

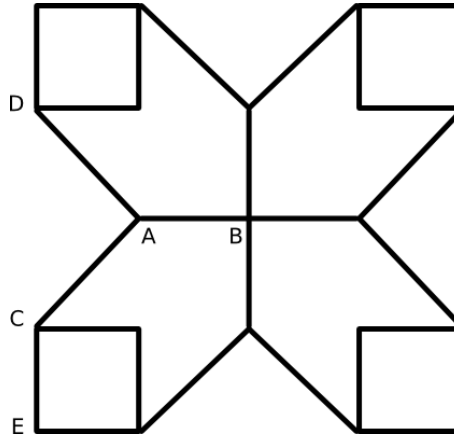
Assignment 5

Reading Assignment:

1. Section 1.9: Time Reversal
2. Section 1.10: Ergodic Theorem

Problems:

1. Two particles X and Y perform independent random walks on the graph shown in the diagram. So, for example, a particle at A jumps to B , C , or D with equal probability $1/3$.



Find the probability that X and Y ever meet at a vertex in the following cases:

- (a) X starts at A and Y starts at B ;
- (b) X starts at A and Y starts at E .

For $I = B, D$, let M_I denote the expected time, when both X and Y start at I , until they are once again both at I . Show that $9M_D = 16M_B$.

2. A professor has N umbrellas. He walks to the office in the morning and walks home in the evening. If it is raining he likes to carry an umbrella and if it is fine he does not. Suppose that it rains on each journey with probability p , independently of past weather. What is the long-run proportion of journeys on which the professor gets wet?
3. Let $\{X_n\}_{n \geq 0}$ be an irreducible Markov chain on I having an invariant distribution π . For $J \subseteq I$ let $\{Y_m\}_{m \geq 0}$ be the Markov chain on J obtained by observing $\{X_n\}_{n \geq 0}$ whilst in J . (See Example 1.4.4.) Show that $\{Y_m\}_{m \geq 0}$ is positive recurrent and find its invariant distribution.