

Simplification Questions Algebraic Answers

Answer 1

$$Z = A \cdot /B \cdot C + A \cdot B \cdot C + /A \cdot B \cdot C + /A \cdot /B \cdot C + A \cdot B \cdot /C + /A \cdot B \cdot /C$$

You may find that you have to rearrange the order of the subterms to get compatible ones next to each other. This one has been arranged so that we can just take successive pairs.

$$\begin{aligned} &= A \cdot C \cdot (/B + B) + /A \cdot C \cdot (B + /B) + B \cdot /C \cdot (/A + A) \\ &= A \cdot C + /A \cdot C + B \cdot /C \end{aligned}$$

Now we can take the two left-most subexpressions and do the same again:

$$\begin{aligned} &= C \cdot (A + /A) + B \cdot /C \\ &= C + B \cdot /C \end{aligned}$$

You can apply the simplification rule to this to get $C + B$, but I'll go the long way round, and show where the simplification rule comes from:

Remember the implied brackets. We can write the above as:

$$= C + (B \cdot /C)$$

You can OR the C with the B, and you can OR the C with the $/C$, and then AND the results together. So you can write the above expression as:

$$\begin{aligned} &= (C + B) \cdot (C + /C) \\ &= (C + B) \cdot 1 \\ &= C + B \end{aligned}$$

It just saves time if you remember the simplification rule.

Answer 2

$$Z = A \cdot /B \cdot C + A \cdot /B \cdot /C + A \cdot B \cdot C + /A \cdot /B \cdot C + /A \cdot /B \cdot /C$$

Here, we can take the first two subexpressions, and the last two subexpressions:

$$\begin{aligned} &= A \cdot /B \cdot (C + /C) + A \cdot B \cdot C + /A \cdot /B \cdot (C + /C) \\ &= A \cdot /B + A \cdot B \cdot C + /A \cdot /B \end{aligned}$$

Rearrange the terms to get the first and third together:

$$\begin{aligned} &= A \cdot /B + /A \cdot /B + A \cdot B \cdot C \\ &= /B \cdot (A + /A) + A \cdot B \cdot C \\ &= /B + A \cdot B \cdot C \end{aligned}$$

If you were to fill in these cells on a Karnaugh map, you would get the same answer, but it is just that when we did it with the map, we grouped the ones differently. To get it in the same form as the K map answer, you can write the above as:

$$\begin{aligned}
 &= \overline{B} + (A \cdot B \cdot C) \\
 &= (\overline{B} + A) \cdot (\overline{B} + B) \cdot (\overline{B} + C) \\
 &= (\overline{B} + A) \cdot (\overline{B} + C) \\
 &= \overline{B} + (A \cdot C)
 \end{aligned}$$

Answer 3

$$Z = A \cdot \overline{B} \cdot C + A \cdot B \cdot C + \overline{A} \cdot B \cdot C + \overline{A} \cdot \overline{B} \cdot \overline{C}$$

Concentrate on the first two subexpressions to start with:

$$\begin{aligned}
 &= A \cdot C \cdot (\overline{B} + B) + \overline{A} \cdot B \cdot C + \overline{A} \cdot \overline{B} \cdot \overline{C} \\
 &= A \cdot C + \overline{A} \cdot B \cdot C + \overline{A} \cdot \overline{B} \cdot \overline{C}
 \end{aligned}$$

Now combine the left-most two subexpressions. Take the common factor C outside the brackets. Don't forget the implied brackets around the $\overline{A} \cdot B$ subexpression. You can't just combine the A with the \overline{A} .

$$= C \cdot (A + (\overline{A} \cdot B)) + \overline{A} \cdot \overline{B} \cdot \overline{C}$$

But, you can apply the simplification rule to $(A + (\overline{A} \cdot B))$ and replace it with $(A + B)$

$$= C \cdot (A + B) + \overline{A} \cdot \overline{B} \cdot \overline{C}$$

And then put the C back into the brackets.

$$= C \cdot A + C \cdot B + \overline{A} \cdot \overline{B} \cdot \overline{C}$$

Answer 4

$$Z = \overline{A} \cdot \overline{B} \cdot C \cdot \overline{D} + \overline{A} \cdot B \cdot C \cdot \overline{D} + \overline{A} \cdot B \cdot C \cdot D + \overline{A} \cdot \overline{B} \cdot C \cdot D + \overline{A} \cdot B \cdot C \cdot D + \overline{A} \cdot \overline{B} \cdot C \cdot D + \overline{A} \cdot B \cdot \overline{C} \cdot D + \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D$$

Luckily we can combine consecutive pairs of subexpressions

$$\begin{aligned}
 &= \overline{A} \cdot C \cdot \overline{D} \cdot (\overline{B} + B) + \overline{A} \cdot C \cdot \overline{D} \cdot (B + \overline{B}) + \overline{A} \cdot C \cdot D \cdot (B + \overline{B}) + \overline{A} \cdot C \cdot D \cdot (B + \overline{B}) \\
 &= \overline{A} \cdot C \cdot \overline{D} + \overline{A} \cdot C \cdot \overline{D} + \overline{A} \cdot C \cdot D + \overline{A} \cdot C \cdot D \\
 &= C \cdot \overline{D} \cdot (\overline{A} + \overline{A}) + \overline{A} \cdot D \cdot (C + C) \\
 &= C \cdot \overline{D} + \overline{A} \cdot D
 \end{aligned}$$

Answer 5

$$\begin{aligned}
 Z &= \overline{A} \cdot \overline{B} \cdot C \cdot \overline{D} + \overline{A} \cdot B \cdot C \cdot \overline{D} + \overline{A} \cdot B \cdot C \cdot D + \overline{A} \cdot \overline{B} \cdot C \cdot D + \overline{A} \cdot B \cdot C \cdot D + \overline{A} \cdot \overline{B} \cdot C \cdot D \\
 &\quad + \overline{A} \cdot B \cdot \overline{C} \cdot D + \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D + \overline{A} \cdot B \cdot C \cdot D + \overline{A} \cdot B \cdot C \cdot D + \overline{A} \cdot B \cdot \overline{C} \cdot D + \overline{A} \cdot B \cdot \overline{C} \cdot D
 \end{aligned}$$

Combine consecutive pairs of subexpressions

$$= \overline{A} \cdot C \cdot \overline{D} \cdot (\overline{B} + B) + \overline{A} \cdot C \cdot \overline{D} \cdot (B + \overline{B}) + \overline{A} \cdot C \cdot D \cdot (B + \overline{B})$$

$$\begin{aligned}
& + /A./C.D.(B+/B) \quad + A.C.D./(B+B) \quad + A./C.D./(B+B) \\
= & A.C./D \quad + /A.C./D \quad + /A.C.D \quad /A./C.D + A.C.D \quad + A./C.D
\end{aligned}$$

Again, combine consecutive pairs of subexpressions

$$\begin{aligned}
= & C./D.(A+/A) \quad + /A.D.(C+/C) \quad + A.D.(C+/C) \\
= & C./D \quad + /A.D \quad + A.D
\end{aligned}$$

Combine last two subexpressions

$$\begin{aligned}
= & C./D \quad + D./(A+A) \\
= & C./D \quad + D
\end{aligned}$$

Use the simplification rule to get

$$= C + D$$

Answer 6

$$\begin{aligned}
Z = & A./B.C./D + A.B.C./D + /A.B.C./D + /A./B.C./D + A.B.C.D + /A.B.C.D \\
& + A.B./C.D + /A.B./C.D + A./B./C./D + A.B./C./D + /A.B./C./D + /A./B./C./D
\end{aligned}$$

Combine consecutive pairs of subexpressions

$$\begin{aligned}
= & A.C./D./(B+B) \quad + /A.C./D.(B+/B) \quad + B.C.D.(A+/A) \\
& + B./C.D.(A+/A) \quad + A./C./D./(B+B) \quad + /A./C./D./(B+B) \\
= & A.C./D \quad + /A.C./D \quad + B.C.D \quad B./C.D + A./C./D \quad + /A./C./D
\end{aligned}$$

Again, combine consecutive pairs of subexpressions

$$\begin{aligned}
= & C./D.(A+/A) \quad + B.D.(C+/C) \quad + /C./D.(A+/A) \\
= & C./D \quad + B.D \quad + /C./D
\end{aligned}$$

Rearrange to get the first and third subexpressions together

$$\begin{aligned}
= & C./D \quad + /C./D \quad + B.D \\
= & /D.(C+/C) \quad + B.D \\
= & /D \quad + B.D
\end{aligned}$$

Use the simplification rule

$$= /D + B$$