Posterior Predictive Distribution

September 28, 2021

1 Notes

- Prior predictive: Before we see the data
  - $p(y) = \int_{\theta \in \Theta} p(y, \theta) d\theta = \int_{\theta \in \Theta} p(y \mid \theta) p(\theta) d\theta$
- Posterior predictive: After we see the data
  - $p(y' \mid y) = \int p(y', \theta, \mid y) d\theta = \int p(y' \mid \theta, y) p(\theta \mid y) d\theta$
  - Notice that $p(y' \mid \theta, y) = p(y' \mid \theta)$ is the likelihood of the new observation, since the new observation y' is independent of the data if we have the model defined by θ. Moreover, $p(\theta \mid y)$ is the posterior distribution. Therefore, the posterior predictive distribution is the product between the likelihood and posterior integrating over the parameter values.