Different hypotheses for value of A:

prior: $Pr[A = 0.1] = 0.3$

$Pr[A = 0.9] = 0.7$

Binary variable B_n has value with likelihood L

$$Pr[B_n | A] = A^{B_n} (1-A)^{(1-B_n)}$$

a) Observe data$_1$

data = $\{1, 0, 1\}$

- Prior probability of observing 0, 1 depends on A

$Pr[B_n = 0 | A = 0.1] = 0.9$ $\quad$ $Pr[B_n = 0 | A = 0.9] = 0.1$

$Pr[B_n = 1 | A = 0.1] = 0.1$ $\quad$ $Pr[B_n = 1 | A = 0.9] = 0.9$

- From Bayes' we know:

$$Pr[A = 0.1 | data_1] = \frac{Pr[data_1 | A = 0.1] \cdot Pr[A = 0.1]}{Pr[data_1]}$$

$$= \frac{Pr[data_1 | A = 0.1] \cdot Pr[A = 0.1]}{Pr[data_1 | A = 0.1] \cdot Pr[A = 0.1] + Pr[data_1 | A = 0.9] Pr[A = 0.9]}$$

- Need probability of observing data

$$Pr[data_1 | A = 0.1] = Pr[B_n = 1 | A = 0.1] \cdot Pr[B_n = 0 | A = 0.1] \cdot Pr[B_n = 1 | A = 0.1]$$

$$= 0.1 \cdot 0.9 \cdot 0.1$$

$$= 0.009$$

$$Pr[data_1 | A = 0.9] = \ldots$$

$$= 0.9 \cdot 0.1 \cdot 0.9$$

$$= 0.081$$

- So we get:

$Pr[A = 0.1 | data_1] \approx 0.04$

$Pr[A = 0.9 | data_1] \approx 0.96$

$Pr[data_1] =$

$= Pr[data_1 | A = 0.1] \cdot Pr[A = 0.1]$

$+ Pr[data_1 | A = 0.9] \cdot Pr[A = 0.9]$

$= 0.0594$

$Pr[A=0.1 \mid data_2] \approx 0.045$

$Pr[A=0.9 \mid data_2] \approx 0.955$

$Pr[A=0.1 \mid data_3] \approx 0.00536$

$Pr[A=0.9 \mid data_3] \approx 0.9947$

Note that the distribution of 1's and 0's changes between (a) and (b), but is the same for (b) and (c)