Assignment 4

Reading Assignment:

1. Section 1.7: Invariant Distributions
2. Section 1.8: Convergence to Equilibrium

Problems:

1. Let $\{X_n\}_{n\geq 0}$ be a simple random walk on $\mathbb{Z}$ with $p_{i,i-1} = q < p = p_{i,i+1}$. Find

$$\gamma_i^0 = \mathrm{E}_0\left(\sum_{n=0}^{T_0-1} \mathbf{1}_{\{X_n=1\}}\right)$$

and verify that

$$\gamma_i^0 = \inf_{\lambda} \lambda_i \quad \forall i$$

where the infimum is taken over all invariant measures λ with $\lambda_0 = 1$.

2. A fair die is thrown repeatedly. Let X_n denote the sum of the first n throws. Find

$$\lim_{n\to\infty} \Pr(X_n \text{ is a multiple of } 13)$$

quoting carefully any general theorems that you use.

3. Each morning a student takes one of the three books he owns from his shelf. The probability that he chooses book i is α_i, where $0 < \alpha < 1$ for $i = 1, 2, 3$ and choices on successive days are independent. In the evening he replaces the book at the left-hand end of the shelf. If p_n denotes the probability that on day n the student finds the books in order $1, 2, 3$ from left to right, show that, irrespective of the initial arrangement of the books, p_n converges as $n \to \infty$, and determine the limit.