

DM566: Data Mining and Machine Learning

Spring term 2022

Exercise 16

Exercise 16-1 Bayesian Inference

Given the following hypothesis for a value A , let the prior probability of each hypothesis be:

$$\Pr[A = 0.1] = 0.3$$

$$\Pr[A = 0.9] = 0.7$$

Let B_n be a binary variable. We define the likelihood as:

$$\Pr[B_n | A] = A^{B_n}(1 - A)^{(1-B_n)}.$$

We make the following observations:

$$data = \{B_1 = 1, B_2 = 0, B_3 = 1\} = \{1, 0, 1\}.$$

a.

Calculate the posterior probabilities $\Pr[A = 0.1 | data]$ and $\Pr[A = 0.9 | data]$, such that the posterior probability is adjusted based on the observations.

b.

Calculate the posterior probabilities when $data = \{1, 0, 1, 1, 0, 0, 1\}$.

c.

Again, calculate the posterior probabilities when $data = \{1, 0, 1, 1, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1\}$.

Comment on how the relationship between the prior on A and the posterior on A is affected by the observations as the size of the dataset increases.