IEMS 313 Project
Phase II

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Sets

STATIONS = set of stations in network

ARCS = set of arcs in network, defined as (i, j) where i, j ∈ *STATIONS* and i ≠ j and $distance_{ij}$ < *max_line_length*

LINES = set of edges in network, defined as (i, j) where i, j ∈ *STATIONS* and i < j

DELIVS = set of deliveries, defined as (s,d) where s, d ∈ *STATIONS*

Parameters

$distance_{ij}$ = distance between station i and station j in miles

load_per_building = maximum number of incoming goods (including what comes from a source) that one reloading building can handle

track_capacity = maximum number of units that can be transported on one track

cost_reload_bldg = cost for building additional reloading buildings

cost_track = cost for building additional tracks where there is already a line

cost_line = cost for paving a new line between two unconnected stations

max_line_length = maximum length of a line

$exist\_reload\_bldgs_i$ = number of existing reloading buildings at station i

$exist\_tracks_{ij}$ = number of undirected tracks that exist between stations i and j

$exist\_line_{ij} = \begin{cases} 1 \ if \ exist\_tracks_{ij} > 0 \\ \quad 0 \ otherwise \end{cases}$

$volume_{sd}$ = number of units in shipment (s,d)

M = big M for number of tracks between any two stations, calculated as

$$\frac{1}{track\_capacity} * \sum_{(s,d) \in DELIVS} volume_{sd}$$

Variables

v_{sdij} = Number of units of shipment from station s to station d that are shipped along arc (i, j)

x_{ij} = Number of tracks along arc (i,j)

y_{ij} = 1 if a line exists between station i and station j, 0 otherwise

z_i = Number of reloading buildings at station i min

$$cost\_reload\_bldg \sum_{i \in STATIONS} z_i - cost\_reload\_bldg \sum_{i \in STATIONS} exist\_reload\_bldgs_i$$

$$+ cost\_track \sum_{(i,j) \in ARCS} distance_{ij} * x_{ij} - cost\_track \sum_{(i,j) \in LINES} distance_{ij} * exist\_tracks_{ij}$$

$$+ cost\_line \sum_{(i,j) \in LINES} distance_{ij} * y_{ij} - cost\_line \sum_{(i,j) \in LINES} distance_{ij} * exist\_line_{ij}$$

s.t. $z_i \geq exist\_reload\_bldgs_i \quad \forall\, i \in STATIONS$
(Can't remove existing reloading buildings)

$x_{ij} + x_{ji} \geq exist\_tracks_{ij} \quad \forall\, (i,j) \in LINES$
(Can't remove existing tracks)

$y_{ij} \geq exist\_line_{ij} \quad \forall\, (i,j) \in LINES$
(Can't remove existing lines)

$\sum_{(s,d) \in DELIVS} v_{sdij} \leq track\_capacity * x_{ij} \quad \forall\, (i,j) \in ARCS$
(Enforce track capacity)

$\left(\sum_{(s,d) \in DELIVS} \sum_{(i,k) \in ARCS} v_{sdik}\right) + \left(\sum_{(k,d) \in DELIVS} volume_{kd}\right) \leq load\_per\_building * z_k \quad \forall\, k \in STATIONS$
(Enforce reloading building capacity)

$\sum_{(i,s) \in ARCS} v_{sdis} - \sum_{(s,j) \in ARCS} v_{sdsj} = -volume_{sd} \quad \forall\, (s,d) \in DELIVS$
(Source sends out all units)

$\sum_{(i,d) \in ARCS} v_{sdid} - \sum_{(d,j) \in ARCS} v_{sddj} = volume_{sd} \quad \forall\, (s,d) \in DELIVS$
(Destination receives all units)

$\sum_{(i,k) \in ARCS} v_{sdik} - \sum_{(k,j) \in ARCS} v_{sdkj} = 0 \quad \forall\, (s,d) \in DELIVS\ \forall\, k \in STATIONS, k \neq s \text{ and } k \neq d$
(All nodes that aren't source or destination have balanced flow)

$x_{ij} \leq M * y_{ij} \quad \forall\, (i,j) \in ARCS, i < j$
(Can only have tracks where line exists)

$x_{ij} \leq M * y_{ji} \quad \forall\, (i,j) \in ARCS, i > j$
(Can only have tracks where line exists)

$v_{sdij} \geq 0$ and integer $\quad \forall (s,d) \in DELIVS \ \ \forall (i,j) \in ARCS$
$x_{ij} \geq 0$ and integer $\quad \forall (i,j) \in ARCS$
y_{ij} binary $\quad \forall (i,j) \in LINES$
$z_i \geq 0$ and integer $\quad \forall i \in STATIONS$

Discussion of Model

Variables

The variable v_{sdij} represents the number of units of the shipment that goes from station s to station d that are sent along the arc (i,j). This variable is required to determine precisely how to distribute the shipments after the network has been completely built.

The variable x_{ij} represents the number of tracks along the arc (i,j). This variable is required to know how many tracks to build from station i to station j.

The variable y_{ij} is a binary value that represents whether or not there is a line between stations i and j. If there is a line between stations i and j, $y_{ij} = 1$. Otherwise, $y_{ij} = 0$. This variable is required to ensure that tracks are only built between two stations if a line exists between those stations.

The variable z_i represents the number of reloading buildings at station i. This variable is required to know how many reloading buildings to build at each station.

Objective

The objective is to minimize the total cost of building a network that will allow all of the shipments to be distributed. The objective function calculates the total cost by summing the cost of all of the reloading buildings, then subtracting the cost of the reloading buildings that already existed, then adding the cost of all of the tracks, then subtracting the cost of all of the tracks that already existing, then adding the cost of paving all of the lines, and finally subtracting the cost of all of the lines that already existed. In other words, the objective is to minimize total cost, where total cost is defined as:

Total cost = (Total cost of reloading buildings – Cost of existing reloading buildings)
+ (Total cost of tracks – Cost of existing tracks)
+ (Total cost of lines – Cost of existing lines)

Constraints

$$z_i \geq exist\_reload\_bldgs_i \quad \forall\, i \in STATIONS$$
$$x_{ij} + x_{ji} \geq exist\_tracks_{ij} \quad \forall\, (i,j) \in LINES$$
$$y_{ij} \geq exist\_line_{ij} \quad \forall\, (i,j) \in LINES$$

The first three constraints ensure that none of the existing reloading buildings, tracks, or lines are removed from the network. That is, the number of reloading buildings, tracks, and lines in the final network must be greater or equal to the number of existing reloading buildings, tracks, and lines, respectively.

$$\sum_{(s,d)\in DELIVS} v_{sdij} \leq track\_capacity * x_{ij} \quad \forall\,(i,j) \in ARCS$$

The next constraint ensures that none of the tracks ship more than *track_capacity* units. For each arc (i,j) the number of units sent along (i,j) is calculated by summing v_{sdij} over all shipments (s,d). The constraint enforces that this sum is less than or equal to the number of tracks from station i to j times the *track_capacity*. In other words, no single track ships more *track_capacity* units.

$$\sum_{(s,d)\in DELIVS} \sum_{(i,k)\in ARCS} v_{sdik} + \sum_{(k,d)\in DELIVS} volume_{kd} \leq load\_per\_building * z_k \quad \forall\, k \in STATIONS$$

The next constraint ensures that none of the reloading buildings handle more than *load_per_building* incoming units (this includes units from a source). For every station, the number of incoming units is calculated by summing the number of units coming into the station over all deliveries and over all arcs that lead to the station, and then adding in the volume of any shipment that originates at the station. This sum must be less than or equal to the number of reloading buildings at the station times *load_per_building*.

$$\sum_{(i,s)\in ARCS} v_{sdis} - \sum_{(s,j)\in ARCS} v_{sdsj} = -volume_{sd} \quad \forall\,(s,d) \in DELIVS$$
$$\sum_{(i,d)\in ARCS} v_{sdid} - \sum_{(d,j)\in ARCS} v_{sddj} = volume_{sd} \quad \forall\,(s,d) \in DELIVS$$
$$\sum_{(i,k)\in ARCS} v_{sdik} - \sum_{(k,j)\in ARCS} v_{sdkj} = 0 \quad \forall\,(s,d) \in DELIVS\ \forall\, k \in STATIONS, k \neq s \text{ and } k \neq d$$

The next three constraints ensure that for each shipment, the source station sends out all units, the destination station receives all units, and every other station in the network has a balanced flow (i.e. the number of units from that shipment coming in is equal to the number of units from that shipment going out).

$$x_{ij} \leq M * y_{ij} \quad \forall\,(i,j) \in ARCS, i < j$$
$$x_{ij} \leq M * y_{ji} \quad \forall\,(i,j) \in ARCS, i > j$$

The last two main constraints ensure that tracks are only built between two stations if a line exists between those stations. It is necessary to have two constraints because y_{ij} is defined for lines (i.e. when i < j) but we want this constraint to apply to all arcs. Therefore, we separate the constraint into two cases, one where i < j and one where i > j. The effect of the constraint is that if $y_{ij} = 1$, then the constraint does not restrict x_{ij}. However, if $y_{ij} = 0$, then the constraint ensures that $x_{ij} = 0$, too.

$v_{sdij} \geq 0$ and integer $\quad \forall (s,d) \in DELIVS \ \ \forall (i,j) \in ARCS$

$x_{ij} \geq 0$ and integer $\quad \forall (i,j) \in ARCS$

y_{ij} binary $\quad \forall (i, j) \in LINES$
$z_i \geq 0$ and integer $\quad \forall i \in STATIONS$

Finally, the variable constraints ensure that

- v_{sdij} (the number of units of shipment (s,d) sent on arc (i,j)) is greater than or equal to zero and integer for every shipment and every arc
- x_{ij} (the number of tracks along the arc (i,j)) is greater than or equal to zero and integer for every arc
- y_{ij} (whether or not there is a line between stations i and j) is binary for every pair of stations
- z_i (the number of reloading buildings at station i) is greater than or equal to zero and integer at every station

We did not research any formulations while creating this model.

<u>AMPL Program Scenario 1 (Reloading buildings cost $1,000,000)</u>

- **300 MIP Simplex Iterations**
- **0 Branch and Bound Nodes**
- **Time CPLEX needs to solve MILP: 0.11 seconds**
- Send 8 units of shipment A along the path 4-6-7. Send the other 3 units along 4-1-3-5-7. This requires no cost.
- Send all 7 units of shipment C along the path 6-5-2. This requires building a new line and track from station 6 directly to station 5 at a cost of $1,697,056.28 and reloading stations at stations 2, 6, and 7, at a cost of $1,000,000 each.
- Send all 3 units of shipment B along the path 3-5-2.
- The total cost is **$4,697,056.28**

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	50	0	1	
2	250	0	1 + 1 = 2	1,000,000
3	100	50	2	
4	0	100	3	
5	200	100	2 + 1 = 3	1,000,000
6	100	200	2 + 1 = 3	1,000,000
7	200	200	3	

Station	Station	# tracks	Costs
1	3	1	
1	4	1	
2	5	1	
3	5	1	
4	6	1	
5	6	1	1,697,056.28
5	7	1	
6	7	1	

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	4	7	11	4-6-7 4-1-3-5-7	8 3
B	3	2	3	3-5-2	3
C	6	2	7	6-5-2	7

This is the same solution as the one provided as an example in the assignment description. That solution was indeed optimal.

AMPL Program Scenario 1 (Reloading buildings cost $5000,000)

- **229 MIP Simplex Iterations**
- **0 Branch and Bound Nodes**
- **Time CPLEX needs to solve MILP: 0.06 seconds**
- Send 8 units of shipment A along the path 4-6-7. Send the other 3 units along 4-1-3-5-7. This requires no cost.
- Send all 7 units of shipment C along the path 6-5-2. This requires building a new line and track from station 6 directly to station 5 at a cost of $1,697,056.28 and reloading stations at stations 2, 6, and 7, at a cost of $1,000,000 each.
- Send all 3 units of shipment B along the path 3-5-2.
- The total cost is **$3,000,000**

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	50	0	1	
2	250	0	1 + 1 = 2	500,000
3	100	50	2	
4	0	100	3	
5	200	100	2 + 1 = 3	500,000
6	100	200	2 + 1 = 3	500,000
7	200	200	3 + 1 = 4	500,000

Station	Station	# tracks	Costs
1	3	1	
1	4	1	
2	5	1	
3	5	1	
4	6	1	
5	7	1 + 1 = 2	500,000
6	7	1 + 1 = 2	500,000

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	4	7	11	4-6-7 4-1-3-5-7	8 3
B	3	2	3	3-5-2	3
C	6	2	7	6-7-5-2	7

AMPL Program Scenario 2, Reloading buildings cost $1,000,000

- **280 MIP Simplex Iterations**
- **0 Branch and Bound Nodes**
- **Time CPLEX needs to solve MILP: 0.08 seconds**
- Send 10 units of shipment A along the path 4-6-7. Send the other 1 unit along 4-1-3-5-7. This requires no cost.
- Send all 6 units of shipment C along the path 4-1-3-5-2. This requires building a new line and track from station 1 directly to station 2 at a cost of ($7,000 + $5,000) * 200 = $2,400,000 and reloading stations at stations 1, 2, and 4, at a cost of $1,000,000 each.
- Send all 3 units of shipment B along the path 3-5-2. This requires building another reloading station at station 4 at a cost of $1,000,000.
- The total cost is **$6,400,000**

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	50	0	1 +1 = 2	1,000,000
2	250	0	1 + 2 = 3	2,000,000
3	100	50	2	
4	0	100	3+1=4	1,000,000
5	200	100	2	
6	100	200	2	
7	200	200	3	

Station	Station	# tracks	Costs
1	3	1	
1	4	1	
2	5	1	
3	5	1	
4	6	1	
5	7	1	
6	7	1	
1	2	1	2,400,000

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	4	7	11	4-6-7 4-1-3-5-7	10 1
B	3	2	3	3-5-2	3
C	4	2	9	4-1-2 4-1-3-5-2	3 6

HEURISTIC Scenario 2, Reloading buildings cost $1,000,000

- Send 10 units of shipment A along the path 4-6-7. Send the other 1 unit along 4-1-3-5-7. This requires no cost.

- Send all 9 units of shipment C along the path 4-1-3-5-2. This requires building a reloading station at stations 1, 2, and 4, at a cost of $1,000,000 each.
- Build a new line and track from station 3 directly to station 2 at a cost of ($7,000 + $5,000) * 158.1139 = $1,897,366.80. Build another reloading station at stations 3 and 2 at a cost of $1,000,000 each.
- The total cost is **$6,897,366.80.**

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	50	0	1+1=2	1,000,000
2	250	0	1+2=3	2,000,000
3	100	50	2+1=3	1,000,000
4	0	100	3+1=4	1,000,000
5	200	100	2	
6	100	200	2	
7	200	200	3	

Station	Station	# tracks	Costs
1	3	1	
1	4	1	
2	5	1	
3	5	1	
4	6	1	
5	7	1	
6	7	1	
3	2	1	1,897,366.80

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	4	7	11	4-6-7 4-1-3-5-7	10 1
B	3	2	3	3-2	3
C	4	2	9	4-1-3-5-2	9

Comparison:
The heuristic performed well, having routes very close to the linear program result. However, it did not aptly account for the high reloading station price, and did not build a new line from station 1 to station 2. As a result, the optimal solution was cheaper by over $400,000, given that less reloading stations had to be built to accommodate Shipment 3.

AMPL Program Scenario 2, Reloading Buildings Cost $500,000

- **237 MIP Simplex Iterations**
- **0 Branch and Bound Nodes**
- **Time CPLEX needs to solve MILP: 0.11 seconds**
- Send 10 units of shipment A along the path 4-6-7. Send the other 1 unit along 4-1-3-5-7. This requires no cost.
- Send all 9 units of shipment C along the path 4-1-3-5-2. This requires building a reloading station at stations 1 and 4, at a cost of $500,000 each.
- Send all 3 units of shipment B along the path 3-5-2. This requires building a new track alongside the existing track between stations 3 and 5 at a cost of $5,000 * 111.8034 = $559017, and between stations 5 and 2 at a cost of $5,000 * 111.8034 = $559017. It also requires building a new reloading station at stations 3, 5, and 2 at a cost of $500,000 each.
- The total cost is **$4,118,034**.

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	50	0	1+1 = 2	500,000
2	250	0	1+2 = 3	1,000,000
3	100	50	2+1 = 3	500,000
4	0	100	3+1 = 4	500,000
5	200	100	2+1 = 3	500,000
6	100	200	2	
7	200	200	3	

Station	Station	# tracks	Costs
1	3	1	
1	4	1	
2	5	1+1 = 2	$559,017
3	5	1+1 = 2	$559,017
4	6	1	
5	7	1	
6	7	1	

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	4	7	11	4-6-7 4-1-3-5-7	10 1
B	3	2	3	3-5-2	3
C	4	2	9	4-1-3-5-2	9

HEURISTIC Scenario 2, Reloading buildings cost $500,000

- Send 10 units of shipment A along the path 4-6-7. Send the other 1 unit along 4-1-3-5-7. This requires no cost.
- Send all 9 units of shipment C along the path 4-1-3-5-2. This requires building a reloading station at stations 1 and 4, at a cost of $500,000 each.

- Send all 3 units of shipment B along the path 3-5-2. This requires building a new track alongside the existing track between stations 3 and 5 at a cost of $5,000 * 111.8034 = $559017, and between stations 5 and 2 at a cost of $5,000 * 111.8034 = $559017. It also requires building a new reloading station at stations 3, 5, and 2 at a cost of $500,000 each.
- The total cost is **$4,118,034**.

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	50	0	1+1=2	500,000
2	250	0	1+2=3	1,000,000
3	100	50	2+1=3	500,000
4	0	100	3+1=4	500,000
5	200	100	2+1=3	500,000
6	100	200	2	
7	200	200	3	

Station	Station	# tracks	Costs
1	3	1	
1	4	1	
2	5	1+1 = 2	$559017
3	5	1+1 = 2	$559017
4	6	1	
5	7	1	
6	7	1	

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	4	7	11	4-6-7 4-1-3-5-7	10 1
B	3	2	3	3-5-2	3
C	4	2	9	4-1-3-5-2	9

<u>Comparison:</u>
Our heuristic performed very well here, obtaining exactly the optimal solution. This is in large part due to low costs of the reloading buildings.

AMPL Program Scenario 3, Reloading Buildings cost $1 million

- **299 MIP Simplex Iterations**
- **0 Branch and Bound Nodes**
- **Time CPLEX needs to solve MILP: 0.09 seconds**
- Send 11 units of shipment A along the path 4-6-7. This requires building new track from 4-6 and from 6-7 along with new a new reloading building at 6 for a total cost of $2,707,017
- Send all 8 units of shipment C along the path 6-3. This requires building a new line and a new track between stations 6 and 3. This costs ($7,000 + $5,000) * 150 = $1,800,00. It also requires building a new reloading stations at station 6, at a cost of $1,000,000.
- Send all 4 units of shipment D along the path 7-5-3-1. This requires building new track from 5-3 at a cost of $559,017.
- Send all 3 units of shipment B along the path 3-5-2. This requires building a new reloading station at station 3 at a cost of $1,000,000.
- The total cost is **$6,566,124.**

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	50	0	1	
2	250	0	1	
3	100	50	2+1 = 3	1,000,000
4	0	100	3	
5	200	100	2	
6	100	200	2+2 = 4	2,000,000
7	200	200	3	

Station	Station	# tracks	Costs
1	3	1	
1	4	1	
2	5	1	
3	5	1 + 1 = 2	559,017
4	6	1 +1 = 2	707,107
5	7	1	
6	7	1 + 1 = 2	500,000
3	6	1	1,800,000

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	4	7	11	4-6-7	11
B	3	2	3	3-5-2	3
C	6	3	8	6-3	8
D	7	1	4	7-5-3-1	4

HEURISTIC Scenario 3, Reloading buildings cost $1 million

- Send 10 units of shipment A along the path 4-6-7. Send the other 1 unit along 4-1-3-5-7. This requires no cost.

- Send all 8 units of shipment C along the path 6-3. This requires building a new line and a new track between stations 6 and 3. This costs ($7,000 + $5,000) * 150 = $1,800,00. It also requires building two new reloading stations at station 6, at a cost of $1,000,000 each.
- Send all 4 units of shipment D along the path 7-5-3-1. This requires building new track alongside the existing track between stations 7 and 5 at a cost of $5,000 * 100 = $500,000, between stations 5 and 3 at a cost of $5,000 * 111.8034 = $559,017, and between stations 3 and 1 at a cost of $5,000 * 70.7107 = $353,553.50. This also requires building a new reloading station at station 3 at a cost of $1,000,000.
- Send all 3 units of shipment B along the path 3-5-2. This requires building a new reloading station at station 3 at a cost of $1,000,000.
- The total cost is **$7,212,570.50**.

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	50	0	1	
2	250	0	1	
3	100	50	2+2 = 4	2,000,000
4	0	100	3	
5	200	100	2	
6	100	200	2+2 = 4	2,000,000
7	200	200	3	

Station	Station	# tracks	Costs
1	3	1+1 = 2	353,553.50
1	4	1	
2	5	1	
3	5	1+1 = 2	559,017
4	6	1	
5	7	1+1 = 2	500,000
6	7	1	
6	3	1	1,800,000

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	4	7	11	4-6-7 4-1-3-5-7	10 1
B	3	2	3	3-5-2	3
C	6	3	8	6-3	8
D	7	1	4	7-5-3-1	4

<u>Comparison:</u>
In this case, our heuristic missed the optimal solution by close to $700,000 because it considers feasible paths before creating track by existing feasible paths. At the start, the heuristic ships 10 units through the 4-6-7 path and only one through the 4-1-3-5-7 path. It would have been more cost effective to simply create more track for the 4-6-7 path instead of forcing the tracks between 1-3-5-7 in that order for one

unit. Luckily, shipment B also utilized that path; however, shipment D ended up costing the network close to 1.5 million dollars more than it would have if shipment A were directed all along the same path.

AMPL Program Scenario 3, Reloading buildings cost $500,00

- **2212 MIP Simplex Iterations**
- **134 Branch and Bound Nodes**
- **Time CPLEX needs to solve MILP: 0.17 seconds**
- Send 8 units of shipment A along the path 4-6-7. Send the other 3 units along 4-1-3-5-7. This requires no cost.
- Send all 8 units of shipment C along the path 6-3. This requires building a new line and a new track between stations 6 and 3. This costs ($7,000 + $5,000) * 150 = $1,800,00. It also requires building two new reloading stations at station 6, at a cost of $500,000.
- Send all 4 units of shipment D along the path 7-6-4-1. This requires building a new reloading station at 1 at a cost of $500,000 and two new tracks from 6-4 and 7-6 at a cost of $707,107 and $500,000, respectively.
- Send all 3 units of shipment B along the path 3-5-2. This requires building a new reloading station at station 3 at a cost of $500,000.
- The total cost is **$5,007,107**

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	50	0	1 + 1 = 2	500,000
2	250	0	1	
3	100	50	2+1 = 3	500,000
4	0	100	3	
5	200	100	2	
6	100	200	2+2 = 4	1,000,000
7	200	200	3	

Station	Station	# tracks	Costs
1	3	1	
1	4	1	
2	5	1	
3	5	1	
4	6	1 + 1 = 2	707,107
5	7	1	
6	7	1 + 1 = 2	500,000
3	6	1	1,800,000

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	4	7	11	4-6-7 4-1-3-5-7	8 3
B	3	2	3	3-5-2	3
C	6	3	8	6-3	8
D	7	1	4	7-6-4-1	4

HEURISTIC Scenario 3, Reloading buildings cost $500,000

- Send 10 units of shipment A along the path 4-6-7. Send the other 1 unit along 4-1-3-5-7. This requires no cost.
- Send all 8 units of shipment C along the path 6-3. This requires building a new line and a new track between stations 6 and 3. This costs ($7,000 + $5,000) * 150 = $1,800,00. It also requires building two new reloading stations at station 6, at a cost of $500,000.
- Send all 4 units of shipment D along the path 7-5-3-1. This requires building new track alongside the existing track between stations 7 and 5 at a cost of $5,000 * 100 = $500,000, between stations 5 and 3 at a cost of $5,000 * 111.8034 = $559,017, and between stations 3 and 1 at a cost of $5,000 * 70.7107 = $353,553.50. This also requires building a new reloading station at station 3 at a cost of $500,000.
- Send all 3 units of shipment B along the path 3-5-2. This requires building a new reloading station at station 3 at a cost of $500,000.
- The total cost is **$5,212,570.50.**

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	50	0	1	
2	250	0	1	
3	100	50	2+2 = 4	1,000,000
4	0	100	3	
5	200	100	2	
6	100	200	2+2 = 4	1,000,000
7	200	200	3	

Station	Station	# tracks	Costs
1	3	1+1 = 2	353,553.50
1	4	1	
2	5	1	
3	5	1+1 = 2	559,017
4	6	1	
5	7	1+1 = 2	500,000
6	7	1	
6	3	1	1,800,000

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	4	7	11	4-6-7 4-1-3-5-7	10 1
B	3	2	3	3-5-2	3
C	6	3	8	6-3	8
D	7	1	4	7-5-3-1	4

<u>Comparison:</u>
In this case, the heuristic again came close to the optimal solution (missing the mark by a little over $200,000) but failed due to putting too much of Shipment A on one path, and not leaving

room for other shipments which could have used the path. This caused a need for 1 more track to be built in the heuristic network than in the optimal network, since 7-5-3-1 became more cost effective than 7-6-4-1 given the organization of Shipment A. This is problematic given the low costs of reloading stations.

AMPL Program Scenario 4, Reloading buildings cost $1,000,000

- **178953 MIP Simplex Iterations**
- **9752 Branch and Bound Nodes**
- **Time CPLEX needs to solve MILP: 5.77 seconds**
- The total cost is $**6,516,884.16**

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	0	0	4 + 1 = 5	$1,000,000
2	50	50	5 + 1 = 6	$1,000,000
3	250	50	4 + 1 = 5	$1,000,000
4	400	50	4	
5	0	100	6	
6	250	100	4	
7	150	150	2	
8	300	150	4	
9	50	200	4	
10	350	200	6 + 1 = 7	$1,000,000
11	150	250	3	
12	100	300	3	
13	400	300	7	
14	250	350	4	

Station	Station	# tracks	Costs
1	2	2	
1	5	1	
2	3	2	
2	5	2 + 1 = 3	$353,553.39
2	7	1	
3	4	3	
4	10	2	
5	9	3	
6	7	1	
6	8	2	
8	10	2	
9	12	2	
10	13	3	
11	12	2	
11	14	2	
13	14	3	
10	3	1	$2,163,330.77

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	1	13	17	1-5-9-12-11-14-13 1-2-7-6-8-10-13 1-2-3-4-10-13 1-2-5-9-12-11-14-13	10 2 3 2
B	5	10	8	5-2-7-6-8-10	8
C	9	4	8	9-5-2-3-4 9-12-11-14-13-10-4	7 1
D	14	3	7	14-13-10-4-3	7
E	13	6	6	13-10-8-6	6
F	10	1	8	10-4-3-2-1 10-3-2-1	2 6

We did not run our heuristic on scenario 4.

AMPL Program Scenario 4, Reloading buildings cost $500,000

- **94924 MIP Simplex Iterations**
- **5374 Branch and Bound Nodes**
- **Time CPLEX needs to solve MILP: 4.11 seconds**
- The total cost is $**3,853,553.39**

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	0	0	4 + 1 = 5	$500,000
2	50	50	5 + 1 = 6	$500,000
3	250	50	4 + 1 = 5	$500,000
4	400	50	4 + 1 = 5	$500,000
5	0	100	6	
6	250	100	4	
7	150	150	2	
8	300	150	4	
9	50	200	4 + 1 = 5	$500,000
10	350	200	6 + 1 = 7	$500,000
11	150	250	3	
12	100	300	3	
13	400	300	7	
14	250	350	4 + 1 = 5	$500,000

Station	Station	# tracks	Costs
1	2	2	
1	5	1	
2	3	2	
2	5	2 + 1 = 3	$353,553.39
2	7	1	
3	4	3	
4	10	2	
5	9	3	
6	7	1	
6	8	2	
8	10	2	
9	12	2	
10	13	3	
11	12	2	
11	14	2	
13	14	3	

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	1	13	17	1-5-9-12-11-14-13 1-2-7-6-8-10-13 1-2-5-9-12-11-14-13	10 3 4
B	5	10	8	5-2-7-6-8-10 5-9-12-11-14-13-10	7 1
C	9	4	8	9-5-2-3-4	8
D	14	3	7	14-13-10-4-3	7
E	13	6	6	13-10-8-6	6
F	10	1	8	10-4-3-2-1	8

We did not run our heuristic on scenario 4.

AMPL Program Scenario 5, Reloading buildings cost $1,000,000

- **2989236 MIP Simplex Iterations**
- **76882 Branch and Bound Nodes**
- **Time CPLEX needs to solve MILP: 121.48 seconds**
- The total cost is $**36,082,247.93**

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	0	0	4 + 1 = 5	$1,000,000
2	50	50	5	
3	250	50	4 + 1 = 5	$1,000,000
4	400	50	4	
5	0	100	6	
6	250	100	4	
7	150	150	2 + 6 = 8	$6,000,000
8	300	150	4	
9	50	200	4 + 3 = 7	$3,000,000
10	350	200	6 + 3 = 9	$3,000,000
11	150	250	3 + 2 = 5	$2,000,000
12	100	300	3	
13	400	300	7	
14	250	350	4	

Station	Station	# tracks	Costs
1	2	2	
1	5	1	
2	3	2	
2	5	2	
2	7	1	
3	4	3	
4	10	2	
5	9	3	
6	7	1	
6	8	2	
8	10	2	
9	12	2	
10	13	3	
11	12	2	
11	14	2	
13	14	3	
3	8	2	$1,900,657.783

4	6	1	$1,897,366.596
5	7	2	$2,687,936.011
6	11	2	$3,064,718.588
7	10	4	$5,566,192.587
8	13	2	$3,064,718.588
7	9	2	$1,900,657.783

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	1	13	17	1-2-3-8-13 1-2-7-10-13 1-2-5-7-10-13 1-2-5-9-12-11-14-13	10 3 1 3
B	5	10	8	5-7-10	8
C	9	4	8	9-5-2-3-4	8
D	14	3	7	14-13-8-3	7
E	13	6	6	13-14-11-6 13-8-6	3 3
F	10	1	8	10-7-5-1	8
G	10	9	4	10-7-5-9 10-13-14-11-12-9	2 2
H	11	6	5	11-6	5
I	9	10	20	9-7-10 9-12-11-14-13-10	18 2
J	4	11	10	4-6-11 4-10-13-14-11	9 1

We did not run our heuristic on scenario 5.

AMPL Program Scenario 5, Reloading buildings cost $500,000

- **145902 MIP Simplex Iterations**
- **6274 Branch and Bound Nodes**
- **Time CPLEX needs to solve MILP: 10.56 seconds**
- The total cost is $**23,316,436.78**

Station	x-coordinate	y-coordinate	# reloading buildings	Costs
1	0	0	4 + 1 = 5	$500,000
2	50	50	5 + 1 = 6	$500,000
3	250	50	4 + 1 = 5	$500,000
4	400	50	4 + 3 = 7	$1,500,000
5	0	100	6	
6	250	100	4	
7	150	150	2	
8	300	150	4	
9	50	200	4 + 6 = 10	$3,000,000
10	350	200	6 + 9 =15	$4,500,000
11	150	250	3 + 8 = 11	$4,000,000
12	100	300	3	
13	400	300	7	
14	250	350	4	

Station	Station	# tracks	Costs
1	2	2	
1	5	1	
2	3	2	
2	5	2	
2	7	1	
3	4	3	
4	10	2 + 1 = 3	$790,569.42
5	9	3	
6	7	1	
6	8	2	
8	10	2	
9	12	2	
10	13	3	
11	12	2	
11	14	2	
13	14	3	
9	11	3	$2,459,674.77

10	11	4	$5,566,192.59

Shipment	Origin	Destination	Total Volume	Path	Path Volume
A	1	13	17	1-2-7-6-8-10-13 1-5-9-11-14-13	8 9
B	5	10	8	5-9-12-11-10 5-9-12-11-14-13-10 5-2-7-6-8-10	6 1 1
C	9	4	8	9-5-2-3-4	8
D	14	3	7	14-13-10-4-3	7
E	13	6	6	13-10-8-6	6
F	10	1	8	10-4-3-2-1	8
G	10	9	4	10-13-14-11-12-9 10-4-3-2-5-9 10-11-12-9	2 1 1
H	11	6	5	11-10-8-6 11-12-9-5-2-7-6	4 1
I	9	10	20	9-11-10 9-12-11-10	18 2
J	4	11	10	4-10-11 4-3-2-5-9-11	9 1

We did not run our heuristic on scenario 5.

Overall Comparison of Heuristic vs Optimal Solution

In Phase I of this project, we created a heuristic and ran it on scenarios 2 and 3. Overall, our heuristic performed fairly well, obtaining results close to the optimal solution, even exactly matching the optimal solution for Scenario 2 when reloading buildings cost $500,000. In each of the other scenarios, the solution found by our heuristic was more expensive than the optimal solution by between $200,000 and $700,000.

Timing Summary

The solving process for scenarios 1, 2, and 3 each took a mere fraction of a second and were often solved with zero branch-and-bound nodes, largely because the network was small and many of the shipments could coexist on the same track. Additionally, the largest shipment by volume could frequently be routed at no cost, and there were no cases where the number of additional reloading buildings at stations or tracks had to be increased by more than 2 to accommodate demand.

Scenarios 4 and 5 required more time to solve because the network was much larger. Scenario 4 required several seconds and thousands of branch-and-bound nodes to solve. Scenario 5 (which had 4 more shipments than scenario 4) required about two minutes to solve when reloading buildings cost $1 million, and about 10 seconds to solve when reloading buildings cost $500,000.

Clearly, as the network and number of shipments grow, the time required to solve the problem increases. Given that scenario 5 still only had 10 shipments, which is a relatively small network in the real world, it is easy to imagine that a network with hundreds or thousands of shipments could require hours or days to find an optimal solution.