Assignment 2

Reading Assignment:

1. Section 1.3: Hitting Times and Absorption Probabilities
2. Section 1.4: Strong Markov Property

Problems:

1. A gambler has \$2 and needs to increase it to \$10 in a hurry. He can play a game with the following rules: a fair coin is tossed; if a player bets on the right side, he wins a sum equal to his stake, and his stake is returned; otherwise he loses his stake. The gambler decides to use a bold strategy in which he stakes all his money if he has \$5 or less, and otherwise stakes just enough to increase his capital, if he wins, to \$10.

   Let $X_0 = 2$ and let X_n be his capital after n throws. Prove that the gambler will achieve his aim with probability 1/5. What is the expected number of tosses until the gambler either achieves his aim or loses his capital?

2. Let $\{X_n\}_{n\geq 0}$ be a Markov chain on $\{0, 1, \ldots\}$ with transition probabilities given by

   $$p_{01} = 1,\ p_{i,i+1} + p_{i,i-1} = 1,\ p_{i,i+1} = \left(\frac{i+1}{i}\right)^2 p_{i,i-1},\ i \geq 1.$$

   Show that if $X_0 = 0$ then the probability that $X_n \geq 1$ for all $n \geq 1$ is $6/\pi^2$. **Hint:** Read section 1.3 carefully.

3. Let $Y_1, Y_2, \ldots$ be independent identically distributed random variables with $\Pr(Y_1 = 1) = \Pr(Y_1 = -1) = 0.5$ and set $X_0 = 1, X_n = X_0 + Y_1 + \cdots + Y_n$ for $n \geq 1$. Define

   $$H_0 = \inf\{n \geq 0 : X_n = 0\}.$$

   Use the strong Markov property to find the generating function $\phi(s) = \mathrm{E}_1\left(s^{H_0}\right)$, $0 \leq s \leq 1$.