Assignment 1

Reading Assignment:

1. Section 1.1: Definition and Basic Properties
2. Section 1.2: Class Structure

Problems:

1. Let $B_1, B_2, \ldots$ be disjoint events with $\bigcup_{n=1}^{\infty} B_n = \Omega$. Show that if A is another event and $\Pr(A|B_n) = p$ for all n then $\Pr(A) = p$.

2. Suppose that $\{X_n\}_{n\geq 0}$ is Markov (λ, P). If $Y_n = X_{kn}$, show that $\{Y_n\}_{n\geq 0}$ is Markov (λ, P^k).

3. Let X_0 be a random variable with values in a countable set I. Let $Y_1, Y_2, \ldots$ be a sequence of independent random variables, uniformly distributed on $[0, 1]$. Suppose we are given a function
$$G : I \times [0, 1] \to I$$
and define inductively
$$X_{n+1} = G(X_n, Y_{n+1}).$$
Show that $\{X_n\}_{n\geq 0}$ is a Markov chain and express its transition matrix P in terms of G. Can all Markov chains be realized in this way? How would you simulate a Markov chain using a computer?

4. A flea hops about at random on the vertices of a triangle, with all jumps equally likely. Find the probability that after n hops the flea is back where it started.

   A second flea also hops about the vertices of a triangle, but this flea is twice as likely to jump clockwise as anticlockwise. What is the probability that after n hops this second flea is back where it started? [Recall that $e^{\pm i\pi/6} = \sqrt{3}/2 \pm i/2$.]

5. A die is 'fixed' so that each time it is rolled the score cannot be the same as the preceding score, all other scores having probability 1/5. If the first score is a 6, what is the probability p that the nth score is a 6? What is the probability that the nth score is a 1?