

Q1

(a) According to service law:

Service demand  $D(j) = U(j)/X(0)$

$X(0) = 1231/(30 \times 60) = 0.6839$

$U(\text{Disk1})$

Service demand at each device:

device	D(j)	utilization
Disk1	754ms	0.516
Disk2	826ms	0.565
Disk3	1028ms	0.703
CPU	895ms	0.612

(b)

Using bottleneck analysis to determine the asymptotic bound on the system:

When there are 40 active terminals

Disk2 has highest service demand 1030ms.

The maximum system throughput is  $1/1030 = 0.97$  jobs/s

According to the throughput bound:

$$X(0) \leq \min \left[ \frac{1}{\max(D_i)}, \frac{N}{\sum_{i=1}^k D_i + \text{thinktime}} \right]$$

Assume  $t_0$  is the crosspoint of the two boundary lines.

$$1/(0.76 + 0.838 + 1.03 + 0.897 + 27)t_0 = 0.97$$

$$1/30.525 \times t_0 = 0.97$$

$$t_0 = 29.6$$

Since  $N = 40 > 29.6$

Therefore the asymptotic bound on the system throughput is 0.97 jobs/s

(c)

When the number of terminal is 40

response time + thinking time = number of terminals / system throughput

Therefore response time =  $40/0.97 - 27 = 14.23$ s

Q2

(a)

According to queuing theory:

The possibility that an incoming arrival is blocked = Probability that there are  $m$  customers in the system.

In this case,  $m = 4$

Service rate =  $1/\text{mean service time} = 1/10$ min

Therefore,

$$\lambda = 20/60 = 1/3 (\text{transactions/min})$$

$$\mu = 1/10 (\text{transactions/min})$$

$$\rho = \frac{\lambda}{\mu} = 10/3$$

$$P_m = \frac{\frac{\rho^m}{m!}}{\sum_{k=0}^m \frac{\rho^k}{k!}}$$

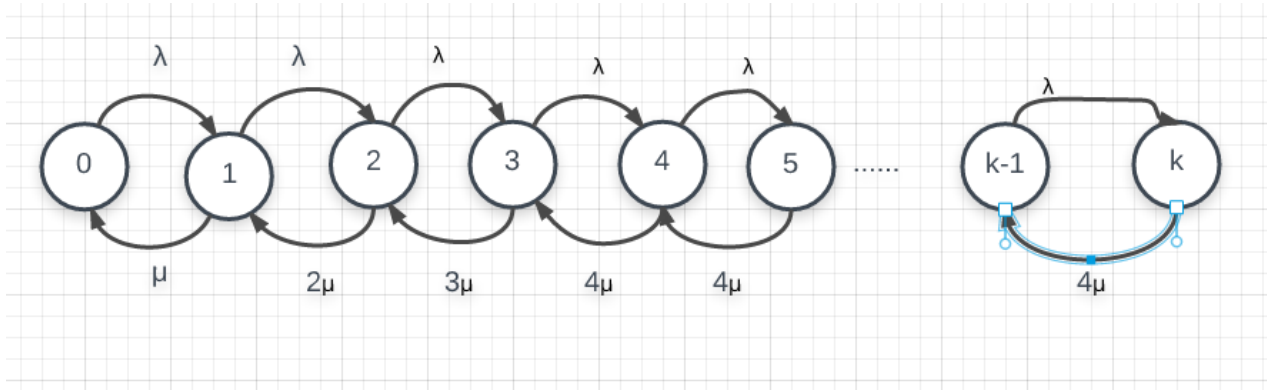
Possibility that an incoming call is rejected  $P_m = 0.2425$

(b)

(i)

Decrease the loss rate less than 50% means the loss rate is less than  $0.2425 \cdot 0.5 = 0.12125$

Formulate Markov chain for 4 operators and M holding slots as follows:



Definition of state:

State=0 :all operators are idle

State=1 only one operators is busy

State=2 only 2 operators are busy

State=3 only 3 operators are busy

State=4 All operators are busy

State=5 All operators are busy, 1 task in queue

...

State=k All operators are busy, k-4 tasks in queue

(ii)

balance equation:

$$\lambda P_0 = \mu P_1$$

$$\lambda P_0 + 2\mu P_2 = (\mu + \lambda) P_1$$

$$\lambda P_1 + 3\mu P_3 = (2\mu + \lambda) P_2$$

$$\lambda P_2 + 4\mu P_4 = (3\mu + \lambda) P_3$$

$$\lambda P_3 + 4\mu P_5 = (4\mu + \lambda) P_4$$

$$\lambda P_4 + 4\mu P_6 = (4\mu + \lambda) P_5$$

...

$$4\mu P_k = \lambda P_{k-1}$$

(iii)

$$k = M + 4$$

$$P_1 = \rho P_0$$

$$P_2 = \rho^2 / 2 P_0$$

$$P_3 = \rho^3 / 6 P_0$$

$$P_4 = \rho^4 / 24 P_0$$

$$P_5 = \rho^4 / 24 * \rho / 4 P_0$$

$$P_6 = \rho^4 / 24 * (\rho / 4)^2 P_0$$

...

$$P_k = \rho^4 / 24 * (\rho / 4)^{k-4} P_0$$

$$(1 + \rho + \rho^2 / 2 + \rho^3 / 6 + \rho^4 / 24 + (\rho^4 / 24) \sum_{i=5}^k ((\rho / 4)^{k-i})) P_0 = 1$$

$$P_0 = \frac{1}{1 + \rho + \rho^2 / 2 + \rho^3 / 6 + \rho^4 / 24 + (\rho^4 / 24) \sum_{i=5}^k ((\rho / 4)^{k-i})}$$

(iv)

Steady state probabilities

$$P_0 = 0.0312$$

$$P_1 = 0.104$$

$$P_2 = 0.173$$

$$P_3 = 0.193$$

$$P_4 = 0.161$$

$$P_5 = 0.134$$

$$P_6 = 0.111$$

$$P_7 = 0.0929$$

Smallest value of M is 3

Using python code to calculate.

(v)

The time an accepted call will be wait :

$$N_{avg} = 3.659$$

$$\text{throughput} = 20/h$$

Using little's law:

$$\text{responsetime} = 3.659 / 20 (1 - P_7) = 0.20h$$

$$\text{mean service time} = 1/6 h$$

$$\text{wating time} = \text{response time} - \text{service time} = 0.035h$$

Q3

(a)

A list of 3-tuple states:

$$(\#CPU1, \#CPU2, \#Disk)$$

$$(3, 0, 0)$$

$$(2, 1, 0)$$

$$(1, 2, 0)$$

$$(0, 3, 0)$$

$$(2, 0, 1)$$

$$(1, 1, 1)$$

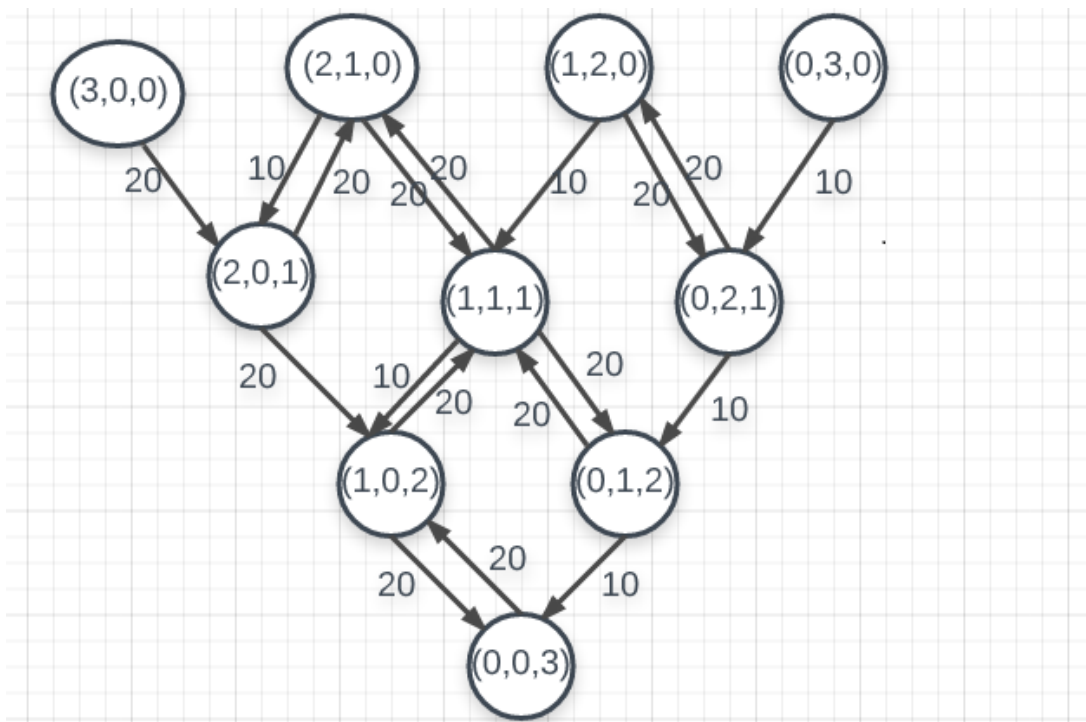
$$(0, 2, 1)$$

$$(1, 0, 2)$$

$$(0, 1, 2)$$

$$(0, 0, 3)$$

The transition rate between states:



(b)

A=

$$\begin{pmatrix}
 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 30 & 0 & 0 & -20 & -20 & 0 & 0 & 0 & 0 \\
 0 & 0 & 30 & 0 & 0 & 0 & -20 & 0 & 0 & 0 \\
 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\
 -20 & -10 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 \\
 0 & -20 & -10 & 0 & 0 & 50 & 0 & -20 & -20 & 0 \\
 0 & 0 & -20 & -10 & 0 & 0 & 30 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & -20 & -10 & 0 & 40 & 0 & -10 \\
 0 & 0 & 0 & 0 & 0 & -20 & -10 & 0 & 30 & 0 \\
 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
 \end{pmatrix} \quad (1)$$

x=

$$\begin{pmatrix}
 P(3,0,0) \\
 P(2,1,0) \\
 P(1,2,0) \\
 P(0,3,0) \\
 P(2,0,1) \\
 P(1,1,1) \\
 P(0,2,1) \\
 P(1,0,2) \\
 P(0,1,2) \\
 P(0,0,3)
 \end{pmatrix} \quad (2)$$

B=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

(3)

Balance equation:

$$A * x = B$$

(c)

$P(3,0,0)=0$   
 $P(2,1,0)=0.1579$   
 $P(1,2,0)=0$   
 $P(0,3,0)=0$   
 $P(2,0,1)=0.0395$   
 $P(1,1,1)=0.1974$   
 $P(0,2,1)=0$   
 $P(1,0,2)=0.2039$   
 $P(0,1,2)=0.1316$   
 $P(0,0,3)=0.2697$

(d)

Throughput=Utilisation\*Service rate

Disk utilisation= $P(2,0,1)+P(1,1,1)+P(0,2,1)+P(1,0,2)+P(0,1,2)+P(0,0,3)=0.8421$

Throughput= $0.8421*20=16.842$  transactions/s

(e)

Use Little's Law

CPU1 utilization= $0.1579 + 0.0395 + 0.1974 + 0.2039 = 0.5987$

CPU1 throughput=utilization\*service rate= $0.5987*20 = 11.974$  transactions/s

$N=(0.1579+0.0395)*2+(0.1974+0.2039)*1=0.7961$

Response time of CPU1:

$R=N/x$  with  $N=0.7961$

$=0.7961/11.974=66\text{ms}$

(f)

Using little's law

mean jobs in Disk= $(0.0395+ 0.1974)*1+(0.2039+0.1316)*2+0.2697*3=1.717$

The time user has to wait:

$R=N/x=1.717/16.842=102\text{ms}$

waiting time=response time -service time= $102-50=52\text{ms}$

