Variational Inference for Dirichlet Process to Stratify Cancer Patients Using DNA Methylation

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Abstract

Variational Inference (VI) is an alternative strategy to Markov Chain Monte Carlo (MCMC) which tends to be faster and easier to scale to larger datasets (Blei et al., 2016). This is especially advantageous in applications that interact with high dimensional data. Previous work has been done on a VI method for Dirichlet Process Mixture Models (DPMMs) based on the well-known stick-breaking process (Blei et al., 2016). In contrast, we propose a similar model, but based on the Chinese Restaurant Process (CRP) instead. This model has fewer parameters to estimate; therefore, we hypothesize that our model will perform faster than the model by Blei et al.

DNA methylation is an epigenetic mark that is associated with transcriptional repression and may be closely related to cancer. A common objective is to identify a latent structure shared across cancers from different tissue types reflecting commonly altered gene pathways. These latent structures stratify cancer patients into functionally similar groups and can inform therapy decisions in clinical applications. This study will apply our VI model on DNA methylation data for stratification.

1 Introduction

1.1 DNA Methylation

Cancers develop via the acquisition of genomic changes. These changes in turn alter the cells harbouring them, leading to changes in cell state (phenotype). One common effect of these mutations is to induce a series of modifications to the genome, which do not alter the encoded DNA but rather the ability of the DNA to be read and processed into protein. These heritable non-genetic changes are broadly referred to as epigenetic changes. According to Baylin and Jones (2011), "Epigenetic alterations are leading candidates for the development of specific markers for cancer detection, diagnosis and prognosis". One such epigenetic change is DNA methylation, which is unambiguously linked with transcriptional repression. When present in promoter regions, DNA methylation correlates negatively with gene expression; furthermore, characteristic changes in DNA methylation have been reported for cancer. Research indicates that gene promoter CpG islands acquire abnormal hypermethylation resulting in transcriptional silencing in cancer (Bock, 2012). It is important to note that 70-80% of CpGs in the human genome are affected by DNA methylation. Therefore, understanding the effects of DNA methylation in cancer can provide valuable information for finding an effective remedy for cancer in humans. The potential relationship between DNA methylation and cancer opens up a new avenue of exploration. Advances in next-generation sequencing and microarray technology allow for analysis on DNA methylation in large samples and genome-wide; currently, DNA methylation is the only epigenetic mark that can be measured reliably. As a result,

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DNA methylation can be profiled accurately using high throughput sequencing and is a potential feature for categorizing cancer patients into functionally similar groups.

1.2 Variational Inference

Variational inference is an alternative strategy to Markov Chain Monte Carlo (MCMC) sampling which tends to be faster and easier to scale to larger datasets (Blei et al., 2016). The key characteristic of variational inference is that it casts Bayesian inference as an optimization problem (Salimans et al., 2015). Variational inference attempts to approximate the posterior with another distribution $q_\theta(z|x)$ by choosing its parameters θ to optimize the evidence lower bound (ELBO) on the marginal likelihood,

$$\begin{aligned}\log p(x) &\geq \log p(x) - D_{KL}(q_\theta(z \mid x)||p(z \mid x)) \\ &= E_{q_\theta(z|x)}(\log p(x,z) - \log q_\theta(z \mid x))\end{aligned}$$

In recent years, there have been many advances in the field of VI, which are summarized in a review by Zhang et al., (2019).

1.3 Dirichlet Process and Chinese Restaurant Process

The Dirichlet process is a stochastic process used for Bayesian nonparametric regression; in particular, for constructing Dirichlet process mixture models (Neal, 2000). A Dirichlet process G is a distribution of distributions consisting of a base distribution G_0 and a positive real number α, and can be written as, $G \sim \text{DP}(G_0, \alpha)$, where G_0 is a continuous distribution such that the probability of any two samples generated from this distribution being equal is zero, whereas G is a discrete distribution consisting of infinitely many number of point masses, so the probability of two samples colliding is non-zero. For any measurable finite k partitions $\{B_i\}_{i=1}^k$,

$$G(B_1), \cdots, G(B_k)) \sim \text{Dir}(\alpha G(B_1), \cdots, \alpha G(B_k))$$

then

$$\Pr[\, X_1, \cdots, X_k \,] = \int \Pr[\, G \,] \prod_{i=1}^{k} \Pr[\, X_i \mid G \,]\, dG$$

which represents the dependencies among $\{X_i\}_{i=1}^k$ by marginalizing out G.

The Chinese restaurant process, stick-breaking process and Polya urn scheme are three common perspectives regarding the Dirichlet Process, and only the first method will be focused on in this project. Assuming a restaurant has infinitely many tables, and let $\{X_i\}_{i=1}^k$ be the customers of the restaurant. Then $\{X_i\}_{i=1}^k$ are partitioned determined by the same table the represented customers sitting at. Considering the behavior of X_n given the previous $n-1$ observations:

$$X_n \mid (X_1 = x_1, X_2 = x_2, \cdots, X_{n-1} = x_{n-1}) = \begin{cases} x_n^* & \text{with probability } \frac{\mid \{j : x_j = n\} \mid}{n-1+\alpha} \\ \text{new draw from } G_0 & \text{with probability } \frac{\alpha}{n-1+\alpha} \end{cases}$$

where $\mid \{j : x_j = n\} \mid$ is the number of times the value x_n occurs in $\{x_1, x_2, \cdots x_n\}$.

The fact is that people are more likely to sit at the tables where many people are already sitting, however, the customers will sit at a new table with probability proportional to α.

1.4 Dirichlet Process Mixture Model

The Dirichlet Process Mixture Model (DPMM) is the resulting hierarchical model of applying Dirichlet Process, which was initially proposed for classifying data points into unbounded number of mixture components. Given a sample from $\{x_1, \cdots, x_N\}$ from the DPMM, the goal is to compute the posterior predictive distribution

$$\Pr[\, X = \hat{x} \mid x_1, \cdots, x_N, \alpha, G_0 \,] = \int \Pr[\, \hat{x} \mid x \,] \Pr[\, x \mid x_1, \cdots, x_N, \alpha, G_0 \,]\, dx,$$

where the posterior distribution $\Pr[\, x \mid x_1, \cdots, x_N \,]$ does not have a closed form. The well-known EM algorithm is used for inference in finite by optimizing likelihood but for the finite mixture models, so MCMC and Variational inference are better options as G is nonparametric.

2 Related Work

Blei and Jordan (2006) provided a Variational inference algorithm for Dirichlet Process Mixture Models based on the stick-breaking process, and also compared the mean convergence time consumption among their proposed variational inference algorithm and MCMC (including Collapsed Gibbs sampler and Blocked Gibbs sampler). Let the base distribution G_0 be

$$\Pr[\, x^* \mid \lambda \,] = h(x^*) \exp(\lambda_1^\top x^* + \lambda_2(-a(x^*)) - a(\lambda))$$

where λs' are hyperparameters, and $a(x^*)$ is a cumulant function. Then the target function for prediction, which is constructed on the stick-breaking process, is

$$\Pr[\, x_n \mid z_n, x_1^*, x_2^*, \cdots] = \prod_{i=1}^{\infty} [h(x_n) \exp({x_i^*}^\top x_n - a(x_i^*))]^{\mathbf{1}[z_n = i]},$$

However, this is too complicated to directly compute with. Variational Inference is brought up as an alternative, along with the mean-field variational approximations assuming the independence of latent variables and a derived coordinate ascent algorithm, resulting in a straightforward expression:

$$\Pr[\, x_{N+1} \mid \; x_1, x_2, \cdots, x_{n-1}, \alpha, \lambda \,] \approx \sum_{t=1}^{T} \mathrm{E}[\, \pi_t(\mathbf{v}) \,] \, \mathrm{E}[\, \Pr[\, x_{N+1} \mid x_t^* \,] \,]$$

where q is an approximation to the predict distribution depending on $x_1, x_2, \cdots, \alpha, \lambda$, and each component v_i in $\mathbf{v}$ follows the beta distribution with parameters $(1, \alpha)$.

3 Methods

3.1 Model

We propose a Variational Inference Dirichlet Process Gaussian Mixture Model based on the Chinese Restaurant Process, to stratify cancer patients based on DNA methylation values. This paper will build upon Blei and Jordan's (2006) work on Variational Inference for Dirichlet Process Mixtures. Our method differs from theirs in that we base our model on a Chinese Restaurant Process as opposed to a stick-breaking process. This Variational Inference model will be implemented in the probabilistic programming language Pyro. See Figure 1 for the graphical models for each variant of the model.

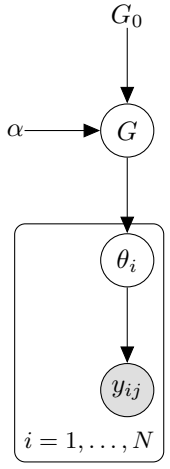

Figure 1: Graphical model for DPMM based on CRP

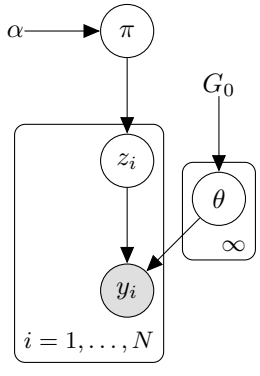

Figure 2: Graphical model for DPMM based on the stick-breaking process

3.2 Plots

We intend to use clustermaps to visualize the main results, as our study is solving a clustering problem. Color bars on the left hand side indicates cancer type and purple squares represent the clusters. The more data points in the cluster, the larger the squares. The intensity of a square describes the proportion of iterations the data point of interest appeared in that cluster. We expect that the clustermaps in this study will resemble Figure 3, because data points should cluster by cancer types.

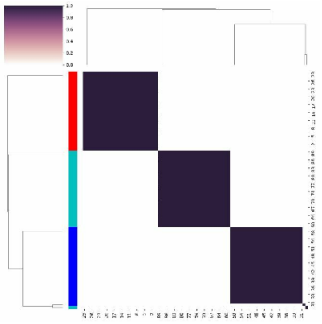

Figure 3: Example of a clustermap. Color labels indicate cancer types: Type I, Type II, Type III

4 Conclusion

Variational Inference (VI) is an alternative strategy to Markov Chain Monte Carlo (MCMC) which tends to be faster and easier to scale to larger datasets (Blei et al., 2016). This is especially advantageous in applications that interact with high dimensional data, such as DNA methylation cluster analysis. We intend to build off the VI for DPMM model based on the stick-breaking process by Blei et al. and implement the same model, based on the Chinese Restaurant Process (CRP); the model will be written in the probabilistic programming language Pyro. We hypothesize that our model will be faster because it contains fewer parameters to estimate. Our model will be applied to a bioinformatic problem - stratifying cancer patients into functionally similar groups using DNA methylation. Consequently, these functionally similar groups can inform therapy decisions in clinical applications.

5 Headings: first level

All headings should be lower case (except for first word and proper nouns), flush left, and bold.

First-level headings should be in 12-point type.

5.1 Headings: second level

Second-level headings should be in 10-point type.

5.1.1 Headings: third level

Third-level headings should be in 10-point type.

Paragraphs There is also a `\paragraph` command available, which sets the heading in bold, flush left, and inline with the text, with the heading followed by 1 em of space.

6 Citations, figures, tables, references

These instructions apply to everyone.

6.1 Citations within the text

The `natbib` package will be loaded for you by default. Citations may be author/year or numeric, as long as you maintain internal consistency. As to the format of the references themselves, any style is acceptable as long as it is used consistently.

The documentation for `natbib` may be found at

`http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf`

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

```
\citet{hasselmo} investigated\dots
```

produces

Hasselmo, et al. (1995) investigated…

If you wish to load the `natbib` package with options, you may add the following before loading the `neurips_2021` package:

```
\PassOptionsToPackage{options}{natbib}
```

If `natbib` clashes with another package you load, you can add the optional argument `nonatbib` when loading the style file:

```
\usepackage[nonatbib]{neurips_2021}
```

As submission is double blind, refer to your own published work in the third person. That is, use "In the previous work of Jones et al. [4]," not "In our previous work [4]." If you cite your other papers that are not widely available (e.g., a journal paper under review), use anonymous author names in the citation, e.g., an author of the form "A. Anonymous."

6.2 Footnotes

Footnotes should be used sparingly. If you do require a footnote, indicate footnotes with a number[1] in the text. Place the footnotes at the bottom of the page on which they appear. Precede the footnote with a horizontal rule of 2 inches (12 picas).

[1] Sample of the first footnote.

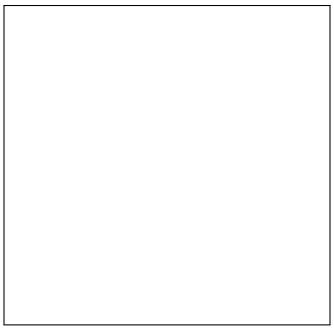

Figure 4: Sample figure caption.

Table 1: Sample table title

	Part	
Name	Description	Size (μm)
Dendrite	Input terminal	$\sim$100
Axon	Output terminal	$\sim$10
Soma	Cell body	up to 10^6

Note that footnotes are properly typeset *after* punctuation marks.[2]

6.3 Figures

All artwork must be neat, clean, and legible. Lines should be dark enough for purposes of reproduction. The figure number and caption always appear after the figure. Place one line space before the figure caption and one line space after the figure. The figure caption should be lower case (except for first word and proper nouns); figures are numbered consecutively.

You may use color figures. However, it is best for the figure captions and the paper body to be legible if the paper is printed in either black/white or in color.

6.4 Tables

All tables must be centered, neat, clean and legible. The table number and title always appear before the table. See Table 1.

Place one line space before the table title, one line space after the table title, and one line space after the table. The table title must be lower case (except for first word and proper nouns); tables are numbered consecutively.

Note that publication-quality tables *do not contain vertical rules.* We strongly suggest the use of the `booktabs` package, which allows for typesetting high-quality, professional tables:

`https://www.ctan.org/pkg/booktabs`

This package was used to typeset Table 1.

7 Final instructions

Do not change any aspects of the formatting parameters in the style files. In particular, do not modify the width or length of the rectangle the text should fit into, and do not change font sizes (except perhaps in the **References** section; see below). Please note that pages should be numbered.

[2] As in this example.

8 Preparing PDF files

Please prepare submission files with paper size "US Letter," and not, for example, "A4."

Fonts were the main cause of problems in the past years. Your PDF file must only contain Type 1 or Embedded TrueType fonts. Here are a few instructions to achieve this.

- You should directly generate PDF files using `pdflatex`.
- You can check which fonts a PDF files uses. In Acrobat Reader, select the menu Files>Document Properties>Fonts and select Show All Fonts. You can also use the program `pdffonts` which comes with `xpdf` and is available out-of-the-box on most Linux machines.
- The IEEE has recommendations for generating PDF files whose fonts are also acceptable for NeurIPS. Please see `http://www.emfield.org/icuwb2010/downloads/IEEE-PDF-SpecV32.pdf`
- `xfig` "patterned" shapes are implemented with bitmap fonts. Use "solid" shapes instead.
- The `\bbold` package almost always uses bitmap fonts. You should use the equivalent AMS Fonts:

  ```
  \usepackage{amsfonts}
  ```

  followed by, e.g., `\mathbb{R}`, `\mathbb{N}`, or `\mathbb{C}` for $\mathbb{R}$, $\mathbb{N}$ or $\mathbb{C}$. You can also use the following workaround for reals, natural and complex:

  ```
  \newcommand{\RR}{I\!\!R} %real numbers
  \newcommand{\Nat}{I\!\!N} %natural numbers
  \newcommand{\CC}{I\!\!\!\!C} %complex numbers
  ```

  Note that `amsfonts` is automatically loaded by the `amssymb` package.

If your file contains type 3 fonts or non embedded TrueType fonts, we will ask you to fix it.

8.1 Margins in LaTeX

Most of the margin problems come from figures positioned by hand using `\special` or other commands. We suggest using the command `\includegraphics` from the `graphicx` package. Always specify the figure width as a multiple of the line width as in the example below:

```
\usepackage[pdftex]{graphicx} ...
\includegraphics[width=0.8\linewidth]{myfile.pdf}
```

See Section 4.4 in the graphics bundle documentation (`http://mirrors.ctan.org/macros/latex/required/graphics/grfguide.pdf`)

A number of width problems arise when LaTeX cannot properly hyphenate a line. Please give LaTeX hyphenation hints using the `\-` command when necessary.

References

References follow the acknowledgments. Use unnumbered first-level heading for the references. Any choice of citation style is acceptable as long as you are consistent. It is permissible to reduce the font size to `small` (9 point) when listing the references. Note that the Reference section does not count towards the page limit.

[1] Alexander, J.A. & Mozer, M.C. (1995) Template-based algorithms for connectionist rule extraction. In G. Tesauro, D.S. Touretzky and T.K. Leen (eds.), *Advances in Neural Information Processing Systems 7*, pp. 609–616. Cambridge, MA: MIT Press.

[2] Bower, J.M. & Beeman, D. (1995) *The Book of GENESIS: Exploring Realistic Neural Models with the GEneral NEural SImulation System.* New York: TELOS/Springer–Verlag.

[3] Hasselmo, M.E., Schnell, E. & Barkai, E. (1995) Dynamics of learning and recall at excitatory recurrent synapses and cholinergic modulation in rat hippocampal region CA3. *Journal of Neuroscience* **15**(7):5249-5262.

Checklist

The checklist follows the references. Please read the checklist guidelines carefully for information on how to answer these questions. For each question, change the default **[TODO]** to [Yes] , [No] , or [N/A] . You are strongly encouraged to include a **justification to your answer**, either by referencing the appropriate section of your paper or providing a brief inline description. For example:

- Did you include the license to the code and datasets? [Yes] See Section 3.
- Did you include the license to the code and datasets? [No] The code and the data are proprietary.
- Did you include the license to the code and datasets? [N/A]

Please do not modify the questions and only use the provided macros for your answers. Note that the Checklist section does not count towards the page limit. In your paper, please delete this instructions block and only keep the Checklist section heading above along with the questions/answers below.

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? **[TODO]**
   (b) Did you describe the limitations of your work? **[TODO]**
   (c) Did you discuss any potential negative societal impacts of your work? **[TODO]**
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? **[TODO]**
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? **[TODO]**
   (b) Did you include complete proofs of all theoretical results? **[TODO]**
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? **[TODO]**
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? **[TODO]**
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? **[TODO]**
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? **[TODO]**
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? **[TODO]**
   (b) Did you mention the license of the assets? **[TODO]**
   (c) Did you include any new assets either in the supplemental material or as a URL? **[TODO]**
   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? **[TODO]**
   (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? **[TODO]**
5. If you used crowdsourcing or conducted research with human subjects...
   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? **[TODO]**
   (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? **[TODO]**
   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? **[TODO]**

A Appendix

Optionally include extra information (complete proofs, additional experiments and plots) in the appendix. This section will often be part of the supplemental material.