

ECE2800J

Programming and Introductory Data Structures

Recursion; Function Pointers; Function Call Mechanism

Learning Objectives:

Understand recursion and know how to write recursive functions

Understand how to write more general code with function pointers

Understand function call mechanism

Outline

- Recursion
- Function Pointers
- Function Call Mechanism

Recursion

- Recursion is a nice way to solve problems
 - “Recursive” just means “refers to itself”.
 - There is (at least) one “trivial” base or “stopping” case.
 - All other cases can be solved by first solving one smaller case, and then combining the solution with a simple step.
- Example: calculate factorial $n!$

$$n! = \begin{cases} 1 & n = 0 \\ n \cdot (n-1)! & n > 0 \end{cases}$$

```
int factorial (int n) {  
    // REQUIRES: n >= 0  
    // EFFECTS:  computes n!  
    if (n == 0) return 1; // base case  
    else return n*factorial(n-1); // recursive step  
}
```

Recursive Helper Function

- Sometimes it is easier to find a recursive solution to a problem if you change the original problem slightly, and then solve that problem using a **recursive helper function**.

```
soln()  
{  
    ...  
    soln_helper();  
    ...  
}
```

```
soln_helper()  
{  
    ...  
    soln_helper();  
    ...  
}
```

Recursive Helper Function

Example

- A palindrome is a string that is equal to itself when you reverse all characters.
 - For example: rotor, racecar

- Write a function to test if a string is a palindrome.

```
bool is_palindrome(string s);  
// EFFECTS: return true if s is  
// a palindrome.
```

Palindrome Example

- If a string is empty, it is a palindrome.
- If a string is of length one, it is a palindrome.
- Given a string of length more than one, it is a palindrome, if
 - its first character equals its last one, **and**
 - the substring without the first and the last characters is a palindrome.
- In order to test whether a **substring** is a palindrome, we define a **helper** function

```
bool is_palindrome_helper(string s,  
    int begin, int end);  
// EFFECTS: return true if the substring  
// of s starting at begin and ending at  
// end is a palindrome.
```

Palindrome Example

```
bool is_palindrome_helper(string s,  
    int begin, int end)  
// EFFECTS: return true if the substring  
// of s starting at begin and ending at  
// end is a palindrome.  
{  
    if(begin >= end) return true;  
    if(s[begin] == s[end])  
        return is_palindrome_helper(s,  
            begin+1, end-1);  
    else return false;  
}
```

Palindrome Example

- With the helper function, `is_palindrome()` can be realized as

```
bool is_palindrome(string s)
// EFFECTS: return true if s is
// a palindrome.
{
    return is_palindrome_helper(s, 0,
                                s.length()-1);
}
```


Outline

- Recursion
- **Function Pointers**
- Function Call Mechanism

Function Pointers

Motivation

- If you were asked to write a function to add all the elements in a list, and another to multiply all the elements in a list, your functions would be almost exactly **the same**.
- Writing almost the exact same function twice is a bad idea!

Why?

1. It's wasteful of your time!!
2. If you find a better way to implement some common parts, you have to change **many different** places; this is prone to error.

Our Example: list_t type

- A list can hold a sequence of zero or more integers.
- There is a recursive definition for the values that a list can take:
 - A valid list is:
either an empty list
or an integer followed by another valid list

Function Pointers

Background on lists

- Here are some examples of valid lists:

```
( 1 2 3 4 ) // a list of four elements  
( 2 5 2 )   // a list of three elements  
( )         // an empty list
```

- There are also several operations that can be applied to lists. We will use the following three:
 - `list_first()` takes a list, and returns the first element (an integer) from the list. **REQUIRES: non-empty list!**
 - `list_rest()` takes a list and returns the list comprising all but the first element. **REQUIRES: non-empty list!**
 - `list_isEmpty()` takes a list and returns the Boolean “true” if the argument is an empty list, and “false” otherwise.

Function Pointers

Using lists

- Suppose we want to write a **recursive** function to find the smallest element in a list.
 - The function requires the input list to be non-empty.

Question: how do you do it **recursively**?

- **Answer:**

`smallest(list) = the element (if list has only a single element)`
`or the minimum of the first element and the smallest element from the rest of the list`

Function Pointers

Using recursion to find the smallest element in a list

```
int smallest(list_t list)
    // REQUIRES: list is not empty
    // EFFECTS:  returns smallest element
    // in the list
{
    int first = list_first(list);
    list_t rest = list_rest(list);
    if(list_isEmpty(rest)) return first;
    int cand = smallest(rest);
    if(first <= cand) return first;
    return cand;
}
```

Function Pointers

Using lists

- Now suppose we want to write a recursive function to find the largest element in a list.
 - The function also requires the input list to be non-empty.
- Recursive definition:
`largest(list)` = the element (if list has only a single element)
or the maximum of the first element and the largest element from the rest of the list

Function Pointers

Using recursion to find the largest element in a list

```
int largest(list_t list)
// REQUIRES: list is not empty
// EFFECTS:  returns largest element
// in the list
{
    int first = list_first(list);
    list_t rest = list_rest(list);
    if(list_isEmpty(rest)) return first;
    int cand = largest(rest);
    if(first >= cand) return first;
    return cand;
}
```


Function Pointers

More Motivation

- `largest` is almost identical to the definition of `smallest`.
- Unsurprisingly, the solution is almost identical, too.
- In fact, the **only** differences between `smallest` and `largest` are:
 1. The names of the function
 2. The comment in the EFFECTS list
 3. The polarity of the comparison: `<=` vs. `>=`
- It is silly to write almost the same function twice!

Function pointers to rescue!

Function Pointers

A first look

- So far, we've only defined functions as entities that can be called. However, functions can also be referred to by **variables**, and passed as **arguments** to functions.
- Suppose there are two functions we want to pick between: `min()` and `max()`. They are defined as follows:

```
int min(int a, int b);  
    // EFFECTS: returns the smaller of a and b.  
int max(int a, int b);  
    // EFFECTS: returns the larger of a and b.
```

Function Pointers

A first look

```
int min(int a, int b);  
    // EFFECTS: returns the smaller of a and b.  
int max(int a, int b);  
    // EFFECTS: returns the larger of a and b.
```

- These two functions have precisely the same type signature:
 - They both take two integers, and return an integer.
- Of course, they do completely different things:
 - One returns a min and one returns a max.
 - **However, from a syntactic point of view, you call either of them the same way.**

Function Pointers

The basic format

- How do you define a **variable** that points to a function that takes two integers, and returns an integer?

- Here's how:

```
int    (*foo) (int, int);
```

- You read this from "inside out". In other words:

<code>foo</code>	“foo”
<code>(*foo)</code>	“is a pointer”
<code>(*foo) (</code>	“to a function”
<code>(*foo) (int, int);</code>	“that takes two integers”
<code>int (*foo) (int, int);</code>	“and returns an integer”

Function Pointers

The basic format

```
int    (*foo) (int, int);
```

- Once we've declared foo, we can **assign** any function with the same type signature to it:

```
foo = min;
```

- Furthermore, after assigning min to foo, we can just call it as follows:

```
foo(3, 5)
```

- ...and we'll get back 3!

Function Pointers v.s. Variable Pointers

- For function pointers, the compiler allows us to **ignore** the “**address-of**” and “**dereference**” operators.

```
int (*foo)(int, int);  
foo = min; // min() is predefined  
foo(5, 3);
```

We don't write:

```
foo = &min;  
(*foo)(5, 3);
```

- In contrast, for variable pointers:

```
int foo;  
int *bar;  
bar = &foo;  
*bar = 2;
```

Function Pointers

Re-write `smallest` in terms of function pointers

```
int compare_help(list_t list, int (*fn)(int, int))
{
    int first = list_first(list);
    list_t rest = list_rest(list);
    if(list_isEmpty(rest)) return first;
    int cand = compare_help(rest, fn);
    return fn(first, cand);
}

int smallest(list_t list)
    // REQUIRES: list is not empty
    // EFFECTS: returns smallest element in list
{
    return compare_help(list, min);
}
```

```
int min(int a, int b);
    // EFFECTS: returns the
    // smaller of a and b.
```

Function Pointers

Re-write `largest` in terms of function pointers

```
int compare_help(list_t list, int (*fn)(int, int))
{
    int first = list_first(list);
    list_t rest = list_rest(list);
    if(list_isEmpty(rest)) return first;
    int cand = compare_help(rest, fn);
    return fn(first, cand);
}

int largest(list_t list)
    // REQUIRES: list is not empty
    // EFFECTS: returns largest element in list
{
    return compare_help(list, max);
}
```

```
int max(int a, int b);
    // EFFECTS: returns the
    // larger of a and b.
```


Outline

- Recursion
- Function Pointers
- **Function Call Mechanism**

Call Stacks

How a function call really works

- When we call a function, the program does following steps:
 1. Evaluate the actual arguments to the function (order is not guaranteed).

Example: `y = add(4-1, 5);`
 2. Create an “**activation record**” (sometimes called a “**stack frame**”) to hold the function's **formal parameters** and **local variables**.
 - When call function `int add(int a, int b)`, system creates an activation record:

`a, b (formal), result (local)`
 3. Copy the actuals' values to the formals' storage space.

`a=3`
`b=5`
 4. Evaluate the function in its local scope.
 5. Replace the function call with the result.

`y=8`
 6. Destroy the activation record.

Call Stacks

How a function call really works

- It is typical to have multiple function calls. How the activation records are maintained?
 - Answer: stored as a **stack**.
- Stack: a set of objects which is modified as **last in first out**.
Example: a stack of plates in a cafeteria
 - Each time you clean a plate, you add it to the top of the stack
 - Each time a new plate is needed, the one at the top is taken **first**



Call Stacks

How a function call really works

- When a function $f()$ is called, its **activation record** is added to the “top” of the stack.
- When the function $f()$ returns, its **activation record** is removed from the “top” of the stack.
- In the meantime, $f()$ may have called **other functions**.
 - **These functions** create corresponding activation records.
 - **These functions** must return (and destroy their corresponding activation records) before $f()$ can return.

Call Stacks

Example

- When a function is called, its **activation record** is added to the “top” of the stack.
- When that function returns, its **activation record** is removed from the “top” of the stack.



double add(double a, double b): a = 1, b = 0, result = 0

double sin(double x): x = 1, result = 0

int main(): x = 1, sinResult = 0

- Note: “top” is placed in quotes, because in reality, stack of activation records grows **down** rather than **up**.

Call Stacks

Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

Call Stacks

Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

Main starts out with an activation record with room only for the local “result”:

main:

result: 0

Call Stacks

Example

```
int plus_one(int x) {  
    return (x+1);  
}  
  
int plus_two(int x) {  
    return (1 + plus_one(x));  
}  
  
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

Then, main calls plus_two,
passing the literal value "0":

main:

result: 0

plus_two:

x: 0

Call Stacks

Example

```
int plus_one(int x) {  
    return (x+1);  
}  
  
int plus_two(int x) {  
    return (1 + plus_one(x));  
}  
  
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

Which in turn calls plus_one:

main:

result: 0

plus_two:

x: 0

plus_one:

x: 0

Call Stacks

Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

plus_one adds one to x,
returning the value 1:

main:

result: 0

plus_two:

x: 0

plus_one:

x: 0



Call Stacks

Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

plus_one's activation record
is destroyed:

main:

result: 0

plus_two:

x: 0



~~plus_one:~~

x: 0

Call Stacks

Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

plus_two adds one to the result,
and returns the value 2:

main:

result: 2



plus_two:

x: 0

Call Stacks

Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

plus_two's activation record
is destroyed:

main:

result: 2

2

plus_two:

x: 0

Call Stacks

Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

main then prints the result:

2

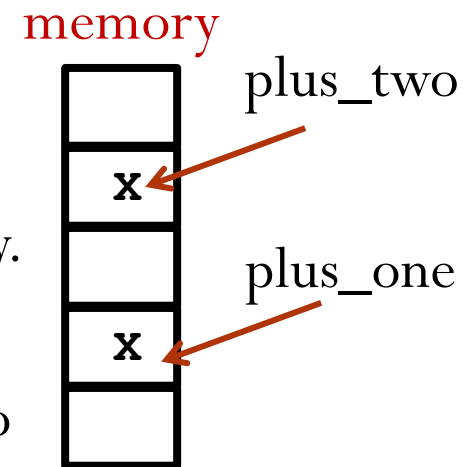
main:

result: 2

Call Stacks

Example: Some things to note

- Even though `plus_one` and `plus_two` both have formal parameters called “x”, there is no problem.
 - These two x’s are at different locations in memory.
 - `plus_one` cannot see `plus_two`'s x.
 - Instead, the **value** of `plus_two`'s x is passed to `plus_one`, and stored in `plus_one`'s x.



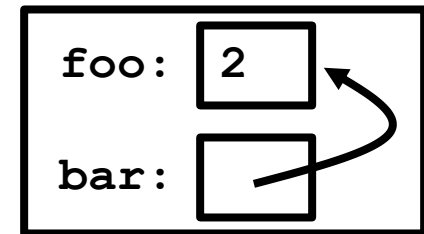
Call Stack

Example: Using Pointers

```
void add_one(int *x) {  
    *x = *x + 1;  
}
```

```
int main() {  
    int foo = 2;  
    int *bar = &foo;  
    add_one(bar);  
    return 0;  
}
```

Activation record of main:



Call Stack

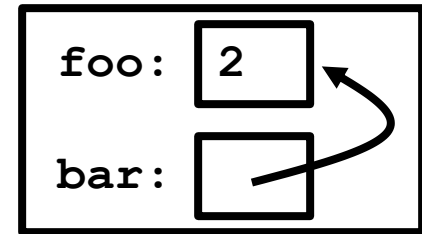
Example: Using Pointers

```
void add_one(int *x) {  
    *x = *x + 1;  
}
```

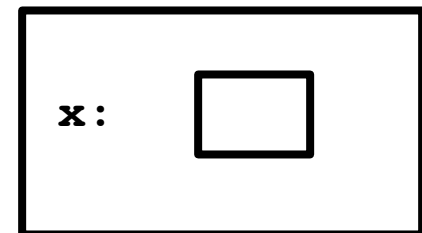
```
int main() {  
    int foo = 2;  
    int *bar = &foo;  
    add_one(bar);  
    return 0;  
}
```

Main calls `add_one`,
creating an activation
record for `add_one`

main:



add_one:



Call Stack

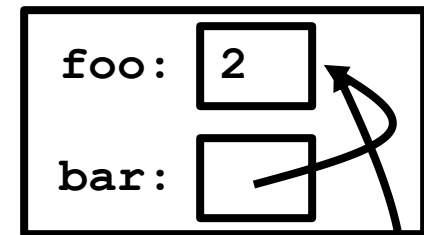
Example: Using Pointers

```
void add_one(int *x) {  
    *x = *x + 1;  
}
```

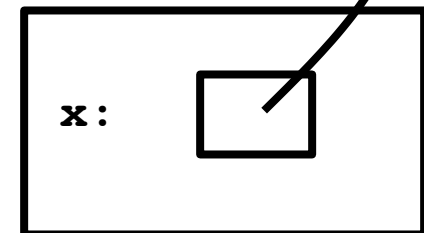
```
int main() {  
    int foo = 2;  
    int *bar = &foo;  
    add_one(bar);  
    return 0;  
}
```

Copy the value of bar to add_one's formal parameter x.

main:



add_one:



Both x and bar point to foo.

Call Stack

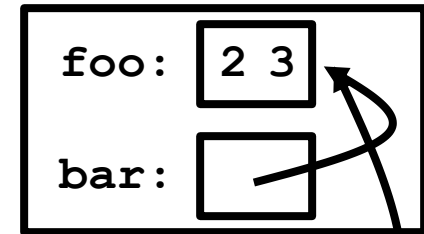
Example: Using Pointers

```
void add_one(int *x) {  
    *x = *x + 1;  
}
```

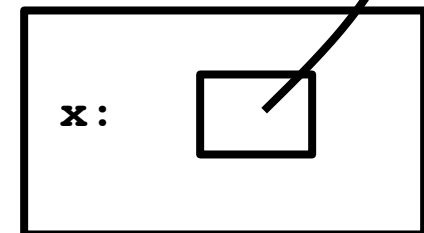
```
int main() {  
    int foo = 2;  
    int *bar = &foo;  
    add_one(bar);  
    return 0;  
}
```

add_one adds 1 to the object pointed to by x.

main:



add_one:



Call Stack

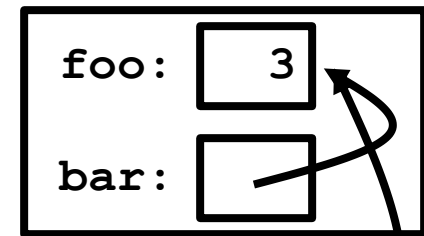
Example: Using Pointers

```
void add_one(int *x) {  
    *x = *x + 1;  
}
```

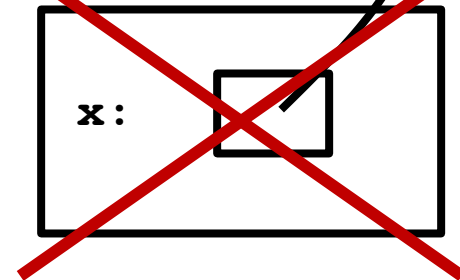
```
int main() {  
    int foo = 2;  
    int *bar = &foo;  
    add_one(bar);  
    return 0;  
}
```

add_one's activation record is destroyed.

main:



add_one:



Call Stack

Example: Recursion

main

x:

- Suppose we call our function as follows:

```
void main()
```

1. {
2. int x;
3. x = factorial(3);
4. }

```
int factorial (int n) {  
1. if (n == 0) return 1;  
2. else return n*factorial(n-1);  
}
```

Call Stack

Example: Recursion

- `main()` calls `factorial` with an argument 3.
- We evaluate the actual argument, create an activation record, and copy the actual value to the formal.

main

x:

factorial

n:

RA: main line #3

RA = "Return Address"

```
int factorial (int n) {  
1. if (n == 0) return 1;  
2. else return n*factorial(n-1);  
}
```

Call Stack

Example: Recursion

- Now we evaluate the body of factorial:
 - n is not zero, so we evaluate the **else** arm of the if statement:
 $\text{return } 3 * \text{factorial}(2)$
 - So, factorial must call factorial. We will create a **new** activation record for a **new** instance of factorial.

main

x:

factorial

n:

RA: main line #3

factorial

n:

RA: factorial line #2

```
int factorial (int n) {  
1. if (n == 0) return 1;  
2. else return n*factorial(n-1);  
}
```

Call Stack

Example: Recursion

- Again, n is not zero, so we evaluate the **else** arm again:

`return 2 * factorial(1)`

- This creates a new activation record for factorial

```
int factorial (int n) {  
1. if (n == 0) return 1;  
2. else return n*factorial(n-1);  
}
```

main

x:

factorial

n:

RA: main line #3

factorial

n:

RA: factorial line #2

factorial

n:

RA: factorial line #2

Call Stack

Example: Recursion

- And again, we evaluate the **else** arm:

return 1*factorial(0)

- This creates a new activation record for factorial

```
int factorial (int n) {  
1. if (n == 0) return 1;  
2. else return n*factorial(n-1);  
}
```

main

x:

factorial

n:

RA: main line #3

factorial

n:

RA: factorial line #2

factorial

n:

RA: factorial line #2

factorial

n:

RA: factorial line #2

Call Stack

Example: Recursion

- In evaluating factorial(0), n is zero, so we evaluate the **if** arm rather than **else** arm.
- Return the value “1”
- Popping the most recent activation record off the stack.

```
int factorial (int n) {  
1. if (n == 0) return 1;  
2. else return n*factorial(n-1);  
}
```

main

x:

factorial

n:

RA: main line #3

factorial

n:

RA: factorial line #2

factorial

n:

RA: factorial line #2

~~factorial~~

~~n:~~

~~RA: factorial line #2~~

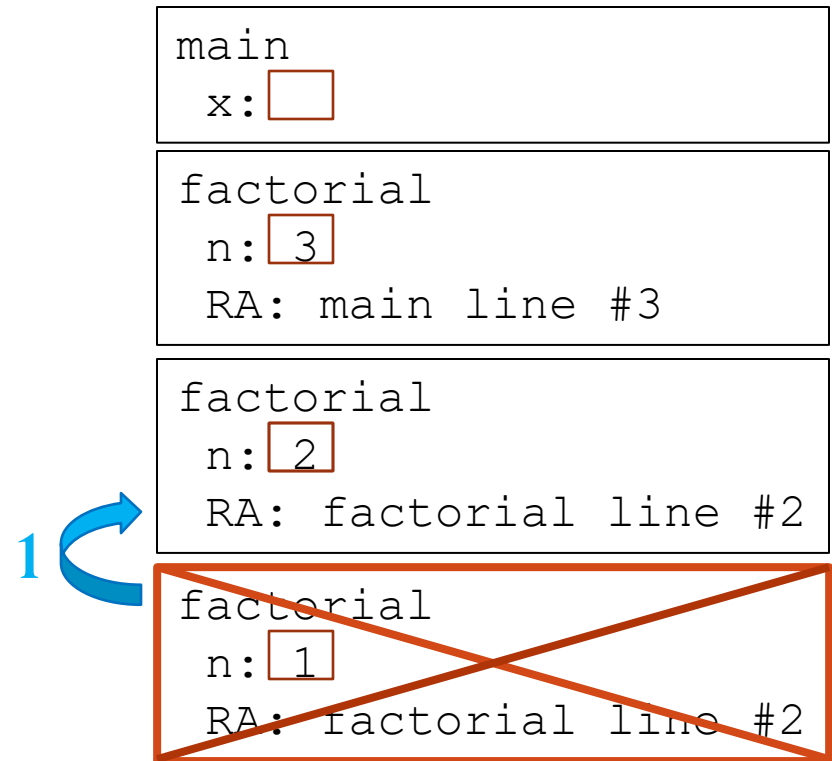
Call Stack

Example: Recursion

- In **factorial(1)**, we called factorial(0) as follows:
 return 1 * factorial(0)
- Now we know the value of factorial(0), so we complete factorial(1):

 return 1 * 1 => return 1;
from factorial(1)

- This pops another activation record off the stack



Call Stack

Example: Recursion

- Now it allows us to complete evaluating **factorial(2)**:

return $2 * \text{factorial}(1)$ \Rightarrow

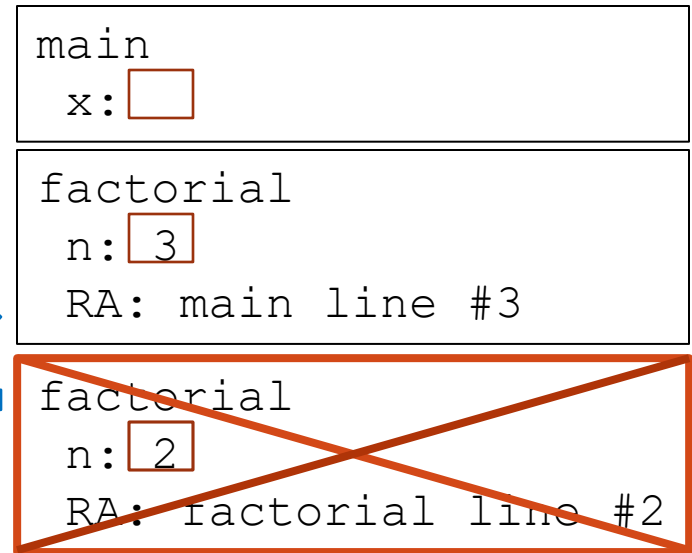
return $2 * 1$ \Rightarrow

return 2

from factorial(2)

- Now pop off another activation record.

2 



Call Stack

Example: Recursion

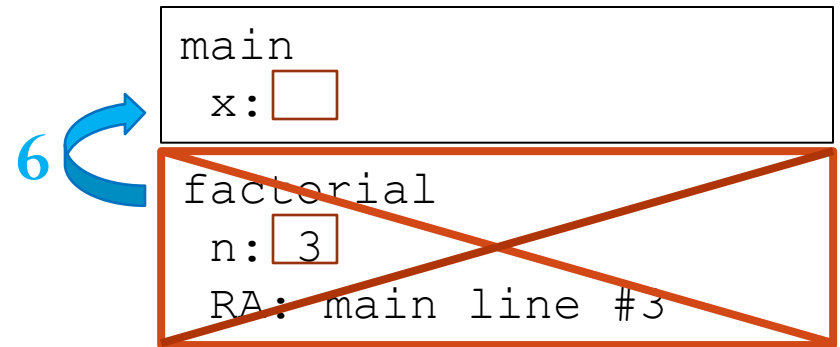
- Now we can complete evaluating **factorial(3)**:

return 3 * factorial(2) =>

return 3 * 2 =>

return 6

- That is the correct answer.
- Don't forget that last pop!





Which Statements Are True?

Select all the correct answers.

- **A.** The number of recursive calls of factorial can be as high as we want.
- **B.** The number of calls of factorial could be just 1.
- **C.** We can change the function factorial so that the number of calls of factorial could be **reduced by 1** in general case.
- **D.** None of the above.

```
int factorial (int n) {  
    if (n == 0) return 1;  
    else return n*factorial(n-1);  
}
```



Reference

- Recursion
 - Problem Solving with C++, 8th Edition, Chapter 14
- Function pointers
 - C++ Primer (4th Edition), Chapter 7.9