p) \\
&\quad \vee KillLeader(p)
\end{aligned}$$

The Next predicate is the predicate which chooses what should happen in the next state. Here we say that for some process  $p$ , we can either check if the leader is alive, handle any messages or kill the leader. Killing the leader is only a function to ensure the proper functioning of the spec.

### 3.7 Invariants and Properties

#### 3.7.1 Invariants

TODO: Confidence 1/10: very very rough draft

$$\begin{aligned}
TypeOK &\triangleq \forall p \in ProcessID : \\
&\quad \wedge State[p].ID \in ProcessID \\
&\quad \wedge State[p].Condition \in \{\text{"Active"}, \text{"Dead"}\} \\
&\quad \wedge State[p].Leader \in ProcessID \\
&\quad \wedge State[p].Participating \in \text{BOOLEAN} \\
&\quad \wedge MessageBox \in [ProcessID \rightarrow Seq(Message)] \\
\\
OneLeader &\triangleq \forall p, q \in ProcessID : (State[p].Leader = p \\
&\quad \wedge State[q].Leader = q \\
&\quad \wedge State[p].Condition \neq \text{"Dead"} \\
&\quad \wedge State[q].Condition \neq \text{"Dead"}) \\
&\quad \Rightarrow (p = q) \\
\\
UniqueID &\triangleq \forall p, q \in ProcessID : (State[p].ID = State[q].ID) \Rightarrow (p = q) \\
\\
Participating &\triangleq \forall p \in ProcessID : State[p].Participating = \text{TRUE} \Rightarrow State[p].Leader \neq p
\end{aligned}$$

#### How the invariants are set up:

The invariants are set up by depending on whether they are checking one process or two processes. If one process is checked, it checks for all processes  $p$ , while if they are checking two processes, they check for all processes  $p$  and  $q$ . Using this way, we make sure that each process gets treated correctly, and that we don't just select a random pair that might work at that moment.

Invariants will be checked to be true in every possible state.

Earlier in the report, we talked about TypeOK, as it is the invariant which makes sure that every variable holds legal values for this specification. It does

this by checking each process in ProcessID (I.e., each process in general) and making sure that they are a part of the correct set. (TODO: FOR PROCESSID THIS IS REDUNDANT). This means that the Condition property should either be Active or Dead, the leader should be a legal process id, participating should be a boolean and the messagebox should be a mapping from processid to a sequence of messages.

The OneLeader invariant makes sure that if there exists two leaders, then those two leaders must be the same process. UniqueID makes sure that no process will have the same id.

Participating makes sure that if a process is participating, it is not the leader itself.

### 3.7.2 Properties

..Explain what properties are...

There are two kinds of temporal properties: safety properties, and liveness properties. Safety properties make sure that the system doesn't do bad things, and liveness makes sure our system always does a good thing.[?]

$ \begin{aligned} \text{ElectionTerminationImpliesSameLeader} &\triangleq \forall p, q \in \text{ProcessID} : \\ &(\text{State}[p].\text{Condition} = \text{"Active"} \\ &\wedge \text{State}[q].\text{Condition} = \text{"Active"} \\ &\wedge \text{State}[p].\text{Participating} = \text{FALSE} \\ &\wedge \text{State}[q].\text{Participating} = \text{FALSE}) \\ &\Rightarrow (\text{State}[p].\text{Leader} = \text{State}[q].\text{Leader}) \end{aligned} $
$ \text{MaxAliveID} \triangleq \text{CHOOSE } id \in \text{ProcessID} : \forall p \in \text{ProcessID} : \text{State}[p].\text{Condition} = \text{"Dead"} \vee id \geq \text{State}[p].ID $
$ \begin{aligned} \text{HighestAliveProcessIsLeader} &\triangleq \forall p \in \text{ProcessID} : \\ &\text{State}[p].\text{Participating} = \text{FALSE} \Rightarrow \\ &(\text{State}[\text{MaxAliveID}].\text{Condition} = \text{"Active"} \Rightarrow \text{State}[p].\text{Leader} = \text{MaxAliveID}) \end{aligned} $
$ \text{ElectionWillEnd} \triangleq \forall p \in \text{ProcessID} : \text{State}[p].\text{Participating} \leadsto (\Diamond(\text{State}[p].\text{Participating} = \text{FALSE})) $
$ \begin{aligned} \text{EventuallyElection} &\triangleq \forall p \in \text{ProcessID} : (\text{State}[p].\text{Condition} = \text{"Active"} \\ &\wedge \text{State}[\text{State}[p].\text{Leader}].\text{Condition} = \text{"Dead"} \\ &\wedge \text{State}[p].\text{Participating} = \text{FALSE}) \\ &\leadsto \Diamond(\text{State}[p].\text{Participating} = \text{TRUE}) \end{aligned} $

ElectionTerminationImpliesSameLeader is a safety property that ensures that when an election is done, everyone has the same leader.

HighestAliveProcessIsLeader is a safety property, tghat ensures that if a process is not participating in an election, then the highest alive process should be the leader.

ElectionWillEnd is a safety property that ensures that ensures that

## Chapter 4

# Ring Algorithm

...Also called LCR

### 4.1 Thought Process

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#### **Similarities to Bully Algorithm**

The Ring Algorithm and the Bully Algorithm are both algorithms for leader election, and thus many of the core functions and models can be reused. These include invariants, properties, processes, messages and more.

## 4.2 Setup

```

CONSTANT  $N$ 

ASSUME  $N \in \text{Nat}$ 

 $\text{ProcessID} \triangleq 1 \dots N$ 

VARIABLES  $\text{State}, \text{MBox}$ 

 $\text{Message} \triangleq \{ \text{"PROBE"}, \text{"SELECTED"} \} \times \text{ProcessID}$ 

 $\text{Init} \triangleq \wedge \text{State} = [p \in \text{ProcessID} \mapsto [$ 
     $\text{ID} \mapsto p,$ 
     $\text{Status} \mapsto \text{"Active"},$ 
     $\text{Leader} \mapsto N,$ 
     $\text{Participating} \mapsto \text{FALSE}]$ 
 $]$ 
 $\wedge \text{MBox} = [p \in \text{ProcessID} \mapsto \langle \rangle]$ 

```

This setup is a lot like the setup of the Bully Algorithm, except the Message function is different. This is solely a syntactic change. In the Ring algorithm each message holds an “ID”, which is usually depicted as PROBE(ID) or SELECTED(ID). Thus, the change is made to reflect this, and also to avoid confusion with how the BullyAlgorithm handles it, where the process is the sender of the message, not the id. The sender isn’t needed in the Ring Algorithm, as it is a ring structure and sends the message to the neighbour.

## 4.3 Helper functions

```

 $\text{HigherIDs}(p) \triangleq \{q \in \text{ProcessID} : q > p\}$ 
 $\text{LowerIDs}(p) \triangleq \{q \in \text{ProcessID} : q < p\}$ 

 $\text{SendMessage}(p, \text{id}, \text{msg}) \triangleq \text{MBox}' = [\text{MBox} \text{ EXCEPT } ![p] = \text{Append}(\text{MBox}[p], \langle \text{msg}, \text{id} \rangle)]$ 

 $\text{MaxAliveID} \triangleq \text{CHOOSE } \text{id} \in \text{ProcessID} : \forall p \in \text{ProcessID} : \text{State}[p].\text{Status} = \text{"Dead"} \vee \text{id} \geq \text{State}[p].\text{ID}$ 

 $\text{Neighbour}(p) \triangleq (p \% \text{MaxAliveID}) + 1$ 

```

The helper functions are quite alike the helper functions in the bully algorithm as well, with the most noticable differences being the SendMessage and Neighbour functions. The reason I decided for having a SendMessage function in the Ring Algorithm and not in Bully Algorithm, is that it aesthetically looks much prettier in the ring algorithm, and less cluttered, while it would only make it look more cluttered in the Bully Algorithm. The neighbour function is a helper function for finding the right-side neighbour of the process. Since the algorithm assumes a ring structure, the right-hand neighbour is just the process id plus one. However, since a leader might die, this is divided with the MaxAliveID, to make sure that if the right hand neighbour is dead, it is sent to the lowest id instead. TODO: this should probably be changed, is in the real algorithm any process can die at any time.

## 4.4 Event Handlers

In the Ring algorithm there are two main events: receiving the Probe message and receiving the Selected message.

$$\begin{aligned}
\text{ProcessProbe}(p, id) &\triangleq \vee \wedge p < id \\
&\quad \wedge \text{SendMessage}(\text{Neighbour}(p), id, \text{"PROBE"}) \\
&\vee \wedge p = id \\
&\quad \wedge \text{State}' = [\text{State} \text{ EXCEPT } ![p].\text{Leader} = id] \\
&\quad \wedge \text{SendMessage}(\text{Neighbour}(p), p, \text{"SELECTED"}) \\
\\
\text{ProcessSelected}(p, id) &\triangleq \vee \wedge p \neq id \\
&\quad \wedge \text{State}' = [\text{State} \text{ EXCEPT } ![p].\text{Leader} = id, \\
&\quad \quad \quad ![p].\text{Participating} = \text{FALSE}] \\
&\vee \wedge p = id \\
&\quad \wedge \text{State}' = [\text{State} \text{ EXCEPT } ![p].\text{Participating} = \text{FALSE}]
\end{aligned}$$

### ProcessProbe

According to the algorithm, in case a process receives a probe message, and the process id is lower than the id in the Probe message, the message is sent on the the next process. We do this using the SnedMessage and Neighbour helper functions. TODO: THIS IS WHERE THE PROBLEM HAPPENS PROBABLY. In the case of the ID being the same as the process ID, we assume that they are the new leader, and a SELECTED message with that process ID is sent, as well as their own leading being set to be themselves. If the process ID is greater than the ID in the message, we do nothing, as is specified by the algorithm.

### ProcessSelected

ProcessSelected simply checks if the process ID is equal to the ID of the Selected message. If so, the process is declared the election is done. Otherwise, the message is sent on, to keep declaring the new leader.

## 4.5 HandleMessages

$$\begin{aligned}
\text{HandleMessages}(p) &\triangleq \wedge \text{State}[p].\text{Status} = \text{"Active"} \\
&\wedge \text{MBox}[p] \neq \langle \rangle \\
&\wedge \vee \wedge \text{Head}(\text{MBox}[p])[1] = \text{"PROBE"} \\
&\quad \wedge \text{ProcessProbe}(\text{Neighbour}(p), \text{Head}(\text{MBox}[p])[2]) \\
&\quad \wedge \text{IF } \text{State}[p].\text{Participating} = \text{FALSE} \text{ THEN } \text{State}' = \\
&\quad \quad [\text{State} \text{ EXCEPT } ![p].\text{Participating} = \text{TRUE}] \text{ ELSE UNCHANGED } \text{State} \\
&\quad \wedge \text{Print}(\text{"We got here"}, \text{TRUE}) \\
&\quad \wedge \text{MBox}' = [\text{MBox} \text{ EXCEPT } ![p] = \text{Tail}(\text{MBox}[p])] \\
&\vee \wedge \text{Head}(\text{MBox}[p])[1] = \text{"SELECTED"} \\
&\quad \wedge \text{ProcessSelected}(p, \text{Head}(\text{MBox}[p])[2]) \\
&\quad \wedge \text{MBox}' = [\text{MBox} \text{ EXCEPT } ![p] = \text{Tail}(\text{MBox}[p])]
\end{aligned}$$

TODO: Change image

HandleMessage is mostly identical to the HandleMessage functionp from the bully algorithm, except with changed messages, and in case of the message being a probe and the process' *participating* field being false, we change it to true.

## 4.6 Checking The Leader Condition

## Chapter 5

# Evaluation



## Chapter 6

# Conclusion

...While the Bully Algorithm worked fine, the Ring Algorithm did not.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.