

Propositions and Rules of Boolean Algebra

There is an order of precedence of operations, just as there is in ordinary arithmetic. In Boolean algebra, it is as follows:

Brackets (evaluated first), then NOT, then AND, then OR.

These four basic rules are all provable:

1: Commutative Proposition

$$A.B = B.A$$

$$A+B = B+A$$

$$A \text{ and } B = B \text{ and } A$$

$$A \text{ or } B = B \text{ or } A$$

2: Distributive Proposition

$$A(B+C) = A.B + A.C$$

$$A \text{ and } (B \text{ or } C) = (A \text{ and } B) \text{ or } (A \text{ and } C)$$

3: Identity Proposition

$$A+0 = A$$

$$A+1 = 1$$

$$A.1 = A$$

$$A.0 = 0$$

$$A \text{ or } 0 = A$$

$$A \text{ or } 1 = 1$$

$$A \text{ and } 1 = A$$

$$A \text{ and } 0 = 0$$

4: Inverse Proposition

$$A+/\!A = 1$$

$$A./\!A = 0$$

$$A \text{ or } \text{NOT } A = 1$$

$$A \text{ and } \text{NOT } A = 0$$

With these propositions a large number of Boolean algebra theorems may be developed, and proved. The most important which we will need are:

1: De Morgan's Theorem

$$\!/(A+B) = \!/\!A./\!B$$

$$\!/(A.B) = \!/\!A+/\!B$$

$$\text{NOT}(A \text{ or } B) = (\text{NOT } A \text{ and } \text{NOT } B)$$

$$\text{NOT}(A \text{ and } B) = (\text{NOT } A \text{ or } \text{NOT } B)$$

2: Simplification Theorem

$$A+A.B = A$$

$$A+/\!A.B = A+B$$

$$A \text{ or } (A \text{ and } B) = A$$

$$A \text{ or } (\text{NOT } A \text{ and } B) = A \text{ or } B$$