Time Series Analysis of DJIA 30 Stocks

Yucheng Wang, Zhuoheng Han, Hongsheng Xie, Yueni Wang
yw6@andrew.cmu.edu,
zhuohenh@andrew.cmu.edu,
hongshex@andrew.cmu.edu,
yueniw@andrew.cmu.edu
Department of Statistics and Data Science,
Carnegie Mellon University

Abstract

In this project, we addressed the problem of the time series model fitting on the stock data. We did the analysis on three out of thirty DJIA stocks, they are Google, Amazon, and Apple. The data exploration was done to check the overall characteristics of the stock price and returns. Then, we fitted ARIMA(AutoRegressive Integrated Moving Average) model and GARCH(generalized autoregressive conditional heteroskedasticity) model for the three stock return sequences, the models were selected by AIC criterion and the significance of the parameters. Due to the heteroskedasticity patterns, for each stock, our final model is a GARCH(1,1) model. The model performance and validity were evaluated by simulation and conditional SD, our models could capture most of the patterns in the stock time series and perform great in simulation. In the end, the multivariate nature of the stocks was confirmed and discussed, we showed that the stock price and returns are correlated positively.

1 Introduction

The Stock and Financial data are highly uncertain and random. The Stock and financial markets are affected by many unpredictable factors and it is usually considered erratic. In practice, it is hard to perform accurate predictions of the stock price, etc., but we can still model the stock price and gain some valuable insight into the stock markets. Due to the stochastic and time-related patterns of the stock markets, time series analysis methods are usually used in analyzing the stock data.

In this project, we are modeling 3 out of 30 stock price sequences from Historical stock data for DJIA 30 companies(2006-01-01 2018-01-01)[1]. For each company, the data is explored by exploratory data analysis, and a GARCH[2](Generalized Autoregressive Conditional Heteroskedasticity) model is built. Further, for each company, the future stock price is predicted by our model. In the end, the relationship and correlation of the 3 stock price sequences are described, and the general patterns of the stock markets are discussed.

We chose three representative companies, AAPL(Apple), GOOGL(Google), and AMZN(Amazon) to conduct our analysis. For each time series sequence, 3018 rows of stock price records are used, Table(1) is an overview of the data we are using.

Date	Open	High	Low	Close	Volume	Name
1/3/2006	211.47	218.05	209.32	217.83	13137450	GOOGL
1/4/2006	222.17	224.7	220.09	222.84	15292353	GOOGL
1/5/2006	223.22	226	220.97	225.85	10815661	GOOGL
1/6/2006	228.66	235.49	226.85	233.06	17759521	GOOGL
1/9/2006	233.44	236.94	230.7	233.68	12795837	GOOGL
1/10/2006	232.44	235.36	231.25	235.11	9104719	GOOGL
1/11/2006	235.87	237.79	234.82	236.05	9008664	GOOGL

Table 1: Dataset Overview

The dataset includes the open price, close price, highest price, and lowest price each day, in our analysis, for simplicity reasons, we are only using the open price data, this is reasonable since the open price is representative of the stock price in a specific day.

2 Method

In our analysis, we are using a similar procedure in analyzing the three time series sequences.

2.1 Data Preprocessing

Firstly, we preprocessed the data by checking for the missing value and creating the time series object in R.

2.2 Data Exploration

In the data exploration part, we analyzed some of the characteristics of the time series by plotting the open price data, log open price data, and the returns(Defined by differencing the

log open price). Then, we used the LOESS method to find out the trend of the time series. After that, we evaluate the ACF and PACF of the returns to find out the autocorrelation of the time series.

2.3 Model Building

In the model building section, we are building models for the returns. According to the result of the ACF and PACF mentioned in the Data Exploration part, we used the function auto.Arima() in the package forecast to fit an ARIMA model, we chose the best model by evaluating the AIC criterion. Here we do not want to fit a model with too many parameters, thus the orders of the ARIMA models are restricted, and the maximum orders of the autoregression and moving average parts are set to 3. After fitting the ARIMA model, the ACF and PACF of the model residuals are evaluated, the potential heteroscedasticity patterns are discovered by QQ-plots of the residuals and the box-pierce test for squared residuals. After confirming the heteroscedasticity patterns of the data, in order to fit the volatility of the stock price, we tried to fit an ARMA-GRACH model or a GRACH model to the data. We fitted multiple numbers of ARMA-GARCH and GRACH models to the data and we find out the best model by evaluating the AIC and the significance of the parameters. The simplicity of the model is highly important in our analysis, thus the number of the parameters is limited in our modeling process.

2.4 Model Evaluation

After choosing the best GARCH(ARMA-GARCH) model, the model result is evaluated by the ACF and PACF plot of the residuals. Further, the prediction and simulation of the model are performed, and the performance of the model is shown by the simulation.

2.5 Multivariate Characteristics

After fitting the univariate GARCH(ARMA-GRACH) models for the three stocks, the multivariate characteristics are examined by the cross-correlation function and the visualization of the returns of the three sequences.

3 Result

3.1 Google

3.1.1 Data Preprocessing

Firstly, no missing value was found in all three datasets. And we transformed the data into a time series object.

3.1.2 Data Exploration

According to the figure 1(red line is the open price, the blue line is the close price, the green line is the lowest price, yellow is the highest price), we find that all the lines with different colors are overlapping with each other, which means we do not need to analyze all the price, instead, we only need to do our analysis on the open price.

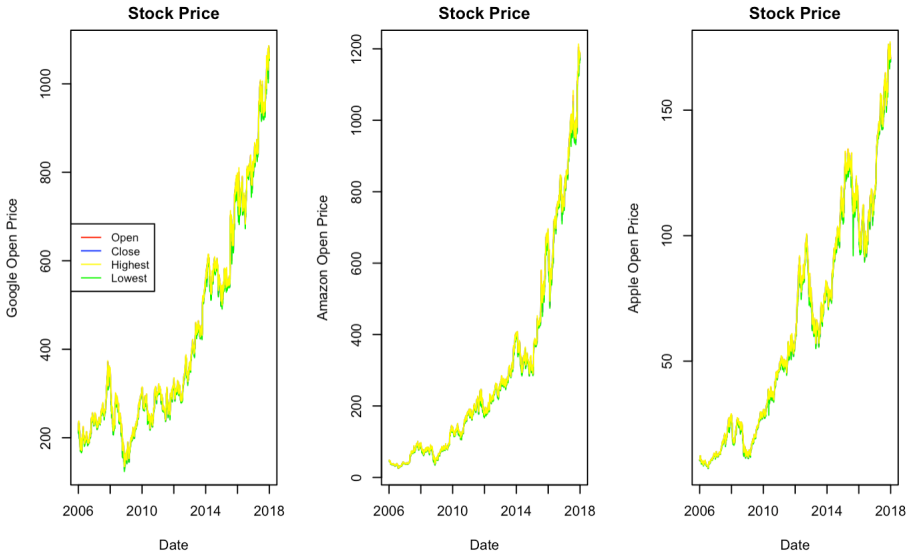


Figure 1: The Open, Close, Highest, Lowest Stock Price

Firstly, we analyze the stock price of Google, figure 2 is the open price of Google, in this figure, we find that the trend of the stock price is exponential, the exponential trend is also shown in figure 3, which is the LOESS of this sequence.

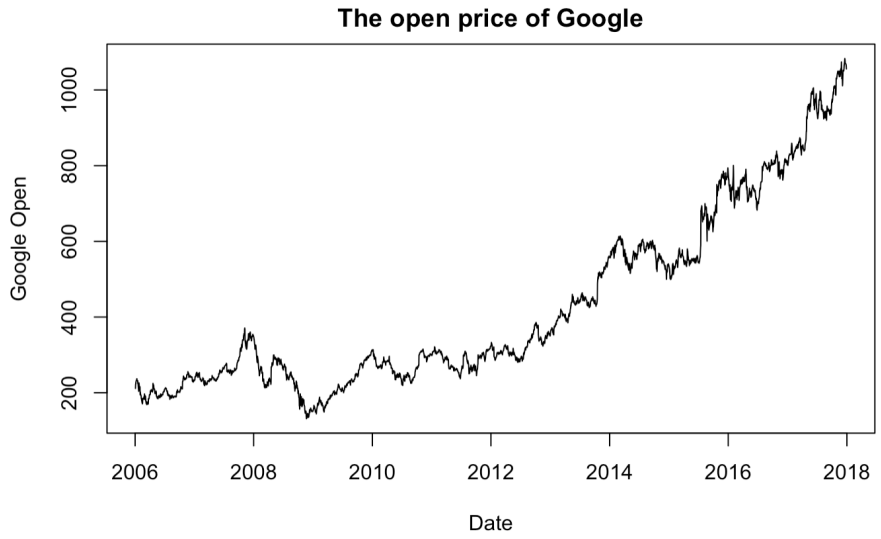


Figure 2: The open price of google

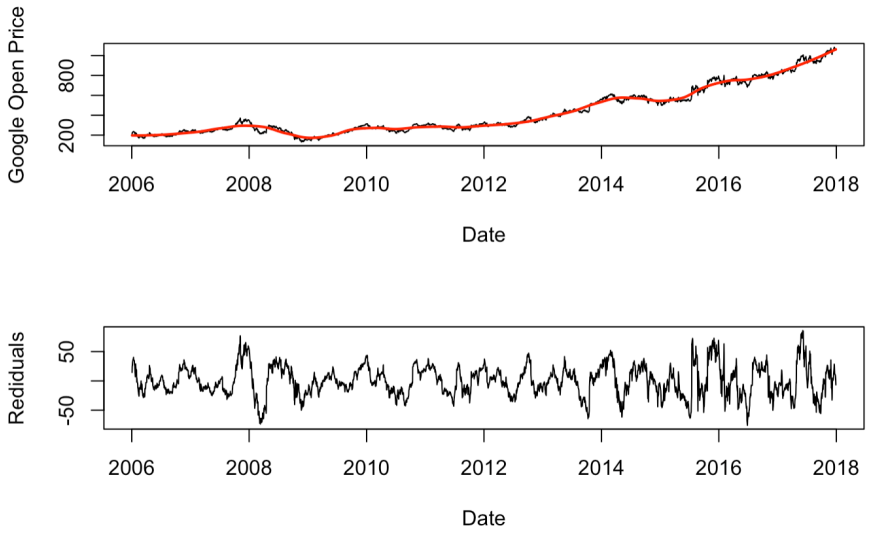


Figure 3: LOESS for Google open price

A constant trend is preferred here, so a natural way here is taking log transformation, figure 4 shows that after taking log transformation, the trend seems to be a constant.

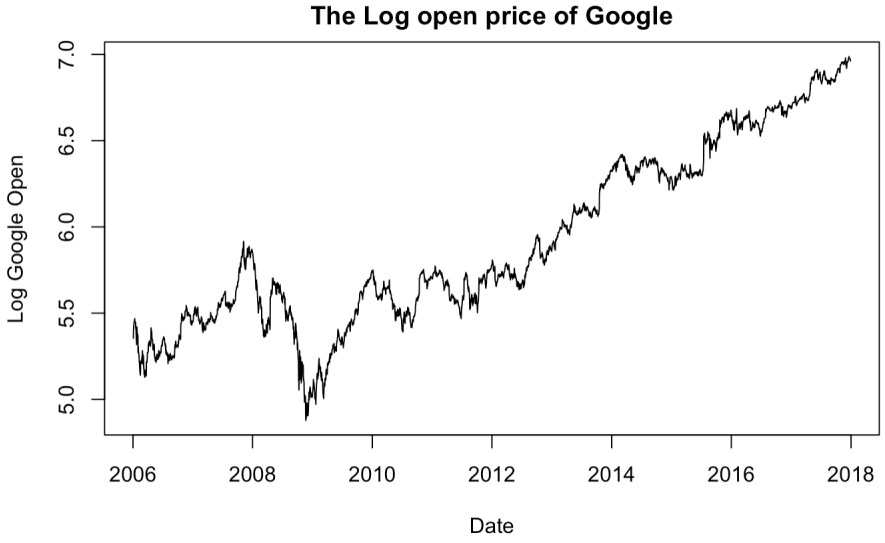


Figure 4: Log Google open price

For a constant trend time series, we can detrend it by differencing. Figure 5 is the returns of Google, obviously, there is no trend in this time series, also, we did not find strong evidence of any seasonal patterns.

3.1.3 Model Building

According to figure 4, although there might be a heteroscedasticity pattern in this time series, we still tried to fit an ARIMA model first.

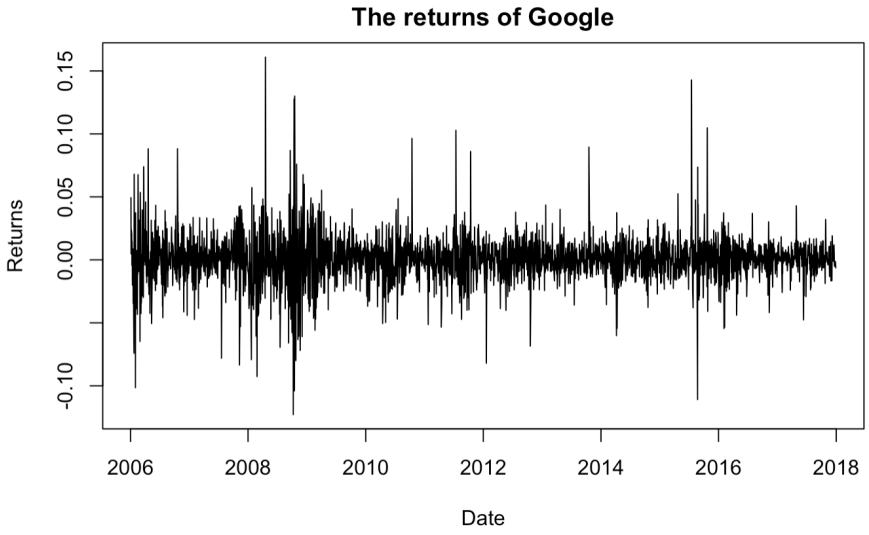


Figure 5: Google returns

Figure 6 is the ACF and PACF plot of the returns of Google, we cannot find the order of the ARMA model by figure 6.

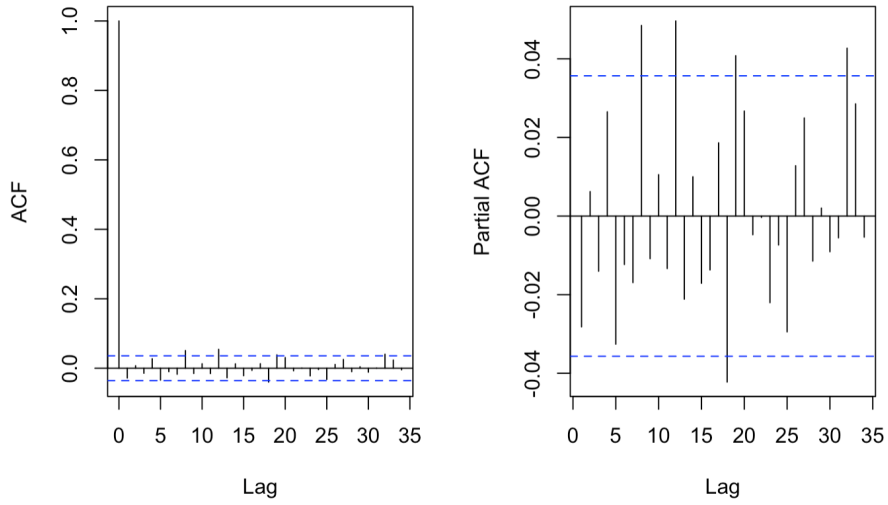


Figure 6: ACF, PACF of Returns of Google

Thus, by auto.arima, we can compare the AIC of the models. Here our best model is ARIMA(1,0,3) with a non-zero mean, which has the lowest AIC of -15542.65. However, as shown in Figure 7 the model does not perform very well. By analyzing the residuals of the ARIMA(1,0,3) model, in figure 8, according to the QQ plot, and the density of the residuals, we find that the residuals are from a distribution with heavier tails, it could be a t distribution. Also, from the ACF and PACF plot, we can see that the residual is not white noise, thus this ARIMA(1,0,3) is not good enough. Also, from the result, we could reasonably doubt that there is a heteroscedasticity pattern in this time series. By the box-pierce test(figure 9) of the squared residuals, we find that the null hypothesis was rejected

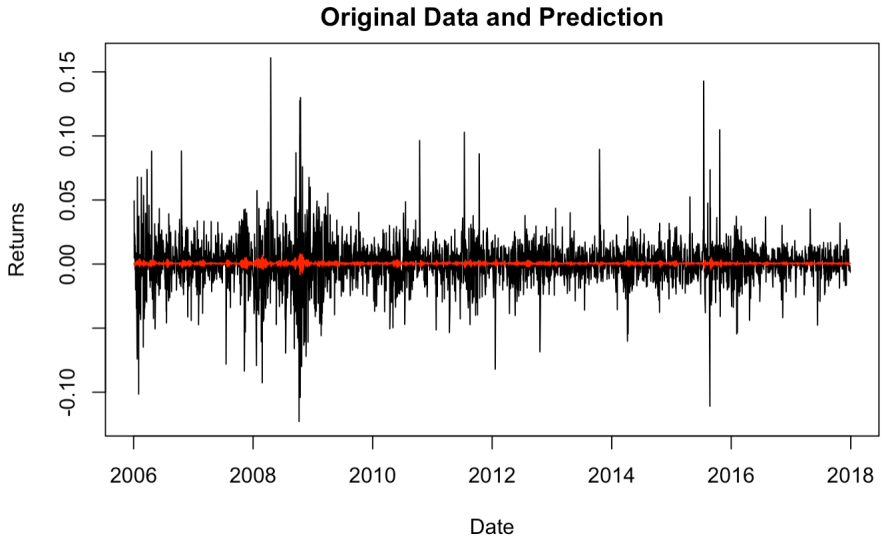


Figure 7: Actual and Prediction returns of Google using ARIMA(1,0,3) Model

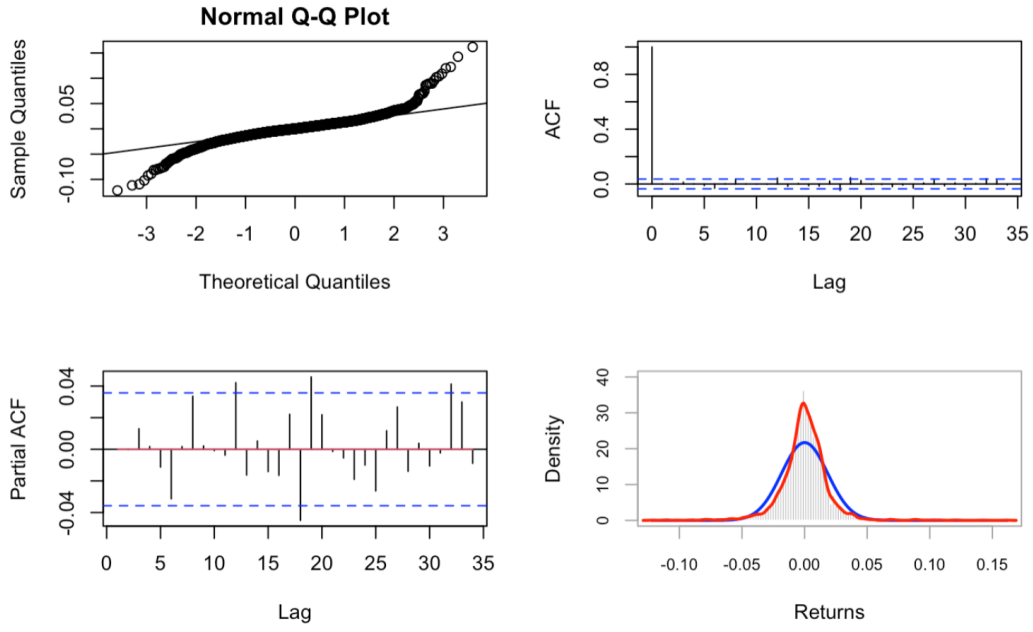


Figure 8: Residuals Analysis of ARIMA(1,0,3) for ARIMA(1,0,3)

Model	AIC	Model	AIC
GARCH(1,0)	-5.23	ARMA(1,3)+GARCH(1,0)	-5.23
GARCH(1,1)	-5.36	ARMA(1,3)+GARCH(1,1)	-5.36
GARCH(2,0)	-5.29	ARMA(1,3)+GARCH(2,0)	-5.29
GARCH(2,1)	-5.36	ARMA(1,3)+GARCH(2,1)	-5.36
GARCH(1,2)	-5.36	ARMA(1,3)+GARCH(1,2)	-5.36
GARCH(2,2)	-5.23	ARMA(1,3)+GARCH(2,2)	-5.36

Table 2: Model Comparison

for all the lags, which means a heteroscedasticity pattern exists.

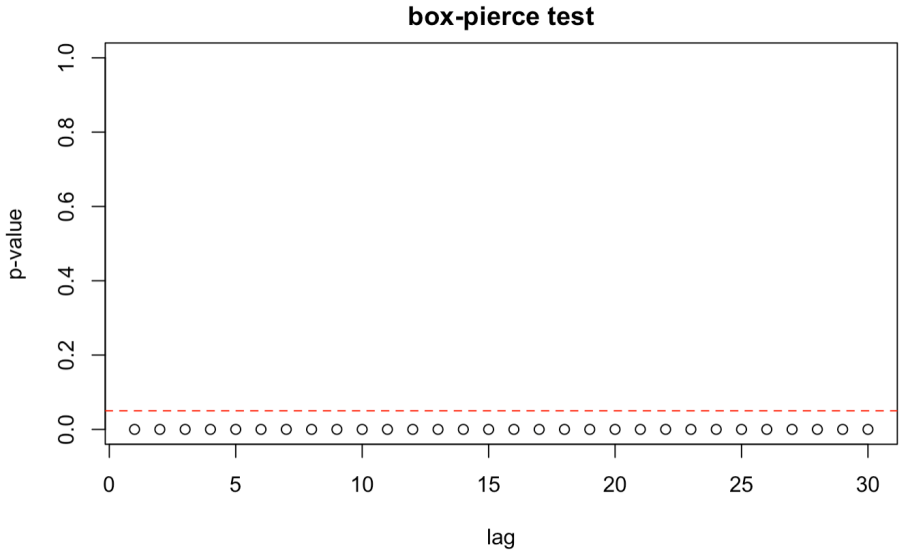


Figure 9: Box-pierce Test

Thus, from here we tried to fit a GARCH model or an ARMA-GARCH model, here we fitted 12 models(table 2), the first 6 models are the GARCH models, and the rest of the models are the ARMA-GARCH model with ARMA(1,3)(ARMA(1,3) is the best model when fitting ARIMA model). According to the AIC of the models, we find that there are many models with the lowest AIC of -5.36, however, except for GARCH(1,1) model, there are always some insignificant parameters in other models, so here our best model is GARCH(1,1) model.

So our final model is shown in figure 10.

Also, we used another way to find out the best GARCH model, we fitted the ARMA model on the sequence of the squared returns using auto.arima function and we find that the best model for the squared returns should be ARMA(2,1), by the definition of the GARCH model, we should pick GARCH(2,1) instead of GARCH(1,1), but we also find that the new parameters are extremely small(10^{-8}) and it is not significant, thus, our best models should still be GARCH(1,1) for the stock returns of Google.

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     6.797e-04   2.795e-04    2.432 0.015009 *
omega  5.700e-06   1.598e-06    3.566 0.000363 ***
alpha1 4.754e-02   1.087e-02    4.374 1.22e-05 ***
beta1  9.346e-01   1.448e-02   64.528  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 10: Summary of GARCH(1,1)

3.1.4 Model Evaluation

The Conditional Standard Deviation is shown in figure 11, according to the plot, we find that the conditional SD shows a similar pattern with the returns, thus the model seems to be a good one.

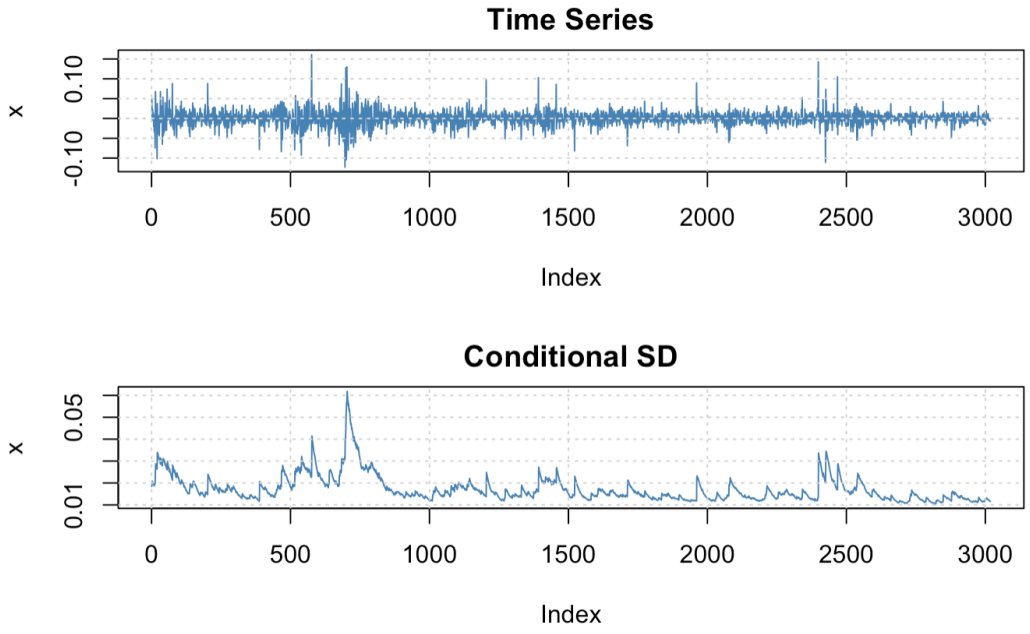


Figure 11: Conditional SD

Further, we did a simulation by using this model, according to figure 12. By this plot, we find that the simulations are all very reasonable, the patterns of the simulations are similar to the original data, and the prediction seems to be convincing. Also, the overall variance was captured by the model very well.

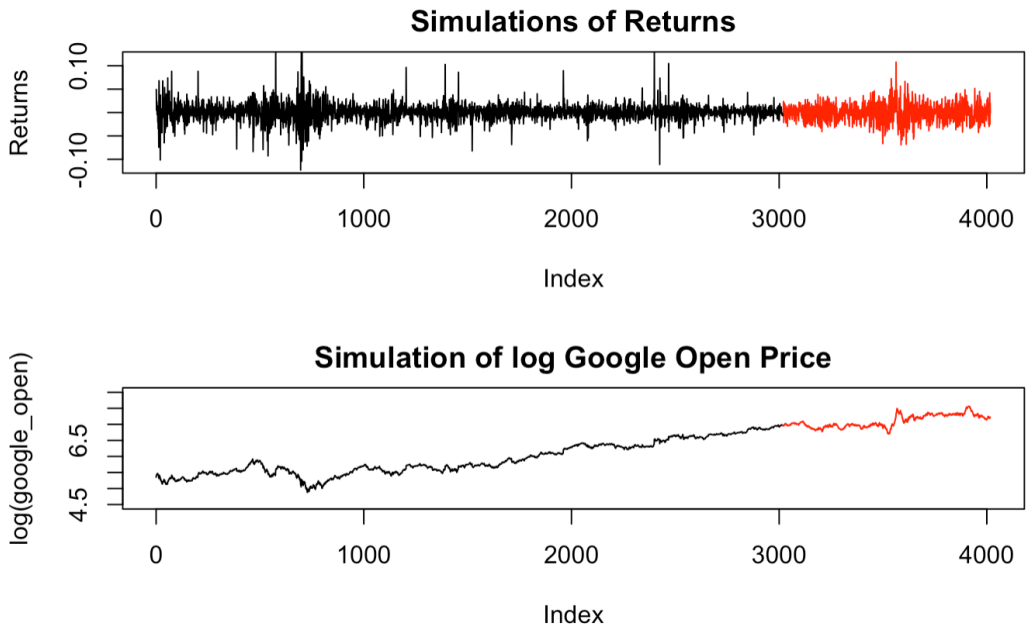


Figure 12: Simulation

3.2 Amazon and Apple

For Amazon and Apple, we used the completely same procedure to fit the model, the model result and evaluations are shown below(complete procedure please refer to the code).

3.2.1 Apple

For the stock returns of Apple, our final model is also a GARCH(1,1) model, which is shown in figure 13.

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     1.623e-03   3.230e-04    5.024 5.06e-07 ***
omega  1.396e-05   2.933e-06    4.758 1.95e-06 ***
alpha1 8.325e-02   1.274e-02    6.534 6.41e-11 ***
beta1  8.861e-01   1.698e-02   52.197  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 13: Model result of Stock Returns of Apple

The condition SD is shown in figure 14, the condition SD fit the returns quite well. Also, by the simulation of the model(figure 15), we find that our model could capture the characteristics of the data very well.

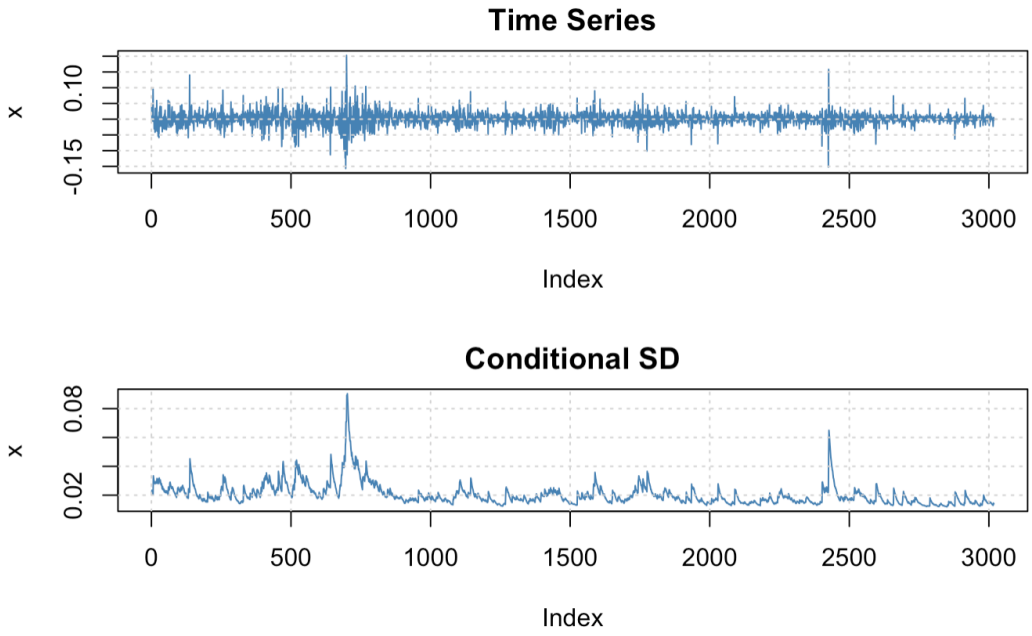


Figure 14: Conditional SD of GARCH(1,1)

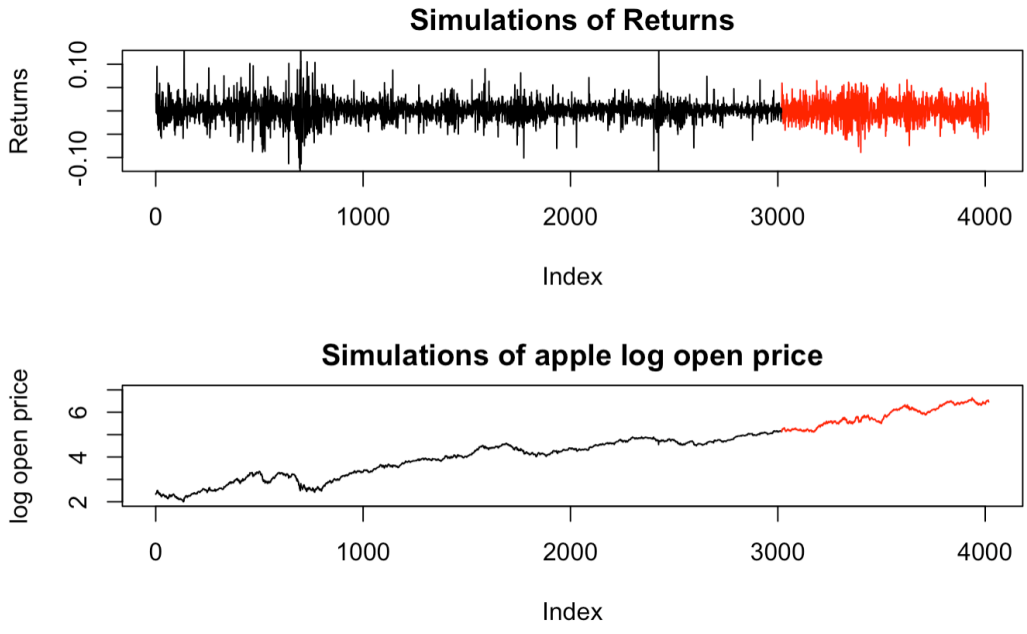


Figure 15: Simulations

3.2.2 Amazon

For the stock returns of Amazon, our final model is also a GARCH(1,1) model, which is shown in figure 16.

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     1.258e-03   3.851e-04    3.266 0.001090 **
omega  2.553e-06   6.794e-07    3.758 0.000171 ***
alpha1 1.652e-02   2.013e-03    8.207 2.22e-16 ***
beta1  9.789e-01   2.682e-03  364.948  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 16: Model result of Stock Returns of Amazon

The condition SD is shown in figure 17, the condition SD fit the returns quite well. Also, by the simulation of the model(figure **??**), we find that our model could capture the

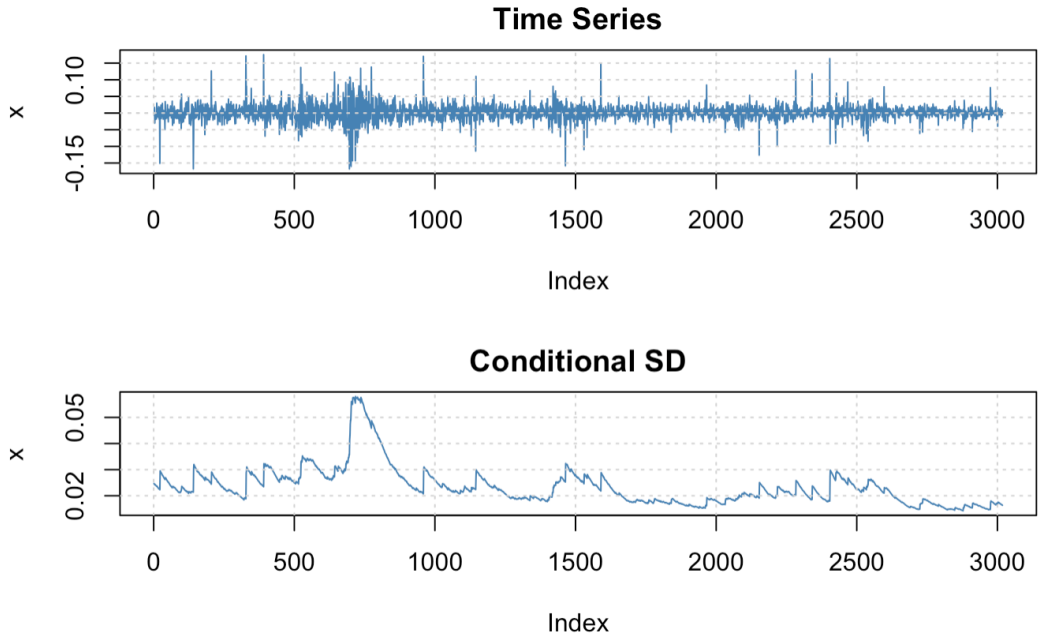


Figure 17: Conditional SD of GARCH(1,1)

characteristics of the data very well.

In the end, we find that the returns of all the three stocks should be fitted by GARCH(1,1) model, it is reasonable for us to think whether the three time series have anything in common, which is the reason why we examine the multivariate characteristics in the next section.

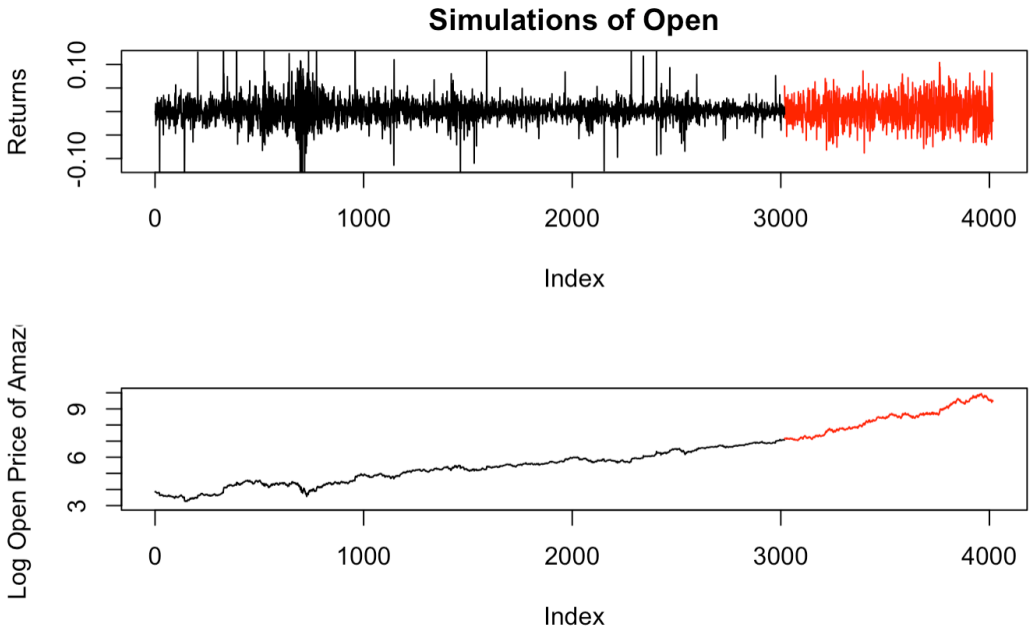


Figure 18: Simulations

3.3 Multivariate Characteristics

Firstly, we plotted the open price for all the three companies in figure 19. We find that although the stocks price of the three companies is quite different, the trends in some specific time interval are the same, which is shown more clearly in figure 20. It is clear that after

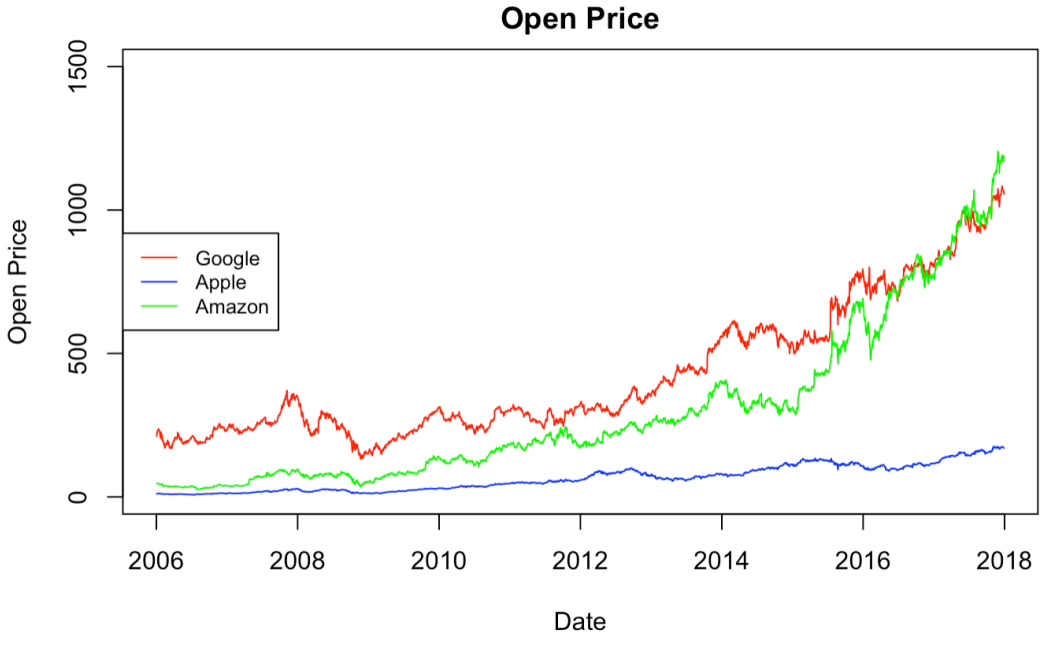


Figure 19: Comparison

normalizing the open price, we find that the three time series are highly correlated that they have similar trends. To show the correlations between time series, we used the Vector AR

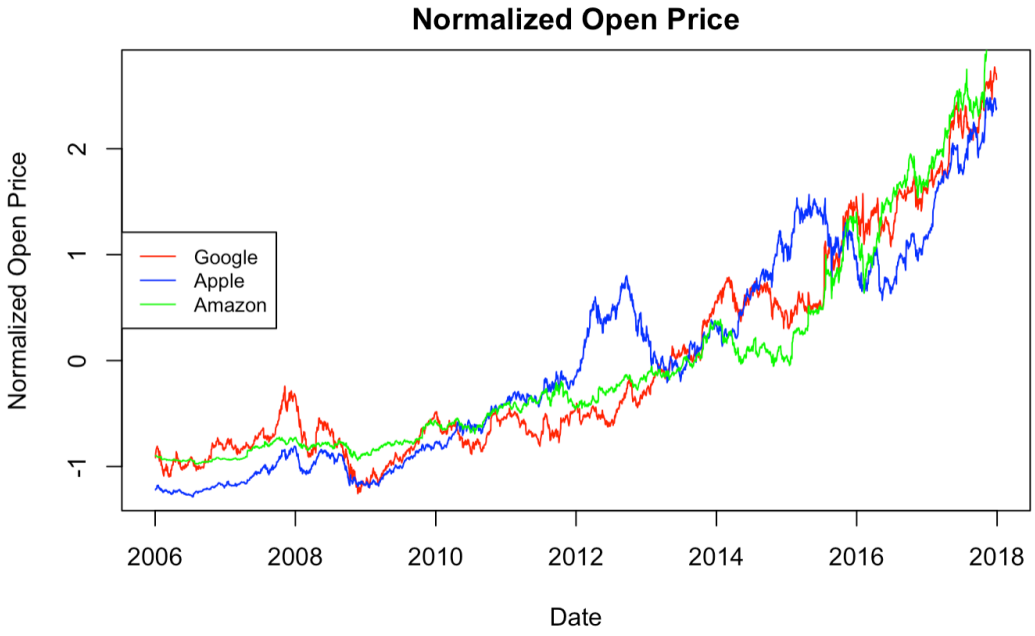


Figure 20: Comparison after Normalized

model to examine the cross-correlation between the stock returns sequences. It might not be the most proper way of model fitting, since heteroscedasticity patterns exist in the time series, but the result in figure 21 still can show the correlation between the stock returns and stock price. According to the plots, we find that the correlation between stock returns

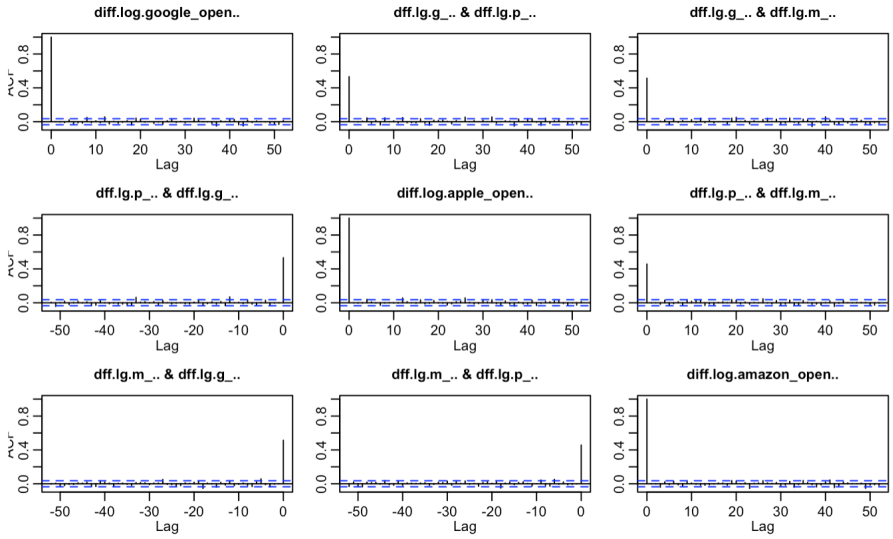


Figure 21: CCFs of Returns

of different companies is positive and all greater than 0.4, which means that the three stock returns positively correlated with each other. It is also true for the open price sequence of the three companies.

4 Discussion

In this project, we fitted three GARCH(1,1) models for the stock returns of Google, Amazon, and Apple. For financial and stock data, strong volatility is usually a significant pattern, which means some simple time series models like ARIMA and ARMA might not be suitable. For this kind of data, fitting an ARCH or GARCH model is usually more reasonable.

By looking at the final model and the conditional SD of each model in figure 11, 14, 17, we can easily find out that the pattern of the conditional SD is very similar to each other, for example, they all have a peak between index 500 and 1000, it means there might be some factors, which could influence all the stocks, these factors could be the economic conditions, international relations, or even wars.

However, we find that although all three stock prices are influenced by some factors, the degrees of the influence are quite different, it is quite reasonable, for instance, during the COVID-19 pandemic, it is reasonable that both the stock prices of Google and Apple decreased. But the stock price of Apple might decrease even more dramatically since the pandemic could be more influential on the supply chain and logistics of Apple.

These kinds of factors cause the high correlations between stock prices, however also make the stock price hard to predict, since some of these factors are hard to predict. Thus, our model could only show an ideal trend in near future, for a long-term prediction, the model performance could be poor, but it is still a good way to simulate possible patterns in the future.

There could be many possible improvements in our analysis. Firstly, we only chose 3 out of 30 stocks and analyzed their relationship, however, there might be some stocks that have completely different features, if we have more time, it would be better to analyze all the 30 stocks and could have a more convincing conclusion. Also, in our model-fitting processes, we only choose some models with relatively low orders and small numbers of parameters, however, it is likely that a more complicated model could have a better performance. Further, when we are calculating the CCFs between the returns sequences, we used the Vector AR model, but it might not be the best choice, since the heteroscedasticity patterns might be omitted.

If we have more time, our next step is to build a more advanced model like a neural network, etc. these kinds of models are more flexible and could possibly capture more hidden features in the time series. Also, it would be good to fit another multivariate time series to have a better view of the multivariate nature of the stock price data.

References

[1] . Djia 30 stock time series, 2022. https://www.kaggle.com/datasets/szrlee/stock-time-series-20050101-to-201712311, Last accessed on 2022-04-15.

[2] . Package 'rugarch', note = https://cran.r-project.org/web/packages/rugarch/rugarch.pdf, last accessed on 2022-04-15,, 2022.

Time series Technical Appendix

4/17/2022

Google

EDA

```
library(fGarch)
```

```
## Loading required package: timeDate
```

```
## Loading required package: timeSeries
```

```
## Loading required package: fBasics
```

```
library(forecast)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

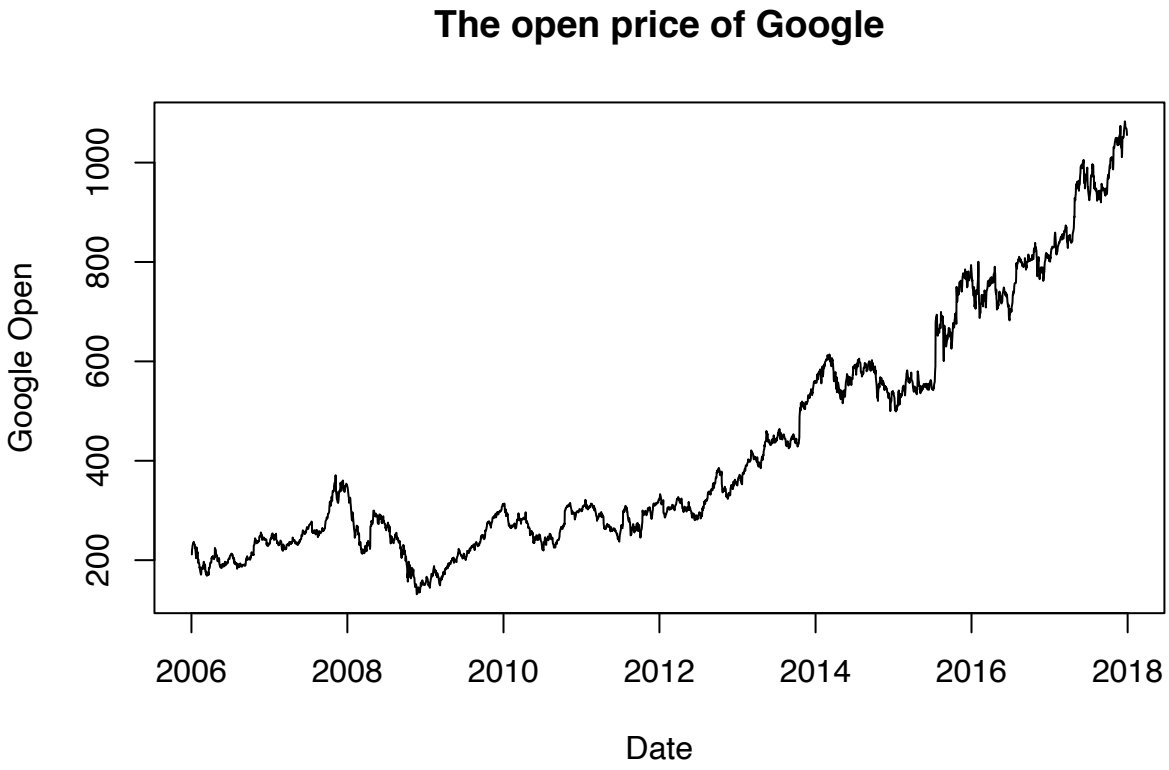
```
library(tseries)
google = read.csv("/Users/wyc/GOOGL_2006-01-01_to_2018-01-01.csv")
```

```
head(google)
```

```
##         Date   Open   High    Low  Close   Volume  Name
## 1 2006-01-03 211.47 218.05 209.32 217.83 13137450 GOOGL
## 2 2006-01-04 222.17 224.70 220.09 222.84 15292353 GOOGL
## 3 2006-01-05 223.22 226.00 220.97 225.85 10815661 GOOGL
## 4 2006-01-06 228.66 235.49 226.85 233.06 17759521 GOOGL
## 5 2006-01-09 233.44 236.94 230.70 233.68 12795837 GOOGL
## 6 2006-01-10 232.44 235.36 231.25 235.11  9104719 GOOGL
```

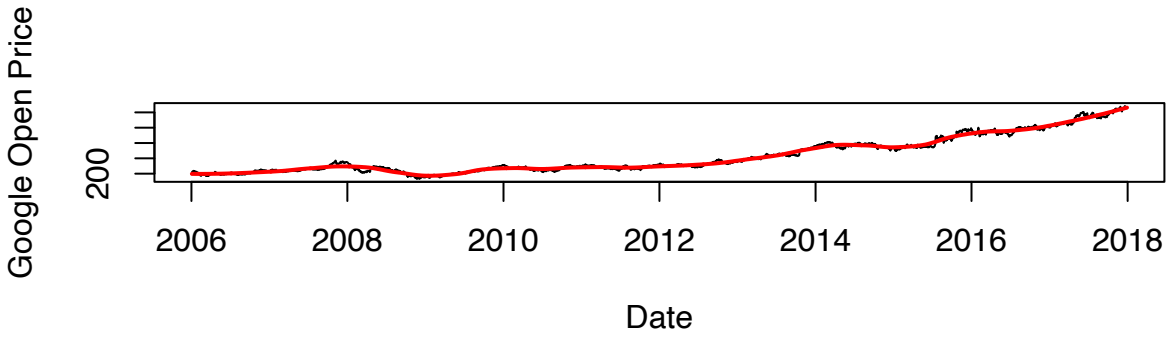
```
google_open = google$Open
google_open = ts(google_open,start = 0)
```

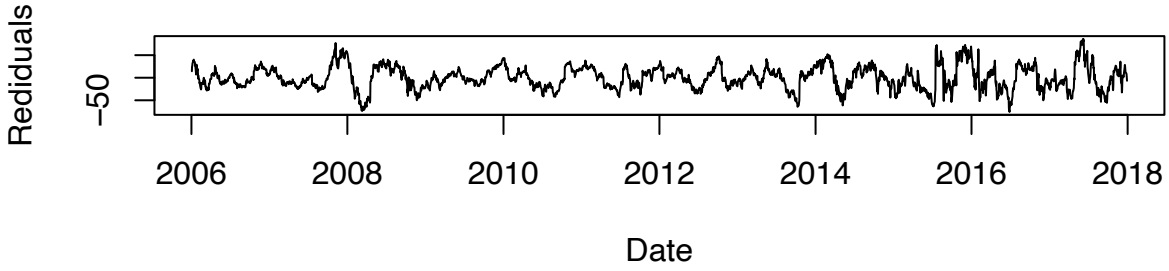
```
plot(y=google_open,x=as.Date(google$Date),type = "l",main="The open price of Google",xlab="Date", ylab=
```



The open price of google

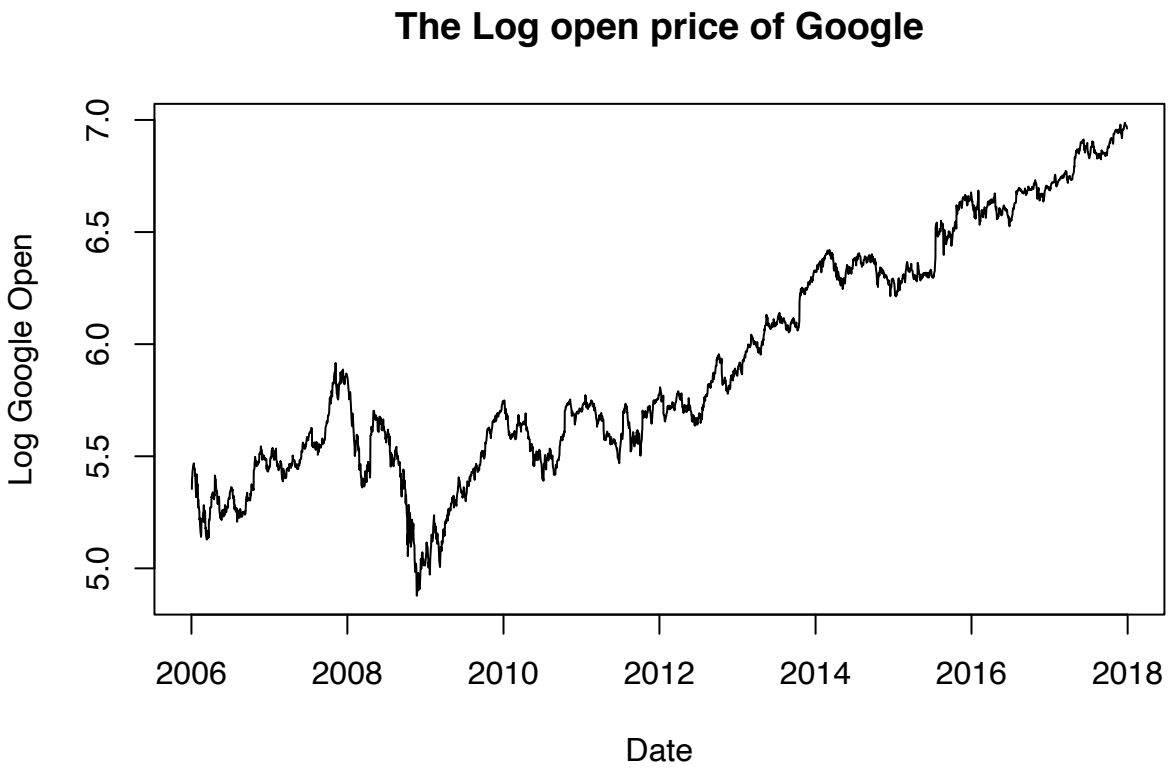
```
par(mfrow=c(2,1))
t.hat <- predict(loess(google_open ~ time(google_open), span=0.17))
plot(y=google_open,as.Date(google$Date),xlab="Date",ylab="Google Open Price",type="l")
lines(x=as.Date(google$Date), y=t.hat, col="red", lwd=2)
plot(y=google_open-t.hat,x=as.Date(google$Date),type="l",ylab = "Rediduals",xlab="Date")
```





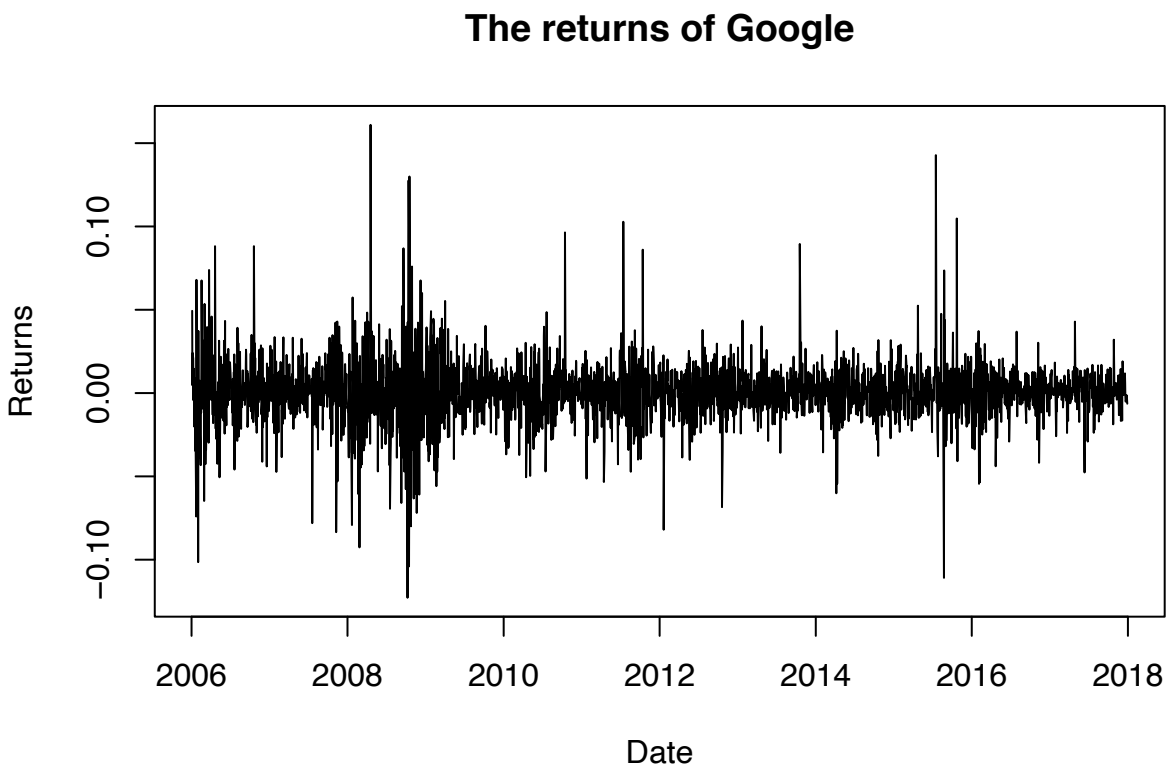
Loess, dipict the trend

```
plot(y=log(google_open),x=as.Date(google$Date),type = "l",main="The Log open price of Google",xlab="Dat
```



Take log to have a constant trend.

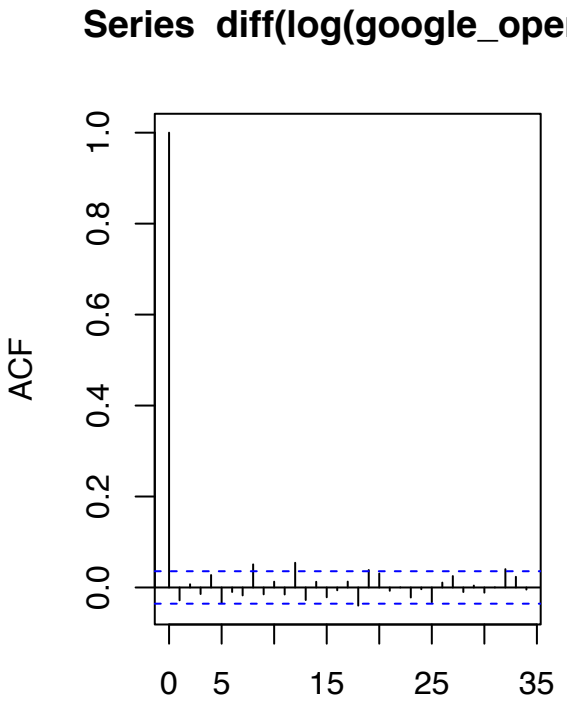
```
plot(y=diff(log(google_open)),x=as.Date(google$Date)[-1],type = "l",main="The returns of Google",xlab="
```

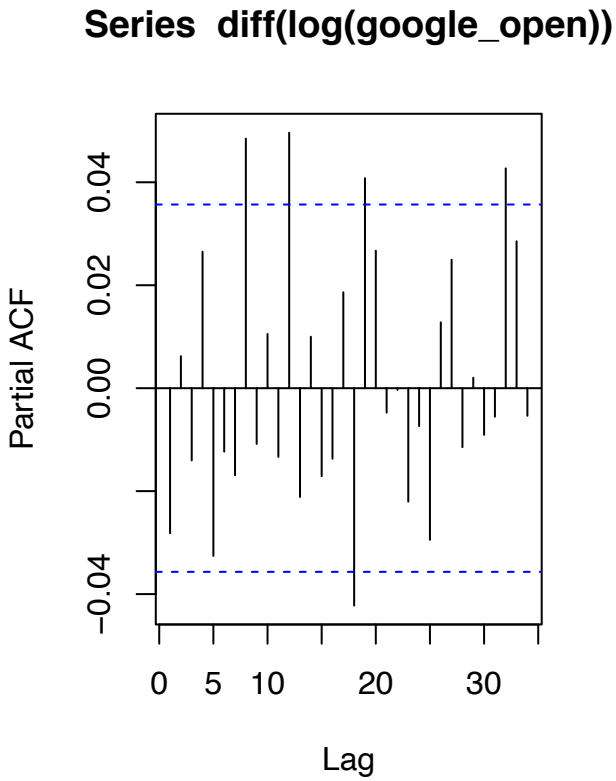


Take one diff, seems no trend, after diff it is return of the stock. Does not seems to have any seasonal pattern.

Model building

```
library(forecast)
par(mfrow=c(1,2))
acf(diff(log(google_open)))
pacf(diff(log(google_open)))
```






```
model = auto.arima(diff(log(google_open)), max.p = 3, max.q = 3,max.order = 10, stationary = T, seasonal
```

```
##
##  ARIMA(0,0,0) with zero mean     : -15537.68
##  ARIMA(0,0,0) with non-zero mean : -15538.2
##  ARIMA(0,0,1) with zero mean     : -15537.91
##  ARIMA(0,0,1) with non-zero mean : -15538.57
##  ARIMA(0,0,2) with zero mean     : -15536.05
##  ARIMA(0,0,2) with non-zero mean : -15536.68
##  ARIMA(0,0,3) with zero mean     : -15534.47
##  ARIMA(0,0,3) with non-zero mean : -15535.16
##  ARIMA(1,0,0) with zero mean     : -15537.94
##  ARIMA(1,0,0) with non-zero mean : -15538.6
##  ARIMA(1,0,1) with zero mean     : -15535.92
##  ARIMA(1,0,1) with non-zero mean : -15536.58
##  ARIMA(1,0,2) with zero mean     : Inf
##  ARIMA(1,0,2) with non-zero mean : Inf
##  ARIMA(1,0,3) with zero mean     : -15541.99
##  ARIMA(1,0,3) with non-zero mean : -15542.65
##  ARIMA(2,0,0) with zero mean     : -15536.09
##  ARIMA(2,0,0) with non-zero mean : -15536.71
##  ARIMA(2,0,1) with zero mean     : Inf
##  ARIMA(2,0,1) with non-zero mean : Inf
##  ARIMA(2,0,2) with zero mean     : Inf
##  ARIMA(2,0,2) with non-zero mean : -15541.85
##  ARIMA(2,0,3) with zero mean     : -15539.23
##  ARIMA(2,0,3) with non-zero mean : Inf
##  ARIMA(3,0,0) with zero mean     : -15534.61
##  ARIMA(3,0,0) with non-zero mean : -15535.3
```

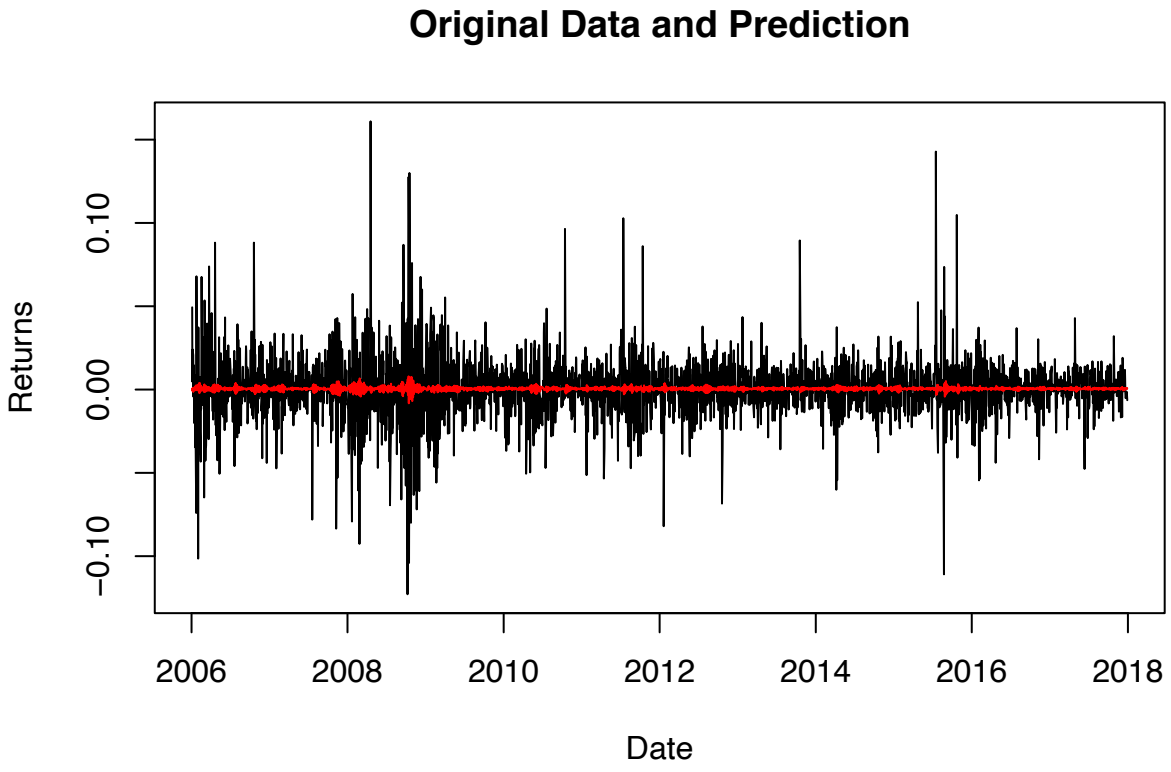
```
##  ARIMA(3,0,1) with zero mean     : -15542.01
##  ARIMA(3,0,1) with non-zero mean : -15541.15
##  ARIMA(3,0,2) with zero mean     : -15539.27
##  ARIMA(3,0,2) with non-zero mean : Inf
##  ARIMA(3,0,3) with zero mean     : Inf
##  ARIMA(3,0,3) with non-zero mean : Inf
##
##
##
##  Best model: ARIMA(1,0,3) with non-zero mean
```

Fit ARIMA model to the return. Hard to figure our the orders of the ARIMA model by acf, pacf, thus auto.arima. arima(1,0,3) best.

```
arima_model = arima(x=diff(log(google_open)),order = c(1,0,3),method='ML')
summary(arima_model)
```

```
##
## Call:
## arima(x = diff(log(google_open)), order = c(1, 0, 3), method = "ML")
##
## Coefficients:
##           ar1     ma1      ma2      ma3  intercept
##       -0.9142  0.8898  -0.0187  -0.0207      5e-04
## s.e.   0.0498  0.0528   0.0241   0.0191      3e-04
##
## sigma^2 estimated as 0.0003382:  log likelihood = 7777.35,  aic = -15542.7
##
## Training set error measures:
##                         ME       RMSE       MAE MPE MAPE      MASE         ACF1
## Training set 2.995613e-05 0.01839033 0.0123407 NaN  Inf 0.6943978 -0.000199247
```

```
plot(x=as.Date(google$Date)[-1],y=diff(log(google_open)),main="Original Data and Prediction",type="l",x
lines(x=as.Date(google$Date)[-1],y=diff(log(google_open))-arima_model$resid,col="red")
```



Bad result.

```
par(mfrow = c(2,2))
qqnorm(arima_model$resid); qqline(arima_model$resid)
acf(arima_model$resid)
pacf(arima_model$resid)
library(MASS)
library(QRM)
```

```
## Loading required package: gsl
## Loading required package: Matrix
## Loading required package: mvtnorm
## Loading required package: numDeriv
##
## Attaching package: 'QRM'
## The following object is masked from 'package:base':
##
##     lbeta
```

```
fit1 <- fitdistr(arima_model$resid, "normal")
para <- fit1$estimate
curve(dnorm(x, para[1], para[2]), col = 2, add = TRUE)
fit2 = fit.st(arima_model$resid)
p_val = rep(0,30)
for(i in 1:30){
  p_val[i] = Box.test(arima_model$residual^2,lag = i)[3]
}

stdret <- residuals(arima_model, standardize = TRUE)
```

```
library(PerformanceAnalytics)
```

```
## Loading required package: xts
```

```
## Loading required package: zoo
```

```
##
## Attaching package: 'zoo'
```

```
## The following object is masked from 'package:timeSeries':
##
##     time<-
```

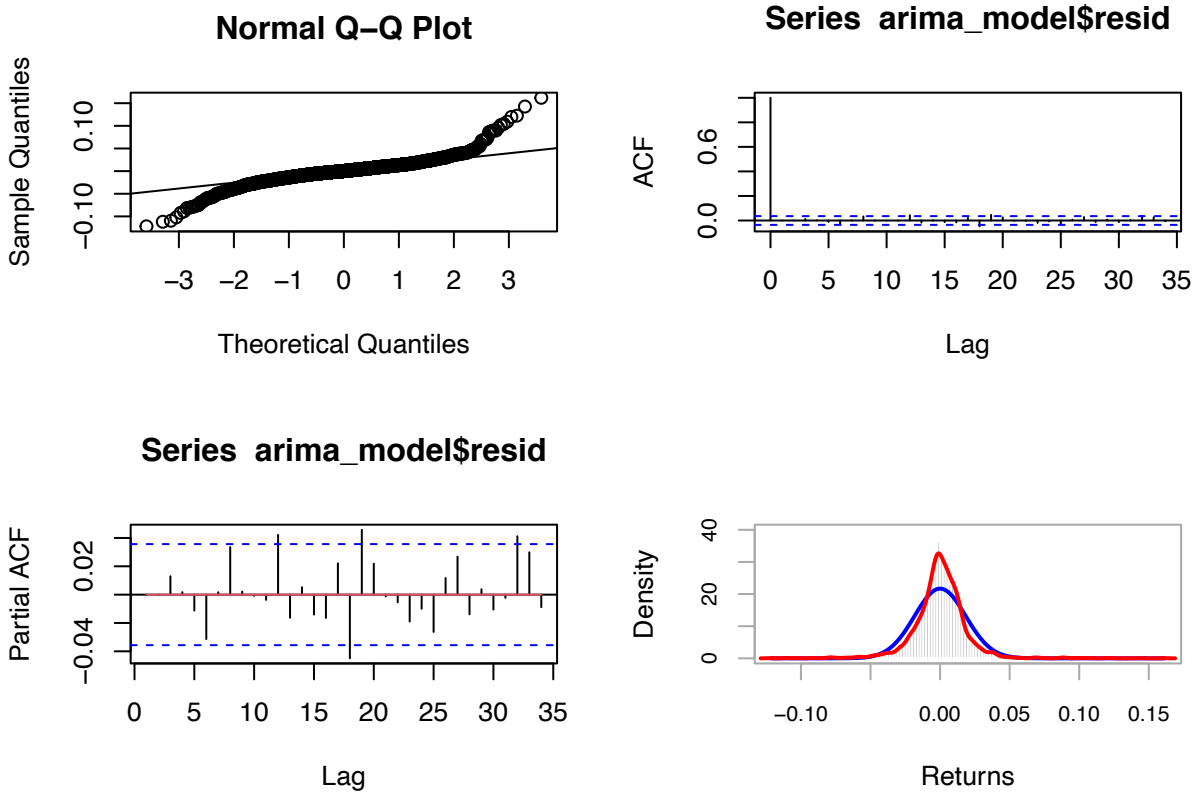
```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
##
## Attaching package: 'PerformanceAnalytics'
```

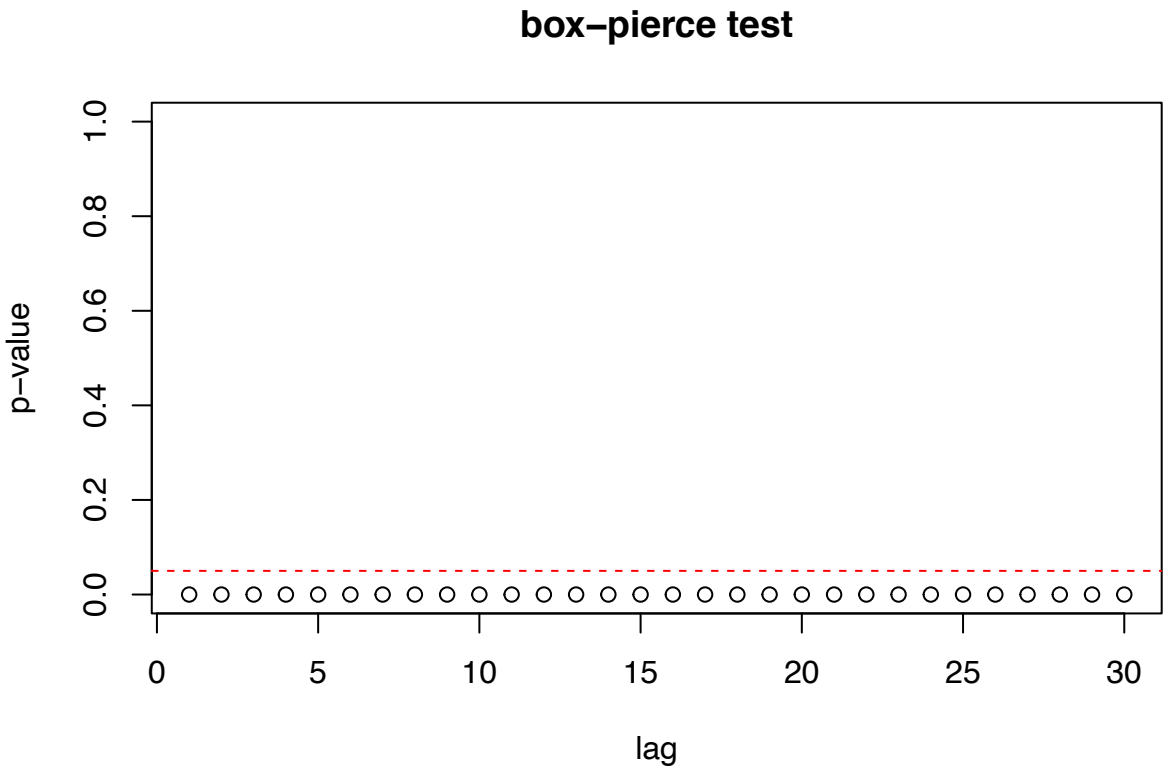
```
## The following objects are masked from 'package:timeDate':
##
##     kurtosis, skewness
```

```
## The following object is masked from 'package:graphics':
##
##     legend
```

```
chart.Histogram(stdret, methods = c("add.normal", "add.density"),
          colorset=c("gray","red","blue"))
```



```
par(mfrow = c(1,1))
plot(y=p_val,x=1:30,xlab = "lag",ylab="p-value",ylim=c(0,1),main="box-pierce test")
abline(h=0.05,col="red",lty=2)
```



qqplot, non-gaussian resid. Heteroscedasticity exist from the test. But we probabily can use arima(1,0,3) as a mean model

```
library(fGarch)
gfit1 = garchFit(~garch(1,0),data = diff(log(google_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(0, 0)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(1, 0)
##  ARMA Order:                 0 0
##  Max ARMA Order:             0
##  GARCH Order:                1 0
##  Max GARCH Order:            1
##  Maximum Order:              1
##  Conditional Dist:           norm
##  h.start:                    2
##  llh.start:                  1
##  Length of Series:           3018
##  Recursion Init:             mci
##  Series Scale:               0.01843157
##
## Parameter Initialization:
##  Initial Parameters:           $params
##  Limits of Transformations:    $U, $V
```

```
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U          V    params includes
##     mu     -0.28901297   0.289013 0.0289013     TRUE
##     omega   0.00000100 100.000000 0.1000000     TRUE
##     alpha1  0.00000001   1.000000 0.1000000     TRUE
##     gamma1 -0.99999999   1.000000 0.1000000    FALSE
##     delta   0.00000000   2.000000 2.0000000    FALSE
##     skew    0.10000000  10.000000 1.0000000    FALSE
##     shape   1.00000000  10.000000 4.0000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1
##      1      2      3
##  Persistence:                  0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     9291.4533: 0.0289013 0.100000 0.100000
##   1:     4238.0669: 0.0288999  1.06643 0.356913
##   2:     4232.0486: 0.0289036  1.07672 0.294744
##   3:     4190.5519: 0.0289049 0.960414 0.246197
##   4:     4163.3714: 0.0289073 0.790822 0.194057
##   5:     4162.7766: 0.0289137 0.752982 0.367401
##   6:     4161.7477: 0.0296584 0.688253 0.312482
##   7:     4159.3113: 0.0311228 0.745539 0.267528
##   8:     4159.2636: 0.0290423 0.743491 0.297738
##   9:     4159.1720: 0.0308710 0.742888 0.285825
##  10:     4159.1709: 0.0301533 0.742912 0.285199
##  11:     4159.1708: 0.0302373 0.742905 0.284855
##  12:     4159.1708: 0.0302317 0.742904 0.284883
##  13:     4159.1708: 0.0302318 0.742903 0.284883
##
## Final Estimate of the Negative LLH:
##  LLH:  -7893.787    norm LLH:  -2.615569
##           mu        omega       alpha1
## 0.0005572194 0.0002523812 0.2848828976
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu        omega        alpha1
## mu     -1.061767e+07      1244623      435.9237
## omega   1.244623e+06 -17615770904 -2096060.1561
## alpha1  4.359237e+02     -2096060    -1054.0637
## attr(,"time")
## Time difference of 0.02591586 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.1256821 secs
```

```
gfit2 = garchFit(~garch(1,1),data = diff(log(google_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.01843157
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U          V    params includes
##     mu     -0.28901297   0.289013 0.0289013     TRUE
##     omega   0.00000100 100.000000 0.1000000     TRUE
##     alpha1  0.00000001   1.000000 0.1000000     TRUE
##     gamma1 -0.99999999   1.000000 0.1000000    FALSE
##     beta1   0.00000001   1.000000 0.8000000     TRUE
##     delta   0.00000000   2.000000 2.0000000    FALSE
##     skew    0.10000000  10.000000 1.0000000    FALSE
##     shape   1.00000000  10.000000 4.0000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1
##      1      2      3      5
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3988.4070: 0.0289013 0.100000 0.100000 0.800000
##   1:     3983.2635: 0.0289021 0.0822374 0.101597 0.792792
##   2:     3972.8937: 0.0289032 0.0811637 0.118334 0.802212
##   3:     3972.4899: 0.0289069 0.0491205 0.138782 0.808140
##   4:     3969.7117: 0.0289106 0.0566692 0.142807 0.825369
##   5:     3963.0300: 0.0289291 0.0500884 0.126184 0.832438
##   6:     3961.7358: 0.0289353 0.0425124 0.115402 0.846449
##   7:     3959.0738: 0.0289613 0.0385342 0.0869166 0.873969
##   8:     3958.8371: 0.0289618 0.0372444 0.0892701 0.876910
```

```
##   9:     3958.3404: 0.0289648 0.0346250 0.0868269 0.878645
##  10:     3957.9655: 0.0289685 0.0340572 0.0850930 0.882182
##  11:     3956.1556: 0.0290280 0.0165492 0.0506765 0.930160
##  12:     3955.1820: 0.0290281 0.0175396 0.0511257 0.930775
##  13:     3955.1704: 0.0290632 0.0175355 0.0508372 0.930800
##  14:     3955.1599: 0.0291345 0.0176412 0.0505974 0.931088
##  15:     3955.1510: 0.0292063 0.0175573 0.0503708 0.931208
##  16:     3955.1007: 0.0303104 0.0171611 0.0485197 0.933328
##  17:     3955.0307: 0.0343514 0.0169696 0.0489060 0.933326
##  18:     3955.0198: 0.0354495 0.0174937 0.0493969 0.932132
##  19:     3955.0086: 0.0365444 0.0167102 0.0474731 0.934595
##  20:     3955.0052: 0.0370364 0.0167693 0.0474705 0.934681
##  21:     3955.0049: 0.0368745 0.0167822 0.0475568 0.934560
##  22:     3955.0049: 0.0368779 0.0167769 0.0475417 0.934580
##  23:     3955.0049: 0.0368778 0.0167773 0.0475428 0.934578
##
## Final Estimate of the Negative LLH:
##  LLH:  -8097.953    norm LLH:  -2.683218
##           mu        omega       alpha1        beta1
## 6.797165e-04 5.699616e-06 4.754280e-02 9.345783e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu         omega        alpha1         beta1
## mu     -12967223.43  3.910124e+08 -2.025057e+04  4.594106e+04
## omega  391012363.26 -7.454003e+12 -1.135732e+09 -1.584795e+09
## alpha1    -20250.57 -1.135732e+09 -2.840852e+05 -3.214280e+05
## beta1      45941.06 -1.584795e+09 -3.214280e+05 -3.993223e+05
## attr(,"time")
## Time difference of 0.03688717 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.1639209 secs
```

```
gfit3 = garchFit(~garch(2,0),data = diff(log(google_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 0)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               2 0
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.01843157
```

```
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U          V    params includes
##     mu     -0.28901297   0.289013 0.0289013     TRUE
##     omega   0.00000100 100.000000 0.1000000     TRUE
##     alpha1  0.00000001   1.000000 0.0500000     TRUE
##     alpha2  0.00000001   1.000000 0.0500000     TRUE
##     gamma1 -0.99999999   1.000000 0.1000000    FALSE
##     gamma2 -0.99999999   1.000000 0.1000000    FALSE
##     delta   0.00000000   2.000000 2.0000000    FALSE
##     skew    0.10000000  10.000000 1.0000000    FALSE
##     shape   1.00000000  10.000000 4.0000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2
##      1      2      3      4
##  Persistence:                 0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8203.9974: 0.0289013 0.100000 0.0500000 0.0500000
##   1:     4243.4835: 0.0289043 0.945200 0.400442 0.453518
##   2:     4226.7519: 0.0293383 0.928977 0.0289811 0.787476
##   3:     4172.9166: 0.0298156 0.486243 1.00000e-08  1.00000
##   4:     4080.4410: 0.0306049 0.588900 0.141034 0.532213
##   5:     4072.2254: 0.0306212 0.529036 0.136951 0.494113
##   6:     4068.2212: 0.0306569 0.525961 0.193409 0.451057
##   7:     4067.4938: 0.0307756 0.543601 0.168786 0.312224
##   8:     4066.4194: 0.0307949 0.568365 0.196585 0.331813
##   9:     4065.7205: 0.0313186 0.535384 0.201465 0.349985
##  10:     4065.6488: 0.0325181 0.538940 0.205088 0.354670
##  11:     4065.5293: 0.0354959 0.542286 0.200622 0.346418
##  12:     4065.5024: 0.0384539 0.530317 0.204191 0.355563
##  13:     4065.4025: 0.0414417 0.535535 0.204437 0.355492
##  14:     4065.3848: 0.0433893 0.538924 0.202005 0.351456
##  15:     4065.3847: 0.0434552 0.538818 0.202213 0.351593
##  16:     4065.3847: 0.0434454 0.538828 0.202206 0.351573
##
## Final Estimate of the Negative LLH:
##  LLH:  -7987.573    norm LLH:  -2.646644
##           mu        omega       alpha1       alpha2
## 0.0008007672 0.0001830523 0.2022059682 0.3515732205
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu        omega        alpha1        alpha2
## mu     -1.241273e+07      1928574    -1648.0944      194.9101
## omega   1.928574e+06 -23194770209 -2852832.0989 -2439986.5136
```

```
## alpha1 -1.648094e+03     -2852832    -1345.5008     -371.8337
## alpha2  1.949101e+02     -2439987     -371.8337     -798.3207
## attr(,"time")
## Time difference of 0.04038811 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.1411591 secs
```

```
gfit4 = garchFit(~garch(2,1),data = diff(log(google_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 1)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               2 1
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.01843157
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U          V    params includes
##     mu     -0.28901297   0.289013 0.0289013     TRUE
##     omega   0.00000100 100.000000 0.1000000     TRUE
##     alpha1  0.00000001   1.000000 0.0500000     TRUE
##     alpha2  0.00000001   1.000000 0.0500000     TRUE
##     gamma1 -0.99999999   1.000000 0.1000000    FALSE
##     gamma2 -0.99999999   1.000000 0.1000000    FALSE
##     beta1   0.00000001   1.000000 0.8000000     TRUE
##     delta   0.00000000   2.000000 2.0000000    FALSE
##     skew    0.10000000  10.000000 1.0000000    FALSE
##     shape   1.00000000  10.000000 4.0000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2  beta1
##      1      2      3      4      7
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
```

```
##
## R coded nlminb Solver:
##
##   0:     3988.4903: 0.0289013 0.100000 0.0500000 0.0500000 0.800000
##   1:     3982.7925: 0.0289022 0.0812051 0.0518990 0.0507098 0.791419
##   2:     3971.0729: 0.0289034 0.0793571 0.0665781 0.0638831 0.797629
##   3:     3968.6409: 0.0289056 0.0606196 0.0737000 0.0683265 0.794558
##   4:     3967.1381: 0.0289144 0.0558263 0.0857801 0.0687709 0.810738
##   5:     3964.2460: 0.0289288 0.0516123 0.0879044 0.0506220 0.819632
##   6:     3962.6050: 0.0289492 0.0501207 0.0803009 0.0231793 0.849804
##   7:     3962.1547: 0.0289494 0.0472054 0.0798800 0.0226517 0.848851
##   8:     3961.6575: 0.0289504 0.0470341 0.0805726 0.0226174 0.851909
##   9:     3961.1109: 0.0289528 0.0434000 0.0799149 0.0203244 0.856442
##  10:     3958.8865: 0.0289640 0.0346805 0.0755742 0.00832408 0.882654
##  11:     3958.1795: 0.0289675 0.0302031 0.0733101 0.00337533 0.892237
##  12:     3958.0756: 0.0289716 0.0284543 0.0711460 1.00000e-08 0.902898
##  13:     3957.1798: 0.0289749 0.0257309 0.0673698 1.00000e-08 0.906292
##  14:     3956.5583: 0.0289895 0.0206139 0.0552703 1.00000e-08 0.922841
##  15:     3956.5418: 0.0289896 0.0206967 0.0553922 5.97258e-05 0.922980
##  16:     3956.5370: 0.0289896 0.0205097 0.0554053 1.00000e-08 0.922983
##  17:     3956.5214: 0.0289985 0.0204975 0.0555476 1.00000e-08 0.923185
##  18:     3956.5125: 0.0290192 0.0202025 0.0556069 1.00000e-08 0.923311
##  19:     3956.5045: 0.0290642 0.0202007 0.0557048 1.00000e-08 0.923531
##  20:     3956.4995: 0.0291096 0.0200978 0.0556680 1.00000e-08 0.923558
##  21:     3956.3970: 0.0329775 0.0188449 0.0545532 1.00000e-08 0.926344
##  22:     3956.3192: 0.0368458 0.0188781 0.0523660 1.00000e-08 0.927665
##  23:     3956.2936: 0.0377198 0.0182198 0.0504413 1.00000e-08 0.930474
##  24:     3956.2815: 0.0368427 0.0175576 0.0491324 1.00000e-08 0.932303
##  25:     3956.2807: 0.0368940 0.0175227 0.0493540 1.00000e-08 0.932139
##  26:     3956.2806: 0.0368974 0.0175263 0.0493028 1.00000e-08 0.932178
##  27:     3956.2806: 0.0368950 0.0175270 0.0493050 1.00000e-08 0.932176
##
## Final Estimate of the Negative LLH:
##  LLH:  -8096.677    norm LLH:  -2.682796
##           mu        omega       alpha1       alpha2        beta1
## 6.800326e-04 5.954314e-06 4.930501e-02 1.000000e-08 9.321758e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu         omega        alpha1        alpha2         beta1
## mu     -12969188.01  3.873228e+08      -20759.3 -1.108793e+04  4.441394e+04
## omega  387322811.89 -6.914025e+12 -1049476238.1 -1.053447e+09 -1.473003e+09
## alpha1    -20759.30 -1.049476e+09     -261640.6 -2.609935e+05 -2.976602e+05
## alpha2    -11087.93 -1.053447e+09     -260993.5 -2.641601e+05 -2.998930e+05
## beta1      44413.94 -1.473003e+09     -297660.2 -2.998930e+05 -3.714942e+05
## attr(,"time")
## Time difference of 0.1039488 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.2786491 secs
```

```
gfit5 = garchFit(~garch(1,2),data = diff(log(google_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 2)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               1 2
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.01843157
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U          V    params includes
##     mu     -0.28901297   0.289013 0.0289013     TRUE
##     omega   0.00000100 100.000000 0.1000000     TRUE
##     alpha1  0.00000001   1.000000 0.1000000     TRUE
##     gamma1 -0.99999999   1.000000 0.1000000    FALSE
##     beta1   0.00000001   1.000000 0.4000000     TRUE
##     beta2   0.00000001   1.000000 0.4000000     TRUE
##     delta   0.00000000   2.000000 2.0000000    FALSE
##     skew    0.10000000  10.000000 1.0000000    FALSE
##     shape   1.00000000  10.000000 4.0000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1  beta2
##      1      2      3      5      6
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3987.6639: 0.0289013 0.100000 0.100000 0.400000 0.400000
##   1:     3983.5599: 0.0289018 0.0847937 0.100313 0.393012 0.393036
##   2:     3970.6871: 0.0289026 0.0815708 0.115381 0.399681 0.399873
##   3:     3967.7577: 0.0289050 0.0502375 0.133145 0.402345 0.403067
##   4:     3965.0695: 0.0289057 0.0567666 0.137530 0.411351 0.412707
##   5:     3958.8789: 0.0289120 0.0462375 0.128446 0.412288 0.419155
##   6:     3957.9904: 0.0289146 0.0392605 0.118931 0.417894 0.427233
##   7:     3956.8157: 0.0289240 0.0401686 0.106336 0.419272 0.435860
```

```
##   8:     3955.8856: 0.0289852 0.0249855 0.0865212 0.423570 0.469323
##   9:     3955.4678: 0.0289852 0.0245499 0.0858581 0.422952 0.468718
##  10:     3955.2427: 0.0289853 0.0255432 0.0852415 0.422858 0.468671
##  11:     3955.2027: 0.0289885 0.0252816 0.0831102 0.422008 0.469076
##  12:     3954.8686: 0.0289930 0.0256899 0.0820306 0.422308 0.471092
##  13:     3954.6932: 0.0290030 0.0248997 0.0787713 0.421749 0.474312
##  14:     3954.5115: 0.0290269 0.0235941 0.0736282 0.421099 0.481987
##  15:     3954.4400: 0.0290701 0.0221307 0.0703295 0.416899 0.489423
##  16:     3954.3395: 0.0291260 0.0225341 0.0706348 0.410441 0.495940
##  17:     3953.1715: 0.0304664 0.0295141 0.0918347 0.238649 0.640294
##  18:     3952.6323: 0.0309467 0.0319703 0.0992535 0.176479 0.692906
##  19:     3952.3245: 0.0312447 0.0334954 0.103857 0.137912 0.725547
##  20:     3952.2922: 0.0312447 0.0338271 0.103909 0.138166 0.725848
##  21:     3952.2804: 0.0312492 0.0336960 0.103723 0.137800 0.726091
##  22:     3952.2659: 0.0312592 0.0338491 0.103643 0.137277 0.726898
##  23:     3952.2457: 0.0312808 0.0337563 0.103580 0.135785 0.728120
##  24:     3952.0894: 0.0318432 0.0354264 0.107141 0.0996938 0.759715
##  25:     3951.5639: 0.0388981 0.0311684 0.0920353 0.106471 0.771094
##  26:     3951.4744: 0.0388981 0.0307297 0.0918409 0.106228 0.770863
##  27:     3951.4429: 0.0388972 0.0295735 0.0920222 0.106918 0.771808
##  28:     3951.2312: 0.0348203 0.0267663 0.0836794 0.0962366 0.792633
##  29:     3951.2283: 0.0339495 0.0270916 0.0845475 0.0959473 0.791819
##  30:     3951.2281: 0.0341076 0.0271666 0.0847104 0.0951312 0.792400
##  31:     3951.2280: 0.0341518 0.0271941 0.0847456 0.0953837 0.792070
##  32:     3951.2280: 0.0341429 0.0271874 0.0847407 0.0953243 0.792146
##  33:     3951.2280: 0.0341426 0.0271874 0.0847403 0.0953235 0.792147
##
## Final Estimate of the Negative LLH:
##  LLH:  -8101.73    norm LLH:  -2.68447
##           mu        omega       alpha1        beta1        beta2
## 6.293020e-04 9.236194e-06 8.474025e-02 9.532348e-02 7.921468e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1         beta1         beta2
## mu     -13179483.581  2.335969e+08 -3.811846e+03      41867.16      42060.94
## omega  233596857.630 -2.558883e+12 -3.870449e+08 -547377198.54 -546738925.64
## alpha1     -3811.846 -3.870449e+08 -9.348172e+04    -109376.58    -109098.21
## beta1      41867.163 -5.473772e+08 -1.093766e+05    -140135.73    -139542.14
## beta2      42060.944 -5.467389e+08 -1.090982e+05    -139542.14    -139435.26
## attr(,"time")
## Time difference of 0.05267906 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.4375098 secs
```

```
gfit6 = garchFit(~garch(2,2),data = diff(log(google_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
```

```
##  Formula Variance:          ~ garch(2, 2)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               2 2
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.01843157
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U          V    params includes
##     mu     -0.28901297   0.289013 0.0289013     TRUE
##     omega   0.00000100 100.000000 0.1000000     TRUE
##     alpha1  0.00000001   1.000000 0.0500000     TRUE
##     alpha2  0.00000001   1.000000 0.0500000     TRUE
##     gamma1 -0.99999999   1.000000 0.1000000    FALSE
##     gamma2 -0.99999999   1.000000 0.1000000    FALSE
##     beta1   0.00000001   1.000000 0.4000000     TRUE
##     beta2   0.00000001   1.000000 0.4000000     TRUE
##     delta   0.00000000   2.000000 2.0000000    FALSE
##     skew    0.10000000  10.000000 1.0000000    FALSE
##     shape   1.00000000  10.000000 4.0000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2  beta1  beta2
##      1      2      3      4      7      8
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3987.0910: 0.0289013 0.100000 0.0500000 0.0500000 0.400000 0.400000
##   1:     3986.8556: 0.0289020 0.0820867 0.0503950 0.0493393 0.390986 0.390978
##   2:     3972.3385: 0.0289022 0.0859952 0.0562887 0.0549129 0.395431 0.395472
##   3:     3963.7382: 0.0289042 0.0702623 0.0725964 0.0684315 0.398458 0.398682
##   4:     3961.8848: 0.0289044 0.0668773 0.0716974 0.0672258 0.396680 0.396916
##   5:     3960.9278: 0.0289061 0.0603893 0.0772363 0.0698013 0.399865 0.400398
##   6:     3960.7151: 0.0289063 0.0584631 0.0765634 0.0687109 0.398946 0.399513
##   7:     3960.4410: 0.0289072 0.0582319 0.0771481 0.0675248 0.400448 0.401222
##   8:     3960.1478: 0.0289090 0.0553515 0.0774311 0.0641160 0.402211 0.403417
##   9:     3958.0627: 0.0289197 0.0456944 0.0785706 0.0441773 0.415572 0.419758
##  10:     3957.4414: 0.0289248 0.0416158 0.0750988 0.0376573 0.418887 0.426892
##  11:     3955.4047: 0.0289714 0.0224075 0.0557039 1.56921e-05 0.436741 0.481180
##  12:     3955.1965: 0.0289715 0.0221841 0.0563501 0.000590296 0.437015 0.481461
```

```
##  13:     3955.0402: 0.0289723 0.0215112 0.0566992 0.000189460 0.436555 0.481398
##  14:     3954.9734: 0.0289791 0.0215337 0.0572799 1.00000e-08 0.436057 0.482880
##  15:     3954.8514: 0.0289974 0.0208966 0.0571181 1.00000e-08 0.433752 0.485193
##  16:     3954.7545: 0.0290401 0.0217390 0.0588088 1.00000e-08 0.428432 0.488648
##  17:     3954.6406: 0.0291223 0.0219997 0.0599113 1.00000e-08 0.418115 0.496903
##  18:     3954.6111: 0.0292555 0.0190069 0.0574515 1.00000e-08 0.402978 0.518593
##  19:     3954.5839: 0.0294015 0.0194353 0.0596801 1.00000e-08 0.382340 0.535302
##  20:     3954.1062: 0.0295467 0.0207411 0.0621043 1.00000e-08 0.362392 0.552768
##  21:     3953.8786: 0.0296762 0.0212242 0.0688414 1.00000e-08 0.341370 0.567875
##  22:     3952.6572: 0.0304071 0.0256203 0.0793977 1.00000e-08 0.223336 0.669796
##  23:     3951.5197: 0.0311588 0.0292489 0.0856074 0.00234439 0.126666 0.753866
##  24:     3951.3701: 0.0311588 0.0296168 0.0858604 0.00268096 0.126969 0.754190
##  25:     3951.3498: 0.0311624 0.0293123 0.0858697 0.00293707 0.126492 0.754518
##  26:     3950.9758: 0.0315232 0.0308729 0.0881883 0.00457142 0.0868904 0.788856
##  27:     3950.9595: 0.0315232 0.0308856 0.0882543 0.00485984 0.0869911 0.788970
##  28:     3950.9463: 0.0315244 0.0306346 0.0881491 0.00497329 0.0868640 0.788891
##  29:     3950.9305: 0.0315388 0.0307161 0.0881538 0.00527760 0.0867066 0.789158
##  30:     3950.9120: 0.0315701 0.0305874 0.0879692 0.00555252 0.0860851 0.789450
##  31:     3950.8855: 0.0316348 0.0307694 0.0878210 0.00610143 0.0850637 0.790286
##  32:     3950.7484: 0.0323005 0.0318851 0.0859870 0.00966011 0.0738459 0.797869
##  33:     3950.6997: 0.0330626 0.0318626 0.0847429 0.0105706 0.0702824 0.802410
##  34:     3950.6403: 0.0348296 0.0298981 0.0808678 0.0110970 0.0660730 0.811589
##  35:     3950.6389: 0.0330595 0.0306455 0.0797029 0.0131225 0.0580735 0.817732
##  36:     3950.6323: 0.0339525 0.0306420 0.0796708 0.0131301 0.0603322 0.815498
##  37:     3950.6310: 0.0346228 0.0303766 0.0804315 0.0122456 0.0621043 0.814266
##  38:     3950.6303: 0.0342658 0.0303607 0.0800181 0.0125419 0.0625416 0.813918
##  39:     3950.6303: 0.0345012 0.0305112 0.0799671 0.0126835 0.0615895 0.814573
##  40:     3950.6301: 0.0344109 0.0304342 0.0800593 0.0125764 0.0620287 0.814265
##  41:     3950.6301: 0.0344076 0.0304319 0.0800465 0.0125761 0.0620420 0.814265
##
## Final Estimate of the Negative LLH:
##  LLH:  -8102.328    norm LLH:  -2.684668
##           mu        omega       alpha1       alpha2        beta1        beta2
## 6.341861e-04 1.033839e-05 8.004652e-02 1.257615e-02 6.204197e-02 8.142647e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1        alpha2         beta1
## mu     -13180604.425  2.214638e+08 -8.016349e+03      12727.46      35539.08
## omega  221463834.666 -2.117503e+12 -3.159626e+08 -331005293.29 -451983967.23
## alpha1     -8016.349 -3.159626e+08 -7.612776e+04     -78661.26     -89005.21
## alpha2     12727.461 -3.310053e+08 -7.866126e+04     -93774.54     -96080.99
## beta1      35539.077 -4.519840e+08 -8.900521e+04     -96080.99    -115241.58
## beta2      36227.198 -4.513455e+08 -8.900857e+04     -94676.41    -114548.96
##                beta2
## mu          36227.20
## omega  -451345474.32
## alpha1     -89008.57
## alpha2     -94676.41
## beta1     -114548.96
## beta2     -114581.39
## attr(,"time")
## Time difference of 0.06761098 secs
##
## --- END OF TRACE ---
```

```
##
##
## Time to Estimate Parameters:
##  Time difference of 0.3077481 secs
```

```
gfit7 = garchFit(~arma(1,3)+garch(1,0),data = diff(log(google_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(1, 3)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(1, 0)
##  ARMA Order:                 1 3
##  Max ARMA Order:             3
##  GARCH Order:                1 0
##  Max GARCH Order:            1
##  Maximum Order:              3
##  Conditional Dist:           norm
##  h.start:                    4
##  llh.start:                  1
##  Length of Series:           3018
##  Recursion Init:             mci
##  Series Scale:               0.01843157
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U          V      params includes
##     mu     -0.28901297   0.289013  0.02913901     TRUE
##     ar1    -0.99999999   1.000000 -0.91616431     TRUE
##     ma1    -0.99999999   1.000000  0.89235197     TRUE
##     ma2    -0.99999999   1.000000 -0.01815199     TRUE
##     ma3    -0.99999999   1.000000 -0.02031633     TRUE
##     omega   0.00000100 100.000000  0.10000000     TRUE
##     alpha1  0.00000001   1.000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.000000  0.10000000    FALSE
##     delta   0.00000000   2.000000  2.00000000    FALSE
##     skew    0.10000000  10.000000  1.00000000    FALSE
##     shape   1.00000000  10.000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1    ma2    ma3  omega alpha1
##      1      2      3      4      5      6      7
##  Persistence:                 0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     9224.4794: 0.0291390 -0.916164 0.892352 -0.0181520 -0.0203163 0.100000 0.100000
##   1:     4269.5237: 0.0291411 -0.942185 0.867846 0.0114806 -0.0549832  1.06312 0.362758
```

```
##   2:     4250.8978: 0.0291415 -0.932166 0.874637 0.00412697 -0.0468538  1.05815 0.360600
##   3:     4226.4579: 0.0291424 -0.913215 0.888491 -0.0101344 -0.0309173  1.04597 0.355371
##   4:     4217.1043: 0.0291443 -0.903052 0.897348 -0.0160470 -0.0209360  1.01896 0.343602
##   5:     4207.1036: 0.0291483 -0.902177 0.884064 0.00290688 -0.0292530 0.999727 0.329165
##   6:     4190.9530: 0.0291607 -0.863094 0.879170 0.0133751 -0.0118436 0.958920 0.295892
##   7:     4171.4067: 0.0292014 -0.821552 0.795465 0.0535688 0.0115887 0.881902 0.249898
##   8:     4165.7367: 0.0292411 -0.748709 0.805217 0.00685830 -0.0690678 0.813522 0.263313
##   9:     4157.4654: 0.0292751 -0.802990 0.738934 -0.00785734 0.00489903 0.743613 0.295406
##  10:     4155.8609: 0.0292774 -0.792693 0.750748 -0.00900216 0.00370242 0.742611 0.294531
##  11:     4154.9275: 0.0292798 -0.793347 0.751743 0.00194777 -0.00756368 0.741719 0.293499
##  12:     4153.0661: 0.0292948 -0.782920 0.775075 0.0181938 0.00105843 0.740041 0.295942
##  13:     4152.7973: 0.0293341 -0.790182 0.790815 0.0442645 -0.00142557 0.740703 0.292503
##  14:     4151.8403: 0.0294059 -0.801708 0.813406 0.0342279 -0.00444118 0.745106 0.277608
##  15:     4150.9988: 0.0296321 -0.850897 0.861953 0.0285003 -0.0121769 0.725588 0.317488
##  16:     4150.9390: 0.0296329 -0.852155 0.861370 0.0296233 -0.0129736 0.725603 0.316967
##  17:     4150.9046: 0.0296352 -0.852710 0.862569 0.0293726 -0.0121241 0.725635 0.315721
##  18:     4150.8432: 0.0296451 -0.856052 0.864617 0.0299865 -0.0126233 0.725509 0.315234
##  19:     4150.3338: 0.0298765 -0.914213 0.921207 0.0239766 -0.0153420 0.720123 0.323661
##  20:     4150.2098: 0.0298961 -0.908081 0.922720 0.0303058 -0.0122850 0.723225 0.313985
##  21:     4150.1837: 0.0301616 -0.910812 0.923814 0.0391950 -0.00789401 0.724039 0.314998
##  22:     4150.0678: 0.0304565 -0.907953 0.921083 0.0330697 -0.0120893 0.725262 0.310926
##  23:     4150.0634: 0.0304575 -0.907704 0.921280 0.0328361 -0.0116380 0.725352 0.310697
##  24:     4150.0592: 0.0304691 -0.907910 0.921008 0.0331095 -0.0116448 0.725458 0.310458
##  25:     4150.0543: 0.0305014 -0.907733 0.921016 0.0330210 -0.0111756 0.725634 0.310082
##  26:     4150.0475: 0.0305742 -0.907861 0.920626 0.0332561 -0.0111582 0.725826 0.309724
##  27:     4149.9056: 0.0348163 -0.903404 0.912965 0.0290831 -0.0120224 0.732040 0.299065
##  28:     4149.8076: 0.0390687 -0.903369 0.914073 0.0376727 -0.00416132 0.734330 0.297418
##  29:     4149.7854: 0.0433169 -0.901649 0.923290 0.0412352 -0.0107355 0.729462 0.292856
##  30:     4149.6039: 0.0475732 -0.906816 0.919382 0.0353992 -0.00801692 0.727575 0.297104
##  31:     4149.5805: 0.0493363 -0.909271 0.921866 0.0362755 -0.00872166 0.732658 0.300279
##  32:     4149.5643: 0.0511104 -0.908817 0.923023 0.0351287 -0.00818445 0.732262 0.300632
##  33:     4149.5160: 0.0528846 -0.909090 0.922479 0.0356610 -0.00851094 0.731041 0.300172
##  34:     4149.4684: 0.0599834 -0.908976 0.922484 0.0354524 -0.00841900 0.728128 0.299503
##  35:     4149.4525: 0.0635240 -0.910022 0.922755 0.0341561 -0.00959514 0.729730 0.301261
##  36:     4149.4511: 0.0640197 -0.909960 0.922766 0.0351214 -0.00870690 0.730252 0.300550
##  37:     4149.4506: 0.0644358 -0.909362 0.921348 0.0343784 -0.00882499 0.730000 0.300046
##  38:     4149.4505: 0.0643721 -0.909731 0.921944 0.0345818 -0.00878032 0.730043 0.300129
##  39:     4149.4505: 0.0643580 -0.909647 0.921882 0.0346011 -0.00878007 0.730043 0.300179
##  40:     4149.4505: 0.0643632 -0.909668 0.921892 0.0345910 -0.00878100 0.730043 0.300162
##  41:     4149.4505: 0.0643632 -0.909668 0.921892 0.0345912 -0.00878097 0.730043 0.300162
##
## Final Estimate of the Negative LLH:
##  LLH:  -7903.507    norm LLH:  -2.61879
##           mu           ar1           ma1           ma2           ma3
##  0.0011863142 -0.9096683055  0.9218922717  0.0345911942 -0.0087809695
##         omega        alpha1
##  0.0002480122  0.3001616360
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         ar1        ma1          ma2         ma3
## mu     -2863805.9685  -7742.7833  -2345.523    1824.4908  -1133.4519
## ar1       -7742.7833 -13732.2048 -12520.033   13023.5242 -11673.8565
## ma1       -2345.5227 -12520.0330 -12237.263   12558.5888 -11137.9071
## ma2        1824.4908  13023.5242  12558.589  -15030.9972  13465.8087
```

```
## ma3       -1133.4519 -11673.8565 -11137.907   13465.8087 -14955.9682
## omega   -292290.1752 406261.1230 323825.017 -475912.2888 748271.1009
## alpha1      737.7785   -187.8256   -152.814     229.8932   -442.8375
##                omega        alpha1
## mu     -2.922902e+05      737.7785
## ar1     4.062611e+05     -187.8256
## ma1     3.238250e+05     -152.8140
## ma2    -4.759123e+05      229.8932
## ma3     7.482711e+05     -442.8375
## omega  -1.800676e+10 -2081560.2207
## alpha1 -2.081560e+06    -1015.5663
## attr(,"time")
## Time difference of 0.071311 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.7666471 secs
```

```
gfit8 = garchFit(~arma(1,3)+garch(1,1),data = diff(log(google_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                1 3
##  Max ARMA Order:            3
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.01843157
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U          V      params includes
##     mu     -0.28901297   0.289013  0.02913901     TRUE
##     ar1    -0.99999999   1.000000 -0.91616431     TRUE
##     ma1    -0.99999999   1.000000  0.89235197     TRUE
##     ma2    -0.99999999   1.000000 -0.01815199     TRUE
##     ma3    -0.99999999   1.000000 -0.02031633     TRUE
##     omega   0.00000100 100.000000  0.10000000     TRUE
##     alpha1  0.00000001   1.000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.000000  0.10000000    FALSE
##     beta1   0.00000001   1.000000  0.80000000     TRUE
```

```
##     delta   0.00000000   2.000000  2.00000000    FALSE
##     skew    0.10000000  10.000000  1.00000000    FALSE
##     shape   1.00000000  10.000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1    ma2    ma3  omega alpha1  beta1
##      1      2      3      4      5      6      7      9
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3989.7231: 0.0291390 -0.916164 0.892352 -0.0181520 -0.0203163 0.100000 0.100000 0.800000
##   1:     3986.5930: 0.0291401 -0.913051 0.894668 -0.0189318 -0.0185565 0.0796301 0.101946 0.792108
##   2:     3972.9882: 0.0291418 -0.909871 0.896826 -0.0188056 -0.0173172 0.0794274 0.120923 0.803211
##   3:     3968.4871: 0.0291445 -0.910443 0.894964 -0.0133304 -0.0205765 0.0607518 0.130941 0.805704
##   4:     3965.4685: 0.0291502 -0.901729 0.899844 -0.0111903 -0.0187226 0.0552805 0.136009 0.824043
##   5:     3962.2255: 0.0291589 -0.897774 0.898125 -0.000773356 -0.0224983 0.0457437 0.125521 0.836588
##   6:     3960.0128: 0.0291696 -0.891123 0.898271 0.00423395 -0.0186726 0.0420118 0.112341 0.851688
##   7:     3959.8546: 0.0291705 -0.891578 0.897378 0.00524508 -0.0190366 0.0425012 0.111166 0.853532
##   8:     3959.5828: 0.0291716 -0.891318 0.897039 0.00573839 -0.0186723 0.0410572 0.109279 0.854519
##   9:     3953.9719: 0.0292194 -0.880625 0.882582 0.0245956 -0.00146776 0.0191787 0.0520212 0.925372
##  10:     3953.1689: 0.0292194 -0.880598 0.882597 0.0245803 -0.00146542 0.0201313 0.0526593 0.926141
##  11:     3953.0482: 0.0292206 -0.880520 0.882417 0.0241959 -0.00154818 0.0192262 0.0525305 0.927076
##  12:     3953.0393: 0.0292207 -0.880477 0.882440 0.0241740 -0.00154812 0.0192363 0.0526347 0.927214
##  13:     3953.0325: 0.0292208 -0.880435 0.882462 0.0241530 -0.00154936 0.0190635 0.0526256 0.927213
##  14:     3953.0186: 0.0292216 -0.880286 0.882465 0.0240550 -0.00176862 0.0190567 0.0526496 0.927436
##  15:     3952.9995: 0.0292233 -0.879962 0.882450 0.0238521 -0.00227964 0.0188333 0.0525214 0.927648
##  16:     3952.9714: 0.0292279 -0.879256 0.882251 0.0234171 -0.00332813 0.0187565 0.0523714 0.928105
##  17:     3952.9496: 0.0292355 -0.878495 0.881428 0.0228578 -0.00388285 0.0185392 0.0521749 0.928348
##  18:     3952.8263: 0.0293626 -0.867776 0.865768 0.0172486 -0.00540237 0.0174738 0.0510374 0.931042
##  19:     3952.6943: 0.0294981 -0.852974 0.853795 0.0157492 -0.0113223 0.0173717 0.0500125 0.931368
##  20:     3952.6920: 0.0295693 -0.845757 0.848214 0.0168800 -0.0126350 0.0177197 0.0500438 0.931910
##  21:     3952.5928: 0.0296074 -0.842451 0.845650 0.0186827 -0.0118908 0.0174057 0.0499604 0.931741
##  22:     3952.5368: 0.0297626 -0.831603 0.835613 0.0223761 -0.00969936 0.0176141 0.0502919 0.931340
##  23:     3952.5193: 0.0299554 -0.821872 0.824792 0.0198144 -0.0112504 0.0179149 0.0501522 0.930506
##  24:     3952.5047: 0.0301972 -0.812164 0.815126 0.0181361 -0.0135398 0.0184421 0.0501153 0.930558
##  25:     3952.4761: 0.0306394 -0.809235 0.816573 0.0207993 -0.0161800 0.0173982 0.0487844 0.932631
##  26:     3952.3863: 0.0314516 -0.821982 0.826379 0.0206369 -0.0109073 0.0170622 0.0484644 0.933281
##  27:     3952.3818: 0.0314517 -0.821979 0.826379 0.0206334 -0.0109019 0.0171150 0.0485043 0.933331
##  28:     3952.3806: 0.0314518 -0.821966 0.826376 0.0206143 -0.0108718 0.0170556 0.0485346 0.933362
##  29:     3952.3803: 0.0314558 -0.821912 0.826318 0.0205904 -0.0108617 0.0170740 0.0485613 0.933396
##  30:     3952.3792: 0.0314598 -0.821859 0.826256 0.0205714 -0.0108618 0.0170473 0.0485520 0.933381
##  31:     3952.3781: 0.0314678 -0.821745 0.826132 0.0205180 -0.0108342 0.0170374 0.0485764 0.933423
##  32:     3952.2751: 0.0334249 -0.796695 0.796386 0.0126892 -0.0133977 0.0177673 0.0492607 0.931819
##  33:     3952.0429: 0.0391430 -0.795093 0.794565 0.0145445 -0.0111926 0.0158523 0.0449581 0.937682
##  34:     3951.7166: 0.0514611 -0.781612 0.779565 0.0169411 -0.0100311 0.0168932 0.0501332 0.931983
##  35:     3951.6778: 0.0637909 -0.779567 0.790115 0.0205255 -0.0106411 0.0174710 0.0498612 0.931651
##  36:     3951.5140: 0.0690621 -0.768252 0.774357 0.0203175 -0.0116643 0.0172834 0.0478255 0.933976
##  37:     3951.4560: 0.0661818 -0.779145 0.780167 0.0167569 -0.0133694 0.0163356 0.0465951 0.935884
##  38:     3951.4516: 0.0652878 -0.771006 0.773335 0.0201925 -0.0107474 0.0165300 0.0467447 0.935556
##  39:     3951.4489: 0.0661739 -0.774700 0.776970 0.0187747 -0.0119326 0.0164635 0.0466524 0.935764
```

```
##  40:     3951.4487: 0.0660812 -0.774698 0.776866 0.0188312 -0.0118839 0.0164554 0.0466401 0.935758
##  41:     3951.4487: 0.0660716 -0.774804 0.776988 0.0188660 -0.0118431 0.0164557 0.0466411 0.935762
##  42:     3951.4487: 0.0660803 -0.774829 0.777021 0.0188681 -0.0118464 0.0164562 0.0466417 0.935760
##
## Final Estimate of the Negative LLH:
##  LLH:  -8101.509    norm LLH:  -2.684397
##           mu           ar1           ma1           ma2           ma3
##  1.217964e-03 -7.748293e-01  7.770207e-01  1.886811e-02 -1.184643e-02
##         omega        alpha1         beta1
##  5.590550e-06  4.664165e-02  9.357605e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1          ma1          ma2           ma3
## mu     -4.078600e+06 -3.811159e+03   -766.95219    1492.9868   -1055.98505
## ar1    -3.811159e+03 -6.204402e+03  -6067.45961    4724.7881   -3753.10279
## ma1    -7.669522e+02 -6.067460e+03  -5971.62263    4599.6282   -3579.14309
## ma2     1.492987e+03  4.724788e+03   4599.62821   -6113.9658    4763.81392
## ma3    -1.055985e+03 -3.753103e+03  -3579.14309    4763.8139   -6199.09035
## omega   2.123706e+08  1.127909e+06 747708.48844 1062176.6872 1280634.02151
## alpha1 -1.139052e+04  2.499277e+00    -29.86862    -222.7370     -25.62588
## beta1   2.504179e+04 -7.268877e+01    -82.93870     180.6158      21.74884
##                omega        alpha1         beta1
## mu      2.123706e+08 -1.139052e+04  2.504179e+04
## ar1     1.127909e+06  2.499277e+00 -7.268877e+01
## ma1     7.477085e+05 -2.986862e+01 -8.293870e+01
## ma2     1.062177e+06 -2.227370e+02  1.806158e+02
## ma3     1.280634e+06 -2.562588e+01  2.174884e+01
## omega  -7.725736e+12 -1.179735e+09 -1.644151e+09
## alpha1 -1.179735e+09 -2.948638e+05 -3.334024e+05
## beta1  -1.644151e+09 -3.334024e+05 -4.141420e+05
## attr(,"time")
## Time difference of 0.110265 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.9115231 secs
```

```
gfit9 = garchFit(~arma(1,3)+garch(2,0),data = diff(log(google_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 0)
##  ARMA Order:                1 3
##  Max ARMA Order:            3
##  GARCH Order:               2 0
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
```

```
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.01843157
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U          V      params includes
##     mu     -0.28901297   0.289013  0.02913901     TRUE
##     ar1    -0.99999999   1.000000 -0.91616431     TRUE
##     ma1    -0.99999999   1.000000  0.89235197     TRUE
##     ma2    -0.99999999   1.000000 -0.01815199     TRUE
##     ma3    -0.99999999   1.000000 -0.02031633     TRUE
##     omega   0.00000100 100.000000  0.10000000     TRUE
##     alpha1  0.00000001   1.000000  0.05000000     TRUE
##     alpha2  0.00000001   1.000000  0.05000000     TRUE
##     gamma1 -0.99999999   1.000000  0.10000000    FALSE
##     gamma2 -0.99999999   1.000000  0.10000000    FALSE
##     delta   0.00000000   2.000000  2.00000000    FALSE
##     skew    0.10000000  10.000000  1.00000000    FALSE
##     shape   1.00000000  10.000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1    ma2    ma3  omega alpha1 alpha2
##      1      2      3      4      5      6      7      8
##  Persistence:                 0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8228.4001: 0.0291390 -0.916164 0.892352 -0.0181520 -0.0203163 0.100000 0.0500000 0.0500000
##   1:     4247.7739: 0.0291432 -0.914390 0.886283 -0.0122731 -0.0276915 0.949629 0.402760 0.441870
##   2:     4243.5990: 0.0291437 -0.912183 0.887722 -0.0136151 -0.0261075 0.944563 0.400320 0.439668
##   3:     4218.3269: 0.0291478 -0.898029 0.896178 -0.0210349 -0.0165760 0.901480 0.379540 0.420999
##   4:     4158.7532: 0.0291594 -0.898778 0.875141 0.00350991 -0.0340991 0.801900 0.329433 0.378858
##   5:     4092.4460: 0.0291889 -0.839478 0.877127 -0.00799913 -0.00570207 0.600767 0.226613 0.296793
##   6:     4090.6290: 0.0292078 -0.837869 0.814007 0.0402303 -0.0504484 0.562942 0.205087 0.289530
##   7:     4069.5092: 0.0292290 -0.804822 0.796861 0.0237366 -0.0217222 0.557066 0.201792 0.295556
##   8:     4067.5947: 0.0293167 -0.815746 0.809284 -0.0133860 -0.0165526 0.552059 0.205814 0.324480
##   9:     4066.8400: 0.0294637 -0.807199 0.770204 -0.0133523 -0.0117695 0.540011 0.204614 0.352229
##  10:     4066.4344: 0.0296175 -0.765187 0.765985 -0.00365773 -0.0316970 0.531348 0.203286 0.365890
##  11:     4066.1942: 0.0296222 -0.764854 0.765303 -0.00653262 -0.0268561 0.532350 0.203475 0.365014
##  12:     4066.0367: 0.0296285 -0.767875 0.760772 -0.00547337 -0.0259480 0.533413 0.203578 0.363867
##  13:     4065.8352: 0.0296853 -0.762262 0.756526 -0.00841451 -0.0218168 0.536697 0.203220 0.357056
##  14:     4065.5564: 0.0298466 -0.752323 0.742574 -0.00510731 -0.0225133 0.541654 0.201576 0.343548
##  15:     4065.3964: 0.0300957 -0.738754 0.732548 0.00280890 -0.0119613 0.539609 0.203068 0.346072
##  16:     4065.1815: 0.0303427 -0.730122 0.715896 -0.00617187 -0.0169440 0.540839 0.200866 0.349071
##  17:     4065.1725: 0.0303491 -0.729402 0.715588 -0.00664580 -0.0159382 0.540932 0.200867 0.348762
##  18:     4065.1650: 0.0303559 -0.729494 0.714380 -0.00605032 -0.0161080 0.541023 0.200864 0.348466
##  19:     4065.1460: 0.0304005 -0.727687 0.713227 -0.00664579 -0.0155030 0.540918 0.201245 0.348066
```

```
##  20:     4064.6093: 0.0335562 -0.648023 0.634907 -0.0143309 -0.0242758 0.548666 0.191828 0.347573
##  21:     4064.0061: 0.0367455 -0.574772 0.556247 -0.0119658 -0.0136870 0.525694 0.209067 0.351418
##  22:     4063.9329: 0.0379531 -0.549237 0.529195 0.00302293 -0.00201143 0.546119 0.208505 0.362738
##  23:     4063.6672: 0.0384854 -0.534673 0.521975 0.00287713 0.00136892 0.536122 0.191481 0.360612
##  24:     4063.5339: 0.0390350 -0.527223 0.512553 0.00145185 -0.00985398 0.531785 0.208175 0.351794
##  25:     4063.3312: 0.0397475 -0.514149 0.500120 0.00108130 -0.00370927 0.535358 0.204944 0.355630
##  26:     4063.3083: 0.0397637 -0.515589 0.496477 8.98529e-05 -0.00607629 0.536669 0.202789 0.354497
##  27:     4063.2704: 0.0398580 -0.512053 0.495917 -0.00161205 -0.00443515 0.537119 0.202011 0.354581
##  28:     4063.2465: 0.0399668 -0.510307 0.492668 -0.000874760 -0.00544730 0.537460 0.201353 0.354566
##  29:     4063.0563: 0.0419296 -0.470075 0.455522 0.00657709 0.00168986 0.536365 0.201465 0.361081
##  30:     4062.7613: 0.0438676 -0.432627 0.415079 0.000226739 -0.00444850 0.527929 0.208891 0.356317
##  31:     4062.5179: 0.0459472 -0.399529 0.385425 -0.00929594 -0.00673059 0.539952 0.189730 0.357224
##  32:     4062.1430: 0.0478677 -0.363174 0.344160 0.00219546 -0.00221474 0.542195 0.198788 0.348191
##  33:     4061.9255: 0.0496387 -0.317277 0.302992 0.00132224 0.00337433 0.535254 0.204864 0.357542
##  34:     4061.7179: 0.0507107 -0.265902 0.244318 0.00331166 -0.00121430 0.527462 0.202975 0.372173
##  35:     4061.3663: 0.0517544 -0.208455 0.192198 0.00684506 0.00341380 0.533349 0.199667 0.353180
##  36:     4061.3212: 0.0517547 -0.209693 0.189067 0.00329420 0.00390769 0.538720 0.201697 0.354559
##  37:     4061.2904: 0.0516934 -0.204006 0.185321 0.00302866 0.00298571 0.536488 0.200421 0.353752
##  38:     4061.2701: 0.0516466 -0.199809 0.179301 0.00258218 0.00279985 0.537934 0.200772 0.354112
##  39:     4060.7446: 0.0493769 -0.0348899 0.0152353 0.0129322 0.0100254 0.537066 0.206243 0.353727
##  40:     4060.2605: 0.0377354 0.203620 -0.226826 0.0106313 0.0152110 0.535483 0.198453 0.359366
##  41:     4060.0601: 0.0249211 0.401390 -0.429213 0.0268571 -0.00408638 0.535635 0.196949 0.335781
##  42:     4059.7733: 0.0199382 0.475141 -0.493635 0.0178832 -0.0124436 0.538876 0.201182 0.354879
##  43:     4059.6289: 0.0213831 0.502001 -0.522386 0.0171265 -0.0101759 0.536126 0.201967 0.356170
##  44:     4059.6084: 0.0198712 0.526262 -0.549328 0.0168512 -0.00715634 0.534506 0.201325 0.356087
##  45:     4059.5900: 0.0189522 0.564762 -0.588789 0.0157875 -0.00982338 0.533702 0.200134 0.357566
##  46:     4059.5843: 0.0189512 0.565619 -0.587902 0.0161940 -0.00991715 0.534321 0.200411 0.357912
##  47:     4059.5830: 0.0189161 0.565937 -0.588517 0.0159454 -0.0103703 0.534534 0.200492 0.357997
##  48:     4059.5816: 0.0184979 0.572475 -0.594981 0.0160720 -0.0106849 0.534525 0.200461 0.358144
##  49:     4059.5814: 0.0183177 0.570498 -0.592707 0.0160598 -0.0105968 0.534945 0.199958 0.358384
##  50:     4059.5811: 0.0181890 0.576023 -0.598412 0.0161843 -0.0108179 0.534846 0.200185 0.358369
##  51:     4059.5811: 0.0182796 0.573895 -0.596258 0.0161358 -0.0107349 0.534740 0.200329 0.358319
##  52:     4059.5811: 0.0182727 0.574150 -0.596532 0.0161620 -0.0107639 0.534789 0.200155 0.358387
##  53:     4059.5811: 0.0182538 0.574433 -0.596806 0.0161587 -0.0107622 0.534797 0.200224 0.358354
##  54:     4059.5811: 0.0182610 0.574314 -0.596688 0.0161575 -0.0107601 0.534785 0.200226 0.358357
##
## Final Estimate of the Negative LLH:
##  LLH:  -7993.377    norm LLH:  -2.648567
##            mu           ar1           ma1           ma2           ma3
##  0.0003365792  0.5743142009 -0.5966883472  0.0161574590 -0.0107600601
##         omega        alpha1        alpha2
##  0.0001816787  0.2002258409  0.3583565590
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu          ar1          ma1         ma2          ma3
## mu     -74386203.731 -80054.09558 -21314.61110 -29120.5713 -47304.36014
## ar1        -80054.096  -3179.09534  -3046.78952  -1841.8786  -1320.22360
## ma1        -21314.611  -3046.78952  -2983.48169  -1746.6866  -1096.56372
## ma2        -29120.571  -1841.87856  -1746.68659  -3052.0299  -2470.18559
## ma3        -47304.360  -1320.22360  -1096.56372  -2470.1856  -5091.42471
## omega     3064344.093 557504.62169 564542.62162 262484.9148 -57107.32745
## alpha1      -3651.460     44.17981     49.43425    111.6758    151.19860
## alpha2       1213.907   -218.99209   -224.29382   -160.5370    -79.12214
##                omega        alpha1        alpha2
```

```
## mu      3.064344e+06 -3.651460e+03  1.213907e+03
## ar1     5.575046e+05  4.417981e+01 -2.189921e+02
## ma1     5.645426e+05  4.943425e+01 -2.242938e+02
## ma2     2.624849e+05  1.116758e+02 -1.605370e+02
## ma3    -5.710733e+04  1.511986e+02 -7.912214e+01
## omega  -2.342495e+10 -2.885808e+06 -2.442839e+06
## alpha1 -2.885808e+06 -1.371441e+03 -3.617312e+02
## alpha2 -2.442839e+06 -3.617312e+02 -7.857238e+02
## attr(,"time")
## Time difference of 0.1125941 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.290958 secs
```

```
gfit10 = garchFit(~arma(1,3)+garch(2,1),data = diff(log(google_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(1, 3)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(2, 1)
##  ARMA Order:                 1 3
##  Max ARMA Order:             3
##  GARCH Order:                2 1
##  Max GARCH Order:            2
##  Maximum Order:              3
##  Conditional Dist:           norm
##  h.start:                    4
##  llh.start:                  1
##  Length of Series:           3018
##  Recursion Init:             mci
##  Series Scale:               0.01843157
##
## Parameter Initialization:
##  Initial Parameters:           $params
##  Limits of Transformations:    $U, $V
##  Which Parameters are Fixed?   $includes
##  Parameter Matrix:
##                      U          V      params includes
##     mu     -0.28901297   0.289013  0.02913901     TRUE
##     ar1    -0.99999999   1.000000 -0.91616431     TRUE
##     ma1    -0.99999999   1.000000  0.89235197     TRUE
##     ma2    -0.99999999   1.000000 -0.01815199     TRUE
##     ma3    -0.99999999   1.000000 -0.02031633     TRUE
##     omega   0.00000100 100.000000  0.10000000     TRUE
##     alpha1  0.00000001   1.000000  0.05000000     TRUE
##     alpha2  0.00000001   1.000000  0.05000000     TRUE
##     gamma1 -0.99999999   1.000000  0.10000000    FALSE
##     gamma2 -0.99999999   1.000000  0.10000000    FALSE
##     beta1   0.00000001   1.000000  0.80000000     TRUE
##     delta   0.00000000   2.000000  2.00000000    FALSE
```

```
##     skew    0.10000000  10.000000  1.00000000    FALSE
##     shape   1.00000000  10.000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1    ma2    ma3  omega alpha1 alpha2  beta1
##      1      2      3      4      5      6      7      8     11
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3990.2648: 0.0291390 -0.916164 0.892352 -0.0181520 -0.0203163 0.100000 0.0500000 0.0500000 0
##   1:     3986.5067: 0.0291402 -0.912737 0.894935 -0.0192802 -0.0182254 0.0786907 0.0524235 0.0505950
##   2:     3971.8354: 0.0291417 -0.909719 0.897051 -0.0195102 -0.0168203 0.0772174 0.0692898 0.0651762
##   3:     3969.6087: 0.0291450 -0.910107 0.895251 -0.0137831 -0.0201566 0.0570251 0.0785359 0.0703012
##   4:     3966.5616: 0.0291557 -0.898474 0.900847 -0.00651213 -0.0202609 0.0553724 0.0861391 0.064745
##   5:     3963.6346: 0.0291689 -0.893307 0.898959 0.00618325 -0.0232668 0.0533850 0.0851657 0.0480805
##   6:     3963.3331: 0.0291697 -0.891887 0.899822 0.00532001 -0.0218132 0.0524100 0.0845265 0.0466492
##   7:     3963.3207: 0.0291707 -0.891399 0.899752 0.00539426 -0.0211366 0.0535907 0.0851443 0.0463133
##   8:     3963.0320: 0.0291709 -0.891321 0.899723 0.00542345 -0.0210179 0.0524567 0.0845887 0.0455636
##   9:     3961.9559: 0.0291837 -0.886440 0.897759 0.00741303 -0.0137729 0.0448069 0.0810709 0.0300375
##  10:     3956.4639: 0.0292110 -0.883877 0.884833 0.0160126 -0.00498652 0.0290418 0.0823777 0.0073609
##  11:     3954.5409: 0.0292522 -0.873454 0.876458 0.0214188 -0.00956611 0.0228716 0.0508886 1.00000e-0
##  12:     3953.8948: 0.0292522 -0.873405 0.876493 0.0213795 -0.00952129 0.0232553 0.0516294 0.0006298
##  13:     3953.5914: 0.0292523 -0.873300 0.876566 0.0212954 -0.00942467 0.0220743 0.0519344 0.0006817
##  14:     3953.3123: 0.0292532 -0.873064 0.876616 0.0210744 -0.00990217 0.0219097 0.0531058 1.00000e-0
##  15:     3952.8019: 0.0292571 -0.872296 0.876711 0.0206104 -0.0116265 0.0193579 0.0514706 1.00000e-08
##  16:     3952.6448: 0.0292668 -0.871826 0.875652 0.0198870 -0.0122534 0.0179494 0.0501788 1.00000e-08
##  17:     3952.5862: 0.0292882 -0.871577 0.872153 0.0196622 -0.00945421 0.0169997 0.0494432 1.00000e-0
##  18:     3952.5101: 0.0293202 -0.868314 0.869442 0.0212610 -0.00844705 0.0173440 0.0496552 1.00000e-0
##  19:     3952.4783: 0.0293491 -0.864323 0.868443 0.0223779 -0.0103261 0.0171473 0.0493097 1.00000e-08
##  20:     3952.4127: 0.0294289 -0.859360 0.861085 0.0200744 -0.0100900 0.0173952 0.0491645 1.00000e-08
##  21:     3952.4025: 0.0294738 -0.857082 0.857480 0.0199609 -0.00874158 0.0172267 0.0490964 1.00000e-0
##  22:     3952.3555: 0.0295347 -0.854313 0.854684 0.0210955 -0.00779620 0.0171566 0.0489711 1.00000e-0
##  23:     3952.3318: 0.0296047 -0.851864 0.852293 0.0198628 -0.00936238 0.0167571 0.0484894 1.00000e-0
##  24:     3952.2918: 0.0297514 -0.846045 0.847217 0.0212526 -0.00868396 0.0173208 0.0492146 1.00000e-0
##  25:     3952.2914: 0.0299521 -0.842138 0.842388 0.0211068 -0.00762548 0.0170788 0.0488334 1.00000e-0
##  26:     3952.2391: 0.0300569 -0.841177 0.840450 0.0194223 -0.00841412 0.0170794 0.0484812 1.00000e-0
##  27:     3952.2019: 0.0303508 -0.838535 0.839767 0.0201853 -0.00988862 0.0165988 0.0480327 1.00000e-0
##  28:     3952.1854: 0.0306183 -0.835856 0.835871 0.0178031 -0.0109236 0.0167256 0.0478878 1.00000e-08
##  29:     3952.1565: 0.0308962 -0.833130 0.832142 0.0167910 -0.0110782 0.0165427 0.0477361 1.00000e-08
##  30:     3952.1276: 0.0311661 -0.830672 0.830665 0.0203720 -0.00853029 0.0166333 0.0480019 1.00000e-0
##  31:     3952.1063: 0.0317536 -0.825390 0.827229 0.0184743 -0.00937963 0.0172937 0.0480225 1.00000e-0
##  32:     3952.0812: 0.0317536 -0.825388 0.827227 0.0184781 -0.00938253 0.0174128 0.0481438 2.19899e-0
##  33:     3952.0770: 0.0317569 -0.825348 0.827177 0.0184717 -0.00939075 0.0172655 0.0481764 1.00000e-0
##  34:     3952.0692: 0.0317666 -0.825240 0.827034 0.0184351 -0.00940043 0.0173177 0.0482597 1.00000e-0
##  35:     3952.0644: 0.0317869 -0.825025 0.826752 0.0183681 -0.00942090 0.0171928 0.0482866 1.00000e-0
##  36:     3952.0585: 0.0318287 -0.824614 0.826214 0.0182368 -0.00945333 0.0172144 0.0483122 1.00000e-0
##  37:     3951.9900: 0.0334658 -0.809462 0.806754 0.0137912 -0.0107465 0.0166664 0.0464317 1.00000e-08
##  38:     3951.9163: 0.0351160 -0.793419 0.789819 0.0109307 -0.0139429 0.0178074 0.0493524 1.00000e-08
##  39:     3951.7592: 0.0386743 -0.783222 0.784454 0.0201062 -0.0145412 0.0170102 0.0472950 1.00000e-08
##  40:     3951.7504: 0.0386743 -0.783191 0.784482 0.0200743 -0.0145061 0.0170485 0.0473902 1.00000e-08
```

```
##  41:     3951.7431: 0.0386744 -0.783154 0.784516 0.0200355 -0.0144632 0.0169274 0.0474068 1.00000e-08
##  42:     3951.7368: 0.0386816 -0.783133 0.784561 0.0199997 -0.0144158 0.0169659 0.0474956 1.00000e-08
##  43:     3951.7305: 0.0386981 -0.783140 0.784618 0.0199726 -0.0143697 0.0168639 0.0474895 1.00000e-08
##  44:     3951.7245: 0.0387315 -0.783178 0.784714 0.0199413 -0.0143053 0.0168941 0.0475583 1.00000e-08
##  45:     3951.7174: 0.0387987 -0.783296 0.784874 0.0199198 -0.0142275 0.0168051 0.0475503 1.00000e-08
##  46:     3951.7080: 0.0389332 -0.783561 0.785176 0.0198994 -0.0141107 0.0168291 0.0476031 1.00000e-08
##  47:     3951.4366: 0.0483113 -0.804342 0.804940 0.0203228 -0.00934145 0.0160571 0.0475675 1.00000e-0
##  48:     3951.3485: 0.0576752 -0.825455 0.830216 0.0138459 -0.0153593 0.0169257 0.0472024 1.00000e-08
##  49:     3951.2207: 0.0669841 -0.792608 0.794758 0.0196205 -0.0112303 0.0180790 0.0484838 1.00000e-08
##  50:     3951.1731: 0.0681051 -0.791568 0.795256 0.0212232 -0.0118064 0.0167160 0.0460797 1.00000e-08
##  51:     3951.1484: 0.0672563 -0.779154 0.781319 0.0181678 -0.0131667 0.0165793 0.0469089 1.00000e-08
##  52:     3951.1437: 0.0662595 -0.782978 0.785008 0.0190146 -0.0113589 0.0163198 0.0463817 1.00000e-08
##  53:     3951.1434: 0.0663417 -0.781140 0.783283 0.0188868 -0.0116480 0.0163876 0.0464720 1.00000e-08
##  54:     3951.1433: 0.0663222 -0.780325 0.782513 0.0189064 -0.0116754 0.0163881 0.0464667 1.00000e-08
##  55:     3951.1433: 0.0662978 -0.780041 0.782232 0.0189197 -0.0116643 0.0163881 0.0464659 1.00000e-08
##  56:     3951.1433: 0.0662960 -0.780028 0.782218 0.0189209 -0.0116633 0.0163875 0.0464650 1.00000e-08
##
## Final Estimate of the Negative LLH:
##  LLH:  -8101.814    norm LLH:  -2.684498
##           mu           ar1           ma1           ma2           ma3
##  1.221939e-03 -7.800277e-01  7.822180e-01  1.892089e-02 -1.166330e-02
##         omega        alpha1        alpha2         beta1
##  5.567202e-06  4.646504e-02  1.000000e-08  9.359745e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1          ma1          ma2           ma3
## mu     -4.053825e+06 -3.802937e+03   -777.47539    1494.2326 -1.055839e+03
## ar1    -3.802937e+03 -6.333532e+03  -6185.58620    4847.7332 -3.872941e+03
## ma1    -7.774754e+02 -6.185586e+03  -6081.29543    4712.1454 -3.686097e+03
## ma2     1.494233e+03  4.847733e+03   4712.14545   -6220.8625  4.873504e+03
## ma3    -1.055839e+03 -3.872941e+03  -3686.09726    4873.5042 -6.304694e+03
## omega   2.119060e+08  1.168828e+06 776631.50711 1027599.5453  1.306228e+06
## alpha1 -1.142640e+04  4.094643e+00    -29.00109    -224.1220 -2.436142e+01
## alpha2 -5.561136e+03 -6.727773e+01   -139.68078    -236.4849  8.901166e+00
## beta1   2.506681e+04 -6.584229e+01    -78.78078     177.1072  2.259360e+01
##                omega        alpha1        alpha2         beta1
## mu      2.119060e+08 -1.142640e+04 -5.561136e+03  2.506681e+04
## ar1     1.168828e+06  4.094643e+00 -6.727773e+01 -6.584229e+01
## ma1     7.766315e+05 -2.900109e+01 -1.396808e+02 -7.878078e+01
## ma2     1.027600e+06 -2.241220e+02 -2.364849e+02  1.771072e+02
## ma3     1.306228e+06 -2.436142e+01  8.901166e+00  2.259360e+01
## omega  -7.785731e+12 -1.188884e+09 -1.190107e+09 -1.655771e+09
## alpha1 -1.188884e+09 -2.971684e+05 -2.947938e+05 -3.357713e+05
## alpha2 -1.190107e+09 -2.947938e+05 -2.965322e+05 -3.374780e+05
## beta1  -1.655771e+09 -3.357713e+05 -3.374780e+05 -4.168484e+05
## attr(,"time")
## Time difference of 0.1364341 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.052546 secs
```

```
gfit11 = garchFit(~arma(1,3)+garch(1,2),data = diff(log(google_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 2)
##  ARMA Order:                1 3
##  Max ARMA Order:            3
##  GARCH Order:               1 2
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.01843157
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U          V      params includes
##     mu     -0.28901297   0.289013  0.02913901     TRUE
##     ar1    -0.99999999   1.000000 -0.91616431     TRUE
##     ma1    -0.99999999   1.000000  0.89235197     TRUE
##     ma2    -0.99999999   1.000000 -0.01815199     TRUE
##     ma3    -0.99999999   1.000000 -0.02031633     TRUE
##     omega   0.00000100 100.000000  0.10000000     TRUE
##     alpha1  0.00000001   1.000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.000000  0.10000000    FALSE
##     beta1   0.00000001   1.000000  0.40000000     TRUE
##     beta2   0.00000001   1.000000  0.40000000     TRUE
##     delta   0.00000000   2.000000  2.00000000    FALSE
##     skew    0.10000000  10.000000  1.00000000    FALSE
##     shape   1.00000000  10.000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1    ma2    ma3  omega alpha1  beta1  beta2
##      1      2      3      4      5      6      7      9     10
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3986.9972: 0.0291390 -0.916164 0.892352 -0.0181520 -0.0203163 0.100000 0.100000 0.400000 0.4
##   1:     3979.5280: 0.0291394 -0.915277 0.893010 -0.0181885 -0.0198132 0.0909871 0.100143 0.395941 0
##   2:     3974.1167: 0.0291401 -0.913906 0.893992 -0.0179818 -0.0191591 0.0870314 0.108484 0.399525 0
##   3:     3973.7785: 0.0291416 -0.912357 0.894855 -0.0163522 -0.0189552 0.0668948 0.114596 0.396869 0
```

```
##   4:     3964.5384: 0.0291419 -0.912080 0.895008 -0.0160914 -0.0189372 0.0721219 0.119792 0.402329 0
##   5:     3961.8809: 0.0291433 -0.911441 0.895064 -0.0140238 -0.0194697 0.0621147 0.122311 0.403510 0
##   6:     3958.0353: 0.0291470 -0.908403 0.896272 -0.00974501 -0.0197388 0.0516535 0.123475 0.415466 0
##   7:     3957.2981: 0.0291524 -0.899589 0.901767 -0.00860594 -0.0152378 0.0390034 0.113884 0.420546 0
##   8:     3954.3700: 0.0291588 -0.902292 0.895284 0.00484392 -0.0213734 0.0333491 0.107353 0.426876 0
##   9:     3953.9496: 0.0291589 -0.901820 0.895638 0.00454822 -0.0209761 0.0348463 0.107582 0.427772 0
##  10:     3953.6027: 0.0291598 -0.900898 0.895894 0.00491127 -0.0204318 0.0340228 0.105995 0.427534 0
##  11:     3953.1750: 0.0291621 -0.899185 0.896117 0.00610854 -0.0193854 0.0342597 0.103575 0.428914 0
##  12:     3952.4478: 0.0291674 -0.896918 0.895649 0.00902490 -0.0174913 0.0318284 0.0977243 0.430610 0
##  13:     3951.8353: 0.0291746 -0.895607 0.894597 0.0111001 -0.0151474 0.0304889 0.0924481 0.433235 0
##  14:     3951.3127: 0.0291870 -0.894010 0.893843 0.0126025 -0.0145947 0.0283147 0.0861460 0.434582 0
##  15:     3950.9016: 0.0292126 -0.890897 0.892770 0.0145886 -0.0151799 0.0274170 0.0814978 0.435059 0
##  16:     3950.7270: 0.0292348 -0.891456 0.888378 0.0139160 -0.0101231 0.0250685 0.0792674 0.434916 0
##  17:     3950.4848: 0.0292729 -0.886940 0.886916 0.0180363 -0.00918202 0.0269539 0.0778634 0.430795 0
##  18:     3950.4750: 0.0292731 -0.886750 0.887050 0.0179373 -0.00905926 0.0266348 0.0776657 0.430694 0
##  19:     3950.4406: 0.0292743 -0.886498 0.887135 0.0179272 -0.00919570 0.0267558 0.0776849 0.430794 0
##  20:     3950.4107: 0.0292773 -0.885884 0.887342 0.0178911 -0.00949880 0.0264266 0.0773790 0.430609 0
##  21:     3950.3643: 0.0292850 -0.884838 0.887479 0.0179713 -0.0104672 0.0263480 0.0771107 0.430299 0
##  22:     3950.3245: 0.0292951 -0.884006 0.887204 0.0178952 -0.0110057 0.0259721 0.0766485 0.429505 0
##  23:     3949.9831: 0.0294983 -0.877552 0.872876 0.0144919 -0.00648246 0.0230048 0.0722424 0.412493 0
##  24:     3949.6252: 0.0297066 -0.864160 0.865990 0.0198059 -0.00969464 0.0221566 0.0688504 0.392163 0
##  25:     3949.4626: 0.0299077 -0.850925 0.858072 0.0257740 -0.0106037 0.0224340 0.0696073 0.371193 0
##  26:     3948.9782: 0.0300988 -0.841406 0.848165 0.0284768 -0.00824561 0.0232483 0.0723031 0.346422 0
##  27:     3948.5661: 0.0304535 -0.833618 0.831304 0.0111390 -0.0183029 0.0247332 0.0719829 0.299865 0
##  28:     3947.9680: 0.0308189 -0.823037 0.821753 0.0108455 -0.0176868 0.0240872 0.0765905 0.249394 0
##  29:     3946.2271: 0.0322542 -0.784113 0.779767 0.0133928 -0.0109174 0.0285288 0.0930789 0.0463743 0
##  30:     3945.9277: 0.0322543 -0.784212 0.779687 0.0135377 -0.0110195 0.0292481 0.0933840 0.0469286 0
##  31:     3945.8325: 0.0322546 -0.784635 0.779348 0.0141827 -0.0114665 0.0290948 0.0929983 0.0470650 0
##  32:     3945.7902: 0.0322773 -0.786082 0.780431 0.0147308 -0.0111752 0.0293962 0.0928590 0.0473683 0
##  33:     3945.7262: 0.0323208 -0.788996 0.783040 0.0152818 -0.0105057 0.0290662 0.0923288 0.0471792 0
##  34:     3945.6527: 0.0324127 -0.794775 0.788423 0.0160776 -0.00979307 0.0291005 0.0916275 0.0473732
##  35:     3945.2762: 0.0335544 -0.853780 0.849912 0.0149381 -0.00984326 0.0266541 0.0848063 0.0470655
##  36:     3945.0820: 0.0342182 -0.877424 0.870138 0.0157256 -0.00429603 0.0278260 0.0864217 0.0511670
##  37:     3945.0314: 0.0353306 -0.880700 0.873851 0.0146251 -0.00508934 0.0275533 0.0876196 0.0556122
##  38:     3944.7767: 0.0362233 -0.862883 0.859104 0.0125733 -0.0131442 0.0279093 0.0887006 0.0593062 0
##  39:     3944.6371: 0.0372377 -0.851581 0.847445 0.0196569 -0.00779647 0.0273992 0.0867484 0.0589178
##  40:     3944.5822: 0.0383806 -0.853236 0.849347 0.0204868 -0.00800330 0.0278479 0.0873997 0.0613097
##  41:     3944.4645: 0.0395220 -0.850745 0.847816 0.0206521 -0.00961167 0.0276181 0.0877328 0.0633198
##  42:     3944.2567: 0.0440674 -0.865414 0.863698 0.0222309 -0.00738434 0.0285476 0.0897320 0.0699393
##  43:     3944.2200: 0.0486433 -0.853043 0.855475 0.0232428 -0.0142595 0.0281879 0.0895300 0.0721113 0
##  44:     3944.0865: 0.0532035 -0.866838 0.870253 0.0252286 -0.0120337 0.0277371 0.0881395 0.0691344 0
##  45:     3944.0356: 0.0577783 -0.854124 0.860417 0.0282157 -0.00909227 0.0276595 0.0871677 0.0716872
##  46:     3943.9651: 0.0623649 -0.864332 0.867189 0.0263646 -0.00751887 0.0275573 0.0861206 0.0721450
##  47:     3943.9550: 0.0649027 -0.881491 0.884714 0.0283087 -0.00517416 0.0270559 0.0854859 0.0716050
##  48:     3943.9473: 0.0647695 -0.874261 0.876766 0.0274180 -0.00608489 0.0273446 0.0857508 0.0724347
##  49:     3943.9471: 0.0646384 -0.874894 0.877594 0.0275019 -0.00601463 0.0273237 0.0856986 0.0723014
##  50:     3943.9471: 0.0646687 -0.874986 0.877642 0.0274879 -0.00600925 0.0273154 0.0857192 0.0723178
##  51:     3943.9471: 0.0646722 -0.874968 0.877628 0.0274907 -0.00601412 0.0273206 0.0857141 0.0723103
##  52:     3943.9471: 0.0646690 -0.874956 0.877618 0.0274907 -0.00601348 0.0273192 0.0857136 0.0723129
##
## Final Estimate of the Negative LLH:
##  LLH:  -8109.011    norm LLH:  -2.686882
##           mu           ar1           ma1           ma2           ma3
##  1.191951e-03 -8.749559e-01  8.776176e-01  2.749073e-02 -6.013479e-03
```

```
##         omega        alpha1         beta1         beta2
##  9.280954e-06  8.571362e-02  7.231292e-02  8.139608e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu          ar1          ma1           ma2          ma3
## mu      -3650600.208   -4486.7109   -1889.9604     1956.8842   -1708.9280
## ar1        -4486.711  -10307.7746   -9474.3643     8562.8772   -7154.2257
## ma1        -1889.960   -9474.3643   -8905.6248     7872.4521   -6340.7397
## ma2         1956.884    8562.8772    7872.4521    -9508.7918    7638.0938
## ma3        -1708.928   -7154.2257   -6340.7397     7638.0938   -8904.1094
## omega  120312404.158 5569413.1720 4749975.6880 -3482759.3299 4988032.2129
## alpha1     -1350.849     185.5510     113.3975     -390.8288     161.6280
## beta1      22208.435    1282.0026    1078.0646    -1207.9147     988.3525
## beta2      21853.354     822.8391     707.0015     -866.9848     731.8499
##                omega        alpha1         beta1         beta2
## mu      1.203124e+08 -1.350849e+03  2.220844e+04  2.185335e+04
## ar1     5.569413e+06  1.855510e+02  1.282003e+03  8.228391e+02
## ma1     4.749976e+06  1.133975e+02  1.078065e+03  7.070015e+02
## ma2    -3.482759e+06 -3.908288e+02 -1.207915e+03 -8.669848e+02
## ma3     4.988032e+06  1.616280e+02  9.883525e+02  7.318499e+02
## omega  -2.527831e+12 -3.809369e+08 -5.413317e+08 -5.386512e+08
## alpha1 -3.809369e+08 -9.157627e+04 -1.074405e+05 -1.069355e+05
## beta1  -5.413317e+08 -1.074405e+05 -1.386683e+05 -1.373865e+05
## beta2  -5.386512e+08 -1.069355e+05 -1.373865e+05 -1.370247e+05
## attr(,"time")
## Time difference of 0.131134 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.8383 secs
```

```
gfit12 = garchFit(~arma(1,3)+garch(2,2),data = diff(log(google_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(1, 3)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(2, 2)
##  ARMA Order:                 1 3
##  Max ARMA Order:             3
##  GARCH Order:                2 2
##  Max GARCH Order:            2
##  Maximum Order:              3
##  Conditional Dist:           norm
##  h.start:                    4
##  llh.start:                  1
##  Length of Series:           3018
##  Recursion Init:             mci
##  Series Scale:               0.01843157
##
## Parameter Initialization:
##  Initial Parameters:          $params
```

```
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U          V      params includes
##     mu     -0.28901297   0.289013  0.02913901     TRUE
##     ar1    -0.99999999   1.000000 -0.91616431     TRUE
##     ma1    -0.99999999   1.000000  0.89235197     TRUE
##     ma2    -0.99999999   1.000000 -0.01815199     TRUE
##     ma3    -0.99999999   1.000000 -0.02031633     TRUE
##     omega   0.00000100 100.000000  0.10000000     TRUE
##     alpha1  0.00000001   1.000000  0.05000000     TRUE
##     alpha2  0.00000001   1.000000  0.05000000     TRUE
##     gamma1 -0.99999999   1.000000  0.10000000    FALSE
##     gamma2 -0.99999999   1.000000  0.10000000    FALSE
##     beta1   0.00000001   1.000000  0.40000000     TRUE
##     beta2   0.00000001   1.000000  0.40000000     TRUE
##     delta   0.00000000   2.000000  2.00000000    FALSE
##     skew    0.10000000  10.000000  1.00000000    FALSE
##     shape   1.00000000  10.000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1    ma2    ma3  omega alpha1 alpha2  beta1  beta2
##      1      2      3      4      5      6      7      8     11     12
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3987.4965: 0.0291390 -0.916164 0.892352 -0.0181520 -0.0203163 0.100000 0.0500000 0.0500000 0
##   1:     3979.4440: 0.0291394 -0.915171 0.893104 -0.0184081 -0.0196950 0.0912098 0.0502754 0.0495638
##   2:     3973.6715: 0.0291401 -0.913867 0.894066 -0.0185580 -0.0189722 0.0877898 0.0575821 0.0557011
##   3:     3973.2235: 0.0291417 -0.912051 0.895204 -0.0177301 -0.0184282 0.0681943 0.0625687 0.0581406
##   4:     3964.1432: 0.0291419 -0.911807 0.895332 -0.0175161 -0.0184311 0.0731963 0.0674877 0.0626149
##   5:     3961.5966: 0.0291443 -0.911318 0.894981 -0.0143560 -0.0197737 0.0637468 0.0707589 0.0624921
##   6:     3960.8620: 0.0291497 -0.902248 0.901199 -0.0145937 -0.0155155 0.0542588 0.0756258 0.0588967
##   7:     3957.1559: 0.0291561 -0.903380 0.896102 -0.00356620 -0.0212124 0.0438946 0.0771070 0.049836
##   8:     3955.2553: 0.0291669 -0.896674 0.896436 0.00307823 -0.0187164 0.0418344 0.0810090 0.0343634
##   9:     3954.9046: 0.0291775 -0.889281 0.897732 0.00747466 -0.0141805 0.0377036 0.0824463 0.0170548
##  10:     3952.9769: 0.0291892 -0.887611 0.894886 0.0101601 -0.00780818 0.0340343 0.0843739 0.0045634
##  11:     3951.4674: 0.0291935 -0.889488 0.892399 0.0106677 -0.00621363 0.0288213 0.0825537 0.0006450
##  12:     3950.9688: 0.0292004 -0.889549 0.891758 0.0128372 -0.00735740 0.0259693 0.0785697 1.00000e-0
##  13:     3950.5049: 0.0292127 -0.886621 0.892960 0.0171385 -0.00881665 0.0238974 0.0725007 1.00000e-0
##  14:     3950.3250: 0.0292363 -0.883365 0.891944 0.0228434 -0.00686780 0.0252986 0.0684679 1.00000e-0
##  15:     3950.0808: 0.0292620 -0.882222 0.887976 0.0236892 -0.00295818 0.0222841 0.0651397 1.00000e-0
##  16:     3950.0731: 0.0292621 -0.882260 0.887927 0.0237417 -0.00301552 0.0223273 0.0652189 1.00000e-0
##  17:     3950.0666: 0.0292622 -0.882300 0.887874 0.0237990 -0.00307848 0.0222121 0.0652133 1.00000e-0
##  18:     3950.0575: 0.0292634 -0.882219 0.887774 0.0238299 -0.00327677 0.0222648 0.0653425 1.00000e-0
##  19:     3950.0449: 0.0292664 -0.881944 0.887603 0.0238215 -0.00363772 0.0221731 0.0654258 1.00000e-0
##  20:     3950.0277: 0.0292727 -0.881320 0.887200 0.0237691 -0.00420625 0.0222490 0.0656515 1.00000e-0
##  21:     3949.6414: 0.0296338 -0.849466 0.856723 0.0198460 -0.0146539 0.0250356 0.0727783 1.00000e-08
##  22:     3947.9771: 0.0309232 -0.751600 0.752010 0.0178434 -0.0123380 0.0269867 0.0792327 1.00000e-08
##  23:     3947.3693: 0.0313218 -0.721161 0.724086 0.0196602 -0.0131897 0.0261870 0.0803122 1.00000e-08
```

```
##  24:     3946.1538: 0.0321144 -0.660334 0.665293 0.0209592 -0.0145046 0.0261566 0.0820067 0.00218992
##  25:     3946.1201: 0.0321145 -0.660380 0.665255 0.0210114 -0.0145313 0.0265536 0.0821668 0.00233497
##  26:     3946.1007: 0.0321166 -0.660058 0.665023 0.0212022 -0.0144581 0.0264836 0.0821995 0.00222890
##  27:     3946.0870: 0.0321206 -0.659631 0.664462 0.0217357 -0.0144168 0.0269515 0.0824184 0.00212230
##  28:     3946.0686: 0.0321297 -0.658094 0.663518 0.0224628 -0.0140342 0.0269404 0.0828724 0.00203709
##  29:     3946.0636: 0.0321388 -0.656572 0.662553 0.0232133 -0.0136608 0.0271761 0.0834503 0.00207552
##  30:     3946.0572: 0.0321218 -0.659919 0.664426 0.0220296 -0.0144280 0.0269714 0.0823938 0.00192247
##  31:     3946.0492: 0.0321223 -0.660155 0.664243 0.0222948 -0.0145472 0.0271434 0.0824924 0.00193042
##  32:     3946.0395: 0.0321243 -0.659898 0.664027 0.0225033 -0.0144838 0.0270945 0.0825722 0.00187030
##  33:     3946.0313: 0.0321286 -0.659386 0.663570 0.0229515 -0.0143601 0.0272323 0.0828741 0.00188442
##  34:     3946.0194: 0.0321371 -0.658194 0.662704 0.0237499 -0.0140466 0.0273235 0.0833611 0.00181766
##  35:     3946.0107: 0.0321457 -0.656997 0.661851 0.0245422 -0.0137221 0.0275527 0.0839254 0.00182926
##  36:     3946.0032: 0.0321548 -0.655812 0.661045 0.0253381 -0.0133648 0.0276371 0.0844274 0.00174226
##  37:     3945.9934: 0.0321664 -0.654954 0.660551 0.0262313 -0.0128662 0.0278776 0.0851387 0.00164639
##  38:     3945.9569: 0.0321579 -0.663761 0.667023 0.0242645 -0.0129604 0.0273937 0.0843219 0.00076992
##  39:     3945.9544: 0.0321582 -0.663827 0.666991 0.0243428 -0.0130121 0.0273040 0.0843262 0.00084863
##  40:     3945.9510: 0.0321602 -0.663818 0.666990 0.0244171 -0.0129730 0.0273691 0.0844191 0.00087087
##  41:     3945.9278: 0.0322260 -0.663032 0.667135 0.0264276 -0.0113813 0.0277737 0.0863521 1.00000e-08
##  42:     3945.8860: 0.0325083 -0.661119 0.665092 0.0252773 -0.0123014 0.0282144 0.0872397 0.00044292
##  43:     3945.8087: 0.0328078 -0.659678 0.663104 0.0233935 -0.0142066 0.0278099 0.0861008 0.00134870
##  44:     3945.7679: 0.0330987 -0.659264 0.664494 0.0260839 -0.0138520 0.0273038 0.0829376 0.00174284
##  45:     3945.7309: 0.0334195 -0.661474 0.664970 0.0254329 -0.0137452 0.0270997 0.0828243 0.00249165
##  46:     3945.6637: 0.0337409 -0.662763 0.664920 0.0243218 -0.0141026 0.0274456 0.0828997 0.00340205
##  47:     3945.3818: 0.0366838 -0.664318 0.664099 0.0170961 -0.0175645 0.0307086 0.0850447 0.00790472
##  48:     3945.2959: 0.0395686 -0.694889 0.686071 0.0239593 -0.00687914 0.0310471 0.0882341 0.00769428
##  49:     3944.7574: 0.0472993 -0.785330 0.777218 0.0260524 -0.00333904 0.0318178 0.0890293 0.00301901
##  50:     3944.5473: 0.0491568 -0.806183 0.797171 0.0249440 -0.00386161 0.0308003 0.0887181 0.00284000
##  51:     3944.3897: 0.0528780 -0.847518 0.836815 0.0229005 -0.00510151 0.0302247 0.0883922 0.00181938
##  52:     3944.3099: 0.0569036 -0.848191 0.845451 0.0238122 -0.00949260 0.0271265 0.0872950 1.00000e-08
##  53:     3944.1136: 0.0609583 -0.848413 0.844369 0.0240662 -0.00398623 0.0290734 0.0820196 0.00787394
##  54:     3943.9590: 0.0609583 -0.848400 0.844388 0.0240737 -0.00400905 0.0295023 0.0822770 0.00810941
##  55:     3943.9481: 0.0609584 -0.848171 0.844698 0.0241537 -0.00431512 0.0293058 0.0824929 0.00770149
##  56:     3943.9372: 0.0609899 -0.848810 0.845324 0.0241413 -0.00433718 0.0294069 0.0825699 0.00771291
##  57:     3943.9256: 0.0610510 -0.849927 0.846767 0.0242010 -0.00462631 0.0292451 0.0827430 0.00733891
##  58:     3943.9121: 0.0611779 -0.852545 0.849233 0.0241208 -0.00462924 0.0293452 0.0828125 0.00727058
##  59:     3943.8945: 0.0614431 -0.857131 0.854105 0.0240708 -0.00484989 0.0291435 0.0830096 0.00682358
##  60:     3943.8380: 0.0632503 -0.867555 0.867368 0.0252549 -0.00484332 0.0290452 0.0830729 0.00654443
##  61:     3943.8314: 0.0637473 -0.871568 0.871083 0.0251997 -0.00502559 0.0290718 0.0830721 0.00633370
##  62:     3943.8212: 0.0642551 -0.868197 0.868796 0.0256578 -0.00568860 0.0287681 0.0833084 0.00609470
##  63:     3943.8188: 0.0644299 -0.873582 0.876545 0.0274692 -0.00616075 0.0285833 0.0836690 0.00526310
##  64:     3943.8148: 0.0646071 -0.871705 0.874741 0.0275337 -0.00611245 0.0288858 0.0839363 0.00536340
##  65:     3943.8130: 0.0651257 -0.871108 0.873213 0.0267256 -0.00625172 0.0286882 0.0836838 0.00550617
##  66:     3943.8126: 0.0646055 -0.871602 0.874041 0.0270492 -0.00608900 0.0286950 0.0835730 0.00552268
##  67:     3943.8126: 0.0647491 -0.871694 0.874143 0.0270417 -0.00611644 0.0287348 0.0837692 0.00544838
##  68:     3943.8126: 0.0648113 -0.871513 0.873927 0.0270069 -0.00616853 0.0287073 0.0836524 0.00548042
##  69:     3943.8126: 0.0647559 -0.871581 0.873998 0.0270152 -0.00613297 0.0287135 0.0836779 0.00548000
##  70:     3943.8126: 0.0647644 -0.871580 0.874001 0.0270180 -0.00613827 0.0287147 0.0836819 0.00547689
##
## Final Estimate of the Negative LLH:
##  LLH:  -8109.145    norm LLH:  -2.686927
##           mu           ar1           ma1           ma2           ma3
##  1.193709e-03 -8.715798e-01  8.740014e-01  2.701797e-02 -6.138274e-03
##         omega        alpha1        alpha2         beta1         beta2
##  9.755032e-06  8.368192e-02  5.476885e-03  5.969707e-02  8.217181e-01
```

```
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu           ar1           ma1           ma2           ma3
## mu      -3667832.859   -4328.15808 -1.742891e+03     1889.2665   -1585.6371
## ar1        -4328.158  -10029.12474 -9.254462e+03     8300.9235   -6961.0871
## ma1        -1742.891   -9254.46242 -8.722730e+03     7653.7090   -6195.1020
## ma2         1889.267    8300.92348  7.653709e+03    -9261.7830    7479.2799
## ma3        -1585.637   -6961.08709 -6.195102e+03     7479.2799   -8749.3649
## omega  117957870.321 5067979.30869  4.323569e+06 -3067730.3025 4460745.7660
## alpha1     -2543.223      42.08812  6.756114e-01     -245.2054      54.8555
## alpha2      9934.874    1922.70673  1.511338e+03    -1999.8886    1424.2135
## beta1      20705.210    1160.32796  9.754447e+02    -1084.5131     889.6818
## beta2      20538.869     702.40575  6.032944e+02     -743.4099     638.5724
##                omega        alpha1        alpha2         beta1         beta2
## mu      1.179579e+08 -2.543223e+03  9.934874e+03  2.070521e+04  2.053887e+04
## ar1     5.067979e+06  4.208812e+01  1.922707e+03  1.160328e+03  7.024058e+02
## ma1     4.323569e+06  6.756114e-01  1.511338e+03  9.754447e+02  6.032944e+02
## ma2    -3.067730e+06 -2.452054e+02 -1.999889e+03 -1.084513e+03 -7.434099e+02
## ma3     4.460746e+06  5.485550e+01  1.424213e+03  8.896818e+02  6.385724e+02
## omega  -2.324175e+12 -3.477772e+08 -3.708173e+08 -4.971303e+08 -4.946097e+08
## alpha1 -3.477772e+08 -8.350038e+04 -8.768791e+04 -9.794929e+04 -9.762285e+04
## alpha2 -3.708173e+08 -8.768791e+04 -1.072733e+05 -1.074321e+05 -1.050565e+05
## beta1  -4.971303e+08 -9.794929e+04 -1.074321e+05 -1.270988e+05 -1.257879e+05
## beta2  -4.946097e+08 -9.762285e+04 -1.050565e+05 -1.257879e+05 -1.255387e+05
## attr(,"time")
## Time difference of 0.221247 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.13415 secs
```

```
summary(gfit1)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x7ffe065a2ba0>
##  [data = diff(log(google_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1
## 0.00055722  0.00025238  0.28488290
##
## Std. Errors:
```

```
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     5.572e-04   3.069e-04    1.816   0.0694 .
## omega  2.524e-04   8.623e-06   29.267  < 2e-16 ***
## alpha1 2.849e-01   3.525e-02    8.081 6.66e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7893.787    normalized:  2.615569
##
## Description:
##  Sun Apr 17 22:14:56 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  10136.18  0
##  Shapiro-Wilk Test  R    W      0.9123439 0
##  Ljung-Box Test     R    Q(10)  22.07873  0.01470852
##  Ljung-Box Test     R    Q(15)  31.66956  0.007138678
##  Ljung-Box Test     R    Q(20)  41.46245  0.003247624
##  Ljung-Box Test     R^2  Q(10)  73.57851  8.990364e-12
##  Ljung-Box Test     R^2  Q(15)  122.6855  0
##  Ljung-Box Test     R^2  Q(20)  151.4308  0
##  LM Arch Test       R    TR^2   79.81818  4.470313e-12
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.229150 -5.223173 -5.229152 -5.227001
```

```
summary(gfit2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7ffe037e9e80>
##  [data = diff(log(google_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 6.7972e-04  5.6996e-06  4.7543e-02  9.3458e-01
##
## Std. Errors:
```

```
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     6.797e-04   2.795e-04    2.432 0.015009 *
## omega  5.700e-06   1.598e-06    3.566 0.000363 ***
## alpha1 4.754e-02   1.087e-02    4.374 1.22e-05 ***
## beta1  9.346e-01   1.448e-02   64.528  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  8097.953    normalized:  2.683218
##
## Description:
##  Sun Apr 17 22:14:56 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  13715.47  0
##  Shapiro-Wilk Test  R    W      0.9226129 0
##  Ljung-Box Test     R    Q(10)  13.88787  0.178168
##  Ljung-Box Test     R    Q(15)  16.47777  0.3510284
##  Ljung-Box Test     R    Q(20)  21.95521  0.3429459
##  Ljung-Box Test     R^2  Q(10)  2.899156  0.9836986
##  Ljung-Box Test     R^2  Q(15)  4.388775  0.9961583
##  Ljung-Box Test     R^2  Q(20)  6.40014   0.9982378
##  LM Arch Test       R    TR^2   3.332717  0.9927043
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.363786 -5.355817 -5.363789 -5.360920
```

```
summary(gfit3)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 0), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ garch(2, 0)
## <environment: 0x7ffe05489458>
##  [data = diff(log(google_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2
## 0.00080077  0.00018305  0.20220597  0.35157322
##
```

```
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     8.008e-04   2.839e-04    2.821  0.00479 **
## omega  1.831e-04   8.696e-06   21.049  < 2e-16 ***
## alpha1 2.022e-01   3.187e-02    6.346 2.22e-10 ***
## alpha2 3.516e-01   4.317e-02    8.143 4.44e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7987.573    normalized:  2.646644
##
## Description:
##  Sun Apr 17 22:14:56 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  7034.448  0
##  Shapiro-Wilk Test  R    W      0.9262855 0
##  Ljung-Box Test     R    Q(10)  18.21878  0.05138337
##  Ljung-Box Test     R    Q(15)  21.47087  0.1224473
##  Ljung-Box Test     R    Q(20)  30.03723  0.06925265
##  Ljung-Box Test     R^2  Q(10)  28.50236  0.001499267
##  Ljung-Box Test     R^2  Q(15)  37.27195  0.001154734
##  Ljung-Box Test     R^2  Q(20)  52.35082  0.000101191
##  LM Arch Test       R    TR^2   34.99212  0.0004696649
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.290638 -5.282669 -5.290642 -5.287773
```

```
summary(gfit4)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 1), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ garch(2, 1)
## <environment: 0x7ffe01e76c70>
##  [data = diff(log(google_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2       beta1
## 6.8003e-04  5.9543e-06  4.9305e-02  1.0000e-08  9.3218e-01
```

```
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     6.800e-04   2.796e-04    2.432  0.01502 *
## omega  5.954e-06   2.060e-06    2.891  0.00384 **
## alpha1 4.931e-02   1.655e-02    2.979  0.00289 **
## alpha2 1.000e-08   1.999e-02    0.000  1.00000
## beta1  9.322e-01   1.938e-02   48.112  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  8096.677    normalized:  2.682796
##
## Description:
##  Sun Apr 17 22:14:57 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  13613.39  0
##  Shapiro-Wilk Test  R    W      0.9224881 0
##  Ljung-Box Test     R    Q(10)  13.80481  0.1820825
##  Ljung-Box Test     R    Q(15)  16.33503  0.3601418
##  Ljung-Box Test     R    Q(20)  21.73294  0.3551762
##  Ljung-Box Test     R^2  Q(10)  2.911749  0.9834252
##  Ljung-Box Test     R^2  Q(15)  4.375489  0.9962233
##  Ljung-Box Test     R^2  Q(20)  6.424273  0.9981896
##  LM Arch Test       R    TR^2   3.337772  0.992653
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.362278 -5.352317 -5.362283 -5.358696
```

```
summary(gfit5)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 2), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 2)
## <environment: 0x7ffe037ef5c0>
##  [data = diff(log(google_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
```

```
##         mu       omega      alpha1       beta1       beta2
## 6.2930e-04  9.2362e-06  8.4740e-02  9.5323e-02  7.9215e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     6.293e-04   2.781e-04    2.263   0.0236 *
## omega  9.236e-06   2.200e-06    4.199 2.69e-05 ***
## alpha1 8.474e-02   1.590e-02    5.331 9.77e-08 ***
## beta1  9.532e-02   4.588e-02    2.078   0.0377 *
## beta2  7.921e-01   4.719e-02   16.787  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  8101.73    normalized:  2.68447
##
## Description:
##  Sun Apr 17 22:14:57 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  12725.94  0
##  Shapiro-Wilk Test  R    W      0.923043  0
##  Ljung-Box Test     R    Q(10)  15.8139   0.1050849
##  Ljung-Box Test     R    Q(15)  18.41576  0.2414388
##  Ljung-Box Test     R    Q(20)  24.34106  0.2278112
##  Ljung-Box Test     R^2  Q(10)  2.72465   0.9871823
##  Ljung-Box Test     R^2  Q(15)  4.054999  0.9975517
##  Ljung-Box Test     R^2  Q(20)  6.171504  0.9986452
##  LM Arch Test       R    TR^2   3.085716  0.994908
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.365626 -5.355665 -5.365631 -5.362044
```

```
summary(gfit6)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 2), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ garch(2, 2)
## <environment: 0x7ffe04a92290>
##  [data = diff(log(google_open))]
##
## Conditional Distribution:
##  norm
```

```
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2       beta1       beta2
## 6.3419e-04  1.0338e-05  8.0047e-02  1.2576e-02  6.2042e-02  8.1426e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     6.342e-04   2.782e-04    2.280 0.022627 *
## omega  1.034e-05   2.781e-06    3.717 0.000202 ***
## alpha1 8.005e-02   1.735e-02    4.614 3.94e-06 ***
## alpha2 1.258e-02   1.192e-02    1.055 0.291532
## beta1  6.204e-02   4.479e-02    1.385 0.165965
## beta2  8.143e-01   4.321e-02   18.845  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  8102.328    normalized:  2.684668
##
## Description:
##  Sun Apr 17 22:14:57 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  12792.29  0
##  Shapiro-Wilk Test  R    W      0.9229275 0
##  Ljung-Box Test     R    Q(10)  15.24977  0.1232176
##  Ljung-Box Test     R    Q(15)  17.81511  0.2725104
##  Ljung-Box Test     R    Q(20)  23.59196  0.2606706
##  Ljung-Box Test     R^2  Q(10)  2.553868  0.9900673
##  Ljung-Box Test     R^2  Q(15)  3.953497  0.9978856
##  Ljung-Box Test     R^2  Q(20)  6.179697  0.9986321
##  LM Arch Test       R    TR^2   2.970175  0.9957509
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.365360 -5.353407 -5.365367 -5.361061
```

```
summary(gfit7)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 3) + garch(1, 0), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ arma(1, 3) + garch(1, 0)
## <environment: 0x7ffe06a560b0>
##  [data = diff(log(google_open))]
```

```
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ma1          ma2          ma3        omega
##  0.00118631  -0.90966831   0.92189227   0.03459119  -0.00878097   0.00024801
##      alpha1
##  0.30016164
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.186e-03   5.945e-04    1.996    0.046 *
## ar1    -9.097e-01   3.346e-02  -27.185  < 2e-16 ***
## ma1     9.219e-01   3.961e-02   23.276  < 2e-16 ***
## ma2     3.459e-02   2.802e-02    1.235    0.217
## ma3    -8.781e-03   1.899e-02   -0.462    0.644
## omega   2.480e-04   8.629e-06   28.741  < 2e-16 ***
## alpha1  3.002e-01   3.671e-02    8.176 2.22e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7903.507    normalized:  2.61879
##
## Description:
##  Sun Apr 17 22:14:58 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  9925.909  0
##  Shapiro-Wilk Test  R    W      0.9136361 0
##  Ljung-Box Test     R    Q(10)  12.50596  0.2526201
##  Ljung-Box Test     R    Q(15)  15.64281  0.4061793
##  Ljung-Box Test     R    Q(20)  32.19497  0.04126459
##  Ljung-Box Test     R^2  Q(10)  71.99955  1.819911e-11
##  Ljung-Box Test     R^2  Q(15)  108.2431  3.330669e-16
##  Ljung-Box Test     R^2  Q(20)  135.255   0
##  LM Arch Test       R    TR^2   73.40159  7.35928e-11
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.232940 -5.218995 -5.232951 -5.227926
```

```
summary(gfit8)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
```

```
##  garchFit(formula = ~arma(1, 3) + garch(1, 1), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ arma(1, 3) + garch(1, 1)
## <environment: 0x7ffe04c8abf8>
##  [data = diff(log(google_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu          ar1          ma1          ma2          ma3        omega
##  1.2180e-03  -7.7483e-01   7.7702e-01   1.8868e-02  -1.1846e-02   5.5905e-06
##      alpha1        beta1
##  4.6642e-02   9.3576e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      1.218e-03   5.158e-04    2.361 0.018211 *
## ar1    -7.748e-01   1.733e-01   -4.470 7.82e-06 ***
## ma1     7.770e-01   1.747e-01    4.449 8.64e-06 ***
## ma2     1.887e-02   2.525e-02    0.747 0.454825
## ma3    -1.185e-02   2.078e-02   -0.570 0.568707
## omega   5.591e-06   1.566e-06    3.569 0.000358 ***
## alpha1  4.664e-02   1.060e-02    4.401 1.08e-05 ***
## beta1   9.358e-01   1.414e-02   66.163  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  8101.509    normalized:  2.684397
##
## Description:
##  Sun Apr 17 22:14:59 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  13864.85  0
##  Shapiro-Wilk Test  R    W      0.9223178 0
##  Ljung-Box Test     R    Q(10)  6.618727  0.7608821
##  Ljung-Box Test     R    Q(15)  8.27309   0.9123679
##  Ljung-Box Test     R    Q(20)  15.00349  0.7762078
##  Ljung-Box Test     R^2  Q(10)  2.86358   0.9844546
##  Ljung-Box Test     R^2  Q(15)  4.460227  0.995794
##  Ljung-Box Test     R^2  Q(20)  6.211546  0.99858
##  LM Arch Test       R    TR^2   3.544794  0.990323
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.363492 -5.347554 -5.363506 -5.357761
```

```
summary(gfit9)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 3) + garch(2, 0), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ arma(1, 3) + garch(2, 0)
## <environment: 0x7ffe02f19390>
##  [data = diff(log(google_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ma1          ma2          ma3        omega
##  0.00033658   0.57431420  -0.59668835   0.01615746  -0.01076006   0.00018168
##      alpha1       alpha2
##  0.20022584   0.35835656
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      3.366e-04   2.280e-04    1.476   0.1400
## ar1     5.743e-01   2.564e-01    2.240   0.0251 *
## ma1    -5.967e-01   2.602e-01   -2.294   0.0218 *
## ma2     1.616e-02   2.787e-02    0.580   0.5621
## ma3    -1.076e-02   2.120e-02   -0.508   0.6117
## omega   1.817e-04   8.833e-06   20.569  < 2e-16 ***
## alpha1  2.002e-01   3.165e-02    6.327 2.50e-10 ***
## alpha2  3.584e-01   4.496e-02    7.970 1.55e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7993.377    normalized:  2.648567
##
## Description:
##  Sun Apr 17 22:15:00 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  6966.181  0
##  Shapiro-Wilk Test  R    W      0.9263083 0
##  Ljung-Box Test     R    Q(10)  18.59114  0.04577379
##  Ljung-Box Test     R    Q(15)  21.75578  0.1143695
##  Ljung-Box Test     R    Q(20)  31.66869  0.04695909
##  Ljung-Box Test     R^2  Q(10)  28.35102  0.001585714
```

```
##  Ljung-Box Test     R^2  Q(15)  36.71727  0.001391526
##  Ljung-Box Test     R^2  Q(20)  49.02026  0.0003054228
##  LM Arch Test       R    TR^2   34.66234  0.0005299288
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.291833 -5.275896 -5.291847 -5.286102
```

```
summary(gfit10)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 3) + garch(2, 1), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ arma(1, 3) + garch(2, 1)
## <environment: 0x7ffe055cace8>
##  [data = diff(log(google_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ma1          ma2          ma3        omega
##  1.2219e-03  -7.8003e-01   7.8222e-01   1.8921e-02  -1.1663e-02   5.5672e-06
##      alpha1       alpha2        beta1
##  4.6465e-02   1.0000e-08   9.3597e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.222e-03   5.187e-04    2.356  0.01848 *
## ar1    -7.800e-01   1.749e-01   -4.461 8.16e-06 ***
## ma1     7.822e-01   1.766e-01    4.431 9.40e-06 ***
## ma2     1.892e-02   2.541e-02    0.745  0.45648
## ma3    -1.166e-02   2.091e-02   -0.558  0.57703
## omega   5.567e-06   2.148e-06    2.592  0.00954 **
## alpha1  4.647e-02   1.584e-02    2.934  0.00335 **
## alpha2  1.000e-08   2.333e-02    0.000  1.00000
## beta1   9.360e-01   2.069e-02   45.234  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  8101.814    normalized:  2.684498
##
## Description:
##  Sun Apr 17 22:15:01 2022 by user:
##
##
```

```
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  13875.51  0
##  Shapiro-Wilk Test  R    W      0.9223427 0
##  Ljung-Box Test     R    Q(10)  6.5325    0.768719
##  Ljung-Box Test     R    Q(15)  8.164114  0.9170346
##  Ljung-Box Test     R    Q(20)  14.9191   0.7810181
##  Ljung-Box Test     R^2  Q(10)  2.86997   0.9843205
##  Ljung-Box Test     R^2  Q(15)  4.468197  0.9957517
##  Ljung-Box Test     R^2  Q(20)  6.220158  0.9985657
##  LM Arch Test       R    TR^2   3.546412  0.9903029
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.363031 -5.345102 -5.363049 -5.356584
```

```
summary(gfit11)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 3) + garch(1, 2), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ arma(1, 3) + garch(1, 2)
## <environment: 0x7ffdea41f648>
##  [data = diff(log(google_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ma1          ma2          ma3        omega
##  1.1920e-03  -8.7496e-01   8.7762e-01   2.7491e-02  -6.0135e-03   9.2810e-06
##      alpha1        beta1        beta2
##  8.5714e-02   7.2313e-02   8.1396e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.192e-03   5.302e-04    2.248   0.0246 *
## ar1    -8.750e-01   7.374e-02  -11.866  < 2e-16 ***
## ma1     8.776e-01   7.622e-02   11.515  < 2e-16 ***
## ma2     2.749e-02   2.502e-02    1.099   0.2719
## ma3    -6.013e-03   2.026e-02   -0.297   0.7666
## omega   9.281e-06   2.178e-06    4.260 2.04e-05 ***
## alpha1  8.571e-02   1.558e-02    5.502 3.75e-08 ***
## beta1   7.231e-02   3.428e-02    2.109   0.0349 *
## beta2   8.140e-01   3.620e-02   22.488  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Log Likelihood:
##  8109.011    normalized:  2.686882
##
## Description:
##  Sun Apr 17 22:15:02 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  12420.78  0
##  Shapiro-Wilk Test  R    W      0.9234955 0
##  Ljung-Box Test     R    Q(10)  5.897926  0.8237649
##  Ljung-Box Test     R    Q(15)  6.582135  0.9682259
##  Ljung-Box Test     R    Q(20)  14.68199  0.7943057
##  Ljung-Box Test     R^2  Q(10)  2.567237  0.9898593
##  Ljung-Box Test     R^2  Q(15)  3.989442  0.9977717
##  Ljung-Box Test     R^2  Q(20)  5.943182  0.9989721
##  LM Arch Test       R    TR^2   3.120098  0.9946351
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.367800 -5.349871 -5.367818 -5.361353
```

```
summary(gfit12)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 3) + garch(2, 2), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ arma(1, 3) + garch(2, 2)
## <environment: 0x7ffdea8f2608>
##  [data = diff(log(google_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ma1          ma2          ma3        omega
##  1.1937e-03  -8.7158e-01   8.7400e-01   2.7018e-02  -6.1383e-03   9.7550e-06
##      alpha1       alpha2        beta1        beta2
##  8.3682e-02   5.4769e-03   5.9697e-02   8.2172e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      1.194e-03   5.293e-04    2.255   0.0241 *
## ar1    -8.716e-01   7.710e-02  -11.305  < 2e-16 ***
## ma1     8.740e-01   7.950e-02   10.994  < 2e-16 ***
```

```
## ma2     2.702e-02   2.522e-02    1.071   0.2840
## ma3    -6.138e-03   2.046e-02   -0.300   0.7641
## omega   9.755e-06   2.507e-06    3.891 9.97e-05 ***
## alpha1  8.368e-02   1.651e-02    5.070 3.98e-07 ***
## alpha2  5.477e-03   1.072e-02    0.511   0.6094
## beta1   5.970e-02   3.870e-02    1.543   0.1229
## beta2   8.217e-01   3.679e-02   22.337  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  8109.145    normalized:  2.686927
##
## Description:
##  Sun Apr 17 22:15:03 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  12497.24  0
##  Shapiro-Wilk Test  R    W      0.9233532 0
##  Ljung-Box Test     R    Q(10)  5.840023  0.8285199
##  Ljung-Box Test     R    Q(15)  6.565055  0.9686158
##  Ljung-Box Test     R    Q(20)  14.50432  0.8040334
##  Ljung-Box Test     R^2  Q(10)  2.451337  0.991566
##  Ljung-Box Test     R^2  Q(15)  3.917126  0.9979962
##  Ljung-Box Test     R^2  Q(20)  5.927733  0.9989916
##  LM Arch Test       R    TR^2   3.033169  0.9953053
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.367227 -5.347305 -5.367249 -5.360063
```

According to the AIC(no big difference) and the significance, we find that generally we should have a ARMA model as mean, here the best model is ARMA(1,3)+GARCH(1,1), but we find that ma2, ma3, not significant. since AIC are similar, and the significance, we choose GARCH(1,1) as our best model, it is simple and reasonable.

```
gfit = garchFit(~garch(1,1),data = diff(log(google_open)))
```

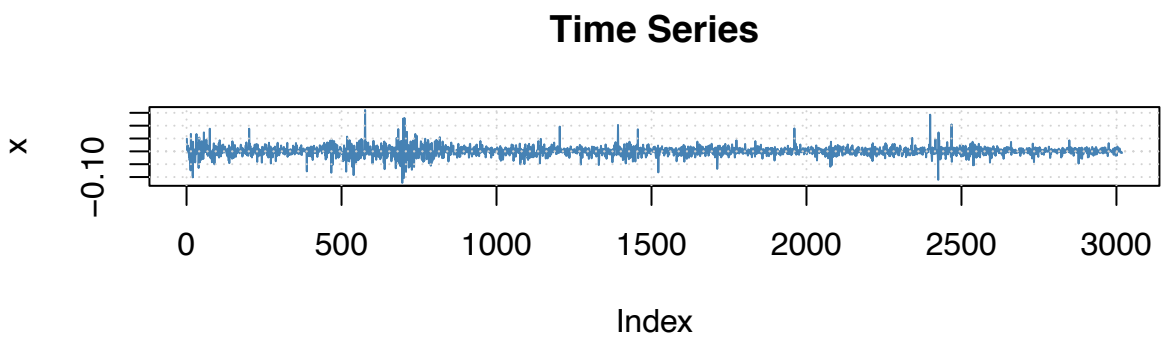
```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(0, 0)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(1, 1)
##  ARMA Order:                 0 0
##  Max ARMA Order:             0
##  GARCH Order:                1 1
##  Max GARCH Order:            1
##  Maximum Order:              1
##  Conditional Dist:           norm
##  h.start:                    2
##  llh.start:                  1
##  Length of Series:           3018
```

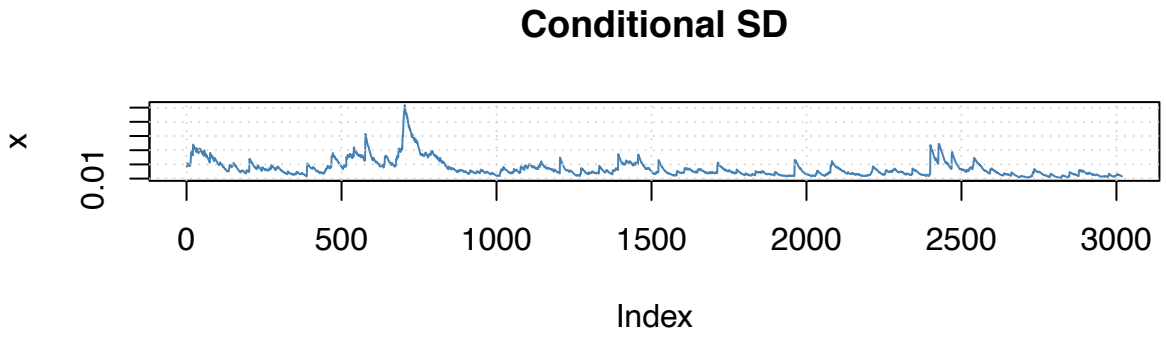
```
##  Recursion Init:            mci
##  Series Scale:              0.01843157
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U          V    params includes
##     mu     -0.28901297   0.289013 0.0289013     TRUE
##     omega   0.00000100 100.000000 0.1000000     TRUE
##     alpha1  0.00000001   1.000000 0.1000000     TRUE
##     gamma1 -0.99999999   1.000000 0.1000000    FALSE
##     beta1   0.00000001   1.000000 0.8000000     TRUE
##     delta   0.00000000   2.000000 2.0000000    FALSE
##     skew    0.10000000  10.000000 1.0000000    FALSE
##     shape   1.00000000  10.000000 4.0000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1
##      1      2      3      5
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3988.4070: 0.0289013 0.100000 0.100000 0.800000
##   1:     3983.2635: 0.0289021 0.0822374 0.101597 0.792792
##   2:     3972.8937: 0.0289032 0.0811637 0.118334 0.802212
##   3:     3972.4899: 0.0289069 0.0491205 0.138782 0.808140
##   4:     3969.7117: 0.0289106 0.0566692 0.142807 0.825369
##   5:     3963.0300: 0.0289291 0.0500884 0.126184 0.832438
##   6:     3961.7358: 0.0289353 0.0425124 0.115402 0.846449
##   7:     3959.0738: 0.0289613 0.0385342 0.0869166 0.873969
##   8:     3958.8371: 0.0289618 0.0372444 0.0892701 0.876910
##   9:     3958.3404: 0.0289648 0.0346250 0.0868269 0.878645
##  10:     3957.9655: 0.0289685 0.0340572 0.0850930 0.882182
##  11:     3956.1556: 0.0290280 0.0165492 0.0506765 0.930160
##  12:     3955.1820: 0.0290281 0.0175396 0.0511257 0.930775
##  13:     3955.1704: 0.0290632 0.0175355 0.0508372 0.930800
##  14:     3955.1599: 0.0291345 0.0176412 0.0505974 0.931088
##  15:     3955.1510: 0.0292063 0.0175573 0.0503708 0.931208
##  16:     3955.1007: 0.0303104 0.0171611 0.0485197 0.933328
##  17:     3955.0307: 0.0343514 0.0169696 0.0489060 0.933326
##  18:     3955.0198: 0.0354495 0.0174937 0.0493969 0.932132
##  19:     3955.0086: 0.0365444 0.0167102 0.0474731 0.934595
##  20:     3955.0052: 0.0370364 0.0167693 0.0474705 0.934681
##  21:     3955.0049: 0.0368745 0.0167822 0.0475568 0.934560
##  22:     3955.0049: 0.0368779 0.0167769 0.0475417 0.934580
##  23:     3955.0049: 0.0368778 0.0167773 0.0475428 0.934578
##
## Final Estimate of the Negative LLH:
```

```
##  LLH:  -8097.953    norm LLH:  -2.683218
##           mu        omega       alpha1        beta1
## 6.797165e-04 5.699616e-06 4.754280e-02 9.345783e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu         omega        alpha1         beta1
## mu     -12967223.43  3.910124e+08 -2.025057e+04  4.594106e+04
## omega  391012363.26 -7.454003e+12 -1.135732e+09 -1.584795e+09
## alpha1    -20250.57 -1.135732e+09 -2.840852e+05 -3.214280e+05
## beta1      45941.06 -1.584795e+09 -3.214280e+05 -3.993223e+05
## attr(,"time")
## Time difference of 0.02717781 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.112267 secs
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
summary(gfit)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = diff(log(google_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7ffe042b8608>
##  [data = diff(log(google_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 6.7972e-04  5.6996e-06  4.7543e-02  9.3458e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     6.797e-04   2.795e-04    2.432 0.015009 *
## omega  5.700e-06   1.598e-06    3.566 0.000363 ***
## alpha1 4.754e-02   1.087e-02    4.374 1.22e-05 ***
## beta1  9.346e-01   1.448e-02   64.528  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Log Likelihood:
##  8097.953    normalized:  2.683218
##
## Description:
##  Sun Apr 17 22:15:04 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  13715.47  0
##  Shapiro-Wilk Test  R    W      0.9226129 0
##  Ljung-Box Test     R    Q(10)  13.88787  0.178168
##  Ljung-Box Test     R    Q(15)  16.47777  0.3510284
##  Ljung-Box Test     R    Q(20)  21.95521  0.3429459
##  Ljung-Box Test     R^2  Q(10)  2.899156  0.9836986
##  Ljung-Box Test     R^2  Q(15)  4.388775  0.9961583
##  Ljung-Box Test     R^2  Q(20)  6.40014   0.9982378
##  LM Arch Test       R    TR^2   3.332717  0.9927043
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.363786 -5.355817 -5.363789 -5.360920
```

```
par(mfrow=c(2,1))
plot(gfit, which = 1)
plot(gfit, which = 2)
```

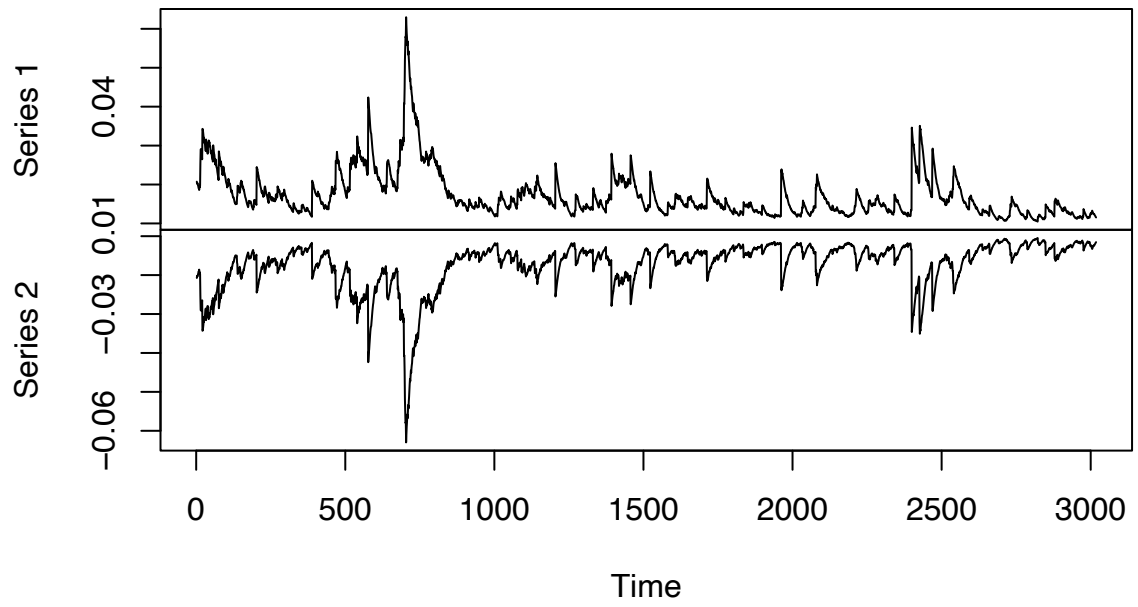





```
gfit = garch(diff(log(google_open)),order = c(1,1))
```

```
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
```

```
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     3.057504e-04     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1 -1.062e+04
##      1    7 -1.062e+04  3.23e-04  4.76e-04  1.6e-04  2.1e+10  1.6e-05  4.93e+06
##      2    8 -1.062e+04  3.53e-05  4.13e-05  1.3e-04  2.0e+00  1.6e-05  3.14e+01
##      3    9 -1.062e+04  2.12e-06  2.16e-06  1.5e-04  2.0e+00  1.6e-05  3.12e+01
##      4   16 -1.066e+04  3.21e-03  4.89e-03  3.9e-01  2.0e+00  6.4e-02  3.11e+01
##      5   19 -1.072e+04  5.96e-03  5.26e-03  6.9e-01  2.0e+00  2.6e-01  2.39e+00
##      6   21 -1.074e+04  1.41e-03  1.35e-03  7.5e-02  2.0e+00  5.1e-02  3.01e+02
##      7   23 -1.077e+04  3.04e-03  2.89e-03  1.2e-01  2.0e+00  1.0e-01  1.76e+04
##      8   24 -1.081e+04  3.72e-03  6.31e-03  1.8e-01  2.0e+00  2.0e-01  1.97e+06
##      9   36 -1.083e+04  1.89e-03  7.05e-03  1.7e-05  2.6e+00  2.2e-05  1.00e+01
##     10   37 -1.084e+04  6.48e-04  4.42e-04  1.6e-05  2.0e+00  2.2e-05  1.34e+00
##     11   38 -1.084e+04  1.09e-04  1.50e-04  1.6e-05  2.0e+00  2.2e-05  5.18e+00
##     12   39 -1.084e+04  1.13e-05  1.26e-05  1.6e-05  2.0e+00  2.2e-05  4.10e+00
##     13   46 -1.085e+04  1.01e-03  1.77e-03  5.3e-02  2.0e+00  7.6e-02  4.19e+00
##     14   47 -1.085e+04  1.39e-04  1.50e-04  1.2e-02  0.0e+00  2.2e-02  1.50e-04
##     15   49 -1.085e+04  4.95e-04  2.45e-04  2.3e-02  0.0e+00  5.1e-02  2.45e-04
##     16   51 -1.086e+04  2.01e-04  2.31e-04  9.7e-03  1.9e+00  2.0e-02  3.39e-03
##     17   54 -1.086e+04  5.51e-04  6.29e-04  3.7e-02  7.8e-01  8.2e-02  2.36e-03
##     18   55 -1.086e+04  2.07e-05  5.11e-04  3.4e-02  8.8e-01  8.2e-02  7.19e-04
##     19   64 -1.087e+04  2.18e-04  4.99e-04  5.5e-07  4.5e+00  1.0e-06  1.30e-03
##     20   65 -1.087e+04  9.59e-06  8.29e-06  5.3e-07  2.0e+00  1.0e-06  1.27e-03
##     21   66 -1.087e+04  2.07e-07  1.86e-07  5.3e-07  2.0e+00  1.0e-06  1.44e-03
##     22   74 -1.087e+04  1.37e-04  3.47e-04  8.6e-03  1.4e+00  1.7e-02  1.43e-03
##     23   75 -1.087e+04  6.76e-05  1.22e-04  7.3e-03  9.4e-01  1.7e-02  2.65e-04
##     24   76 -1.087e+04  2.88e-06  2.55e-06  8.2e-04  0.0e+00  1.7e-03  2.55e-06
##     25   77 -1.087e+04  3.35e-07  3.97e-07  6.9e-04  0.0e+00  1.5e-03  3.97e-07
##     26   79 -1.087e+04  4.59e-09  1.85e-08  8.8e-05  7.0e-01  2.1e-04  2.53e-08
##     27  106 -1.087e+04 -1.67e-15  1.72e-14  1.1e-14  6.2e+04  2.1e-14  9.37e-09
##
##  ***** FALSE CONVERGENCE *****
##
##  FUNCTION    -1.086794e+04   RELDX        1.131e-14
##  FUNC. EVALS     106         GRAD. EVALS      27
##  PRELDF       1.723e-14      NPRELDF      9.373e-09
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    5.933201e-06     1.000e+00    -8.885e+03
##      2    5.028872e-02     1.000e+00    -3.148e+00
##      3    9.313829e-01     1.000e+00    -2.445e+00
plot(predict(gfit))
```




```
library(rugarch)
```

```
## Loading required package: parallel
```

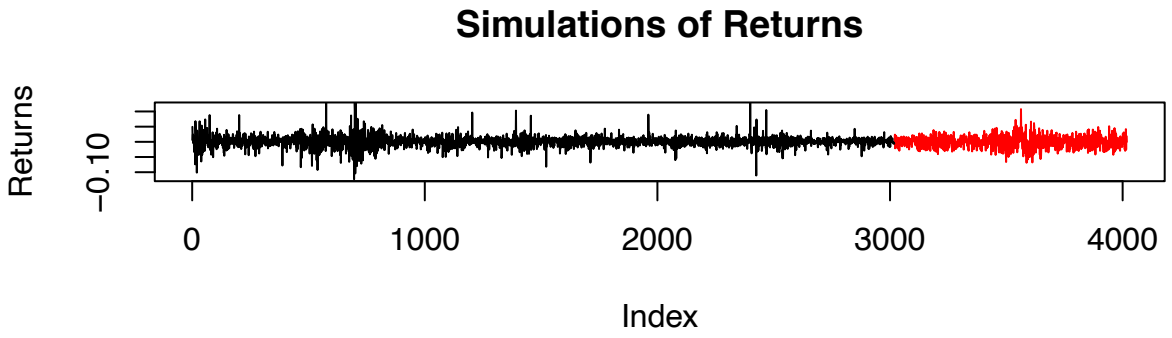
```
##
## Attaching package: 'rugarch'
```

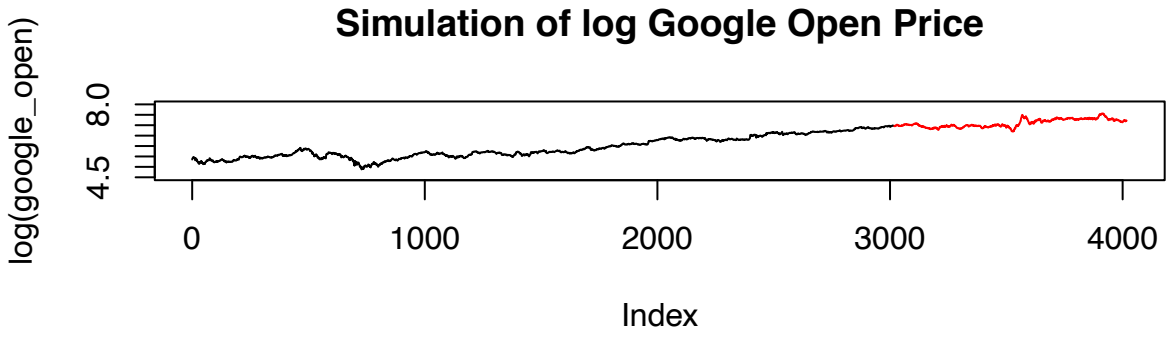
```
## The following objects are masked from 'package:fBasics':
##
##     qgh, qnig
```

```
## The following object is masked from 'package:stats':
##
##     sigma
```

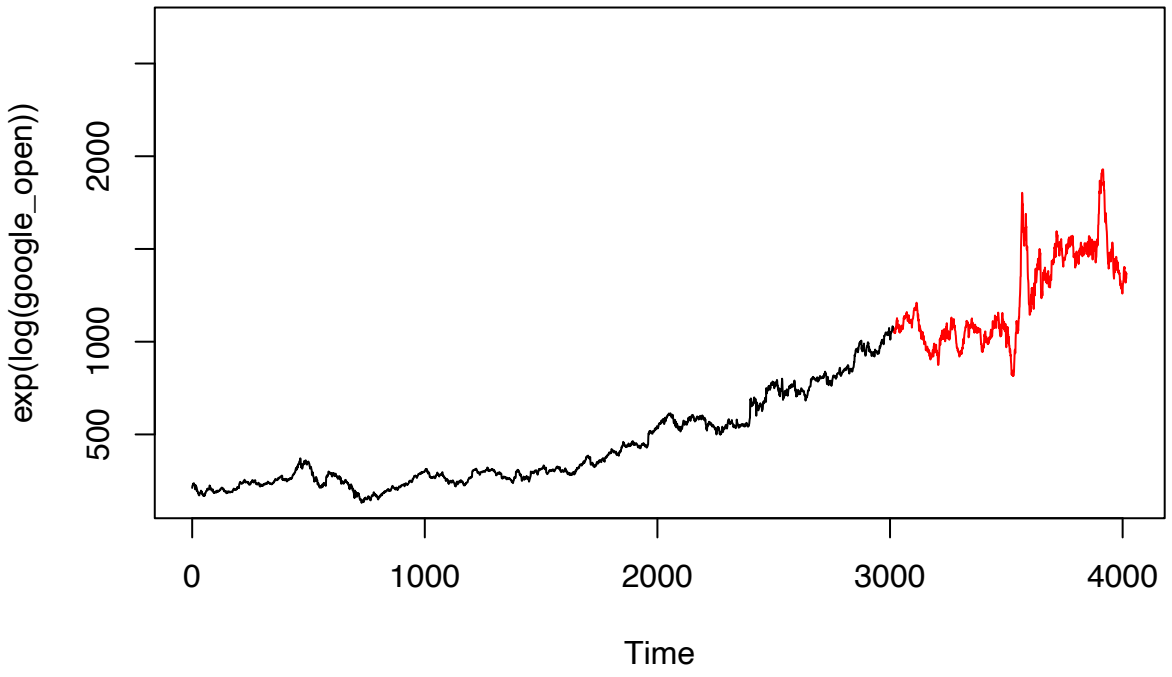
```
default_spec <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1,1)))
default_garch <- ugarchfit(spec = default_spec, data = diff(log(google_open)))
set.seed(135)
sim = ugarchsim(default_garch,n.sim=1000)
sim_rst = sim@simulation$seriesSim[,1]
par(mfrow=c(2,1))
matplot(y = sim@simulation$seriesSim[,1],x=3018:4017, type= "l",
       main = "Simulations of Returns",col =  "red",xlim=c(0,4020),ylab="Returns",xlab="Index",ylim=c(-0
lines(diff(log(google_open)))

sim = rep(0,1000)
first = log(google_open)[3018]
for(i in 1:1000){
  if(i==1){
    sim[i]=first+sim_rst[1]
  }
  else{
```

```
  sim[i]=sim[i-1]+sim_rst[i]}
}

plot(log(google_open),xlim = c(0,4020),ylim = c(4.5,8),main="Simulation of log Google Open Price",xlab=
lines(x=3018:4017,y=sim,col="red",type="l")
```

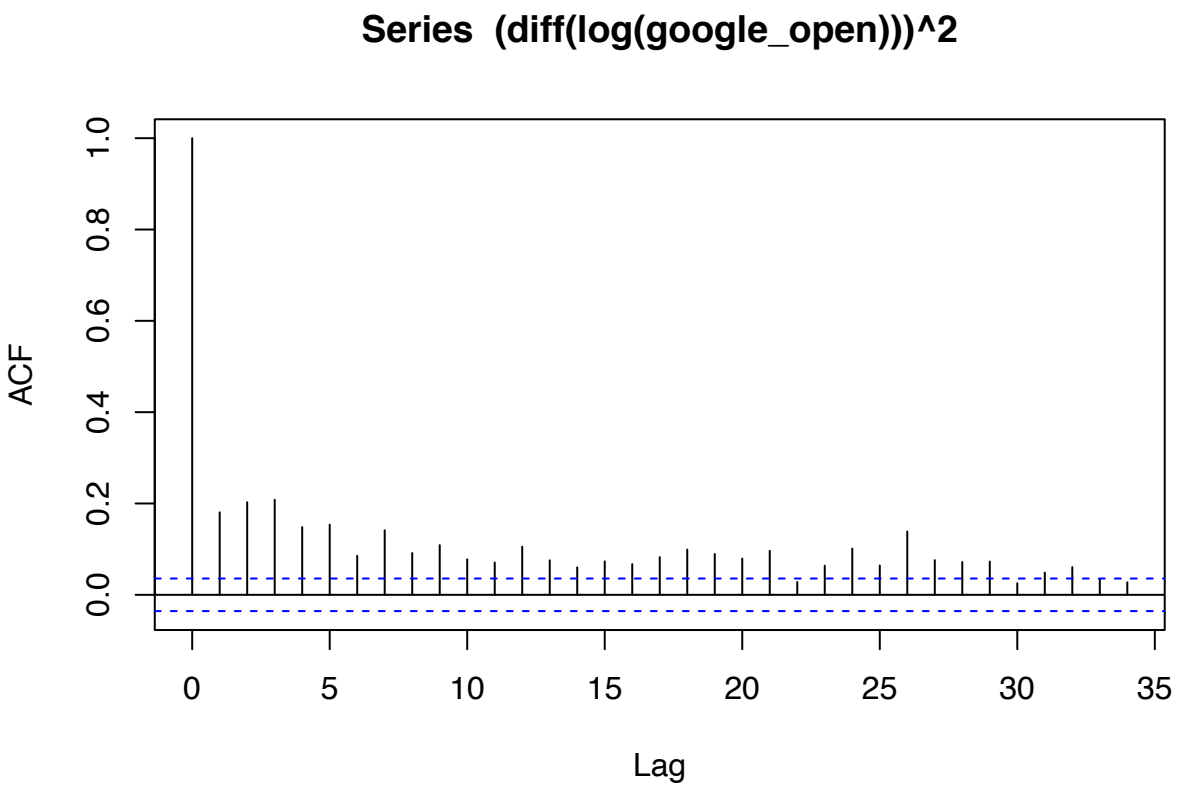




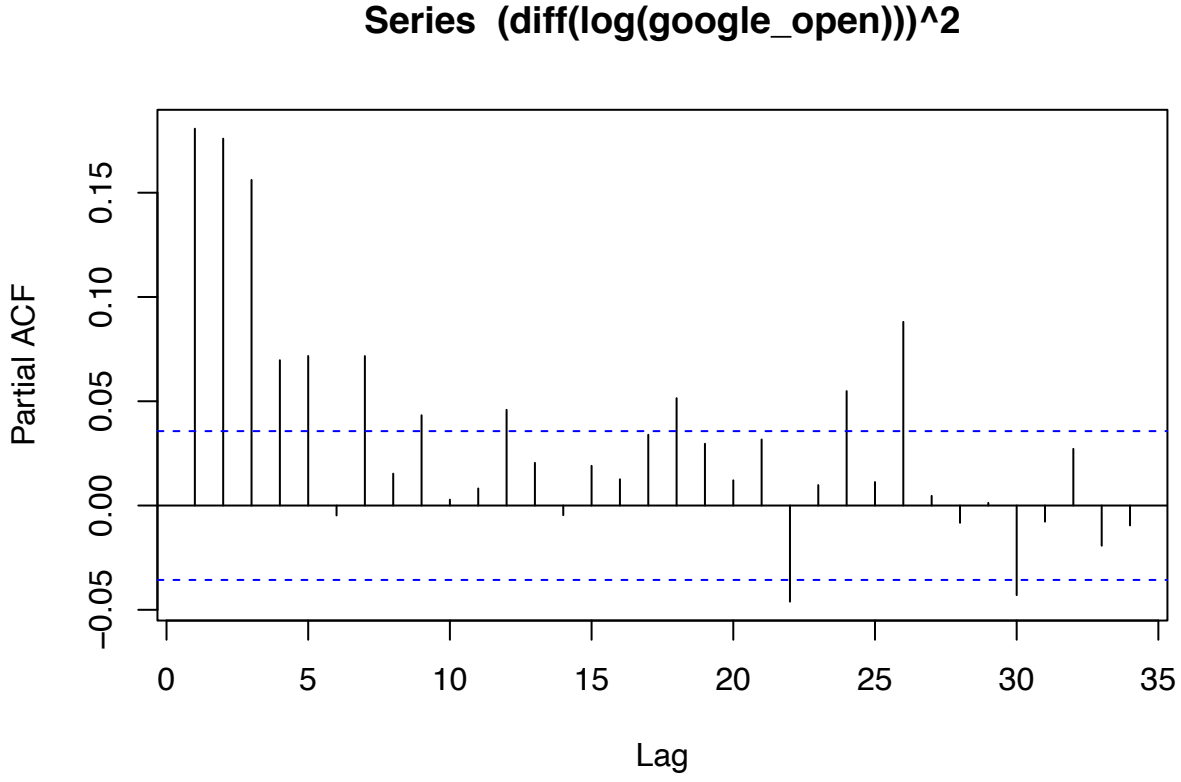

```
plot(exp(log(google_open)),xlim = c(0,4020),ylim=c(150,2700))
lines(x=3018:4017,y=exp(sim),col="red",type="l")
```



```
acf((diff(log(google_open)))^2)
```




```
pacf((diff(log(google_open)))^2)
```



```
model = auto.arima(diff(log(google_open))^2, max.p = 2, max.q = 2,max.order = 10, stationary = T, seasor
```

```
##
##  ARIMA(0,0,0) with zero mean     : -32000.44
##  ARIMA(0,0,0) with non-zero mean : -32248.44
##  ARIMA(0,0,1) with zero mean     : -32131.1
##  ARIMA(0,0,1) with non-zero mean : -32321.56
##  ARIMA(0,0,2) with zero mean     : -32227.95
##  ARIMA(0,0,2) with non-zero mean : -32385.61
##  ARIMA(1,0,0) with zero mean     : -32186.48
##  ARIMA(1,0,0) with non-zero mean : -32346.69
##  ARIMA(1,0,1) with zero mean     : -32525.89
##  ARIMA(1,0,1) with non-zero mean : -32548.26
##  ARIMA(1,0,2) with zero mean     : -32523.9
##  ARIMA(1,0,2) with non-zero mean : -32549.2
##  ARIMA(2,0,0) with zero mean     : -32332.73
##  ARIMA(2,0,0) with non-zero mean : -32439.55
##  ARIMA(2,0,1) with zero mean     : -32523.92
##  ARIMA(2,0,1) with non-zero mean : -32549.25
##  ARIMA(2,0,2) with zero mean     : Inf
##  ARIMA(2,0,2) with non-zero mean : -32547.23
##
##
##
##  Best model: ARIMA(2,0,1) with non-zero mean
```

garch(1,1) is better

apple

EDA

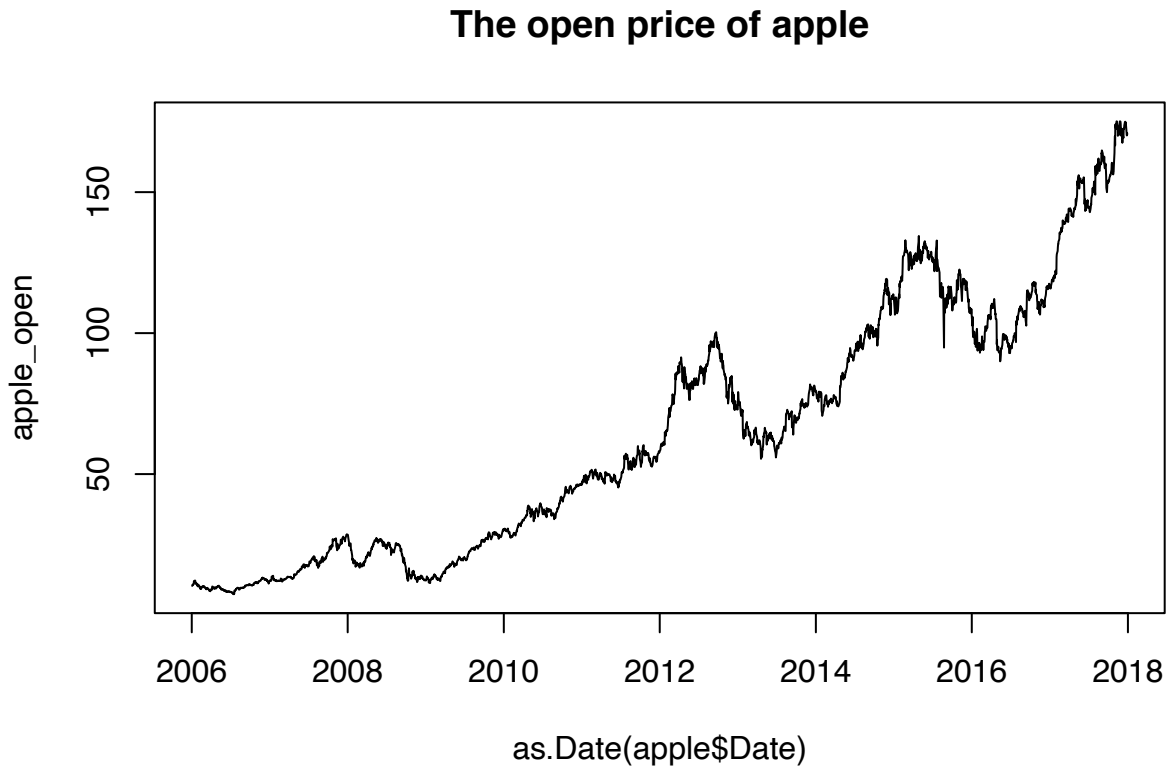
```
apple = read.csv("/Users/wyc/AAPL_2006-01-01_to_2018-01-01.csv")
```

```
head(apple)
```

```
##         Date  Open  High   Low Close    Volume Name
## 1 2006-01-03 10.34 10.68 10.32 10.68 201853036 AAPL
## 2 2006-01-04 10.73 10.85 10.64 10.71 155225609 AAPL
## 3 2006-01-05 10.69 10.70 10.54 10.63 112396081 AAPL
## 4 2006-01-06 10.75 10.96 10.65 10.90 176139334 AAPL
## 5 2006-01-09 10.96 11.03 10.82 10.86 168861224 AAPL
## 6 2006-01-10 10.89 11.70 10.83 11.55 570088246 AAPL
```

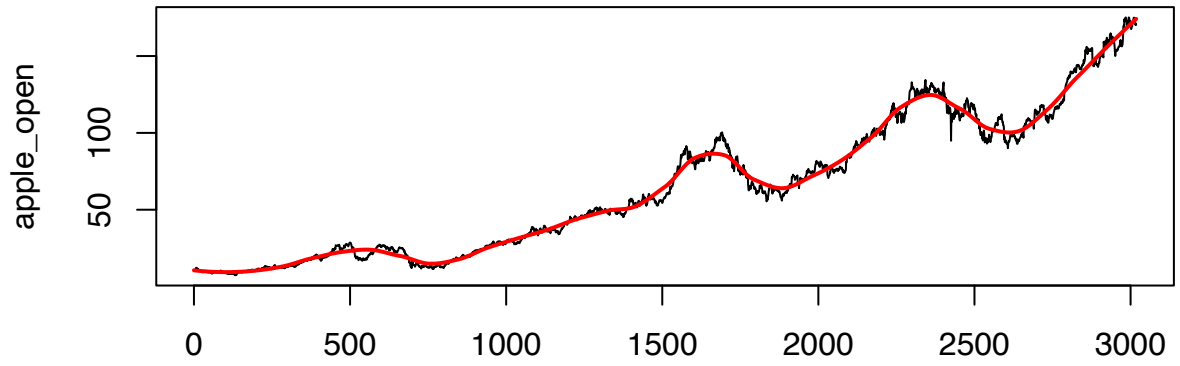
```
apple_open = apple$Open
apple_open = ts(apple_open,start = 0)
```

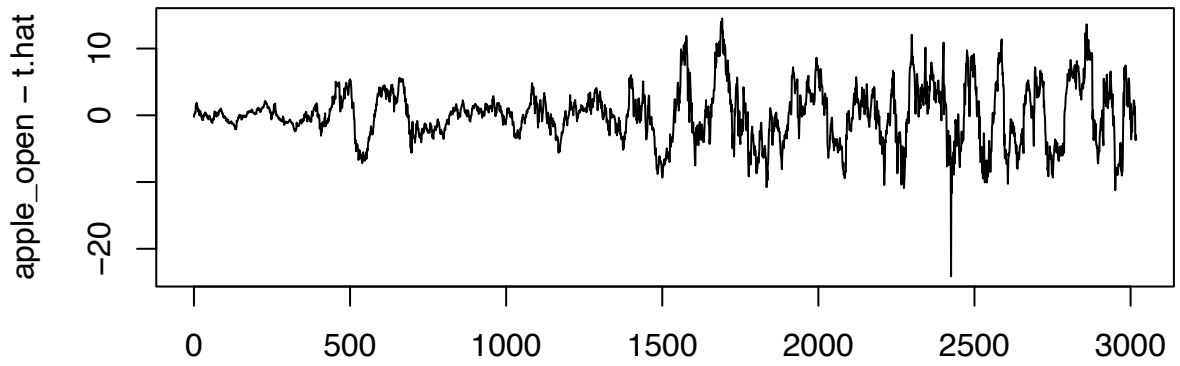
```
plot(y=apple_open,x=as.Date(apple$Date),type = "l",main="The open price of apple")
```



The open price of apple

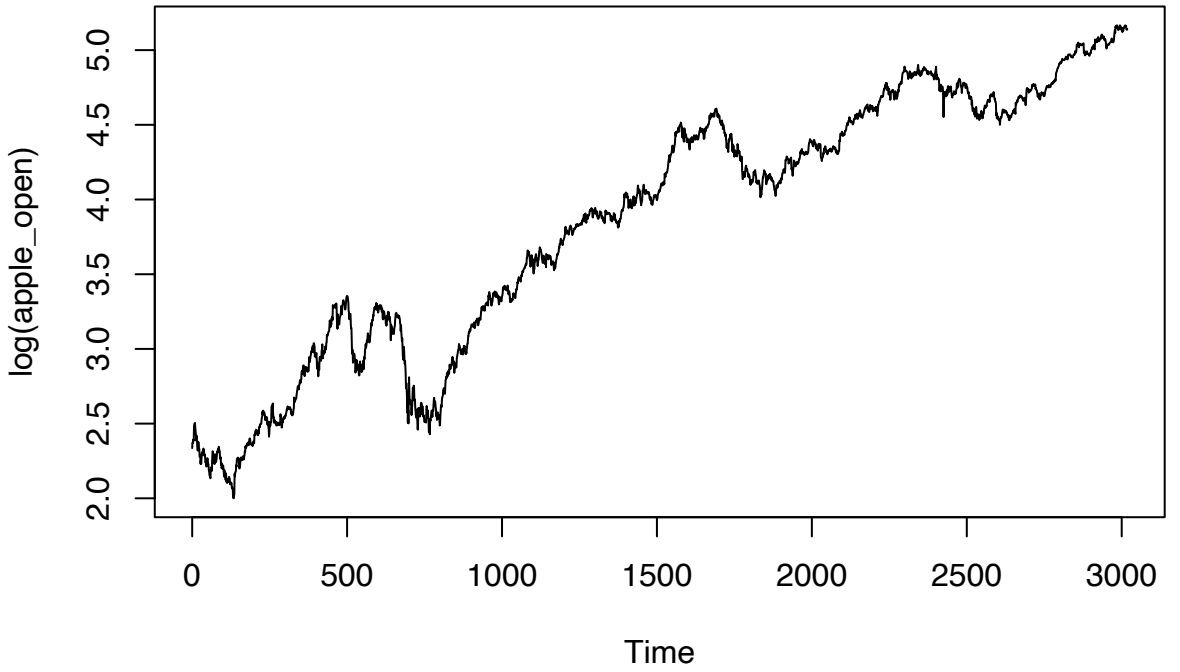
```
par(mfrow=c(2,1), mar = c(2, 4, 2, 2))
t.hat <- predict(loess(apple_open ~ time(apple_open), span=0.17))
plot(apple_open)
lines(as.numeric(time(apple_open)), t.hat, col="red", lwd=2)
plot(apple_open-t.hat)
```





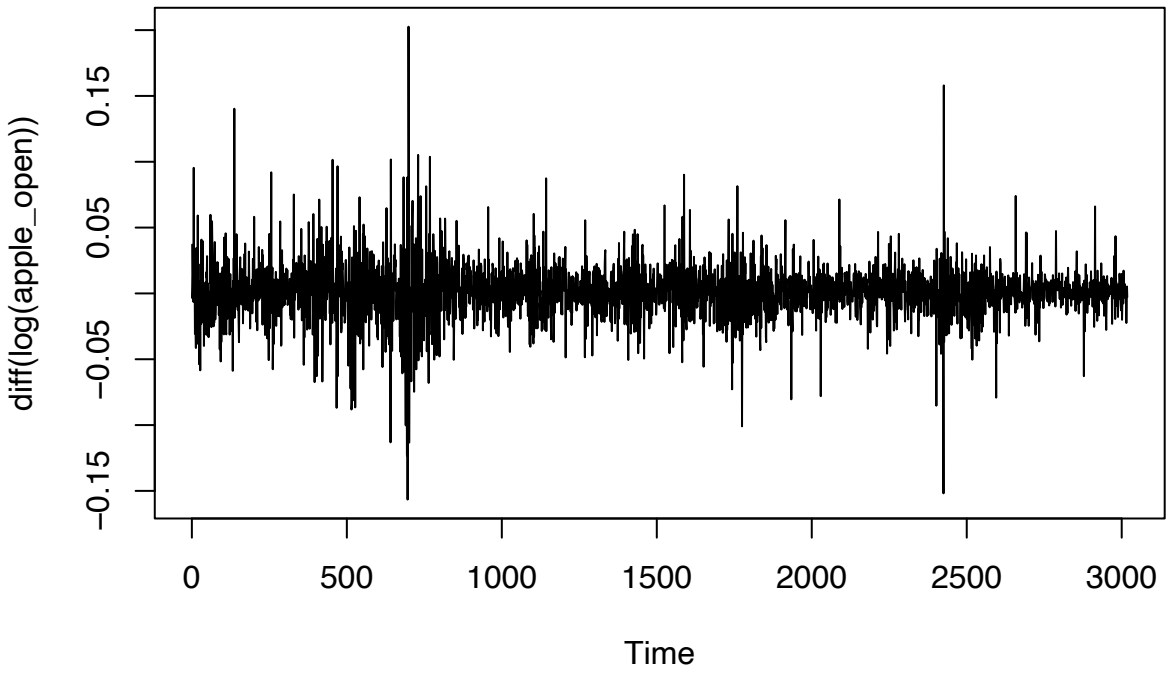
Loess, dipict the trend

```
plot(log(apple_open))
```

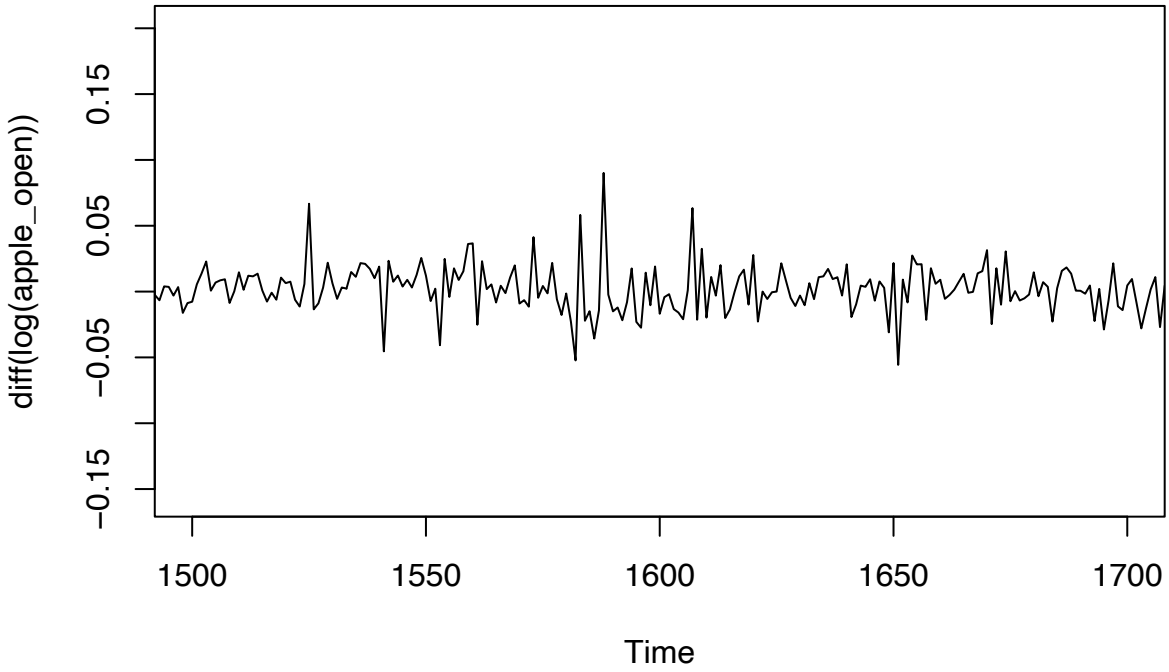


Take log to have a constant trend.

```
plot(diff(log(apple_open)))
```



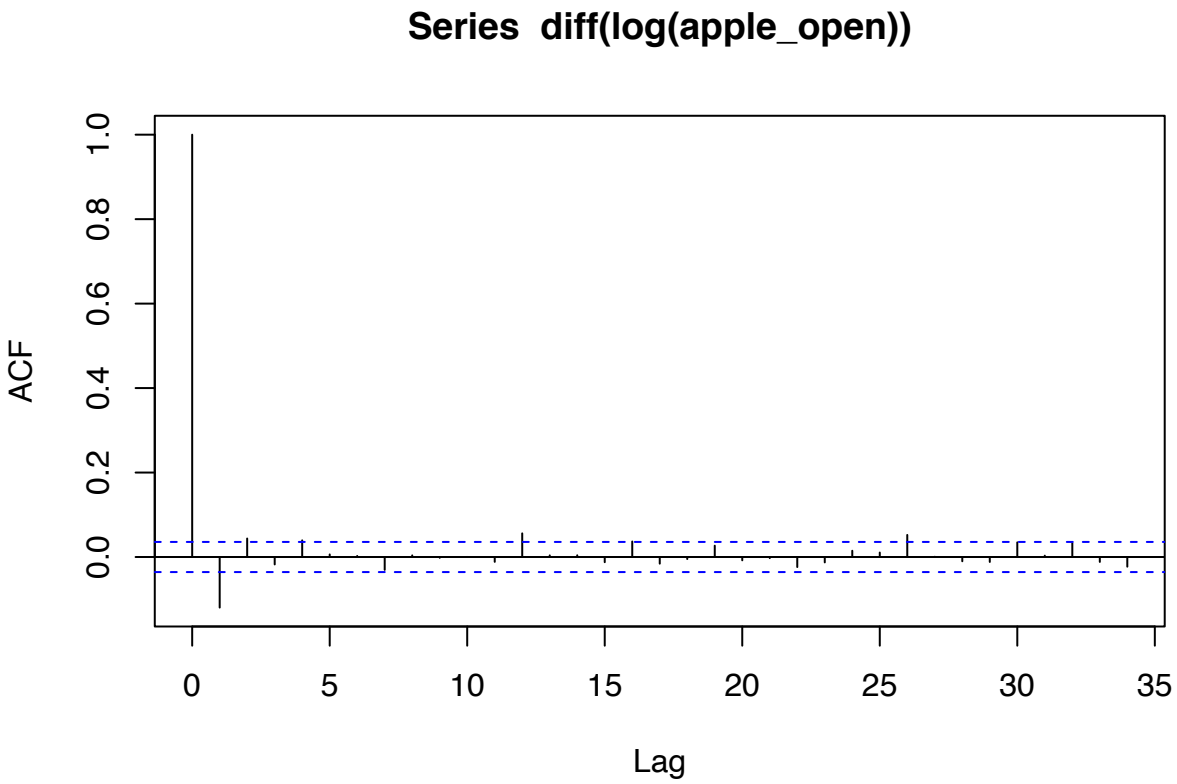

```
plot(diff(log(apple_open)),xlim=c(1500,1700))
```



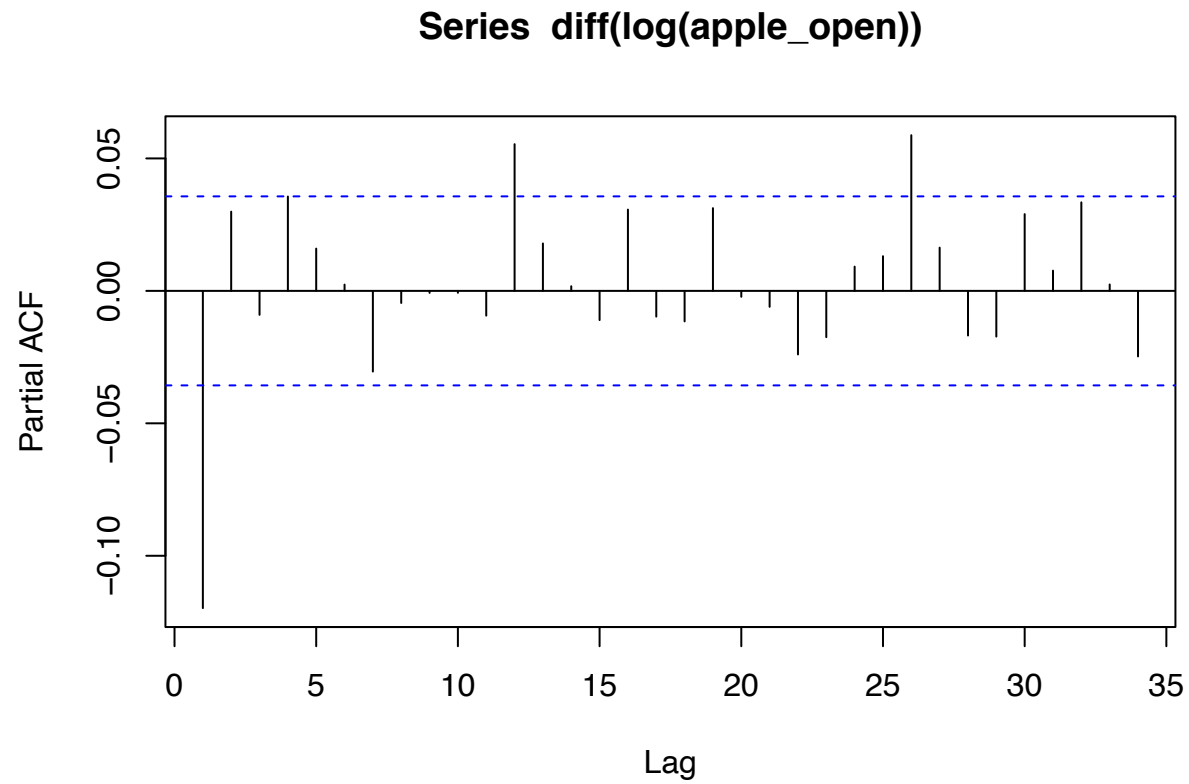
Take one diff, seems no trend, after diff it is return of the stock. Does not seems to have any seasonal pattern.

Model building

```
library(forecast)
acf(diff(log(apple_open)))
```




```
pacf(diff(log(apple_open)))
```




```
model = auto.arima(diff(log(apple_open)), max.p = 3, max.q = 3,max.order = 10, stationary = T, seasonal
```

```
##
```

```
##  ARIMA(0,0,0) with zero mean     : -14465.06
##  ARIMA(0,0,0) with non-zero mean : -14468.43
##  ARIMA(0,0,1) with zero mean     : -14502.26
##  ARIMA(0,0,1) with non-zero mean : -14507.12
##  ARIMA(0,0,2) with zero mean     : -14505.67
##  ARIMA(0,0,2) with non-zero mean : -14510.03
##  ARIMA(0,0,3) with zero mean     : -14503.81
##  ARIMA(0,0,3) with non-zero mean : -14508.26
##  ARIMA(1,0,0) with zero mean     : -14505.25
##  ARIMA(1,0,0) with non-zero mean : -14510.05
##  ARIMA(1,0,1) with zero mean     : -14507.05
##  ARIMA(1,0,1) with non-zero mean : -14511.47
##  ARIMA(1,0,2) with zero mean     : -14505.74
##  ARIMA(1,0,2) with non-zero mean : -14510.61
##  ARIMA(1,0,3) with zero mean     : -14505.65
##  ARIMA(1,0,3) with non-zero mean : -14509.69
##  ARIMA(2,0,0) with zero mean     : -14506.35
##  ARIMA(2,0,0) with non-zero mean : -14510.74
##  ARIMA(2,0,1) with zero mean     : Inf
##  ARIMA(2,0,1) with non-zero mean : Inf
##  ARIMA(2,0,2) with zero mean     : Inf
##  ARIMA(2,0,2) with non-zero mean : Inf
##  ARIMA(2,0,3) with zero mean     : -14507.06
##  ARIMA(2,0,3) with non-zero mean : -14509.37
##  ARIMA(3,0,0) with zero mean     : -14504.49
##  ARIMA(3,0,0) with non-zero mean : -14508.99
##  ARIMA(3,0,1) with zero mean     : Inf
##  ARIMA(3,0,1) with non-zero mean : Inf
##  ARIMA(3,0,2) with zero mean     : -14507.48
##  ARIMA(3,0,2) with non-zero mean : Inf
##  ARIMA(3,0,3) with zero mean     : -14518.51
##  ARIMA(3,0,3) with non-zero mean : -14522.72
##
##
##
##  Best model: ARIMA(3,0,3) with non-zero mean
```

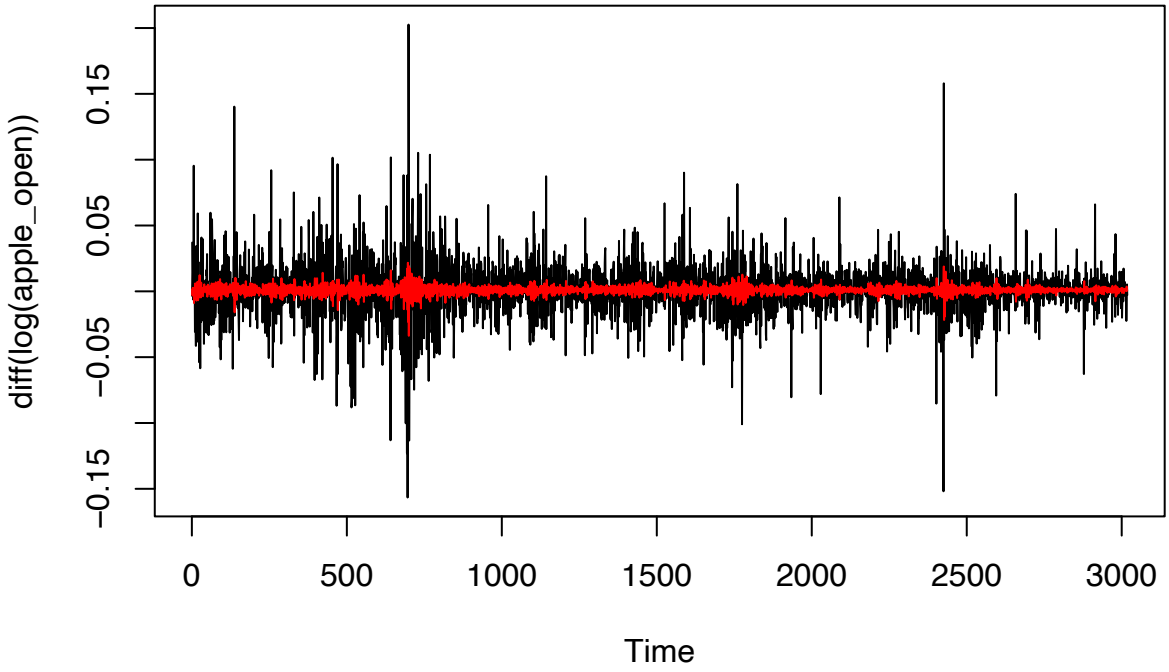
Fit ARIMA model to the return. Hard to figure our the orders of the ARIMA model by acf, pacf, thus auto.arima. arima(3,0,3) best.

```
arima_model = arima(x=diff(log(apple_open)),order = c(3,0,3),method='ML')
summary(arima_model)
```

```
##
## Call:
## arima(x = diff(log(apple_open)), order = c(3, 0, 3), method = "ML")
##
## Coefficients:
##          ar1     ar2      ar3      ma1     ma2     ma3  intercept
##       0.3838  0.0014  -0.6765  -0.4934  0.0808  0.6394      1e-03
## s.e.     NaN     NaN      NaN      NaN     NaN     NaN      4e-04
##
## sigma^2 estimated as 0.0004744:  log likelihood = 7266.6,  aic = -14517.21
##
## Training set error measures:
```

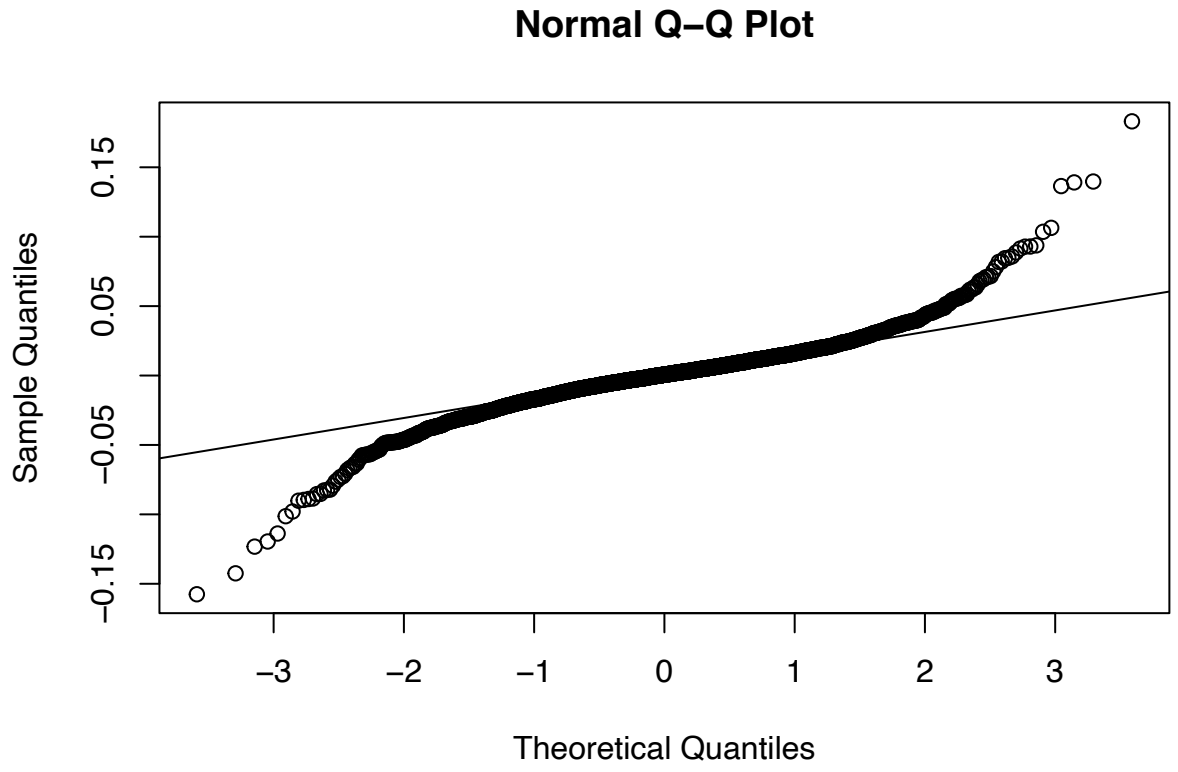
```
##                         ME       RMSE        MAE MPE MAPE      MASE
## Training set -2.716412e-05 0.02178111 0.01489763 NaN  Inf 0.6746955
##                      ACF1
## Training set -0.009253137
```

```
plot(diff(log(apple_open)))
lines(diff(log(apple_open))-arima_model$resid,col="red")
```

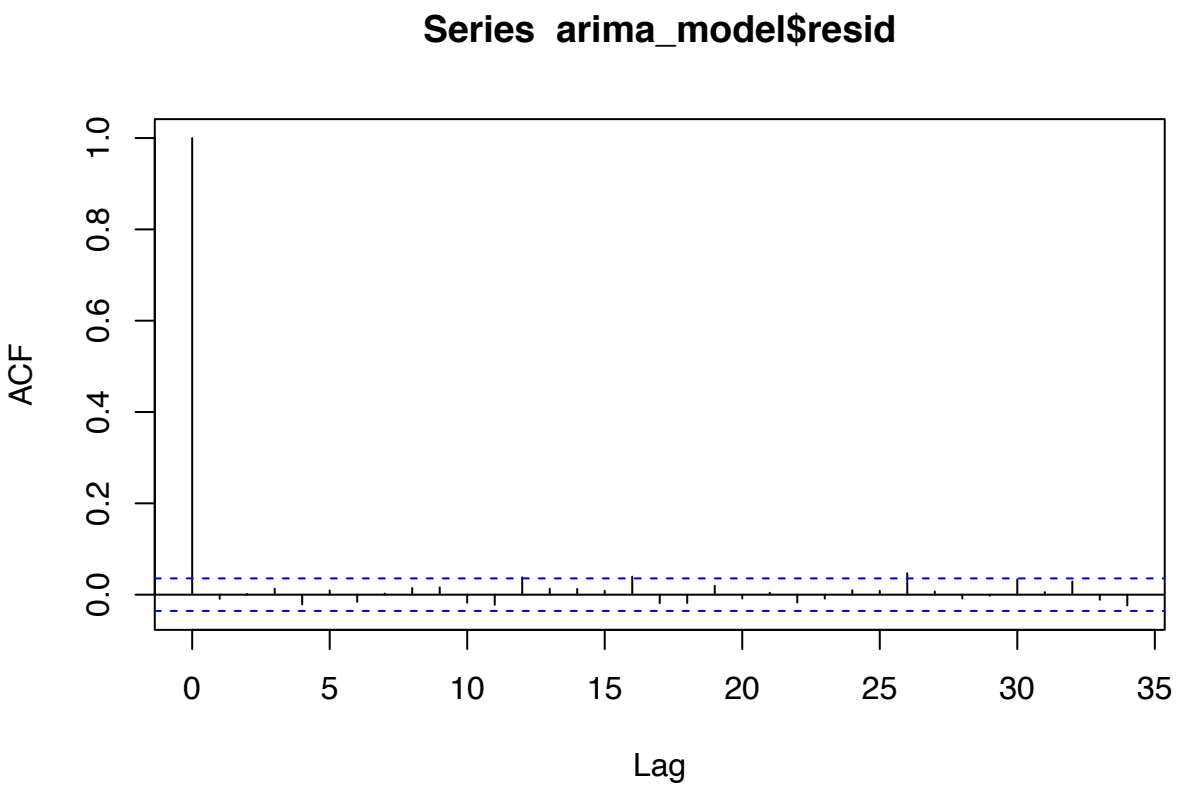


Bad result.

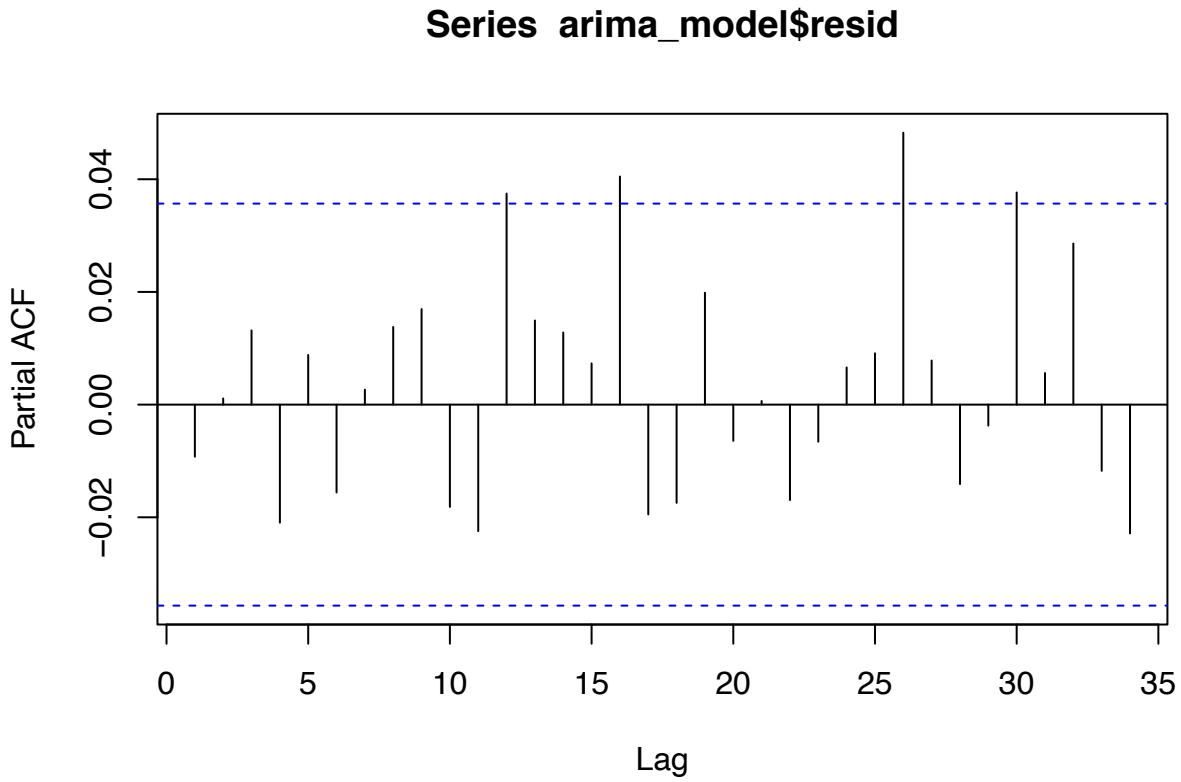
```
qqnorm(arima_model$resid); qqline(arima_model$resid)
```



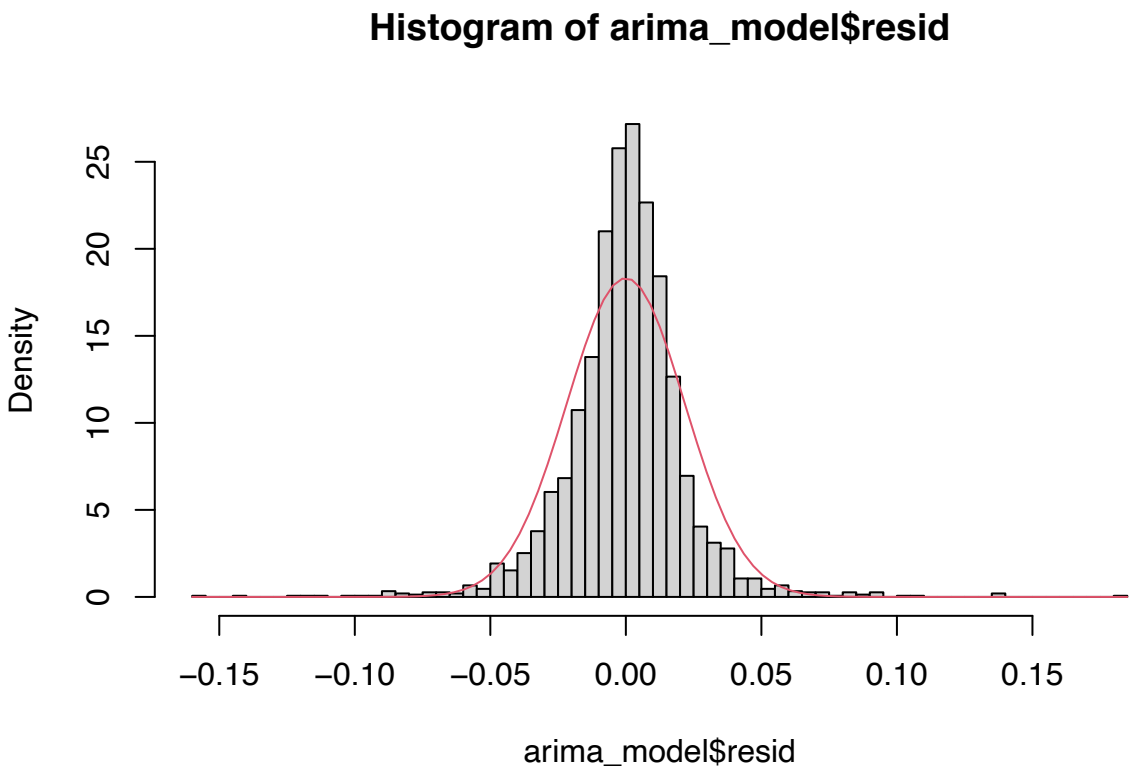

```
acf(arima_model$resid)
```



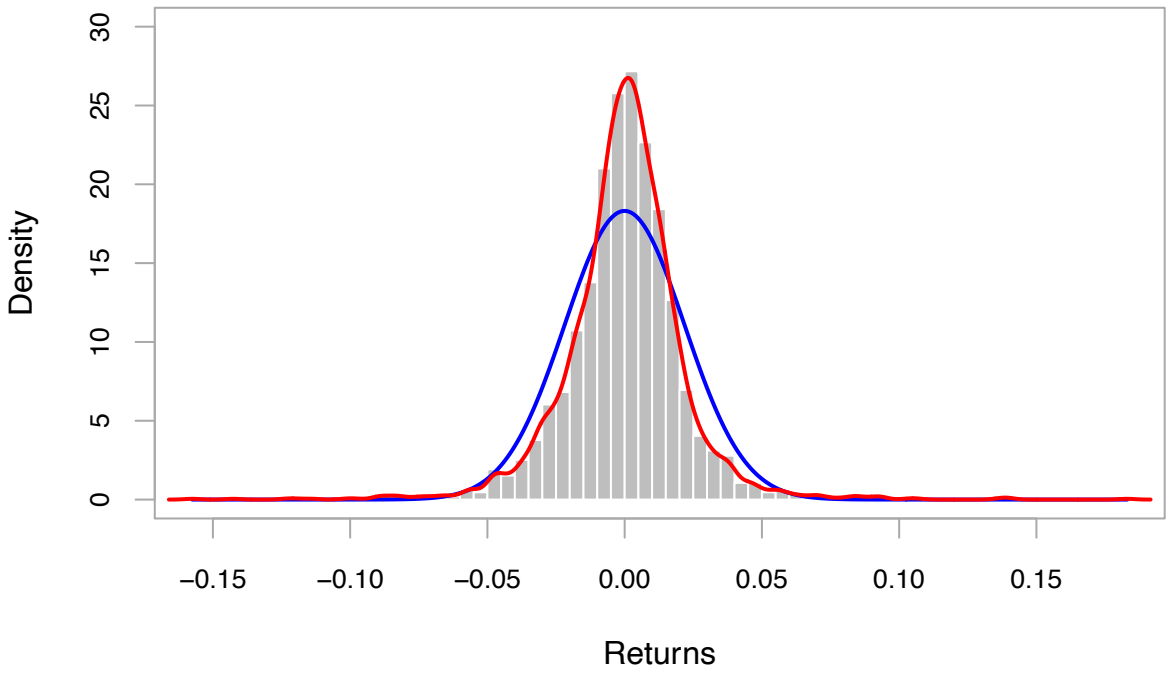

```
pacf(arima_model$resid)
```




```
hist(arima_model$resid,breaks=100,prob=TRUE)
library(MASS)
library(QRM)
fit1 <- fitdistr(arima_model$resid, "normal")
para <- fit1$estimate
curve(dnorm(x, para[1], para[2]), col = 2, add = TRUE)
```




```
fit2 = fit.st(arima_model$resid)
p_val = rep(0,30)
for(i in 1:30){
  p_val[i] = Box.test(arima_model$residual^2,lag = i)[3]
}
stdret <- residuals(arima_model, standardize = TRUE)
library(PerformanceAnalytics)
chart.Histogram(stdret, methods = c("add.normal", "add.density"),
          colorset=c("gray","red","blue"))
```



qqplot, non-gaussian resid. Heteroscedasticity exist from the test. But we probabily can use arima(1,0,3) as a mean model

```
library(fGarch)
gfit1 = garchFit(~garch(1,0),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(0, 0)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(1, 0)
##  ARMA Order:                 0 0
##  Max ARMA Order:             0
##  GARCH Order:                1 0
##  Max GARCH Order:            1
##  Maximum Order:              1
##  Conditional Dist:           norm
##  h.start:                    2
##  llh.start:                  1
##  Length of Series:           3018
##  Recursion Init:             mci
##  Series Scale:               0.02200558
##
## Parameter Initialization:
##  Initial Parameters:           $params
##  Limits of Transformations:    $U, $V
##  Which Parameters are Fixed?   $includes
##  Parameter Matrix:
##                       U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
```

```
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1
##      1      2      3
##  Persistence:                  0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8769.9924: 0.0422032 0.100000 0.100000
##   1:     4192.8649: 0.0422200  1.06555 0.360205
##   2:     4187.0253: 0.0422850  1.07055 0.313632
##   3:     4099.4719: 0.0423551 0.722546 0.174131
##   4:     4097.0384: 0.0526972 0.666718 0.452342
##   5:     4091.1308: 0.0598495 0.640521 0.376614
##   6:     4089.0333: 0.0598858 0.707656 0.333555
##   7:     4088.2787: 0.0616389 0.687482 0.268520
##   8:     4087.6268: 0.0648078 0.695407 0.294229
##   9:     4087.5816: 0.0614502 0.690471 0.297089
##  10:     4087.5779: 0.0623418 0.689101 0.298807
##  11:     4087.5775: 0.0622191 0.689574 0.300152
##  12:     4087.5772: 0.0623408 0.689447 0.299577
##  13:     4087.5772: 0.0623374 0.689444 0.299576
##
## Final Estimate of the Negative LLH:
##  LLH:  -7430.496    norm LLH:  -2.46206
##           mu        omega       alpha1
## 0.0013717719 0.0003338605 0.2995755543
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu       omega       alpha1
## mu     -8098781.334    -5550259     5013.757
## omega  -5550259.400 -9837324600 -1503391.752
## alpha1     5013.757    -1503392    -1245.668
## attr(,"time")
## Time difference of 0.0304811 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.1334391 secs
```

```
gfit2 = garchFit(~garch(1,1),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
```

```
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1   0.00000001   1.0000000 0.80000000     TRUE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1
##      1      2      3      5
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3956.9207: 0.0422032 0.100000 0.100000 0.800000
##   1:     3951.7454: 0.0422071 0.0774769 0.0988866 0.788793
##   2:     3934.4556: 0.0422156 0.0757252 0.121400 0.799936
##   3:     3931.8868: 0.0422329 0.0533463 0.131919 0.804679
##   4:     3928.4349: 0.0422803 0.0503244 0.132421 0.829649
##   5:     3924.0138: 0.0423708 0.0387574 0.117234 0.845930
##   6:     3921.8326: 0.0424971 0.0368799 0.0993199 0.863272
##   7:     3921.8056: 0.0424982 0.0364912 0.0991010 0.863258
##   8:     3921.7765: 0.0425019 0.0364931 0.0989519 0.864145
##   9:     3921.6685: 0.0425120 0.0355007 0.0980850 0.865360
##  10:     3921.3766: 0.0425691 0.0331671 0.0951336 0.871955
##  11:     3921.2451: 0.0428320 0.0316229 0.0916603 0.874450
##  12:     3920.9999: 0.0431374 0.0323339 0.0900277 0.876443
##  13:     3920.3794: 0.0467049 0.0291091 0.0801406 0.887425
##  14:     3920.0006: 0.0503196 0.0264704 0.0800513 0.893026
##  15:     3919.6174: 0.0539395 0.0269423 0.0814479 0.889132
```

```
##  16:     3919.2279: 0.0575542 0.0297721 0.0844164 0.884473
##  17:     3918.9919: 0.0611769 0.0298267 0.0857209 0.882454
##  18:     3918.8595: 0.0647962 0.0279662 0.0850417 0.886469
##  19:     3918.6968: 0.0684184 0.0292835 0.0857577 0.884142
##  20:     3918.5939: 0.0720387 0.0284228 0.0828569 0.886756
##  21:     3918.5827: 0.0740275 0.0288860 0.0833149 0.886017
##  22:     3918.5822: 0.0737546 0.0288236 0.0832628 0.886087
##  23:     3918.5822: 0.0737460 0.0288183 0.0832446 0.886109
##  24:     3918.5822: 0.0737480 0.0288192 0.0832491 0.886104
##
## Final Estimate of the Negative LLH:
##  LLH:  -7599.491    norm LLH:  -2.518055
##           mu        omega       alpha1        beta1
## 1.622869e-03 1.395559e-05 8.324908e-02 8.861041e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1         beta1
## mu      -9627339.967 -5.917409e+07 -3.096645e+03     -23001.54
## omega  -59174092.978 -1.332843e+12 -2.589564e+08 -386654965.12
## alpha1     -3096.645 -2.589564e+08 -9.236301e+04    -104269.78
## beta1     -23001.542 -3.866550e+08 -1.042698e+05    -135854.84
## attr(,"time")
## Time difference of 0.02871919 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.2021661 secs
```

```
gfit3 = garchFit(~garch(2,0),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 0)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               2 0
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
```

```
##                      U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.05000000     TRUE
##     alpha2  0.00000001   1.0000000 0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     gamma2 -0.99999999   1.0000000 0.10000000    FALSE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2
##      1      2      3      4
##  Persistence:                  0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8013.8444: 0.0422032 0.100000 0.0500000 0.0500000
##   1:     4235.0428: 0.0422167 0.955183 0.399164 0.433075
##   2:     4234.9209: 0.0439077  1.13462 1.00000e-08 0.324208
##   3:     4179.9374: 0.0439308  1.01028 0.184292 0.281526
##   4:     4085.2239: 0.0439689 0.816791 0.102750 0.196908
##   5:     4063.9278: 0.0439974 0.733245 0.112458 0.181179
##   6:     4045.7339: 0.0440894 0.576916 0.181337 0.191200
##   7:     4045.4413: 0.0443827 0.551375 0.168213 0.359768
##   8:     4043.1114: 0.0471312 0.551521 0.181078 0.305774
##   9:     4042.8316: 0.0487970 0.579174 0.199391 0.237481
##  10:     4042.7432: 0.0488864 0.561234 0.189843 0.244902
##  11:     4042.4888: 0.0495596 0.569261 0.195315 0.256027
##  12:     4042.4028: 0.0504463 0.564691 0.192311 0.257161
##  13:     4042.0603: 0.0605859 0.567328 0.192953 0.258843
##  14:     4042.0431: 0.0607475 0.563567 0.192588 0.262860
##  15:     4042.0413: 0.0607584 0.563370 0.191245 0.262748
##  16:     4042.0401: 0.0608096 0.563953 0.191167 0.263073
##  17:     4042.0283: 0.0623975 0.564441 0.191428 0.261650
##  18:     4042.0250: 0.0630975 0.563607 0.190522 0.264092
##  19:     4042.0244: 0.0635821 0.563913 0.190741 0.263217
##  20:     4042.0244: 0.0635214 0.563877 0.190713 0.263324
##  21:     4042.0244: 0.0635214 0.563877 0.190713 0.263323
##
## Final Estimate of the Negative LLH:
##  LLH:  -7476.049    norm LLH:  -2.477153
##           mu        omega       alpha1       alpha2
## 0.0013978248 0.0002730548 0.1907131039 0.2633233749
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu        omega        alpha1        alpha2
## mu     -8704457.875     -1339815     2511.5902    -2262.9842
## omega  -1339815.302 -11402654233 -1868359.6552 -1904044.7449
## alpha1     2511.590     -1868360    -1607.1570     -599.7291
```

```
## alpha2    -2262.984     -1904045     -599.7291    -1034.9452
## attr(,"time")
## Time difference of 0.04619503 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.2178218 secs
```

```
gfit4 = garchFit(~garch(2,1),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(0, 0)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(2, 1)
##  ARMA Order:                 0 0
##  Max ARMA Order:             0
##  GARCH Order:                2 1
##  Max GARCH Order:            2
##  Maximum Order:              2
##  Conditional Dist:           norm
##  h.start:                    3
##  llh.start:                  1
##  Length of Series:           3018
##  Recursion Init:             mci
##  Series Scale:               0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.05000000     TRUE
##     alpha2  0.00000001   1.0000000 0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     gamma2 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1   0.00000001   1.0000000 0.80000000     TRUE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2  beta1
##      1      2      3      4      7
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
```

```
## R coded nlminb Solver:
##
##   0:     3961.2284: 0.0422032 0.100000 0.0500000 0.0500000 0.800000
##   1:     3958.7303: 0.0422074 0.0752517 0.0508888 0.0474489 0.787032
##   2:     3940.8029: 0.0422089 0.0813216 0.0591294 0.0546774 0.793355
##   3:     3932.0767: 0.0422233 0.0573959 0.0781163 0.0657755 0.795219
##   4:     3931.2299: 0.0422251 0.0595147 0.0793128 0.0664944 0.797239
##   5:     3930.7493: 0.0422464 0.0558698 0.0797298 0.0610600 0.801784
##   6:     3926.2015: 0.0424423 0.0342303 0.0940236 0.0237442 0.854616
##   7:     3925.6732: 0.0425097 0.0365274 0.0975882 0.00883740 0.851554
##   8:     3924.6840: 0.0426063 0.0417997 0.109169 1.00000e-08 0.853123
##   9:     3923.7059: 0.0432666 0.0393731 0.107985 1.00000e-08 0.853286
##  10:     3923.4405: 0.0439217 0.0376607 0.106225 1.00000e-08 0.855562
##  11:     3923.1058: 0.0452577 0.0372157 0.106062 1.00000e-08 0.858035
##  12:     3922.3192: 0.0520852 0.0396470 0.104634 1.00000e-08 0.854596
##  13:     3921.1775: 0.0588999 0.0346079 0.102210 1.00000e-08 0.863892
##  14:     3919.8580: 0.0655327 0.0274224 0.0784649 1.00000e-08 0.893495
##  15:     3919.6429: 0.0655328 0.0267654 0.0781439 1.00000e-08 0.893024
##  16:     3919.6399: 0.0655687 0.0267854 0.0781811 1.00000e-08 0.892847
##  17:     3919.4514: 0.0723369 0.0290940 0.0830677 1.00000e-08 0.885912
##  18:     3919.4384: 0.0738143 0.0285713 0.0816341 1.00000e-08 0.887716
##  19:     3919.4375: 0.0741907 0.0282842 0.0819157 1.00000e-08 0.887867
##  20:     3919.4368: 0.0740823 0.0284085 0.0818148 1.00000e-08 0.887780
##  21:     3919.4368: 0.0740822 0.0284066 0.0818116 1.00000e-08 0.887784
##
## Final Estimate of the Negative LLH:
##  LLH:  -7598.637    norm LLH:  -2.517772
##           mu        omega       alpha1       alpha2        beta1
## 0.0016302210 0.0000137558 0.0818116153 0.0000000100 0.8877843641
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1        alpha2         beta1
## mu      -9613940.331 -6.062416e+07 -2.483909e+03 -6.614305e+03     -23546.25
## omega  -60624156.394 -1.369698e+12 -2.672092e+08 -2.716151e+08 -397608426.50
## alpha1     -2483.909 -2.672092e+08 -9.566311e+04 -9.466930e+04    -107548.36
## alpha2     -6614.305 -2.716151e+08 -9.466930e+04 -9.693189e+04    -109998.16
## beta1     -23546.254 -3.976084e+08 -1.075484e+05 -1.099982e+05    -139633.64
## attr(,"time")
## Time difference of 0.0579071 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.1825788 secs
```

```
gfit5 = garchFit(~garch(1,2),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 2)
##  ARMA Order:                0 0
```

```
##  Max ARMA Order:            0
##  GARCH Order:               1 2
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1   0.00000001   1.0000000 0.40000000     TRUE
##     beta2   0.00000001   1.0000000 0.40000000     TRUE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1  beta2
##      1      2      3      5      6
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3956.8872: 0.0422032 0.100000 0.100000 0.400000 0.400000
##   1:     3944.4022: 0.0422047 0.0899069 0.0990812 0.394347 0.394326
##   2:     3930.5546: 0.0422168 0.0708371 0.128410 0.401932 0.402054
##   3:     3926.2249: 0.0422183 0.0652001 0.126804 0.398988 0.399111
##   4:     3924.7939: 0.0422249 0.0613912 0.130817 0.402168 0.402417
##   5:     3924.5394: 0.0422479 0.0489040 0.126811 0.406113 0.406764
##   6:     3922.2763: 0.0422651 0.0496447 0.126677 0.411005 0.411962
##   7:     3921.5553: 0.0422987 0.0453213 0.122980 0.413825 0.415238
##   8:     3921.4807: 0.0423361 0.0427864 0.120541 0.418012 0.419821
##   9:     3920.9024: 0.0424131 0.0393035 0.115364 0.419856 0.422276
##  10:     3920.5802: 0.0425509 0.0384857 0.110819 0.422493 0.425845
##  11:     3920.4351: 0.0427756 0.0366745 0.107407 0.424469 0.427969
##  12:     3920.3682: 0.0430753 0.0365126 0.107359 0.425429 0.428529
##  13:     3919.8684: 0.0494113 0.0404036 0.104671 0.424470 0.425780
##  14:     3919.3054: 0.0557481 0.0364204 0.104887 0.427443 0.424848
##  15:     3918.5601: 0.0620872 0.0372335 0.105286 0.430190 0.425172
##  16:     3918.1922: 0.0684265 0.0356065 0.103573 0.431686 0.425226
```

```
##  17:     3918.1473: 0.0703854 0.0342206 0.0999622 0.437687 0.425034
##  18:     3918.1120: 0.0723591 0.0352058 0.102920 0.436156 0.422559
##  19:     3918.1111: 0.0723591 0.0350692 0.102881 0.436102 0.422505
##  20:     3918.1099: 0.0723642 0.0350979 0.102902 0.436103 0.422608
##  21:     3918.1090: 0.0723736 0.0349333 0.102913 0.436076 0.422771
##  22:     3918.1083: 0.0723958 0.0349342 0.102877 0.435875 0.423086
##  23:     3918.1051: 0.0726614 0.0347056 0.102367 0.433386 0.426256
##  24:     3918.1034: 0.0734446 0.0346357 0.102314 0.438042 0.421780
##  25:     3918.1031: 0.0735297 0.0346299 0.102405 0.431823 0.427901
##  26:     3918.1030: 0.0735061 0.0345066 0.102109 0.433927 0.426215
##  27:     3918.1030: 0.0735103 0.0345082 0.102106 0.433899 0.426243
##
## Final Estimate of the Negative LLH:
##  LLH:  -7599.97    norm LLH:  -2.518214
##           mu        omega       alpha1        beta1        beta2
## 1.617636e-03 1.671045e-05 1.021062e-01 4.338989e-01 4.262428e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu         omega        alpha1         beta1         beta2
## mu      -9610550.87     -48863677      -1575.93     -19652.92     -20443.96
## omega  -48863677.20 -881944093578 -175074538.79 -256547881.81 -258405751.82
## alpha1     -1575.93    -175074539     -62923.67     -70122.85     -70414.40
## beta1     -19652.92    -256547882     -70122.85     -89968.56     -90553.97
## beta2     -20443.96    -258405752     -70414.40     -90553.97     -91159.84
## attr(,"time")
## Time difference of 0.1355062 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.395247 secs
```

```
gfit6 = garchFit(~garch(2,2),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 2)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               2 2
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
```

```
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega    0.00000100 100.0000000 0.10000000     TRUE
##     alpha1   0.00000001   1.0000000 0.05000000     TRUE
##     alpha2   0.00000001   1.0000000 0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     gamma2 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1    0.00000001   1.0000000 0.40000000     TRUE
##     beta2    0.00000001   1.0000000 0.40000000     TRUE
##     delta    0.00000000   2.0000000 2.00000000    FALSE
##     skew     0.10000000  10.0000000 1.00000000    FALSE
##     shape    1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2  beta1  beta2
##      1      2      3      4      7      8
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3960.6913: 0.0422032 0.100000 0.0500000 0.0500000 0.400000 0.400000
##   1:     3947.3819: 0.0422046 0.0899484 0.0499530 0.0486347 0.394177 0.394149
##   2:     3938.4302: 0.0422076 0.0847254 0.0594796 0.0556292 0.395589 0.395594
##   3:     3933.9601: 0.0422099 0.0769976 0.0611815 0.0559336 0.392803 0.392822
##   4:     3930.8744: 0.0422128 0.0755256 0.0674731 0.0606278 0.395443 0.395541
##   5:     3927.2047: 0.0422325 0.0585260 0.0701322 0.0583491 0.397503 0.398169
##   6:     3923.6486: 0.0422739 0.0526635 0.0773971 0.0550156 0.407156 0.409133
##   7:     3921.8303: 0.0423413 0.0443887 0.0834983 0.0422042 0.410699 0.414394
##   8:     3921.0437: 0.0424415 0.0440009 0.0928681 0.0284155 0.413345 0.419074
##   9:     3920.6575: 0.0426328 0.0362319 0.0912554 0.0197297 0.420522 0.429182
##  10:     3920.4951: 0.0430409 0.0387686 0.0970754 0.00674444 0.419727 0.432273
##  11:     3920.4373: 0.0430419 0.0387471 0.0974641 0.00712085 0.420022 0.432573
##  12:     3920.3966: 0.0430432 0.0380874 0.0975455 0.00717106 0.419906 0.432463
##  13:     3920.3606: 0.0430945 0.0381004 0.0979393 0.00743950 0.420174 0.432793
##  14:     3920.3188: 0.0432080 0.0375942 0.0978348 0.00730562 0.420096 0.432883
##  15:     3920.2690: 0.0434368 0.0373666 0.0976624 0.00736065 0.420462 0.433618
##  16:     3919.8234: 0.0472911 0.0345927 0.0926035 0.00484694 0.422494 0.440607
##  17:     3919.0195: 0.0559337 0.0365915 0.105475 1.00000e-08 0.414899 0.441651
##  18:     3918.8532: 0.0559340 0.0359588 0.105147 1.00000e-08 0.414491 0.441241
##  19:     3918.8437: 0.0559430 0.0354705 0.104737 0.000421814 0.414805 0.441579
##  20:     3918.8390: 0.0559797 0.0356325 0.104517 0.000441534 0.414854 0.441667
##  21:     3918.4298: 0.0653488 0.0391064 0.109096 0.00552273 0.404739 0.439241
##  22:     3918.3835: 0.0704733 0.0433194 0.0897288 0.0341474 0.267672 0.560516
##  23:     3918.0850: 0.0773202 0.0421824 0.0909213 0.0314087 0.138140 0.692341
##  24:     3917.8552: 0.0741699 0.0476345 0.0976055 0.0425125 1.00000e-08 0.809316
##  25:     3917.7988: 0.0684704 0.0514103 0.0977391 0.0543287 1.00000e-08 0.795580
##  26:     3917.5363: 0.0731001 0.0477183 0.0907990 0.0495884 1.00000e-08 0.807717
##  27:     3917.5251: 0.0717268 0.0469709 0.0905311 0.0479185 1.00000e-08 0.810485
```

```
##  28:     3917.5248: 0.0716262 0.0469499 0.0903986 0.0480473 1.00000e-08 0.810570
##  29:     3917.5248: 0.0716195 0.0469293 0.0903693 0.0480391 1.00000e-08 0.810618
##  30:     3917.5248: 0.0716208 0.0469281 0.0903732 0.0480394 1.00000e-08 0.810617
##
## Final Estimate of the Negative LLH:
##  LLH:  -7600.549    norm LLH:  -2.518406
##           mu        omega       alpha1       alpha2        beta1        beta2
## 1.576058e-03 2.272472e-05 9.037316e-02 4.803941e-02 1.000000e-08 8.106174e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1        alpha2         beta1
## mu      -9659577.167     -24991465     -9847.364      15489.38 -5.113561e+03
## omega  -24991464.987 -482265199776 -91567817.591 -101567676.68 -1.400820e+08
## alpha1     -9847.364     -91567818    -35180.553     -30918.81 -3.630537e+04
## alpha2     15489.381    -101567677    -30918.813     -42046.27 -4.158898e+04
## beta1      -5113.561    -140082027    -36305.368     -41588.98 -4.937719e+04
## beta2     -14190.714    -140571816    -36891.420     -40643.96 -4.936468e+04
##                beta2
## mu         -14190.71
## omega  -140571815.56
## alpha1     -36891.42
## alpha2     -40643.96
## beta1      -49364.68
## beta2      -49430.15
## attr(,"time")
## Time difference of 0.07136202 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.2710299 secs
```

```
gfit7 = garchFit(~arma(3,3)+garch(1,0),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(3, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 0)
##  ARMA Order:                3 3
##  Max ARMA Order:            3
##  GARCH Order:               1 0
##  Max GARCH Order:           1
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
```

```
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V       params includes
##     mu     -0.42203167   0.4220317  0.043417533     TRUE
##     ar1    -0.99999999   1.0000000  0.383923685     TRUE
##     ar2    -0.99999999   1.0000000  0.001167706     TRUE
##     ar3    -0.99999999   1.0000000 -0.677235319     TRUE
##     ma1    -0.99999999   1.0000000 -0.493566554     TRUE
##     ma2    -0.99999999   1.0000000  0.081168715     TRUE
##     ma3    -0.99999999   1.0000000  0.639903273     TRUE
##     omega   0.00000100 100.0000000  0.100000000     TRUE
##     alpha1  0.00000001   1.0000000  0.100000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.100000000    FALSE
##     delta   0.00000000   2.0000000  2.000000000    FALSE
##     skew    0.10000000  10.0000000  1.000000000    FALSE
##     shape   1.00000000  10.0000000  4.000000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ar2    ar3    ma1    ma2    ma3  omega alpha1
##      1      2      3      4      5      6      7      8      9
##  Persistence:                  0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8808.9972: 0.0434175 0.383924 0.00116771 -0.677235 -0.493567 0.0811687 0.639903 0.100000 0.1
##   1:     4196.0868: 0.0434297 0.393256 0.000842162 -0.663050 -0.483274 0.0871635 0.655953  1.06675 0
##   2:     4183.3448: 0.0434334 0.392619 -0.0117915 -0.673248 -0.487657 0.0691313 0.642101  1.05244 0.3
##   3:     4171.7508: 0.0434397 0.400685 -0.00621403 -0.666164 -0.477889 0.0773075 0.649285  1.02852 0
##   4:     4155.4200: 0.0434629 0.396612 -0.0294663 -0.664841 -0.488917 0.0537036 0.652112 0.981212 0.3
##   5:     4137.1689: 0.0435751 0.442718 -0.0315323 -0.669913 -0.430951 0.0494991 0.625159 0.901864 0.1
##   6:     4120.0968: 0.0436433 0.387100 -0.0250984 -0.599680 -0.467351 0.0492673 0.664810 0.832687 0.1
##   7:     4103.8961: 0.0437268 0.367576 0.00438334 -0.614572 -0.452120 0.0257745 0.557880 0.780479 0.1
##   8:     4093.8167: 0.0437771 0.385205 0.0262726 -0.576042 -0.426785 0.0400121 0.574863 0.755898 0.23
##   9:     4087.3106: 0.0438566 0.378196 0.00178117 -0.556927 -0.422325 0.00639393 0.565036 0.736365 0
##  10:     4085.9300: 0.0440462 0.358554 0.0249175 -0.561654 -0.438605 0.0315330 0.550311 0.696391 0.29
##  11:     4084.6436: 0.0440809 0.372121 0.0140640 -0.555094 -0.427728 0.0232879 0.557263 0.695406 0.29
##  12:     4084.5847: 0.0440885 0.371627 0.0124305 -0.555411 -0.428733 0.0218229 0.557090 0.695269 0.29
##  13:     4084.5668: 0.0441100 0.372571 0.0117299 -0.554358 -0.428659 0.0221876 0.558764 0.694956 0.29
##  14:     4084.5474: 0.0441478 0.373238 0.0101161 -0.555474 -0.429555 0.0219465 0.558525 0.694890 0.28
##  15:     4084.5234: 0.0441933 0.374508 0.00959291 -0.555699 -0.429985 0.0233567 0.559475 0.694863 0.1
##  16:     4081.8256: 0.0496185 0.488266 -0.101587 -0.630628 -0.536445 0.133665 0.626167 0.705748 0.304
##  17:     4081.5184: 0.0496227 0.485986 -0.101821 -0.628993 -0.539162 0.133239 0.627483 0.704433 0.303
##  18:     4081.4612: 0.0496320 0.486711 -0.0996517 -0.628143 -0.538562 0.134901 0.626820 0.701769 0.30
##  19:     4081.2371: 0.0496752 0.486610 -0.100497 -0.627996 -0.540537 0.133587 0.624900 0.699965 0.299
##  20:     4081.1368: 0.0497571 0.488412 -0.100106 -0.626745 -0.542019 0.135205 0.624438 0.698523 0.297
##  21:     4081.0286: 0.0499518 0.492549 -0.101761 -0.625932 -0.546950 0.136671 0.622427 0.697141 0.295
##  22:     4080.6433: 0.0517554 0.535800 -0.111618 -0.607147 -0.587110 0.155478 0.603647 0.694560 0.295
##  23:     4080.1555: 0.0536662 0.558927 -0.146052 -0.590876 -0.613730 0.190364 0.581842 0.691507 0.290
##  24:     4079.7025: 0.0554878 0.576596 -0.178088 -0.563844 -0.632574 0.227194 0.553279 0.688234 0.288
##  25:     4079.2323: 0.0573172 0.601545 -0.208844 -0.542223 -0.662253 0.259873 0.529965 0.690122 0.290
```

```
##  26:     4078.9110: 0.0590513 0.627910 -0.240441 -0.522287 -0.688897 0.295120 0.505443 0.697477 0.287
##  27:     4078.6174: 0.0605952 0.650530 -0.274508 -0.497532 -0.713438 0.329776 0.476577 0.688236 0.298
##  28:     4078.5618: 0.0612165 0.663362 -0.285343 -0.492586 -0.726026 0.344007 0.473456 0.686294 0.300
##  29:     4078.4642: 0.0617699 0.675500 -0.298516 -0.487767 -0.738856 0.356253 0.465305 0.685149 0.303
##  30:     4078.3719: 0.0615581 0.671150 -0.297640 -0.479777 -0.734987 0.355883 0.458722 0.687053 0.304
##  31:     4078.3622: 0.0615594 0.671036 -0.297788 -0.479393 -0.735304 0.355776 0.459072 0.686988 0.304
##  32:     4078.3547: 0.0615612 0.671475 -0.297865 -0.479306 -0.735034 0.355724 0.458936 0.686902 0.303
##  33:     4078.3466: 0.0615824 0.671878 -0.298647 -0.478807 -0.735829 0.355990 0.458746 0.686798 0.303
##  34:     4078.3380: 0.0616055 0.672675 -0.299082 -0.478382 -0.736116 0.356764 0.458435 0.686719 0.303
##  35:     4078.3307: 0.0616286 0.673227 -0.299857 -0.478071 -0.736773 0.357125 0.457973 0.686642 0.303
##  36:     4078.2257: 0.0625948 0.695985 -0.323909 -0.464394 -0.760010 0.383580 0.442863 0.684151 0.307
##  37:     4078.0612: 0.0631478 0.717073 -0.349392 -0.445492 -0.780595 0.410806 0.420254 0.684242 0.309
##  38:     4077.6201: 0.0627505 0.759660 -0.407339 -0.400670 -0.828116 0.474477 0.372063 0.690428 0.290
##  39:     4077.4290: 0.0624026 0.809997 -0.464562 -0.359082 -0.881039 0.533975 0.326136 0.689158 0.304
##  40:     4077.1588: 0.0616493 0.860965 -0.520681 -0.317144 -0.931009 0.595528 0.282070 0.685090 0.311
##  41:     4076.8915: 0.0577922 0.896332 -0.563813 -0.291188 -0.964554 0.638627 0.252451 0.679868 0.301
##  42:     4076.8774: 0.0573648 0.909565 -0.581765 -0.277859 -0.980035 0.656066 0.239979 0.680004 0.301
##  43:     4076.7286: 0.0572065 0.917213 -0.590342 -0.271416 -0.986161 0.665566 0.232643 0.680271 0.301
##  44:     4076.7049: 0.0570886 0.924229 -0.599946 -0.264756 -0.992948 0.674084 0.225113 0.680826 0.301
##  45:     4076.6192: 0.0569286 0.931167 -0.608694 -0.257853 -0.999770 0.683729 0.218555 0.680879 0.303
##  46:     4076.4741: 0.0600946 0.932653 -0.624900 -0.244520 -1.00000 0.694719 0.208785 0.684692 0.299
##  47:     4076.4244: 0.0597092 0.936043 -0.623871 -0.243779 -1.00000 0.690475 0.211009 0.689400 0.294
##  48:     4076.3937: 0.0598331 0.941506 -0.632272 -0.238031 -1.00000 0.691944 0.210356 0.685635 0.297
##  49:     4076.3916: 0.0592902 0.941458 -0.632667 -0.237635 -1.00000 0.692298 0.210043 0.685954 0.297
##  50:     4076.3908: 0.0592445 0.940729 -0.631778 -0.238046 -1.00000 0.692317 0.209859 0.686618 0.297
##  51:     4076.3907: 0.0591237 0.940729 -0.631940 -0.238156 -1.00000 0.692433 0.209913 0.686631 0.296
##  52:     4076.3907: 0.0591620 0.940810 -0.631977 -0.238016 -1.00000 0.692375 0.209872 0.686549 0.297
##  53:     4076.3907: 0.0591709 0.940792 -0.631950 -0.238043 -1.00000 0.692378 0.209881 0.686567 0.297
##  54:     4076.3906: 0.0591676 0.940793 -0.631955 -0.238041 -1.00000 0.692379 0.209880 0.686566 0.297
##
## Final Estimate of the Negative LLH:
##  LLH:  -7441.683    norm LLH:  -2.465766
##            mu           ar1           ar2           ar3           ma1
##  0.0013020170  0.9407933718 -0.6319546400 -0.2380414440 -0.9999999900
##           ma2           ma3         omega        alpha1
##  0.6923787685  0.2098804821  0.0003324665  0.2971907102
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu          ar1          ar2          ar3          ma1
## mu     -9787121.481   -2767.4950   -2190.2742   -4893.5937   19874.7783
## ar1       -2767.495  -29695.7644  -19882.5476    5113.3220  -37341.7961
## ar2       -2190.274  -19882.5476  -32338.7972  -19525.3604  -28809.4305
## ar3       -4893.594    5113.3220  -19525.3604  -30823.7374    2137.1783
## ma1       19874.778  -37341.7961  -28809.4305    2137.1783  -50164.2300
## ma2       16106.787  -20528.7945  -39213.0231  -27401.5868  -32379.5246
## ma3        4253.739   11426.1669  -19404.3104  -37434.1356    9792.0167
## omega  -5776979.629 -200734.9889 -313836.2886 -308656.6727 -264985.7324
## alpha1     5267.044     156.5169     305.0518     272.1389     234.5865
##                ma2          ma3         omega        alpha1
## mu       16106.7867    4253.7391   -5776979.6     5267.0444
## ar1     -20528.7945   11426.1669    -200735.0      156.5169
## ar2     -39213.0231  -19404.3104    -313836.3      305.0518
## ar3     -27401.5868  -37434.1356    -308656.7      272.1389
## ma1     -32379.5246    9792.0167    -264985.7      234.5865
```

```
## ma2     -51835.3263  -31830.4175     -384409.8      366.4577
## ma3     -31830.4175  -51278.6463     -287162.2      251.3284
## omega  -384409.8125 -287162.2329 -9949660210.8 -1516564.9464
## alpha1     366.4577     251.3284    -1516564.9    -1240.3266
## attr(,"time")
## Time difference of 0.157423 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.60919 secs
```

```
gfit8 = garchFit(~arma(3,3)+garch(1,1),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(3, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                3 3
##  Max ARMA Order:            3
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V       params includes
##     mu     -0.42203167   0.4220317  0.043417533     TRUE
##     ar1    -0.99999999   1.0000000  0.383923685     TRUE
##     ar2    -0.99999999   1.0000000  0.001167706     TRUE
##     ar3    -0.99999999   1.0000000 -0.677235319     TRUE
##     ma1    -0.99999999   1.0000000 -0.493566554     TRUE
##     ma2    -0.99999999   1.0000000  0.081168715     TRUE
##     ma3    -0.99999999   1.0000000  0.639903273     TRUE
##     omega   0.00000100 100.0000000  0.100000000     TRUE
##     alpha1  0.00000001   1.0000000  0.100000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.100000000    FALSE
##     beta1   0.00000001   1.0000000  0.800000000     TRUE
##     delta   0.00000000   2.0000000  2.000000000    FALSE
##     skew    0.10000000  10.0000000  1.000000000    FALSE
##     shape   1.00000000  10.0000000  4.000000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ar2    ar3    ma1    ma2    ma3  omega alpha1  beta1
```

```
##      1      2      3      4      5      6      7      8      9     11
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3955.7931: 0.0434175 0.383924 0.00116771 -0.677235 -0.493567 0.0811687 0.639903 0.100000 0.1
##   1:     3944.5501: 0.0434198 0.386423 0.00178859 -0.676974 -0.490916 0.0819537 0.639928 0.0883284 0
##   2:     3930.9237: 0.0434339 0.398730 0.00336258 -0.675650 -0.478572 0.0841863 0.640314 0.0722748 0
##   3:     3927.3605: 0.0434361 0.398663 0.00219042 -0.675533 -0.479068 0.0827716 0.640322 0.0656532 0
##   4:     3925.8081: 0.0434423 0.399117 -0.000212281 -0.674859 -0.479611 0.0799392 0.640827 0.0642255
##   5:     3924.3408: 0.0434593 0.401062 -0.00397345 -0.671546 -0.479794 0.0758329 0.644304 0.0523983 0
##   6:     3920.7531: 0.0434819 0.405975 -0.00562834 -0.667240 -0.476793 0.0743410 0.649034 0.0527085 0
##   7:     3918.8177: 0.0435088 0.412372 -0.00863903 -0.668252 -0.472693 0.0702660 0.647686 0.0477015 0
##   8:     3917.1115: 0.0435441 0.412955 -0.00937722 -0.663741 -0.474510 0.0688607 0.652544 0.0432161 0
##   9:     3916.8675: 0.0435457 0.413008 -0.00972864 -0.663909 -0.474638 0.0683884 0.652343 0.0417349 0
##  10:     3916.6648: 0.0435513 0.413188 -0.00999241 -0.664106 -0.474829 0.0679201 0.652120 0.0421720 0
##  11:     3916.2948: 0.0435663 0.413673 -0.00988052 -0.663932 -0.475153 0.0677439 0.652370 0.0403492 0
##  12:     3915.7409: 0.0435987 0.414806 -0.00917972 -0.663015 -0.475650 0.0680773 0.653568 0.0391245 0
##  13:     3915.0235: 0.0436920 0.415805 -0.00926668 -0.668871 -0.478977 0.0655580 0.647061 0.0358599 0
##  14:     3914.8581: 0.0436940 0.415983 -0.00897055 -0.668229 -0.478850 0.0659435 0.647763 0.0350891 0
##  15:     3914.7165: 0.0437000 0.416223 -0.00884735 -0.667732 -0.478863 0.0660430 0.648248 0.0357981 0
##  16:     3914.4217: 0.0437294 0.417003 -0.00902536 -0.667031 -0.479401 0.0655242 0.648773 0.0346193 0
##  17:     3914.0061: 0.0437944 0.418400 -0.00900309 -0.666397 -0.480759 0.0650238 0.649316 0.0337679 0
##  18:     3913.5202: 0.0439513 0.421557 -0.00761702 -0.668157 -0.483210 0.0662901 0.648078 0.0302006 0
##  19:     3913.2209: 0.0441890 0.427659 -0.0120136 -0.665046 -0.487001 0.0622533 0.651998 0.0315203 0
##  20:     3913.1257: 0.0442909 0.428807 -0.0108571 -0.664668 -0.489134 0.0639820 0.652882 0.0286822 0
##  21:     3913.0293: 0.0444165 0.430623 -0.00970058 -0.665570 -0.490747 0.0660604 0.652237 0.0292639 0
##  22:     3912.8694: 0.0445372 0.433572 -0.00941257 -0.666161 -0.490246 0.0678257 0.651327 0.0296262 0
##  23:     3912.6868: 0.0446896 0.434177 -0.0100868 -0.666502 -0.493522 0.0688206 0.651344 0.0288811 0
##  24:     3912.6796: 0.0447901 0.434624 -0.0121165 -0.664905 -0.495684 0.0683175 0.653298 0.0288137 0
##  25:     3912.6671: 0.0448633 0.435361 -0.0128522 -0.664794 -0.496284 0.0687957 0.653382 0.0274121 0
##  26:     3912.5084: 0.0449004 0.435820 -0.0131181 -0.664789 -0.496397 0.0691121 0.653244 0.0279156 0
##  27:     3912.4666: 0.0449836 0.437040 -0.0135891 -0.664835 -0.496443 0.0699201 0.652806 0.0275514 0
##  28:     3912.4014: 0.0451647 0.438666 -0.0148558 -0.664752 -0.497936 0.0714031 0.652303 0.0275079 0
##  29:     3912.1114: 0.0464849 0.450237 -0.0231699 -0.663157 -0.511442 0.0811307 0.649106 0.0256341 0
##  30:     3911.8553: 0.0478474 0.460907 -0.0312867 -0.657050 -0.521628 0.0895717 0.641403 0.0271122 0
##  31:     3911.6429: 0.0492084 0.470841 -0.0417816 -0.648223 -0.527865 0.0978469 0.633759 0.0265216 0
##  32:     3911.3271: 0.0505740 0.479106 -0.0508972 -0.640313 -0.535097 0.108424 0.625481 0.0273091 0.0
##  33:     3911.1017: 0.0519090 0.482575 -0.0581624 -0.635435 -0.544598 0.123406 0.615880 0.0271348 0.0
##  34:     3910.6408: 0.0550621 0.495190 -0.0910410 -0.617808 -0.557003 0.149603 0.599877 0.0282133 0.0
##  35:     3910.6325: 0.0550630 0.495276 -0.0909667 -0.617774 -0.556947 0.149680 0.599910 0.0282439 0.0
##  36:     3910.6257: 0.0550680 0.495374 -0.0909345 -0.617721 -0.556956 0.149738 0.599897 0.0280314 0.0
##  37:     3910.6175: 0.0550858 0.495591 -0.0909469 -0.617570 -0.557088 0.149828 0.599782 0.0281043 0.0
##  38:     3910.6071: 0.0551232 0.496012 -0.0910050 -0.617264 -0.557398 0.149982 0.599521 0.0279521 0.0
##  39:     3910.3014: 0.0577385 0.522559 -0.0977098 -0.596802 -0.581192 0.158638 0.579828 0.0288954 0.0
##  40:     3910.0121: 0.0604201 0.542556 -0.121358 -0.591761 -0.598601 0.183292 0.569064 0.0281579 0.08
##  41:     3909.6648: 0.0630955 0.559906 -0.141309 -0.580135 -0.619491 0.207214 0.555455 0.0293372 0.08
##  42:     3909.2747: 0.0657332 0.576635 -0.160400 -0.565680 -0.641162 0.231332 0.540076 0.0286972 0.07
##  43:     3909.2319: 0.0677394 0.565784 -0.180021 -0.526805 -0.634844 0.249658 0.503320 0.0265274 0.07
##  44:     3909.2136: 0.0677407 0.565888 -0.180239 -0.527084 -0.634918 0.249426 0.503179 0.0264130 0.07
```

```
##  45:     3909.2082: 0.0677586 0.566099 -0.180460 -0.527128 -0.635067 0.249566 0.503168 0.0261349 0.0
##  46:     3909.1952: 0.0677788 0.566323 -0.180685 -0.527143 -0.635231 0.249753 0.503169 0.0262747 0.0
##  47:     3909.1846: 0.0678176 0.566871 -0.181233 -0.527368 -0.635578 0.249977 0.503138 0.0260558 0.0
##  48:     3909.1644: 0.0679021 0.567761 -0.182117 -0.527340 -0.636270 0.250820 0.503166 0.0262437 0.0
##  49:     3909.0014: 0.0692585 0.581810 -0.196033 -0.526541 -0.647605 0.264539 0.503368 0.0270684 0.0
##  50:     3908.5101: 0.0729613 0.621484 -0.229327 -0.521164 -0.689825 0.304324 0.490324 0.0283488 0.0
##  51:     3908.2278: 0.0763904 0.657636 -0.274871 -0.500635 -0.723336 0.346607 0.465905 0.0253379 0.0
##  52:     3908.1556: 0.0786374 0.699001 -0.322242 -0.464197 -0.767394 0.396993 0.429370 0.0246178 0.0
##  53:     3908.1044: 0.0819676 0.727775 -0.360541 -0.431489 -0.799055 0.440609 0.391306 0.0256218 0.0
##  54:     3908.1027: 0.0819675 0.727773 -0.360532 -0.431463 -0.799069 0.440636 0.391373 0.0252743 0.0
##  55:     3908.0874: 0.0819675 0.727774 -0.360529 -0.431453 -0.799073 0.440649 0.391403 0.0254388 0.0
##  56:     3908.0832: 0.0819669 0.727784 -0.360507 -0.431344 -0.799144 0.440788 0.391780 0.0253684 0.0
##  57:     3908.0820: 0.0819497 0.727884 -0.360603 -0.431390 -0.799208 0.440816 0.391804 0.0254006 0.0
##  58:     3908.0529: 0.0806848 0.734093 -0.364736 -0.432044 -0.803097 0.444172 0.392698 0.0253920 0.0
##  59:     3908.0492: 0.0795186 0.740158 -0.370360 -0.432766 -0.810992 0.452373 0.390362 0.0260638 0.0
##  60:     3908.0308: 0.0794985 0.720525 -0.347273 -0.447636 -0.789613 0.425797 0.408461 0.0259013 0.0
##  61:     3908.0266: 0.0794005 0.722375 -0.346736 -0.446037 -0.791324 0.425771 0.407012 0.0254535 0.0
##  62:     3908.0259: 0.0802866 0.712285 -0.338243 -0.453349 -0.780929 0.415947 0.415132 0.0255923 0.0
##  63:     3908.0248: 0.0797597 0.717544 -0.342773 -0.449638 -0.786278 0.421036 0.411031 0.0256657 0.0
##  64:     3908.0247: 0.0797617 0.717979 -0.343338 -0.449265 -0.786775 0.421674 0.410602 0.0256045 0.0
##  65:     3908.0247: 0.0797815 0.717550 -0.342869 -0.449587 -0.786325 0.421179 0.410946 0.0256095 0.0
##  66:     3908.0247: 0.0797870 0.717670 -0.343016 -0.449497 -0.786435 0.421307 0.410869 0.0256192 0.0
##  67:     3908.0247: 0.0797789 0.717697 -0.343041 -0.449477 -0.786473 0.421348 0.410836 0.0256139 0.0
##  68:     3908.0247: 0.0797807 0.717682 -0.343023 -0.449488 -0.786455 0.421327 0.410850 0.0256144 0.0
##
## Final Estimate of the Negative LLH:
##  LLH:  -7610.049    norm LLH:  -2.521554
##             mu           ar1           ar2           ar3           ma1
##  1.755621e-03  7.176819e-01 -3.430226e-01 -4.494884e-01 -7.864554e-01
##            ma2           ma3         omega        alpha1         beta1
##  4.213269e-01  4.108502e-01  1.240366e-05  7.547838e-02  8.967329e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu           ar1           ar2           ar3           ma1
## mu      -8778436.268    -9590.2266 -1.110832e+04   -13842.0544     4010.0926
## ar1        -9590.227   -18596.5353 -1.130205e+04     3528.6057   -23464.6582
## ar2       -11108.316   -11302.0541 -1.880055e+04   -11171.7649   -15180.3195
## ar3       -13842.054     3528.6057 -1.117176e+04   -18764.2665     3934.1766
## ma1         4010.093   -23464.6582 -1.518032e+04     3934.1766   -30775.2685
## ma2         3270.248   -13118.3997 -2.248200e+04   -13853.7454   -18258.6267
## ma3         1308.114     5953.5466 -1.196731e+04   -22275.8456     7167.3692
## omega  -56525529.659 -5924837.2197 -5.347651e+06 -1992376.3629 -7489879.0121
## alpha1     -2566.922     -170.0272  8.779146e+01      172.6496     -123.7039
## beta1     -23325.500    -1863.0505 -1.297678e+03     -476.7194    -2078.4805
##                   ma2           ma3         omega        alpha1         beta1
## mu         3270.2484     1308.1141 -5.652553e+07 -2.566922e+03 -2.332550e+04
## ar1      -13118.3997     5953.5466 -5.924837e+06 -1.700272e+02 -1.863051e+03
## ar2      -22482.0046   -11967.3140 -5.347651e+06  8.779146e+01 -1.297678e+03
## ar3      -13853.7454   -22275.8456 -1.992376e+06  1.726496e+02 -4.767194e+02
## ma1      -18258.6267     7167.3692 -7.489879e+06 -1.237039e+02 -2.078481e+03
## ma2      -28257.8397   -16128.8211 -6.798140e+06  1.985569e+02 -1.418447e+03
## ma3      -16128.8211   -28600.2155 -2.167349e+06  3.038422e+02 -3.861917e+02
## omega  -6798139.6577 -2167349.4285 -1.627748e+12 -3.209956e+08 -4.706908e+08
## alpha1      198.5569      303.8422 -3.209956e+08 -1.139353e+05 -1.277115e+05
```

```
## beta1     -1418.4470     -386.1917 -4.706908e+08 -1.277115e+05 -1.642918e+05
## attr(,"time")
## Time difference of 0.211962 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.545471 secs
```

```
gfit9 = garchFit(~arma(3,3)+garch(2,0),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(3, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 0)
##  ARMA Order:                3 3
##  Max ARMA Order:            3
##  GARCH Order:               2 0
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V       params includes
##     mu     -0.42203167   0.4220317  0.043417533     TRUE
##     ar1    -0.99999999   1.0000000  0.383923685     TRUE
##     ar2    -0.99999999   1.0000000  0.001167706     TRUE
##     ar3    -0.99999999   1.0000000 -0.677235319     TRUE
##     ma1    -0.99999999   1.0000000 -0.493566554     TRUE
##     ma2    -0.99999999   1.0000000  0.081168715     TRUE
##     ma3    -0.99999999   1.0000000  0.639903273     TRUE
##     omega   0.00000100 100.0000000  0.100000000     TRUE
##     alpha1  0.00000001   1.0000000  0.050000000     TRUE
##     alpha2  0.00000001   1.0000000  0.050000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.100000000    FALSE
##     gamma2 -0.99999999   1.0000000  0.100000000    FALSE
##     delta   0.00000000   2.0000000  2.000000000    FALSE
##     skew    0.10000000  10.0000000  1.000000000    FALSE
##     shape   1.00000000  10.0000000  4.000000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ar2    ar3    ma1    ma2    ma3  omega alpha1 alpha2
##      1      2      3      4      5      6      7      8      9     10
##  Persistence:                 0.1
```

```
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8016.8556: 0.0434175 0.383924 0.00116771 -0.677235 -0.493567 0.0811687 0.639903 0.100000 0.0
##   1:     4237.9503: 0.0434297 0.401973 0.00532910 -0.661969 -0.476024 0.0884707 0.654014 0.954555 0.4
##   2:     4218.2748: 0.0434339 0.397390 -0.00953607 -0.669823 -0.484828 0.0673586 0.641781 0.937290 0
##   3:     4179.3403: 0.0434624 0.419935 -0.00110398 -0.642239 -0.462758 0.0795066 0.667810 0.851212 0
##   4:     4076.6254: 0.0435931 0.457333 -0.0280116 -0.599419 -0.421942 -0.00614562 0.622154 0.701397 0
##   5:     4057.8536: 0.0436037 0.437614 -0.0228298 -0.588030 -0.441757 -0.00152699 0.632599 0.680236 0
##   6:     4039.9695: 0.0436497 0.407493 0.000834016 -0.585331 -0.471472 0.0269715 0.640823 0.591967 0
##   7:     4033.8428: 0.0436593 0.418701 -0.00613808 -0.607392 -0.458791 0.0176168 0.615270 0.587439 0
##   8:     4032.8536: 0.0438326 0.413359 0.00827458 -0.588467 -0.449468 0.0242812 0.613531 0.564846 0.1
##   9:     4032.3714: 0.0440650 0.408967 -0.0151765 -0.585547 -0.450616 0.0105678 0.624881 0.545913 0.1
##  10:     4031.2967: 0.0444555 0.403979 -0.00953032 -0.606804 -0.453986 0.0317387 0.626728 0.547438 0
##  11:     4031.1453: 0.0444672 0.406845 -0.0100138 -0.605806 -0.451796 0.0320169 0.629047 0.548417 0.1
##  12:     4031.0694: 0.0444797 0.407129 -0.0130906 -0.606786 -0.452913 0.0290891 0.629451 0.549086 0.1
##  13:     4030.9352: 0.0445227 0.409356 -0.0128087 -0.607556 -0.452320 0.0309422 0.632531 0.549705 0.1
##  14:     4030.7687: 0.0446321 0.411711 -0.0155800 -0.613160 -0.455187 0.0311951 0.636747 0.550595 0.1
##  15:     4030.0101: 0.0453516 0.429685 -0.0218918 -0.645827 -0.472145 0.0436508 0.666642 0.554318 0.1
##  16:     4029.7356: 0.0461566 0.460294 -0.0200691 -0.665066 -0.492301 0.0369584 0.681216 0.555839 0.1
##  17:     4029.6910: 0.0464150 0.468770 -0.0159224 -0.666118 -0.499610 0.0391122 0.688096 0.553169 0.1
##  18:     4029.4167: 0.0465631 0.469588 -0.0180142 -0.668154 -0.501278 0.0389871 0.685560 0.549945 0.1
##  19:     4029.3702: 0.0467958 0.470223 -0.0213646 -0.667798 -0.501260 0.0427670 0.685421 0.548297 0.1
##  20:     4029.3348: 0.0470496 0.472544 -0.0237704 -0.667762 -0.504515 0.0442210 0.685435 0.548718 0.1
##  21:     4029.2893: 0.0472964 0.476237 -0.0240468 -0.667739 -0.507230 0.0458398 0.684757 0.549471 0.1
##  22:     4029.2620: 0.0475508 0.479131 -0.0260091 -0.668334 -0.510311 0.0473815 0.684024 0.550036 0.1
##  23:     4029.2187: 0.0478030 0.482196 -0.0277618 -0.668083 -0.512714 0.0498726 0.684162 0.550030 0.1
##  24:     4029.0517: 0.0515724 0.517056 -0.0557524 -0.657099 -0.548264 0.0798481 0.669813 0.547206 0.1
##  25:     4029.0142: 0.0553038 0.541205 -0.0948250 -0.641141 -0.566626 0.117770 0.647591 0.551065 0.16
##  26:     4028.2916: 0.0570782 0.556410 -0.103830 -0.631345 -0.587085 0.133008 0.636594 0.547803 0.179
##  27:     4028.1849: 0.0588257 0.562341 -0.114728 -0.613971 -0.597235 0.148211 0.615304 0.551135 0.180
##  28:     4028.0235: 0.0606431 0.571241 -0.133138 -0.600203 -0.609436 0.167275 0.603209 0.554281 0.183
##  29:     4027.8975: 0.0623920 0.590510 -0.148168 -0.595823 -0.626166 0.184797 0.595253 0.550013 0.185
##  30:     4027.8843: 0.0623930 0.590180 -0.148555 -0.596007 -0.626676 0.184235 0.594963 0.549914 0.185
##  31:     4027.8762: 0.0624210 0.590532 -0.148628 -0.595705 -0.626801 0.184766 0.594902 0.549839 0.185
##  32:     4027.8701: 0.0624781 0.591019 -0.149160 -0.595448 -0.627496 0.185162 0.594235 0.549519 0.185
##  33:     4027.8597: 0.0626014 0.592136 -0.150273 -0.594588 -0.628490 0.186727 0.593504 0.549497 0.185
##  34:     4027.7740: 0.0658233 0.620323 -0.184323 -0.576399 -0.656875 0.222665 0.570542 0.551241 0.183
##  35:     4027.7689: 0.0658241 0.619889 -0.183985 -0.575614 -0.657724 0.222683 0.571268 0.551093 0.183
##  36:     4027.7418: 0.0658575 0.620432 -0.184349 -0.575607 -0.657819 0.222868 0.570281 0.550985 0.183
##  37:     4027.7354: 0.0658922 0.620900 -0.184260 -0.574948 -0.658248 0.223440 0.569961 0.550801 0.183
##  38:     4027.7261: 0.0659312 0.621224 -0.184770 -0.574702 -0.658791 0.223688 0.569289 0.550702 0.183
##  39:     4027.7178: 0.0660114 0.622101 -0.185341 -0.573782 -0.659712 0.224814 0.568470 0.550503 0.184
##  40:     4027.7064: 0.0661763 0.623677 -0.187297 -0.572463 -0.661626 0.226782 0.566617 0.550332 0.184
##  41:     4027.6794: 0.0668890 0.630831 -0.194852 -0.566588 -0.669284 0.235725 0.560069 0.550760 0.185
##  42:     4027.6715: 0.0677167 0.630909 -0.200806 -0.557392 -0.670807 0.241969 0.551816 0.550287 0.185
##  43:     4027.6712: 0.0677174 0.631115 -0.200660 -0.557562 -0.670823 0.241896 0.551569 0.550250 0.185
##  44:     4027.6689: 0.0677276 0.631186 -0.200668 -0.557407 -0.671075 0.242041 0.551654 0.550267 0.185
##  45:     4027.6681: 0.0677498 0.631518 -0.200766 -0.557449 -0.671528 0.242128 0.551335 0.550266 0.185
##  46:     4027.6660: 0.0677742 0.631823 -0.200905 -0.557248 -0.671802 0.242520 0.551260 0.550276 0.185
```

```
##  47:     4027.6642: 0.0678212 0.632421 -0.201291 -0.557134 -0.672598 0.243022 0.550784 0.550197 0.185
##  48:     4027.6617: 0.0679149 0.633494 -0.202360 -0.556503 -0.673727 0.244454 0.550032 0.550079 0.185
##  49:     4027.6559: 0.0682243 0.638123 -0.207156 -0.553783 -0.679165 0.250380 0.546006 0.551581 0.185
##  50:     4027.6512: 0.0685269 0.641951 -0.212334 -0.551223 -0.683853 0.256878 0.541974 0.550273 0.187
##  51:     4027.6504: 0.0685271 0.642065 -0.212204 -0.551121 -0.683860 0.256900 0.542046 0.550256 0.187
##  52:     4027.6494: 0.0685325 0.642128 -0.212270 -0.551131 -0.683956 0.256814 0.541950 0.550240 0.187
##  53:     4027.6485: 0.0685465 0.642356 -0.212256 -0.550994 -0.684076 0.256902 0.541949 0.550208 0.187
##  54:     4027.6475: 0.0685783 0.642641 -0.212507 -0.550907 -0.684355 0.257005 0.541762 0.550163 0.187
##  55:     4027.6461: 0.0686440 0.643239 -0.212852 -0.550629 -0.684803 0.257405 0.541581 0.550107 0.187
##  56:     4027.6426: 0.0687613 0.643859 -0.213126 -0.550519 -0.685235 0.257503 0.541390 0.550728 0.187
##  57:     4027.6419: 0.0688942 0.644819 -0.213920 -0.550050 -0.686211 0.258293 0.541047 0.550531 0.187
##  58:     4027.6411: 0.0690269 0.645849 -0.214765 -0.549735 -0.687032 0.259064 0.540492 0.550326 0.188
##  59:     4027.6405: 0.0691646 0.646702 -0.215354 -0.549448 -0.687930 0.259861 0.540090 0.550248 0.188
##  60:     4027.6400: 0.0691362 0.646007 -0.214438 -0.549949 -0.687461 0.259295 0.540384 0.550643 0.188
##  61:     4027.6398: 0.0691898 0.646507 -0.214935 -0.549704 -0.688067 0.259813 0.540001 0.550766 0.188
##  62:     4027.6395: 0.0692284 0.648073 -0.216585 -0.548595 -0.689598 0.261577 0.538759 0.551000 0.188
##  63:     4027.6395: 0.0693577 0.647592 -0.215875 -0.549174 -0.689073 0.260836 0.539363 0.551002 0.188
##  64:     4027.6395: 0.0693348 0.648017 -0.216355 -0.548843 -0.689540 0.261397 0.538959 0.551001 0.188
##  65:     4027.6395: 0.0693405 0.648077 -0.216437 -0.548768 -0.689596 0.261474 0.538895 0.551018 0.188
##  66:     4027.6395: 0.0693395 0.648071 -0.216425 -0.548782 -0.689588 0.261462 0.538906 0.551013 0.188
##
## Final Estimate of the Negative LLH:
##  LLH:  -7490.434    norm LLH:  -2.48192
##             mu           ar1           ar2           ar3           ma1
##  0.0015258566  0.6480712395 -0.2164253173 -0.5487817101 -0.6895884902
##            ma2           ma3         omega        alpha1        alpha2
##  0.2614618977  0.5389064563  0.0002668258  0.1865130188  0.2840915277
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu          ar1          ar2           ar3          ma1
## mu     -7179631.351   -5783.7773   -8845.0159   -6039.70985   -2641.5075
## ar1       -5783.777  -21668.6277  -13250.8112    3737.35658  -26737.8273
## ar2       -8845.016  -13250.8112  -21660.6342  -13536.98800  -15000.6709
## ar3       -6039.710    3737.3566  -13536.9880  -22823.74263    7253.4542
## ma1       -2641.508  -26737.8273  -15000.6709    7253.45417  -36024.0889
## ma2       -4111.911  -14626.0624  -25486.4395  -17000.40808  -19236.8110
## ma3        4098.757    6848.5582  -16836.9405  -30360.61821   10719.2432
## omega    444976.392 -430030.9921 -423276.3867 -243168.86595 -299383.4802
## alpha1     2411.211     103.0299     242.5301      60.77065     127.7571
## alpha2    -2781.594     169.7704     142.6160     111.06639     116.3374
##                 ma2          ma3         omega        alpha1        alpha2
## mu       -4111.9112    4098.75730  4.449764e+05  2.411211e+03    -2781.5939
## ar1     -14626.0624    6848.55817 -4.300310e+05  1.030299e+02      169.7704
## ar2     -25486.4395  -16836.94045 -4.232764e+05  2.425301e+02      142.6160
## ar3     -17000.4081  -30360.61821 -2.431689e+05  6.077065e+01      111.0664
## ma1     -19236.8110   10719.24321 -2.993835e+05  1.277571e+02      116.3374
## ma2     -34835.2384  -24631.04838 -3.822655e+05  2.012581e+02      132.1385
## ma3     -24631.0484  -43858.69184 -3.347791e+05 -1.997174e+01      157.5463
## omega  -382265.4720 -334779.09667 -1.174652e+10 -1.911506e+06 -1873441.3763
## alpha1     201.2581     -19.97174 -1.911506e+06 -1.656336e+03     -575.6375
## alpha2     132.1385     157.54633 -1.873441e+06 -5.756375e+02     -988.8458
## attr(,"time")
## Time difference of 0.2143631 secs
##
```

```
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.546987 secs
```

```
gfit10 = garchFit(~arma(3,3)+garch(2,1),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(3, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 1)
##  ARMA Order:                3 3
##  Max ARMA Order:            3
##  GARCH Order:               2 1
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V       params includes
##     mu     -0.42203167   0.4220317  0.043417533     TRUE
##     ar1    -0.99999999   1.0000000  0.383923685     TRUE
##     ar2    -0.99999999   1.0000000  0.001167706     TRUE
##     ar3    -0.99999999   1.0000000 -0.677235319     TRUE
##     ma1    -0.99999999   1.0000000 -0.493566554     TRUE
##     ma2    -0.99999999   1.0000000  0.081168715     TRUE
##     ma3    -0.99999999   1.0000000  0.639903273     TRUE
##     omega   0.00000100 100.0000000  0.100000000     TRUE
##     alpha1  0.00000001   1.0000000  0.050000000     TRUE
##     alpha2  0.00000001   1.0000000  0.050000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.100000000    FALSE
##     gamma2 -0.99999999   1.0000000  0.100000000    FALSE
##     beta1   0.00000001   1.0000000  0.800000000     TRUE
##     delta   0.00000000   2.0000000  2.000000000    FALSE
##     skew    0.10000000  10.0000000  1.000000000    FALSE
##     shape   1.00000000  10.0000000  4.000000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ar2    ar3    ma1    ma2    ma3  omega alpha1 alpha2  beta1
##      1      2      3      4      5      6      7      8      9     10     13
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
```

```
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3958.1844: 0.0434175 0.383924 0.00116771 -0.677235 -0.493567 0.0811687 0.639903 0.100000 0.0
##   1:     3946.8657: 0.0434196 0.386144 0.00167039 -0.677052 -0.491218 0.0818008 0.639873 0.0894201 0
##   2:     3932.5196: 0.0434288 0.394535 0.00273994 -0.676314 -0.482741 0.0832829 0.639929 0.0747810 0
##   3:     3929.1092: 0.0434305 0.395016 0.00227544 -0.676186 -0.482474 0.0827530 0.640018 0.0699031 0
##   4:     3927.0384: 0.0434418 0.398517 -0.000506524 -0.675073 -0.480353 0.0796145 0.640905 0.0591034
##   5:     3923.9535: 0.0434852 0.406821 -0.00989874 -0.668059 -0.477341 0.0691453 0.648057 0.0580230 0
##   6:     3922.0529: 0.0435319 0.419245 -0.00957543 -0.659464 -0.467674 0.0700246 0.657243 0.0515974 0
##   7:     3919.5497: 0.0435928 0.420536 -0.0182138 -0.664118 -0.471324 0.0557780 0.649316 0.0445455 0
##   8:     3918.5362: 0.0435944 0.420651 -0.0175704 -0.663404 -0.471173 0.0565322 0.650090 0.0470279 0
##   9:     3917.6809: 0.0436022 0.419685 -0.0166250 -0.662088 -0.472414 0.0572304 0.651310 0.0443912 0
##  10:     3916.9882: 0.0436239 0.418619 -0.0144918 -0.660609 -0.474295 0.0587461 0.652657 0.0432561 0
##  11:     3915.3095: 0.0436742 0.420817 -0.0112018 -0.663173 -0.474028 0.0606180 0.649781 0.0378677 0
##  12:     3913.7350: 0.0439088 0.420449 -0.0111946 -0.660400 -0.486665 0.0571847 0.654671 0.0244456 0
##  13:     3913.2678: 0.0439197 0.425667 -0.00721943 -0.660537 -0.480417 0.0619604 0.654243 0.0292587 0
##  14:     3912.8644: 0.0439204 0.425576 -0.00737572 -0.660638 -0.480573 0.0617646 0.654133 0.0302377 0
##  15:     3912.6069: 0.0439262 0.425593 -0.00748333 -0.660735 -0.480762 0.0615987 0.653983 0.0288527 0
##  16:     3912.3083: 0.0440189 0.425934 -0.00761874 -0.661447 -0.482061 0.0623140 0.652650 0.0288978 0
##  17:     3912.1800: 0.0441911 0.428833 -0.00919103 -0.661146 -0.483867 0.0617499 0.652457 0.0268529 0
##  18:     3912.0479: 0.0443882 0.431520 -0.0110135 -0.660301 -0.487187 0.0615849 0.653710 0.0276591 0
##  19:     3911.9326: 0.0445558 0.433205 -0.0101704 -0.662720 -0.490016 0.0645912 0.652624 0.0267450 0
##  20:     3911.8969: 0.0445603 0.433034 -0.0104192 -0.662771 -0.490386 0.0643426 0.652599 0.0260917 0
##  21:     3911.8939: 0.0445610 0.433032 -0.0104317 -0.662770 -0.490412 0.0643367 0.652605 0.0262284 0
##  22:     3911.8905: 0.0445665 0.433106 -0.0104714 -0.662780 -0.490494 0.0643588 0.652623 0.0261526 0
##  23:     3911.8875: 0.0445771 0.433194 -0.0105801 -0.662785 -0.490723 0.0643632 0.652679 0.0261387 0
##  24:     3911.7089: 0.0454177 0.446068 -0.0160907 -0.664337 -0.501186 0.0686175 0.655019 0.0254706 0
##  25:     3911.5631: 0.0462966 0.455042 -0.0210180 -0.666027 -0.512108 0.0763701 0.655148 0.0267948 0
##  26:     3911.4147: 0.0472394 0.463259 -0.0270453 -0.665791 -0.519556 0.0844670 0.651507 0.0252739 0
##  27:     3911.3006: 0.0481976 0.470264 -0.0335013 -0.663269 -0.525939 0.0921911 0.646561 0.0252760 0
##  28:     3910.9784: 0.0491320 0.473139 -0.0399915 -0.654670 -0.530074 0.0984658 0.638305 0.0254510 0
##  29:     3910.9633: 0.0491361 0.473030 -0.0399450 -0.654385 -0.530225 0.0985991 0.638556 0.0259924 0
##  30:     3910.9527: 0.0491638 0.473147 -0.0401620 -0.654209 -0.530380 0.0988309 0.638408 0.0257809 0
##  31:     3910.9432: 0.0491929 0.473282 -0.0403861 -0.654031 -0.530537 0.0990713 0.638241 0.0258718 0
##  32:     3909.0800: 0.0708615 0.579321 -0.207307 -0.530589 -0.643080 0.277438 0.506662 0.0231527 0.0
##  33:     3908.4370: 0.0787171 0.625711 -0.256035 -0.490736 -0.698450 0.331870 0.452377 0.0255010 0.0
##  34:     3908.1506: 0.0787172 0.625670 -0.256002 -0.490607 -0.698520 0.331923 0.452558 0.0243812 0.0
##  35:     3908.0428: 0.0787376 0.626376 -0.256382 -0.490930 -0.698828 0.332002 0.453327 0.0249730 0.0
##  36:     3907.9642: 0.0787761 0.627662 -0.257060 -0.491576 -0.699714 0.332408 0.455263 0.0250172 0.0
##  37:     3907.8636: 0.0788920 0.630468 -0.258902 -0.493895 -0.701593 0.333804 0.457637 0.0256831 0.0
##  38:     3907.7578: 0.0791441 0.635752 -0.261710 -0.498577 -0.706698 0.337639 0.461267 0.0257011 0.0
##  39:     3907.7217: 0.0794168 0.648569 -0.252785 -0.500180 -0.723861 0.339077 0.459578 0.0253662 0.0
##  40:     3907.5719: 0.0798773 0.657529 -0.267031 -0.502228 -0.730229 0.348592 0.462859 0.0268604 0.0
##  41:     3907.5688: 0.0798774 0.657606 -0.267030 -0.502222 -0.730146 0.348600 0.462856 0.0271051 0.0
##  42:     3907.5579: 0.0798855 0.657805 -0.267045 -0.502119 -0.730210 0.348590 0.462788 0.0269550 0.0
##  43:     3907.5536: 0.0799010 0.658356 -0.267120 -0.501896 -0.730142 0.348560 0.462666 0.0269993 0.0
##  44:     3907.5480: 0.0799401 0.659101 -0.267307 -0.501383 -0.730633 0.348628 0.462305 0.0269117 0.0
##  45:     3907.5453: 0.0799767 0.659663 -0.267847 -0.500916 -0.731095 0.349086 0.461887 0.0269928 0.0
##  46:     3907.5410: 0.0800260 0.660175 -0.269577 -0.500530 -0.731806 0.350798 0.461240 0.0268582 0.0
##  47:     3907.5344: 0.0801365 0.662813 -0.271321 -0.497754 -0.733748 0.352042 0.459120 0.0270568 0.0
##  48:     3907.5327: 0.0801370 0.662957 -0.271545 -0.497591 -0.733638 0.351894 0.459335 0.0270999 0.0
##  49:     3907.5237: 0.0801380 0.663241 -0.271798 -0.497355 -0.733855 0.352086 0.459150 0.0269591 0.0
```

```
##  50:     3907.5206: 0.0801625 0.663773 -0.272341 -0.497011 -0.734214 0.352457 0.458885 0.0269891 0.0
##  51:     3907.5166: 0.0802119 0.664640 -0.273296 -0.496318 -0.735037 0.353404 0.458172 0.0269184 0.0
##  52:     3907.4922: 0.0807369 0.675337 -0.288322 -0.484163 -0.746016 0.368202 0.446081 0.0268518 0.0
##  53:     3907.4743: 0.0810123 0.690648 -0.302840 -0.476415 -0.759577 0.382515 0.437599 0.0270412 0.0
##  54:     3907.4730: 0.0810123 0.690629 -0.302854 -0.476380 -0.759594 0.382515 0.437646 0.0269709 0.0
##  55:     3907.4720: 0.0810104 0.690622 -0.302874 -0.476350 -0.759606 0.382519 0.437675 0.0270092 0.0
##  56:     3907.4710: 0.0810029 0.690632 -0.302919 -0.476308 -0.759624 0.382540 0.437687 0.0269502 0.0
##  57:     3907.4701: 0.0809868 0.690667 -0.303006 -0.476240 -0.759655 0.382590 0.437679 0.0269751 0.0
##  58:     3907.4688: 0.0809544 0.690751 -0.303179 -0.476120 -0.759721 0.382707 0.437620 0.0269203 0.0
##  59:     3907.4671: 0.0808903 0.690942 -0.303542 -0.475881 -0.759870 0.382980 0.437456 0.0269329 0.0
##  60:     3907.4507: 0.0798556 0.694783 -0.310414 -0.471582 -0.763181 0.388864 0.433550 0.0266540 0.0
##  61:     3907.4461: 0.0800124 0.703645 -0.322698 -0.464188 -0.774081 0.403734 0.423444 0.0262004 0.0
##  62:     3907.4385: 0.0793215 0.713367 -0.331948 -0.456547 -0.783577 0.413380 0.416176 0.0258911 0.0
##  63:     3907.4353: 0.0793433 0.716946 -0.338551 -0.453336 -0.787214 0.419387 0.413004 0.0261000 0.0
##  64:     3907.4254: 0.0797902 0.716393 -0.341169 -0.451274 -0.785911 0.420227 0.412207 0.0260073 0.0
##  65:     3907.4224: 0.0797980 0.715996 -0.340423 -0.450859 -0.784898 0.419018 0.412108 0.0259215 0.0
##  66:     3907.4222: 0.0797604 0.716650 -0.341224 -0.450323 -0.785560 0.419844 0.411540 0.0258969 0.0
##  67:     3907.4189: 0.0787539 0.734173 -0.362569 -0.435535 -0.803067 0.441682 0.395932 0.0258965 0.0
##  68:     3907.4186: 0.0788143 0.735640 -0.364487 -0.434095 -0.804529 0.443492 0.394568 0.0258397 0.0
##  69:     3907.4186: 0.0788593 0.737150 -0.366498 -0.432942 -0.806064 0.445578 0.393243 0.0258315 0.0
##  70:     3907.4186: 0.0788145 0.737373 -0.366846 -0.432627 -0.806292 0.445916 0.392952 0.0258373 0.0
##  71:     3907.4186: 0.0788112 0.737354 -0.366794 -0.432667 -0.806269 0.445863 0.392991 0.0258360 0.0
##  72:     3907.4186: 0.0788140 0.737343 -0.366783 -0.432676 -0.806258 0.445851 0.393001 0.0258360 0.0
##
## Final Estimate of the Negative LLH:
##  LLH:  -7610.655    norm LLH:  -2.521754
##             mu           ar1           ar2           ar3           ma1
##  1.734349e-03  7.373432e-01 -3.667833e-01 -4.326759e-01 -8.062575e-01
##            ma2           ma3         omega        alpha1        alpha2
##  4.458511e-01  3.930013e-01  1.251097e-05  7.613239e-02  1.000000e-08
##         beta1
##  8.958451e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu           ar1           ar2           ar3           ma1
## mu      -9009918.826    -9958.4386 -1.150995e+04 -1.428761e+04     4009.1437
## ar1        -9958.439   -19428.6975 -1.184432e+04  3.713608e+03   -24553.4388
## ar2       -11509.955   -11844.3236 -1.963560e+04 -1.170608e+04   -15876.1455
## ar3       -14287.608     3713.6081 -1.170608e+04 -1.960228e+04     4197.6198
## ma1         4009.144   -24553.4388 -1.587615e+04  4.197620e+03   -32264.5655
## ma2         3191.706   -13712.3767 -2.349727e+04 -1.454963e+04   -19034.7653
## ma3         1212.996     6322.9772 -1.256967e+04 -2.339688e+04     7718.2095
## omega  -56707990.691 -6043124.8077 -5.472824e+06 -2.019727e+06 -7644802.4858
## alpha1     -2446.413     -168.8928  9.201717e+01  1.791706e+02     -122.3649
## alpha2     -4230.306     -586.7910 -2.243126e+02 -3.711409e-02     -567.7168
## beta1     -23336.858    -1884.1444 -1.308282e+03 -4.671332e+02    -2107.3125
##                   ma2           ma3         omega        alpha1        alpha2
## mu         3191.7063     1212.9959 -5.670799e+07 -2.446413e+03 -4.230306e+03
## ar1      -13712.3767     6322.9772 -6.043125e+06 -1.688928e+02 -5.867910e+02
## ar2      -23497.2673   -12569.6698 -5.472824e+06  9.201717e+01 -2.243126e+02
## ar3      -14549.6301   -23396.8757 -2.019727e+06  1.791706e+02 -3.711409e-02
## ma1      -19034.7653     7718.2095 -7.644802e+06 -1.223649e+02 -5.677168e+02
## ma2      -29523.1435   -16961.4341 -6.957330e+06  2.038675e+02 -1.254376e+02
## ma3      -16961.4341   -30197.8474 -2.176974e+06  3.076863e+02  1.622218e+02
```

```
## omega  -6957329.8296 -2176974.4471 -1.603758e+12 -3.156285e+08 -3.195103e+08
## alpha1      203.8675      307.6863 -3.156285e+08 -1.118971e+05 -1.108121e+05
## alpha2     -125.4376      162.2218 -3.195103e+08 -1.108121e+05 -1.128710e+05
## beta1     -1428.8177     -361.2056 -4.631936e+08 -1.254891e+05 -1.277841e+05
##                beta1
## mu     -2.333686e+04
## ar1    -1.884144e+03
## ar2    -1.308282e+03
## ar3    -4.671332e+02
## ma1    -2.107313e+03
## ma2    -1.428818e+03
## ma3    -3.612056e+02
## omega  -4.631936e+08
## alpha1 -1.254891e+05
## alpha2 -1.277841e+05
## beta1  -1.615604e+05
## attr(,"time")
## Time difference of 0.21591 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.563431 secs
```

```
gfit11 = garchFit(~arma(3,3)+garch(1,2),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(3, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 2)
##  ARMA Order:                3 3
##  Max ARMA Order:            3
##  GARCH Order:               1 2
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V       params includes
##     mu     -0.42203167   0.4220317  0.043417533     TRUE
##     ar1    -0.99999999   1.0000000  0.383923685     TRUE
##     ar2    -0.99999999   1.0000000  0.001167706     TRUE
##     ar3    -0.99999999   1.0000000 -0.677235319     TRUE
```

```
##     ma1    -0.99999999   1.0000000 -0.493566554     TRUE
##     ma2    -0.99999999   1.0000000  0.081168715     TRUE
##     ma3    -0.99999999   1.0000000  0.639903273     TRUE
##     omega   0.00000100 100.0000000  0.100000000     TRUE
##     alpha1  0.00000001   1.0000000  0.100000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.100000000    FALSE
##     beta1   0.00000001   1.0000000  0.400000000     TRUE
##     beta2   0.00000001   1.0000000  0.400000000     TRUE
##     delta   0.00000000   2.0000000  2.000000000    FALSE
##     skew    0.10000000  10.0000000  1.000000000    FALSE
##     shape   1.00000000  10.0000000  4.000000000    FALSE
##   Index List of Parameters to be Optimized:
##     mu    ar1    ar2    ar3    ma1    ma2    ma3  omega alpha1  beta1  beta2
##      1      2      3      4      5      6      7      8      9     11     12
##   Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3953.4301: 0.0434175 0.383924 0.00116771 -0.677235 -0.493567 0.0811687 0.639903 0.100000 0.1
##   1:     3941.0925: 0.0434191 0.385540 0.00159850 -0.677095 -0.491840 0.0817112 0.639878 0.0907292 0
##   2:     3921.0255: 0.0434355 0.400044 0.00440645 -0.675803 -0.476879 0.0854222 0.639831 0.0630796 0
##   3:     3919.6351: 0.0434378 0.399506 0.00286544 -0.675751 -0.477913 0.0835352 0.639691 0.0593355 0
##   4:     3918.8318: 0.0434410 0.399209 0.00133868 -0.675436 -0.478767 0.0817197 0.639862 0.0596360 0
##   5:     3917.1716: 0.0434522 0.399872 -0.00168726 -0.673558 -0.479506 0.0783698 0.641608 0.0518071 0
##   6:     3915.6875: 0.0434692 0.402535 -0.00355511 -0.670371 -0.478320 0.0766173 0.644921 0.0502496 0
##   7:     3914.5601: 0.0434903 0.406780 -0.00562372 -0.669260 -0.475754 0.0742897 0.645892 0.0464387 0
##   8:     3913.9059: 0.0435312 0.410669 -0.00743812 -0.668853 -0.474534 0.0717218 0.646203 0.0440927 0
##   9:     3913.3960: 0.0435740 0.410791 -0.00708210 -0.664076 -0.476696 0.0719977 0.651365 0.0398239 0
##  10:     3912.9056: 0.0437008 0.416160 -0.00947488 -0.666317 -0.477538 0.0668954 0.647322 0.0397507 0
##  11:     3912.8276: 0.0437022 0.416137 -0.00945049 -0.666043 -0.477638 0.0669371 0.647618 0.0403036 0
##  12:     3912.7658: 0.0437044 0.416126 -0.00944219 -0.665731 -0.477770 0.0669553 0.647953 0.0398531 0
##  13:     3912.6449: 0.0437279 0.417020 -0.00932538 -0.665410 -0.477932 0.0668186 0.648117 0.0402501 0
##  14:     3912.4735: 0.0437861 0.418693 -0.00882677 -0.665313 -0.478805 0.0667604 0.647969 0.0390652 0
##  15:     3912.2427: 0.0439255 0.420891 -0.00769965 -0.665640 -0.482578 0.0669093 0.647499 0.0365652 0
##  16:     3911.8807: 0.0442403 0.431324 -0.0111183 -0.661949 -0.485689 0.0627653 0.650475 0.0382079 0
##  17:     3911.7563: 0.0442516 0.430534 -0.0107839 -0.660822 -0.486938 0.0632510 0.651792 0.0374693 0
##  18:     3911.7187: 0.0442523 0.430530 -0.0107876 -0.660831 -0.486970 0.0632517 0.651786 0.0371348 0
##  19:     3911.6772: 0.0442634 0.430520 -0.0108060 -0.660954 -0.487375 0.0633072 0.651721 0.0369511 0
##  20:     3911.5430: 0.0443316 0.431027 -0.0103373 -0.661721 -0.488496 0.0648907 0.651202 0.0360607 0
##  21:     3911.4090: 0.0445323 0.432773 -0.0109885 -0.662777 -0.491642 0.0678157 0.650868 0.0352559 0
##  22:     3911.3279: 0.0447654 0.434911 -0.0136017 -0.662871 -0.494741 0.0696982 0.651242 0.0342651 0
##  23:     3911.2434: 0.0450131 0.437027 -0.0164043 -0.662445 -0.496431 0.0719138 0.650335 0.0344719 0
##  24:     3911.1717: 0.0452620 0.439074 -0.0184541 -0.661228 -0.497586 0.0740199 0.648214 0.0344919 0
##  25:     3910.7037: 0.0482776 0.466371 -0.0321065 -0.641173 -0.529168 0.0933263 0.627619 0.0326880 0
##  26:     3910.4837: 0.0513323 0.481577 -0.0615509 -0.624378 -0.545916 0.124184 0.608576 0.0327762 0.1
##  27:     3909.5352: 0.0543390 0.494487 -0.0973073 -0.615997 -0.562563 0.159839 0.597047 0.0309761 0.0
##  28:     3909.0515: 0.0571909 0.526769 -0.0962555 -0.592106 -0.596697 0.170401 0.568117 0.0342748 0.0
##  29:     3909.0429: 0.0571927 0.527144 -0.0963628 -0.592322 -0.596254 0.170429 0.567911 0.0344807 0.0
##  30:     3908.9897: 0.0572173 0.527413 -0.0965721 -0.592185 -0.596325 0.170586 0.567704 0.0340694 0.0
##  31:     3908.9659: 0.0572769 0.528061 -0.0971038 -0.591903 -0.596520 0.171005 0.567251 0.0341228 0.0
```

```
##  32:     3908.9322: 0.0574001 0.529101 -0.0981059 -0.591225 -0.597319 0.171937 0.566496 0.0338589 0.0
##  33:     3907.4188: 0.0697571 0.624990 -0.199582 -0.531310 -0.690571 0.274682 0.501202 0.0322475 0.09
##  34:     3907.0775: 0.0739502 0.638504 -0.240398 -0.506726 -0.714480 0.321927 0.470127 0.0326794 0.09
##  35:     3907.0427: 0.0739506 0.638889 -0.240243 -0.506781 -0.714004 0.322178 0.470082 0.0331577 0.09
##  36:     3906.9750: 0.0739744 0.639151 -0.240372 -0.506582 -0.713987 0.322510 0.469832 0.0326907 0.09
##  37:     3906.9490: 0.0740328 0.639981 -0.240833 -0.506497 -0.714443 0.323440 0.469574 0.0328546 0.09
##  38:     3906.9246: 0.0741041 0.641967 -0.241803 -0.507747 -0.716280 0.325204 0.470310 0.0325986 0.09
##  39:     3906.9045: 0.0740712 0.643911 -0.242803 -0.509293 -0.718118 0.326632 0.471169 0.0327103 0.09
##  40:     3906.8842: 0.0741780 0.643650 -0.244536 -0.504183 -0.717860 0.328227 0.466202 0.0320429 0.09
##  41:     3906.8610: 0.0739979 0.646936 -0.246767 -0.507590 -0.719878 0.329599 0.468589 0.0324832 0.09
##  42:     3906.8494: 0.0740077 0.651125 -0.249787 -0.510520 -0.723496 0.332822 0.470705 0.0323986 0.09
##  43:     3906.8118: 0.0742412 0.653053 -0.252459 -0.508303 -0.725519 0.336065 0.468541 0.0325804 0.09
##  44:     3906.7978: 0.0744580 0.654635 -0.255253 -0.505382 -0.727302 0.339180 0.465733 0.0323740 0.09
##  45:     3906.7187: 0.0749466 0.668358 -0.278563 -0.489819 -0.742260 0.360993 0.449179 0.0321539 0.09
##  46:     3906.6687: 0.0757778 0.684327 -0.300009 -0.477780 -0.758598 0.383974 0.436125 0.0325039 0.09
##  47:     3906.6296: 0.0763617 0.703614 -0.323015 -0.469613 -0.777665 0.407469 0.426003 0.0325334 0.09
##  48:     3906.5399: 0.0768076 0.715449 -0.343121 -0.445737 -0.789265 0.426396 0.403905 0.0320884 0.09
##  49:     3906.5156: 0.0768077 0.715478 -0.343127 -0.445747 -0.789242 0.426392 0.403898 0.0322851 0.09
##  50:     3906.5110: 0.0768080 0.715656 -0.343185 -0.445819 -0.789107 0.426346 0.403862 0.0321063 0.09
##  51:     3906.5093: 0.0768003 0.715922 -0.343438 -0.445641 -0.789365 0.426613 0.403660 0.0321070 0.09
##  52:     3906.5070: 0.0767868 0.716433 -0.343951 -0.445268 -0.789897 0.427180 0.403229 0.0319972 0.09
##  53:     3906.4819: 0.0764387 0.732294 -0.361211 -0.432767 -0.806289 0.445769 0.389413 0.0311613 0.09
##  54:     3906.4199: 0.0757586 0.782929 -0.421078 -0.391300 -0.854240 0.504315 0.347547 0.0318238 0.09
##  55:     3906.4091: 0.0758237 0.827379 -0.480759 -0.351105 -0.901776 0.567654 0.303115 0.0314672 0.09
##  56:     3906.3822: 0.0727114 0.866500 -0.527588 -0.321252 -0.939669 0.614593 0.271775 0.0313801 0.09
##  57:     3906.3692: 0.0703655 0.900235 -0.579220 -0.286200 -0.971124 0.662456 0.238070 0.0314228 0.09
##  58:     3906.3351: 0.0683301 0.928519 -0.611597 -0.261644 -1.00000 0.696245 0.211424 0.0316001 0.095
##  59:     3906.3241: 0.0699748 0.927907 -0.611063 -0.262580 -1.00000 0.696310 0.212276 0.0316376 0.090
##  60:     3906.3210: 0.0698357 0.928152 -0.613246 -0.261721 -1.00000 0.698234 0.211365 0.0315004 0.090
##  61:     3906.3202: 0.0698659 0.928286 -0.613483 -0.261709 -1.00000 0.698199 0.211488 0.0316018 0.090
##  62:     3906.3200: 0.0697771 0.928372 -0.613627 -0.261592 -1.00000 0.698170 0.211518 0.0315671 0.090
##  63:     3906.3200: 0.0697905 0.928378 -0.613347 -0.261746 -1.00000 0.697956 0.211661 0.0315927 0.090
##  64:     3906.3200: 0.0697954 0.928370 -0.613471 -0.261684 -1.00000 0.698054 0.211599 0.0315842 0.090
##  65:     3906.3200: 0.0697919 0.928371 -0.613469 -0.261680 -1.00000 0.698052 0.211598 0.0315838 0.090
##
## Final Estimate of the Negative LLH:
##  LLH:  -7611.753    norm LLH:  -2.522118
##           mu           ar1           ar2           ar3           ma1
##  1.535812e-03  9.283714e-01 -6.134688e-01 -2.616802e-01 -1.000000e+00
##           ma2           ma3         omega        alpha1         beta1
##  6.980518e-01  2.115979e-01  1.529433e-05  9.627748e-02  4.163074e-01
##         beta2
##  4.523853e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu           ar1           ar2           ar3           ma1
## mu     -1.156796e+07   -14331.2005 -1.696628e+04   -19528.4091     6259.5921
## ar1    -1.433120e+04   -33602.0028 -2.090437e+04     7145.9766   -43480.2307
## ar2    -1.696628e+04   -20904.3710 -3.407257e+04   -20523.1283   -29538.4139
## ar3    -1.952841e+04     7145.9766 -2.052313e+04   -33787.8095     6873.7089
## ma1     6.259592e+03   -43480.2307 -2.953841e+04     6873.7089   -59908.9851
## ma2     3.233099e+03   -23464.7359 -4.227822e+04   -27865.7718   -34839.9289
## ma3    -5.707341e+02    13539.4304 -2.217744e+04   -42406.5937    16336.0052
## omega  -5.151144e+07 -5238881.5253 -5.820564e+06 -2662109.2942 -6780687.8540
```

```
## alpha1 -1.152992e+03     -247.7189  6.719419e+01      225.0402     -224.2156
## beta1  -2.244777e+04    -1610.8172 -1.330452e+03     -469.7604    -1909.6176
## beta2  -2.323829e+04    -1690.0771 -1.400733e+03     -508.5106    -2014.3427
##                 ma2           ma3         omega        alpha1         beta1
## mu         3233.0988     -570.7341 -5.151144e+07 -1.152992e+03 -2.244777e+04
## ar1      -23464.7359    13539.4304 -5.238882e+06 -2.477189e+02 -1.610817e+03
## ar2      -42278.2179   -22177.4403 -5.820564e+06  6.719419e+01 -1.330452e+03
## ar3      -27865.7718   -42406.5937 -2.662109e+06  2.250402e+02 -4.697604e+02
## ma1      -34839.9289    16336.0052 -6.780688e+06 -2.242156e+02 -1.909618e+03
## ma2      -54752.2985   -32370.6436 -7.536482e+06  1.696021e+02 -1.521607e+03
## ma3      -32370.6436   -57965.9507 -3.114952e+06  3.398605e+02 -3.561462e+02
## omega  -7536482.1217 -3114951.8506 -1.008551e+12 -2.024640e+08 -2.921539e+08
## alpha1      169.6021      339.8605 -2.024640e+08 -7.202013e+04 -7.999923e+04
## beta1     -1521.6067     -356.1462 -2.921539e+08 -7.999923e+04 -1.017177e+05
## beta2     -1612.5164     -395.4739 -2.940951e+08 -8.029320e+04 -1.023235e+05
##                beta2
## mu     -2.323829e+04
## ar1    -1.690077e+03
## ar2    -1.400733e+03
## ar3    -5.085106e+02
## ma1    -2.014343e+03
## ma2    -1.612516e+03
## ma3    -3.954739e+02
## omega  -2.940951e+08
## alpha1 -8.029320e+04
## beta1  -1.023235e+05
## beta2  -1.029454e+05
## attr(,"time")
## Time difference of 0.707377 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 3.436935 secs
```

```
gfit12 = garchFit(~arma(3,3)+garch(2,2),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(3, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 2)
##  ARMA Order:                3 3
##  Max ARMA Order:            3
##  GARCH Order:               2 2
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
```

```
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V       params includes
##     mu     -0.42203167   0.4220317  0.043417533     TRUE
##     ar1    -0.99999999   1.0000000  0.383923685     TRUE
##     ar2    -0.99999999   1.0000000  0.001167706     TRUE
##     ar3    -0.99999999   1.0000000 -0.677235319     TRUE
##     ma1    -0.99999999   1.0000000 -0.493566554     TRUE
##     ma2    -0.99999999   1.0000000  0.081168715     TRUE
##     ma3    -0.99999999   1.0000000  0.639903273     TRUE
##     omega   0.00000100 100.0000000  0.100000000     TRUE
##     alpha1  0.00000001   1.0000000  0.050000000     TRUE
##     alpha2  0.00000001   1.0000000  0.050000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.100000000    FALSE
##     gamma2 -0.99999999   1.0000000  0.100000000    FALSE
##     beta1   0.00000001   1.0000000  0.400000000     TRUE
##     beta2   0.00000001   1.0000000  0.400000000     TRUE
##     delta   0.00000000   2.0000000  2.000000000    FALSE
##     skew    0.10000000  10.0000000  1.000000000    FALSE
##     shape   1.00000000  10.0000000  4.000000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ar2    ar3    ma1    ma2    ma3  omega alpha1 alpha2  beta1
##      1      2      3      4      5      6      7      8      9     10     13
##  beta2
##     14
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3956.0088: 0.0434175 0.383924 0.00116771 -0.677235 -0.493567 0.0811687 0.639903 0.100000 0.0
##   1:     3943.2643: 0.0434190 0.385438 0.00152714 -0.677087 -0.491956 0.0816287 0.639899 0.0911336 0
##   2:     3924.9263: 0.0434281 0.393817 0.00297282 -0.676199 -0.483314 0.0835851 0.640022 0.0732756 0
##   3:     3921.2476: 0.0434294 0.394168 0.00257358 -0.676105 -0.483140 0.0831332 0.640078 0.0689300 0
##   4:     3918.7542: 0.0434419 0.397675 -0.000629925 -0.674968 -0.481154 0.0795555 0.640924 0.0562381
##   5:     3918.1210: 0.0435027 0.409028 -0.0120743 -0.664344 -0.476552 0.0673830 0.651710 0.0548480 0
##   6:     3914.7583: 0.0435365 0.417387 -0.0124516 -0.659972 -0.470158 0.0672947 0.656197 0.0492735 0
##   7:     3913.6297: 0.0435516 0.416577 -0.0148487 -0.662923 -0.472288 0.0635293 0.652650 0.0499514 0
##   8:     3913.3249: 0.0435736 0.416084 -0.0143417 -0.661007 -0.474106 0.0638570 0.654952 0.0456180 0
##   9:     3913.1783: 0.0435988 0.417877 -0.0148362 -0.662845 -0.473815 0.0626783 0.653057 0.0475287 0
##  10:     3913.0345: 0.0435998 0.417874 -0.0147779 -0.662758 -0.473867 0.0627380 0.653163 0.0467346 0
##  11:     3912.9371: 0.0436078 0.417899 -0.0143426 -0.662131 -0.474230 0.0631756 0.653926 0.0458137 0
##  12:     3912.7560: 0.0436336 0.419199 -0.0138258 -0.662819 -0.474222 0.0633432 0.653343 0.0435288 0
##  13:     3912.5180: 0.0436658 0.419589 -0.0136223 -0.663516 -0.475527 0.0630553 0.652763 0.0430971 0
##  14:     3912.3576: 0.0437091 0.419584 -0.0133325 -0.663198 -0.477613 0.0628373 0.653059 0.0425085 0
##  15:     3912.1466: 0.0437582 0.421441 -0.0126822 -0.662950 -0.477816 0.0631390 0.653139 0.0414913 0
##  16:     3912.0602: 0.0438124 0.423053 -0.0122664 -0.663360 -0.478378 0.0633854 0.652708 0.0393823 0
```

```
##  17:     3911.8810: 0.0439055 0.424102 -0.0121014 -0.663791 -0.480703 0.0637745 0.652309 0.0388078 0
##  18:     3911.7531: 0.0440175 0.425533 -0.0122940 -0.662697 -0.483003 0.0642178 0.653189 0.0388831 0
##  19:     3911.6539: 0.0441305 0.427081 -0.0128610 -0.663326 -0.484585 0.0649855 0.652608 0.0384435 0
##  20:     3911.5776: 0.0442503 0.428458 -0.0132467 -0.664085 -0.486394 0.0660579 0.651786 0.0371510 0
##  21:     3911.4939: 0.0443956 0.430619 -0.0138995 -0.663554 -0.487901 0.0673010 0.651792 0.0368179 0
##  22:     3911.4404: 0.0445418 0.432589 -0.0152786 -0.662913 -0.489233 0.0682519 0.651790 0.0367084 0
##  23:     3911.2636: 0.0453319 0.437603 -0.0195129 -0.663968 -0.500390 0.0770703 0.646979 0.0362377 0
##  24:     3911.2376: 0.0453341 0.437811 -0.0193223 -0.663711 -0.500206 0.0773330 0.647245 0.0363703 0
##  25:     3911.2185: 0.0453498 0.438060 -0.0194526 -0.663528 -0.500255 0.0774607 0.647292 0.0360857 0
##  26:     3911.2007: 0.0453685 0.438344 -0.0196563 -0.663345 -0.500332 0.0775624 0.647316 0.0361988 0
##  27:     3911.1791: 0.0454082 0.438933 -0.0201013 -0.662969 -0.500507 0.0777633 0.647352 0.0359922 0
##  28:     3911.0674: 0.0458232 0.444153 -0.0254134 -0.660237 -0.503118 0.0791097 0.646226 0.0358463 0
##  29:     3910.6092: 0.0478526 0.459811 -0.0363223 -0.650282 -0.520750 0.0984323 0.630864 0.0351483 0
##  30:     3910.6083: 0.0478546 0.459698 -0.0364034 -0.650077 -0.520952 0.0983741 0.631088 0.0348724 0
##  31:     3910.5807: 0.0478550 0.459681 -0.0364204 -0.650032 -0.520989 0.0983636 0.631136 0.0350557 0
##  32:     3910.5627: 0.0478584 0.459568 -0.0365411 -0.649742 -0.521240 0.0982869 0.631450 0.0351369 0
##  33:     3910.5479: 0.0478976 0.459844 -0.0368202 -0.649476 -0.521623 0.0986239 0.631279 0.0348948 0
##  34:     3910.5315: 0.0479377 0.460135 -0.0370988 -0.649233 -0.521992 0.0989809 0.631071 0.0349354 0
##  35:     3910.1897: 0.0502836 0.477516 -0.0526121 -0.636995 -0.542291 0.120998 0.616412 0.0314260 0.0
##  36:     3909.9699: 0.0526555 0.491584 -0.0755978 -0.626203 -0.558843 0.144996 0.603899 0.0336501 0.0
##  37:     3909.3354: 0.0550430 0.507638 -0.0949742 -0.612588 -0.576557 0.166487 0.589138 0.0339694 0.0
##  38:     3908.9025: 0.0573480 0.522349 -0.107202 -0.588183 -0.591638 0.179818 0.563607 0.0337841 0.08
##  39:     3908.5135: 0.0596747 0.536724 -0.134384 -0.582945 -0.606368 0.205648 0.554808 0.0323127 0.09
##  40:     3908.1735: 0.0620127 0.557107 -0.156142 -0.577682 -0.626771 0.228590 0.547194 0.0335177 0.09
##  41:     3907.8874: 0.0643226 0.581079 -0.173548 -0.570881 -0.650633 0.248674 0.538777 0.0342538 0.09
##  42:     3907.6732: 0.0664769 0.580719 -0.201723 -0.553085 -0.654343 0.276785 0.519531 0.0330803 0.09
##  43:     3907.6368: 0.0664791 0.581172 -0.201970 -0.553410 -0.654115 0.276237 0.518929 0.0329952 0.09
##  44:     3907.6228: 0.0664793 0.581213 -0.201970 -0.553399 -0.654098 0.276213 0.518917 0.0328349 0.09
##  45:     3907.6115: 0.0664822 0.581653 -0.201963 -0.553270 -0.653918 0.275962 0.518811 0.0327963 0.09
##  46:     3907.5957: 0.0665234 0.582147 -0.202223 -0.552997 -0.654387 0.276266 0.518561 0.0326398 0.09
##  47:     3907.3414: 0.0705384 0.622575 -0.227294 -0.529744 -0.695103 0.308196 0.495071 0.0350955 0.10
##  48:     3906.9564: 0.0745685 0.653706 -0.267403 -0.510696 -0.726439 0.348417 0.470055 0.0336096 0.10
##  49:     3906.8430: 0.0750823 0.657441 -0.268595 -0.494878 -0.730883 0.351187 0.454092 0.0337726 0.09
##  50:     3906.7421: 0.0755047 0.667114 -0.276443 -0.493068 -0.739634 0.360072 0.451613 0.0331034 0.09
##  51:     3906.7103: 0.0759496 0.676113 -0.286898 -0.487360 -0.749009 0.371056 0.445716 0.0330363 0.09
##  52:     3906.6397: 0.0762694 0.684842 -0.298317 -0.478362 -0.758636 0.383006 0.436825 0.0322676 0.09
##  53:     3906.5924: 0.0765276 0.695687 -0.310687 -0.472092 -0.769435 0.395964 0.428984 0.0324490 0.09
##  54:     3906.5726: 0.0767910 0.705783 -0.323222 -0.463610 -0.779072 0.408340 0.420210 0.0324361 0.09
##  55:     3906.5257: 0.0769987 0.716098 -0.335947 -0.454886 -0.788852 0.420940 0.411303 0.0321209 0.09
##  56:     3906.5058: 0.0770150 0.724634 -0.346902 -0.446404 -0.798498 0.432132 0.402323 0.0319495 0.09
##  57:     3906.4326: 0.0769025 0.766207 -0.399625 -0.410640 -0.837503 0.483084 0.366567 0.0312509 0.09
##  58:     3906.4059: 0.0756049 0.803833 -0.447401 -0.375992 -0.876375 0.532767 0.329774 0.0313514 0.09
##  59:     3906.3968: 0.0733731 0.831083 -0.480798 -0.351521 -0.902595 0.565290 0.304903 0.0313191 0.09
##  60:     3906.3922: 0.0719273 0.865213 -0.527076 -0.321967 -0.936447 0.611178 0.274891 0.0315070 0.09
##  61:     3906.3613: 0.0706620 0.900233 -0.578201 -0.285356 -0.971897 0.662123 0.237276 0.0318532 0.09
##  62:     3906.3486: 0.0702361 0.910165 -0.591525 -0.276201 -0.981898 0.675464 0.227164 0.0317672 0.09
##  63:     3906.3467: 0.0699339 0.920553 -0.604480 -0.266202 -0.992378 0.688766 0.216740 0.0319003 0.09
##  64:     3906.3426: 0.0698817 0.921997 -0.606345 -0.264807 -0.993858 0.690695 0.215324 0.0318114 0.09
##  65:     3906.3345: 0.0697593 0.922646 -0.607809 -0.264266 -0.994572 0.692191 0.214781 0.0319177 0.09
##  66:     3906.3328: 0.0697025 0.924076 -0.609710 -0.262930 -0.996047 0.694088 0.213350 0.0318876 0.09
##  67:     3906.3254: 0.0696467 0.927812 -0.613724 -0.260464 -1.00000 0.698742 0.210266 0.0318866 0.090
##  68:     3906.3211: 0.0699177 0.927893 -0.611888 -0.262474 -1.00000 0.697336 0.211808 0.0316351 0.090
##  69:     3906.3204: 0.0698340 0.928429 -0.614078 -0.261678 -1.00000 0.698495 0.211618 0.0315310 0.090
##  70:     3906.3202: 0.0697861 0.928500 -0.613560 -0.261581 -1.00000 0.697940 0.211635 0.0316502 0.090
```

```
##  71:     3906.3201: 0.0698043 0.928479 -0.613748 -0.261610 -1.00000 0.698149 0.211630 0.0316017 0.090
##  72:     3906.3201: 0.0698046 0.928482 -0.613746 -0.261611 -1.00000 0.698157 0.211627 0.0316102 0.090
##  73:     3906.3201: 0.0698045 0.928483 -0.613744 -0.261611 -1.00000 0.698157 0.211627 0.0316038 0.090
##  74:     3906.3201: 0.0698040 0.928489 -0.613732 -0.261611 -1.00000 0.698155 0.211622 0.0316052 0.090
##  75:     3906.3201: 0.0698026 0.928490 -0.613717 -0.261608 -1.00000 0.698141 0.211622 0.0316054 0.090
##  76:     3906.3201: 0.0698013 0.928492 -0.613702 -0.261605 -1.00000 0.698127 0.211622 0.0316092 0.090
##  77:     3906.3201: 0.0697990 0.928494 -0.613677 -0.261600 -1.00000 0.698105 0.211623 0.0316122 0.090
##  78:     3906.3201: 0.0698013 0.928479 -0.613696 -0.261602 -1.00000 0.698132 0.211608 0.0316045 0.090
##  79:     3906.3201: 0.0698013 0.928479 -0.613695 -0.261602 -1.00000 0.698133 0.211608 0.0316055 0.090
##  80:     3906.3201: 0.0698013 0.928478 -0.613694 -0.261601 -1.00000 0.698133 0.211607 0.0316048 0.090
##  81:     3906.3201: 0.0698013 0.928478 -0.613694 -0.261601 -1.00000 0.698133 0.211607 0.0316054 0.090
##  82:     3906.3201: 0.0698013 0.928477 -0.613693 -0.261601 -1.00000 0.698133 0.211606 0.0316046 0.090
##  83:     3906.3201: 0.0698012 0.928476 -0.613691 -0.261601 -1.00000 0.698133 0.211605 0.0316052 0.090
##  84:     3906.3200: 0.0697958 0.928427 -0.613618 -0.261578 -1.00000 0.698112 0.211558 0.0316067 0.090
##  85:     3906.3200: 0.0698048 0.928390 -0.613639 -0.261591 -1.00000 0.698170 0.211537 0.0315971 0.090
##  86:     3906.3200: 0.0698052 0.928402 -0.613627 -0.261587 -1.00000 0.698147 0.211543 0.0315995 0.090
##  87:     3906.3200: 0.0698003 0.928404 -0.613626 -0.261588 -1.00000 0.698143 0.211546 0.0316004 0.090
##
## Final Estimate of the Negative LLH:
##  LLH:  -7611.753    norm LLH:  -2.522118
##           mu           ar1           ar2           ar3           ma1
##  1.535997e-03  9.284043e-01 -6.136257e-01 -2.615880e-01 -1.000000e+00
##           ma2           ma3         omega        alpha1        alpha2
##  6.981431e-01  2.115462e-01  1.530236e-05  9.631766e-02  1.000000e-08
##         beta1         beta2
##  4.160670e-01  4.525696e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1           ar2           ar3           ma1
## mu     -1.156710e+07   -14330.0323 -1.697685e+04   -19542.3627     6271.4320
## ar1    -1.433003e+04   -33616.6756 -2.091270e+04     7152.1817   -43500.2675
## ar2    -1.697685e+04   -20912.7007 -3.408816e+04   -20531.2975   -29571.2506
## ar3    -1.954236e+04     7152.1817 -2.053130e+04   -33802.2778     6854.5484
## ma1     6.271432e+03   -43500.2675 -2.957125e+04     6854.5484   -59950.9257
## ma2     3.236037e+03   -23474.0549 -4.230785e+04   -27889.5817   -34894.7589
## ma3    -5.781241e+02    13549.5021 -2.217810e+04   -42414.4319    16312.1999
## omega  -5.147407e+07 -5232121.9132 -5.819452e+06 -2666712.5258 -6772203.8092
## alpha1 -1.152755e+03     -247.8113  6.696483e+01      225.0257     -224.1225
## alpha2 -6.399300e+03     -591.1977 -1.822217e+02      201.8476     -511.8246
## beta1  -2.243634e+04    -1609.7603 -1.330259e+03     -470.3091    -1908.4995
## beta2  -2.322629e+04    -1688.9952 -1.400565e+03     -509.1349    -2013.2212
##                  ma2           ma3         omega        alpha1        alpha2
## mu      3.236037e+03     -578.1241 -5.147407e+07 -1.152755e+03 -6.399300e+03
## ar1    -2.347405e+04    13549.5021 -5.232122e+06 -2.478113e+02 -5.911977e+02
## ar2    -4.230785e+04   -22178.1007 -5.819452e+06  6.696483e+01 -1.822217e+02
## ar3    -2.788958e+04   -42414.4319 -2.666713e+06  2.250257e+02  2.018476e+02
## ma1    -3.489476e+04    16312.1999 -6.772204e+06 -2.241225e+02 -5.118246e+02
## ma2    -5.481339e+04   -32386.8869 -7.535161e+06  1.694494e+02  2.218629e+01
## ma3    -3.238689e+04   -57958.7196 -3.121039e+06  3.397435e+02  4.286370e+02
## omega  -7.535161e+06 -3121039.0904 -1.007653e+12 -2.022763e+08 -2.097591e+08
## alpha1  1.694494e+02      339.7435 -2.022763e+08 -7.195277e+04 -7.264855e+04
## alpha2  2.218629e+01      428.6370 -2.097591e+08 -7.264855e+04 -7.638229e+04
## beta1  -1.521714e+03     -357.1275 -2.919051e+08 -7.992812e+04 -8.334514e+04
## beta2  -1.612708e+03     -396.5811 -2.938448e+08 -8.022188e+04 -8.354448e+04
```

```
##                beta1         beta2
## mu     -2.243634e+04 -2.322629e+04
## ar1    -1.609760e+03 -1.688995e+03
## ar2    -1.330259e+03 -1.400565e+03
## ar3    -4.703091e+02 -5.091349e+02
## ma1    -1.908500e+03 -2.013221e+03
## ma2    -1.521714e+03 -1.612708e+03
## ma3    -3.571275e+02 -3.965811e+02
## omega  -2.919051e+08 -2.938448e+08
## alpha1 -7.992812e+04 -8.022188e+04
## alpha2 -8.334514e+04 -8.354448e+04
## beta1  -1.016340e+05 -1.022393e+05
## beta2  -1.022393e+05 -1.028608e+05
## attr(,"time")
## Time difference of 1.293721 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 8.564199 secs
```

```
summary(gfit1)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x7ffe04298950>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1
## 0.00137177  0.00033386  0.29957555
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.372e-03   3.522e-04    3.895 9.82e-05 ***
## omega  3.339e-04   1.118e-05   29.873  < 2e-16 ***
## alpha1 2.996e-01   3.144e-02    9.529  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
```

```
##  7430.496    normalized:  2.46206
##
## Description:
##  Sun Apr 17 22:15:19 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2964.63   0
##  Shapiro-Wilk Test  R    W      0.9455149 0
##  Ljung-Box Test     R    Q(10)  14.3517   0.1575454
##  Ljung-Box Test     R    Q(15)  19.70402  0.1835833
##  Ljung-Box Test     R    Q(20)  25.66359  0.1772121
##  Ljung-Box Test     R^2  Q(10)  100.8347  0
##  Ljung-Box Test     R^2  Q(15)  176.7088  0
##  Ljung-Box Test     R^2  Q(20)  203.5581  0
##  LM Arch Test       R    TR^2   101.3707  3.330669e-16
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.922131 -4.916155 -4.922133 -4.919982
```

```
summary(gfit2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7ffe06008c50>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 1.6229e-03  1.3956e-05  8.3249e-02  8.8610e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.623e-03   3.230e-04    5.024 5.06e-07 ***
## omega  1.396e-05   2.933e-06    4.758 1.95e-06 ***
## alpha1 8.325e-02   1.274e-02    6.534 6.41e-11 ***
## beta1  8.861e-01   1.698e-02   52.197  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Log Likelihood:
##  7599.491    normalized:  2.518055
##
## Description:
##  Sun Apr 17 22:15:19 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2413.331  0
##  Shapiro-Wilk Test  R    W      0.9556894 0
##  Ljung-Box Test     R    Q(10)  23.61874  0.00867931
##  Ljung-Box Test     R    Q(15)  24.89168  0.0514172
##  Ljung-Box Test     R    Q(20)  26.83161  0.1400683
##  Ljung-Box Test     R^2  Q(10)  6.134568  0.8038335
##  Ljung-Box Test     R^2  Q(15)  7.945045  0.925967
##  Ljung-Box Test     R^2  Q(20)  12.15692  0.910569
##  LM Arch Test       R    TR^2   6.950795  0.860848
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.033460 -5.025491 -5.033464 -5.030595
```

```
summary(gfit3)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 0), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(2, 0)
## <environment: 0x7ffe032ec8d8>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2
## 0.00139782  0.00027305  0.19071310  0.26332337
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0013978   0.0003393    4.120 3.79e-05 ***
## omega  0.0002730   0.0000116   23.545  < 2e-16 ***
## alpha1 0.1907131   0.0290607    6.563 5.29e-11 ***
## alpha2 0.2633234   0.0391462    6.727 1.74e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Log Likelihood:
##  7476.049    normalized:  2.477153
##
## Description:
##  Sun Apr 17 22:15:19 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2426.505  0
##  Shapiro-Wilk Test  R    W      0.9500436 0
##  Ljung-Box Test     R    Q(10)  11.04426  0.35408
##  Ljung-Box Test     R    Q(15)  13.80287  0.5405297
##  Ljung-Box Test     R    Q(20)  15.79841  0.7290494
##  Ljung-Box Test     R^2  Q(10)  54.92043  3.267996e-08
##  Ljung-Box Test     R^2  Q(15)  104.9732  1.44329e-15
##  Ljung-Box Test     R^2  Q(20)  117.5986  7.771561e-16
##  LM Arch Test       R    TR^2   61.75385  1.079099e-08
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.951656 -4.943687 -4.951660 -4.948791
```

```
summary(gfit4)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 1), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(2, 1)
## <environment: 0x7ffe06bdf660>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2       beta1
## 1.6302e-03  1.3756e-05  8.1812e-02  1.0000e-08  8.8778e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.630e-03   3.236e-04    5.038 4.71e-07 ***
## omega  1.376e-05   3.825e-06    3.596 0.000323 ***
## alpha1 8.181e-02   1.773e-02    4.613 3.96e-06 ***
## alpha2 1.000e-08   2.598e-02    0.000 1.000000
## beta1  8.878e-01   2.452e-02   36.204  < 2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7598.637    normalized:  2.517772
##
## Description:
##  Sun Apr 17 22:15:19 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2433.143  0
##  Shapiro-Wilk Test  R    W      0.955488  0
##  Ljung-Box Test     R    Q(10)  23.79947  0.008150813
##  Ljung-Box Test     R    Q(15)  25.08415  0.04882468
##  Ljung-Box Test     R    Q(20)  27.01846  0.1347453
##  Ljung-Box Test     R^2  Q(10)  6.071797  0.8091955
##  Ljung-Box Test     R^2  Q(15)  7.859352  0.9292975
##  Ljung-Box Test     R^2  Q(20)  11.9935   0.9162996
##  LM Arch Test       R    TR^2   6.943438  0.8613287
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.032231 -5.022270 -5.032237 -5.028649
```

```
summary(gfit5)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 2), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 2)
## <environment: 0x7ffde8118fd0>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       beta2
## 0.00161764  0.00001671  0.10210624  0.43389889  0.42624283
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.618e-03   3.236e-04    4.999 5.77e-07 ***
## omega  1.671e-05   3.827e-06    4.367 1.26e-05 ***
## alpha1 1.021e-01   1.809e-02    5.644 1.66e-08 ***
```

```
## beta1  4.339e-01   3.005e-01    1.444    0.149
## beta2  4.262e-01   2.874e-01    1.483    0.138
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7599.97    normalized:  2.518214
##
## Description:
##  Sun Apr 17 22:15:20 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2458.816  0
##  Shapiro-Wilk Test  R    W      0.9552953 0
##  Ljung-Box Test     R    Q(10)  23.68236  0.008489648
##  Ljung-Box Test     R    Q(15)  25.06931  0.04902035
##  Ljung-Box Test     R    Q(20)  27.31732  0.1265701
##  Ljung-Box Test     R^2  Q(10)  4.371237  0.9290499
##  Ljung-Box Test     R^2  Q(15)  6.153824  0.9770464
##  Ljung-Box Test     R^2  Q(20)  10.25717  0.9632655
##  LM Arch Test       R    TR^2   5.051767  0.9562275
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.033115 -5.023154 -5.033120 -5.029533
```

```
summary(gfit6)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 2), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(2, 2)
## <environment: 0x7ffdea4ce068>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2       beta1       beta2
## 1.5761e-03  2.2725e-05  9.0373e-02  4.8039e-02  1.0000e-08  8.1062e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.576e-03   3.193e-04    4.936 7.99e-07 ***
```

```
## omega  2.272e-05   4.989e-06    4.555 5.24e-06 ***
## alpha1 9.037e-02   1.225e-02    7.375 1.65e-13 ***
## alpha2 4.804e-02         NaN      NaN      NaN
## beta1  1.000e-08         NaN      NaN      NaN
## beta2  8.106e-01         NaN      NaN      NaN
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7600.549    normalized:  2.518406
##
## Description:
##  Sun Apr 17 22:15:20 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2319.783  0
##  Shapiro-Wilk Test  R    W      0.9563619 0
##  Ljung-Box Test     R    Q(10)  24.05746  0.007449338
##  Ljung-Box Test     R    Q(15)  25.38136  0.04505045
##  Ljung-Box Test     R    Q(20)  27.57852  0.1197582
##  Ljung-Box Test     R^2  Q(10)  5.199817  0.8774364
##  Ljung-Box Test     R^2  Q(15)  6.64619   0.9667347
##  Ljung-Box Test     R^2  Q(20)  10.82893  0.9505388
##  LM Arch Test       R    TR^2   6.190534  0.9061741
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.032835 -5.020882 -5.032843 -5.028537
```

```
summary(gfit7)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(3, 3) + garch(1, 0), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ arma(3, 3) + garch(1, 0)
## <environment: 0x7ffe0429da60>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ar2          ar3          ma1          ma2
##  0.00130202   0.94079337  -0.63195464  -0.23804144  -0.99999999   0.69237877
##         ma3        omega       alpha1
##  0.20988048   0.00033247   0.29719071
##
## Std. Errors:
```

```
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.302e-03   2.838e-04    4.588 4.48e-06 ***
## ar1     9.408e-01         NaN      NaN      NaN
## ar2    -6.320e-01         NaN      NaN      NaN
## ar3    -2.380e-01         NaN      NaN      NaN
## ma1    -1.000e+00         NaN      NaN      NaN
## ma2     6.924e-01         NaN      NaN      NaN
## ma3     2.099e-01         NaN      NaN      NaN
## omega   3.325e-04   1.105e-05   30.097  < 2e-16 ***
## alpha1  2.972e-01   3.136e-02    9.476  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7441.683    normalized:  2.465766
##
## Description:
##  Sun Apr 17 22:15:22 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  3024.769  0
##  Shapiro-Wilk Test  R    W      0.9448529 0
##  Ljung-Box Test     R    Q(10)  5.909785  0.8227849
##  Ljung-Box Test     R    Q(15)  11.40206  0.7236071
##  Ljung-Box Test     R    Q(20)  16.3524   0.6945352
##  Ljung-Box Test     R^2  Q(10)  97.80357  1.110223e-16
##  Ljung-Box Test     R^2  Q(15)  174.9526  0
##  Ljung-Box Test     R^2  Q(20)  200.138   0
##  LM Arch Test       R    TR^2   101.3389  3.330669e-16
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.925568 -4.907639 -4.925586 -4.919121
```

```
summary(gfit8)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(3, 3) + garch(1, 1), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ arma(3, 3) + garch(1, 1)
## <environment: 0x7ffdea466d50>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
```

```
##
## Coefficient(s):
##          mu          ar1          ar2          ar3          ma1          ma2
##  1.7556e-03   7.1768e-01  -3.4302e-01  -4.4949e-01  -7.8646e-01   4.2133e-01
##         ma3        omega       alpha1        beta1
##  4.1085e-01   1.2404e-05   7.5478e-02   8.9673e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.756e-03   4.366e-04    4.021 5.80e-05 ***
## ar1     7.177e-01   2.590e-01    2.771  0.00559 **
## ar2    -3.430e-01   3.139e-01   -1.093  0.27443
## ar3    -4.495e-01   2.205e-01   -2.038  0.04154 *
## ma1    -7.865e-01   2.644e-01   -2.974  0.00294 **
## ma2     4.213e-01   3.276e-01    1.286  0.19845
## ma3     4.109e-01   2.374e-01    1.731  0.08354 .
## omega   1.240e-05   2.706e-06    4.584 4.57e-06 ***
## alpha1  7.548e-02   1.189e-02    6.347 2.20e-10 ***
## beta1   8.967e-01   1.596e-02   56.196  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7610.049    normalized:  2.521554
##
## Description:
##  Sun Apr 17 22:15:23 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2597.793  0
##  Shapiro-Wilk Test  R    W      0.9539386 0
##  Ljung-Box Test     R    Q(10)  4.083457  0.9435035
##  Ljung-Box Test     R    Q(15)  7.6856    0.935768
##  Ljung-Box Test     R    Q(20)  9.502234  0.9763265
##  Ljung-Box Test     R^2  Q(10)  4.963435  0.8936072
##  Ljung-Box Test     R^2  Q(15)  6.815394  0.9625713
##  Ljung-Box Test     R^2  Q(20)  10.80479  0.9511285
##  LM Arch Test       R    TR^2   5.861159  0.9229184
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.036480 -5.016559 -5.036502 -5.029317
```

```
summary(gfit9)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
```

```
##  garchFit(formula = ~arma(3, 3) + garch(2, 0), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ arma(3, 3) + garch(2, 0)
## <environment: 0x7ffdebbd5868>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ar2          ar3          ma1          ma2
##  0.00152586   0.64807124  -0.21642532  -0.54878171  -0.68958849   0.26146190
##         ma3        omega       alpha1       alpha2
##  0.53890646   0.00026683   0.18651302   0.28409153
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.526e-03   3.999e-04    3.815 0.000136 ***
## ar1     6.481e-01   1.458e-01    4.444 8.83e-06 ***
## ar2    -2.164e-01   1.782e-01   -1.215 0.224465
## ar3    -5.488e-01   1.293e-01   -4.243 2.20e-05 ***
## ma1    -6.896e-01   1.572e-01   -4.388 1.15e-05 ***
## ma2     2.615e-01   1.972e-01    1.326 0.184796
## ma3     5.389e-01   1.467e-01    3.674 0.000239 ***
## omega   2.668e-04   1.149e-05   23.233  < 2e-16 ***
## alpha1  1.865e-01   2.863e-02    6.515 7.28e-11 ***
## alpha2  2.841e-01   4.029e-02    7.051 1.78e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7490.434    normalized:  2.48192
##
## Description:
##  Sun Apr 17 22:15:25 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2374.294  0
##  Shapiro-Wilk Test  R    W      0.9503444 0
##  Ljung-Box Test     R    Q(10)  1.208155  0.9995933
##  Ljung-Box Test     R    Q(15)  6.149589  0.9771239
##  Ljung-Box Test     R    Q(20)  7.381926  0.9952312
##  Ljung-Box Test     R^2  Q(10)  51.66705  1.315019e-07
##  Ljung-Box Test     R^2  Q(15)  99.91508  1.354472e-14
##  Ljung-Box Test     R^2  Q(20)  112.4027  7.105427e-15
##  LM Arch Test       R    TR^2   59.17867  3.185104e-08
##
## Information Criterion Statistics:
```

```
##       AIC       BIC       SIC      HQIC
## -4.957213 -4.937291 -4.957235 -4.950049
```

```
summary(gfit10)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(3, 3) + garch(2, 1), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ arma(3, 3) + garch(2, 1)
## <environment: 0x7ffdec50a528>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ar2          ar3          ma1          ma2
##  1.7343e-03   7.3734e-01  -3.6678e-01  -4.3268e-01  -8.0626e-01   4.4585e-01
##         ma3        omega       alpha1       alpha2        beta1
##  3.9300e-01   1.2511e-05   7.6132e-02   1.0000e-08   8.9585e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      1.734e-03   4.531e-04    3.828 0.000129 ***
## ar1     7.373e-01   2.856e-01    2.582 0.009821 **
## ar2    -3.668e-01   3.505e-01   -1.046 0.295341
## ar3    -4.327e-01   2.460e-01   -1.759 0.078603 .
## ma1    -8.063e-01   2.912e-01   -2.769 0.005629 **
## ma2     4.459e-01   3.644e-01    1.224 0.221077
## ma3     3.930e-01   2.639e-01    1.489 0.136385
## omega   1.251e-05   3.493e-06    3.581 0.000342 ***
## alpha1  7.613e-02   1.816e-02    4.193 2.75e-05 ***
## alpha2  1.000e-08   2.570e-02    0.000 1.000000
## beta1   8.958e-01   2.264e-02   39.577  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7610.655    normalized:  2.521754
##
## Description:
##  Sun Apr 17 22:15:26 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2582.423  0
```

```
##  Shapiro-Wilk Test  R    W      0.9540882 0
##  Ljung-Box Test     R    Q(10)  4.233411  0.9362037
##  Ljung-Box Test     R    Q(15)  7.79244   0.9318341
##  Ljung-Box Test     R    Q(20)  9.60433   0.9747924
##  Ljung-Box Test     R^2  Q(10)  5.031755  0.8890467
##  Ljung-Box Test     R^2  Q(15)  6.879477  0.9609082
##  Ljung-Box Test     R^2  Q(20)  10.90898  0.9485489
##  LM Arch Test       R    TR^2   5.885438  0.9217465
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.036219 -5.014305 -5.036246 -5.028339
```

```
summary(gfit11)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(3, 3) + garch(1, 2), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ arma(3, 3) + garch(1, 2)
## <environment: 0x7ffdec0fe458>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ar2          ar3          ma1          ma2
##  1.5358e-03   9.2837e-01  -6.1347e-01  -2.6168e-01  -1.0000e+00   6.9805e-01
##         ma3        omega       alpha1        beta1        beta2
##  2.1160e-01   1.5294e-05   9.6277e-02   4.1631e-01   4.5239e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.536e-03   2.290e-04    6.706 2.01e-11 ***
## ar1     9.284e-01         NaN      NaN      NaN
## ar2    -6.135e-01         NaN      NaN      NaN
## ar3    -2.617e-01         NaN      NaN      NaN
## ma1    -1.000e+00         NaN      NaN      NaN
## ma2     6.981e-01         NaN      NaN      NaN
## ma3     2.116e-01         NaN      NaN      NaN
## omega   1.529e-05   3.692e-06    4.143 3.43e-05 ***
## alpha1  9.628e-02   1.874e-02    5.139 2.77e-07 ***
## beta1   4.163e-01   3.871e-01    1.075    0.282
## beta2   4.524e-01   3.712e-01    1.219    0.223
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Log Likelihood:
##  7611.753    normalized:  2.522118
##
## Description:
##  Sun Apr 17 22:15:30 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2597.582  0
##  Shapiro-Wilk Test  R    W      0.9540093 0
##  Ljung-Box Test     R    Q(10)  6.50768   0.770961
##  Ljung-Box Test     R    Q(15)  9.835726  0.8299439
##  Ljung-Box Test     R    Q(20)  11.88103  0.9201099
##  Ljung-Box Test     R^2  Q(10)  3.536478  0.9658452
##  Ljung-Box Test     R^2  Q(15)  5.27005   0.9895825
##  Ljung-Box Test     R^2  Q(20)  9.170993  0.9808408
##  LM Arch Test       R    TR^2   4.181233  0.9799395
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.036947 -5.015033 -5.036974 -5.029067
```

```
summary(gfit12)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(3, 3) + garch(2, 2), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ arma(3, 3) + garch(2, 2)
## <environment: 0x7ffe03312718>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ar2          ar3          ma1          ma2
##  1.5360e-03   9.2840e-01  -6.1363e-01  -2.6159e-01  -1.0000e+00   6.9814e-01
##         ma3        omega       alpha1       alpha2        beta1        beta2
##  2.1155e-01   1.5302e-05   9.6318e-02   1.0000e-08   4.1607e-01   4.5257e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.536e-03   2.286e-04    6.718 1.84e-11 ***
## ar1     9.284e-01         NaN      NaN      NaN
## ar2    -6.136e-01         NaN      NaN      NaN
## ar3    -2.616e-01         NaN      NaN      NaN
```

```
## ma1    -1.000e+00         NaN      NaN      NaN
## ma2     6.981e-01         NaN      NaN      NaN
## ma3     2.115e-01         NaN      NaN      NaN
## omega   1.530e-05   1.154e-06   13.264  < 2e-16 ***
## alpha1  9.632e-02   1.788e-02    5.388 7.12e-08 ***
## alpha2  1.000e-08         NaN      NaN      NaN
## beta1   4.161e-01         NaN      NaN      NaN
## beta2   4.526e-01         NaN      NaN      NaN
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7611.753    normalized:  2.522118
##
## Description:
##  Sun Apr 17 22:15:38 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2597.437  0
##  Shapiro-Wilk Test  R    W      0.9540103 0
##  Ljung-Box Test     R    Q(10)  6.511249  0.770639
##  Ljung-Box Test     R    Q(15)  9.83811   0.8297977
##  Ljung-Box Test     R    Q(20)  11.88293  0.9200464
##  Ljung-Box Test     R^2  Q(10)  3.536604  0.9658409
##  Ljung-Box Test     R^2  Q(15)  5.270639  0.9895764
##  Ljung-Box Test     R^2  Q(20)  9.171513  0.9808342
##  LM Arch Test       R    TR^2   4.181312  0.9799378
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.036285 -5.012379 -5.036316 -5.027688
```

According to the AIC(no big difference) and the significance, we find that generally we should have a ARMA model as mean, here the best model is GARCH(1,1)

```
gfit = garchFit(~garch(1,1),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
```

```
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega    0.00000100 100.0000000 0.10000000     TRUE
##     alpha1   0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1    0.00000001   1.0000000 0.80000000     TRUE
##     delta    0.00000000   2.0000000 2.00000000    FALSE
##     skew     0.10000000  10.0000000 1.00000000    FALSE
##     shape    1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1
##      1      2      3      5
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3956.9207: 0.0422032 0.100000 0.100000 0.800000
##   1:     3951.7454: 0.0422071 0.0774769 0.0988866 0.788793
##   2:     3934.4556: 0.0422156 0.0757252 0.121400 0.799936
##   3:     3931.8868: 0.0422329 0.0533463 0.131919 0.804679
##   4:     3928.4349: 0.0422803 0.0503244 0.132421 0.829649
##   5:     3924.0138: 0.0423708 0.0387574 0.117234 0.845930
##   6:     3921.8326: 0.0424971 0.0368799 0.0993199 0.863272
##   7:     3921.8056: 0.0424982 0.0364912 0.0991010 0.863258
##   8:     3921.7765: 0.0425019 0.0364931 0.0989519 0.864145
##   9:     3921.6685: 0.0425120 0.0355007 0.0980850 0.865360
##  10:     3921.3766: 0.0425691 0.0331671 0.0951336 0.871955
##  11:     3921.2451: 0.0428320 0.0316229 0.0916603 0.874450
##  12:     3920.9999: 0.0431374 0.0323339 0.0900277 0.876443
##  13:     3920.3794: 0.0467049 0.0291091 0.0801406 0.887425
##  14:     3920.0006: 0.0503196 0.0264704 0.0800513 0.893026
##  15:     3919.6174: 0.0539395 0.0269423 0.0814479 0.889132
##  16:     3919.2279: 0.0575542 0.0297721 0.0844164 0.884473
##  17:     3918.9919: 0.0611769 0.0298267 0.0857209 0.882454
##  18:     3918.8595: 0.0647962 0.0279662 0.0850417 0.886469
##  19:     3918.6968: 0.0684184 0.0292835 0.0857577 0.884142
##  20:     3918.5939: 0.0720387 0.0284228 0.0828569 0.886756
##  21:     3918.5827: 0.0740275 0.0288860 0.0833149 0.886017
##  22:     3918.5822: 0.0737546 0.0288236 0.0832628 0.886087
##  23:     3918.5822: 0.0737460 0.0288183 0.0832446 0.886109
##  24:     3918.5822: 0.0737480 0.0288192 0.0832491 0.886104
##
## Final Estimate of the Negative LLH:
```

```
##  LLH:  -7599.491    norm LLH:  -2.518055
##           mu        omega       alpha1        beta1
## 1.622869e-03 1.395559e-05 8.324908e-02 8.861041e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1         beta1
## mu      -9627339.967 -5.917409e+07 -3.096645e+03     -23001.54
## omega  -59174092.978 -1.332843e+12 -2.589564e+08 -386654965.12
## alpha1     -3096.645 -2.589564e+08 -9.236301e+04    -104269.78
## beta1     -23001.542 -3.866550e+08 -1.042698e+05    -135854.84
## attr(,"time")
## Time difference of 0.08687186 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.608 secs
```

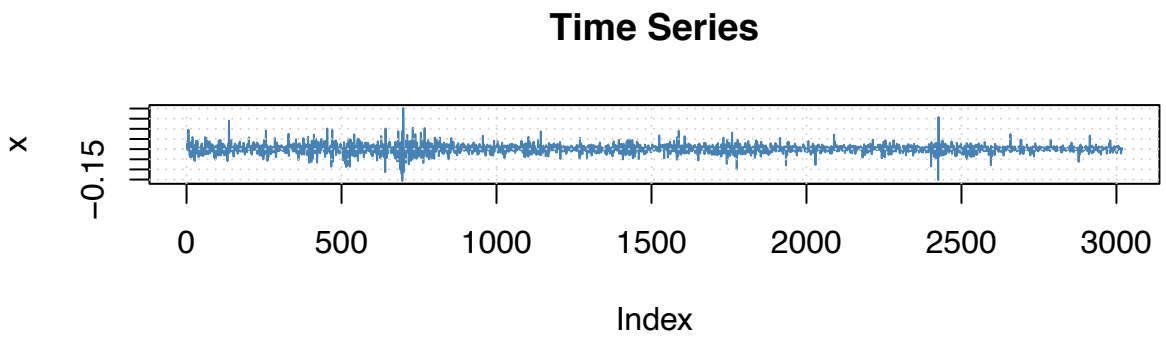
```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

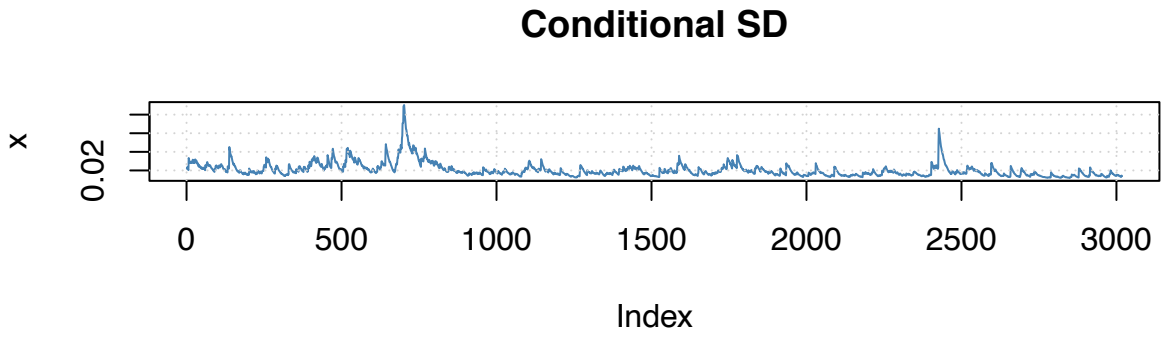
```
summary(gfit)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7ffe04ac1fb0>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 1.6229e-03  1.3956e-05  8.3249e-02  8.8610e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.623e-03   3.230e-04    5.024 5.06e-07 ***
## omega  1.396e-05   2.933e-06    4.758 1.95e-06 ***
## alpha1 8.325e-02   1.274e-02    6.534 6.41e-11 ***
## beta1  8.861e-01   1.698e-02   52.197  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Log Likelihood:
##  7599.491    normalized:  2.518055
##
## Description:
##  Sun Apr 17 22:15:40 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2413.331  0
##  Shapiro-Wilk Test  R    W      0.9556894 0
##  Ljung-Box Test     R    Q(10)  23.61874  0.00867931
##  Ljung-Box Test     R    Q(15)  24.89168  0.0514172
##  Ljung-Box Test     R    Q(20)  26.83161  0.1400683
##  Ljung-Box Test     R^2  Q(10)  6.134568  0.8038335
##  Ljung-Box Test     R^2  Q(15)  7.945045  0.925967
##  Ljung-Box Test     R^2  Q(20)  12.15692  0.910569
##  LM Arch Test       R    TR^2   6.950795  0.860848
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.033460 -5.025491 -5.033464 -5.030595
```

```
par(mfrow=c(2,1))
plot(gfit, which = 1)
plot(gfit, which = 2)
```

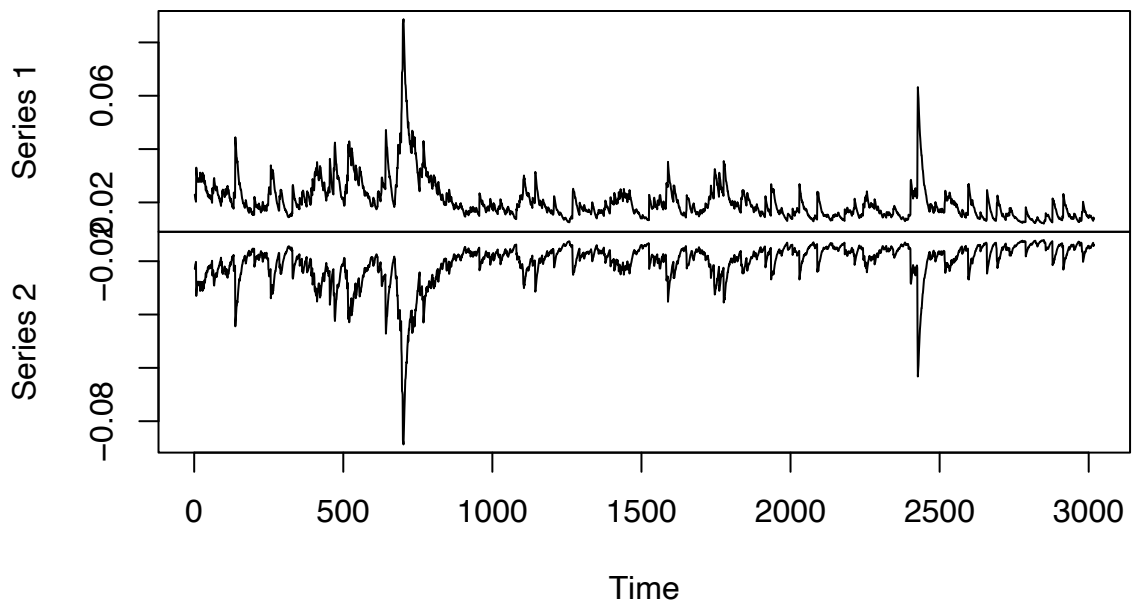




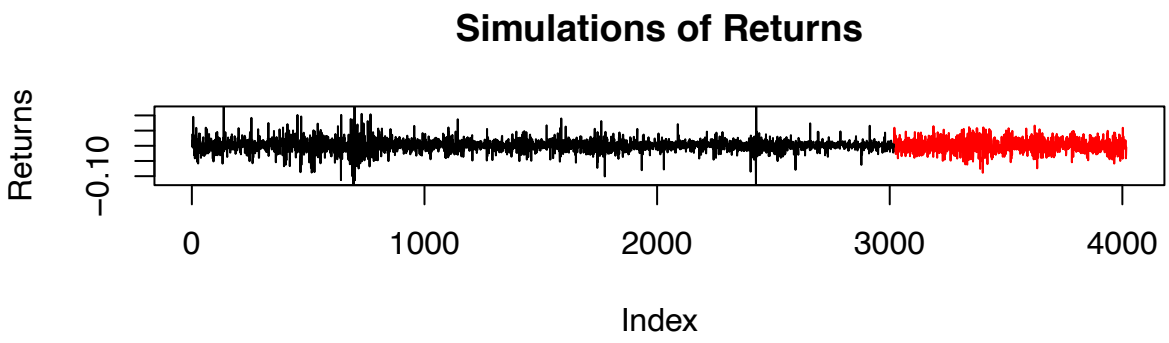

```
gfit = garch(diff(log(apple_open)),order = c(1,1))
```

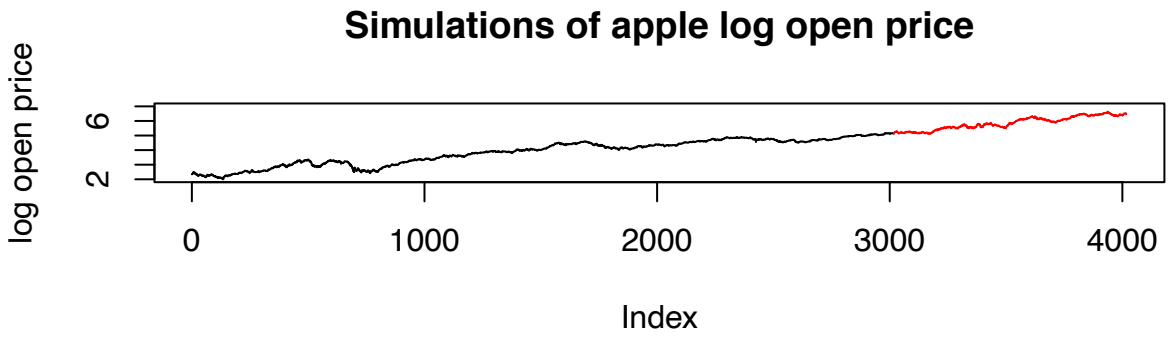
```
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
```

```
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     4.358212e-04     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1 -1.012e+04
##      1    7 -1.012e+04  8.37e-04  1.41e-03  4.3e-04  7.8e+09  4.3e-05  5.53e+06
##      2    8 -1.012e+04  1.52e-05  1.85e-05  4.2e-04  2.0e+00  4.3e-05  7.04e+01
##      3   14 -1.016e+04  3.64e-03  5.11e-03  3.0e-01  2.0e+00  4.4e-02  6.97e+01
##      4   20 -1.016e+04  1.94e-05  4.18e-05  3.8e-05  9.2e+00  7.2e-06  8.16e-01
##      5   21 -1.016e+04  3.70e-07  3.65e-07  3.8e-05  2.0e+00  7.2e-06  8.31e-01
##      6   30 -1.020e+04  3.36e-03  2.99e-03  6.9e-01  2.0e+00  2.4e-01  8.29e-01
##      7   32 -1.026e+04  6.25e-03  7.84e-03  4.5e-01  2.0e+00  4.7e-01  1.58e+01
##      8   42 -1.029e+04  2.49e-03  2.92e-02  2.9e-05  3.0e+00  4.4e-05  7.62e-02
##      9   44 -1.032e+04  2.93e-03  1.70e-03  1.2e-05  2.0e+00  2.2e-05  2.93e-02
##     10   45 -1.032e+04  4.07e-04  2.51e-03  1.4e-05  2.1e+00  2.2e-05  8.43e-01
##     11   46 -1.033e+04  9.82e-04  1.67e-03  1.2e-05  2.0e+00  2.2e-05  9.42e-02
##     12   47 -1.033e+04  1.13e-04  8.71e-05  1.4e-05  2.0e+00  2.2e-05  1.79e-01
##     13   48 -1.033e+04  8.98e-06  9.18e-06  1.4e-05  2.0e+00  2.2e-05  2.21e-01
##     14   55 -1.035e+04  1.70e-03  3.38e-03  5.5e-02  2.0e+00  8.9e-02  2.17e-01
##     15   66 -1.035e+04  2.34e-04  1.01e-03  3.2e-06  2.6e+00  5.4e-06  1.34e-03
##     16   67 -1.035e+04  1.62e-04  1.31e-04  2.8e-06  2.0e+00  5.4e-06  3.88e-04
##     17   68 -1.035e+04  6.08e-06  6.49e-06  2.8e-06  2.0e+00  5.4e-06  1.92e-04
##     18   69 -1.035e+04  2.09e-07  2.58e-07  2.7e-06  2.0e+00  5.4e-06  1.90e-04
##     19   75 -1.035e+04  1.15e-04  1.28e-04  2.7e-03  7.1e-01  5.5e-03  1.90e-04
##     20   78 -1.036e+04  2.56e-04  1.88e-04  1.4e-02  0.0e+00  3.3e-02  1.88e-04
##     21   79 -1.036e+04  9.25e-05  8.23e-05  1.4e-02  0.0e+00  2.9e-02  8.23e-05
##     22   80 -1.036e+04  3.18e-06  2.04e-05  1.3e-03  0.0e+00  3.1e-03  2.04e-05
##     23   81 -1.036e+04  8.09e-06  8.90e-06  1.1e-03  0.0e+00  2.0e-03  8.90e-06
##     24   82 -1.036e+04  1.33e-07  1.49e-07  3.4e-04  0.0e+00  8.1e-04  1.49e-07
##     25   83 -1.036e+04  5.91e-08  2.05e-08  4.7e-05  0.0e+00  8.4e-05  2.05e-08
##     26   96 -1.036e+04 -4.21e-15  4.10e-15  6.5e-15  2.2e+05  1.2e-14  1.52e-09
##
##  ***** FALSE CONVERGENCE *****
##
##  FUNCTION    -1.035796e+04   RELDX        6.540e-15
##  FUNC. EVALS      96         GRAD. EVALS      26
##  PRELDF       4.102e-15      NPRELDF      1.521e-09
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    1.327962e-05     1.000e+00    -3.640e+03
##      2    7.830574e-02     1.000e+00    -5.732e-01
##      3    8.923816e-01     1.000e+00    -7.191e-01
plot(predict(gfit))
```

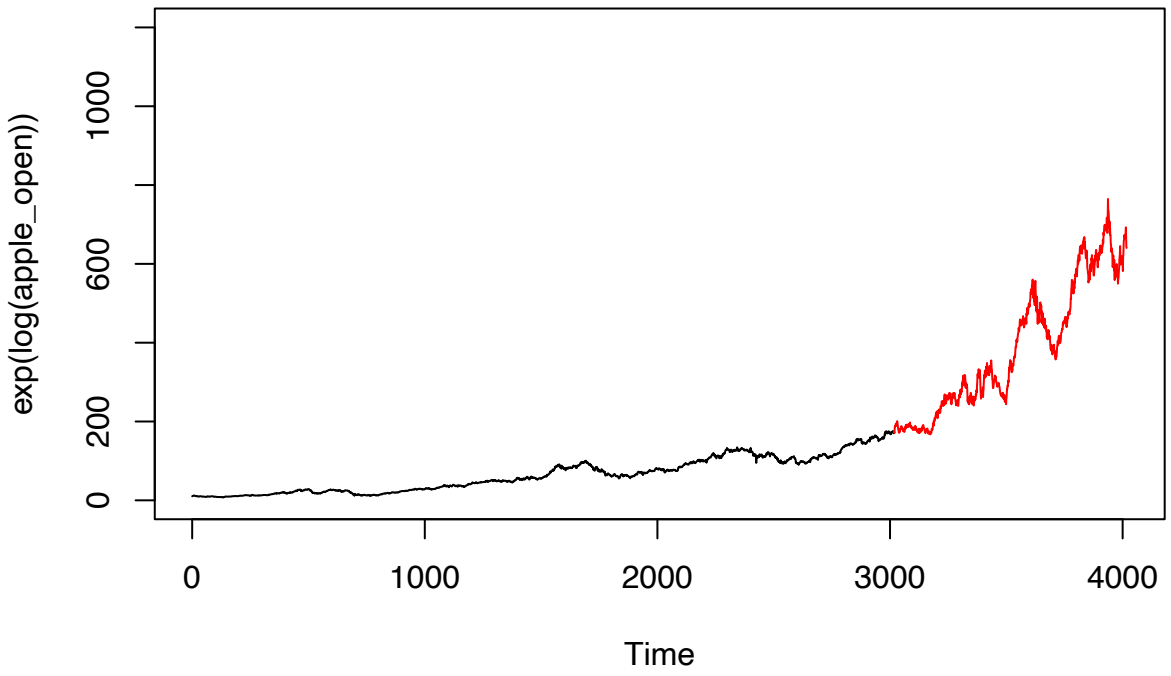



```
library(rugarch)
par(mfrow=c(2,1))
default_spec <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1,1)))
default_garch <- ugarchfit(spec = default_spec, data = diff(log(apple_open)))
set.seed(116)
sim = ugarchsim(default_garch,n.sim=1000)
sim_rst = sim@simulation$seriesSim[,1]
matplot(y = sim@simulation$seriesSim[,1],x=3018:4017, type = "l",
       main = "Simulations of Returns",col = "red",xlim=c(0,4020),ylim=c(-0.12,0.12),xlab="Index",ylab="
lines(diff(log(apple_open)))


sim = rep(0,1000)
first = log(apple_open)[3018]
for(i in 1:1000){
  if(i==1){
    sim[i]=first+sim_rst[1]
  }
  else{
  sim[i]=sim[i-1]+sim_rst[i]}
}

plot(log(apple_open),xlim = c(0,4020),ylim = c(2,7),main = "Simulations of apple log open price",xlab="I
lines(x=3018:4017,y=sim,col="red",type="l")
```

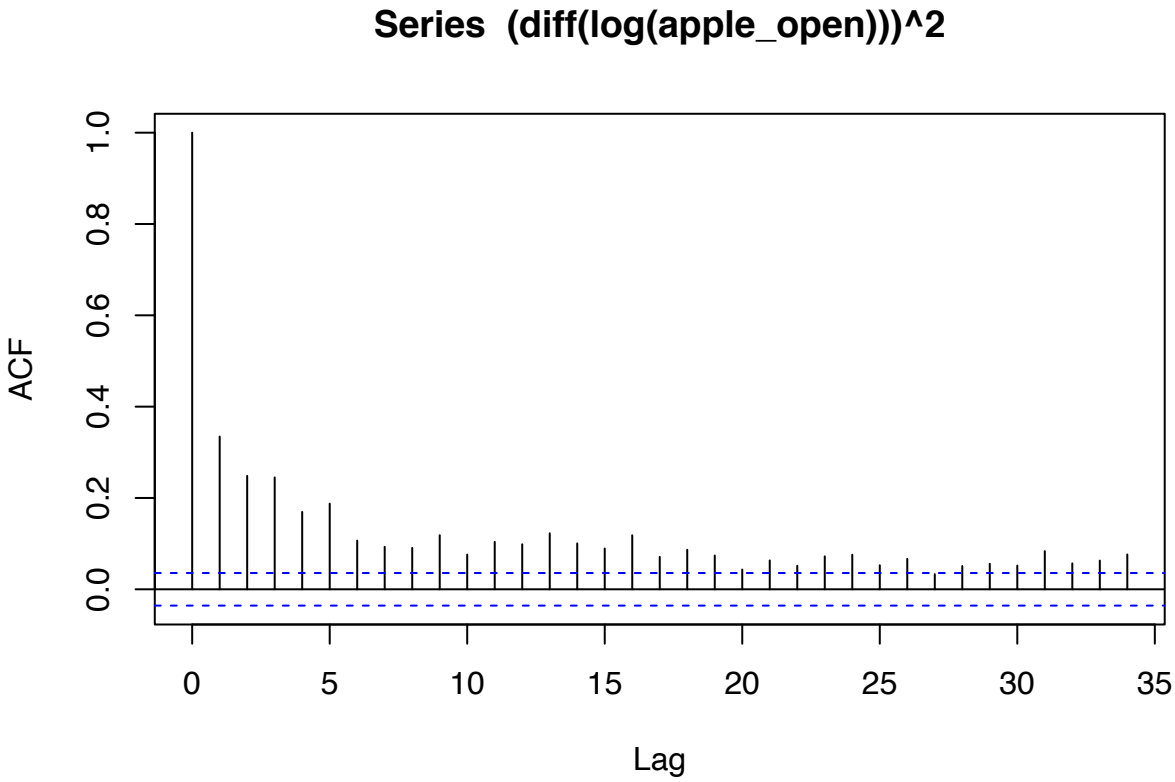




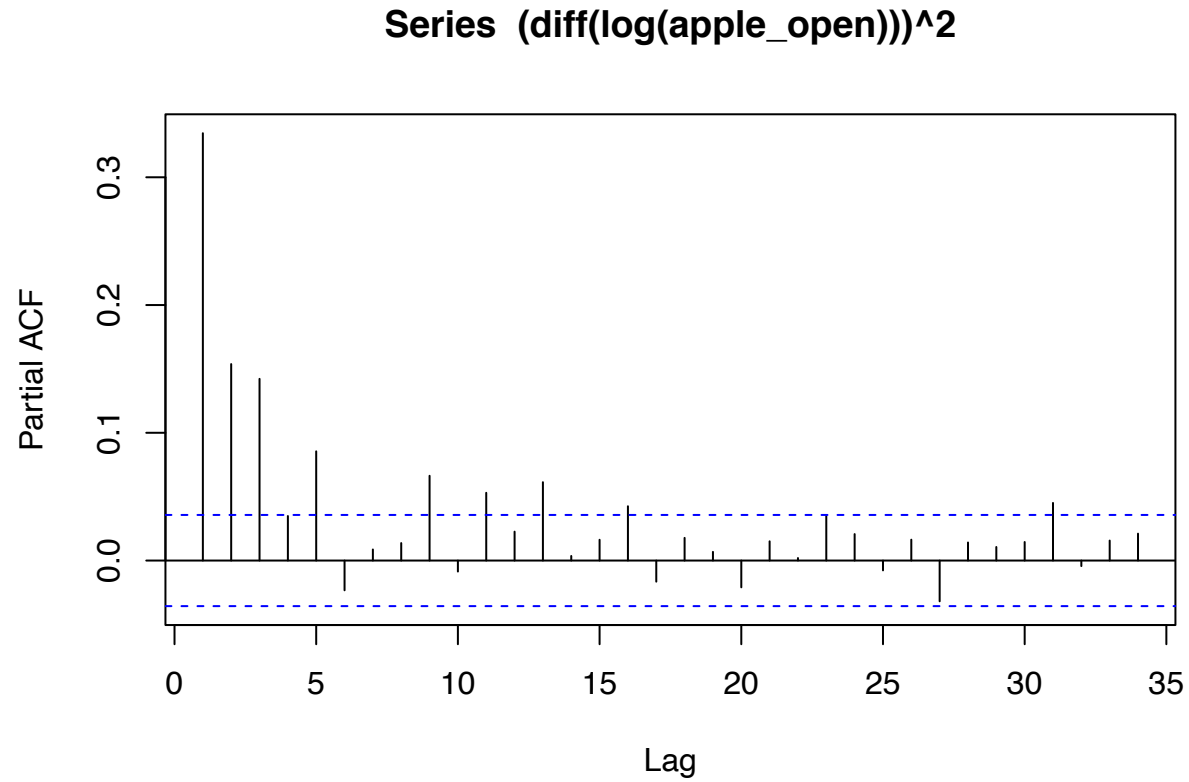

```
plot(exp(log(apple_open)),xlim = c(0,4020),ylim=c(0,1200))
lines(x=3018:4017,y=exp(sim),col="red",type="l")
```




```
acf((diff(log(apple_open)))^2)
```




```
pacf((diff(log(apple_open)))^2)
```




```
model = auto.arima(diff(log(apple_open))^2, max.p = 3, max.q = 3,max.order = 10, stationary = T, seasona
```

```
##
```

```
##  ARIMA(0,0,0) with zero mean     : -30186.96
##  ARIMA(0,0,0) with non-zero mean : -30465.39
##  ARIMA(0,0,1) with zero mean     : -30548.83
##  ARIMA(0,0,1) with non-zero mean : -30735.09
##  ARIMA(0,0,2) with zero mean     : -30648.91
##  ARIMA(0,0,2) with non-zero mean : -30801.59
##  ARIMA(0,0,3) with zero mean     : -30758.6
##  ARIMA(0,0,3) with non-zero mean : -30883.06
##  ARIMA(1,0,0) with zero mean     : -30692.65
##  ARIMA(1,0,0) with non-zero mean : -30821.45
##  ARIMA(1,0,1) with zero mean     : -30919.87
##  ARIMA(1,0,1) with non-zero mean : -30961.44
##  ARIMA(1,0,2) with zero mean     : -30932.75
##  ARIMA(1,0,2) with non-zero mean : -30963.94
##  ARIMA(1,0,3) with zero mean     : -30933.66
##  ARIMA(1,0,3) with non-zero mean : -30962.73
##  ARIMA(2,0,0) with zero mean     : -30801.14
##  ARIMA(2,0,0) with non-zero mean : -30891.64
##  ARIMA(2,0,1) with zero mean     : -30936.09
##  ARIMA(2,0,1) with non-zero mean : -30963.79
##  ARIMA(2,0,2) with zero mean     : -30944.21
##  ARIMA(2,0,2) with non-zero mean : -30981.97
##  ARIMA(2,0,3) with zero mean     : -30944.34
##  ARIMA(2,0,3) with non-zero mean : -30980.11
##  ARIMA(3,0,0) with zero mean     : -30885.58
##  ARIMA(3,0,0) with non-zero mean : -30951.27
##  ARIMA(3,0,1) with zero mean     : -30946.82
##  ARIMA(3,0,1) with non-zero mean : -30962.13
##  ARIMA(3,0,2) with zero mean     : -30945.91
##  ARIMA(3,0,2) with non-zero mean : -30980.13
##  ARIMA(3,0,3) with zero mean     : Inf
##  ARIMA(3,0,3) with non-zero mean : -30980.47
##
##
##
##  Best model: ARIMA(2,0,2) with non-zero mean
```

garch(1,1) is better

apple

EDA

```
apple = read.csv("/Users/wyc/AAPL_2006-01-01_to_2018-01-01.csv")
```

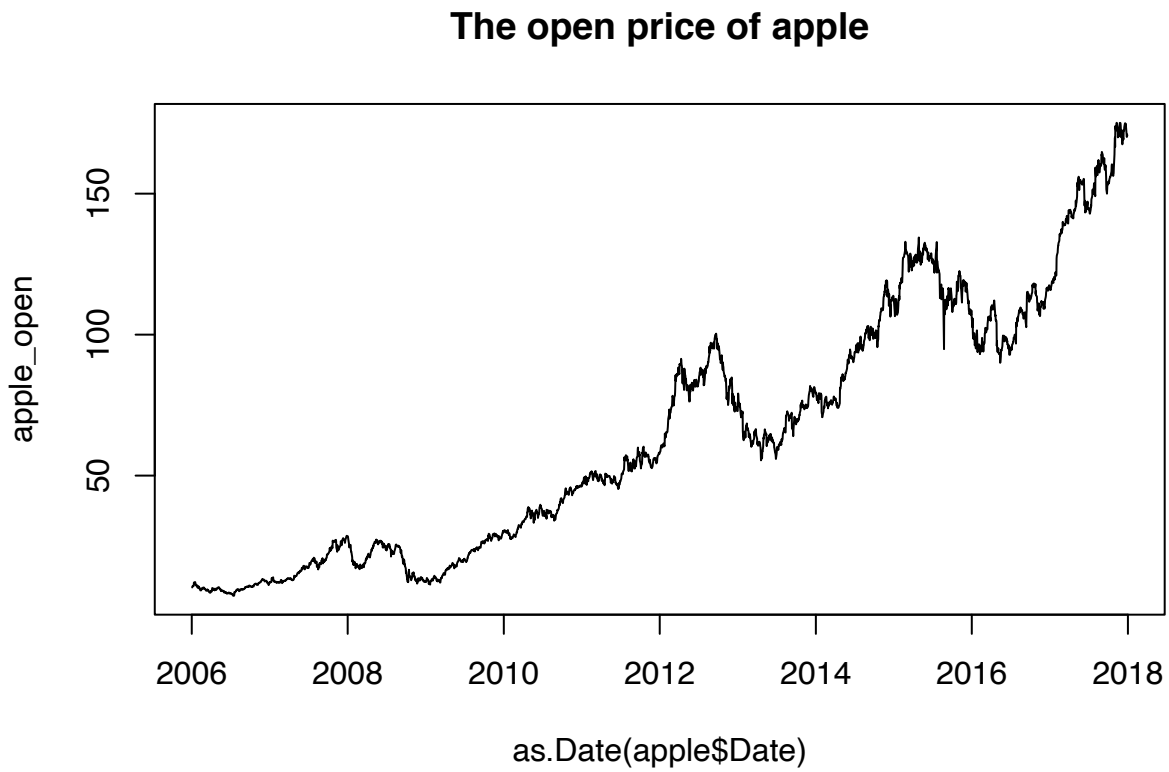
```
head(apple)
```

```
##         Date  Open  High   Low Close    Volume Name
## 1 2006-01-03 10.34 10.68 10.32 10.68 201853036 AAPL
## 2 2006-01-04 10.73 10.85 10.64 10.71 155225609 AAPL
## 3 2006-01-05 10.69 10.70 10.54 10.63 112396081 AAPL
## 4 2006-01-06 10.75 10.96 10.65 10.90 176139334 AAPL
## 5 2006-01-09 10.96 11.03 10.82 10.86 168861224 AAPL
```

```
## 6 2006-01-10 10.89 11.70 10.83 11.55 570088246 AAPL
```

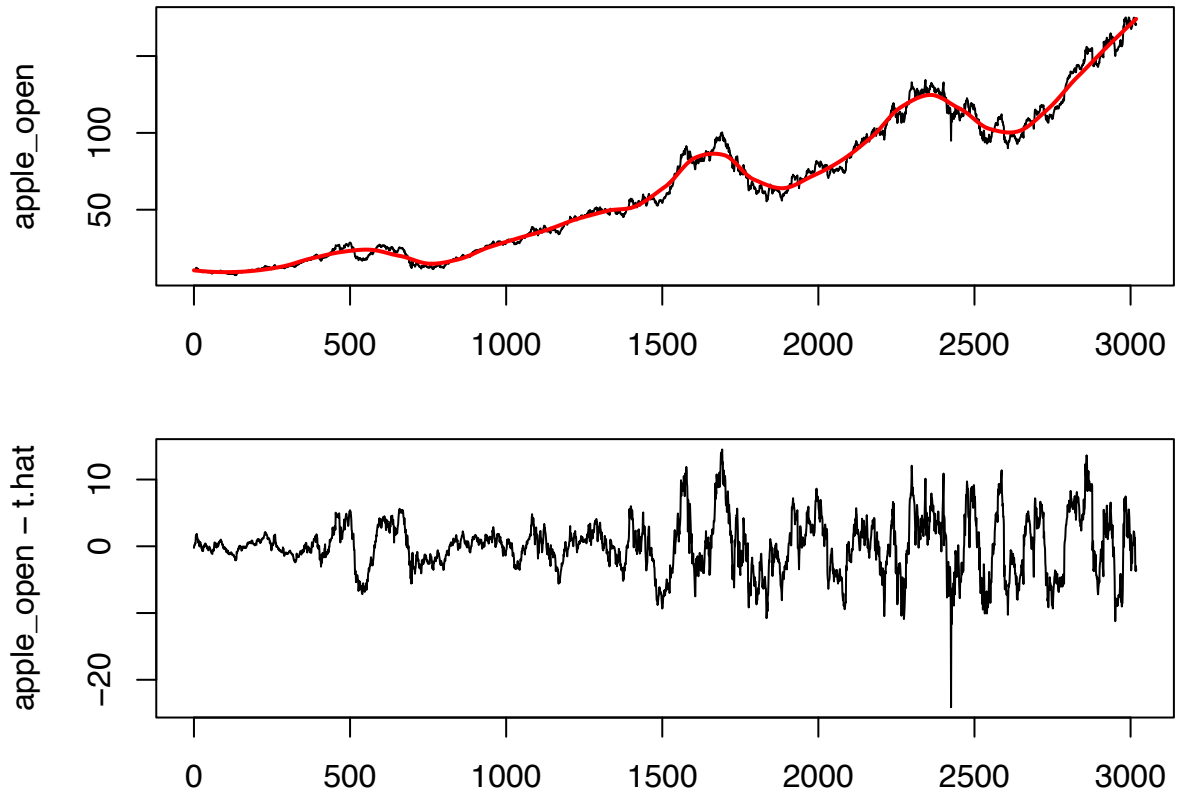
```
apple_open = apple$Open
apple_open = ts(apple_open,start = 0)
```

```
plot(y=apple_open,x=as.Date(apple$Date),type = "l",main="The open price of apple")
```



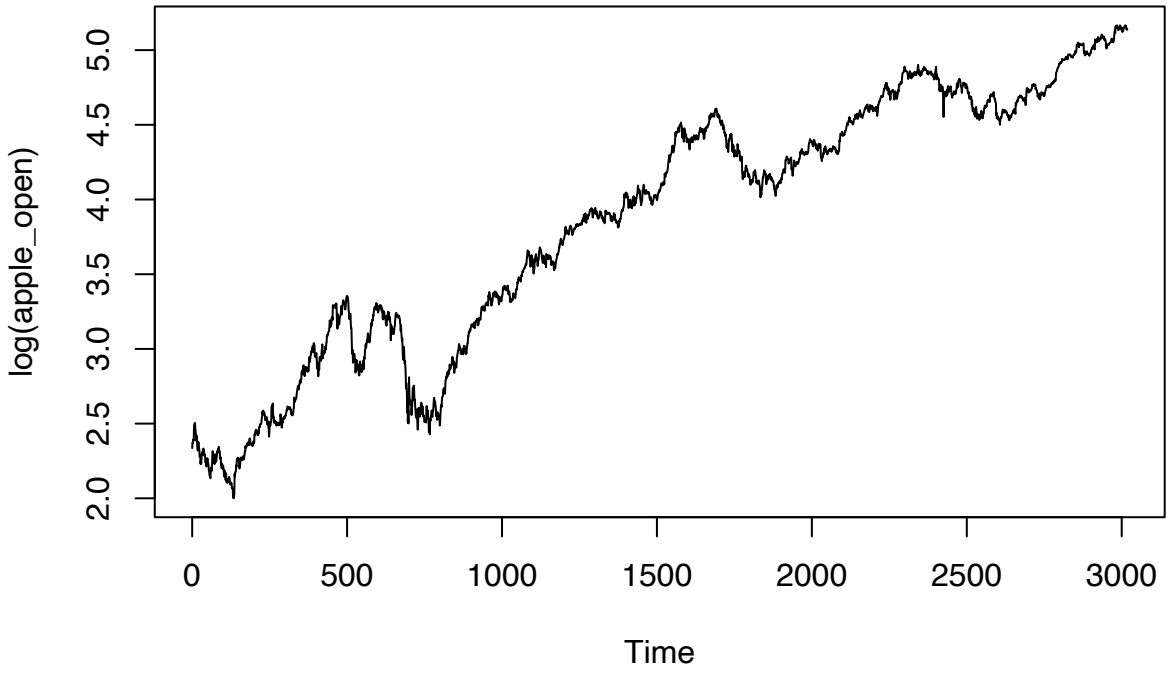
The open price of apple

```
par(mfrow=c(2,1), mar = c(2, 4, 2, 2))
t.hat <- predict(loess(apple_open ~ time(apple_open), span=0.17))
plot(apple_open)
lines(as.numeric(time(apple_open)), t.hat, col="red", lwd=2)
plot(apple_open-t.hat)
```



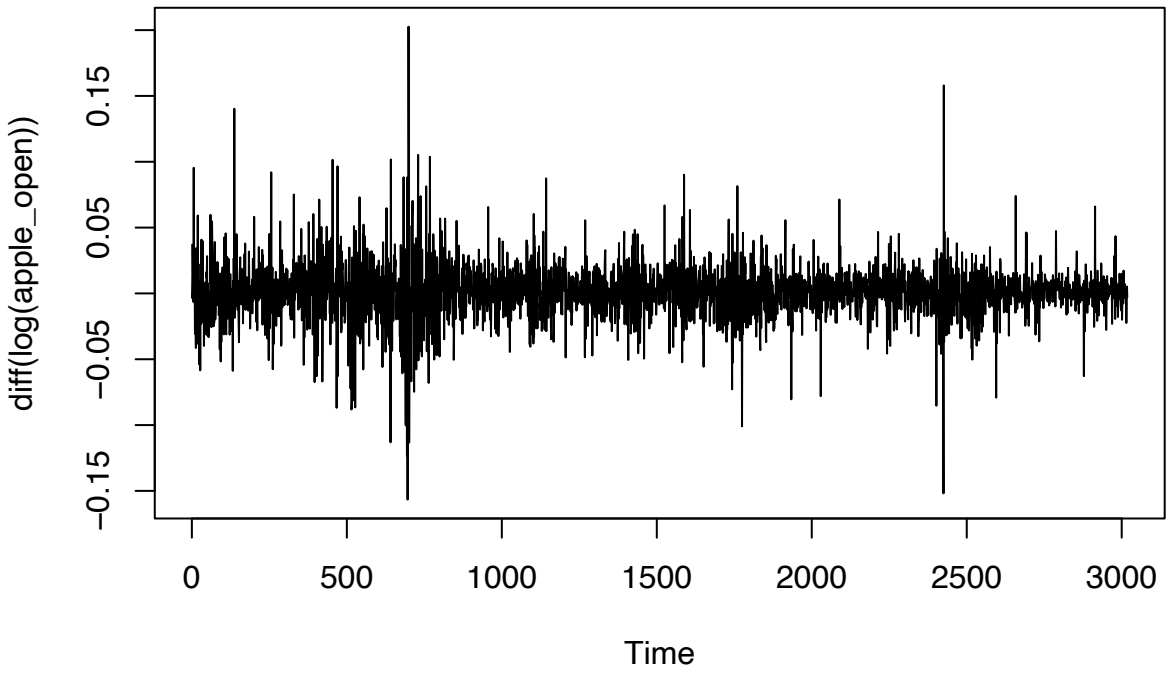
Loess, dipict the trend

```
plot(log(apple_open))
```

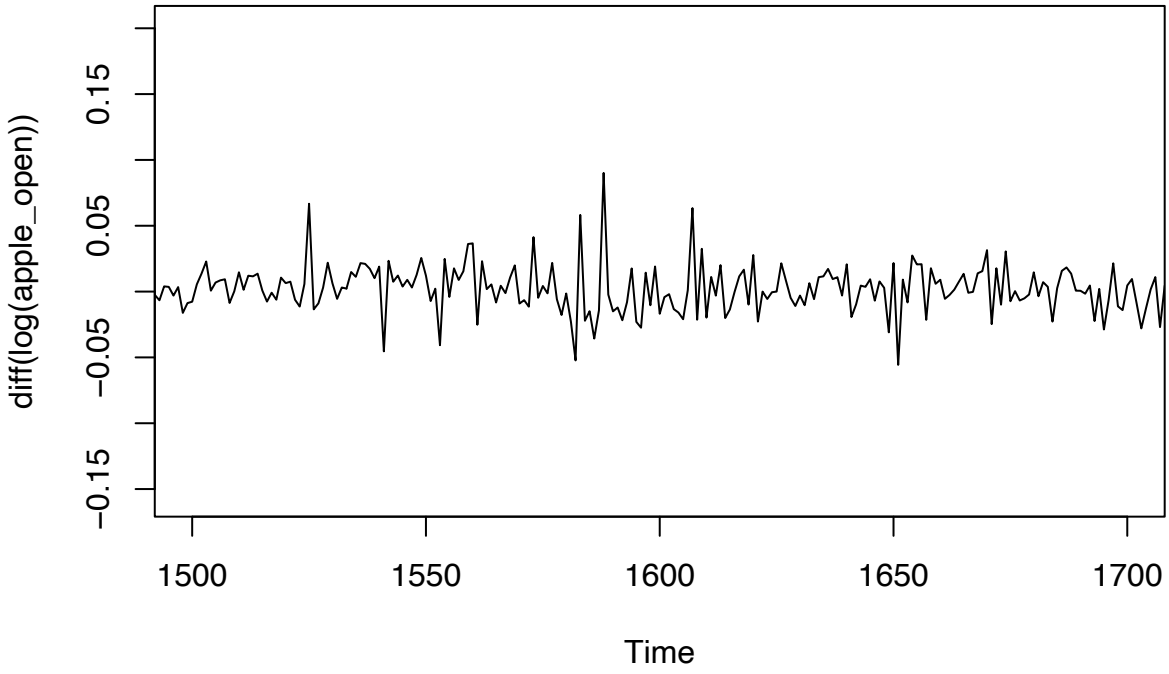


Take log to have a constant trend.

```
plot(diff(log(apple_open)))
```



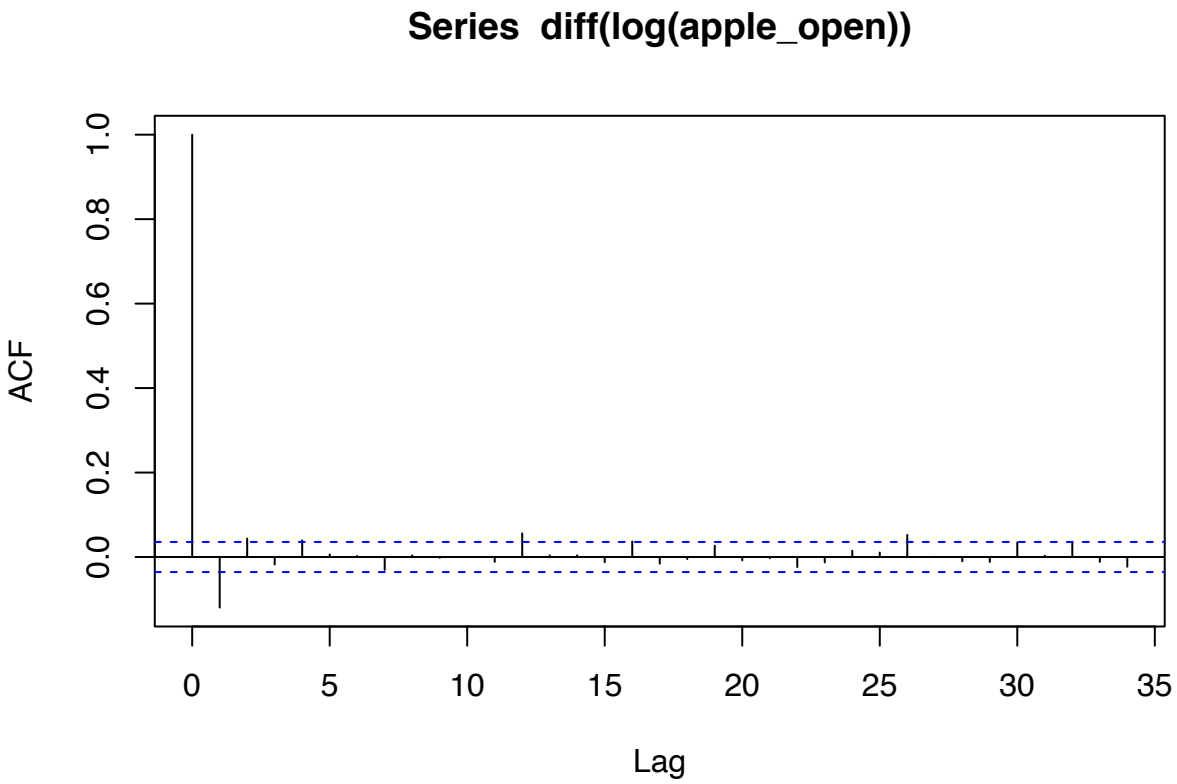

```
plot(diff(log(apple_open)),xlim=c(1500,1700))
```



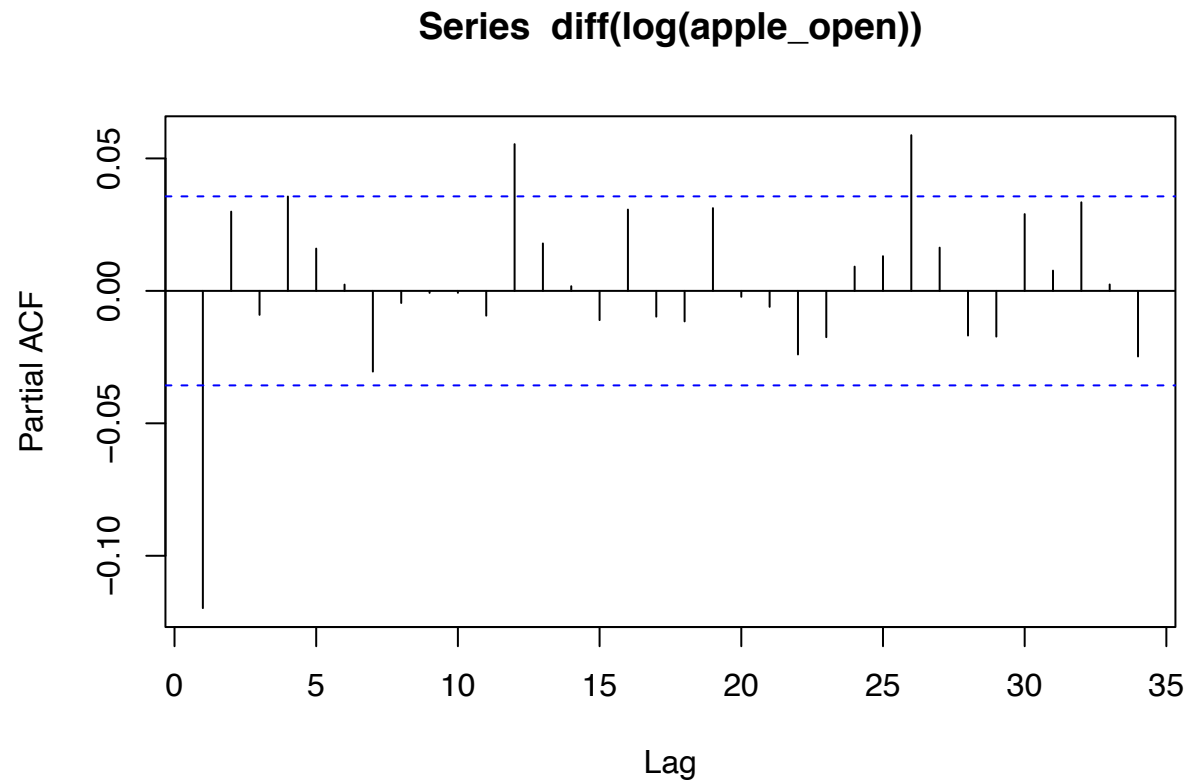
Take one diff, seems no trend, after diff it is return of the stock. Does not seems to have any seasonal pattern.

Model building

```
library(forecast)
acf(diff(log(apple_open)))
```




```
pacf(diff(log(apple_open)))
```




```
model = auto.arima(diff(log(apple_open)), max.p = 3, max.q = 3,max.order = 10, stationary = T, seasonal
```

```
##
```

```
##  ARIMA(0,0,0) with zero mean     : -14465.06
##  ARIMA(0,0,0) with non-zero mean : -14468.43
##  ARIMA(0,0,1) with zero mean     : -14502.26
##  ARIMA(0,0,1) with non-zero mean : -14507.12
##  ARIMA(0,0,2) with zero mean     : -14505.67
##  ARIMA(0,0,2) with non-zero mean : -14510.03
##  ARIMA(0,0,3) with zero mean     : -14503.81
##  ARIMA(0,0,3) with non-zero mean : -14508.26
##  ARIMA(1,0,0) with zero mean     : -14505.25
##  ARIMA(1,0,0) with non-zero mean : -14510.05
##  ARIMA(1,0,1) with zero mean     : -14507.05
##  ARIMA(1,0,1) with non-zero mean : -14511.47
##  ARIMA(1,0,2) with zero mean     : -14505.74
##  ARIMA(1,0,2) with non-zero mean : -14510.61
##  ARIMA(1,0,3) with zero mean     : -14505.65
##  ARIMA(1,0,3) with non-zero mean : -14509.69
##  ARIMA(2,0,0) with zero mean     : -14506.35
##  ARIMA(2,0,0) with non-zero mean : -14510.74
##  ARIMA(2,0,1) with zero mean     : Inf
##  ARIMA(2,0,1) with non-zero mean : Inf
##  ARIMA(2,0,2) with zero mean     : Inf
##  ARIMA(2,0,2) with non-zero mean : Inf
##  ARIMA(2,0,3) with zero mean     : -14507.06
##  ARIMA(2,0,3) with non-zero mean : -14509.37
##  ARIMA(3,0,0) with zero mean     : -14504.49
##  ARIMA(3,0,0) with non-zero mean : -14508.99
##  ARIMA(3,0,1) with zero mean     : Inf
##  ARIMA(3,0,1) with non-zero mean : Inf
##  ARIMA(3,0,2) with zero mean     : -14507.48
##  ARIMA(3,0,2) with non-zero mean : Inf
##  ARIMA(3,0,3) with zero mean     : -14518.51
##  ARIMA(3,0,3) with non-zero mean : -14522.72
##
##
##
##  Best model: ARIMA(3,0,3) with non-zero mean
```

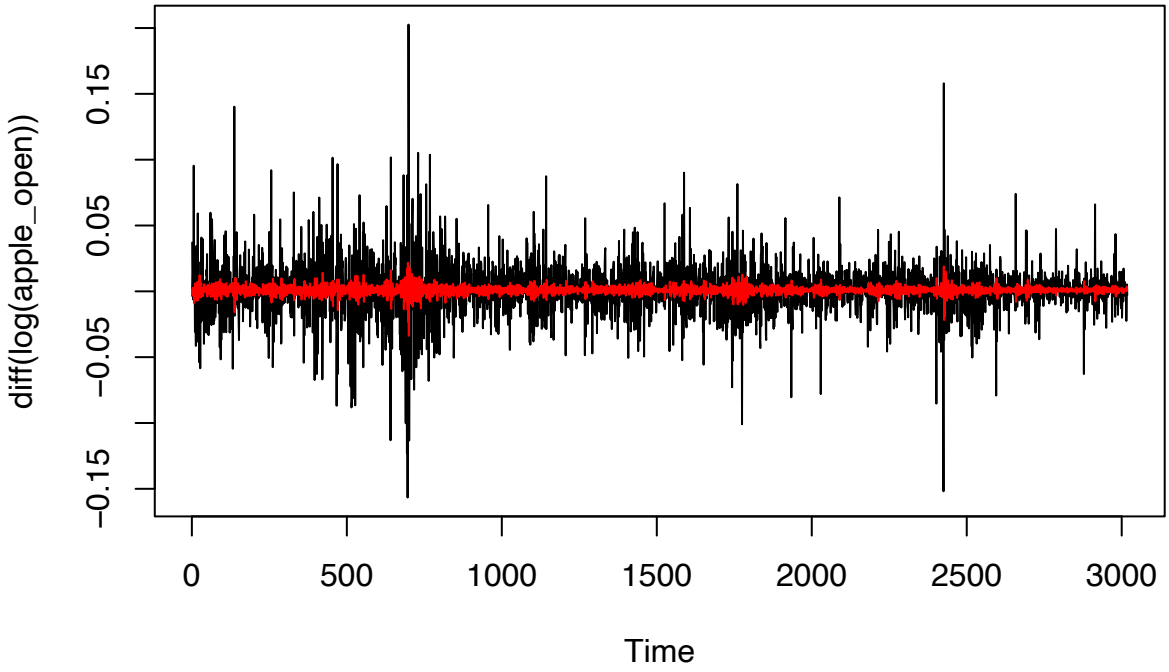
Fit ARIMA model to the return. Hard to figure our the orders of the ARIMA model by acf, pacf, thus auto.arima. arima(3,0,3) best.

```
arima_model = arima(x=diff(log(apple_open)),order = c(3,0,3),method='ML')
summary(arima_model)
```

```
##
## Call:
## arima(x = diff(log(apple_open)), order = c(3, 0, 3), method = "ML")
##
## Coefficients:
##          ar1     ar2      ar3      ma1     ma2     ma3  intercept
##       0.3838  0.0014  -0.6765  -0.4934  0.0808  0.6394      1e-03
## s.e.     NaN     NaN      NaN      NaN     NaN     NaN      4e-04
##
## sigma^2 estimated as 0.0004744:  log likelihood = 7266.6,  aic = -14517.21
##
## Training set error measures:
```

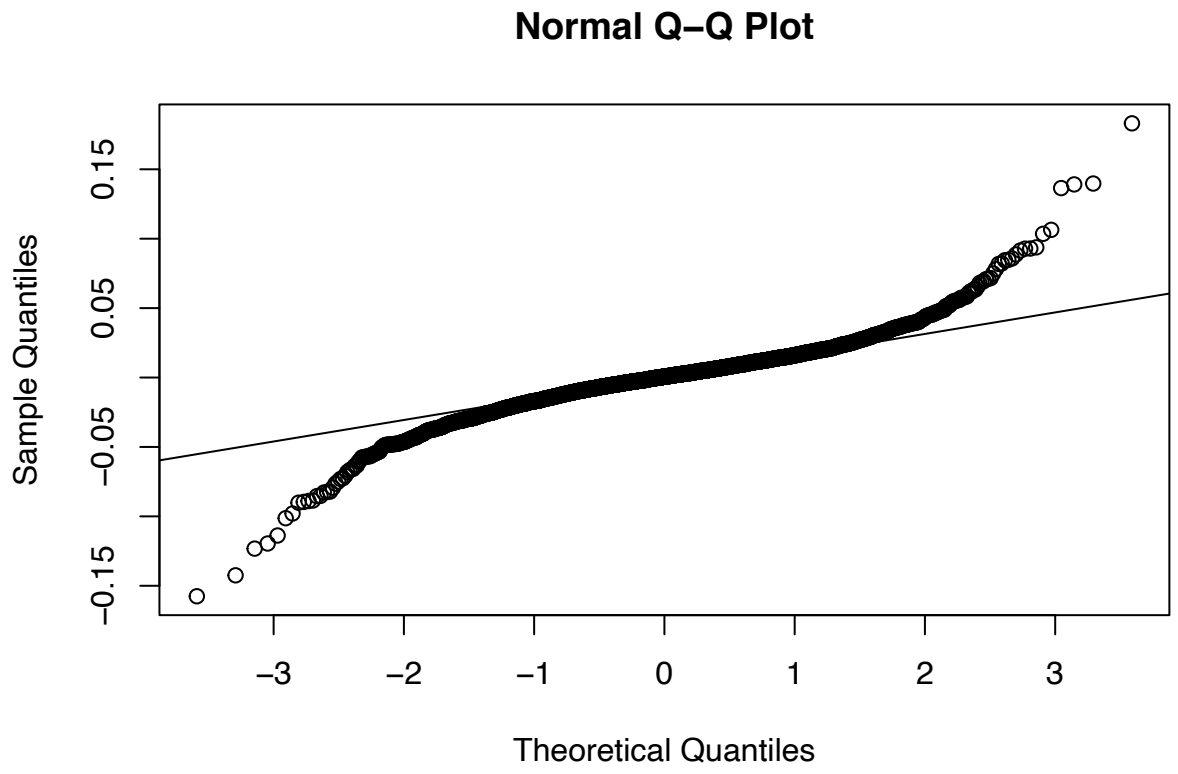
```
##                         ME       RMSE        MAE MPE MAPE      MASE
## Training set -2.716412e-05 0.02178111 0.01489763 NaN  Inf 0.6746955
##                      ACF1
## Training set -0.009253137
```

```
plot(diff(log(apple_open)))
lines(diff(log(apple_open))-arima_model$resid,col="red")
```

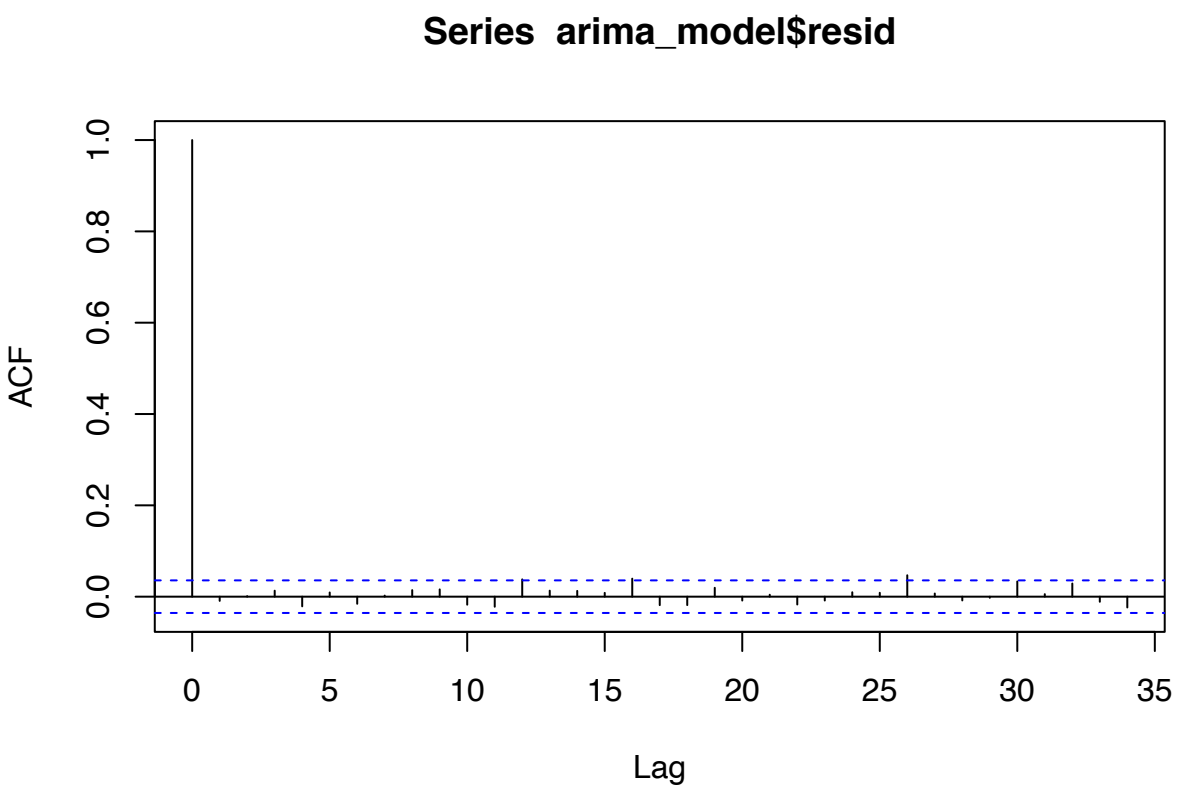


Bad result.

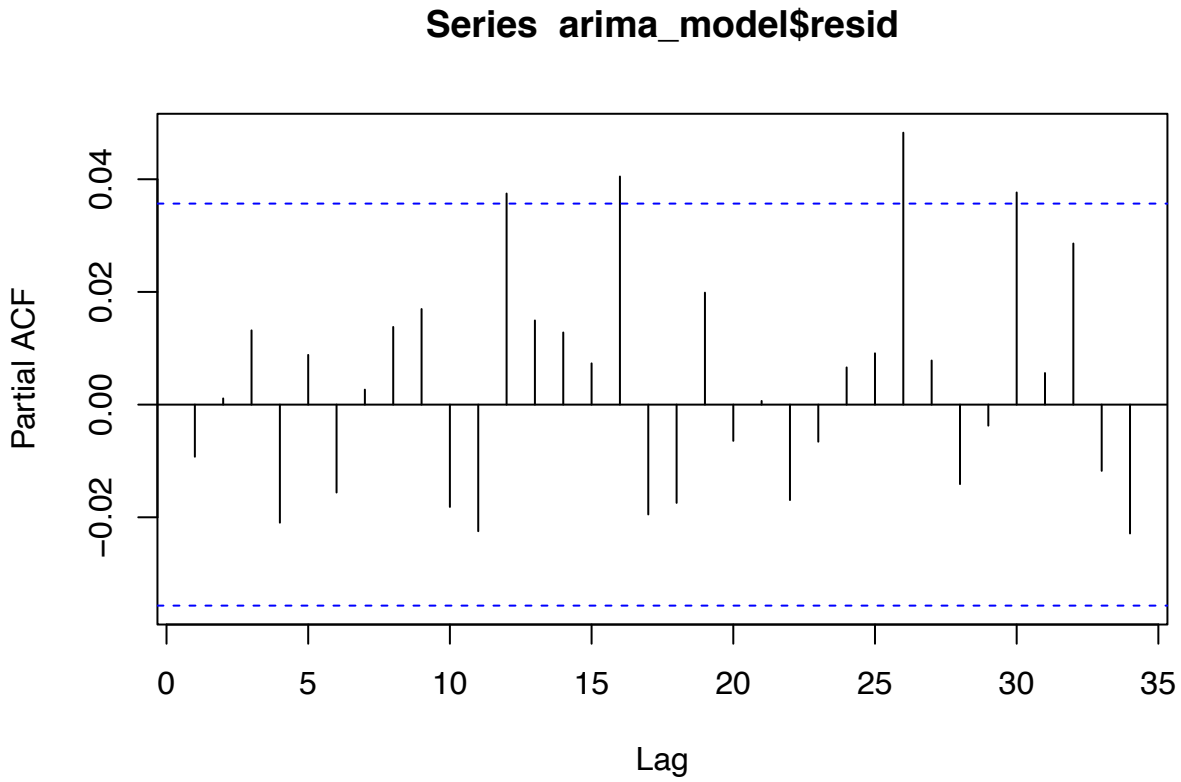
```
qqnorm(arima_model$resid); qqline(arima_model$resid)
```



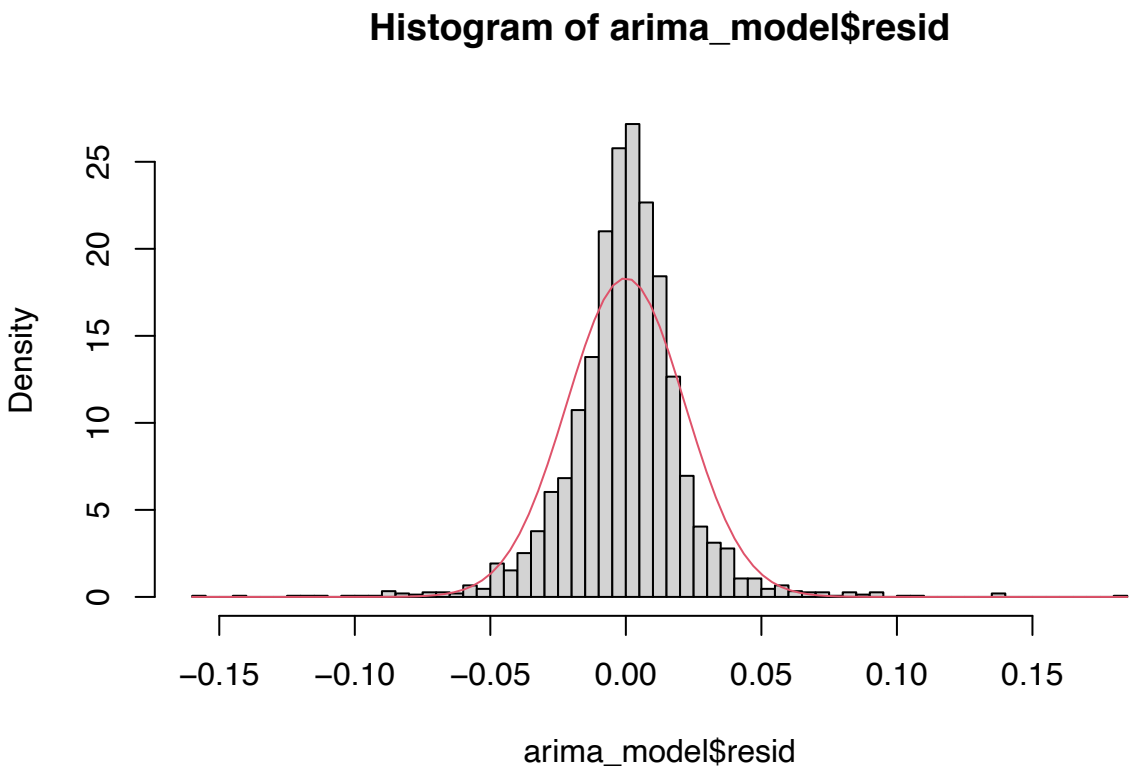

```
acf(arima_model$resid)
```



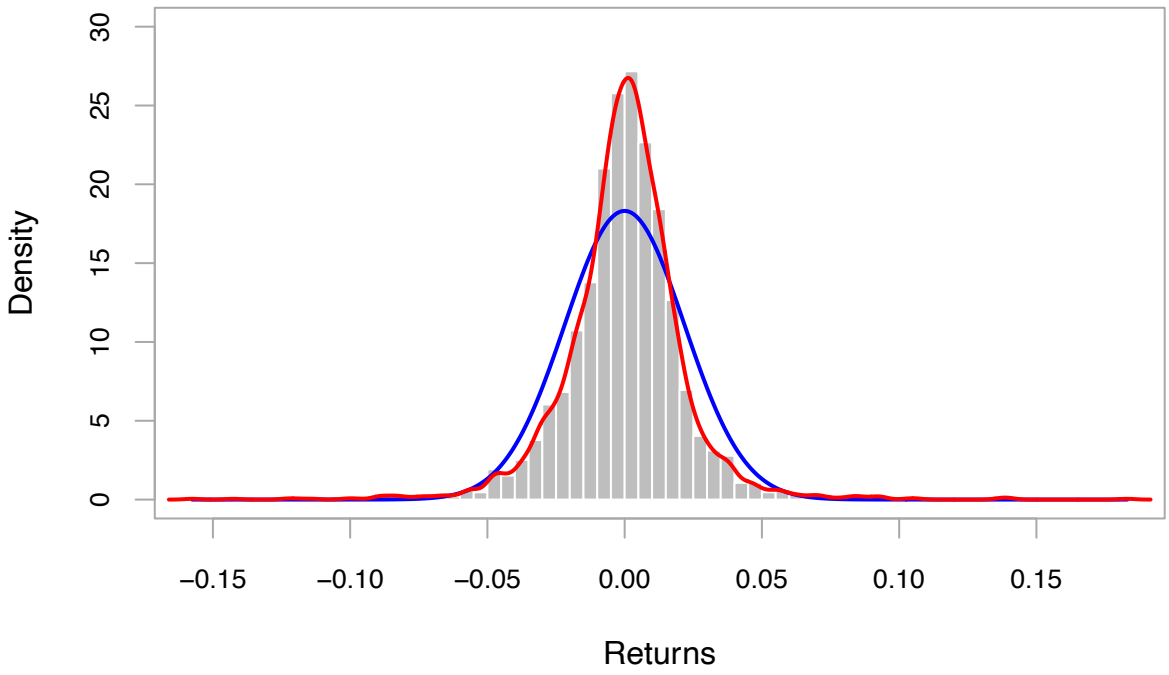

```
pacf(arima_model$resid)
```




```
hist(arima_model$resid,breaks=100,prob=TRUE)
library(MASS)
library(QRM)
fit1 <- fitdistr(arima_model$resid, "normal")
para <- fit1$estimate
curve(dnorm(x, para[1], para[2]), col = 2, add = TRUE)
```




```
fit2 = fit.st(arima_model$resid)
p_val = rep(0,30)
for(i in 1:30){
  p_val[i] = Box.test(arima_model$residual^2,lag = i)[3]
}
stdret <- residuals(arima_model, standardize = TRUE)
library(PerformanceAnalytics)
chart.Histogram(stdret, methods = c("add.normal", "add.density"),
          colorset=c("gray","red","blue"))
```



qqplot, non-gaussian resid. Heteroscedasticity exist from the test. But we probabily can use arima(1,0,3) as a mean model

```
library(fGarch)
gfit1 = garchFit(~garch(1,0),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(0, 0)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(1, 0)
##  ARMA Order:                 0 0
##  Max ARMA Order:             0
##  GARCH Order:                1 0
##  Max GARCH Order:            1
##  Maximum Order:              1
##  Conditional Dist:           norm
##  h.start:                    2
##  llh.start:                  1
##  Length of Series:           3018
##  Recursion Init:             mci
##  Series Scale:               0.02200558
##
## Parameter Initialization:
##  Initial Parameters:           $params
##  Limits of Transformations:    $U, $V
##  Which Parameters are Fixed?   $includes
##  Parameter Matrix:
##                       U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
```

```
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1
##      1      2      3
##  Persistence:                  0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8769.9924: 0.0422032 0.100000 0.100000
##   1:     4192.8649: 0.0422200  1.06555 0.360205
##   2:     4187.0253: 0.0422850  1.07055 0.313632
##   3:     4099.4719: 0.0423551 0.722546 0.174131
##   4:     4097.0384: 0.0526972 0.666718 0.452342
##   5:     4091.1308: 0.0598495 0.640521 0.376614
##   6:     4089.0333: 0.0598858 0.707656 0.333555
##   7:     4088.2787: 0.0616389 0.687482 0.268520
##   8:     4087.6268: 0.0648078 0.695407 0.294229
##   9:     4087.5816: 0.0614502 0.690471 0.297089
##  10:     4087.5779: 0.0623418 0.689101 0.298807
##  11:     4087.5775: 0.0622191 0.689574 0.300152
##  12:     4087.5772: 0.0623408 0.689447 0.299577
##  13:     4087.5772: 0.0623374 0.689444 0.299576
##
## Final Estimate of the Negative LLH:
##  LLH:  -7430.496    norm LLH:  -2.46206
##           mu        omega       alpha1
## 0.0013717719 0.0003338605 0.2995755543
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu       omega       alpha1
## mu     -8098781.334    -5550259     5013.757
## omega  -5550259.400 -9837324600 -1503391.752
## alpha1     5013.757    -1503392    -1245.668
## attr(,"time")
## Time difference of 0.01343513 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.06005192 secs
```

```
gfit2 = garchFit(~garch(1,1),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
```

```
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1   0.00000001   1.0000000 0.80000000     TRUE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1
##      1      2      3      5
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3956.9207: 0.0422032 0.100000 0.100000 0.800000
##   1:     3951.7454: 0.0422071 0.0774769 0.0988866 0.788793
##   2:     3934.4556: 0.0422156 0.0757252 0.121400 0.799936
##   3:     3931.8868: 0.0422329 0.0533463 0.131919 0.804679
##   4:     3928.4349: 0.0422803 0.0503244 0.132421 0.829649
##   5:     3924.0138: 0.0423708 0.0387574 0.117234 0.845930
##   6:     3921.8326: 0.0424971 0.0368799 0.0993199 0.863272
##   7:     3921.8056: 0.0424982 0.0364912 0.0991010 0.863258
##   8:     3921.7765: 0.0425019 0.0364931 0.0989519 0.864145
##   9:     3921.6685: 0.0425120 0.0355007 0.0980850 0.865360
##  10:     3921.3766: 0.0425691 0.0331671 0.0951336 0.871955
##  11:     3921.2451: 0.0428320 0.0316229 0.0916603 0.874450
##  12:     3920.9999: 0.0431374 0.0323339 0.0900277 0.876443
##  13:     3920.3794: 0.0467049 0.0291091 0.0801406 0.887425
##  14:     3920.0006: 0.0503196 0.0264704 0.0800513 0.893026
##  15:     3919.6174: 0.0539395 0.0269423 0.0814479 0.889132
```

```
##  16:     3919.2279: 0.0575542 0.0297721 0.0844164 0.884473
##  17:     3918.9919: 0.0611769 0.0298267 0.0857209 0.882454
##  18:     3918.8595: 0.0647962 0.0279662 0.0850417 0.886469
##  19:     3918.6968: 0.0684184 0.0292835 0.0857577 0.884142
##  20:     3918.5939: 0.0720387 0.0284228 0.0828569 0.886756
##  21:     3918.5827: 0.0740275 0.0288860 0.0833149 0.886017
##  22:     3918.5822: 0.0737546 0.0288236 0.0832628 0.886087
##  23:     3918.5822: 0.0737460 0.0288183 0.0832446 0.886109
##  24:     3918.5822: 0.0737480 0.0288192 0.0832491 0.886104
##
## Final Estimate of the Negative LLH:
##  LLH:  -7599.491    norm LLH:  -2.518055
##           mu        omega       alpha1        beta1
## 1.622869e-03 1.395559e-05 8.324908e-02 8.861041e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1         beta1
## mu      -9627339.967 -5.917409e+07 -3.096645e+03     -23001.54
## omega  -59174092.978 -1.332843e+12 -2.589564e+08 -386654965.12
## alpha1     -3096.645 -2.589564e+08 -9.236301e+04    -104269.78
## beta1     -23001.542 -3.866550e+08 -1.042698e+05    -135854.84
## attr(,"time")
## Time difference of 0.0252521 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.100929 secs
```

```
gfit3 = garchFit(~garch(2,0),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 0)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               2 0
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
```

```
##                      U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.05000000     TRUE
##     alpha2  0.00000001   1.0000000 0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     gamma2 -0.99999999   1.0000000 0.10000000    FALSE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2
##      1      2      3      4
##  Persistence:                  0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8013.8444: 0.0422032 0.100000 0.0500000 0.0500000
##   1:     4235.0428: 0.0422167 0.955183 0.399164 0.433075
##   2:     4234.9209: 0.0439077  1.13462 1.00000e-08 0.324208
##   3:     4179.9374: 0.0439308  1.01028 0.184292 0.281526
##   4:     4085.2239: 0.0439689 0.816791 0.102750 0.196908
##   5:     4063.9278: 0.0439974 0.733245 0.112458 0.181179
##   6:     4045.7339: 0.0440894 0.576916 0.181337 0.191200
##   7:     4045.4413: 0.0443827 0.551375 0.168213 0.359768
##   8:     4043.1114: 0.0471312 0.551521 0.181078 0.305774
##   9:     4042.8316: 0.0487970 0.579174 0.199391 0.237481
##  10:     4042.7432: 0.0488864 0.561234 0.189843 0.244902
##  11:     4042.4888: 0.0495596 0.569261 0.195315 0.256027
##  12:     4042.4028: 0.0504463 0.564691 0.192311 0.257161
##  13:     4042.0603: 0.0605859 0.567328 0.192953 0.258843
##  14:     4042.0431: 0.0607475 0.563567 0.192588 0.262860
##  15:     4042.0413: 0.0607584 0.563370 0.191245 0.262748
##  16:     4042.0401: 0.0608096 0.563953 0.191167 0.263073
##  17:     4042.0283: 0.0623975 0.564441 0.191428 0.261650
##  18:     4042.0250: 0.0630975 0.563607 0.190522 0.264092
##  19:     4042.0244: 0.0635821 0.563913 0.190741 0.263217
##  20:     4042.0244: 0.0635214 0.563877 0.190713 0.263324
##  21:     4042.0244: 0.0635214 0.563877 0.190713 0.263323
##
## Final Estimate of the Negative LLH:
##  LLH:  -7476.049    norm LLH:  -2.477153
##           mu        omega       alpha1       alpha2
## 0.0013978248 0.0002730548 0.1907131039 0.2633233749
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu        omega        alpha1        alpha2
## mu     -8704457.875     -1339815     2511.5902    -2262.9842
## omega  -1339815.302 -11402654233 -1868359.6552 -1904044.7449
## alpha1     2511.590     -1868360    -1607.1570     -599.7291
```

```
## alpha2    -2262.984     -1904045     -599.7291    -1034.9452
## attr(,"time")
## Time difference of 0.02861404 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.123333 secs
```

```
gfit4 = garchFit(~garch(2,1),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(0, 0)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(2, 1)
##  ARMA Order:                 0 0
##  Max ARMA Order:             0
##  GARCH Order:                2 1
##  Max GARCH Order:            2
##  Maximum Order:              2
##  Conditional Dist:           norm
##  h.start:                    3
##  llh.start:                  1
##  Length of Series:           3018
##  Recursion Init:             mci
##  Series Scale:               0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.05000000     TRUE
##     alpha2  0.00000001   1.0000000 0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     gamma2 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1   0.00000001   1.0000000 0.80000000     TRUE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2  beta1
##      1      2      3      4      7
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
```

```
## R coded nlminb Solver:
##
##   0:     3961.2284: 0.0422032 0.100000 0.0500000 0.0500000 0.800000
##   1:     3958.7303: 0.0422074 0.0752517 0.0508888 0.0474489 0.787032
##   2:     3940.8029: 0.0422089 0.0813216 0.0591294 0.0546774 0.793355
##   3:     3932.0767: 0.0422233 0.0573959 0.0781163 0.0657755 0.795219
##   4:     3931.2299: 0.0422251 0.0595147 0.0793128 0.0664944 0.797239
##   5:     3930.7493: 0.0422464 0.0558698 0.0797298 0.0610600 0.801784
##   6:     3926.2015: 0.0424423 0.0342303 0.0940236 0.0237442 0.854616
##   7:     3925.6732: 0.0425097 0.0365274 0.0975882 0.00883740 0.851554
##   8:     3924.6840: 0.0426063 0.0417997 0.109169 1.00000e-08 0.853123
##   9:     3923.7059: 0.0432666 0.0393731 0.107985 1.00000e-08 0.853286
##  10:     3923.4405: 0.0439217 0.0376607 0.106225 1.00000e-08 0.855562
##  11:     3923.1058: 0.0452577 0.0372157 0.106062 1.00000e-08 0.858035
##  12:     3922.3192: 0.0520852 0.0396470 0.104634 1.00000e-08 0.854596
##  13:     3921.1775: 0.0588999 0.0346079 0.102210 1.00000e-08 0.863892
##  14:     3919.8580: 0.0655327 0.0274224 0.0784649 1.00000e-08 0.893495
##  15:     3919.6429: 0.0655328 0.0267654 0.0781439 1.00000e-08 0.893024
##  16:     3919.6399: 0.0655687 0.0267854 0.0781811 1.00000e-08 0.892847
##  17:     3919.4514: 0.0723369 0.0290940 0.0830677 1.00000e-08 0.885912
##  18:     3919.4384: 0.0738143 0.0285713 0.0816341 1.00000e-08 0.887716
##  19:     3919.4375: 0.0741907 0.0282842 0.0819157 1.00000e-08 0.887867
##  20:     3919.4368: 0.0740823 0.0284085 0.0818148 1.00000e-08 0.887780
##  21:     3919.4368: 0.0740822 0.0284066 0.0818116 1.00000e-08 0.887784
##
## Final Estimate of the Negative LLH:
##  LLH:  -7598.637    norm LLH:  -2.517772
##           mu        omega       alpha1       alpha2        beta1
## 0.0016302210 0.0000137558 0.0818116153 0.0000000100 0.8877843641
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1        alpha2         beta1
## mu      -9613940.331 -6.062416e+07 -2.483909e+03 -6.614305e+03     -23546.25
## omega  -60624156.394 -1.369698e+12 -2.672092e+08 -2.716151e+08 -397608426.50
## alpha1     -2483.909 -2.672092e+08 -9.566311e+04 -9.466930e+04    -107548.36
## alpha2     -6614.305 -2.716151e+08 -9.466930e+04 -9.693189e+04    -109998.16
## beta1     -23546.254 -3.976084e+08 -1.075484e+05 -1.099982e+05    -139633.64
## attr(,"time")
## Time difference of 0.04656005 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.1624081 secs
```

```
gfit5 = garchFit(~garch(1,2),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 2)
##  ARMA Order:                0 0
```

```
##  Max ARMA Order:            0
##  GARCH Order:               1 2
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1   0.00000001   1.0000000 0.40000000     TRUE
##     beta2   0.00000001   1.0000000 0.40000000     TRUE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1  beta2
##      1      2      3      5      6
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3956.8872: 0.0422032 0.100000 0.100000 0.400000 0.400000
##   1:     3944.4022: 0.0422047 0.0899069 0.0990812 0.394347 0.394326
##   2:     3930.5546: 0.0422168 0.0708371 0.128410 0.401932 0.402054
##   3:     3926.2249: 0.0422183 0.0652001 0.126804 0.398988 0.399111
##   4:     3924.7939: 0.0422249 0.0613912 0.130817 0.402168 0.402417
##   5:     3924.5394: 0.0422479 0.0489040 0.126811 0.406113 0.406764
##   6:     3922.2763: 0.0422651 0.0496447 0.126677 0.411005 0.411962
##   7:     3921.5553: 0.0422987 0.0453213 0.122980 0.413825 0.415238
##   8:     3921.4807: 0.0423361 0.0427864 0.120541 0.418012 0.419821
##   9:     3920.9024: 0.0424131 0.0393035 0.115364 0.419856 0.422276
##  10:     3920.5802: 0.0425509 0.0384857 0.110819 0.422493 0.425845
##  11:     3920.4351: 0.0427756 0.0366745 0.107407 0.424469 0.427969
##  12:     3920.3682: 0.0430753 0.0365126 0.107359 0.425429 0.428529
##  13:     3919.8684: 0.0494113 0.0404036 0.104671 0.424470 0.425780
##  14:     3919.3054: 0.0557481 0.0364204 0.104887 0.427443 0.424848
##  15:     3918.5601: 0.0620872 0.0372335 0.105286 0.430190 0.425172
##  16:     3918.1922: 0.0684265 0.0356065 0.103573 0.431686 0.425226
```

```
##  17:     3918.1473: 0.0703854 0.0342206 0.0999622 0.437687 0.425034
##  18:     3918.1120: 0.0723591 0.0352058 0.102920 0.436156 0.422559
##  19:     3918.1111: 0.0723591 0.0350692 0.102881 0.436102 0.422505
##  20:     3918.1099: 0.0723642 0.0350979 0.102902 0.436103 0.422608
##  21:     3918.1090: 0.0723736 0.0349333 0.102913 0.436076 0.422771
##  22:     3918.1083: 0.0723958 0.0349342 0.102877 0.435875 0.423086
##  23:     3918.1051: 0.0726614 0.0347056 0.102367 0.433386 0.426256
##  24:     3918.1034: 0.0734446 0.0346357 0.102314 0.438042 0.421780
##  25:     3918.1031: 0.0735297 0.0346299 0.102405 0.431823 0.427901
##  26:     3918.1030: 0.0735061 0.0345066 0.102109 0.433927 0.426215
##  27:     3918.1030: 0.0735103 0.0345082 0.102106 0.433899 0.426243
##
## Final Estimate of the Negative LLH:
##  LLH:  -7599.97    norm LLH:  -2.518214
##           mu        omega       alpha1        beta1        beta2
## 1.617636e-03 1.671045e-05 1.021062e-01 4.338989e-01 4.262428e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu         omega        alpha1         beta1         beta2
## mu      -9610550.87     -48863677      -1575.93     -19652.92     -20443.96
## omega  -48863677.20 -881944093578 -175074538.79 -256547881.81 -258405751.82
## alpha1     -1575.93    -175074539     -62923.67     -70122.85     -70414.40
## beta1     -19652.92    -256547882     -70122.85     -89968.56     -90553.97
## beta2     -20443.96    -258405752     -70414.40     -90553.97     -91159.84
## attr(,"time")
## Time difference of 0.03672981 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.1437609 secs
```

```
gfit6 = garchFit(~garch(2,2),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 2)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               2 2
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
```

```
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega    0.00000100 100.0000000 0.10000000     TRUE
##     alpha1   0.00000001   1.0000000 0.05000000     TRUE
##     alpha2   0.00000001   1.0000000 0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     gamma2 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1    0.00000001   1.0000000 0.40000000     TRUE
##     beta2    0.00000001   1.0000000 0.40000000     TRUE
##     delta    0.00000000   2.0000000 2.00000000    FALSE
##     skew     0.10000000  10.0000000 1.00000000    FALSE
##     shape    1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2  beta1  beta2
##      1      2      3      4      7      8
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3960.6913: 0.0422032 0.100000 0.0500000 0.0500000 0.400000 0.400000
##   1:     3947.3819: 0.0422046 0.0899484 0.0499530 0.0486347 0.394177 0.394149
##   2:     3938.4302: 0.0422076 0.0847254 0.0594796 0.0556292 0.395589 0.395594
##   3:     3933.9601: 0.0422099 0.0769976 0.0611815 0.0559336 0.392803 0.392822
##   4:     3930.8744: 0.0422128 0.0755256 0.0674731 0.0606278 0.395443 0.395541
##   5:     3927.2047: 0.0422325 0.0585260 0.0701322 0.0583491 0.397503 0.398169
##   6:     3923.6486: 0.0422739 0.0526635 0.0773971 0.0550156 0.407156 0.409133
##   7:     3921.8303: 0.0423413 0.0443887 0.0834983 0.0422042 0.410699 0.414394
##   8:     3921.0437: 0.0424415 0.0440009 0.0928681 0.0284155 0.413345 0.419074
##   9:     3920.6575: 0.0426328 0.0362319 0.0912554 0.0197297 0.420522 0.429182
##  10:     3920.4951: 0.0430409 0.0387686 0.0970754 0.00674444 0.419727 0.432273
##  11:     3920.4373: 0.0430419 0.0387471 0.0974641 0.00712085 0.420022 0.432573
##  12:     3920.3966: 0.0430432 0.0380874 0.0975455 0.00717106 0.419906 0.432463
##  13:     3920.3606: 0.0430945 0.0381004 0.0979393 0.00743950 0.420174 0.432793
##  14:     3920.3188: 0.0432080 0.0375942 0.0978348 0.00730562 0.420096 0.432883
##  15:     3920.2690: 0.0434368 0.0373666 0.0976624 0.00736065 0.420462 0.433618
##  16:     3919.8234: 0.0472911 0.0345927 0.0926035 0.00484694 0.422494 0.440607
##  17:     3919.0195: 0.0559337 0.0365915 0.105475 1.00000e-08 0.414899 0.441651
##  18:     3918.8532: 0.0559340 0.0359588 0.105147 1.00000e-08 0.414491 0.441241
##  19:     3918.8437: 0.0559430 0.0354705 0.104737 0.000421814 0.414805 0.441579
##  20:     3918.8390: 0.0559797 0.0356325 0.104517 0.000441534 0.414854 0.441667
##  21:     3918.4298: 0.0653488 0.0391064 0.109096 0.00552273 0.404739 0.439241
##  22:     3918.3835: 0.0704733 0.0433194 0.0897288 0.0341474 0.267672 0.560516
##  23:     3918.0850: 0.0773202 0.0421824 0.0909213 0.0314087 0.138140 0.692341
##  24:     3917.8552: 0.0741699 0.0476345 0.0976055 0.0425125 1.00000e-08 0.809316
##  25:     3917.7988: 0.0684704 0.0514103 0.0977391 0.0543287 1.00000e-08 0.795580
##  26:     3917.5363: 0.0731001 0.0477183 0.0907990 0.0495884 1.00000e-08 0.807717
##  27:     3917.5251: 0.0717268 0.0469709 0.0905311 0.0479185 1.00000e-08 0.810485
```

```
##  28:     3917.5248: 0.0716262 0.0469499 0.0903986 0.0480473 1.00000e-08 0.810570
##  29:     3917.5248: 0.0716195 0.0469293 0.0903693 0.0480391 1.00000e-08 0.810618
##  30:     3917.5248: 0.0716208 0.0469281 0.0903732 0.0480394 1.00000e-08 0.810617
##
## Final Estimate of the Negative LLH:
##  LLH:  -7600.549    norm LLH:  -2.518406
##           mu        omega       alpha1       alpha2        beta1        beta2
## 1.576058e-03 2.272472e-05 9.037316e-02 4.803941e-02 1.000000e-08 8.106174e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1        alpha2         beta1
## mu      -9659577.167     -24991465     -9847.364      15489.38 -5.113561e+03
## omega  -24991464.987 -482265199776 -91567817.591 -101567676.68 -1.400820e+08
## alpha1     -9847.364     -91567818    -35180.553     -30918.81 -3.630537e+04
## alpha2     15489.381    -101567677    -30918.813     -42046.27 -4.158898e+04
## beta1      -5113.561    -140082027    -36305.368     -41588.98 -4.937719e+04
## beta2     -14190.714    -140571816    -36891.420     -40643.96 -4.936468e+04
##                beta2
## mu         -14190.71
## omega  -140571815.56
## alpha1     -36891.42
## alpha2     -40643.96
## beta1      -49364.68
## beta2      -49430.15
## attr(,"time")
## Time difference of 0.0660131 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.2020631 secs
```

```
gfit7 = garchFit(~arma(3,3)+garch(1,0),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(3, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 0)
##  ARMA Order:                3 3
##  Max ARMA Order:            3
##  GARCH Order:               1 0
##  Max GARCH Order:           1
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
```

```
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V       params includes
##     mu     -0.42203167   0.4220317  0.043417533     TRUE
##     ar1    -0.99999999   1.0000000  0.383923685     TRUE
##     ar2    -0.99999999   1.0000000  0.001167706     TRUE
##     ar3    -0.99999999   1.0000000 -0.677235319     TRUE
##     ma1    -0.99999999   1.0000000 -0.493566554     TRUE
##     ma2    -0.99999999   1.0000000  0.081168715     TRUE
##     ma3    -0.99999999   1.0000000  0.639903273     TRUE
##     omega   0.00000100 100.0000000  0.100000000     TRUE
##     alpha1  0.00000001   1.0000000  0.100000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.100000000    FALSE
##     delta   0.00000000   2.0000000  2.000000000    FALSE
##     skew    0.10000000  10.0000000  1.000000000    FALSE
##     shape   1.00000000  10.0000000  4.000000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ar2    ar3    ma1    ma2    ma3  omega alpha1
##      1      2      3      4      5      6      7      8      9
##  Persistence:                  0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8808.9972: 0.0434175 0.383924 0.00116771 -0.677235 -0.493567 0.0811687 0.639903 0.100000 0.1
##   1:     4196.0868: 0.0434297 0.393256 0.000842162 -0.663050 -0.483274 0.0871635 0.655953  1.06675 0
##   2:     4183.3448: 0.0434334 0.392619 -0.0117915 -0.673248 -0.487657 0.0691313 0.642101  1.05244 0.3
##   3:     4171.7508: 0.0434397 0.400685 -0.00621403 -0.666164 -0.477889 0.0773075 0.649285  1.02852 0
##   4:     4155.4200: 0.0434629 0.396612 -0.0294663 -0.664841 -0.488917 0.0537036 0.652112 0.981212 0.3
##   5:     4137.1689: 0.0435751 0.442718 -0.0315323 -0.669913 -0.430951 0.0494991 0.625159 0.901864 0.1
##   6:     4120.0968: 0.0436433 0.387100 -0.0250984 -0.599680 -0.467351 0.0492673 0.664810 0.832687 0.1
##   7:     4103.8961: 0.0437268 0.367576 0.00438334 -0.614572 -0.452120 0.0257745 0.557880 0.780479 0.1
##   8:     4093.8167: 0.0437771 0.385205 0.0262726 -0.576042 -0.426785 0.0400121 0.574863 0.755898 0.23
##   9:     4087.3106: 0.0438566 0.378196 0.00178117 -0.556927 -0.422325 0.00639393 0.565036 0.736365 0
##  10:     4085.9300: 0.0440462 0.358554 0.0249175 -0.561654 -0.438605 0.0315330 0.550311 0.696391 0.29
##  11:     4084.6436: 0.0440809 0.372121 0.0140640 -0.555094 -0.427728 0.0232879 0.557263 0.695406 0.29
##  12:     4084.5847: 0.0440885 0.371627 0.0124305 -0.555411 -0.428733 0.0218229 0.557090 0.695269 0.29
##  13:     4084.5668: 0.0441100 0.372571 0.0117299 -0.554358 -0.428659 0.0221876 0.558764 0.694956 0.29
##  14:     4084.5474: 0.0441478 0.373238 0.0101161 -0.555474 -0.429555 0.0219465 0.558525 0.694890 0.28
##  15:     4084.5234: 0.0441933 0.374508 0.00959291 -0.555699 -0.429985 0.0233567 0.559475 0.694863 0.1
##  16:     4081.8256: 0.0496185 0.488266 -0.101587 -0.630628 -0.536445 0.133665 0.626167 0.705748 0.304
##  17:     4081.5184: 0.0496227 0.485986 -0.101821 -0.628993 -0.539162 0.133239 0.627483 0.704433 0.303
##  18:     4081.4612: 0.0496320 0.486711 -0.0996517 -0.628143 -0.538562 0.134901 0.626820 0.701769 0.30
##  19:     4081.2371: 0.0496752 0.486610 -0.100497 -0.627996 -0.540537 0.133587 0.624900 0.699965 0.299
##  20:     4081.1368: 0.0497571 0.488412 -0.100106 -0.626745 -0.542019 0.135205 0.624438 0.698523 0.297
##  21:     4081.0286: 0.0499518 0.492549 -0.101761 -0.625932 -0.546950 0.136671 0.622427 0.697141 0.295
##  22:     4080.6433: 0.0517554 0.535800 -0.111618 -0.607147 -0.587110 0.155478 0.603647 0.694560 0.295
##  23:     4080.1555: 0.0536662 0.558927 -0.146052 -0.590876 -0.613730 0.190364 0.581842 0.691507 0.290
##  24:     4079.7025: 0.0554878 0.576596 -0.178088 -0.563844 -0.632574 0.227194 0.553279 0.688234 0.288
##  25:     4079.2323: 0.0573172 0.601545 -0.208844 -0.542223 -0.662253 0.259873 0.529965 0.690122 0.290
```

```
##  26:     4078.9110: 0.0590513 0.627910 -0.240441 -0.522287 -0.688897 0.295120 0.505443 0.697477 0.287
##  27:     4078.6174: 0.0605952 0.650530 -0.274508 -0.497532 -0.713438 0.329776 0.476577 0.688236 0.298
##  28:     4078.5618: 0.0612165 0.663362 -0.285343 -0.492586 -0.726026 0.344007 0.473456 0.686294 0.300
##  29:     4078.4642: 0.0617699 0.675500 -0.298516 -0.487767 -0.738856 0.356253 0.465305 0.685149 0.303
##  30:     4078.3719: 0.0615581 0.671150 -0.297640 -0.479777 -0.734987 0.355883 0.458722 0.687053 0.304
##  31:     4078.3622: 0.0615594 0.671036 -0.297788 -0.479393 -0.735304 0.355776 0.459072 0.686988 0.304
##  32:     4078.3547: 0.0615612 0.671475 -0.297865 -0.479306 -0.735034 0.355724 0.458936 0.686902 0.303
##  33:     4078.3466: 0.0615824 0.671878 -0.298647 -0.478807 -0.735829 0.355990 0.458746 0.686798 0.303
##  34:     4078.3380: 0.0616055 0.672675 -0.299082 -0.478382 -0.736116 0.356764 0.458435 0.686719 0.303
##  35:     4078.3307: 0.0616286 0.673227 -0.299857 -0.478071 -0.736773 0.357125 0.457973 0.686642 0.303
##  36:     4078.2257: 0.0625948 0.695985 -0.323909 -0.464394 -0.760010 0.383580 0.442863 0.684151 0.307
##  37:     4078.0612: 0.0631478 0.717073 -0.349392 -0.445492 -0.780595 0.410806 0.420254 0.684242 0.309
##  38:     4077.6201: 0.0627505 0.759660 -0.407339 -0.400670 -0.828116 0.474477 0.372063 0.690428 0.290
##  39:     4077.4290: 0.0624026 0.809997 -0.464562 -0.359082 -0.881039 0.533975 0.326136 0.689158 0.304
##  40:     4077.1588: 0.0616493 0.860965 -0.520681 -0.317144 -0.931009 0.595528 0.282070 0.685090 0.311
##  41:     4076.8915: 0.0577922 0.896332 -0.563813 -0.291188 -0.964554 0.638627 0.252451 0.679868 0.301
##  42:     4076.8774: 0.0573648 0.909565 -0.581765 -0.277859 -0.980035 0.656066 0.239979 0.680004 0.301
##  43:     4076.7286: 0.0572065 0.917213 -0.590342 -0.271416 -0.986161 0.665566 0.232643 0.680271 0.301
##  44:     4076.7049: 0.0570886 0.924229 -0.599946 -0.264756 -0.992948 0.674084 0.225113 0.680826 0.301
##  45:     4076.6192: 0.0569286 0.931167 -0.608694 -0.257853 -0.999770 0.683729 0.218555 0.680879 0.303
##  46:     4076.4741: 0.0600946 0.932653 -0.624900 -0.244520 -1.00000 0.694719 0.208785 0.684692 0.299
##  47:     4076.4244: 0.0597092 0.936043 -0.623871 -0.243779 -1.00000 0.690475 0.211009 0.689400 0.294
##  48:     4076.3937: 0.0598331 0.941506 -0.632272 -0.238031 -1.00000 0.691944 0.210356 0.685635 0.297
##  49:     4076.3916: 0.0592902 0.941458 -0.632667 -0.237635 -1.00000 0.692298 0.210043 0.685954 0.297
##  50:     4076.3908: 0.0592445 0.940729 -0.631778 -0.238046 -1.00000 0.692317 0.209859 0.686618 0.297
##  51:     4076.3907: 0.0591237 0.940729 -0.631940 -0.238156 -1.00000 0.692433 0.209913 0.686631 0.296
##  52:     4076.3907: 0.0591620 0.940810 -0.631977 -0.238016 -1.00000 0.692375 0.209872 0.686549 0.297
##  53:     4076.3907: 0.0591709 0.940792 -0.631950 -0.238043 -1.00000 0.692378 0.209881 0.686567 0.297
##  54:     4076.3906: 0.0591676 0.940793 -0.631955 -0.238041 -1.00000 0.692379 0.209880 0.686566 0.297
##
## Final Estimate of the Negative LLH:
##  LLH:  -7441.683    norm LLH:  -2.465766
##            mu           ar1           ar2           ar3           ma1
##  0.0013020170  0.9407933718 -0.6319546400 -0.2380414440 -0.9999999900
##           ma2           ma3         omega        alpha1
##  0.6923787685  0.2098804821  0.0003324665  0.2971907102
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu          ar1          ar2          ar3          ma1
## mu     -9787121.481   -2767.4950   -2190.2742   -4893.5937   19874.7783
## ar1       -2767.495  -29695.7644  -19882.5476    5113.3220  -37341.7961
## ar2       -2190.274  -19882.5476  -32338.7972  -19525.3604  -28809.4305
## ar3       -4893.594    5113.3220  -19525.3604  -30823.7374    2137.1783
## ma1       19874.778  -37341.7961  -28809.4305    2137.1783  -50164.2300
## ma2       16106.787  -20528.7945  -39213.0231  -27401.5868  -32379.5246
## ma3        4253.739   11426.1669  -19404.3104  -37434.1356    9792.0167
## omega  -5776979.629 -200734.9889 -313836.2886 -308656.6727 -264985.7324
## alpha1     5267.044     156.5169     305.0518     272.1389     234.5865
##                 ma2          ma3         omega        alpha1
## mu       16106.7867    4253.7391   -5776979.6     5267.0444
## ar1     -20528.7945   11426.1669    -200735.0      156.5169
## ar2     -39213.0231  -19404.3104    -313836.3      305.0518
## ar3     -27401.5868  -37434.1356    -308656.7      272.1389
## ma1     -32379.5246    9792.0167    -264985.7      234.5865
```

```
## ma2     -51835.3263  -31830.4175     -384409.8      366.4577
## ma3     -31830.4175  -51278.6463     -287162.2      251.3284
## omega  -384409.8125 -287162.2329 -9949660210.8 -1516564.9464
## alpha1     366.4577     251.3284    -1516564.9    -1240.3266
## attr(,"time")
## Time difference of 0.1379118 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.382553 secs
```

```
gfit8 = garchFit(~arma(3,3)+garch(1,1),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(3, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                3 3
##  Max ARMA Order:            3
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V       params includes
##     mu     -0.42203167   0.4220317  0.043417533     TRUE
##     ar1    -0.99999999   1.0000000  0.383923685     TRUE
##     ar2    -0.99999999   1.0000000  0.001167706     TRUE
##     ar3    -0.99999999   1.0000000 -0.677235319     TRUE
##     ma1    -0.99999999   1.0000000 -0.493566554     TRUE
##     ma2    -0.99999999   1.0000000  0.081168715     TRUE
##     ma3    -0.99999999   1.0000000  0.639903273     TRUE
##     omega   0.00000100 100.0000000  0.100000000     TRUE
##     alpha1  0.00000001   1.0000000  0.100000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.100000000    FALSE
##     beta1   0.00000001   1.0000000  0.800000000     TRUE
##     delta   0.00000000   2.0000000  2.000000000    FALSE
##     skew    0.10000000  10.0000000  1.000000000    FALSE
##     shape   1.00000000  10.0000000  4.000000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ar2    ar3    ma1    ma2    ma3  omega alpha1  beta1
```

```
##      1      2      3      4      5      6      7      8      9     11
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3955.7931: 0.0434175 0.383924 0.00116771 -0.677235 -0.493567 0.0811687 0.639903 0.100000 0.1
##   1:     3944.5501: 0.0434198 0.386423 0.00178859 -0.676974 -0.490916 0.0819537 0.639928 0.0883284 0
##   2:     3930.9237: 0.0434339 0.398730 0.00336258 -0.675650 -0.478572 0.0841863 0.640314 0.0722748 0
##   3:     3927.3605: 0.0434361 0.398663 0.00219042 -0.675533 -0.479068 0.0827716 0.640322 0.0656532 0
##   4:     3925.8081: 0.0434423 0.399117 -0.000212281 -0.674859 -0.479611 0.0799392 0.640827 0.0642255
##   5:     3924.3408: 0.0434593 0.401062 -0.00397345 -0.671546 -0.479794 0.0758329 0.644304 0.0523983 0
##   6:     3920.7531: 0.0434819 0.405975 -0.00562834 -0.667240 -0.476793 0.0743410 0.649034 0.0527085 0
##   7:     3918.8177: 0.0435088 0.412372 -0.00863903 -0.668252 -0.472693 0.0702660 0.647686 0.0477015 0
##   8:     3917.1115: 0.0435441 0.412955 -0.00937722 -0.663741 -0.474510 0.0688607 0.652544 0.0432161 0
##   9:     3916.8675: 0.0435457 0.413008 -0.00972864 -0.663909 -0.474638 0.0683884 0.652343 0.0417349 0
##  10:     3916.6648: 0.0435513 0.413188 -0.00999241 -0.664106 -0.474829 0.0679201 0.652120 0.0421720 0
##  11:     3916.2948: 0.0435663 0.413673 -0.00988052 -0.663932 -0.475153 0.0677439 0.652370 0.0403492 0
##  12:     3915.7409: 0.0435987 0.414806 -0.00917972 -0.663015 -0.475650 0.0680773 0.653568 0.0391245 0
##  13:     3915.0235: 0.0436920 0.415805 -0.00926668 -0.668871 -0.478977 0.0655580 0.647061 0.0358599 0
##  14:     3914.8581: 0.0436940 0.415983 -0.00897055 -0.668229 -0.478850 0.0659435 0.647763 0.0350891 0
##  15:     3914.7165: 0.0437000 0.416223 -0.00884735 -0.667732 -0.478863 0.0660430 0.648248 0.0357981 0
##  16:     3914.4217: 0.0437294 0.417003 -0.00902536 -0.667031 -0.479401 0.0655242 0.648773 0.0346193 0
##  17:     3914.0061: 0.0437944 0.418400 -0.00900309 -0.666397 -0.480759 0.0650238 0.649316 0.0337679 0
##  18:     3913.5202: 0.0439513 0.421557 -0.00761702 -0.668157 -0.483210 0.0662901 0.648078 0.0302006 0
##  19:     3913.2209: 0.0441890 0.427659 -0.0120136 -0.665046 -0.487001 0.0622533 0.651998 0.0315203 0
##  20:     3913.1257: 0.0442909 0.428807 -0.0108571 -0.664668 -0.489134 0.0639820 0.652882 0.0286822 0
##  21:     3913.0293: 0.0444165 0.430623 -0.00970058 -0.665570 -0.490747 0.0660604 0.652237 0.0292639 0
##  22:     3912.8694: 0.0445372 0.433572 -0.00941257 -0.666161 -0.490246 0.0678257 0.651327 0.0296262 0
##  23:     3912.6868: 0.0446896 0.434177 -0.0100868 -0.666502 -0.493522 0.0688206 0.651344 0.0288811 0
##  24:     3912.6796: 0.0447901 0.434624 -0.0121165 -0.664905 -0.495684 0.0683175 0.653298 0.0288137 0
##  25:     3912.6671: 0.0448633 0.435361 -0.0128522 -0.664794 -0.496284 0.0687957 0.653382 0.0274121 0
##  26:     3912.5084: 0.0449004 0.435820 -0.0131181 -0.664789 -0.496397 0.0691121 0.653244 0.0279156 0
##  27:     3912.4666: 0.0449836 0.437040 -0.0135891 -0.664835 -0.496443 0.0699201 0.652806 0.0275514 0
##  28:     3912.4014: 0.0451647 0.438666 -0.0148558 -0.664752 -0.497936 0.0714031 0.652303 0.0275079 0
##  29:     3912.1114: 0.0464849 0.450237 -0.0231699 -0.663157 -0.511442 0.0811307 0.649106 0.0256341 0
##  30:     3911.8553: 0.0478474 0.460907 -0.0312867 -0.657050 -0.521628 0.0895717 0.641403 0.0271122 0
##  31:     3911.6429: 0.0492084 0.470841 -0.0417816 -0.648223 -0.527865 0.0978469 0.633759 0.0265216 0
##  32:     3911.3271: 0.0505740 0.479106 -0.0508972 -0.640313 -0.535097 0.108424 0.625481 0.0273091 0.0
##  33:     3911.1017: 0.0519090 0.482575 -0.0581624 -0.635435 -0.544598 0.123406 0.615880 0.0271348 0.0
##  34:     3910.6408: 0.0550621 0.495190 -0.0910410 -0.617808 -0.557003 0.149603 0.599877 0.0282133 0.0
##  35:     3910.6325: 0.0550630 0.495276 -0.0909667 -0.617774 -0.556947 0.149680 0.599910 0.0282439 0.0
##  36:     3910.6257: 0.0550680 0.495374 -0.0909345 -0.617721 -0.556956 0.149738 0.599897 0.0280314 0.0
##  37:     3910.6175: 0.0550858 0.495591 -0.0909469 -0.617570 -0.557088 0.149828 0.599782 0.0281043 0.0
##  38:     3910.6071: 0.0551232 0.496012 -0.0910050 -0.617264 -0.557398 0.149982 0.599521 0.0279521 0.0
##  39:     3910.3014: 0.0577385 0.522559 -0.0977098 -0.596802 -0.581192 0.158638 0.579828 0.0288954 0.0
##  40:     3910.0121: 0.0604201 0.542556 -0.121358 -0.591761 -0.598601 0.183292 0.569064 0.0281579 0.08
##  41:     3909.6648: 0.0630955 0.559906 -0.141309 -0.580135 -0.619491 0.207214 0.555455 0.0293372 0.08
##  42:     3909.2747: 0.0657332 0.576635 -0.160400 -0.565680 -0.641162 0.231332 0.540076 0.0286972 0.07
##  43:     3909.2319: 0.0677394 0.565784 -0.180021 -0.526805 -0.634844 0.249658 0.503320 0.0265274 0.07
##  44:     3909.2136: 0.0677407 0.565888 -0.180239 -0.527084 -0.634918 0.249426 0.503179 0.0264130 0.07
```

```
##  45:     3909.2082: 0.0677586 0.566099 -0.180460 -0.527128 -0.635067 0.249566 0.503168 0.0261349 0.0
##  46:     3909.1952: 0.0677788 0.566323 -0.180685 -0.527143 -0.635231 0.249753 0.503169 0.0262747 0.0
##  47:     3909.1846: 0.0678176 0.566871 -0.181233 -0.527368 -0.635578 0.249977 0.503138 0.0260558 0.0
##  48:     3909.1644: 0.0679021 0.567761 -0.182117 -0.527340 -0.636270 0.250820 0.503166 0.0262437 0.0
##  49:     3909.0014: 0.0692585 0.581810 -0.196033 -0.526541 -0.647605 0.264539 0.503368 0.0270684 0.0
##  50:     3908.5101: 0.0729613 0.621484 -0.229327 -0.521164 -0.689825 0.304324 0.490324 0.0283488 0.0
##  51:     3908.2278: 0.0763904 0.657636 -0.274871 -0.500635 -0.723336 0.346607 0.465905 0.0253379 0.0
##  52:     3908.1556: 0.0786374 0.699001 -0.322242 -0.464197 -0.767394 0.396993 0.429370 0.0246178 0.0
##  53:     3908.1044: 0.0819676 0.727775 -0.360541 -0.431489 -0.799055 0.440609 0.391306 0.0256218 0.0
##  54:     3908.1027: 0.0819675 0.727773 -0.360532 -0.431463 -0.799069 0.440636 0.391373 0.0252743 0.0
##  55:     3908.0874: 0.0819675 0.727774 -0.360529 -0.431453 -0.799073 0.440649 0.391403 0.0254388 0.0
##  56:     3908.0832: 0.0819669 0.727784 -0.360507 -0.431344 -0.799144 0.440788 0.391780 0.0253684 0.0
##  57:     3908.0820: 0.0819497 0.727884 -0.360603 -0.431390 -0.799208 0.440816 0.391804 0.0254006 0.0
##  58:     3908.0529: 0.0806848 0.734093 -0.364736 -0.432044 -0.803097 0.444172 0.392698 0.0253920 0.0
##  59:     3908.0492: 0.0795186 0.740158 -0.370360 -0.432766 -0.810992 0.452373 0.390362 0.0260638 0.0
##  60:     3908.0308: 0.0794985 0.720525 -0.347273 -0.447636 -0.789613 0.425797 0.408461 0.0259013 0.0
##  61:     3908.0266: 0.0794005 0.722375 -0.346736 -0.446037 -0.791324 0.425771 0.407012 0.0254535 0.0
##  62:     3908.0259: 0.0802866 0.712285 -0.338243 -0.453349 -0.780929 0.415947 0.415132 0.0255923 0.0
##  63:     3908.0248: 0.0797597 0.717544 -0.342773 -0.449638 -0.786278 0.421036 0.411031 0.0256657 0.0
##  64:     3908.0247: 0.0797617 0.717979 -0.343338 -0.449265 -0.786775 0.421674 0.410602 0.0256045 0.0
##  65:     3908.0247: 0.0797815 0.717550 -0.342869 -0.449587 -0.786325 0.421179 0.410946 0.0256095 0.0
##  66:     3908.0247: 0.0797870 0.717670 -0.343016 -0.449497 -0.786435 0.421307 0.410869 0.0256192 0.0
##  67:     3908.0247: 0.0797789 0.717697 -0.343041 -0.449477 -0.786473 0.421348 0.410836 0.0256139 0.0
##  68:     3908.0247: 0.0797807 0.717682 -0.343023 -0.449488 -0.786455 0.421327 0.410850 0.0256144 0.0
##
## Final Estimate of the Negative LLH:
##  LLH:  -7610.049    norm LLH:  -2.521554
##             mu           ar1           ar2           ar3           ma1
##  1.755621e-03  7.176819e-01 -3.430226e-01 -4.494884e-01 -7.864554e-01
##            ma2           ma3         omega        alpha1         beta1
##  4.213269e-01  4.108502e-01  1.240366e-05  7.547838e-02  8.967329e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1           ar2           ar3           ma1
## mu      -8778436.268    -9590.2266 -1.110832e+04   -13842.0544     4010.0926
## ar1        -9590.227   -18596.5353 -1.130205e+04     3528.6057   -23464.6582
## ar2       -11108.316   -11302.0541 -1.880055e+04   -11171.7649   -15180.3195
## ar3       -13842.054     3528.6057 -1.117176e+04   -18764.2665     3934.1766
## ma1         4010.093   -23464.6582 -1.518032e+04     3934.1766   -30775.2685
## ma2         3270.248   -13118.3997 -2.248200e+04   -13853.7454   -18258.6267
## ma3         1308.114     5953.5466 -1.196731e+04   -22275.8456     7167.3692
## omega  -56525529.659 -5924837.2197 -5.347651e+06 -1992376.3629 -7489879.0121
## alpha1     -2566.922     -170.0272  8.779146e+01      172.6496     -123.7039
## beta1     -23325.500    -1863.0505 -1.297678e+03     -476.7194    -2078.4805
##                  ma2           ma3         omega        alpha1         beta1
## mu         3270.2484     1308.1141 -5.652553e+07 -2.566922e+03 -2.332550e+04
## ar1      -13118.3997     5953.5466 -5.924837e+06 -1.700272e+02 -1.863051e+03
## ar2      -22482.0046   -11967.3140 -5.347651e+06  8.779146e+01 -1.297678e+03
## ar3      -13853.7454   -22275.8456 -1.992376e+06  1.726496e+02 -4.767194e+02
## ma1      -18258.6267     7167.3692 -7.489879e+06 -1.237039e+02 -2.078481e+03
## ma2      -28257.8397   -16128.8211 -6.798140e+06  1.985569e+02 -1.418447e+03
## ma3      -16128.8211   -28600.2155 -2.167349e+06  3.038422e+02 -3.861917e+02
## omega  -6798139.6577 -2167349.4285 -1.627748e+12 -3.209956e+08 -4.706908e+08
## alpha1      198.5569      303.8422 -3.209956e+08 -1.139353e+05 -1.277115e+05
```

```
## beta1     -1418.4470     -386.1917 -4.706908e+08 -1.277115e+05 -1.642918e+05
## attr(,"time")
## Time difference of 0.1665041 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.297319 secs
```

```
gfit9 = garchFit(~arma(3,3)+garch(2,0),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(3, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 0)
##  ARMA Order:                3 3
##  Max ARMA Order:            3
##  GARCH Order:               2 0
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V       params includes
##     mu     -0.42203167   0.4220317  0.043417533     TRUE
##     ar1    -0.99999999   1.0000000  0.383923685     TRUE
##     ar2    -0.99999999   1.0000000  0.001167706     TRUE
##     ar3    -0.99999999   1.0000000 -0.677235319     TRUE
##     ma1    -0.99999999   1.0000000 -0.493566554     TRUE
##     ma2    -0.99999999   1.0000000  0.081168715     TRUE
##     ma3    -0.99999999   1.0000000  0.639903273     TRUE
##     omega   0.00000100 100.0000000  0.100000000     TRUE
##     alpha1  0.00000001   1.0000000  0.050000000     TRUE
##     alpha2  0.00000001   1.0000000  0.050000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.100000000    FALSE
##     gamma2 -0.99999999   1.0000000  0.100000000    FALSE
##     delta   0.00000000   2.0000000  2.000000000    FALSE
##     skew    0.10000000  10.0000000  1.000000000    FALSE
##     shape   1.00000000  10.0000000  4.000000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ar2    ar3    ma1    ma2    ma3  omega alpha1 alpha2
##      1      2      3      4      5      6      7      8      9     10
##  Persistence:                 0.1
```

```
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8016.8556: 0.0434175 0.383924 0.00116771 -0.677235 -0.493567 0.0811687 0.639903 0.100000 0.0
##   1:     4237.9503: 0.0434297 0.401973 0.00532910 -0.661969 -0.476024 0.0884707 0.654014 0.954555 0.4
##   2:     4218.2748: 0.0434339 0.397390 -0.00953607 -0.669823 -0.484828 0.0673586 0.641781 0.937290 0
##   3:     4179.3403: 0.0434624 0.419935 -0.00110398 -0.642239 -0.462758 0.0795066 0.667810 0.851212 0
##   4:     4076.6254: 0.0435931 0.457333 -0.0280116 -0.599419 -0.421942 -0.00614562 0.622154 0.701397 0
##   5:     4057.8536: 0.0436037 0.437614 -0.0228298 -0.588030 -0.441757 -0.00152699 0.632599 0.680236 0
##   6:     4039.9695: 0.0436497 0.407493 0.000834016 -0.585331 -0.471472 0.0269715 0.640823 0.591967 0
##   7:     4033.8428: 0.0436593 0.418701 -0.00613808 -0.607392 -0.458791 0.0176168 0.615270 0.587439 0
##   8:     4032.8536: 0.0438326 0.413359 0.00827458 -0.588467 -0.449468 0.0242812 0.613531 0.564846 0.1
##   9:     4032.3714: 0.0440650 0.408967 -0.0151765 -0.585547 -0.450616 0.0105678 0.624881 0.545913 0.1
##  10:     4031.2967: 0.0444555 0.403979 -0.00953032 -0.606804 -0.453986 0.0317387 0.626728 0.547438 0
##  11:     4031.1453: 0.0444672 0.406845 -0.0100138 -0.605806 -0.451796 0.0320169 0.629047 0.548417 0.1
##  12:     4031.0694: 0.0444797 0.407129 -0.0130906 -0.606786 -0.452913 0.0290891 0.629451 0.549086 0.1
##  13:     4030.9352: 0.0445227 0.409356 -0.0128087 -0.607556 -0.452320 0.0309422 0.632531 0.549705 0.1
##  14:     4030.7687: 0.0446321 0.411711 -0.0155800 -0.613160 -0.455187 0.0311951 0.636747 0.550595 0.1
##  15:     4030.0101: 0.0453516 0.429685 -0.0218918 -0.645827 -0.472145 0.0436508 0.666642 0.554318 0.1
##  16:     4029.7356: 0.0461566 0.460294 -0.0200691 -0.665066 -0.492301 0.0369584 0.681216 0.555839 0.1
##  17:     4029.6910: 0.0464150 0.468770 -0.0159224 -0.666118 -0.499610 0.0391122 0.688096 0.553169 0.1
##  18:     4029.4167: 0.0465631 0.469588 -0.0180142 -0.668154 -0.501278 0.0389871 0.685560 0.549945 0.1
##  19:     4029.3702: 0.0467958 0.470223 -0.0213646 -0.667798 -0.501260 0.0427670 0.685421 0.548297 0.1
##  20:     4029.3348: 0.0470496 0.472544 -0.0237704 -0.667762 -0.504515 0.0442210 0.685435 0.548718 0.1
##  21:     4029.2893: 0.0472964 0.476237 -0.0240468 -0.667739 -0.507230 0.0458398 0.684757 0.549471 0.1
##  22:     4029.2620: 0.0475508 0.479131 -0.0260091 -0.668334 -0.510311 0.0473815 0.684024 0.550036 0.1
##  23:     4029.2187: 0.0478030 0.482196 -0.0277618 -0.668083 -0.512714 0.0498726 0.684162 0.550030 0.1
##  24:     4029.0517: 0.0515724 0.517056 -0.0557524 -0.657099 -0.548264 0.0798481 0.669813 0.547206 0.1
##  25:     4029.0142: 0.0553038 0.541205 -0.0948250 -0.641141 -0.566626 0.117770 0.647591 0.551065 0.16
##  26:     4028.2916: 0.0570782 0.556410 -0.103830 -0.631345 -0.587085 0.133008 0.636594 0.547803 0.179
##  27:     4028.1849: 0.0588257 0.562341 -0.114728 -0.613971 -0.597235 0.148211 0.615304 0.551135 0.180
##  28:     4028.0235: 0.0606431 0.571241 -0.133138 -0.600203 -0.609436 0.167275 0.603209 0.554281 0.183
##  29:     4027.8975: 0.0623920 0.590510 -0.148168 -0.595823 -0.626166 0.184797 0.595253 0.550013 0.185
##  30:     4027.8843: 0.0623930 0.590180 -0.148555 -0.596007 -0.626676 0.184235 0.594963 0.549914 0.185
##  31:     4027.8762: 0.0624210 0.590532 -0.148628 -0.595705 -0.626801 0.184766 0.594902 0.549839 0.185
##  32:     4027.8701: 0.0624781 0.591019 -0.149160 -0.595448 -0.627496 0.185162 0.594235 0.549519 0.185
##  33:     4027.8597: 0.0626014 0.592136 -0.150273 -0.594588 -0.628490 0.186727 0.593504 0.549497 0.185
##  34:     4027.7740: 0.0658233 0.620323 -0.184323 -0.576399 -0.656875 0.222665 0.570542 0.551241 0.183
##  35:     4027.7689: 0.0658241 0.619889 -0.183985 -0.575614 -0.657724 0.222683 0.571268 0.551093 0.183
##  36:     4027.7418: 0.0658575 0.620432 -0.184349 -0.575607 -0.657819 0.222868 0.570281 0.550985 0.183
##  37:     4027.7354: 0.0658922 0.620900 -0.184260 -0.574948 -0.658248 0.223440 0.569961 0.550801 0.183
##  38:     4027.7261: 0.0659312 0.621224 -0.184770 -0.574702 -0.658791 0.223688 0.569289 0.550702 0.183
##  39:     4027.7178: 0.0660114 0.622101 -0.185341 -0.573782 -0.659712 0.224814 0.568470 0.550503 0.184
##  40:     4027.7064: 0.0661763 0.623677 -0.187297 -0.572463 -0.661626 0.226782 0.566617 0.550332 0.184
##  41:     4027.6794: 0.0668890 0.630831 -0.194852 -0.566588 -0.669284 0.235725 0.560069 0.550760 0.185
##  42:     4027.6715: 0.0677167 0.630909 -0.200806 -0.557392 -0.670807 0.241969 0.551816 0.550287 0.185
##  43:     4027.6712: 0.0677174 0.631115 -0.200660 -0.557562 -0.670823 0.241896 0.551569 0.550250 0.185
##  44:     4027.6689: 0.0677276 0.631186 -0.200668 -0.557407 -0.671075 0.242041 0.551654 0.550267 0.185
##  45:     4027.6681: 0.0677498 0.631518 -0.200766 -0.557449 -0.671528 0.242128 0.551335 0.550266 0.185
##  46:     4027.6660: 0.0677742 0.631823 -0.200905 -0.557248 -0.671802 0.242520 0.551260 0.550276 0.185
```

```
##  47:     4027.6642: 0.0678212 0.632421 -0.201291 -0.557134 -0.672598 0.243022 0.550784 0.550197 0.185
##  48:     4027.6617: 0.0679149 0.633494 -0.202360 -0.556503 -0.673727 0.244454 0.550032 0.550079 0.185
##  49:     4027.6559: 0.0682243 0.638123 -0.207156 -0.553783 -0.679165 0.250380 0.546006 0.551581 0.185
##  50:     4027.6512: 0.0685269 0.641951 -0.212334 -0.551223 -0.683853 0.256878 0.541974 0.550273 0.187
##  51:     4027.6504: 0.0685271 0.642065 -0.212204 -0.551121 -0.683860 0.256900 0.542046 0.550256 0.187
##  52:     4027.6494: 0.0685325 0.642128 -0.212270 -0.551131 -0.683956 0.256814 0.541950 0.550240 0.187
##  53:     4027.6485: 0.0685465 0.642356 -0.212256 -0.550994 -0.684076 0.256902 0.541949 0.550208 0.187
##  54:     4027.6475: 0.0685783 0.642641 -0.212507 -0.550907 -0.684355 0.257005 0.541762 0.550163 0.187
##  55:     4027.6461: 0.0686440 0.643239 -0.212852 -0.550629 -0.684803 0.257405 0.541581 0.550107 0.187
##  56:     4027.6426: 0.0687613 0.643859 -0.213126 -0.550519 -0.685235 0.257503 0.541390 0.550728 0.187
##  57:     4027.6419: 0.0688942 0.644819 -0.213920 -0.550050 -0.686211 0.258293 0.541047 0.550531 0.187
##  58:     4027.6411: 0.0690269 0.645849 -0.214765 -0.549735 -0.687032 0.259064 0.540492 0.550326 0.188
##  59:     4027.6405: 0.0691646 0.646702 -0.215354 -0.549448 -0.687930 0.259861 0.540090 0.550248 0.188
##  60:     4027.6400: 0.0691362 0.646007 -0.214438 -0.549949 -0.687461 0.259295 0.540384 0.550643 0.188
##  61:     4027.6398: 0.0691898 0.646507 -0.214935 -0.549704 -0.688067 0.259813 0.540001 0.550766 0.188
##  62:     4027.6395: 0.0692284 0.648073 -0.216585 -0.548595 -0.689598 0.261577 0.538759 0.551000 0.188
##  63:     4027.6395: 0.0693577 0.647592 -0.215875 -0.549174 -0.689073 0.260836 0.539363 0.551002 0.188
##  64:     4027.6395: 0.0693348 0.648017 -0.216355 -0.548843 -0.689540 0.261397 0.538959 0.551001 0.188
##  65:     4027.6395: 0.0693405 0.648077 -0.216437 -0.548768 -0.689596 0.261474 0.538895 0.551018 0.188
##  66:     4027.6395: 0.0693395 0.648071 -0.216425 -0.548782 -0.689588 0.261462 0.538906 0.551013 0.188
##
## Final Estimate of the Negative LLH:
##  LLH:  -7490.434    norm LLH:  -2.48192
##             mu           ar1           ar2           ar3           ma1
##  0.0015258566  0.6480712395 -0.2164253173 -0.5487817101 -0.6895884902
##            ma2           ma3         omega        alpha1        alpha2
##  0.2614618977  0.5389064563  0.0002668258  0.1865130188  0.2840915277
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu          ar1          ar2           ar3          ma1
## mu     -7179631.351   -5783.7773   -8845.0159   -6039.70985   -2641.5075
## ar1       -5783.777  -21668.6277  -13250.8112    3737.35658  -26737.8273
## ar2       -8845.016  -13250.8112  -21660.6342  -13536.98800  -15000.6709
## ar3       -6039.710    3737.3566  -13536.9880  -22823.74263    7253.4542
## ma1       -2641.508  -26737.8273  -15000.6709    7253.45417  -36024.0889
## ma2       -4111.911  -14626.0624  -25486.4395  -17000.40808  -19236.8110
## ma3        4098.757    6848.5582  -16836.9405  -30360.61821   10719.2432
## omega    444976.392 -430030.9921 -423276.3867 -243168.86595 -299383.4802
## alpha1     2411.211     103.0299     242.5301      60.77065     127.7571
## alpha2    -2781.594     169.7704     142.6160     111.06639     116.3374
##                 ma2          ma3         omega        alpha1        alpha2
## mu       -4111.9112    4098.75730  4.449764e+05  2.411211e+03    -2781.5939
## ar1     -14626.0624    6848.55817 -4.300310e+05  1.030299e+02      169.7704
## ar2     -25486.4395  -16836.94045 -4.232764e+05  2.425301e+02      142.6160
## ar3     -17000.4081  -30360.61821 -2.431689e+05  6.077065e+01      111.0664
## ma1     -19236.8110   10719.24321 -2.993835e+05  1.277571e+02      116.3374
## ma2     -34835.2384  -24631.04838 -3.822655e+05  2.012581e+02      132.1385
## ma3     -24631.0484  -43858.69184 -3.347791e+05 -1.997174e+01      157.5463
## omega  -382265.4720 -334779.09667 -1.174652e+10 -1.911506e+06 -1873441.3763
## alpha1     201.2581     -19.97174 -1.911506e+06 -1.656336e+03     -575.6375
## alpha2     132.1385     157.54633 -1.873441e+06 -5.756375e+02     -988.8458
## attr(,"time")
## Time difference of 0.2430248 secs
##
```

```
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.718891 secs
```

```
gfit10 = garchFit(~arma(3,3)+garch(2,1),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(3, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 1)
##  ARMA Order:                3 3
##  Max ARMA Order:            3
##  GARCH Order:               2 1
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V       params includes
##     mu     -0.42203167   0.4220317  0.043417533     TRUE
##     ar1    -0.99999999   1.0000000  0.383923685     TRUE
##     ar2    -0.99999999   1.0000000  0.001167706     TRUE
##     ar3    -0.99999999   1.0000000 -0.677235319     TRUE
##     ma1    -0.99999999   1.0000000 -0.493566554     TRUE
##     ma2    -0.99999999   1.0000000  0.081168715     TRUE
##     ma3    -0.99999999   1.0000000  0.639903273     TRUE
##     omega   0.00000100 100.0000000  0.100000000     TRUE
##     alpha1  0.00000001   1.0000000  0.050000000     TRUE
##     alpha2  0.00000001   1.0000000  0.050000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.100000000    FALSE
##     gamma2 -0.99999999   1.0000000  0.100000000    FALSE
##     beta1   0.00000001   1.0000000  0.800000000     TRUE
##     delta   0.00000000   2.0000000  2.000000000    FALSE
##     skew    0.10000000  10.0000000  1.000000000    FALSE
##     shape   1.00000000  10.0000000  4.000000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ar2    ar3    ma1    ma2    ma3  omega alpha1 alpha2  beta1
##      1      2      3      4      5      6      7      8      9     10     13
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
```

```
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3958.1844: 0.0434175 0.383924 0.00116771 -0.677235 -0.493567 0.0811687 0.639903 0.100000 0.0
##   1:     3946.8657: 0.0434196 0.386144 0.00167039 -0.677052 -0.491218 0.0818008 0.639873 0.0894201 0
##   2:     3932.5196: 0.0434288 0.394535 0.00273994 -0.676314 -0.482741 0.0832829 0.639929 0.0747810 0
##   3:     3929.1092: 0.0434305 0.395016 0.00227544 -0.676186 -0.482474 0.0827530 0.640018 0.0699031 0
##   4:     3927.0384: 0.0434418 0.398517 -0.000506524 -0.675073 -0.480353 0.0796145 0.640905 0.0591034
##   5:     3923.9535: 0.0434852 0.406821 -0.00989874 -0.668059 -0.477341 0.0691453 0.648057 0.0580230 0
##   6:     3922.0529: 0.0435319 0.419245 -0.00957543 -0.659464 -0.467674 0.0700246 0.657243 0.0515974 0
##   7:     3919.5497: 0.0435928 0.420536 -0.0182138 -0.664118 -0.471324 0.0557780 0.649316 0.0445455 0
##   8:     3918.5362: 0.0435944 0.420651 -0.0175704 -0.663404 -0.471173 0.0565322 0.650090 0.0470279 0
##   9:     3917.6809: 0.0436022 0.419685 -0.0166250 -0.662088 -0.472414 0.0572304 0.651310 0.0443912 0
##  10:     3916.9882: 0.0436239 0.418619 -0.0144918 -0.660609 -0.474295 0.0587461 0.652657 0.0432561 0
##  11:     3915.3095: 0.0436742 0.420817 -0.0112018 -0.663173 -0.474028 0.0606180 0.649781 0.0378677 0
##  12:     3913.7350: 0.0439088 0.420449 -0.0111946 -0.660400 -0.486665 0.0571847 0.654671 0.0244456 0
##  13:     3913.2678: 0.0439197 0.425667 -0.00721943 -0.660537 -0.480417 0.0619604 0.654243 0.0292587 0
##  14:     3912.8644: 0.0439204 0.425576 -0.00737572 -0.660638 -0.480573 0.0617646 0.654133 0.0302377 0
##  15:     3912.6069: 0.0439262 0.425593 -0.00748333 -0.660735 -0.480762 0.0615987 0.653983 0.0288527 0
##  16:     3912.3083: 0.0440189 0.425934 -0.00761874 -0.661447 -0.482061 0.0623140 0.652650 0.0288978 0
##  17:     3912.1800: 0.0441911 0.428833 -0.00919103 -0.661146 -0.483867 0.0617499 0.652457 0.0268529 0
##  18:     3912.0479: 0.0443882 0.431520 -0.0110135 -0.660301 -0.487187 0.0615849 0.653710 0.0276591 0
##  19:     3911.9326: 0.0445558 0.433205 -0.0101704 -0.662720 -0.490016 0.0645912 0.652624 0.0267450 0
##  20:     3911.8969: 0.0445603 0.433034 -0.0104192 -0.662771 -0.490386 0.0643426 0.652599 0.0260917 0
##  21:     3911.8939: 0.0445610 0.433032 -0.0104317 -0.662770 -0.490412 0.0643367 0.652605 0.0262284 0
##  22:     3911.8905: 0.0445665 0.433106 -0.0104714 -0.662780 -0.490494 0.0643588 0.652623 0.0261526 0
##  23:     3911.8875: 0.0445771 0.433194 -0.0105801 -0.662785 -0.490723 0.0643632 0.652679 0.0261387 0
##  24:     3911.7089: 0.0454177 0.446068 -0.0160907 -0.664337 -0.501186 0.0686175 0.655019 0.0254706 0
##  25:     3911.5631: 0.0462966 0.455042 -0.0210180 -0.666027 -0.512108 0.0763701 0.655148 0.0267948 0
##  26:     3911.4147: 0.0472394 0.463259 -0.0270453 -0.665791 -0.519556 0.0844670 0.651507 0.0252739 0
##  27:     3911.3006: 0.0481976 0.470264 -0.0335013 -0.663269 -0.525939 0.0921911 0.646561 0.0252760 0
##  28:     3910.9784: 0.0491320 0.473139 -0.0399915 -0.654670 -0.530074 0.0984658 0.638305 0.0254510 0
##  29:     3910.9633: 0.0491361 0.473030 -0.0399450 -0.654385 -0.530225 0.0985991 0.638556 0.0259924 0
##  30:     3910.9527: 0.0491638 0.473147 -0.0401620 -0.654209 -0.530380 0.0988309 0.638408 0.0257809 0
##  31:     3910.9432: 0.0491929 0.473282 -0.0403861 -0.654031 -0.530537 0.0990713 0.638241 0.0258718 0
##  32:     3909.0800: 0.0708615 0.579321 -0.207307 -0.530589 -0.643080 0.277438 0.506662 0.0231527 0.0
##  33:     3908.4370: 0.0787171 0.625711 -0.256035 -0.490736 -0.698450 0.331870 0.452377 0.0255010 0.0
##  34:     3908.1506: 0.0787172 0.625670 -0.256002 -0.490607 -0.698520 0.331923 0.452558 0.0243812 0.0
##  35:     3908.0428: 0.0787376 0.626376 -0.256382 -0.490930 -0.698828 0.332002 0.453327 0.0249730 0.0
##  36:     3907.9642: 0.0787761 0.627662 -0.257060 -0.491576 -0.699714 0.332408 0.455263 0.0250172 0.0
##  37:     3907.8636: 0.0788920 0.630468 -0.258902 -0.493895 -0.701593 0.333804 0.457637 0.0256831 0.0
##  38:     3907.7578: 0.0791441 0.635752 -0.261710 -0.498577 -0.706698 0.337639 0.461267 0.0257011 0.0
##  39:     3907.7217: 0.0794168 0.648569 -0.252785 -0.500180 -0.723861 0.339077 0.459578 0.0253662 0.0
##  40:     3907.5719: 0.0798773 0.657529 -0.267031 -0.502228 -0.730229 0.348592 0.462859 0.0268604 0.0
##  41:     3907.5688: 0.0798774 0.657606 -0.267030 -0.502222 -0.730146 0.348600 0.462856 0.0271051 0.0
##  42:     3907.5579: 0.0798855 0.657805 -0.267045 -0.502119 -0.730210 0.348590 0.462788 0.0269550 0.0
##  43:     3907.5536: 0.0799010 0.658356 -0.267120 -0.501896 -0.730142 0.348560 0.462666 0.0269993 0.0
##  44:     3907.5480: 0.0799401 0.659101 -0.267307 -0.501383 -0.730633 0.348628 0.462305 0.0269117 0.0
##  45:     3907.5453: 0.0799767 0.659663 -0.267847 -0.500916 -0.731095 0.349086 0.461887 0.0269928 0.0
##  46:     3907.5410: 0.0800260 0.660175 -0.269577 -0.500530 -0.731806 0.350798 0.461240 0.0268582 0.0
##  47:     3907.5344: 0.0801365 0.662813 -0.271321 -0.497754 -0.733748 0.352042 0.459120 0.0270568 0.0
##  48:     3907.5327: 0.0801370 0.662957 -0.271545 -0.497591 -0.733638 0.351894 0.459335 0.0270999 0.0
##  49:     3907.5237: 0.0801380 0.663241 -0.271798 -0.497355 -0.733855 0.352086 0.459150 0.0269591 0.0
```

```
##  50:     3907.5206: 0.0801625 0.663773 -0.272341 -0.497011 -0.734214 0.352457 0.458885 0.0269891 0.0
##  51:     3907.5166: 0.0802119 0.664640 -0.273296 -0.496318 -0.735037 0.353404 0.458172 0.0269184 0.0
##  52:     3907.4922: 0.0807369 0.675337 -0.288322 -0.484163 -0.746016 0.368202 0.446081 0.0268518 0.0
##  53:     3907.4743: 0.0810123 0.690648 -0.302840 -0.476415 -0.759577 0.382515 0.437599 0.0270412 0.0
##  54:     3907.4730: 0.0810123 0.690629 -0.302854 -0.476380 -0.759594 0.382515 0.437646 0.0269709 0.0
##  55:     3907.4720: 0.0810104 0.690622 -0.302874 -0.476350 -0.759606 0.382519 0.437675 0.0270092 0.0
##  56:     3907.4710: 0.0810029 0.690632 -0.302919 -0.476308 -0.759624 0.382540 0.437687 0.0269502 0.0
##  57:     3907.4701: 0.0809868 0.690667 -0.303006 -0.476240 -0.759655 0.382590 0.437679 0.0269751 0.0
##  58:     3907.4688: 0.0809544 0.690751 -0.303179 -0.476120 -0.759721 0.382707 0.437620 0.0269203 0.0
##  59:     3907.4671: 0.0808903 0.690942 -0.303542 -0.475881 -0.759870 0.382980 0.437456 0.0269329 0.0
##  60:     3907.4507: 0.0798556 0.694783 -0.310414 -0.471582 -0.763181 0.388864 0.433550 0.0266540 0.0
##  61:     3907.4461: 0.0800124 0.703645 -0.322698 -0.464188 -0.774081 0.403734 0.423444 0.0262004 0.0
##  62:     3907.4385: 0.0793215 0.713367 -0.331948 -0.456547 -0.783577 0.413380 0.416176 0.0258911 0.0
##  63:     3907.4353: 0.0793433 0.716946 -0.338551 -0.453336 -0.787214 0.419387 0.413004 0.0261000 0.0
##  64:     3907.4254: 0.0797902 0.716393 -0.341169 -0.451274 -0.785911 0.420227 0.412207 0.0260073 0.0
##  65:     3907.4224: 0.0797980 0.715996 -0.340423 -0.450859 -0.784898 0.419018 0.412108 0.0259215 0.0
##  66:     3907.4222: 0.0797604 0.716650 -0.341224 -0.450323 -0.785560 0.419844 0.411540 0.0258969 0.0
##  67:     3907.4189: 0.0787539 0.734173 -0.362569 -0.435535 -0.803067 0.441682 0.395932 0.0258965 0.0
##  68:     3907.4186: 0.0788143 0.735640 -0.364487 -0.434095 -0.804529 0.443492 0.394568 0.0258397 0.0
##  69:     3907.4186: 0.0788593 0.737150 -0.366498 -0.432942 -0.806064 0.445578 0.393243 0.0258315 0.0
##  70:     3907.4186: 0.0788145 0.737373 -0.366846 -0.432627 -0.806292 0.445916 0.392952 0.0258373 0.0
##  71:     3907.4186: 0.0788112 0.737354 -0.366794 -0.432667 -0.806269 0.445863 0.392991 0.0258360 0.0
##  72:     3907.4186: 0.0788140 0.737343 -0.366783 -0.432676 -0.806258 0.445851 0.393001 0.0258360 0.0
##
## Final Estimate of the Negative LLH:
##  LLH:  -7610.655    norm LLH:  -2.521754
##             mu           ar1           ar2           ar3           ma1
##  1.734349e-03  7.373432e-01 -3.667833e-01 -4.326759e-01 -8.062575e-01
##            ma2           ma3         omega        alpha1        alpha2
##  4.458511e-01  3.930013e-01  1.251097e-05  7.613239e-02  1.000000e-08
##         beta1
##  8.958451e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu           ar1           ar2           ar3           ma1
## mu      -9009918.826    -9958.4386 -1.150995e+04 -1.428761e+04     4009.1437
## ar1        -9958.439   -19428.6975 -1.184432e+04  3.713608e+03   -24553.4388
## ar2       -11509.955   -11844.3236 -1.963560e+04 -1.170608e+04   -15876.1455
## ar3       -14287.608     3713.6081 -1.170608e+04 -1.960228e+04     4197.6198
## ma1         4009.144   -24553.4388 -1.587615e+04  4.197620e+03   -32264.5655
## ma2         3191.706   -13712.3767 -2.349727e+04 -1.454963e+04   -19034.7653
## ma3         1212.996     6322.9772 -1.256967e+04 -2.339688e+04     7718.2095
## omega  -56707990.691 -6043124.8077 -5.472824e+06 -2.019727e+06 -7644802.4858
## alpha1     -2446.413     -168.8928  9.201717e+01  1.791706e+02     -122.3649
## alpha2     -4230.306     -586.7910 -2.243126e+02 -3.711409e-02     -567.7168
## beta1     -23336.858    -1884.1444 -1.308282e+03 -4.671332e+02    -2107.3125
##                   ma2           ma3         omega        alpha1        alpha2
## mu         3191.7063     1212.9959 -5.670799e+07 -2.446413e+03 -4.230306e+03
## ar1      -13712.3767     6322.9772 -6.043125e+06 -1.688928e+02 -5.867910e+02
## ar2      -23497.2673   -12569.6698 -5.472824e+06  9.201717e+01 -2.243126e+02
## ar3      -14549.6301   -23396.8757 -2.019727e+06  1.791706e+02 -3.711409e-02
## ma1      -19034.7653     7718.2095 -7.644802e+06 -1.223649e+02 -5.677168e+02
## ma2      -29523.1435   -16961.4341 -6.957330e+06  2.038675e+02 -1.254376e+02
## ma3      -16961.4341   -30197.8474 -2.176974e+06  3.076863e+02  1.622218e+02
```

```
## omega  -6957329.8296 -2176974.4471 -1.603758e+12 -3.156285e+08 -3.195103e+08
## alpha1      203.8675      307.6863 -3.156285e+08 -1.118971e+05 -1.108121e+05
## alpha2     -125.4376      162.2218 -3.195103e+08 -1.108121e+05 -1.128710e+05
## beta1     -1428.8177     -361.2056 -4.631936e+08 -1.254891e+05 -1.277841e+05
##                beta1
## mu     -2.333686e+04
## ar1    -1.884144e+03
## ar2    -1.308282e+03
## ar3    -4.671332e+02
## ma1    -2.107313e+03
## ma2    -1.428818e+03
## ma3    -3.612056e+02
## omega  -4.631936e+08
## alpha1 -1.254891e+05
## alpha2 -1.277841e+05
## beta1  -1.615604e+05
## attr(,"time")
## Time difference of 0.2293122 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 2.347088 secs
```

```
gfit11 = garchFit(~arma(3,3)+garch(1,2),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(3, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 2)
##  ARMA Order:                3 3
##  Max ARMA Order:            3
##  GARCH Order:               1 2
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V       params includes
##     mu     -0.42203167   0.4220317  0.043417533     TRUE
##     ar1    -0.99999999   1.0000000  0.383923685     TRUE
##     ar2    -0.99999999   1.0000000  0.001167706     TRUE
##     ar3    -0.99999999   1.0000000 -0.677235319     TRUE
```

```
##     ma1    -0.99999999   1.0000000 -0.493566554     TRUE
##     ma2    -0.99999999   1.0000000  0.081168715     TRUE
##     ma3    -0.99999999   1.0000000  0.639903273     TRUE
##     omega   0.00000100 100.0000000  0.100000000     TRUE
##     alpha1  0.00000001   1.0000000  0.100000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.100000000    FALSE
##     beta1   0.00000001   1.0000000  0.400000000     TRUE
##     beta2   0.00000001   1.0000000  0.400000000     TRUE
##     delta   0.00000000   2.0000000  2.000000000    FALSE
##     skew    0.10000000  10.0000000  1.000000000    FALSE
##     shape   1.00000000  10.0000000  4.000000000    FALSE
##   Index List of Parameters to be Optimized:
##     mu    ar1    ar2    ar3    ma1    ma2    ma3  omega alpha1  beta1  beta2
##      1      2      3      4      5      6      7      8      9     11     12
##   Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3953.4301: 0.0434175 0.383924 0.00116771 -0.677235 -0.493567 0.0811687 0.639903 0.100000 0.1
##   1:     3941.0925: 0.0434191 0.385540 0.00159850 -0.677095 -0.491840 0.0817112 0.639878 0.0907292 0
##   2:     3921.0255: 0.0434355 0.400044 0.00440645 -0.675803 -0.476879 0.0854222 0.639831 0.0630796 0
##   3:     3919.6351: 0.0434378 0.399506 0.00286544 -0.675751 -0.477913 0.0835352 0.639691 0.0593355 0
##   4:     3918.8318: 0.0434410 0.399209 0.00133868 -0.675436 -0.478767 0.0817197 0.639862 0.0596360 0
##   5:     3917.1716: 0.0434522 0.399872 -0.00168726 -0.673558 -0.479506 0.0783698 0.641608 0.0518071 0
##   6:     3915.6875: 0.0434692 0.402535 -0.00355511 -0.670371 -0.478320 0.0766173 0.644921 0.0502496 0
##   7:     3914.5601: 0.0434903 0.406780 -0.00562372 -0.669260 -0.475754 0.0742897 0.645892 0.0464387 0
##   8:     3913.9059: 0.0435312 0.410669 -0.00743812 -0.668853 -0.474534 0.0717218 0.646203 0.0440927 0
##   9:     3913.3960: 0.0435740 0.410791 -0.00708210 -0.664076 -0.476696 0.0719977 0.651365 0.0398239 0
##  10:     3912.9056: 0.0437008 0.416160 -0.00947488 -0.666317 -0.477538 0.0668954 0.647322 0.0397507 0
##  11:     3912.8276: 0.0437022 0.416137 -0.00945049 -0.666043 -0.477638 0.0669371 0.647618 0.0403036 0
##  12:     3912.7658: 0.0437044 0.416126 -0.00944219 -0.665731 -0.477770 0.0669553 0.647953 0.0398531 0
##  13:     3912.6449: 0.0437279 0.417020 -0.00932538 -0.665410 -0.477932 0.0668186 0.648117 0.0402501 0
##  14:     3912.4735: 0.0437861 0.418693 -0.00882677 -0.665313 -0.478805 0.0667604 0.647969 0.0390652 0
##  15:     3912.2427: 0.0439255 0.420891 -0.00769965 -0.665640 -0.482578 0.0669093 0.647499 0.0365652 0
##  16:     3911.8807: 0.0442403 0.431324 -0.0111183 -0.661949 -0.485689 0.0627653 0.650475 0.0382079 0
##  17:     3911.7563: 0.0442516 0.430534 -0.0107839 -0.660822 -0.486938 0.0632510 0.651792 0.0374693 0
##  18:     3911.7187: 0.0442523 0.430530 -0.0107876 -0.660831 -0.486970 0.0632517 0.651786 0.0371348 0
##  19:     3911.6772: 0.0442634 0.430520 -0.0108060 -0.660954 -0.487375 0.0633072 0.651721 0.0369511 0
##  20:     3911.5430: 0.0443316 0.431027 -0.0103373 -0.661721 -0.488496 0.0648907 0.651202 0.0360607 0
##  21:     3911.4090: 0.0445323 0.432773 -0.0109885 -0.662777 -0.491642 0.0678157 0.650868 0.0352559 0
##  22:     3911.3279: 0.0447654 0.434911 -0.0136017 -0.662871 -0.494741 0.0696982 0.651242 0.0342651 0
##  23:     3911.2434: 0.0450131 0.437027 -0.0164043 -0.662445 -0.496431 0.0719138 0.650335 0.0344719 0
##  24:     3911.1717: 0.0452620 0.439074 -0.0184541 -0.661228 -0.497586 0.0740199 0.648214 0.0344919 0
##  25:     3910.7037: 0.0482776 0.466371 -0.0321065 -0.641173 -0.529168 0.0933263 0.627619 0.0326880 0
##  26:     3910.4837: 0.0513323 0.481577 -0.0615509 -0.624378 -0.545916 0.124184 0.608576 0.0327762 0.1
##  27:     3909.5352: 0.0543390 0.494487 -0.0973073 -0.615997 -0.562563 0.159839 0.597047 0.0309761 0.0
##  28:     3909.0515: 0.0571909 0.526769 -0.0962555 -0.592106 -0.596697 0.170401 0.568117 0.0342748 0.0
##  29:     3909.0429: 0.0571927 0.527144 -0.0963628 -0.592322 -0.596254 0.170429 0.567911 0.0344807 0.0
##  30:     3908.9897: 0.0572173 0.527413 -0.0965721 -0.592185 -0.596325 0.170586 0.567704 0.0340694 0.0
##  31:     3908.9659: 0.0572769 0.528061 -0.0971038 -0.591903 -0.596520 0.171005 0.567251 0.0341228 0.0
```

```
##  32:     3908.9322: 0.0574001 0.529101 -0.0981059 -0.591225 -0.597319 0.171937 0.566496 0.0338589 0.0
##  33:     3907.4188: 0.0697571 0.624990 -0.199582 -0.531310 -0.690571 0.274682 0.501202 0.0322475 0.09
##  34:     3907.0775: 0.0739502 0.638504 -0.240398 -0.506726 -0.714480 0.321927 0.470127 0.0326794 0.09
##  35:     3907.0427: 0.0739506 0.638889 -0.240243 -0.506781 -0.714004 0.322178 0.470082 0.0331577 0.09
##  36:     3906.9750: 0.0739744 0.639151 -0.240372 -0.506582 -0.713987 0.322510 0.469832 0.0326907 0.09
##  37:     3906.9490: 0.0740328 0.639981 -0.240833 -0.506497 -0.714443 0.323440 0.469574 0.0328546 0.09
##  38:     3906.9246: 0.0741041 0.641967 -0.241803 -0.507747 -0.716280 0.325204 0.470310 0.0325986 0.09
##  39:     3906.9045: 0.0740712 0.643911 -0.242803 -0.509293 -0.718118 0.326632 0.471169 0.0327103 0.09
##  40:     3906.8842: 0.0741780 0.643650 -0.244536 -0.504183 -0.717860 0.328227 0.466202 0.0320429 0.09
##  41:     3906.8610: 0.0739979 0.646936 -0.246767 -0.507590 -0.719878 0.329599 0.468589 0.0324832 0.09
##  42:     3906.8494: 0.0740077 0.651125 -0.249787 -0.510520 -0.723496 0.332822 0.470705 0.0323986 0.09
##  43:     3906.8118: 0.0742412 0.653053 -0.252459 -0.508303 -0.725519 0.336065 0.468541 0.0325804 0.09
##  44:     3906.7978: 0.0744580 0.654635 -0.255253 -0.505382 -0.727302 0.339180 0.465733 0.0323740 0.09
##  45:     3906.7187: 0.0749466 0.668358 -0.278563 -0.489819 -0.742260 0.360993 0.449179 0.0321539 0.09
##  46:     3906.6687: 0.0757778 0.684327 -0.300009 -0.477780 -0.758598 0.383974 0.436125 0.0325039 0.09
##  47:     3906.6296: 0.0763617 0.703614 -0.323015 -0.469613 -0.777665 0.407469 0.426003 0.0325334 0.09
##  48:     3906.5399: 0.0768076 0.715449 -0.343121 -0.445737 -0.789265 0.426396 0.403905 0.0320884 0.09
##  49:     3906.5156: 0.0768077 0.715478 -0.343127 -0.445747 -0.789242 0.426392 0.403898 0.0322851 0.09
##  50:     3906.5110: 0.0768080 0.715656 -0.343185 -0.445819 -0.789107 0.426346 0.403862 0.0321063 0.09
##  51:     3906.5093: 0.0768003 0.715922 -0.343438 -0.445641 -0.789365 0.426613 0.403660 0.0321070 0.09
##  52:     3906.5070: 0.0767868 0.716433 -0.343951 -0.445268 -0.789897 0.427180 0.403229 0.0319972 0.09
##  53:     3906.4819: 0.0764387 0.732294 -0.361211 -0.432767 -0.806289 0.445769 0.389413 0.0311613 0.09
##  54:     3906.4199: 0.0757586 0.782929 -0.421078 -0.391300 -0.854240 0.504315 0.347547 0.0318238 0.09
##  55:     3906.4091: 0.0758237 0.827379 -0.480759 -0.351105 -0.901776 0.567654 0.303115 0.0314672 0.09
##  56:     3906.3822: 0.0727114 0.866500 -0.527588 -0.321252 -0.939669 0.614593 0.271775 0.0313801 0.09
##  57:     3906.3692: 0.0703655 0.900235 -0.579220 -0.286200 -0.971124 0.662456 0.238070 0.0314228 0.09
##  58:     3906.3351: 0.0683301 0.928519 -0.611597 -0.261644 -1.00000 0.696245 0.211424 0.0316001 0.095
##  59:     3906.3241: 0.0699748 0.927907 -0.611063 -0.262580 -1.00000 0.696310 0.212276 0.0316376 0.090
##  60:     3906.3210: 0.0698357 0.928152 -0.613246 -0.261721 -1.00000 0.698234 0.211365 0.0315004 0.090
##  61:     3906.3202: 0.0698659 0.928286 -0.613483 -0.261709 -1.00000 0.698199 0.211488 0.0316018 0.090
##  62:     3906.3200: 0.0697771 0.928372 -0.613627 -0.261592 -1.00000 0.698170 0.211518 0.0315671 0.090
##  63:     3906.3200: 0.0697905 0.928378 -0.613347 -0.261746 -1.00000 0.697956 0.211661 0.0315927 0.090
##  64:     3906.3200: 0.0697954 0.928370 -0.613471 -0.261684 -1.00000 0.698054 0.211599 0.0315842 0.090
##  65:     3906.3200: 0.0697919 0.928371 -0.613469 -0.261680 -1.00000 0.698052 0.211598 0.0315838 0.090
##
## Final Estimate of the Negative LLH:
##  LLH:  -7611.753    norm LLH:  -2.522118
##           mu           ar1           ar2           ar3           ma1
##  1.535812e-03  9.283714e-01 -6.134688e-01 -2.616802e-01 -1.000000e+00
##           ma2           ma3         omega        alpha1         beta1
##  6.980518e-01  2.115979e-01  1.529433e-05  9.627748e-02  4.163074e-01
##         beta2
##  4.523853e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu           ar1           ar2           ar3           ma1
## mu     -1.156796e+07   -14331.2005 -1.696628e+04   -19528.4091     6259.5921
## ar1    -1.433120e+04   -33602.0028 -2.090437e+04     7145.9766   -43480.2307
## ar2    -1.696628e+04   -20904.3710 -3.407257e+04   -20523.1283   -29538.4139
## ar3    -1.952841e+04     7145.9766 -2.052313e+04   -33787.8095     6873.7089
## ma1     6.259592e+03   -43480.2307 -2.953841e+04     6873.7089   -59908.9851
## ma2     3.233099e+03   -23464.7359 -4.227822e+04   -27865.7718   -34839.9289
## ma3    -5.707341e+02    13539.4304 -2.217744e+04   -42406.5937    16336.0052
## omega  -5.151144e+07 -5238881.5253 -5.820564e+06 -2662109.2942 -6780687.8540
```

```
## alpha1 -1.152992e+03     -247.7189  6.719419e+01      225.0402     -224.2156
## beta1  -2.244777e+04    -1610.8172 -1.330452e+03     -469.7604    -1909.6176
## beta2  -2.323829e+04    -1690.0771 -1.400733e+03     -508.5106    -2014.3427
##                 ma2           ma3         omega        alpha1         beta1
## mu         3233.0988     -570.7341 -5.151144e+07 -1.152992e+03 -2.244777e+04
## ar1      -23464.7359    13539.4304 -5.238882e+06 -2.477189e+02 -1.610817e+03
## ar2      -42278.2179   -22177.4403 -5.820564e+06  6.719419e+01 -1.330452e+03
## ar3      -27865.7718   -42406.5937 -2.662109e+06  2.250402e+02 -4.697604e+02
## ma1      -34839.9289    16336.0052 -6.780688e+06 -2.242156e+02 -1.909618e+03
## ma2      -54752.2985   -32370.6436 -7.536482e+06  1.696021e+02 -1.521607e+03
## ma3      -32370.6436   -57965.9507 -3.114952e+06  3.398605e+02 -3.561462e+02
## omega  -7536482.1217 -3114951.8506 -1.008551e+12 -2.024640e+08 -2.921539e+08
## alpha1      169.6021      339.8605 -2.024640e+08 -7.202013e+04 -7.999923e+04
## beta1     -1521.6067     -356.1462 -2.921539e+08 -7.999923e+04 -1.017177e+05
## beta2     -1612.5164     -395.4739 -2.940951e+08 -8.029320e+04 -1.023235e+05
##                beta2
## mu     -2.323829e+04
## ar1    -1.690077e+03
## ar2    -1.400733e+03
## ar3    -5.085106e+02
## ma1    -2.014343e+03
## ma2    -1.612516e+03
## ma3    -3.954739e+02
## omega  -2.940951e+08
## alpha1 -8.029320e+04
## beta1  -1.023235e+05
## beta2  -1.029454e+05
## attr(,"time")
## Time difference of 0.281548 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.311374 secs
```

```
gfit12 = garchFit(~arma(3,3)+garch(2,2),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(3, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 2)
##  ARMA Order:                3 3
##  Max ARMA Order:            3
##  GARCH Order:               2 2
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02200558
```

```
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                       U           V       params includes
##     mu     -0.42203167   0.4220317  0.043417533     TRUE
##     ar1    -0.99999999   1.0000000  0.383923685     TRUE
##     ar2    -0.99999999   1.0000000  0.001167706     TRUE
##     ar3    -0.99999999   1.0000000 -0.677235319     TRUE
##     ma1    -0.99999999   1.0000000 -0.493566554     TRUE
##     ma2    -0.99999999   1.0000000  0.081168715     TRUE
##     ma3    -0.99999999   1.0000000  0.639903273     TRUE
##     omega   0.00000100 100.0000000  0.100000000     TRUE
##     alpha1  0.00000001   1.0000000  0.050000000     TRUE
##     alpha2  0.00000001   1.0000000  0.050000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.100000000    FALSE
##     gamma2 -0.99999999   1.0000000  0.100000000    FALSE
##     beta1   0.00000001   1.0000000  0.400000000     TRUE
##     beta2   0.00000001   1.0000000  0.400000000     TRUE
##     delta   0.00000000   2.0000000  2.000000000    FALSE
##     skew    0.10000000  10.0000000  1.000000000    FALSE
##     shape   1.00000000  10.0000000  4.000000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ar2    ar3    ma1    ma2    ma3  omega alpha1 alpha2  beta1
##      1      2      3      4      5      6      7      8      9     10     13
##  beta2
##     14
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3956.0088: 0.0434175 0.383924 0.00116771 -0.677235 -0.493567 0.0811687 0.639903 0.100000 0.0
##   1:     3943.2643: 0.0434190 0.385438 0.00152714 -0.677087 -0.491956 0.0816287 0.639899 0.0911336 0
##   2:     3924.9263: 0.0434281 0.393817 0.00297282 -0.676199 -0.483314 0.0835851 0.640022 0.0732756 0
##   3:     3921.2476: 0.0434294 0.394168 0.00257358 -0.676105 -0.483140 0.0831332 0.640078 0.0689300 0
##   4:     3918.7542: 0.0434419 0.397675 -0.000629925 -0.674968 -0.481154 0.0795555 0.640924 0.0562381
##   5:     3918.1210: 0.0435027 0.409028 -0.0120743 -0.664344 -0.476552 0.0673830 0.651710 0.0548480 0
##   6:     3914.7583: 0.0435365 0.417387 -0.0124516 -0.659972 -0.470158 0.0672947 0.656197 0.0492735 0
##   7:     3913.6297: 0.0435516 0.416577 -0.0148487 -0.662923 -0.472288 0.0635293 0.652650 0.0499514 0
##   8:     3913.3249: 0.0435736 0.416084 -0.0143417 -0.661007 -0.474106 0.0638570 0.654952 0.0456180 0
##   9:     3913.1783: 0.0435988 0.417877 -0.0148362 -0.662845 -0.473815 0.0626783 0.653057 0.0475287 0
##  10:     3913.0345: 0.0435998 0.417874 -0.0147779 -0.662758 -0.473867 0.0627380 0.653163 0.0467346 0
##  11:     3912.9371: 0.0436078 0.417899 -0.0143426 -0.662131 -0.474230 0.0631756 0.653926 0.0458137 0
##  12:     3912.7560: 0.0436336 0.419199 -0.0138258 -0.662819 -0.474222 0.0633432 0.653343 0.0435288 0
##  13:     3912.5180: 0.0436658 0.419589 -0.0136223 -0.663516 -0.475527 0.0630553 0.652763 0.0430971 0
##  14:     3912.3576: 0.0437091 0.419584 -0.0133325 -0.663198 -0.477613 0.0628373 0.653059 0.0425085 0
##  15:     3912.1466: 0.0437582 0.421441 -0.0126822 -0.662950 -0.477816 0.0631390 0.653139 0.0414913 0
##  16:     3912.0602: 0.0438124 0.423053 -0.0122664 -0.663360 -0.478378 0.0633854 0.652708 0.0393823 0
```

```
##  17:     3911.8810: 0.0439055 0.424102 -0.0121014 -0.663791 -0.480703 0.0637745 0.652309 0.0388078 0
##  18:     3911.7531: 0.0440175 0.425533 -0.0122940 -0.662697 -0.483003 0.0642178 0.653189 0.0388831 0
##  19:     3911.6539: 0.0441305 0.427081 -0.0128610 -0.663326 -0.484585 0.0649855 0.652608 0.0384435 0
##  20:     3911.5776: 0.0442503 0.428458 -0.0132467 -0.664085 -0.486394 0.0660579 0.651786 0.0371510 0
##  21:     3911.4939: 0.0443956 0.430619 -0.0138995 -0.663554 -0.487901 0.0673010 0.651792 0.0368179 0
##  22:     3911.4404: 0.0445418 0.432589 -0.0152786 -0.662913 -0.489233 0.0682519 0.651790 0.0367084 0
##  23:     3911.2636: 0.0453319 0.437603 -0.0195129 -0.663968 -0.500390 0.0770703 0.646979 0.0362377 0
##  24:     3911.2376: 0.0453341 0.437811 -0.0193223 -0.663711 -0.500206 0.0773330 0.647245 0.0363703 0
##  25:     3911.2185: 0.0453498 0.438060 -0.0194526 -0.663528 -0.500255 0.0774607 0.647292 0.0360857 0
##  26:     3911.2007: 0.0453685 0.438344 -0.0196563 -0.663345 -0.500332 0.0775624 0.647316 0.0361988 0
##  27:     3911.1791: 0.0454082 0.438933 -0.0201013 -0.662969 -0.500507 0.0777633 0.647352 0.0359922 0
##  28:     3911.0674: 0.0458232 0.444153 -0.0254134 -0.660237 -0.503118 0.0791097 0.646226 0.0358463 0
##  29:     3910.6092: 0.0478526 0.459811 -0.0363223 -0.650282 -0.520750 0.0984323 0.630864 0.0351483 0
##  30:     3910.6083: 0.0478546 0.459698 -0.0364034 -0.650077 -0.520952 0.0983741 0.631088 0.0348724 0
##  31:     3910.5807: 0.0478550 0.459681 -0.0364204 -0.650032 -0.520989 0.0983636 0.631136 0.0350557 0
##  32:     3910.5627: 0.0478584 0.459568 -0.0365411 -0.649742 -0.521240 0.0982869 0.631450 0.0351369 0
##  33:     3910.5479: 0.0478976 0.459844 -0.0368202 -0.649476 -0.521623 0.0986239 0.631279 0.0348948 0
##  34:     3910.5315: 0.0479377 0.460135 -0.0370988 -0.649233 -0.521992 0.0989809 0.631071 0.0349354 0
##  35:     3910.1897: 0.0502836 0.477516 -0.0526121 -0.636995 -0.542291 0.120998 0.616412 0.0314260 0.0
##  36:     3909.9699: 0.0526555 0.491584 -0.0755978 -0.626203 -0.558843 0.144996 0.603899 0.0336501 0.0
##  37:     3909.3354: 0.0550430 0.507638 -0.0949742 -0.612588 -0.576557 0.166487 0.589138 0.0339694 0.0
##  38:     3908.9025: 0.0573480 0.522349 -0.107202 -0.588183 -0.591638 0.179818 0.563607 0.0337841 0.08
##  39:     3908.5135: 0.0596747 0.536724 -0.134384 -0.582945 -0.606368 0.205648 0.554808 0.0323127 0.09
##  40:     3908.1735: 0.0620127 0.557107 -0.156142 -0.577682 -0.626771 0.228590 0.547194 0.0335177 0.09
##  41:     3907.8874: 0.0643226 0.581079 -0.173548 -0.570881 -0.650633 0.248674 0.538777 0.0342538 0.09
##  42:     3907.6732: 0.0664769 0.580719 -0.201723 -0.553085 -0.654343 0.276785 0.519531 0.0330803 0.09
##  43:     3907.6368: 0.0664791 0.581172 -0.201970 -0.553410 -0.654115 0.276237 0.518929 0.0329952 0.09
##  44:     3907.6228: 0.0664793 0.581213 -0.201970 -0.553399 -0.654098 0.276213 0.518917 0.0328349 0.09
##  45:     3907.6115: 0.0664822 0.581653 -0.201963 -0.553270 -0.653918 0.275962 0.518811 0.0327963 0.09
##  46:     3907.5957: 0.0665234 0.582147 -0.202223 -0.552997 -0.654387 0.276266 0.518561 0.0326398 0.09
##  47:     3907.3414: 0.0705384 0.622575 -0.227294 -0.529744 -0.695103 0.308196 0.495071 0.0350955 0.10
##  48:     3906.9564: 0.0745685 0.653706 -0.267403 -0.510696 -0.726439 0.348417 0.470055 0.0336096 0.10
##  49:     3906.8430: 0.0750823 0.657441 -0.268595 -0.494878 -0.730883 0.351187 0.454092 0.0337726 0.09
##  50:     3906.7421: 0.0755047 0.667114 -0.276443 -0.493068 -0.739634 0.360072 0.451613 0.0331034 0.09
##  51:     3906.7103: 0.0759496 0.676113 -0.286898 -0.487360 -0.749009 0.371056 0.445716 0.0330363 0.09
##  52:     3906.6397: 0.0762694 0.684842 -0.298317 -0.478362 -0.758636 0.383006 0.436825 0.0322676 0.09
##  53:     3906.5924: 0.0765276 0.695687 -0.310687 -0.472092 -0.769435 0.395964 0.428984 0.0324490 0.09
##  54:     3906.5726: 0.0767910 0.705783 -0.323222 -0.463610 -0.779072 0.408340 0.420210 0.0324361 0.09
##  55:     3906.5257: 0.0769987 0.716098 -0.335947 -0.454886 -0.788852 0.420940 0.411303 0.0321209 0.09
##  56:     3906.5058: 0.0770150 0.724634 -0.346902 -0.446404 -0.798498 0.432132 0.402323 0.0319495 0.09
##  57:     3906.4326: 0.0769025 0.766207 -0.399625 -0.410640 -0.837503 0.483084 0.366567 0.0312509 0.09
##  58:     3906.4059: 0.0756049 0.803833 -0.447401 -0.375992 -0.876375 0.532767 0.329774 0.0313514 0.09
##  59:     3906.3968: 0.0733731 0.831083 -0.480798 -0.351521 -0.902595 0.565290 0.304903 0.0313191 0.09
##  60:     3906.3922: 0.0719273 0.865213 -0.527076 -0.321967 -0.936447 0.611178 0.274891 0.0315070 0.09
##  61:     3906.3613: 0.0706620 0.900233 -0.578201 -0.285356 -0.971897 0.662123 0.237276 0.0318532 0.09
##  62:     3906.3486: 0.0702361 0.910165 -0.591525 -0.276201 -0.981898 0.675464 0.227164 0.0317672 0.09
##  63:     3906.3467: 0.0699339 0.920553 -0.604480 -0.266202 -0.992378 0.688766 0.216740 0.0319003 0.09
##  64:     3906.3426: 0.0698817 0.921997 -0.606345 -0.264807 -0.993858 0.690695 0.215324 0.0318114 0.09
##  65:     3906.3345: 0.0697593 0.922646 -0.607809 -0.264266 -0.994572 0.692191 0.214781 0.0319177 0.09
##  66:     3906.3328: 0.0697025 0.924076 -0.609710 -0.262930 -0.996047 0.694088 0.213350 0.0318876 0.09
##  67:     3906.3254: 0.0696467 0.927812 -0.613724 -0.260464 -1.00000 0.698742 0.210266 0.0318866 0.090
##  68:     3906.3211: 0.0699177 0.927893 -0.611888 -0.262474 -1.00000 0.697336 0.211808 0.0316351 0.090
##  69:     3906.3204: 0.0698340 0.928429 -0.614078 -0.261678 -1.00000 0.698495 0.211618 0.0315310 0.090
##  70:     3906.3202: 0.0697861 0.928500 -0.613560 -0.261581 -1.00000 0.697940 0.211635 0.0316502 0.090
```

```
##  71:     3906.3201: 0.0698043 0.928479 -0.613748 -0.261610 -1.00000 0.698149 0.211630 0.0316017 0.090
##  72:     3906.3201: 0.0698046 0.928482 -0.613746 -0.261611 -1.00000 0.698157 0.211627 0.0316102 0.090
##  73:     3906.3201: 0.0698045 0.928483 -0.613744 -0.261611 -1.00000 0.698157 0.211627 0.0316038 0.090
##  74:     3906.3201: 0.0698040 0.928489 -0.613732 -0.261611 -1.00000 0.698155 0.211622 0.0316052 0.090
##  75:     3906.3201: 0.0698026 0.928490 -0.613717 -0.261608 -1.00000 0.698141 0.211622 0.0316054 0.090
##  76:     3906.3201: 0.0698013 0.928492 -0.613702 -0.261605 -1.00000 0.698127 0.211622 0.0316092 0.090
##  77:     3906.3201: 0.0697990 0.928494 -0.613677 -0.261600 -1.00000 0.698105 0.211623 0.0316122 0.090
##  78:     3906.3201: 0.0698013 0.928479 -0.613696 -0.261602 -1.00000 0.698132 0.211608 0.0316045 0.090
##  79:     3906.3201: 0.0698013 0.928479 -0.613695 -0.261602 -1.00000 0.698133 0.211608 0.0316055 0.090
##  80:     3906.3201: 0.0698013 0.928478 -0.613694 -0.261601 -1.00000 0.698133 0.211607 0.0316048 0.090
##  81:     3906.3201: 0.0698013 0.928478 -0.613694 -0.261601 -1.00000 0.698133 0.211607 0.0316054 0.090
##  82:     3906.3201: 0.0698013 0.928477 -0.613693 -0.261601 -1.00000 0.698133 0.211606 0.0316046 0.090
##  83:     3906.3201: 0.0698012 0.928476 -0.613691 -0.261601 -1.00000 0.698133 0.211605 0.0316052 0.090
##  84:     3906.3200: 0.0697958 0.928427 -0.613618 -0.261578 -1.00000 0.698112 0.211558 0.0316067 0.090
##  85:     3906.3200: 0.0698048 0.928390 -0.613639 -0.261591 -1.00000 0.698170 0.211537 0.0315971 0.090
##  86:     3906.3200: 0.0698052 0.928402 -0.613627 -0.261587 -1.00000 0.698147 0.211543 0.0315995 0.090
##  87:     3906.3200: 0.0698003 0.928404 -0.613626 -0.261588 -1.00000 0.698143 0.211546 0.0316004 0.090
##
## Final Estimate of the Negative LLH:
##  LLH:  -7611.753    norm LLH:  -2.522118
##           mu           ar1           ar2           ar3           ma1
##  1.535997e-03  9.284043e-01 -6.136257e-01 -2.615880e-01 -1.000000e+00
##          ma2           ma3         omega        alpha1        alpha2
##  6.981431e-01  2.115462e-01  1.530236e-05  9.631766e-02  1.000000e-08
##         beta1         beta2
##  4.160670e-01  4.525696e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1           ar2           ar3           ma1
## mu     -1.156710e+07   -14330.0323 -1.697685e+04   -19542.3627     6271.4320
## ar1    -1.433003e+04   -33616.6756 -2.091270e+04     7152.1817   -43500.2675
## ar2    -1.697685e+04   -20912.7007 -3.408816e+04   -20531.2975   -29571.2506
## ar3    -1.954236e+04     7152.1817 -2.053130e+04   -33802.2778     6854.5484
## ma1     6.271432e+03   -43500.2675 -2.957125e+04     6854.5484   -59950.9257
## ma2     3.236037e+03   -23474.0549 -4.230785e+04   -27889.5817   -34894.7589
## ma3    -5.781241e+02    13549.5021 -2.217810e+04   -42414.4319    16312.1999
## omega  -5.147407e+07 -5232121.9132 -5.819452e+06 -2666712.5258 -6772203.8092
## alpha1 -1.152755e+03     -247.8113  6.696483e+01      225.0257     -224.1225
## alpha2 -6.399300e+03     -591.1977 -1.822217e+02      201.8476     -511.8246
## beta1  -2.243634e+04    -1609.7603 -1.330259e+03     -470.3091    -1908.4995
## beta2  -2.322629e+04    -1688.9952 -1.400565e+03     -509.1349    -2013.2212
##                  ma2           ma3         omega        alpha1        alpha2
## mu      3.236037e+03     -578.1241 -5.147407e+07 -1.152755e+03 -6.399300e+03
## ar1    -2.347405e+04    13549.5021 -5.232122e+06 -2.478113e+02 -5.911977e+02
## ar2    -4.230785e+04   -22178.1007 -5.819452e+06  6.696483e+01 -1.822217e+02
## ar3    -2.788958e+04   -42414.4319 -2.666713e+06  2.250257e+02  2.018476e+02
## ma1    -3.489476e+04    16312.1999 -6.772204e+06 -2.241225e+02 -5.118246e+02
## ma2    -5.481339e+04   -32386.8869 -7.535161e+06  1.694494e+02  2.218629e+01
## ma3    -3.238689e+04   -57958.7196 -3.121039e+06  3.397435e+02  4.286370e+02
## omega  -7.535161e+06 -3121039.0904 -1.007653e+12 -2.022763e+08 -2.097591e+08
## alpha1  1.694494e+02      339.7435 -2.022763e+08 -7.195277e+04 -7.264855e+04
## alpha2  2.218629e+01      428.6370 -2.097591e+08 -7.264855e+04 -7.638229e+04
## beta1  -1.521714e+03     -357.1275 -2.919051e+08 -7.992812e+04 -8.334514e+04
## beta2  -1.612708e+03     -396.5811 -2.938448e+08 -8.022188e+04 -8.354448e+04
```

```
##                beta1         beta2
## mu     -2.243634e+04 -2.322629e+04
## ar1    -1.609760e+03 -1.688995e+03
## ar2    -1.330259e+03 -1.400565e+03
## ar3    -4.703091e+02 -5.091349e+02
## ma1    -1.908500e+03 -2.013221e+03
## ma2    -1.521714e+03 -1.612708e+03
## ma3    -3.571275e+02 -3.965811e+02
## omega  -2.919051e+08 -2.938448e+08
## alpha1 -7.992812e+04 -8.022188e+04
## alpha2 -8.334514e+04 -8.354448e+04
## beta1  -1.016340e+05 -1.022393e+05
## beta2  -1.022393e+05 -1.028608e+05
## attr(,"time")
## Time difference of 0.311347 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 1.85738 secs
```

```
summary(gfit1)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x7ffe04455438>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1
## 0.00137177  0.00033386  0.29957555
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.372e-03   3.522e-04    3.895 9.82e-05 ***
## omega  3.339e-04   1.118e-05   29.873  < 2e-16 ***
## alpha1 2.996e-01   3.144e-02    9.529  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
```

```
##  7430.496    normalized:  2.46206
##
## Description:
##  Sun Apr 17 22:16:02 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2964.63   0
##  Shapiro-Wilk Test  R    W      0.9455149 0
##  Ljung-Box Test     R    Q(10)  14.3517   0.1575454
##  Ljung-Box Test     R    Q(15)  19.70402  0.1835833
##  Ljung-Box Test     R    Q(20)  25.66359  0.1772121
##  Ljung-Box Test     R^2  Q(10)  100.8347  0
##  Ljung-Box Test     R^2  Q(15)  176.7088  0
##  Ljung-Box Test     R^2  Q(20)  203.5581  0
##  LM Arch Test       R    TR^2   101.3707  3.330669e-16
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.922131 -4.916155 -4.922133 -4.919982
```

```
summary(gfit2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7ffe032f5838>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 1.6229e-03  1.3956e-05  8.3249e-02  8.8610e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.623e-03   3.230e-04    5.024 5.06e-07 ***
## omega  1.396e-05   2.933e-06    4.758 1.95e-06 ***
## alpha1 8.325e-02   1.274e-02    6.534 6.41e-11 ***
## beta1  8.861e-01   1.698e-02   52.197  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Log Likelihood:
##  7599.491    normalized:  2.518055
##
## Description:
##  Sun Apr 17 22:16:02 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2413.331  0
##  Shapiro-Wilk Test  R    W      0.9556894 0
##  Ljung-Box Test     R    Q(10)  23.61874  0.00867931
##  Ljung-Box Test     R    Q(15)  24.89168  0.0514172
##  Ljung-Box Test     R    Q(20)  26.83161  0.1400683
##  Ljung-Box Test     R^2  Q(10)  6.134568  0.8038335
##  Ljung-Box Test     R^2  Q(15)  7.945045  0.925967
##  Ljung-Box Test     R^2  Q(20)  12.15692  0.910569
##  LM Arch Test       R    TR^2   6.950795  0.860848
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.033460 -5.025491 -5.033464 -5.030595
```

```
summary(gfit3)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 0), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(2, 0)
## <environment: 0x7ffdea87a3f8>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2
## 0.00139782  0.00027305  0.19071310  0.26332337
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0013978   0.0003393    4.120 3.79e-05 ***
## omega  0.0002730   0.0000116   23.545  < 2e-16 ***
## alpha1 0.1907131   0.0290607    6.563 5.29e-11 ***
## alpha2 0.2633234   0.0391462    6.727 1.74e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Log Likelihood:
##  7476.049    normalized:  2.477153
##
## Description:
##  Sun Apr 17 22:16:02 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2426.505  0
##  Shapiro-Wilk Test  R    W      0.9500436 0
##  Ljung-Box Test     R    Q(10)  11.04426  0.35408
##  Ljung-Box Test     R    Q(15)  13.80287  0.5405297
##  Ljung-Box Test     R    Q(20)  15.79841  0.7290494
##  Ljung-Box Test     R^2  Q(10)  54.92043  3.267996e-08
##  Ljung-Box Test     R^2  Q(15)  104.9732  1.44329e-15
##  Ljung-Box Test     R^2  Q(20)  117.5986  7.771561e-16
##  LM Arch Test       R    TR^2   61.75385  1.079099e-08
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.951656 -4.943687 -4.951660 -4.948791
```

```
summary(gfit4)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 1), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(2, 1)
## <environment: 0x7ffdea205cf8>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2       beta1
## 1.6302e-03  1.3756e-05  8.1812e-02  1.0000e-08  8.8778e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.630e-03   3.236e-04    5.038 4.71e-07 ***
## omega  1.376e-05   3.825e-06    3.596 0.000323 ***
## alpha1 8.181e-02   1.773e-02    4.613 3.96e-06 ***
## alpha2 1.000e-08   2.598e-02    0.000 1.000000
## beta1  8.878e-01   2.452e-02   36.204  < 2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7598.637    normalized:  2.517772
##
## Description:
##  Sun Apr 17 22:16:03 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2433.143  0
##  Shapiro-Wilk Test  R    W      0.955488  0
##  Ljung-Box Test     R    Q(10)  23.79947  0.008150813
##  Ljung-Box Test     R    Q(15)  25.08415  0.04882468
##  Ljung-Box Test     R    Q(20)  27.01846  0.1347453
##  Ljung-Box Test     R^2  Q(10)  6.071797  0.8091955
##  Ljung-Box Test     R^2  Q(15)  7.859352  0.9292975
##  Ljung-Box Test     R^2  Q(20)  11.9935   0.9162996
##  LM Arch Test       R    TR^2   6.943438  0.8613287
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.032231 -5.022270 -5.032237 -5.028649
```

```
summary(gfit5)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 2), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 2)
## <environment: 0x7ffe06987000>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       beta2
## 0.00161764  0.00001671  0.10210624  0.43389889  0.42624283
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.618e-03   3.236e-04    4.999 5.77e-07 ***
## omega  1.671e-05   3.827e-06    4.367 1.26e-05 ***
## alpha1 1.021e-01   1.809e-02    5.644 1.66e-08 ***
```

```
## beta1  4.339e-01   3.005e-01    1.444    0.149
## beta2  4.262e-01   2.874e-01    1.483    0.138
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7599.97    normalized:  2.518214
##
## Description:
##  Sun Apr 17 22:16:03 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2458.816  0
##  Shapiro-Wilk Test  R    W      0.9552953 0
##  Ljung-Box Test     R    Q(10)  23.68236  0.008489648
##  Ljung-Box Test     R    Q(15)  25.06931  0.04902035
##  Ljung-Box Test     R    Q(20)  27.31732  0.1265701
##  Ljung-Box Test     R^2  Q(10)  4.371237  0.9290499
##  Ljung-Box Test     R^2  Q(15)  6.153824  0.9770464
##  Ljung-Box Test     R^2  Q(20)  10.25717  0.9632655
##  LM Arch Test       R    TR^2   5.051767  0.9562275
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.033115 -5.023154 -5.033120 -5.029533
```

```
summary(gfit6)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 2), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(2, 2)
## <environment: 0x7ffde8363588>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2       beta1       beta2
## 1.5761e-03  2.2725e-05  9.0373e-02  4.8039e-02  1.0000e-08  8.1062e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.576e-03   3.193e-04    4.936 7.99e-07 ***
```

```
## omega  2.272e-05   4.989e-06    4.555 5.24e-06 ***
## alpha1 9.037e-02   1.225e-02    7.375 1.65e-13 ***
## alpha2 4.804e-02         NaN      NaN      NaN
## beta1  1.000e-08         NaN      NaN      NaN
## beta2  8.106e-01         NaN      NaN      NaN
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7600.549    normalized:  2.518406
##
## Description:
##  Sun Apr 17 22:16:03 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2319.783  0
##  Shapiro-Wilk Test  R    W      0.9563619 0
##  Ljung-Box Test     R    Q(10)  24.05746  0.007449338
##  Ljung-Box Test     R    Q(15)  25.38136  0.04505045
##  Ljung-Box Test     R    Q(20)  27.57852  0.1197582
##  Ljung-Box Test     R^2  Q(10)  5.199817  0.8774364
##  Ljung-Box Test     R^2  Q(15)  6.64619   0.9667347
##  Ljung-Box Test     R^2  Q(20)  10.82893  0.9505388
##  LM Arch Test       R    TR^2   6.190534  0.9061741
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.032835 -5.020882 -5.032843 -5.028537
```

```
summary(gfit7)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(3, 3) + garch(1, 0), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ arma(3, 3) + garch(1, 0)
## <environment: 0x7ffe05f8b5b8>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ar2          ar3          ma1          ma2
##  0.00130202   0.94079337  -0.63195464  -0.23804144  -0.99999999   0.69237877
##         ma3        omega       alpha1
##  0.20988048   0.00033247   0.29719071
##
## Std. Errors:
```

```
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.302e-03   2.838e-04    4.588 4.48e-06 ***
## ar1     9.408e-01         NaN      NaN      NaN
## ar2    -6.320e-01         NaN      NaN      NaN
## ar3    -2.380e-01         NaN      NaN      NaN
## ma1    -1.000e+00         NaN      NaN      NaN
## ma2     6.924e-01         NaN      NaN      NaN
## ma3     2.099e-01         NaN      NaN      NaN
## omega   3.325e-04   1.105e-05   30.097  < 2e-16 ***
## alpha1  2.972e-01   3.136e-02    9.476  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7441.683    normalized:  2.465766
##
## Description:
##  Sun Apr 17 22:16:04 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  3024.769  0
##  Shapiro-Wilk Test  R    W      0.9448529 0
##  Ljung-Box Test     R    Q(10)  5.909785  0.8227849
##  Ljung-Box Test     R    Q(15)  11.40206  0.7236071
##  Ljung-Box Test     R    Q(20)  16.3524   0.6945352
##  Ljung-Box Test     R^2  Q(10)  97.80357  1.110223e-16
##  Ljung-Box Test     R^2  Q(15)  174.9526  0
##  Ljung-Box Test     R^2  Q(20)  200.138   0
##  LM Arch Test       R    TR^2   101.3389  3.330669e-16
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.925568 -4.907639 -4.925586 -4.919121
```

```
summary(gfit8)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(3, 3) + garch(1, 1), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ arma(3, 3) + garch(1, 1)
## <environment: 0x7ffde8116898>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
```

```
##
## Coefficient(s):
##          mu          ar1          ar2          ar3          ma1          ma2
##  1.7556e-03   7.1768e-01  -3.4302e-01  -4.4949e-01  -7.8646e-01   4.2133e-01
##         ma3        omega       alpha1        beta1
##  4.1085e-01   1.2404e-05   7.5478e-02   8.9673e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.756e-03   4.366e-04    4.021 5.80e-05 ***
## ar1     7.177e-01   2.590e-01    2.771  0.00559 **
## ar2    -3.430e-01   3.139e-01   -1.093  0.27443
## ar3    -4.495e-01   2.205e-01   -2.038  0.04154 *
## ma1    -7.865e-01   2.644e-01   -2.974  0.00294 **
## ma2     4.213e-01   3.276e-01    1.286  0.19845
## ma3     4.109e-01   2.374e-01    1.731  0.08354 .
## omega   1.240e-05   2.706e-06    4.584 4.57e-06 ***
## alpha1  7.548e-02   1.189e-02    6.347 2.20e-10 ***
## beta1   8.967e-01   1.596e-02   56.196  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7610.049    normalized:  2.521554
##
## Description:
##  Sun Apr 17 22:16:06 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2597.793  0
##  Shapiro-Wilk Test  R    W      0.9539386 0
##  Ljung-Box Test     R    Q(10)  4.083457  0.9435035
##  Ljung-Box Test     R    Q(15)  7.6856    0.935768
##  Ljung-Box Test     R    Q(20)  9.502234  0.9763265
##  Ljung-Box Test     R^2  Q(10)  4.963435  0.8936072
##  Ljung-Box Test     R^2  Q(15)  6.815394  0.9625713
##  Ljung-Box Test     R^2  Q(20)  10.80479  0.9511285
##  LM Arch Test       R    TR^2   5.861159  0.9229184
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.036480 -5.016559 -5.036502 -5.029317
```

```
summary(gfit9)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
```

```
##  garchFit(formula = ~arma(3, 3) + garch(2, 0), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ arma(3, 3) + garch(2, 0)
## <environment: 0x7ffde8fdb4d8>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ar2          ar3          ma1          ma2
##  0.00152586   0.64807124  -0.21642532  -0.54878171  -0.68958849   0.26146190
##         ma3        omega       alpha1       alpha2
##  0.53890646   0.00026683   0.18651302   0.28409153
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.526e-03   3.999e-04    3.815 0.000136 ***
## ar1     6.481e-01   1.458e-01    4.444 8.83e-06 ***
## ar2    -2.164e-01   1.782e-01   -1.215 0.224465
## ar3    -5.488e-01   1.293e-01   -4.243 2.20e-05 ***
## ma1    -6.896e-01   1.572e-01   -4.388 1.15e-05 ***
## ma2     2.615e-01   1.972e-01    1.326 0.184796
## ma3     5.389e-01   1.467e-01    3.674 0.000239 ***
## omega   2.668e-04   1.149e-05   23.233  < 2e-16 ***
## alpha1  1.865e-01   2.863e-02    6.515 7.28e-11 ***
## alpha2  2.841e-01   4.029e-02    7.051 1.78e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7490.434    normalized:  2.48192
##
## Description:
##  Sun Apr 17 22:16:07 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2374.294  0
##  Shapiro-Wilk Test  R    W      0.9503444 0
##  Ljung-Box Test     R    Q(10)  1.208155  0.9995933
##  Ljung-Box Test     R    Q(15)  6.149589  0.9771239
##  Ljung-Box Test     R    Q(20)  7.381926  0.9952312
##  Ljung-Box Test     R^2  Q(10)  51.66705  1.315019e-07
##  Ljung-Box Test     R^2  Q(15)  99.91508  1.354472e-14
##  Ljung-Box Test     R^2  Q(20)  112.4027  7.105427e-15
##  LM Arch Test       R    TR^2   59.17867  3.185104e-08
##
## Information Criterion Statistics:
```

```
##       AIC       BIC       SIC      HQIC
## -4.957213 -4.937291 -4.957235 -4.950049
```

```
summary(gfit10)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(3, 3) + garch(2, 1), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ arma(3, 3) + garch(2, 1)
## <environment: 0x7ffdedde6d30>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ar2          ar3          ma1          ma2
##  1.7343e-03   7.3734e-01  -3.6678e-01  -4.3268e-01  -8.0626e-01   4.4585e-01
##         ma3        omega       alpha1       alpha2        beta1
##  3.9300e-01   1.2511e-05   7.6132e-02   1.0000e-08   8.9585e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      1.734e-03   4.531e-04    3.828 0.000129 ***
## ar1     7.373e-01   2.856e-01    2.582 0.009821 **
## ar2    -3.668e-01   3.505e-01   -1.046 0.295341
## ar3    -4.327e-01   2.460e-01   -1.759 0.078603 .
## ma1    -8.063e-01   2.912e-01   -2.769 0.005629 **
## ma2     4.459e-01   3.644e-01    1.224 0.221077
## ma3     3.930e-01   2.639e-01    1.489 0.136385
## omega   1.251e-05   3.493e-06    3.581 0.000342 ***
## alpha1  7.613e-02   1.816e-02    4.193 2.75e-05 ***
## alpha2  1.000e-08   2.570e-02    0.000 1.000000
## beta1   8.958e-01   2.264e-02   39.577  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7610.655    normalized:  2.521754
##
## Description:
##  Sun Apr 17 22:16:10 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2582.423  0
```

```
##  Shapiro-Wilk Test  R    W      0.9540882 0
##  Ljung-Box Test     R    Q(10)  4.233411  0.9362037
##  Ljung-Box Test     R    Q(15)  7.79244   0.9318341
##  Ljung-Box Test     R    Q(20)  9.60433   0.9747924
##  Ljung-Box Test     R^2  Q(10)  5.031755  0.8890467
##  Ljung-Box Test     R^2  Q(15)  6.879477  0.9609082
##  Ljung-Box Test     R^2  Q(20)  10.90898  0.9485489
##  LM Arch Test       R    TR^2   5.885438  0.9217465
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.036219 -5.014305 -5.036246 -5.028339
```

```
summary(gfit11)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(3, 3) + garch(1, 2), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ arma(3, 3) + garch(1, 2)
## <environment: 0x7ffdedebd9a8>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ar2          ar3          ma1          ma2
##  1.5358e-03   9.2837e-01  -6.1347e-01  -2.6168e-01  -1.0000e+00   6.9805e-01
##         ma3        omega       alpha1        beta1        beta2
##  2.1160e-01   1.5294e-05   9.6277e-02   4.1631e-01   4.5239e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      1.536e-03   2.290e-04    6.706 2.01e-11 ***
## ar1     9.284e-01         NaN      NaN      NaN
## ar2    -6.135e-01         NaN      NaN      NaN
## ar3    -2.617e-01         NaN      NaN      NaN
## ma1    -1.000e+00         NaN      NaN      NaN
## ma2     6.981e-01         NaN      NaN      NaN
## ma3     2.116e-01         NaN      NaN      NaN
## omega   1.529e-05   3.692e-06    4.143 3.43e-05 ***
## alpha1  9.628e-02   1.874e-02    5.139 2.77e-07 ***
## beta1   4.163e-01   3.871e-01    1.075    0.282
## beta2   4.524e-01   3.712e-01    1.219    0.223
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Log Likelihood:
##  7611.753    normalized:  2.522118
##
## Description:
##  Sun Apr 17 22:16:11 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2597.582  0
##  Shapiro-Wilk Test  R    W      0.9540093 0
##  Ljung-Box Test     R    Q(10)  6.50768   0.770961
##  Ljung-Box Test     R    Q(15)  9.835726  0.8299439
##  Ljung-Box Test     R    Q(20)  11.88103  0.9201099
##  Ljung-Box Test     R^2  Q(10)  3.536478  0.9658452
##  Ljung-Box Test     R^2  Q(15)  5.27005   0.9895825
##  Ljung-Box Test     R^2  Q(20)  9.170993  0.9808408
##  LM Arch Test       R    TR^2   4.181233  0.9799395
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.036947 -5.015033 -5.036974 -5.029067
```

```
summary(gfit12)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(3, 3) + garch(2, 2), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ arma(3, 3) + garch(2, 2)
## <environment: 0x7ffdf01e1468>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ar2          ar3          ma1          ma2
##  1.5360e-03   9.2840e-01  -6.1363e-01  -2.6159e-01  -1.0000e+00   6.9814e-01
##         ma3        omega       alpha1       alpha2        beta1        beta2
##  2.1155e-01   1.5302e-05   9.6318e-02   1.0000e-08   4.1607e-01   4.5257e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.536e-03   2.286e-04    6.718 1.84e-11 ***
## ar1     9.284e-01         NaN      NaN      NaN
## ar2    -6.136e-01         NaN      NaN      NaN
## ar3    -2.616e-01         NaN      NaN      NaN
```

```
## ma1    -1.000e+00         NaN      NaN      NaN
## ma2     6.981e-01         NaN      NaN      NaN
## ma3     2.115e-01         NaN      NaN      NaN
## omega   1.530e-05   1.154e-06   13.264  < 2e-16 ***
## alpha1  9.632e-02   1.788e-02    5.388 7.12e-08 ***
## alpha2  1.000e-08         NaN      NaN      NaN
## beta1   4.161e-01         NaN      NaN      NaN
## beta2   4.526e-01         NaN      NaN      NaN
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7611.753    normalized:  2.522118
##
## Description:
##  Sun Apr 17 22:16:13 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2597.437  0
##  Shapiro-Wilk Test  R    W      0.9540103 0
##  Ljung-Box Test     R    Q(10)  6.511249  0.770639
##  Ljung-Box Test     R    Q(15)  9.83811   0.8297977
##  Ljung-Box Test     R    Q(20)  11.88293  0.9200464
##  Ljung-Box Test     R^2  Q(10)  3.536604  0.9658409
##  Ljung-Box Test     R^2  Q(15)  5.270639  0.9895764
##  Ljung-Box Test     R^2  Q(20)  9.171513  0.9808342
##  LM Arch Test       R    TR^2   4.181312  0.9799378
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.036285 -5.012379 -5.036316 -5.027688
```

According to the AIC(no big difference) and the significance, we find that generally we should have a ARMA model as mean, here the best model is GARCH(1,1)

```
gfit = garchFit(~garch(1,1),data = diff(log(apple_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
```

```
##  Series Scale:              0.02200558
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.42203167   0.4220317 0.04220317     TRUE
##     omega    0.00000100 100.0000000 0.10000000     TRUE
##     alpha1   0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1    0.00000001   1.0000000 0.80000000     TRUE
##     delta    0.00000000   2.0000000 2.00000000    FALSE
##     skew     0.10000000  10.0000000 1.00000000    FALSE
##     shape    1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1
##      1      2      3      5
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3956.9207: 0.0422032 0.100000 0.100000 0.800000
##   1:     3951.7454: 0.0422071 0.0774769 0.0988866 0.788793
##   2:     3934.4556: 0.0422156 0.0757252 0.121400 0.799936
##   3:     3931.8868: 0.0422329 0.0533463 0.131919 0.804679
##   4:     3928.4349: 0.0422803 0.0503244 0.132421 0.829649
##   5:     3924.0138: 0.0423708 0.0387574 0.117234 0.845930
##   6:     3921.8326: 0.0424971 0.0368799 0.0993199 0.863272
##   7:     3921.8056: 0.0424982 0.0364912 0.0991010 0.863258
##   8:     3921.7765: 0.0425019 0.0364931 0.0989519 0.864145
##   9:     3921.6685: 0.0425120 0.0355007 0.0980850 0.865360
##  10:     3921.3766: 0.0425691 0.0331671 0.0951336 0.871955
##  11:     3921.2451: 0.0428320 0.0316229 0.0916603 0.874450
##  12:     3920.9999: 0.0431374 0.0323339 0.0900277 0.876443
##  13:     3920.3794: 0.0467049 0.0291091 0.0801406 0.887425
##  14:     3920.0006: 0.0503196 0.0264704 0.0800513 0.893026
##  15:     3919.6174: 0.0539395 0.0269423 0.0814479 0.889132
##  16:     3919.2279: 0.0575542 0.0297721 0.0844164 0.884473
##  17:     3918.9919: 0.0611769 0.0298267 0.0857209 0.882454
##  18:     3918.8595: 0.0647962 0.0279662 0.0850417 0.886469
##  19:     3918.6968: 0.0684184 0.0292835 0.0857577 0.884142
##  20:     3918.5939: 0.0720387 0.0284228 0.0828569 0.886756
##  21:     3918.5827: 0.0740275 0.0288860 0.0833149 0.886017
##  22:     3918.5822: 0.0737546 0.0288236 0.0832628 0.886087
##  23:     3918.5822: 0.0737460 0.0288183 0.0832446 0.886109
##  24:     3918.5822: 0.0737480 0.0288192 0.0832491 0.886104
##
## Final Estimate of the Negative LLH:
```

```
##  LLH:  -7599.491    norm LLH:  -2.518055
##           mu        omega       alpha1        beta1
## 1.622869e-03 1.395559e-05 8.324908e-02 8.861041e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1         beta1
## mu      -9627339.967 -5.917409e+07 -3.096645e+03     -23001.54
## omega  -59174092.978 -1.332843e+12 -2.589564e+08 -386654965.12
## alpha1     -3096.645 -2.589564e+08 -9.236301e+04    -104269.78
## beta1     -23001.542 -3.866550e+08 -1.042698e+05    -135854.84
## attr(,"time")
## Time difference of 0.02533293 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.1103091 secs
```

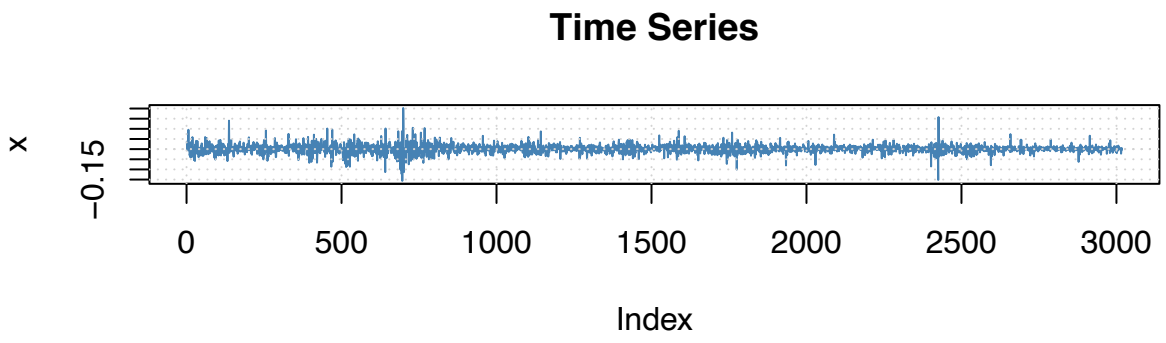
```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

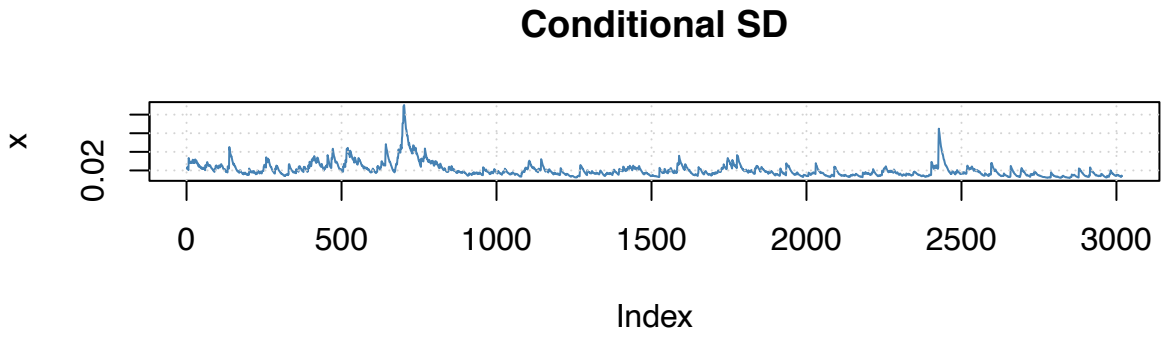
```
summary(gfit)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = diff(log(apple_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7ffdecafe728>
##  [data = diff(log(apple_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 1.6229e-03  1.3956e-05  8.3249e-02  8.8610e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.623e-03   3.230e-04    5.024 5.06e-07 ***
## omega  1.396e-05   2.933e-06    4.758 1.95e-06 ***
## alpha1 8.325e-02   1.274e-02    6.534 6.41e-11 ***
## beta1  8.861e-01   1.698e-02   52.197  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Log Likelihood:
##  7599.491    normalized:  2.518055
##
## Description:
##  Sun Apr 17 22:16:13 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  2413.331  0
##  Shapiro-Wilk Test  R    W      0.9556894 0
##  Ljung-Box Test     R    Q(10)  23.61874  0.00867931
##  Ljung-Box Test     R    Q(15)  24.89168  0.0514172
##  Ljung-Box Test     R    Q(20)  26.83161  0.1400683
##  Ljung-Box Test     R^2  Q(10)  6.134568  0.8038335
##  Ljung-Box Test     R^2  Q(15)  7.945045  0.925967
##  Ljung-Box Test     R^2  Q(20)  12.15692  0.910569
##  LM Arch Test       R    TR^2   6.950795  0.860848
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.033460 -5.025491 -5.033464 -5.030595
```

```
par(mfrow=c(2,1))
plot(gfit, which = 1)
plot(gfit, which = 2)
```

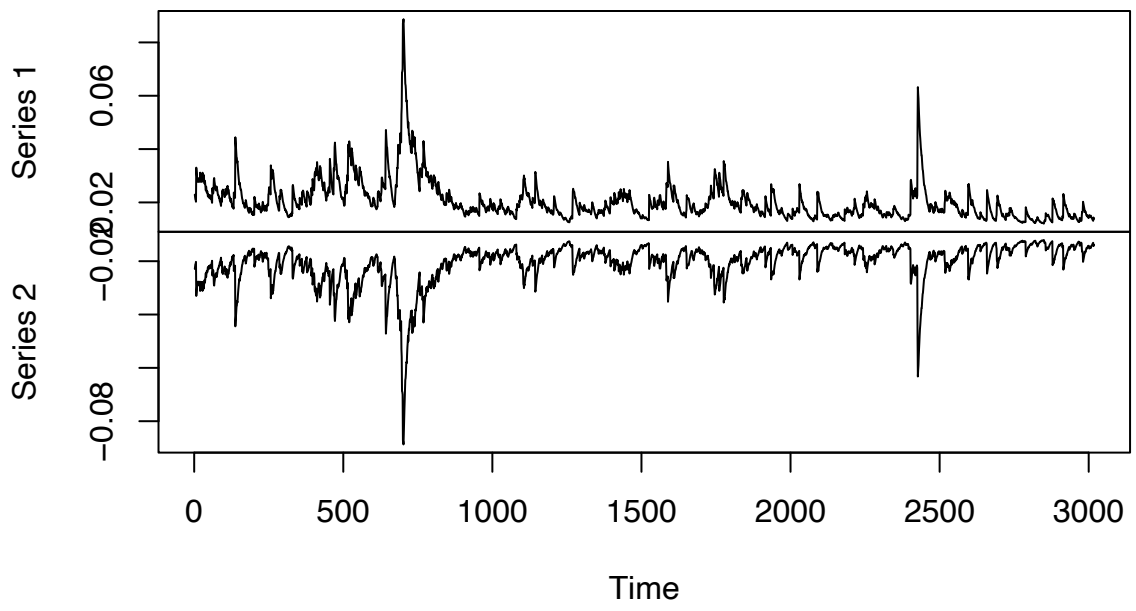




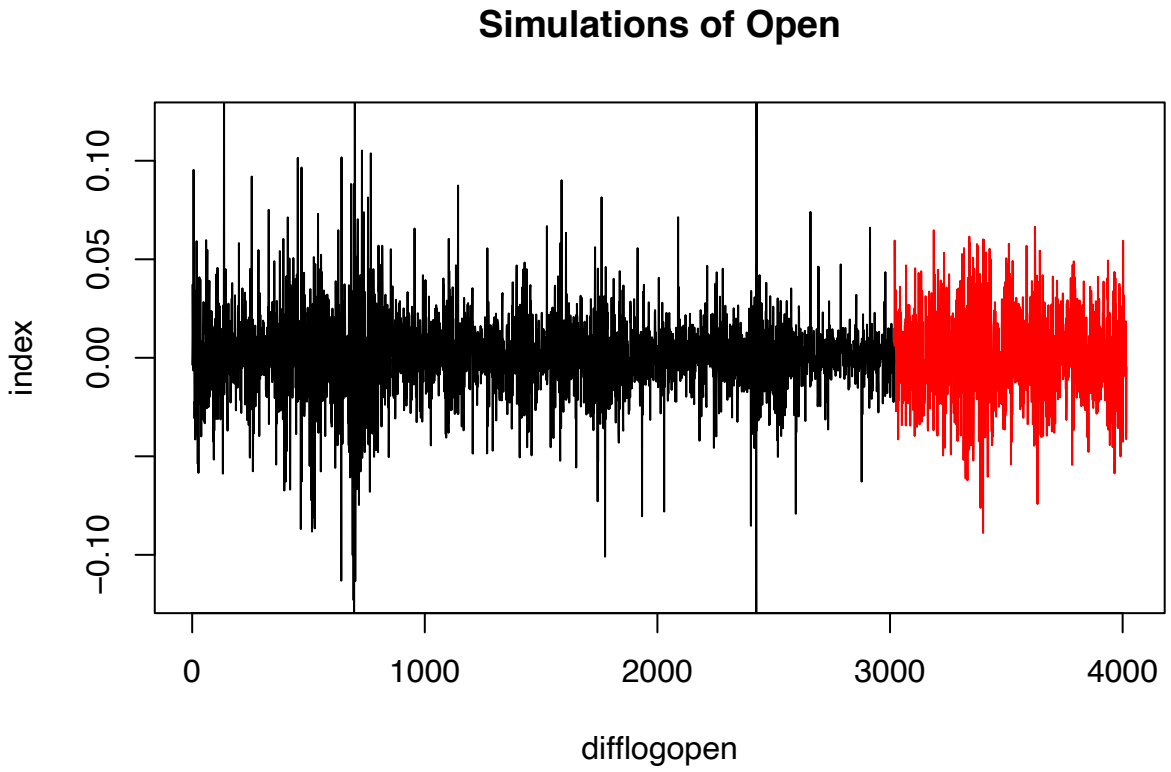

```
gfit = garch(diff(log(apple_open)),order = c(1,1))
```

```
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
```

```
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     4.358212e-04     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1 -1.012e+04
##      1    7 -1.012e+04  8.37e-04  1.41e-03  4.3e-04  7.8e+09  4.3e-05  5.53e+06
##      2    8 -1.012e+04  1.52e-05  1.85e-05  4.2e-04  2.0e+00  4.3e-05  7.04e+01
##      3   14 -1.016e+04  3.64e-03  5.11e-03  3.0e-01  2.0e+00  4.4e-02  6.97e+01
##      4   20 -1.016e+04  1.94e-05  4.18e-05  3.8e-05  9.2e+00  7.2e-06  8.16e-01
##      5   21 -1.016e+04  3.70e-07  3.65e-07  3.8e-05  2.0e+00  7.2e-06  8.31e-01
##      6   30 -1.020e+04  3.36e-03  2.99e-03  6.9e-01  2.0e+00  2.4e-01  8.29e-01
##      7   32 -1.026e+04  6.25e-03  7.84e-03  4.5e-01  2.0e+00  4.7e-01  1.58e+01
##      8   42 -1.029e+04  2.49e-03  2.92e-02  2.9e-05  3.0e+00  4.4e-05  7.62e-02
##      9   44 -1.032e+04  2.93e-03  1.70e-03  1.2e-05  2.0e+00  2.2e-05  2.93e-02
##     10   45 -1.032e+04  4.07e-04  2.51e-03  1.4e-05  2.1e+00  2.2e-05  8.43e-01
##     11   46 -1.033e+04  9.82e-04  1.67e-03  1.2e-05  2.0e+00  2.2e-05  9.42e-02
##     12   47 -1.033e+04  1.13e-04  8.71e-05  1.4e-05  2.0e+00  2.2e-05  1.79e-01
##     13   48 -1.033e+04  8.98e-06  9.18e-06  1.4e-05  2.0e+00  2.2e-05  2.21e-01
##     14   55 -1.035e+04  1.70e-03  3.38e-03  5.5e-02  2.0e+00  8.9e-02  2.17e-01
##     15   66 -1.035e+04  2.34e-04  1.01e-03  3.2e-06  2.6e+00  5.4e-06  1.34e-03
##     16   67 -1.035e+04  1.62e-04  1.31e-04  2.8e-06  2.0e+00  5.4e-06  3.88e-04
##     17   68 -1.035e+04  6.08e-06  6.49e-06  2.8e-06  2.0e+00  5.4e-06  1.92e-04
##     18   69 -1.035e+04  2.09e-07  2.58e-07  2.7e-06  2.0e+00  5.4e-06  1.90e-04
##     19   75 -1.035e+04  1.15e-04  1.28e-04  2.7e-03  7.1e-01  5.5e-03  1.90e-04
##     20   78 -1.036e+04  2.56e-04  1.88e-04  1.4e-02  0.0e+00  3.3e-02  1.88e-04
##     21   79 -1.036e+04  9.25e-05  8.23e-05  1.4e-02  0.0e+00  2.9e-02  8.23e-05
##     22   80 -1.036e+04  3.18e-06  2.04e-05  1.3e-03  0.0e+00  3.1e-03  2.04e-05
##     23   81 -1.036e+04  8.09e-06  8.90e-06  1.1e-03  0.0e+00  2.0e-03  8.90e-06
##     24   82 -1.036e+04  1.33e-07  1.49e-07  3.4e-04  0.0e+00  8.1e-04  1.49e-07
##     25   83 -1.036e+04  5.91e-08  2.05e-08  4.7e-05  0.0e+00  8.4e-05  2.05e-08
##     26   96 -1.036e+04 -4.21e-15  4.10e-15  6.5e-15  2.2e+05  1.2e-14  1.52e-09
##
##  ***** FALSE CONVERGENCE *****
##
##  FUNCTION    -1.035796e+04   RELDX        6.540e-15
##  FUNC. EVALS      96         GRAD. EVALS      26
##  PRELDF       4.102e-15      NPRELDF      1.521e-09
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    1.327962e-05     1.000e+00    -3.640e+03
##      2    7.830574e-02     1.000e+00    -5.732e-01
##      3    8.923816e-01     1.000e+00    -7.191e-01
plot(predict(gfit))
```

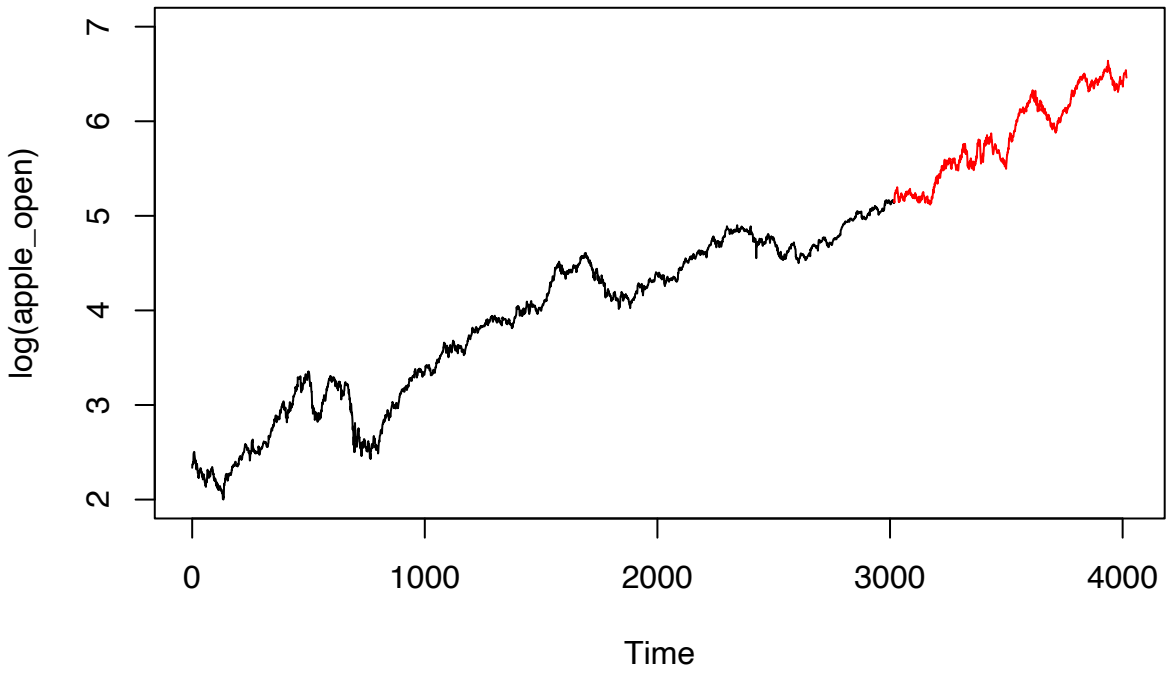



```
library(rugarch)
default_spec <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1,1)))
default_garch <- ugarchfit(spec = default_spec, data = diff(log(apple_open)))
set.seed(116)
sim = ugarchsim(default_garch,n.sim=1000)
sim_rst = sim@simulation$seriesSim[,1]
matplot(y = sim@simulation$seriesSim[,1],x=3018:4017, type = "l",
       main = "Simulations of Open",col = "red",xlim=c(0,4020),ylab="index",xlab="difflogopen",ylim=c(-
lines(diff(log(apple_open)))
```

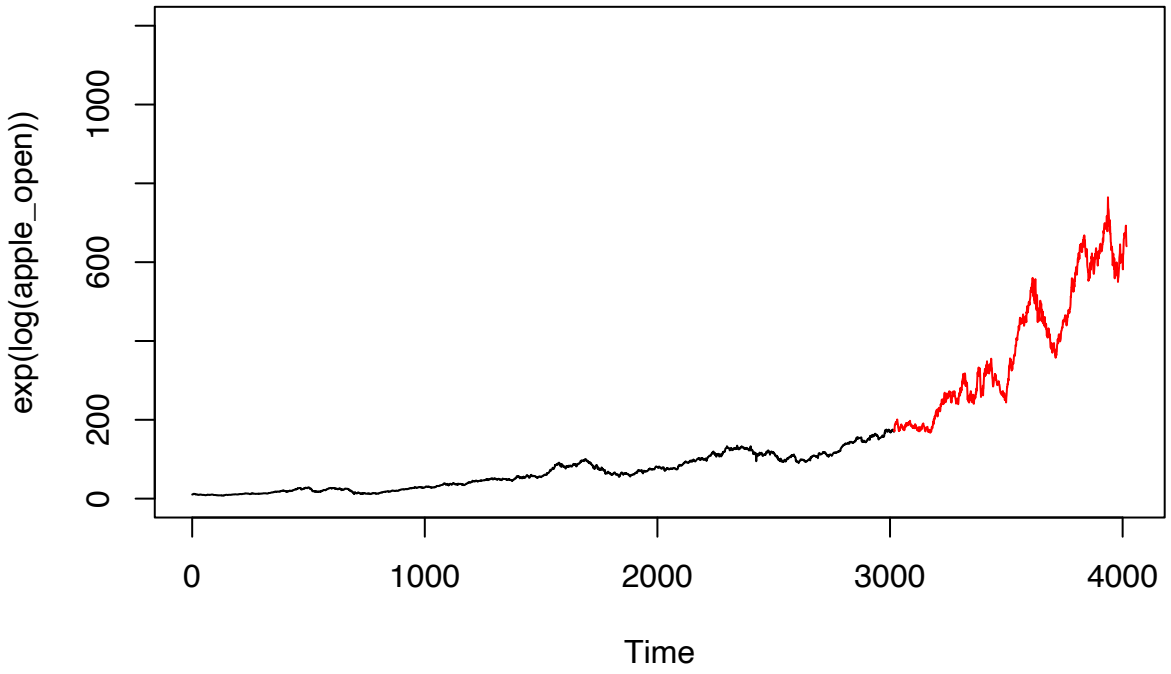



```
sim = rep(0,1000)
first = log(apple_open)[3018]
for(i in 1:1000){
  if(i==1){
    sim[i]=first+sim_rst[1]
  }
  else{
  sim[i]=sim[i-1]+sim_rst[i]}
}
```

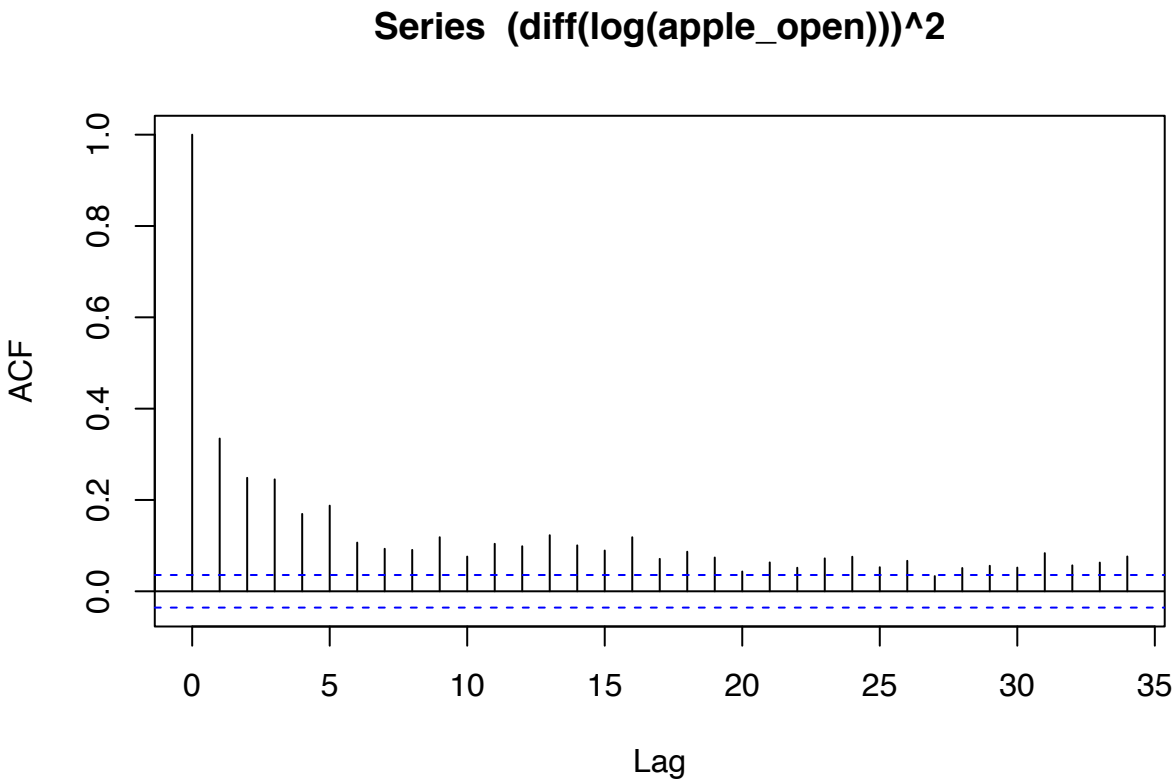
```
plot(log(apple_open),xlim = c(0,4020),ylim = c(2,7))
lines(x=3018:4017,y=sim,col="red",type="l")
```



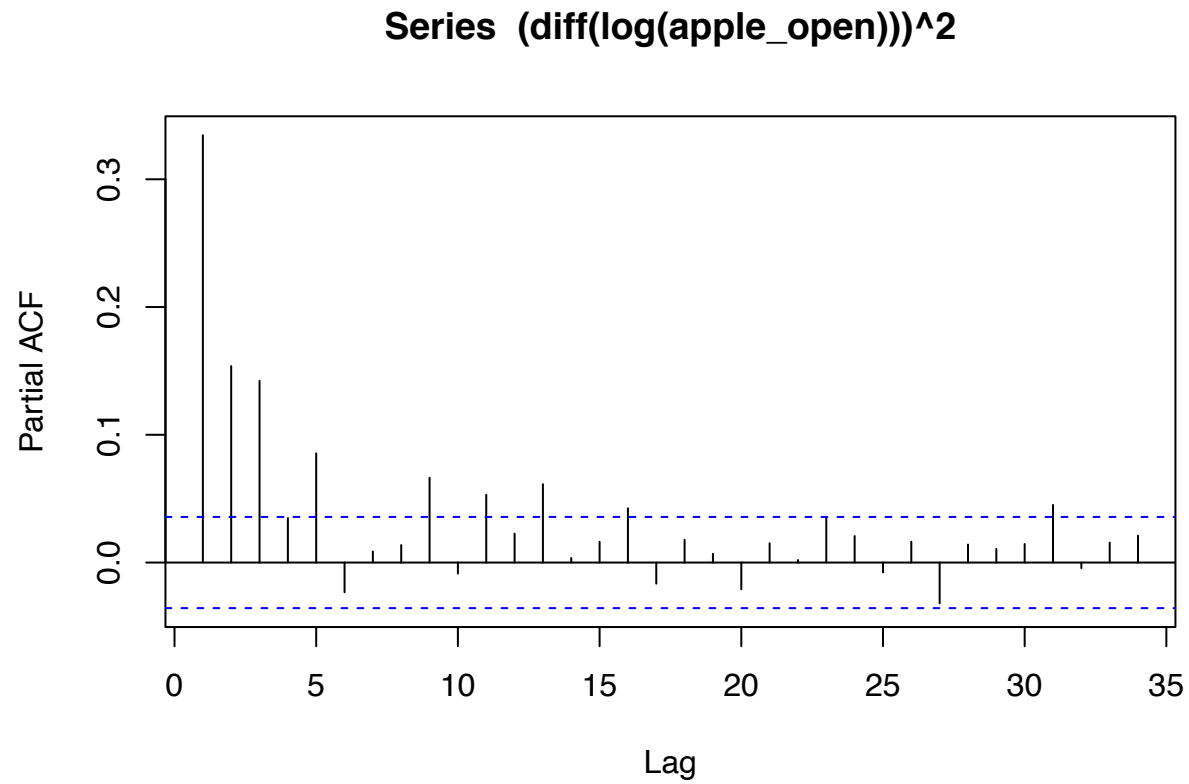

```
plot(exp(log(apple_open)),xlim = c(0,4020),ylim=c(0,1200))
lines(x=3018:4017,y=exp(sim),col="red",type="l")
```




```
acf((diff(log(apple_open)))^2)
```




```
pacf((diff(log(apple_open)))^2)
```




```
model = auto.arima(diff(log(apple_open))^2, max.p = 3, max.q = 3,max.order = 10, stationary = T, seasona
```

```
##
```

```
##  ARIMA(0,0,0) with zero mean     : -30186.96
##  ARIMA(0,0,0) with non-zero mean : -30465.39
##  ARIMA(0,0,1) with zero mean     : -30548.83
##  ARIMA(0,0,1) with non-zero mean : -30735.09
##  ARIMA(0,0,2) with zero mean     : -30648.91
##  ARIMA(0,0,2) with non-zero mean : -30801.59
##  ARIMA(0,0,3) with zero mean     : -30758.6
##  ARIMA(0,0,3) with non-zero mean : -30883.06
##  ARIMA(1,0,0) with zero mean     : -30692.65
##  ARIMA(1,0,0) with non-zero mean : -30821.45
##  ARIMA(1,0,1) with zero mean     : -30919.87
##  ARIMA(1,0,1) with non-zero mean : -30961.44
##  ARIMA(1,0,2) with zero mean     : -30932.75
##  ARIMA(1,0,2) with non-zero mean : -30963.94
##  ARIMA(1,0,3) with zero mean     : -30933.66
##  ARIMA(1,0,3) with non-zero mean : -30962.73
##  ARIMA(2,0,0) with zero mean     : -30801.14
##  ARIMA(2,0,0) with non-zero mean : -30891.64
##  ARIMA(2,0,1) with zero mean     : -30936.09
##  ARIMA(2,0,1) with non-zero mean : -30963.79
##  ARIMA(2,0,2) with zero mean     : -30944.21
##  ARIMA(2,0,2) with non-zero mean : -30981.97
##  ARIMA(2,0,3) with zero mean     : -30944.34
##  ARIMA(2,0,3) with non-zero mean : -30980.11
##  ARIMA(3,0,0) with zero mean     : -30885.58
##  ARIMA(3,0,0) with non-zero mean : -30951.27
##  ARIMA(3,0,1) with zero mean     : -30946.82
##  ARIMA(3,0,1) with non-zero mean : -30962.13
##  ARIMA(3,0,2) with zero mean     : -30945.91
##  ARIMA(3,0,2) with non-zero mean : -30980.13
##  ARIMA(3,0,3) with zero mean     : Inf
##  ARIMA(3,0,3) with non-zero mean : -30980.47
##
##
##
##  Best model: ARIMA(2,0,2) with non-zero mean
```

garch(1,1) is better

amazon

EDA

```
amazon = read.csv("/Users/wyc/AMZN_2006-01-01_to_2018-01-01.csv")
```

```
head(amazon)
```

```
##         Date  Open  High   Low Close  Volume Name
## 1 2006-01-03 47.47 47.85 46.25 47.58 7582127 AMZN
## 2 2006-01-04 47.48 47.73 46.69 47.25 7440914 AMZN
## 3 2006-01-05 47.16 48.20 47.11 47.65 5417258 AMZN
## 4 2006-01-06 47.97 48.58 47.32 47.87 6154285 AMZN
## 5 2006-01-09 46.55 47.10 46.40 47.08 8945056 AMZN
```

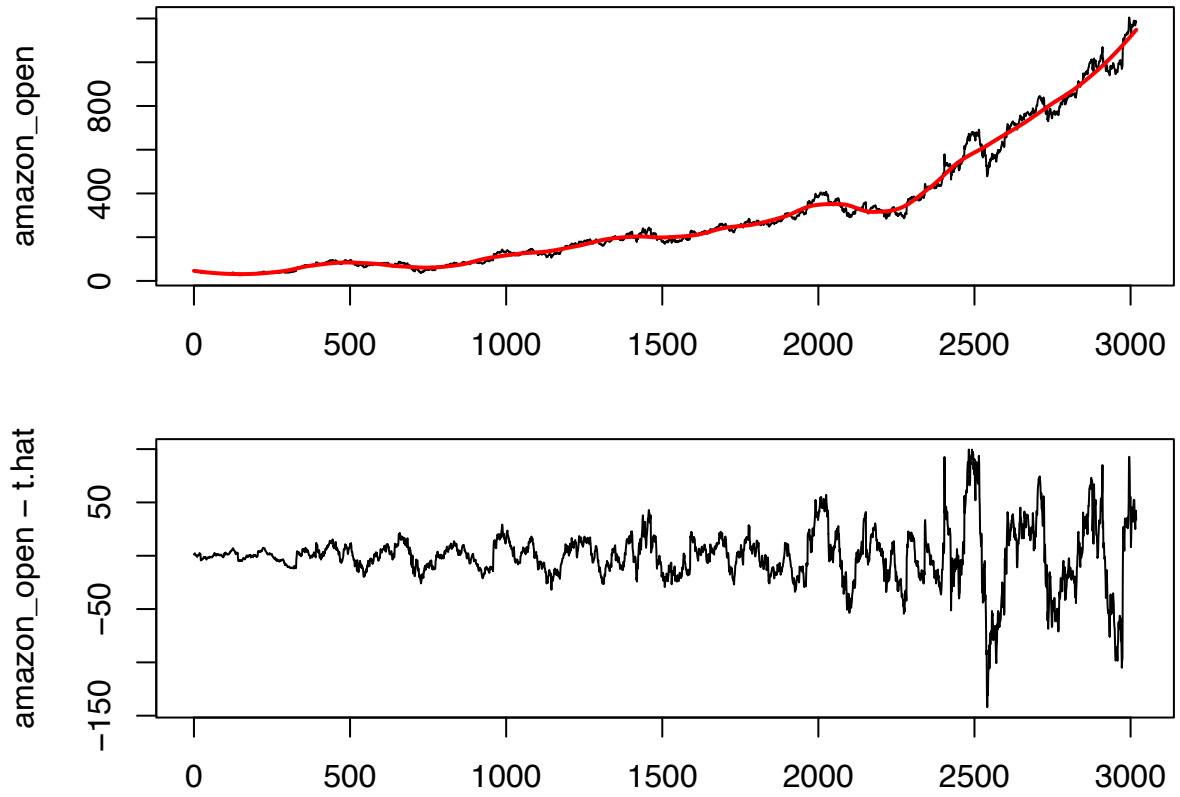
```
## 6 2006-01-10 46.41 46.75 45.36 45.65 9686957 AMZN
```

```
amazon_open = amazon$Open
amazon_open = ts(amazon_open,start = 0)
```

```
plot(y=amazon_open,x=as.Date(amazon$Date),type = "l",main="The open price of amazon")
```

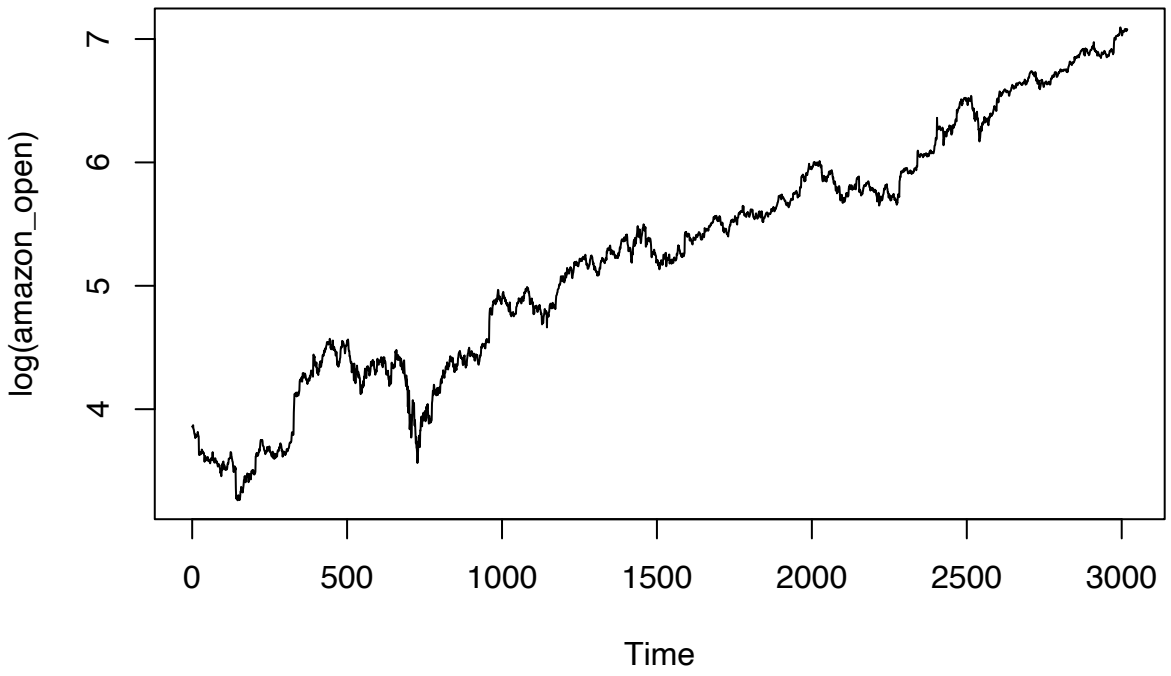
The open price of amazon

```
par(mfrow=c(2,1), mar = c(2, 4, 2, 2))
t.hat <- predict(loess(amazon_open ~ time(amazon_open), span=0.17))
plot(amazon_open)
lines(as.numeric(time(amazon_open)), t.hat, col="red", lwd=2)
plot(amazon_open-t.hat)
```



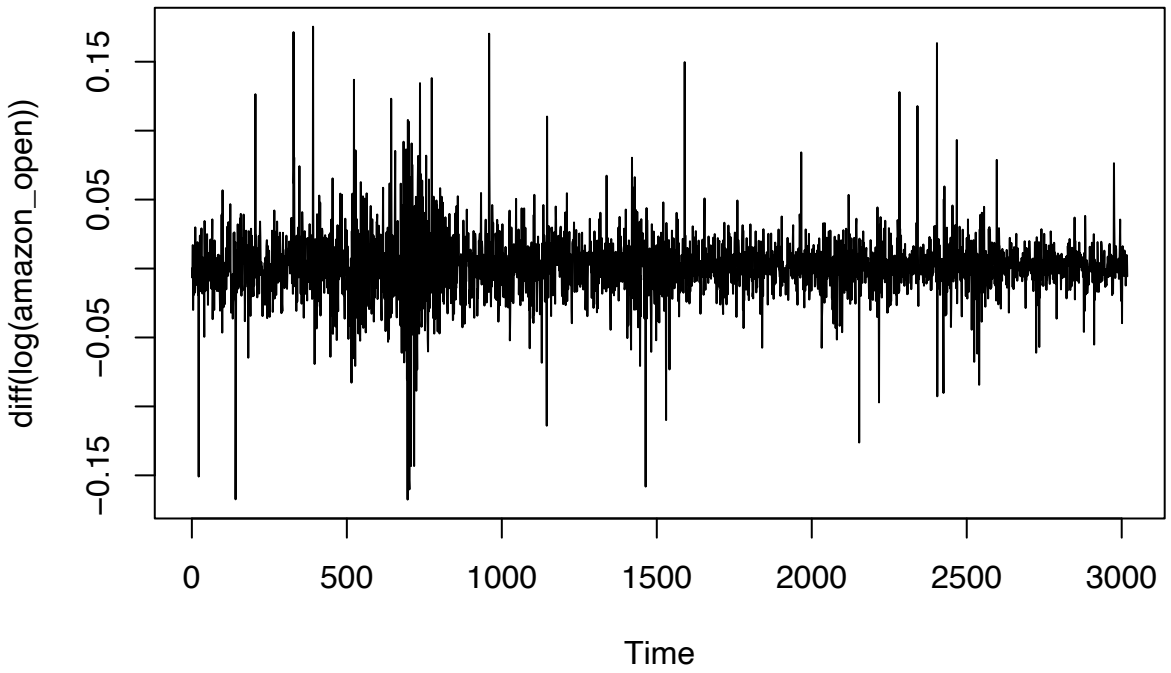
Loess, dipict the trend

```
plot(log(amazon_open))
```

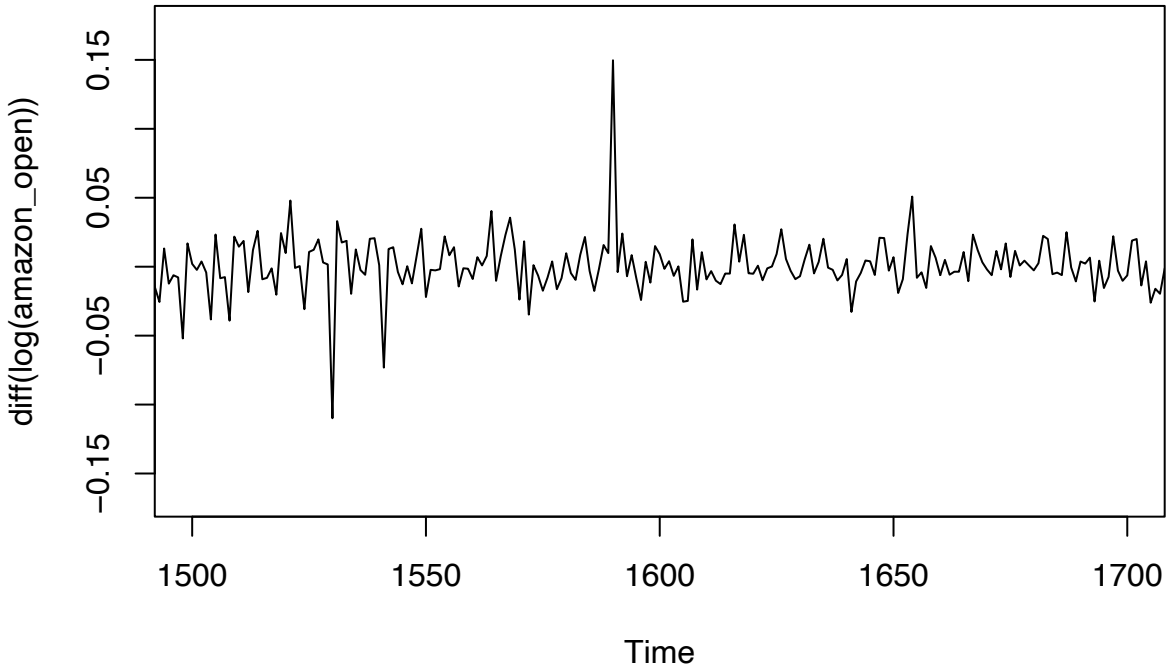


Take log to have a constant trend.

```
plot(diff(log(amazon_open)))
```



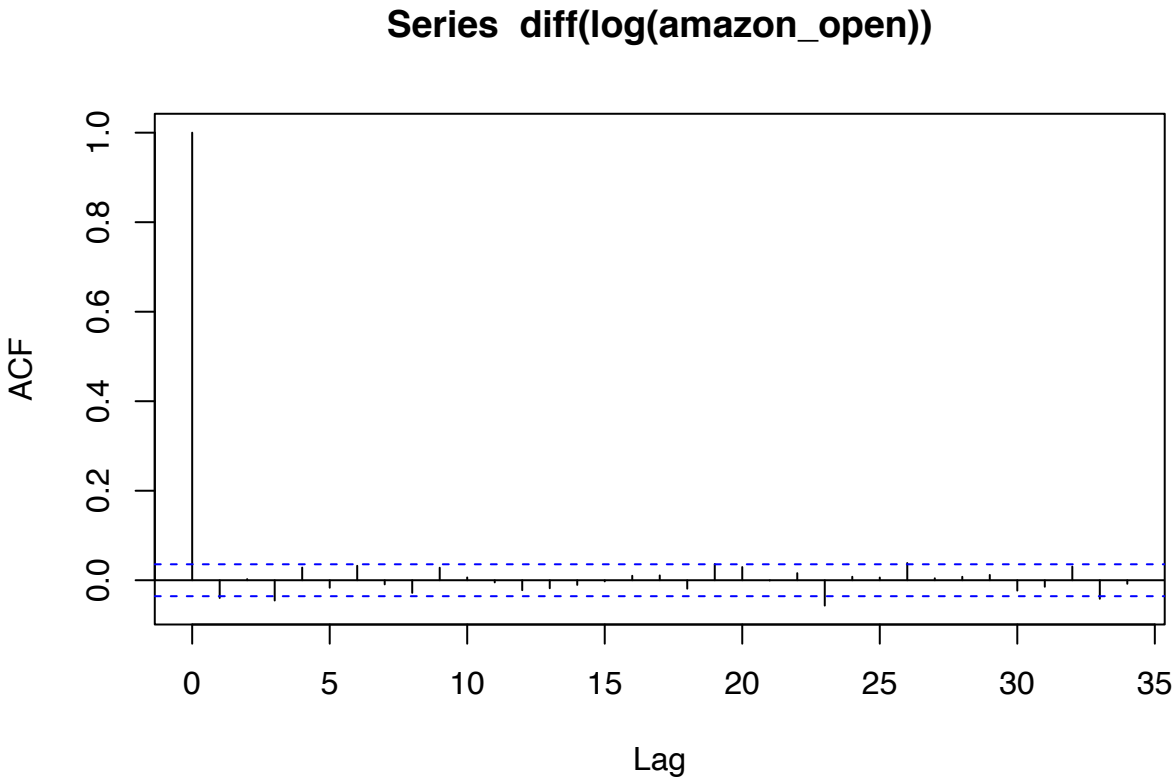

```
plot(diff(log(amazon_open)),xlim=c(1500,1700))
```



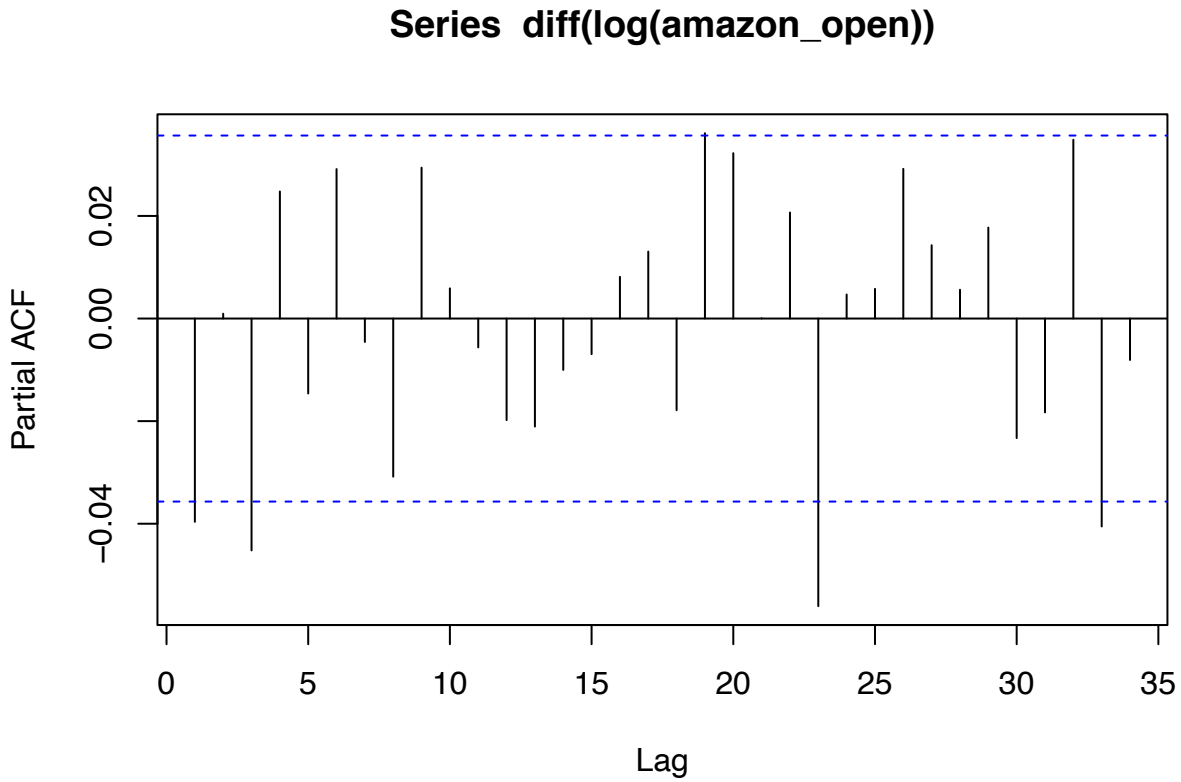
Take one diff, seems no trend, after diff it is return of the stock. Does not seems to have any seasonal pattern.

Model building

```
library(forecast)
acf(diff(log(amazon_open)))
```




```
pacf(diff(log(amazon_open)))
```




```
model = auto.arima(diff(log(amazon_open)), max.p = 3, max.q = 3,max.order = 10, stationary = T, seasonal
```

```
##
```

```
##  ARIMA(0,0,0) with zero mean     : -13764.58
##  ARIMA(0,0,0) with non-zero mean : -13768.18
##  ARIMA(0,0,1) with zero mean     : -13766.85
##  ARIMA(0,0,1) with non-zero mean : -13770.92
##  ARIMA(0,0,2) with zero mean     : -13764.85
##  ARIMA(0,0,2) with non-zero mean : -13768.92
##  ARIMA(0,0,3) with zero mean     : -13767.65
##  ARIMA(0,0,3) with non-zero mean : -13772.22
##  ARIMA(1,0,0) with zero mean     : -13766.87
##  ARIMA(1,0,0) with non-zero mean : -13770.92
##  ARIMA(1,0,1) with zero mean     : Inf
##  ARIMA(1,0,1) with non-zero mean : Inf
##  ARIMA(1,0,2) with zero mean     : Inf
##  ARIMA(1,0,2) with non-zero mean : Inf
##  ARIMA(1,0,3) with zero mean     : -13769.96
##  ARIMA(1,0,3) with non-zero mean : -13774.38
##  ARIMA(2,0,0) with zero mean     : -13764.89
##  ARIMA(2,0,0) with non-zero mean : -13768.92
##  ARIMA(2,0,1) with zero mean     : Inf
##  ARIMA(2,0,1) with non-zero mean : Inf
##  ARIMA(2,0,2) with zero mean     : -13766.11
##  ARIMA(2,0,2) with non-zero mean : -13770.45
##  ARIMA(2,0,3) with zero mean     : Inf
##  ARIMA(2,0,3) with non-zero mean : Inf
##  ARIMA(3,0,0) with zero mean     : -13768.5
##  ARIMA(3,0,0) with non-zero mean : -13773.08
##  ARIMA(3,0,1) with zero mean     : -13769.75
##  ARIMA(3,0,1) with non-zero mean : -13774.14
##  ARIMA(3,0,2) with zero mean     : Inf
##  ARIMA(3,0,2) with non-zero mean : -13772.11
##  ARIMA(3,0,3) with zero mean     : Inf
##  ARIMA(3,0,3) with non-zero mean : Inf
##
##
##
##  Best model: ARIMA(1,0,3) with non-zero mean
```

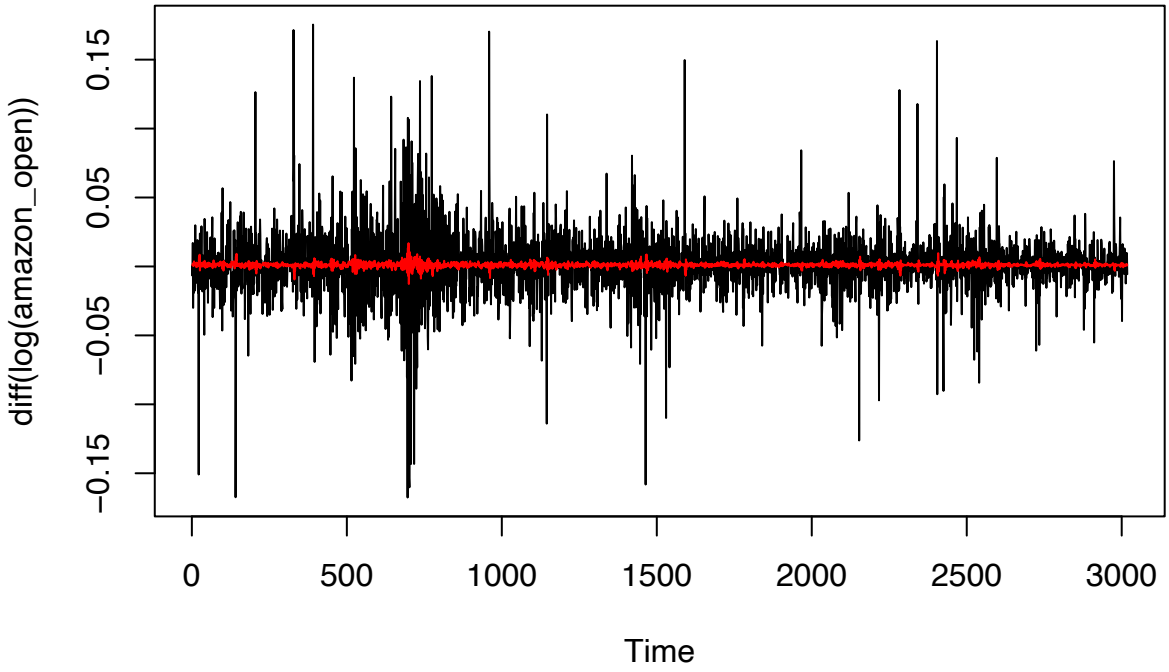
Fit ARIMA model to the return. Hard to figure our the orders of the ARIMA model by acf, pacf, thus auto.arima. arima(1,0,3) best.

```
arima_model = arima(x=diff(log(amazon_open)),order = c(1,0,3),method='ML')
summary(arima_model)
```

```
##
## Call:
## arima(x = diff(log(amazon_open)), order = c(1, 0, 3), method = "ML")
##
## Coefficients:
##           ar1     ma1      ma2      ma3  intercept
##       -0.5963  0.5589  -0.0231  -0.0472     0.0011
## s.e.   0.1557  0.1560   0.0214   0.0192     0.0004
##
## sigma^2 estimated as 0.0006076:  log likelihood = 6893.2,  aic = -13774.41
##
## Training set error measures:
```

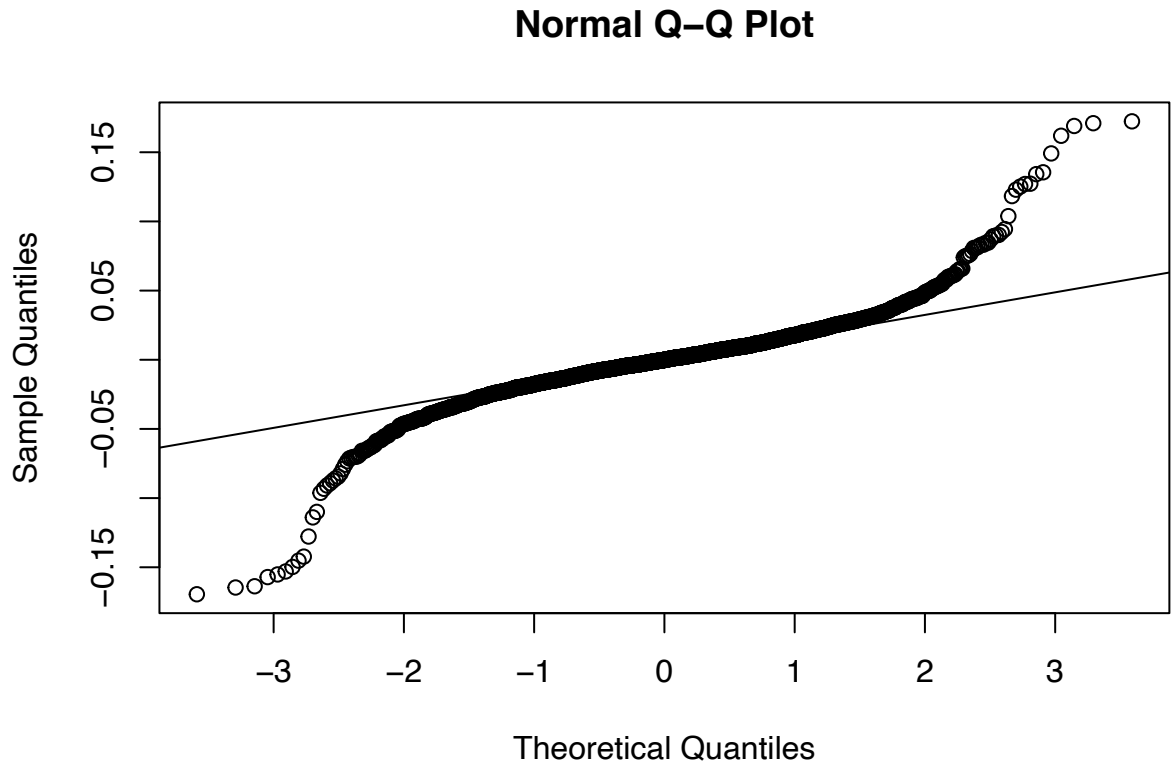
```
##                         ME       RMSE        MAE  MPE MAPE     MASE
## Training set -2.797932e-06 0.02465018 0.01619789 -Inf  Inf 0.686119
##                       ACF1
## Training set -0.0002903583
```

```
plot(diff(log(amazon_open)))
lines(diff(log(amazon_open))-arima_model$resid,col="red")
```

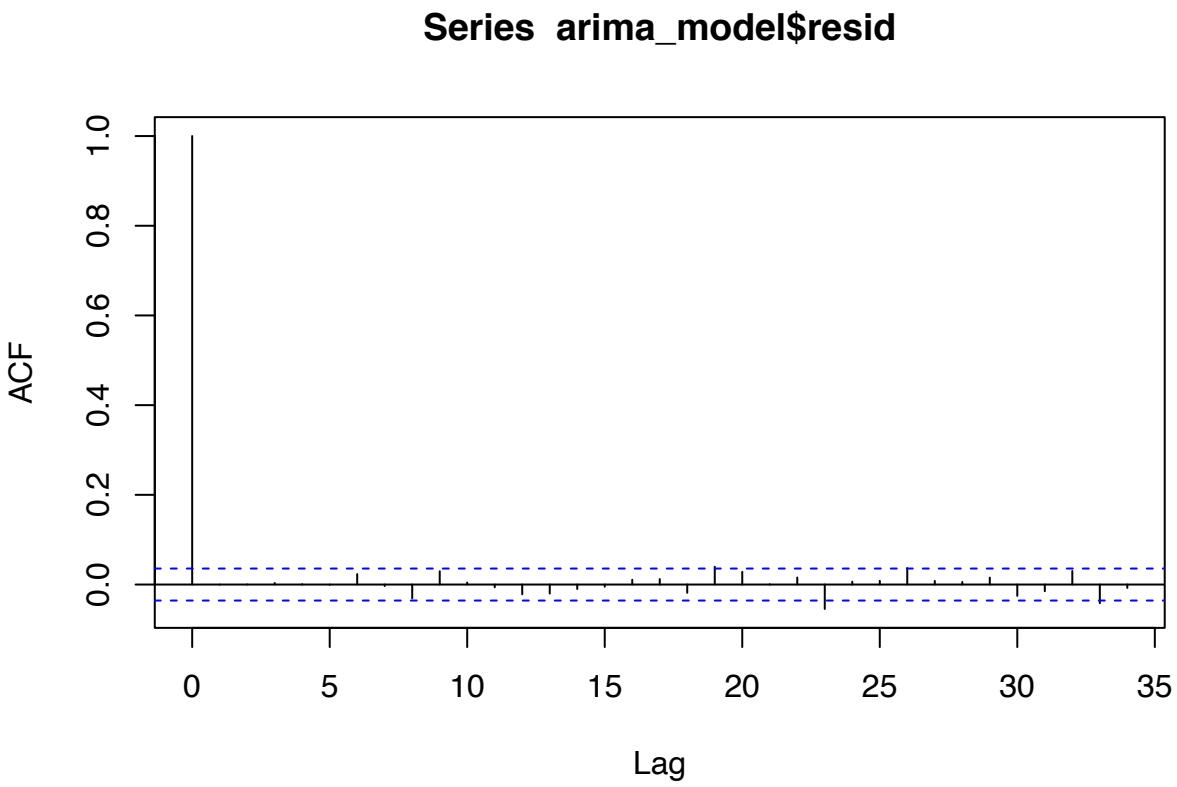


Bad result.

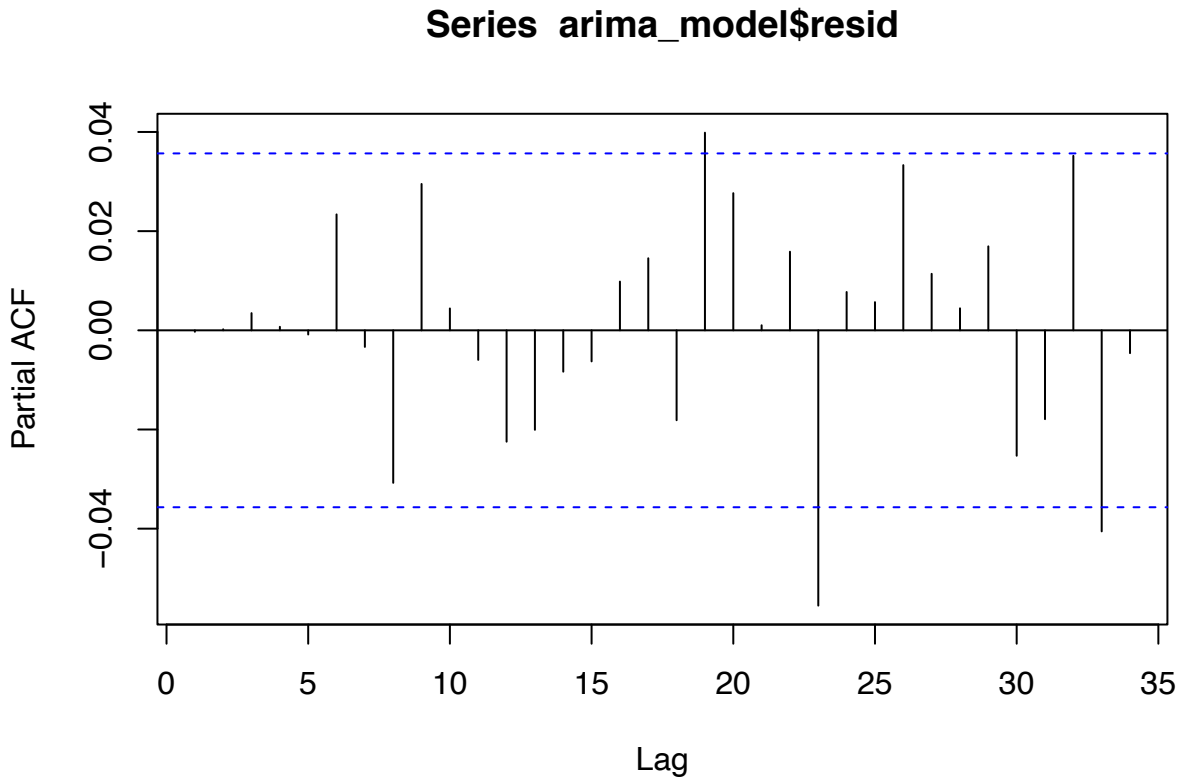
```
qqnorm(arima_model$resid); qqline(arima_model$resid)
```



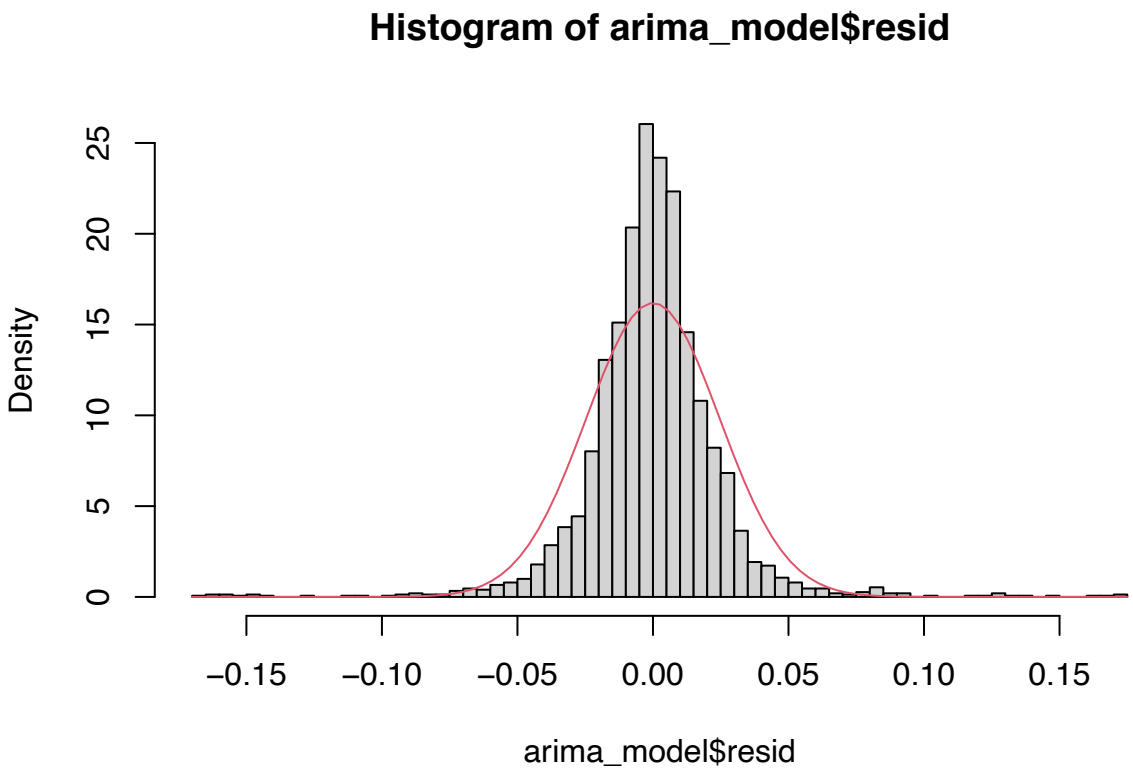

```
acf(arima_model$resid)
```



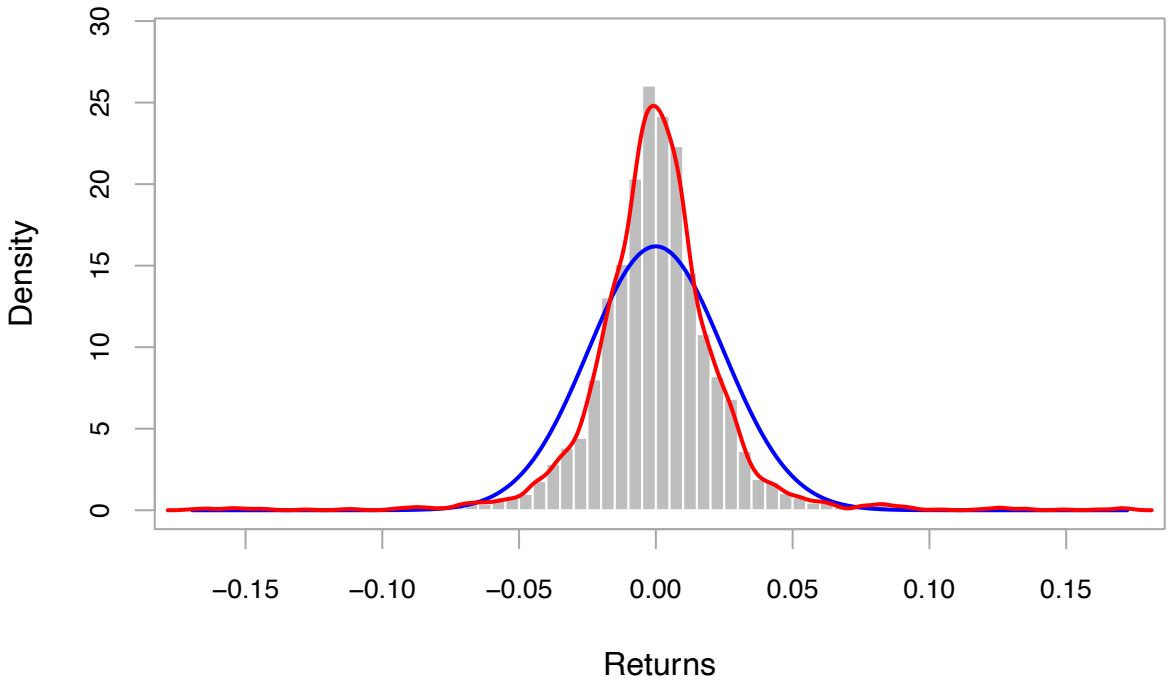

```
pacf(arima_model$resid)
```




```
hist(arima_model$resid,breaks=100,prob=TRUE)
library(MASS)
library(QRM)
fit1 <- fitdistr(arima_model$resid, "normal")
para <- fit1$estimate
curve(dnorm(x, para[1], para[2]), col = 2, add = TRUE)
```




```
fit2 = fit.st(arima_model$resid)
p_val = rep(0,30)
for(i in 1:30){
  p_val[i] = Box.test(arima_model$residual^2,lag = i)[3]
}
stdret <- residuals(arima_model, standardize = TRUE)
library(PerformanceAnalytics)
chart.Histogram(stdret, methods = c("add.normal", "add.density"),
          colorset=c("gray","red","blue"))
```



qqplot, non-gaussian resid. Heteroscedasticity exist from the test. But we probabily can use arima(1,0,3) as a mean model

```
library(fGarch)
gfit1 = garchFit(~garch(1,0),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                 arma
##  Formula Mean:               ~ arma(0, 0)
##  GARCH Model:                garch
##  Formula Variance:           ~ garch(1, 0)
##  ARMA Order:                 0 0
##  Max ARMA Order:             0
##  GARCH Order:                1 0
##  Max GARCH Order:            1
##  Maximum Order:              1
##  Conditional Dist:           norm
##  h.start:                    2
##  llh.start:                  1
##  Length of Series:           3018
##  Recursion Init:             mci
##  Series Scale:               0.02471247
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.43108945   0.4310894 0.04310894     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.10000000     TRUE
```

```
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1
##      1      2      3
##  Persistence:                  0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     9412.5260: 0.0431089 0.100000 0.100000
##   1:     4229.2486: 0.0431014  1.07154 0.336893
##   2:     4223.2143: 0.0430934  1.07925 0.280629
##   3:     4185.6529: 0.0430920 0.974552 0.236601
##   4:     4156.6370: 0.0430894 0.805449 0.181189
##   5:     4156.5738: 0.0430830 0.741121 0.347104
##   6:     4154.1192: 0.0405701 0.726484 0.281497
##   7:     4153.3783: 0.0432113 0.770790 0.234594
##   8:     4153.1704: 0.0395275 0.761343 0.257513
##   9:     4153.1667: 0.0433621 0.759286 0.257935
##  10:     4153.1552: 0.0411748 0.759059 0.253915
##  11:     4153.1539: 0.0413636 0.759850 0.254694
##  12:     4153.1539: 0.0414081 0.759728 0.254699
##  13:     4153.1539: 0.0414028 0.759734 0.254691
##
## Final Estimate of the Negative LLH:
##  LLH:  -7014.796    norm LLH:  -2.324319
##           mu        omega       alpha1
## 0.0010231662 0.0004639741 0.2546912139
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu       omega       alpha1
## mu     -5933634.787     1264494    -2459.249
## omega   1264493.913 -5421866300 -1118785.205
## alpha1    -2459.249    -1118785    -1193.596
## attr(,"time")
## Time difference of 0.02928591 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.117857 secs
```

```
gfit2 = garchFit(~garch(1,1),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
```

```
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02471247
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.43108945   0.4310894 0.04310894     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1   0.00000001   1.0000000 0.80000000     TRUE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1
##      1      2      3      5
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     4042.9458: 0.0431089 0.100000 0.100000 0.800000
##   1:     4042.4973: 0.0431115 0.0954600 0.0999818 0.801181
##   2:     4041.8609: 0.0431140 0.0947808 0.101781 0.805460
##   3:     4040.7912: 0.0431203 0.0869021 0.101822 0.810553
##   4:     4038.6661: 0.0431352 0.0783656 0.0998654 0.827147
##   5:     4034.2588: 0.0431736 0.0624904 0.0719497 0.861564
##   6:     4034.2388: 0.0431815 0.0593079 0.0744078 0.870187
##   7:     4032.8380: 0.0431847 0.0547550 0.0731000 0.870628
##   8:     4032.4272: 0.0431905 0.0524926 0.0731374 0.874812
##   9:     4031.9603: 0.0432072 0.0471452 0.0697264 0.881895
##  10:     4031.1275: 0.0432385 0.0465096 0.0637570 0.889243
##  11:     4030.7681: 0.0432850 0.0435581 0.0574993 0.895687
##  12:     4030.0072: 0.0433501 0.0393592 0.0557322 0.903905
##  13:     4029.6089: 0.0434257 0.0340884 0.0530551 0.911153
##  14:     4027.5678: 0.0437053 0.0256380 0.0410100 0.931861
```

```
##  15:     4026.9140: 0.0437113 0.0222427 0.0381367 0.939629
##  16:     4024.0604: 0.0438437 0.0158058 0.0299452 0.953862
##  17:     4022.7838: 0.0438856 0.0136363 0.0272785 0.958313
##  18:     4020.7412: 0.0439695 0.00929387 0.0219554 0.967220
##  19:     4020.1500: 0.0439906 0.00857662 0.0209301 0.969811
##  20:     4019.0131: 0.0440855 0.00405146 0.0150598 0.979964
##  21:     4018.8344: 0.0440855 0.00418688 0.0152110 0.980095
##  22:     4018.7596: 0.0440840 0.00415944 0.0153064 0.979877
##  23:     4018.7084: 0.0440810 0.00425126 0.0156942 0.979614
##  24:     4018.6620: 0.0440743 0.00445672 0.0160962 0.978774
##  25:     4018.6320: 0.0440850 0.00414197 0.0157388 0.979577
##  26:     4018.5878: 0.0440851 0.00394387 0.0159577 0.979590
##  27:     4018.5827: 0.0440851 0.00398366 0.0159931 0.979620
##  28:     4018.5756: 0.0440848 0.00397482 0.0160159 0.979564
##  29:     4018.5721: 0.0440847 0.00397086 0.0161267 0.979511
##  30:     4018.5644: 0.0440838 0.00402591 0.0162391 0.979301
##  31:     4018.5613: 0.0440845 0.00411101 0.0163703 0.979111
##  32:     4018.5603: 0.0440943 0.00412180 0.0164059 0.979028
##  33:     4018.5592: 0.0441048 0.00413508 0.0164207 0.979018
##  34:     4018.5361: 0.0455334 0.00441944 0.0168915 0.978235
##  35:     4018.4640: 0.0506278 0.00417157 0.0164932 0.978906
##  36:     4018.4638: 0.0509037 0.00418039 0.0165236 0.978868
##  37:     4018.4638: 0.0509034 0.00418057 0.0165232 0.978869
##  38:     4018.4638: 0.0509026 0.00418039 0.0165229 0.978869
##
## Final Estimate of the Negative LLH:
##  LLH:  -7149.486    norm LLH:  -2.368948
##           mu        omega       alpha1        beta1
## 1.257929e-03 2.552992e-06 1.652295e-02 9.788691e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu         omega        alpha1         beta1
## mu      -6745240.65 -9.536841e+07 -3.347588e+04 -6.165248e+04
## omega  -95368410.02 -2.021762e+13 -6.523022e+09 -8.263471e+09
## alpha1    -33475.88 -6.523022e+09 -2.997924e+06 -3.236610e+06
## beta1     -61652.48 -8.263471e+09 -3.236610e+06 -3.880869e+06
## attr(,"time")
## Time difference of 0.03389907 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.3666141 secs
```

```
gfit3 = garchFit(~garch(2,0),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 0)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
```

```
##  GARCH Order:               2 0
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02471247
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.43108945   0.4310894 0.04310894     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.05000000     TRUE
##     alpha2  0.00000001   1.0000000 0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     gamma2 -0.99999999   1.0000000 0.10000000    FALSE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2
##      1      2      3      4
##  Persistence:                  0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8794.5386: 0.0431089 0.100000 0.0500000 0.0500000
##   1:     4274.0316: 0.0431271  1.00316 0.362797 0.344033
##   2:     4246.5627: 0.0435935  1.18632 0.0876904 1.00000e-08
##   3:     4227.0404: 0.0438843 0.955680 0.0623755 0.442855
##   4:     4146.0871: 0.0441225 0.615823 0.110237 0.227365
##   5:     4140.9332: 0.0442304 0.742385 0.214283 0.205552
##   6:     4138.0513: 0.0450182 0.614353 0.287272 0.132943
##   7:     4128.5040: 0.0457981 0.729492 0.172086 0.111182
##   8:     4128.2484: 0.0458321 0.714054 0.170116 0.105665
##   9:     4128.1763: 0.0462139 0.713004 0.181911 0.113052
##  10:     4128.1131: 0.0468699 0.708133 0.179866 0.109342
##  11:     4127.9923: 0.0510938 0.711607 0.172878 0.113353
##  12:     4127.9873: 0.0530198 0.705725 0.188769 0.110549
##  13:     4127.9225: 0.0539816 0.706338 0.180147 0.111019
##  14:     4127.8950: 0.0560389 0.709050 0.179831 0.113514
##  15:     4127.8846: 0.0581015 0.708180 0.179058 0.112938
##  16:     4127.8841: 0.0580537 0.708063 0.177928 0.113789
##  17:     4127.8840: 0.0580355 0.708379 0.178044 0.113398
```

```
##  18:     4127.8840: 0.0580484 0.708261 0.178152 0.113456
##  19:     4127.8840: 0.0580460 0.708276 0.178119 0.113459
##  20:     4127.8840: 0.0580460 0.708276 0.178120 0.113458
##
## Final Estimate of the Negative LLH:
##  LLH:  -7040.066    norm LLH:  -2.332693
##           mu        omega       alpha1       alpha2
## 0.0014344604 0.0004325486 0.1781203559 0.1134579891
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu       omega        alpha1        alpha2
## mu     -5979342.333    -3639974    -2690.6913    12592.2551
## omega  -3639974.200 -5720501408 -1265944.1370 -1437917.9199
## alpha1    -2690.691    -1265944    -1510.6554     -662.8769
## alpha2    12592.255    -1437918     -662.8769    -2159.4763
## attr(,"time")
## Time difference of 0.04768801 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.254621 secs
```

```
gfit4 = garchFit(~garch(2,1),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 1)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               2 1
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02471247
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.43108945   0.4310894 0.04310894     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.05000000     TRUE
##     alpha2  0.00000001   1.0000000 0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
```

```
##     gamma2 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1   0.00000001   1.0000000 0.80000000     TRUE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2  beta1
##      1      2      3      4      7
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     4043.0385: 0.0431089 0.100000 0.0500000 0.0500000 0.800000
##   1:     4042.4232: 0.0431107 0.0956723 0.0511177 0.0495781 0.799622
##   2:     4041.9558: 0.0431125 0.0955013 0.0541514 0.0511997 0.802526
##   3:     4040.8468: 0.0431179 0.0873681 0.0574786 0.0506107 0.804430
##   4:     4039.9355: 0.0431263 0.0844275 0.0600501 0.0490827 0.812403
##   5:     4034.2949: 0.0432124 0.0559152 0.0501893 0.00608585 0.880768
##   6:     4033.5381: 0.0432127 0.0561189 0.0517852 0.00746780 0.881844
##   7:     4033.0483: 0.0432131 0.0538611 0.0521534 0.00750053 0.881195
##   8:     4032.3097: 0.0432172 0.0520274 0.0537177 0.00770622 0.885289
##   9:     4031.1895: 0.0432266 0.0458259 0.0538218 0.00492421 0.891941
##  10:     4030.3289: 0.0432365 0.0416058 0.0549939 0.00269247 0.900082
##  11:     4029.9454: 0.0432413 0.0390663 0.0552857 0.000709147 0.902962
##  12:     4029.6764: 0.0432472 0.0375449 0.0552398 1.00000e-08 0.906730
##  13:     4029.2540: 0.0432602 0.0330836 0.0527559 1.00000e-08 0.913210
##  14:     4028.8890: 0.0432985 0.0298982 0.0501101 1.00000e-08 0.920297
##  15:     4025.2270: 0.0435781 0.0180811 0.0350838 1.00000e-08 0.946432
##  16:     4025.1557: 0.0435781 0.0179008 0.0348568 1.00000e-08 0.946355
##  17:     4025.0832: 0.0435783 0.0181233 0.0344740 1.00000e-08 0.946837
##  18:     4025.0147: 0.0436329 0.0179013 0.0342364 1.00000e-08 0.946776
##  19:     4024.9686: 0.0437448 0.0180109 0.0341198 1.00000e-08 0.947075
##  20:     4024.9121: 0.0439701 0.0178346 0.0338744 1.00000e-08 0.947137
##  21:     4024.8378: 0.0444212 0.0179014 0.0337073 1.00000e-08 0.947515
##  22:     4024.7265: 0.0453238 0.0176721 0.0333209 1.00000e-08 0.947760
##  23:     4024.2046: 0.0536481 0.0173240 0.0314696 1.00000e-08 0.950150
##  24:     4024.1428: 0.0536481 0.0170891 0.0313685 1.00000e-08 0.950105
##  25:     4024.0794: 0.0536418 0.0169288 0.0314556 1.00000e-08 0.950648
##  26:     4024.0169: 0.0535920 0.0167161 0.0313126 1.00000e-08 0.950640
##  27:     4023.9306: 0.0534952 0.0165204 0.0312525 1.00000e-08 0.951358
##  28:     4023.8360: 0.0532918 0.0162979 0.0309880 1.00000e-08 0.951457
##  29:     4023.5638: 0.0528944 0.0155520 0.0304832 1.00000e-08 0.953414
##  30:     4022.8520: 0.0489052 0.0146152 0.0268043 1.00000e-08 0.957138
##  31:     4022.7481: 0.0489052 0.0142065 0.0268561 1.00000e-08 0.957164
##  32:     4022.6625: 0.0489178 0.0142389 0.0270350 1.00000e-08 0.957393
##  33:     4022.5852: 0.0489498 0.0138818 0.0270536 1.00000e-08 0.957452
##  34:     4022.5179: 0.0490201 0.0139100 0.0271806 1.00000e-08 0.957671
##  35:     4022.4515: 0.0491618 0.0135869 0.0271548 1.00000e-08 0.957772
##  36:     4022.3835: 0.0494464 0.0135954 0.0272177 1.00000e-08 0.958027
##  37:     4022.3001: 0.0500157 0.0132626 0.0271030 1.00000e-08 0.958244
```

```
##  38:     4022.1861: 0.0511546 0.0131892 0.0270052 1.00000e-08 0.958709
##  39:     4021.6868: 0.0608770 0.0117481 0.0250798 1.00000e-08 0.961689
##  40:     4020.0567: 0.0511837 0.00434472 0.0181344 1.00000e-08 0.976351
##  41:     4018.6592: 0.0511837 0.00474773 0.0183413 0.000149049 0.976624
##  42:     4018.5260: 0.0511866 0.00470615 0.0182634 1.00000e-08 0.976580
##  43:     4018.4787: 0.0512024 0.00477969 0.0182605 1.00000e-08 0.976665
##  44:     4018.4596: 0.0512348 0.00475893 0.0181268 1.00000e-08 0.976708
##  45:     4018.4362: 0.0513004 0.00475491 0.0180414 1.00000e-08 0.976876
##  46:     4018.3573: 0.0519170 0.00422067 0.0171961 1.00000e-08 0.978223
##  47:     4018.3260: 0.0512967 0.00436955 0.0166319 1.00000e-08 0.978461
##  48:     4018.3125: 0.0506764 0.00416488 0.0164725 1.00000e-08 0.978949
##  49:     4018.3106: 0.0509502 0.00420926 0.0166049 1.00000e-08 0.978756
##  50:     4018.3105: 0.0509473 0.00420335 0.0165801 1.00000e-08 0.978786
##  51:     4018.3105: 0.0509367 0.00420376 0.0165814 1.00000e-08 0.978784
##  52:     4018.3105: 0.0509398 0.00420370 0.0165814 1.00000e-08 0.978784
##
## Final Estimate of the Negative LLH:
##  LLH:  -7149.64    norm LLH:  -2.368999
##           mu        omega       alpha1       alpha2        beta1
## 1.258848e-03 2.567223e-06 1.658135e-02 1.000000e-08 9.787844e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu         omega        alpha1        alpha2         beta1
## mu      -6746310.01 -9.477090e+07 -3.351838e+04 -3.068566e+04 -6.142782e+04
## omega  -94770901.19 -2.005666e+13 -6.469598e+09 -6.410214e+09 -8.198557e+09
## alpha1    -33518.38 -6.469598e+09 -2.972922e+06 -2.901258e+06 -3.210111e+06
## alpha2    -30685.66 -6.410214e+09 -2.901258e+06 -2.870541e+06 -3.182253e+06
## beta1     -61427.82 -8.198557e+09 -3.210111e+06 -3.182253e+06 -3.848717e+06
## attr(,"time")
## Time difference of 0.1175921 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.4998171 secs
```

```
gfit5 = garchFit(~garch(1,2),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 2)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               1 2
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
```

```
##  Series Scale:              0.02471247
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.43108945   0.4310894 0.04310894     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1   0.00000001   1.0000000 0.40000000     TRUE
##     beta2   0.00000001   1.0000000 0.40000000     TRUE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1  beta2
##      1      2      3      5      6
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     4035.7051: 0.0431089 0.100000 0.100000 0.400000 0.400000
##   1:     4034.9298: 0.0431094 0.0972764 0.0994508 0.398835 0.398912
##   2:     4034.0031: 0.0431135 0.0909049 0.104570 0.403187 0.404038
##   3:     4033.0312: 0.0431251 0.0715777 0.103710 0.408067 0.411105
##   4:     4030.0633: 0.0431420 0.0661602 0.0924939 0.418448 0.424707
##   5:     4027.5190: 0.0431781 0.0500099 0.0620873 0.434325 0.446765
##   6:     4021.0801: 0.0432223 0.0256721 0.0459756 0.454725 0.472369
##   7:     4018.5162: 0.0432359 0.0174955 0.0405651 0.461365 0.480620
##   8:     4017.6252: 0.0432394 0.0148951 0.0387970 0.462725 0.482394
##   9:     4016.5678: 0.0432475 0.0102586 0.0356267 0.466884 0.487496
##  10:     4016.4687: 0.0432476 0.0103393 0.0348064 0.466687 0.487321
##  11:     4016.1778: 0.0432471 0.0111904 0.0348575 0.466628 0.487185
##  12:     4016.0232: 0.0432480 0.0107160 0.0343114 0.466941 0.487542
##  13:     4015.8529: 0.0432497 0.00994502 0.0336338 0.467866 0.488584
##  14:     4015.4762: 0.0432524 0.0116773 0.0306631 0.468185 0.488349
##  15:     4015.4321: 0.0432524 0.0115164 0.0306256 0.468147 0.488314
##  16:     4015.3982: 0.0432524 0.0114533 0.0306913 0.468248 0.488421
##  17:     4015.3433: 0.0432527 0.0111640 0.0305611 0.468340 0.488523
##  18:     4014.5431: 0.0432613 0.00884952 0.0273357 0.471233 0.491555
##  19:     4014.5237: 0.0432613 0.00872219 0.0273382 0.471229 0.491554
##  20:     4014.5048: 0.0432618 0.00871092 0.0273806 0.471289 0.491656
##  21:     4014.4768: 0.0432645 0.00856251 0.0272834 0.471236 0.491820
##  22:     4014.4343: 0.0432705 0.00845358 0.0271337 0.471204 0.492273
##  23:     4014.2291: 0.0433361 0.00735204 0.0251269 0.469957 0.496412
##  24:     4014.2220: 0.0433361 0.00738297 0.0252403 0.470053 0.496510
##  25:     4014.1848: 0.0433380 0.00728686 0.0252222 0.469916 0.496563
```

```
##  26:     4014.1645: 0.0433418 0.00720352 0.0253620 0.469804 0.496853
##  27:     4014.1407: 0.0433512 0.00711920 0.0253509 0.469277 0.497292
##  28:     4014.1273: 0.0433700 0.00712229 0.0254100 0.468324 0.498292
##  29:     4014.1091: 0.0434078 0.00706809 0.0254508 0.466338 0.500212
##  30:     4013.3539: 0.0465538 0.00798279 0.0284388 0.300869 0.661758
##  31:     4013.3442: 0.0478827 0.00622129 0.0252220 0.278217 0.689304
##  32:     4013.2462: 0.0485626 0.00602796 0.0259318 0.265640 0.701614
##  33:     4013.1688: 0.0490616 0.00678986 0.0272613 0.231091 0.733993
##  34:     4013.1576: 0.0507448 0.00684210 0.0273687 0.242967 0.721963
##  35:     4013.1446: 0.0492377 0.00685484 0.0277059 0.247244 0.717357
##  36:     4013.1439: 0.0495425 0.00684027 0.0278134 0.242694 0.721823
##  37:     4013.1438: 0.0494643 0.00685328 0.0278123 0.243528 0.720984
##  38:     4013.1438: 0.0494637 0.00685199 0.0278101 0.243532 0.720983
##  39:     4013.1438: 0.0494639 0.00685199 0.0278101 0.243527 0.720987
##
## Final Estimate of the Negative LLH:
##  LLH:  -7154.806    norm LLH:  -2.370711
##           mu        omega       alpha1        beta1        beta2
## 1.222376e-03 4.184550e-06 2.781014e-02 2.435271e-01 7.209875e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu         omega        alpha1         beta1         beta2
## mu      -6781219.37 -3.966583e+07 -1.700169e+04 -2.409575e+04      -25524.4
## omega  -39665830.17 -7.026312e+12 -2.300212e+09 -2.938015e+09 -2943787437.2
## alpha1    -17001.69 -2.300212e+09 -1.062711e+06 -1.152745e+06    -1153785.1
## beta1     -24095.75 -2.938015e+09 -1.152745e+06 -1.381303e+06    -1383588.8
## beta2     -25524.40 -2.943787e+09 -1.153785e+06 -1.383589e+06    -1385943.4
## attr(,"time")
## Time difference of 0.1189611 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.4218349 secs
```

```
gfit6 = garchFit(~garch(2,2),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 2)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               2 2
##  Max GARCH Order:           2
##  Maximum Order:             2
##  Conditional Dist:          norm
##  h.start:                   3
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02471247
```

```
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.43108945   0.4310894 0.04310894     TRUE
##     omega    0.00000100 100.0000000 0.10000000     TRUE
##     alpha1   0.00000001   1.0000000 0.05000000     TRUE
##     alpha2   0.00000001   1.0000000 0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     gamma2 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1    0.00000001   1.0000000 0.40000000     TRUE
##     beta2    0.00000001   1.0000000 0.40000000     TRUE
##     delta    0.00000000   2.0000000 2.00000000    FALSE
##     skew     0.10000000  10.0000000 1.00000000    FALSE
##     shape    1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1 alpha2  beta1  beta2
##      1      2      3      4      7      8
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     4037.7690: 0.0431089 0.100000 0.0500000 0.0500000 0.400000 0.400000
##   1:     4036.7371: 0.0431095 0.0971632 0.0499537 0.0491218 0.398585 0.398644
##   2:     4035.7785: 0.0431124 0.0935605 0.0557786 0.0510152 0.400344 0.400751
##   3:     4035.2974: 0.0431214 0.0801944 0.0620250 0.0474407 0.400508 0.402017
##   4:     4032.6690: 0.0431366 0.0776533 0.0661902 0.0390507 0.407742 0.411262
##   5:     4029.2064: 0.0431877 0.0611947 0.0678407 0.00229724 0.425983 0.436093
##   6:     4028.1472: 0.0432855 0.0366255 0.0493712 1.00000e-08 0.446704 0.469447
##   7:     4023.6665: 0.0433922 0.00902344 0.0239354 1.00000e-08 0.461975 0.499893
##   8:     4023.6238: 0.0434862 0.0317184 0.0536949 1.00000e-08 0.432231 0.483012
##   9:     4021.8679: 0.0435993 0.0204267 0.0444738 1.00000e-08 0.434209 0.503517
##  10:     4018.3181: 0.0436542 0.0165532 0.0421919 1.00000e-08 0.431558 0.509955
##  11:     4017.5095: 0.0437056 0.0131642 0.0397259 1.00000e-08 0.429553 0.516832
##  12:     4016.6137: 0.0437516 0.0110660 0.0372268 1.00000e-08 0.427945 0.524311
##  13:     4015.2167: 0.0437381 0.0117915 0.0301584 1.00000e-08 0.427640 0.528741
##  14:     4014.6055: 0.0437882 0.00869174 0.0286061 1.00000e-08 0.426459 0.536195
##  15:     4014.4558: 0.0438111 0.00707950 0.0272400 1.00000e-08 0.425239 0.538932
##  16:     4013.8632: 0.0438416 0.00651551 0.0265551 1.00000e-08 0.424185 0.542298
##  17:     4013.8335: 0.0438849 0.00599845 0.0260005 1.00000e-08 0.422246 0.545194
##  18:     4013.7898: 0.0439396 0.00634343 0.0261399 1.00000e-08 0.419567 0.547387
##  19:     4013.7863: 0.0439396 0.00633402 0.0260961 1.00000e-08 0.419542 0.547364
##  20:     4013.7825: 0.0439401 0.00636866 0.0260915 1.00000e-08 0.419531 0.547405
##  21:     4013.7794: 0.0439415 0.00636995 0.0260478 1.00000e-08 0.419447 0.547457
##  22:     4013.7759: 0.0439445 0.00639745 0.0260404 1.00000e-08 0.419315 0.547624
##  23:     4013.7720: 0.0439505 0.00638838 0.0260022 1.00000e-08 0.419009 0.547920
##  24:     4013.7662: 0.0439625 0.00639849 0.0259808 1.00000e-08 0.418430 0.548553
```

```
##  25:     4013.6281: 0.0448010 0.00542788 0.0252184 1.00000e-08 0.376738 0.592015
##  26:     4013.1806: 0.0474124 0.00641361 0.0278716 1.00000e-08 0.243453 0.721564
##  27:     4013.1763: 0.0474125 0.00646480 0.0278613 1.00000e-08 0.243461 0.721572
##  28:     4013.1728: 0.0474131 0.00646543 0.0278255 1.00000e-08 0.243437 0.721544
##  29:     4013.1696: 0.0474172 0.00650759 0.0278194 1.00000e-08 0.243460 0.721541
##  30:     4013.1669: 0.0474260 0.00650977 0.0277858 1.00000e-08 0.243467 0.721494
##  31:     4013.1644: 0.0474440 0.00654589 0.0277754 1.00000e-08 0.243532 0.721449
##  32:     4013.1620: 0.0474802 0.00654765 0.0277402 1.00000e-08 0.243627 0.721327
##  33:     4013.1597: 0.0475525 0.00657791 0.0277197 1.00000e-08 0.243855 0.721125
##  34:     4013.1571: 0.0476974 0.00657895 0.0276768 1.00000e-08 0.244269 0.720702
##  35:     4013.1541: 0.0479877 0.00660285 0.0276520 1.00000e-08 0.245075 0.719925
##  36:     4013.1505: 0.0485712 0.00661131 0.0276579 1.00000e-08 0.246293 0.718662
##  37:     4013.1472: 0.0487632 0.00670191 0.0275680 1.00000e-08 0.242898 0.721994
##  38:     4013.1450: 0.0490402 0.00676198 0.0276806 1.00000e-08 0.242057 0.722682
##  39:     4013.1439: 0.0493122 0.00682631 0.0277763 1.00000e-08 0.243122 0.721449
##  40:     4013.1438: 0.0494997 0.00684802 0.0277941 1.00000e-08 0.244086 0.720451
##  41:     4013.1438: 0.0494689 0.00685710 0.0278239 1.00000e-08 0.243206 0.721290
##  42:     4013.1438: 0.0494608 0.00685183 0.0278097 1.00000e-08 0.243537 0.720978
##  43:     4013.1438: 0.0494645 0.00685195 0.0278101 1.00000e-08 0.243526 0.720988
##
## Final Estimate of the Negative LLH:
##  LLH:  -7154.806    norm LLH:  -2.370711
##           mu        omega       alpha1       alpha2        beta1        beta2
## 1.222390e-03 4.184529e-06 2.781009e-02 1.000000e-08 2.435262e-01 7.209884e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu         omega        alpha1        alpha2         beta1
## mu      -6781219.523 -3.966402e+07 -1.700119e+04  3.640331e+03 -2.409515e+04
## omega  -39664015.107 -7.026343e+12 -2.300222e+09 -2.309297e+09 -2.938028e+09
## alpha1    -17001.195 -2.300222e+09 -1.062715e+06 -1.051157e+06 -1.152751e+06
## alpha2      3640.331 -2.309297e+09 -1.051157e+06 -1.069036e+06 -1.158211e+06
## beta1     -24095.152 -2.938028e+09 -1.152751e+06 -1.158211e+06 -1.381310e+06
## beta2     -25523.804 -2.943800e+09 -1.153790e+06 -1.158429e+06 -1.383596e+06
##                beta2
## mu          -25523.8
## omega  -2943800483.3
## alpha1    -1153790.4
## alpha2    -1158428.9
## beta1     -1383595.9
## beta2     -1385950.4
## attr(,"time")
## Time difference of 0.1021531 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.4898622 secs
```

```
gfit7 = garchFit(~arma(1,3)+garch(1,0),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 3)
```

```
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 0)
##  ARMA Order:                1 3
##  Max ARMA Order:            3
##  GARCH Order:               1 0
##  Max GARCH Order:           1
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02471247
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V      params includes
##     mu     -0.43108945   0.4310894  0.04309277     TRUE
##     ar1    -0.99999999   1.0000000 -0.60590245     TRUE
##     ma1    -0.99999999   1.0000000  0.56853128     TRUE
##     ma2    -0.99999999   1.0000000 -0.02341180     TRUE
##     ma3    -0.99999999   1.0000000 -0.04691329     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1    ma2    ma3  omega alpha1
##      1      2      3      4      5      6      7
##  Persistence:                 0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     9459.6629: 0.0430928 -0.605902 0.568531 -0.0234118 -0.0469133 0.100000 0.100000
##   1:     4229.1800: 0.0430933 -0.589291 0.584998 -0.0296602 -0.0316384  1.07204 0.333074
##   2:     4225.0746: 0.0431156 -0.601191 0.573239 0.00312693 -0.0319094  1.07451 0.299330
##   3:     4185.8585: 0.0431414 -0.559700 0.612362 -0.0307884 0.00492762 0.907219 0.220249
##   4:     4182.4082: 0.0432757 -0.537350 0.504788 -0.0349947 -0.145798 0.836011 0.230839
##   5:     4158.0620: 0.0433336 -0.520430 0.492823 -0.0424735 -0.0494842 0.823156 0.222397
##   6:     4153.2624: 0.0433862 -0.507197 0.509091 0.0134245 -0.00500462 0.757169 0.212326
##   7:     4152.5278: 0.0434207 -0.506255 0.510653 0.0144208 -0.0200505 0.778877 0.247835
##   8:     4152.3126: 0.0434341 -0.501508 0.515573 0.00617135 -0.0102433 0.771301 0.245452
##   9:     4152.0462: 0.0434639 -0.508381 0.508481 0.00897779 -0.0116703 0.762009 0.254388
##  10:     4151.9799: 0.0435185 -0.503912 0.513274 0.0101360 -0.0121699 0.759886 0.256584
##  11:     4151.9772: 0.0436419 -0.504744 0.513310 0.0144031 -0.0107788 0.761947 0.251872
```

```
##  12:     4151.9744: 0.0437202 -0.504769 0.514348 0.0120155 -0.00913572 0.760918 0.251708
##  13:     4151.9572: 0.0438199 -0.506459 0.514284 0.0123977 -0.0106926 0.759775 0.253044
##  14:     4151.9523: 0.0439489 -0.507078 0.516067 0.0115193 -0.0113129 0.759610 0.253459
##  15:     4151.9338: 0.0447542 -0.515173 0.522167 0.0109904 -0.00893053 0.759114 0.253570
##  16:     4151.9140: 0.0455362 -0.522991 0.528845 0.0128603 -0.0119649 0.761488 0.251364
##  17:     4151.8724: 0.0463271 -0.529041 0.537786 0.0126129 -0.00988944 0.762073 0.251657
##  18:     4151.8416: 0.0471030 -0.536789 0.544996 0.0131496 -0.0100737 0.759344 0.255423
##  19:     4151.8137: 0.0478905 -0.544395 0.552281 0.0107677 -0.0110378 0.759610 0.252850
##  20:     4151.7940: 0.0486745 -0.553184 0.559050 0.0117863 -0.0117833 0.760646 0.251656
##  21:     4151.7635: 0.0494470 -0.560386 0.568131 0.0117800 -0.0107946 0.761590 0.250848
##  22:     4151.6676: 0.0558920 -0.634353 0.637642 0.0134158 -0.00846759 0.767717 0.245402
##  23:     4151.5540: 0.0622196 -0.706463 0.713098 0.0155616 -0.00616218 0.762483 0.261342
##  24:     4151.5306: 0.0644899 -0.713313 0.720282 0.00877400 -0.00734690 0.758896 0.258444
##  25:     4151.5138: 0.0666946 -0.699578 0.716974 0.0191837 -0.00988206 0.759327 0.255214
##  26:     4151.4776: 0.0666958 -0.701718 0.714859 0.0191122 -0.00936901 0.759085 0.255223
##  27:     4151.4595: 0.0667938 -0.701942 0.715392 0.0175548 -0.00818241 0.758995 0.255061
##  28:     4151.4482: 0.0670078 -0.705213 0.716787 0.0172308 -0.00849794 0.758937 0.254924
##  29:     4151.4390: 0.0674422 -0.710189 0.721455 0.0158674 -0.00839300 0.758948 0.254630
##  30:     4151.4287: 0.0695807 -0.719818 0.730280 0.0147190 -0.00925845 0.758815 0.254177
##  31:     4151.4248: 0.0717871 -0.719138 0.727338 0.0145991 -0.00698774 0.758560 0.254203
##  32:     4151.4194: 0.0713899 -0.755364 0.762467 0.0139658 -0.00624536 0.759766 0.253869
##  33:     4151.4110: 0.0732067 -0.775369 0.783845 0.0144151 -0.00622997 0.759348 0.254085
##  34:     4151.3575: 0.0765668 -0.828371 0.838637 0.0153459 -0.00491272 0.758645 0.254760
##  35:     4149.6804: 0.0862991 -0.985324  1.00000 0.0165515 0.00158326 0.756556 0.257169
##  36:     4148.7081: 0.0862991 -0.985882 0.998557 0.0179853 0.000154919 0.756543 0.257200
##  37:     4148.3460: 0.0862675 -0.983638 0.997798 0.0176848 0.000632402 0.756487 0.257342
##  38:     4148.0570: 0.0861648 -0.981855 0.993743 0.0184071 0.000441231 0.756385 0.257631
##  39:     4147.7714: 0.0860483 -0.979094 0.991143 0.0172801 0.00214506 0.756278 0.257933
##  40:     4147.6511: 0.0859118 -0.977764 0.987455 0.0172395 0.00272301 0.756087 0.258374
##  41:     4147.6087: 0.0857230 -0.976438 0.985683 0.0167155 0.00345119 0.755602 0.259286
##  42:     4147.5239: 0.0838934 -0.975642 0.988470 0.0172853 0.000302768 0.749634 0.269680
##  43:     4147.5212: 0.0842136 -0.975239 0.987233 0.0169121 0.000601798 0.749344 0.269949
##  44:     4147.5208: 0.0845392 -0.975203 0.987251 0.0167507 0.000409514 0.749132 0.270124
##  45:     4147.5208: 0.0848551 -0.975176 0.987256 0.0166378 0.000277857 0.749088 0.270121
##  46:     4147.5207: 0.0848747 -0.975177 0.987256 0.0166435 0.000283224 0.749091 0.270102
##  47:     4147.5207: 0.0848794 -0.975177 0.987258 0.0166484 0.000285856 0.749092 0.270099
##
## Final Estimate of the Negative LLH:
##  LLH:  -7020.429    norm LLH:  -2.326186
##            mu           ar1           ma1           ma2           ma3
##  0.0020975792 -0.9751774672  0.9872584657  0.0166484251  0.0002858561
##         omega        alpha1
##  0.0004574754  0.2700991140
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu         ar1          ma1         ma2         ma3
## mu     -1511479.7962  -9290.3587    -395.6947    800.6965  -1292.4164
## ar1       -9290.3587 -77690.2511  -67954.6427  68136.4524 -67942.7444
## ma1         -395.6947 -67954.6427  -68909.5259  69066.3867 -68867.9144
## ma2          800.6965  68136.4524   69066.3867 -71256.7483  71112.6482
## ma3        -1292.4164 -67942.7444  -68867.9144  71112.6482 -73960.9705
## omega     624583.4062  62056.1163 -104566.4684 -70044.6014 101225.9605
## alpha1     -1132.0706   -103.9739     116.0317    109.4983   -200.4146
##                omega        alpha1
```

```
## mu      6.245834e+05    -1132.0706
## ar1     6.205612e+04     -103.9739
## ma1    -1.045665e+05      116.0317
## ma2    -7.004460e+04      109.4983
## ma3     1.012260e+05     -200.4146
## omega  -5.498893e+09 -1123489.8975
## alpha1 -1.123490e+06    -1104.0024
## attr(,"time")
## Time difference of 0.1231561 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.851907 secs
```

```
gfit8 = garchFit(~arma(1,3)+garch(1,1),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                1 3
##  Max ARMA Order:            3
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02471247
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V      params includes
##     mu     -0.43108945   0.4310894  0.04309277     TRUE
##     ar1    -0.99999999   1.0000000 -0.60590245     TRUE
##     ma1    -0.99999999   1.0000000  0.56853128     TRUE
##     ma2    -0.99999999   1.0000000 -0.02341180     TRUE
##     ma3    -0.99999999   1.0000000 -0.04691329     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     beta1   0.00000001   1.0000000  0.80000000     TRUE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
```

```
##     mu    ar1    ma1    ma2    ma3  omega alpha1  beta1
##      1      2      3      4      5      6      7      9
##  Persistence:                  0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     4044.7427: 0.0430928 -0.605902 0.568531 -0.0234118 -0.0469133 0.100000 0.100000 0.800000
##   1:     4042.8580: 0.0431020 -0.598660 0.575543 -0.0268666 -0.0425723 0.0905030 0.0995514 0.803908
##   2:     4041.5850: 0.0431114 -0.592475 0.581560 -0.0294662 -0.0389231 0.0903996 0.103559 0.815198
##   3:     4039.3375: 0.0431246 -0.586815 0.587091 -0.0307229 -0.0357044 0.0791262 0.100842 0.820581
##   4:     4034.2486: 0.0431787 -0.588581 0.584875 -0.0163922 -0.0384555 0.0611362 0.0840663 0.855585
##   5:     4034.1619: 0.0431799 -0.588528 0.584923 -0.0164446 -0.0382515 0.0616600 0.0838421 0.856734
##   6:     4034.0482: 0.0431812 -0.588457 0.584987 -0.0164898 -0.0380337 0.0608223 0.0828941 0.856920
##   7:     4033.8191: 0.0431845 -0.588193 0.585228 -0.0165959 -0.0374828 0.0607377 0.0818361 0.859201
##   8:     4032.0249: 0.0432214 -0.584401 0.588715 -0.0181472 -0.0310828 0.0507504 0.0677622 0.878796
##   9:     4031.8418: 0.0432219 -0.584580 0.588530 -0.0180030 -0.0311525 0.0516582 0.0683539 0.879914
##  10:     4031.6960: 0.0432248 -0.585191 0.587896 -0.0183780 -0.0307364 0.0506079 0.0679709 0.880329
##  11:     4031.5259: 0.0432313 -0.586624 0.586411 -0.0189160 -0.0301198 0.0500171 0.0677501 0.882500
##  12:     4031.2350: 0.0432477 -0.589780 0.583145 -0.0214149 -0.0286641 0.0488409 0.0651598 0.884238
##  13:     4031.1044: 0.0432700 -0.589878 0.582894 -0.0244309 -0.0278830 0.0454133 0.0663683 0.888360
##  14:     4030.8367: 0.0432903 -0.591618 0.581020 -0.0236448 -0.0298222 0.0450005 0.0614866 0.890620
##  15:     4030.7946: 0.0433013 -0.592269 0.580303 -0.0239684 -0.0304121 0.0462826 0.0611138 0.892733
##  16:     4030.5280: 0.0433094 -0.592429 0.580096 -0.0241569 -0.0307074 0.0453495 0.0602044 0.892700
##  17:     4030.4124: 0.0433136 -0.592603 0.579899 -0.0233672 -0.0310565 0.0440739 0.0608477 0.894899
##  18:     4030.1093: 0.0433603 -0.593671 0.578543 -0.0244127 -0.0325778 0.0426691 0.0573862 0.897948
##  19:     4029.9268: 0.0434216 -0.594068 0.577692 -0.0245234 -0.0329802 0.0416984 0.0545739 0.902294
##  20:     4029.5570: 0.0434976 -0.591682 0.579413 -0.0234929 -0.0314554 0.0391324 0.0537100 0.905179
##  21:     4029.4766: 0.0436649 -0.590008 0.579760 -0.0223299 -0.0260270 0.0355121 0.0521364 0.912852
##  22:     4028.6822: 0.0438273 -0.592162 0.576397 -0.0237357 -0.0222533 0.0317100 0.0470778 0.918922
##  23:     4027.4697: 0.0443128 -0.590538 0.574211 -0.0434999 -0.0476326 0.0224553 0.0325684 0.941435
##  24:     4027.1374: 0.0443129 -0.590577 0.574172 -0.0434310 -0.0476229 0.0228713 0.0332622 0.942117
##  25:     4026.8665: 0.0443238 -0.590751 0.573934 -0.0435573 -0.0475492 0.0220627 0.0328723 0.942504
##  26:     4026.5392: 0.0443471 -0.591139 0.573407 -0.0438088 -0.0473652 0.0213889 0.0327439 0.944298
##  27:     4024.9800: 0.0445093 -0.593418 0.570131 -0.0463035 -0.0459872 0.0171044 0.0274332 0.953323
##  28:     4024.0176: 0.0445643 -0.594371 0.568891 -0.0475442 -0.0457726 0.0150326 0.0259756 0.957027
##  29:     4021.8777: 0.0446724 -0.596250 0.566450 -0.0498252 -0.0452922 0.0108839 0.0235517 0.964702
##  30:     4021.1326: 0.0447187 -0.597054 0.565408 -0.0508151 -0.0450930 0.00882725 0.0221774 0.967694
##  31:     4020.3247: 0.0447659 -0.597862 0.564356 -0.0518445 -0.0448879 0.00739640 0.0210487 0.971100
##  32:     4019.8720: 0.0448134 -0.598673 0.563300 -0.0528784 -0.0446765 0.00560056 0.0196368 0.974207
##  33:     4019.8448: 0.0448315 -0.598964 0.562918 -0.0532086 -0.0445687 0.00524697 0.0192357 0.975556
##  34:     4019.6581: 0.0448413 -0.599091 0.562749 -0.0532862 -0.0444640 0.00481889 0.0188693 0.976033
##  35:     4019.1353: 0.0449759 -0.598424 0.563299 -0.0461703 -0.0393643 0.00302228 0.0165109 0.980537
##  36:     4019.0263: 0.0449759 -0.598428 0.563295 -0.0461499 -0.0393663 0.00305651 0.0163171 0.980397
##  37:     4018.9057: 0.0449771 -0.598402 0.563324 -0.0459988 -0.0392865 0.00321011 0.0163169 0.980456
##  38:     4018.8060: 0.0449805 -0.598330 0.563406 -0.0456102 -0.0390740 0.00320149 0.0161846 0.980399
##  39:     4018.6841: 0.0449874 -0.598177 0.563578 -0.0448129 -0.0386309 0.00333085 0.0160996 0.980466
##  40:     4018.5062: 0.0450014 -0.597863 0.563932 -0.0432098 -0.0377310 0.00331627 0.0158821 0.980493
##  41:     4018.2469: 0.0450295 -0.597225 0.564654 -0.0400011 -0.0359284 0.00342905 0.0156270 0.980720
##  42:     4018.0521: 0.0450582 -0.596570 0.565418 -0.0367854 -0.0341606 0.00336842 0.0153300 0.980841
##  43:     4017.8605: 0.0450886 -0.595865 0.566305 -0.0335510 -0.0325110 0.00344595 0.0151892 0.981035
```

```
##  44:     4017.5460: 0.0451741 -0.593994 0.568964 -0.0259183 -0.0293029 0.00342161 0.0149720 0.981092
##  45:     4017.4282: 0.0454195 -0.591968 0.573158 -0.0215957 -0.0318780 0.00374543 0.0163175 0.979759
##  46:     4017.3225: 0.0457997 -0.591771 0.571270 -0.0203892 -0.0322098 0.00362095 0.0160108 0.979857
##  47:     4017.1277: 0.0461732 -0.591247 0.568961 -0.0189532 -0.0318396 0.00367634 0.0157151 0.980280
##  48:     4017.0619: 0.0465336 -0.589149 0.566283 -0.0178818 -0.0312088 0.00361480 0.0155154 0.980435
##  49:     4016.6024: 0.0520957 -0.539733 0.521146 -0.00876757 -0.0231867 0.00361032 0.0155646 0.98046
##  50:     4016.2428: 0.0580127 -0.507293 0.484238 -0.00600336 -0.0266465 0.00370097 0.0161347 0.97986
##  51:     4016.0859: 0.0639832 -0.475243 0.451853 -0.00905500 -0.0291709 0.00408768 0.0160541 0.97935
##  52:     4016.0555: 0.0702677 -0.473428 0.448924 -0.0115499 -0.0303876 0.00347441 0.0154342 0.980677
##  53:     4015.9525: 0.0750892 -0.537497 0.517079 -0.00808559 -0.0288528 0.00358513 0.0156992 0.98029
##  54:     4015.9474: 0.0773189 -0.583407 0.561072 -0.00715184 -0.0261976 0.00368586 0.0155577 0.98047
##  55:     4015.8724: 0.0786398 -0.603126 0.579762 -0.00814249 -0.0243837 0.00370103 0.0157641 0.98011
##  56:     4015.8650: 0.0794061 -0.610474 0.586668 -0.00895379 -0.0233533 0.00378288 0.0159884 0.97986
##  57:     4015.8584: 0.0808641 -0.630992 0.606699 -0.00814786 -0.0218019 0.00381224 0.0159371 0.97987
##  58:     4015.8582: 0.0827931 -0.614223 0.592099 -0.00589005 -0.0205519 0.00391139 0.0160177 0.97962
##  59:     4015.8532: 0.0830910 -0.631242 0.609062 -0.00671387 -0.0207045 0.00389735 0.0160226 0.97964
##  60:     4015.8518: 0.0820293 -0.631638 0.608543 -0.00720321 -0.0212084 0.00384962 0.0159625 0.97978
##  61:     4015.8510: 0.0826028 -0.637832 0.615035 -0.00731151 -0.0209615 0.00386429 0.0159965 0.97972
##  62:     4015.8510: 0.0825593 -0.636018 0.613308 -0.00722566 -0.0210144 0.00386615 0.0159913 0.97972
##  63:     4015.8510: 0.0825433 -0.636194 0.613437 -0.00724834 -0.0209997 0.00386492 0.0159924 0.97972
##  64:     4015.8510: 0.0825513 -0.636331 0.613578 -0.00725051 -0.0210006 0.00386498 0.0159924 0.97972
##
## Final Estimate of the Negative LLH:
##  LLH:  -7152.099    norm LLH:  -2.369814
##            mu           ar1           ma1           ma2           ma3
##  2.040048e-03 -6.363308e-01  6.135779e-01 -7.250511e-03 -2.100060e-02
##         omega        alpha1         beta1
##  2.360366e-06  1.599238e-02  9.797282e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu           ar1           ma1           ma2         ma3
## mu     -2.686922e+06   -2765.62511     612.96929  2.703493e+02    448.8653
## ar1    -2.765625e+03   -4443.89919   -4313.96489  2.727102e+03  -1719.5883
## ma1     6.129693e+02   -4313.96489   -4206.93485  2.543294e+03  -1513.1278
## ma2     2.703493e+02    2727.10174    2543.29443 -4.207479e+03   2639.1729
## ma3     4.488653e+02   -1719.58834   -1513.12777  2.639173e+03  -4210.0272
## omega  -6.524302e+07 2949614.65525 3166144.74603 -5.309019e+05 925618.3492
## alpha1 -2.227736e+04     -14.45431      11.71039  3.503469e-01    347.0025
## beta1  -4.103886e+04    -611.13055    -464.86405  1.273589e+03   -605.5652
##                omega        alpha1         beta1
## mu     -6.524302e+07 -2.227736e+04 -4.103886e+04
## ar1     2.949615e+06 -1.445431e+01 -6.111305e+02
## ma1     3.166145e+06  1.171039e+01 -4.648640e+02
## ma2    -5.309019e+05  3.503469e-01  1.273589e+03
## ma3     9.256183e+05  3.470025e+02 -6.055652e+02
## omega  -2.221768e+13 -7.153579e+09 -8.997198e+09
## alpha1 -7.153579e+09 -3.274831e+06 -3.530498e+06
## beta1  -8.997198e+09 -3.530498e+06 -4.222212e+06
## attr(,"time")
## Time difference of 0.1140978 secs
##
## --- END OF TRACE ---
##
##
```

```
## Time to Estimate Parameters:
##  Time difference of 0.60695 secs
```

```
gfit9 = garchFit(~arma(1,3)+garch(2,0),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 0)
##  ARMA Order:                1 3
##  Max ARMA Order:            3
##  GARCH Order:               2 0
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02471247
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V      params includes
##     mu     -0.43108945   0.4310894  0.04309277     TRUE
##     ar1    -0.99999999   1.0000000 -0.60590245     TRUE
##     ma1    -0.99999999   1.0000000  0.56853128     TRUE
##     ma2    -0.99999999   1.0000000 -0.02341180     TRUE
##     ma3    -0.99999999   1.0000000 -0.04691329     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.05000000     TRUE
##     alpha2  0.00000001   1.0000000  0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     gamma2 -0.99999999   1.0000000  0.10000000    FALSE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1    ma2    ma3  omega alpha1 alpha2
##      1      2      3      4      5      6      7      8
##  Persistence:                  0.1
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     8802.7518: 0.0430928 -0.605902 0.568531 -0.0234118 -0.0469133 0.100000 0.0500000 0.0500000
##   1:     4274.8993: 0.0431104 -0.606896 0.567463 -0.0161514 -0.0438796  1.00331 0.360539 0.345858
```

```
##   2:     4265.2823: 0.0431550 -0.592047 0.580608 -0.0189716 -0.0213956  1.01194 0.337206 0.314661
##   3:     4175.3733: 0.0432012 -0.605425 0.566911 0.00314245 -0.0287308 0.858513 0.254279 0.221536
##   4:     4148.9620: 0.0432269 -0.593375 0.577765 -0.00454792 -0.0170928 0.803306 0.221611 0.184809
##   5:     4129.5553: 0.0432962 -0.598744 0.571871 0.00793170 -0.0198847 0.692743 0.155772 0.101750
##   6:     4127.9906: 0.0433253 -0.587897 0.581817 -0.00255035 -0.0106778 0.707019 0.166938 0.115115
##   7:     4127.8020: 0.0436790 -0.587369 0.575435 0.0101759 -0.0129140 0.711299 0.186585 0.0996503
##   8:     4127.6158: 0.0438429 -0.586467 0.574155 0.00318416 -0.0130585 0.705398 0.186294 0.110175
##   9:     4127.5727: 0.0439078 -0.587687 0.572501 0.00421362 -0.0148975 0.707611 0.181148 0.110456
##  10:     4127.5645: 0.0439796 -0.585217 0.574257 0.00174996 -0.0125428 0.710364 0.177806 0.110710
##  11:     4127.5254: 0.0441398 -0.586455 0.571623 0.00351268 -0.0141319 0.712976 0.175177 0.109850
##  12:     4127.5080: 0.0443516 -0.584179 0.572212 0.00287126 -0.0160153 0.710732 0.175924 0.111631
##  13:     4127.4955: 0.0445427 -0.585712 0.569092 0.00173047 -0.0131037 0.709861 0.175764 0.111949
##  14:     4127.4718: 0.0447858 -0.583019 0.569975 0.00202406 -0.0144173 0.710573 0.175311 0.112135
##  15:     4127.4670: 0.0448048 -0.583559 0.569300 0.00254264 -0.0148496 0.710584 0.175296 0.111990
##  16:     4127.4627: 0.0448986 -0.582846 0.569253 0.00190679 -0.0141471 0.710710 0.175041 0.111901
##  17:     4127.1975: 0.0511001 -0.564406 0.543526 -0.00212392 -0.0133511 0.700905 0.185444 0.111294
##  18:     4126.7561: 0.0572773 -0.536755 0.527776 0.0108228 -0.0121500 0.711853 0.176403 0.112837
##  19:     4126.6590: 0.0593654 -0.536242 0.514411 -0.00260007 -0.0181025 0.713822 0.170621 0.108123
##  20:     4126.4906: 0.0615658 -0.527100 0.510009 -0.00106781 -0.0152584 0.709081 0.171932 0.115710
##  21:     4126.3795: 0.0638002 -0.520110 0.503667 0.00284132 -0.0162733 0.710167 0.173506 0.113525
##  22:     4126.3049: 0.0660531 -0.513793 0.499041 0.00299707 -0.0137170 0.710503 0.172888 0.114339
##  23:     4126.2237: 0.0683017 -0.511559 0.493866 0.00179523 -0.0170811 0.706094 0.173153 0.118276
##  24:     4126.2194: 0.0685747 -0.509998 0.496392 0.000244730 -0.0180749 0.708690 0.173505 0.116967
##  25:     4126.1957: 0.0688773 -0.512262 0.495363 0.00214761 -0.0172472 0.709016 0.172815 0.116257
##  26:     4126.1859: 0.0692004 -0.512451 0.496801 0.00217511 -0.0156608 0.708929 0.172558 0.116203
##  27:     4126.1332: 0.0717864 -0.519662 0.504994 -0.00220676 -0.0204007 0.708813 0.173116 0.116465
##  28:     4126.0804: 0.0743583 -0.532032 0.510465 0.00127394 -0.0183255 0.709224 0.172219 0.116151
##  29:     4125.9802: 0.0768967 -0.541662 0.524472 -0.000265385 -0.0174477 0.709707 0.171858 0.116700
##  30:     4125.9660: 0.0792449 -0.559575 0.544305 0.00184227 -0.0170112 0.703222 0.176610 0.113654
##  31:     4125.8822: 0.0816898 -0.575548 0.560406 0.00229550 -0.0160736 0.706886 0.175247 0.117184
##  32:     4125.8277: 0.0870674 -0.604516 0.587868 0.00351066 -0.0161479 0.705971 0.171029 0.118535
##  33:     4125.8078: 0.0925822 -0.586266 0.566635 -0.00145938 -0.0154048 0.705276 0.170956 0.119805
##  34:     4125.7895: 0.0944363 -0.598922 0.583386 0.000811148 -0.0165524 0.710233 0.173051 0.116422
##  35:     4125.7711: 0.0945315 -0.633160 0.617298 -0.000572383 -0.0151627 0.708236 0.173327 0.117977
##  36:     4125.7659: 0.0945763 -0.629329 0.612721 0.000952556 -0.0137524 0.707335 0.172359 0.118377
##  37:     4125.7656: 0.0948546 -0.629536 0.613210 0.00157729 -0.0140351 0.707068 0.172608 0.118967
##  38:     4125.7652: 0.0951263 -0.630720 0.614507 0.00151303 -0.0139023 0.707196 0.172629 0.118699
##  39:     4125.7651: 0.0954051 -0.631653 0.615408 0.00142401 -0.0138291 0.707299 0.172644 0.118469
##  40:     4125.7651: 0.0953602 -0.631430 0.615236 0.00145003 -0.0137948 0.707326 0.172598 0.118497
##  41:     4125.7651: 0.0953725 -0.631494 0.615279 0.00144071 -0.0138131 0.707309 0.172618 0.118495
##  42:     4125.7651: 0.0953726 -0.631501 0.615286 0.00144157 -0.0138124 0.707310 0.172618 0.118494
##
## Final Estimate of the Negative LLH:
##  LLH:  -7042.185    norm LLH:  -2.333395
##            mu           ar1           ma1           ma2           ma3
##  0.0023568920 -0.6315013935  0.6152863614  0.0014415656 -0.0138123985
##         omega        alpha1        alpha2
##  0.0004319588  0.1726176154  0.1184938416
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu        ar1        ma1         ma2         ma3
## mu     -2323419.5766 -4009.7969  -546.4246   1552.3630   -107.5929
## ar1       -4009.7969 -3058.4405 -2956.1637   1910.8309  -1387.9193
## ma1        -546.4246 -2956.1637 -2890.5741   1789.1119  -1211.9026
```

```
## ma2        1552.3630  1910.8309  1789.1119  -3200.1588   2284.6935
## ma3        -107.5929 -1387.9193 -1211.9026   2284.6935  -3871.1146
## omega  -2495090.5190 15683.7337 17327.8251 -90512.1692 -20301.7516
## alpha1    -1337.6067   177.0584   180.1091   -108.5039    148.7182
## alpha2     7626.3819  -268.4011  -277.0005    380.6559   -136.3703
##                omega        alpha1        alpha2
## mu     -2.495091e+06    -1337.6067     7626.3819
## ar1     1.568373e+04      177.0584     -268.4011
## ma1     1.732783e+04      180.1091     -277.0005
## ma2    -9.051217e+04     -108.5039      380.6559
## ma3    -2.030175e+04      148.7182     -136.3703
## omega  -5.728475e+09 -1280223.1247 -1443289.9489
## alpha1 -1.280223e+06    -1553.8020     -649.2513
## alpha2 -1.443290e+06     -649.2513    -2039.0592
## attr(,"time")
## Time difference of 0.203624 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 2.022299 secs
```

```
gfit10 = garchFit(~arma(1,3)+garch(2,1),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 1)
##  ARMA Order:                1 3
##  Max ARMA Order:            3
##  GARCH Order:               2 1
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02471247
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V      params includes
##     mu     -0.43108945   0.4310894  0.04309277     TRUE
##     ar1    -0.99999999   1.0000000 -0.60590245     TRUE
##     ma1    -0.99999999   1.0000000  0.56853128     TRUE
##     ma2    -0.99999999   1.0000000 -0.02341180     TRUE
##     ma3    -0.99999999   1.0000000 -0.04691329     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
```

```
##     alpha1  0.00000001   1.0000000  0.05000000     TRUE
##     alpha2  0.00000001   1.0000000  0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     gamma2 -0.99999999   1.0000000  0.10000000    FALSE
##     beta1   0.00000001   1.0000000  0.80000000     TRUE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1    ma2    ma3  omega alpha1 alpha2  beta1
##      1      2      3      4      5      6      7      8     11
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     4045.3933: 0.0430928 -0.605902 0.568531 -0.0234118 -0.0469133 0.100000 0.0500000 0.0500000 0
##   1:     4044.8685: 0.0431008 -0.599376 0.574776 -0.0266919 -0.0427705 0.0883565 0.0533390 0.0478809
##   2:     4041.4223: 0.0431092 -0.593334 0.580576 -0.0295339 -0.0389288 0.0878015 0.0621171 0.0514277
##   3:     4039.7968: 0.0431251 -0.586899 0.586789 -0.0311163 -0.0347047 0.0778286 0.0644134 0.0465025
##   4:     4036.7541: 0.0431713 -0.587800 0.585480 -0.0208497 -0.0347428 0.0742540 0.0551602 0.0252287
##   5:     4036.6130: 0.0431729 -0.587935 0.585335 -0.0206551 -0.0347035 0.0743129 0.0574885 0.0269375
##   6:     4035.8681: 0.0431768 -0.588247 0.584989 -0.0198839 -0.0347448 0.0712492 0.0570648 0.0253726
##   7:     4035.1369: 0.0431864 -0.588874 0.584279 -0.0181659 -0.0347082 0.0681657 0.0586388 0.0241496
##   8:     4033.7648: 0.0432069 -0.589605 0.583359 -0.0152474 -0.0341853 0.0605107 0.0581828 0.0181368
##   9:     4026.4397: 0.0433823 -0.583881 0.587322 -0.0128790 -0.0182293 0.0174067 0.0363041 1.00000e-0
##  10:     4023.7988: 0.0434108 -0.584801 0.586088 -0.0130046 -0.0169166 0.00898972 0.0292210 1.00000e-
##  11:     4023.4203: 0.0434109 -0.584841 0.586047 -0.0129735 -0.0169416 0.00886992 0.0287592 1.00000e-
##  12:     4023.1040: 0.0434109 -0.584892 0.585996 -0.0129339 -0.0169737 0.00932151 0.0285402 1.00000e-
##  13:     4022.7203: 0.0434136 -0.585389 0.585507 -0.0129963 -0.0168013 0.00905898 0.0278009 1.00000e-
##  14:     4022.1232: 0.0434203 -0.586471 0.584456 -0.0133251 -0.0162163 0.00901193 0.0271921 1.00000e-
##  15:     4020.0462: 0.0434901 -0.595811 0.575599 -0.0178617 -0.00991003 0.00393601 0.0203077 1.00000e
##  16:     4019.4552: 0.0434901 -0.595817 0.575593 -0.0178532 -0.00991846 0.00426727 0.0203505 1.00000e
##  17:     4019.1630: 0.0434901 -0.595832 0.575577 -0.0178295 -0.00994167 0.00434389 0.0200232 1.00000e
##  18:     4018.9722: 0.0434921 -0.595955 0.575479 -0.0174321 -0.0104254 0.00469459 0.0200063 1.00000e-
##  19:     4018.6523: 0.0434968 -0.596157 0.575341 -0.0165441 -0.0115320 0.00464607 0.0199202 1.00000e-
##  20:     4018.3482: 0.0435076 -0.596176 0.575487 -0.0148506 -0.0139086 0.00492517 0.0198784 1.00000e-
##  21:     4017.9742: 0.0435347 -0.594284 0.577786 -0.0126576 -0.0182952 0.00447058 0.0191149 1.00000e-
##  22:     4017.8407: 0.0435952 -0.591774 0.580741 -0.00991074 -0.0208911 0.00415854 0.0182124 1.00000e
##  23:     4017.6238: 0.0436586 -0.594205 0.578461 -0.00545983 -0.0209023 0.00433565 0.0188793 1.00000e
##  24:     4017.3336: 0.0438002 -0.592702 0.579555 -0.00734521 -0.0185757 0.00420943 0.0174302 1.00000e
##  25:     4017.2858: 0.0438002 -0.592706 0.579551 -0.00734216 -0.0185782 0.00416333 0.0173689 1.00000e
##  26:     4017.2484: 0.0438003 -0.592735 0.579522 -0.00731920 -0.0185968 0.00432216 0.0172232 1.00000e
##  27:     4017.2111: 0.0438086 -0.592971 0.579271 -0.00725649 -0.0187535 0.00429014 0.0172072 1.00000e
##  28:     4017.1820: 0.0438176 -0.593207 0.579019 -0.00717395 -0.0189036 0.00434436 0.0172488 1.00000e
##  29:     4017.1107: 0.0438650 -0.594071 0.578063 -0.00687487 -0.0195503 0.00434706 0.0172782 1.00000e
##  30:     4017.0187: 0.0440093 -0.593763 0.577875 -0.00699188 -0.0203145 0.00402876 0.0166554 1.00000e
##  31:     4017.0108: 0.0441507 -0.593227 0.577385 -0.00593865 -0.0192889 0.00388591 0.0164469 1.00000e
##  32:     4016.9676: 0.0443001 -0.592929 0.576442 -0.00545003 -0.0187312 0.00396364 0.0165550 1.00000e
##  33:     4016.9398: 0.0445434 -0.592651 0.574664 -0.00869159 -0.0216621 0.00401461 0.0166202 1.00000e
##  34:     4016.8652: 0.0451381 -0.590153 0.572060 -0.00540362 -0.0200165 0.00410975 0.0166927 1.00000e
```

```
##  35:     4016.8355: 0.0457430 -0.587462 0.569210 -0.00383556 -0.0186004 0.00397825 0.0164228 1.00000e
##  36:     4016.7632: 0.0463495 -0.584333 0.566442 -0.00462561 -0.0190425 0.00384049 0.0159864 1.00000e
##  37:     4016.4598: 0.0534796 -0.541751 0.529379 0.00241772 -0.0249767 0.00358345 0.0159553 1.00000e-
##  38:     4016.0872: 0.0606210 -0.505097 0.486589 0.00214843 -0.0217109 0.00371053 0.0158062 1.00000e-
##  39:     4016.0385: 0.0633067 -0.491191 0.469857 0.00237474 -0.0207335 0.00397311 0.0165641 1.00000e-
##  40:     4015.8747: 0.0659995 -0.476866 0.455652 -0.00331687 -0.0246298 0.00373524 0.0163239 1.00000e
##  41:     4015.8708: 0.0662973 -0.476211 0.455902 -0.00322657 -0.0248139 0.00401026 0.0162321 1.00000e
##  42:     4015.8553: 0.0668954 -0.476161 0.455952 -0.00251770 -0.0242432 0.00403349 0.0163302 1.00000e
##  43:     4015.8258: 0.0674918 -0.477428 0.456209 -0.00278661 -0.0238326 0.00393575 0.0162257 1.00000e
##  44:     4015.8008: 0.0698511 -0.484796 0.462790 -0.00407600 -0.0237868 0.00380187 0.0161637 1.00000e
##  45:     4015.7847: 0.0721625 -0.495505 0.473027 -0.00422743 -0.0233106 0.00397632 0.0164570 1.00000e
##  46:     4015.7544: 0.0735352 -0.527668 0.505364 -0.00481683 -0.0228683 0.00391133 0.0162585 1.00000e
##  47:     4015.7459: 0.0746938 -0.557540 0.533529 -0.00587835 -0.0227027 0.00391624 0.0162792 1.00000e
##  48:     4015.7241: 0.0762211 -0.581162 0.557917 -0.00696672 -0.0229848 0.00392702 0.0162312 1.00000e
##  49:     4015.7135: 0.0781761 -0.593771 0.571684 -0.00576348 -0.0214173 0.00392841 0.0161492 1.00000e
##  50:     4015.7062: 0.0795121 -0.621041 0.598225 -0.00608422 -0.0207585 0.00389283 0.0160426 1.00000e
##  51:     4015.7028: 0.0804372 -0.631547 0.608967 -0.00710064 -0.0213545 0.00389377 0.0160506 1.00000e
##  52:     4015.7018: 0.0815655 -0.632697 0.610080 -0.00695977 -0.0210862 0.00390916 0.0160823 1.00000e
##  53:     4015.6993: 0.0826958 -0.632805 0.609999 -0.00729130 -0.0212737 0.00389084 0.0160481 1.00000e
##  54:     4015.6992: 0.0826067 -0.636923 0.614209 -0.00729111 -0.0210059 0.00388033 0.0160449 1.00000e
##  55:     4015.6992: 0.0825634 -0.635480 0.612736 -0.00728945 -0.0210853 0.00388264 0.0160426 1.00000e
##  56:     4015.6992: 0.0825799 -0.635613 0.612867 -0.00728647 -0.0210784 0.00388368 0.0160455 1.00000e
##  57:     4015.6992: 0.0825764 -0.635604 0.612859 -0.00728778 -0.0210790 0.00388315 0.0160444 1.00000e
##
## Final Estimate of the Negative LLH:
##  LLH:  -7152.251    norm LLH:  -2.369864
##           mu           ar1           ma1           ma2           ma3
##  2.040667e-03 -6.356035e-01  6.128593e-01 -7.287778e-03 -2.107901e-02
##         omega        alpha1        alpha2         beta1
##  2.371464e-06  1.604444e-02  1.000000e-08  9.796556e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1          ma1           ma2         ma3
## mu     -2.690015e+06   -2768.70156     616.3153  2.729762e+02    453.2475
## ar1    -2.768702e+03   -4436.06447   -4306.7609  2.719379e+03  -1712.7171
## ma1     6.163153e+02   -4306.76092   -4200.3112  2.536296e+03  -1507.1370
## ma2     2.729762e+02    2719.37920    2536.2955 -4.200852e+03   2632.1396
## ma3     4.532475e+02   -1712.71713   -1507.1370  2.632140e+03  -4203.3776
## omega  -6.493195e+07 2951775.43049 3167168.4728 -5.287839e+05 945902.5688
## alpha1 -2.232838e+04     -14.75446      11.6119  7.146465e-01    346.4952
## alpha2 -2.052297e+04     310.44620     334.4169 -1.723646e+02    466.8949
## beta1  -4.095968e+04    -607.39520    -461.8422  1.272183e+03   -589.3327
##                omega        alpha1        alpha2         beta1
## mu     -6.493195e+07 -2.232838e+04 -2.052297e+04 -4.095968e+04
## ar1     2.951775e+06 -1.475446e+01  3.104462e+02 -6.073952e+02
## ma1     3.167168e+06  1.161190e+01  3.344169e+02 -4.618422e+02
## ma2    -5.287839e+05  7.146465e-01 -1.723646e+02  1.272183e+03
## ma3     9.459026e+05  3.464952e+02  4.668949e+02 -5.893327e+02
## omega  -2.206196e+13 -7.101657e+09 -7.032596e+09 -8.934596e+09
## alpha1 -7.101657e+09 -3.250339e+06 -3.173098e+06 -3.504742e+06
## alpha2 -7.032596e+09 -3.173098e+06 -3.138946e+06 -3.472287e+06
## beta1  -8.934596e+09 -3.504742e+06 -3.472287e+06 -4.190932e+06
## attr(,"time")
## Time difference of 0.1902928 secs
```

```
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.8881209 secs
```

```
gfit11 = garchFit(~arma(1,3)+garch(1,2),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 2)
##  ARMA Order:                1 3
##  Max ARMA Order:            3
##  GARCH Order:               1 2
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02471247
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V      params includes
##     mu     -0.43108945   0.4310894  0.04309277     TRUE
##     ar1    -0.99999999   1.0000000 -0.60590245     TRUE
##     ma1    -0.99999999   1.0000000  0.56853128     TRUE
##     ma2    -0.99999999   1.0000000 -0.02341180     TRUE
##     ma3    -0.99999999   1.0000000 -0.04691329     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     beta1   0.00000001   1.0000000  0.40000000     TRUE
##     beta2   0.00000001   1.0000000  0.40000000     TRUE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1    ma2    ma3  omega alpha1  beta1  beta2
##      1      2      3      4      5      6      7      9     10
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
```

```
## R coded nlminb Solver:
##
##   0:     4036.9355: 0.0430928 -0.605902 0.568531 -0.0234118 -0.0469133 0.100000 0.100000 0.400000 0.4
##   1:     4036.0333: 0.0430937 -0.605125 0.569284 -0.0237907 -0.0464096 0.0971484 0.0993697 0.398815 0
##   2:     4034.9700: 0.0430984 -0.601618 0.572687 -0.0254307 -0.0441641 0.0943434 0.102616 0.402236 0
##   3:     4033.0434: 0.0431106 -0.595085 0.579124 -0.0275776 -0.0399478 0.0805721 0.103141 0.403956 0
##   4:     4030.7969: 0.0431345 -0.592666 0.581737 -0.0235725 -0.0383392 0.0746059 0.0972870 0.412808 0
##   5:     4025.7737: 0.0432593 -0.596798 0.578022 0.00194134 -0.0356069 0.0346030 0.0495909 0.446093 0
##   6:     4025.5805: 0.0432674 -0.597919 0.576871 0.00281277 -0.0351950 0.0332191 0.0508028 0.450406 0
##   7:     4022.7359: 0.0432708 -0.598382 0.576385 0.00300046 -0.0349443 0.0300109 0.0495181 0.450305 0
##   8:     4020.9116: 0.0432848 -0.600743 0.573864 0.00304645 -0.0336503 0.0223829 0.0471713 0.455641 0
##   9:     4020.6045: 0.0433252 -0.603802 0.569037 -0.0145333 -0.0206374 0.00124321 0.0260891 0.475956
##  10:     4018.6536: 0.0433252 -0.603800 0.569038 -0.0145311 -0.0206391 0.00190749 0.0261571 0.476127
##  11:     4017.5868: 0.0433252 -0.603796 0.569042 -0.0145270 -0.0206419 0.00198443 0.0257187 0.475734
##  12:     4015.6186: 0.0433271 -0.603305 0.569581 -0.0141109 -0.0206537 0.00291723 0.0250849 0.475773
##  13:     4014.1711: 0.0433340 -0.602008 0.570908 -0.0137484 -0.0203651 0.00323147 0.0230321 0.475571
##  14:     4013.2235: 0.0433445 -0.601174 0.571657 -0.0120953 -0.0220341 0.00420438 0.0234201 0.475243
##  15:     4012.8747: 0.0433574 -0.600839 0.571818 -0.0103024 -0.0235833 0.00448980 0.0242751 0.474319
##  16:     4012.6718: 0.0433794 -0.599480 0.572875 -0.0104187 -0.0222643 0.00452144 0.0229133 0.474466
##  17:     4012.6499: 0.0433795 -0.599479 0.572875 -0.0104134 -0.0222664 0.00456055 0.0228469 0.474399
##  18:     4012.6280: 0.0433805 -0.599466 0.572874 -0.0103547 -0.0222762 0.00465389 0.0228801 0.474383
##  19:     4012.6035: 0.0433834 -0.599414 0.572883 -0.0101914 -0.0223013 0.00468955 0.0229024 0.474239
##  20:     4012.5730: 0.0433900 -0.599278 0.572921 -0.00985779 -0.0223427 0.00482445 0.0230277 0.474031
##  21:     4012.5363: 0.0434053 -0.598941 0.573034 -0.00923413 -0.0223481 0.00490634 0.0231948 0.473534
##  22:     4012.5012: 0.0434430 -0.598338 0.573100 -0.00823760 -0.0220534 0.00505349 0.0234280 0.472558
##  23:     4012.4792: 0.0434866 -0.598072 0.572769 -0.00768460 -0.0217966 0.00505687 0.0234330 0.471456
##  24:     4012.3352: 0.0441845 -0.591266 0.570366 -0.00459576 -0.0249602 0.00507052 0.0235759 0.454361
##  25:     4012.0522: 0.0455722 -0.583167 0.560346 0.00282018 -0.0185582 0.00531519 0.0238974 0.419615
##  26:     4012.0459: 0.0455722 -0.583166 0.560347 0.00281797 -0.0185579 0.00534712 0.0239133 0.419630
##  27:     4012.0446: 0.0455728 -0.583157 0.560350 0.00280976 -0.0185558 0.00535652 0.0239067 0.419595
##  28:     4012.0429: 0.0455746 -0.583140 0.560343 0.00280653 -0.0185514 0.00537467 0.0239160 0.419551
##  29:     4012.0410: 0.0455784 -0.583107 0.560329 0.00280034 -0.0185424 0.00537825 0.0239153 0.419438
##  30:     4011.7039: 0.0478814 -0.565258 0.549675 0.00234030 -0.0134736 0.00607605 0.0259277 0.359812
##  31:     4011.2793: 0.0501777 -0.554234 0.532621 -0.00524846 -0.0195067 0.00683485 0.0274277 0.300429
##  32:     4011.2402: 0.0526911 -0.539032 0.515727 -0.00316883 -0.0157616 0.00648449 0.0269547 0.245487
##  33:     4011.1622: 0.0539498 -0.531368 0.507139 -0.00102585 -0.0134617 0.00609669 0.0269591 0.220736
##  34:     4011.0782: 0.0557831 -0.520271 0.494181 0.00538288 -0.0106427 0.00662665 0.0290309 0.215318
##  35:     4010.8565: 0.0572420 -0.511495 0.486616 0.00154022 -0.0140100 0.00662687 0.0277338 0.236967
##  36:     4010.8479: 0.0572420 -0.511495 0.486616 0.00154121 -0.0140143 0.00659509 0.0277091 0.236943
##  37:     4010.8426: 0.0572423 -0.511472 0.486641 0.00157361 -0.0141556 0.00664880 0.0275814 0.236980
##  38:     4010.8405: 0.0572592 -0.511377 0.486548 0.00157230 -0.0141681 0.00662569 0.0275670 0.236914
##  39:     4010.8392: 0.0572763 -0.511283 0.486456 0.00157002 -0.0141816 0.00662957 0.0275691 0.236878
##  40:     4010.6470: 0.0631527 -0.479202 0.454460 0.000345853 -0.0169945 0.00610074 0.0270496 0.222436
##  41:     4010.5991: 0.0679511 -0.463254 0.440268 -0.00273133 -0.0256113 0.00625878 0.0272263 0.286211
##  42:     4010.4981: 0.0735629 -0.505537 0.482378 -0.000756528 -0.0227109 0.00670837 0.0274351 0.268877
##  43:     4010.4172: 0.0757279 -0.597415 0.575459 -0.00252921 -0.0222547 0.00678222 0.0276135 0.241680
##  44:     4010.3864: 0.0805104 -0.663365 0.640535 -0.00265498 -0.0183319 0.00644696 0.0272776 0.232370
##  45:     4010.3799: 0.0802626 -0.643666 0.621280 -0.00231926 -0.0187089 0.00641185 0.0273121 0.223381
##  46:     4010.3794: 0.0804851 -0.640964 0.618620 -0.00259076 -0.0189664 0.00643770 0.0273068 0.226379
##  47:     4010.3794: 0.0805617 -0.641211 0.618958 -0.00251071 -0.0189892 0.00643785 0.0273181 0.225951
##  48:     4010.3794: 0.0805725 -0.641343 0.619061 -0.00253072 -0.0189688 0.00643670 0.0273162 0.225984
##  49:     4010.3794: 0.0805713 -0.641337 0.619058 -0.00252840 -0.0189690 0.00643647 0.0273154 0.226007
##
## Final Estimate of the Negative LLH:
```

```
##  LLH:  -7157.571    norm LLH:  -2.371627
##           mu           ar1           ma1           ma2           ma3
##  1.991116e-03 -6.413373e-01  6.190581e-01 -2.528396e-03 -1.896903e-02
##         omega        alpha1         beta1         beta2
##  3.930794e-06  2.731537e-02  2.260070e-01  7.394353e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                   mu           ar1           ma1           ma2           ma3
## mu     -2.656094e+06 -2.914075e+03     365.16400  4.568609e+02      20.35940
## ar1    -2.914075e+03 -4.391840e+03   -4250.06741  2.844343e+03   -1702.77446
## ma1     3.651640e+02 -4.250067e+03   -4134.21563  2.644078e+03   -1474.04418
## ma2     4.568609e+02  2.844343e+03    2644.07761 -4.372900e+03    2618.69089
## ma3     2.035940e+01 -1.702774e+03   -1474.04418  2.618691e+03   -4167.30787
## omega  -2.741297e+07  2.643688e+06 2697968.54276 -1.765819e+06 1491061.61886
## alpha1 -1.081622e+04  1.457873e+02     152.65092 -2.140594e+02     166.53592
## beta1  -1.572191e+04  3.359301e+01      77.64434  7.418389e+00     -10.61832
## beta2  -1.665455e+04  1.092113e+00      49.61024  7.357186e+01     -44.92823
##                omega        alpha1         beta1         beta2
## mu     -2.741297e+07 -1.081622e+04 -1.572191e+04 -1.665455e+04
## ar1     2.643688e+06  1.457873e+02  3.359301e+01  1.092113e+00
## ma1     2.697969e+06  1.526509e+02  7.764434e+01  4.961024e+01
## ma2    -1.765819e+06 -2.140594e+02  7.418389e+00  7.357186e+01
## ma3     1.491062e+06  1.665359e+02 -1.061832e+01 -4.492823e+01
## omega  -7.462253e+12 -2.443217e+09 -3.102756e+09 -3.108523e+09
## alpha1 -2.443217e+09 -1.125002e+06 -1.219732e+06 -1.220798e+06
## beta1  -3.102756e+09 -1.219732e+06 -1.458160e+06 -1.460503e+06
## beta2  -3.108523e+09 -1.220798e+06 -1.460503e+06 -1.462915e+06
## attr(,"time")
## Time difference of 0.155015 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.5936081 secs
```

```
gfit12 = garchFit(~arma(1,3)+garch(2,2),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(1, 3)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(2, 2)
##  ARMA Order:                1 3
##  Max ARMA Order:            3
##  GARCH Order:               2 2
##  Max GARCH Order:           2
##  Maximum Order:             3
##  Conditional Dist:          norm
##  h.start:                   4
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02471247
```

```
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V      params includes
##     mu     -0.43108945   0.4310894  0.04309277     TRUE
##     ar1    -0.99999999   1.0000000 -0.60590245     TRUE
##     ma1    -0.99999999   1.0000000  0.56853128     TRUE
##     ma2    -0.99999999   1.0000000 -0.02341180     TRUE
##     ma3    -0.99999999   1.0000000 -0.04691329     TRUE
##     omega   0.00000100 100.0000000  0.10000000     TRUE
##     alpha1  0.00000001   1.0000000  0.05000000     TRUE
##     alpha2  0.00000001   1.0000000  0.05000000     TRUE
##     gamma1 -0.99999999   1.0000000  0.10000000    FALSE
##     gamma2 -0.99999999   1.0000000  0.10000000    FALSE
##     beta1   0.00000001   1.0000000  0.40000000     TRUE
##     beta2   0.00000001   1.0000000  0.40000000     TRUE
##     delta   0.00000000   2.0000000  2.00000000    FALSE
##     skew    0.10000000  10.0000000  1.00000000    FALSE
##     shape   1.00000000  10.0000000  4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu    ar1    ma1    ma2    ma3  omega alpha1 alpha2  beta1  beta2
##      1      2      3      4      5      6      7      8     11     12
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     4039.2329: 0.0430928 -0.605902 0.568531 -0.0234118 -0.0469133 0.100000 0.0500000 0.0500000 0
##   1:     4038.1101: 0.0430937 -0.605197 0.569201 -0.0238075 -0.0464114 0.0971549 0.0499381 0.0490097
##   2:     4036.8850: 0.0430977 -0.602430 0.571838 -0.0253265 -0.0444462 0.0945995 0.0552044 0.0505239
##   3:     4035.1966: 0.0431088 -0.596647 0.577402 -0.0280566 -0.0402012 0.0827973 0.0604496 0.0470296
##   4:     4032.8705: 0.0431301 -0.592170 0.581803 -0.0281032 -0.0364726 0.0816590 0.0634100 0.0382817
##   5:     4029.1498: 0.0431940 -0.596551 0.577367 -0.0145182 -0.0373570 0.0622037 0.0705390 0.0127177
##   6:     4028.8990: 0.0432071 -0.592200 0.581529 -0.0176701 -0.0335289 0.0563636 0.0700255 0.0073184
##   7:     4027.0996: 0.0432248 -0.593367 0.580317 -0.0158446 -0.0332747 0.0536338 0.0692607 0.0015989
##   8:     4022.0830: 0.0432940 -0.597920 0.575729 -0.0116622 -0.0317986 0.0258406 0.0538321 1.00000e-0
##   9:     4021.9241: 0.0433712 -0.596658 0.569267 -0.0133908 -0.0272252 0.0254150 0.0532300 1.00000e-0
##  10:     4021.7280: 0.0432944 -0.597818 0.575826 -0.0117146 -0.0317391 0.0257407 0.0529047 1.00000e-0
##  11:     4021.6310: 0.0432949 -0.597713 0.575923 -0.0117903 -0.0316323 0.0254330 0.0517359 1.00000e-0
##  12:     4021.4163: 0.0433043 -0.597534 0.575172 -0.0120161 -0.0310572 0.0258364 0.0517209 1.00000e-0
##  13:     4021.2926: 0.0433257 -0.597114 0.573487 -0.0125347 -0.0297448 0.0255781 0.0508948 1.00000e-0
##  14:     4021.2048: 0.0433706 -0.596355 0.569761 -0.0135491 -0.0270774 0.0258573 0.0507241 1.00000e-0
##  15:     4021.0995: 0.0433262 -0.597012 0.573575 -0.0125761 -0.0296942 0.0256031 0.0503770 1.00000e-0
##  16:     4021.0538: 0.0433057 -0.597290 0.575373 -0.0121437 -0.0308783 0.0257701 0.0502006 1.00000e-0
##  17:     4020.9303: 0.0433269 -0.596912 0.573644 -0.0126322 -0.0296062 0.0255242 0.0497744 1.00000e-0
##  18:     4020.7085: 0.0433439 -0.596471 0.572454 -0.0130781 -0.0285257 0.0254521 0.0487679 1.00000e-0
##  19:     4020.6117: 0.0433227 -0.596796 0.574261 -0.0126121 -0.0297702 0.0252092 0.0486051 1.00000e-0
##  20:     4020.4919: 0.0433411 -0.596377 0.572911 -0.0130663 -0.0286252 0.0245209 0.0475686 1.00000e-0
```

```
##  21:     4020.3108: 0.0432986 -0.597024 0.576541 -0.0121352 -0.0311133 0.0245790 0.0475516 1.00000e-0
##  22:     4020.1225: 0.0432594 -0.597525 0.580056 -0.0113313 -0.0333019 0.0241468 0.0467305 1.00000e-0
##  23:     4020.0367: 0.0432177 -0.598168 0.583610 -0.0104216 -0.0357292 0.0236697 0.0463849 1.00000e-0
##  24:     4019.8830: 0.0433015 -0.596858 0.576533 -0.0122983 -0.0307274 0.0237931 0.0462692 1.00000e-0
##  25:     4019.6530: 0.0432234 -0.598028 0.583280 -0.0106323 -0.0351753 0.0228756 0.0453194 1.00000e-0
##  26:     4019.5620: 0.0432239 -0.598021 0.583279 -0.0106757 -0.0350482 0.0227519 0.0450478 1.00000e-0
##  27:     4019.4569: 0.0432306 -0.597917 0.582714 -0.0108314 -0.0346275 0.0224777 0.0448347 1.00000e-0
##  28:     4019.3058: 0.0432410 -0.597756 0.581891 -0.0111018 -0.0338775 0.0222127 0.0443960 1.00000e-0
##  29:     4019.1434: 0.0432711 -0.597288 0.579346 -0.0117711 -0.0320812 0.0219318 0.0441727 1.00000e-0
##  30:     4019.0111: 0.0433306 -0.596339 0.574401 -0.0131066 -0.0284601 0.0216401 0.0436693 1.00000e-0
##  31:     4018.7497: 0.0432099 -0.598188 0.584721 -0.0104554 -0.0355100 0.0207443 0.0429407 1.00000e-0
##  32:     4018.7324: 0.0432102 -0.598200 0.584703 -0.0104769 -0.0354204 0.0202243 0.0424960 1.00000e-0
##  33:     4018.6062: 0.0432051 -0.598284 0.585139 -0.0103692 -0.0356972 0.0203641 0.0425391 1.00000e-0
##  34:     4018.4788: 0.0431984 -0.598429 0.585720 -0.0102514 -0.0359377 0.0195916 0.0418020 1.00000e-0
##  35:     4018.3596: 0.0432236 -0.598043 0.583572 -0.0108028 -0.0344642 0.0199154 0.0420254 1.00000e-0
##  36:     4018.0422: 0.0431812 -0.598781 0.587241 -0.00991954 -0.0366132 0.0180625 0.0402197 1.00000e-
##  37:     4018.0374: 0.0430797 -0.600357 0.595876 -0.00773042 -0.0424668 0.0177340 0.0397771 6.18515e-
##  38:     4017.8437: 0.0431303 -0.599604 0.591553 -0.00883527 -0.0394912 0.0177933 0.0398518 1.00000e-
##  39:     4017.7977: 0.0432317 -0.598057 0.582906 -0.0110385 -0.0335812 0.0186366 0.0405201 1.00000e-0
##  40:     4017.7703: 0.0432647 -0.597544 0.580101 -0.0117510 -0.0316691 0.0184218 0.0404408 1.00000e-0
##  41:     4017.6230: 0.0432317 -0.598066 0.582916 -0.0110369 -0.0335548 0.0182947 0.0402447 1.00000e-0
##  42:     4017.5328: 0.0431682 -0.599122 0.588333 -0.00966908 -0.0370382 0.0167378 0.0387844 1.00000e-
##  43:     4017.1401: 0.0433023 -0.597064 0.576948 -0.0125618 -0.0292141 0.0173672 0.0392568 1.00000e-0
##  44:     4017.1025: 0.0433025 -0.597061 0.576950 -0.0125574 -0.0291949 0.0173805 0.0392051 1.00000e-0
##  45:     4017.0459: 0.0433053 -0.597017 0.576717 -0.0126140 -0.0290264 0.0171836 0.0390592 1.00000e-0
##  46:     4016.9894: 0.0433114 -0.596922 0.576223 -0.0127319 -0.0286594 0.0171343 0.0389466 1.00000e-0
##  47:     4016.9330: 0.0432976 -0.597132 0.577449 -0.0123830 -0.0294700 0.0169150 0.0387766 1.00000e-0
##  48:     4016.8678: 0.0433266 -0.596685 0.575017 -0.0129859 -0.0277621 0.0168784 0.0386636 1.00000e-0
##  49:     4016.8087: 0.0432688 -0.597578 0.579953 -0.0117143 -0.0311354 0.0165739 0.0384168 1.00000e-0
##  50:     4016.1199: 0.0433465 -0.596619 0.573627 -0.0131605 -0.0260273 0.0127836 0.0339193 1.00000e-0
##  51:     4014.7049: 0.0434616 -0.594893 0.563974 -0.0155472 -0.0191971 0.0112294 0.0324579 1.00000e-0
##  52:     4014.6693: 0.0435772 -0.593155 0.554287 -0.0179340 -0.0123421 0.0105833 0.0315589 1.00000e-0
##  53:     4014.5717: 0.0434623 -0.594858 0.564007 -0.0154694 -0.0192044 0.0113769 0.0324470 1.00000e-0
##  54:     4014.5377: 0.0434624 -0.594854 0.564011 -0.0154628 -0.0192101 0.0112729 0.0323611 1.00000e-0
##  55:     4014.4860: 0.0434629 -0.594816 0.564046 -0.0153899 -0.0192412 0.0111992 0.0321439 1.00000e-0
##  56:     4014.4068: 0.0434724 -0.594627 0.564162 -0.0146395 -0.0188134 0.0110992 0.0319364 1.00000e-0
##  57:     4014.3246: 0.0434932 -0.594226 0.564372 -0.0131755 -0.0180043 0.0110926 0.0316116 1.00000e-0
##  58:     4014.2328: 0.0435562 -0.593339 0.564381 -0.0106801 -0.0165136 0.0110778 0.0311645 1.00000e-0
##  59:     4014.1822: 0.0436476 -0.592537 0.563386 -0.0105825 -0.0164087 0.0111074 0.0311093 1.00000e-0
##  60:     4014.1082: 0.0438267 -0.591109 0.561126 -0.0116961 -0.0169258 0.0109153 0.0311062 1.00000e-0
##  61:     4013.6762: 0.0459434 -0.568690 0.545871 0.000397190 -0.00816479 0.0106689 0.0324913 1.00000e
##  62:     4013.0572: 0.0480983 -0.549417 0.526299 -0.000119455 -0.0105532 0.0104252 0.0321075 1.00000e
##  63:     4012.4833: 0.0502620 -0.530366 0.505693 -0.00292073 -0.0136651 0.0105510 0.0326721 1.00000e-
##  64:     4012.0393: 0.0524925 -0.510926 0.484307 -0.00662748 -0.0172717 0.0100970 0.0330412 1.00000e-
##  65:     4011.8005: 0.0550957 -0.487482 0.463599 0.000418173 -0.0160822 0.0108501 0.0320179 1.00000e-
##  66:     4011.4358: 0.0583518 -0.462786 0.433843 -0.00738707 -0.0212597 0.00935299 0.0323686 1.00000e
##  67:     4011.4053: 0.0583520 -0.462756 0.433875 -0.00730560 -0.0213229 0.00923572 0.0321936 1.00000e
##  68:     4011.3735: 0.0583648 -0.462676 0.433814 -0.00728257 -0.0213394 0.00913979 0.0321196 1.00000e
##  69:     4011.3523: 0.0583909 -0.462505 0.433705 -0.00721016 -0.0213949 0.00909678 0.0320228 1.00000e
##  70:     4011.3238: 0.0584452 -0.462174 0.433448 -0.00713291 -0.0214549 0.00900036 0.0319315 1.00000e
##  71:     4010.8153: 0.0611486 -0.446004 0.420741 -0.00359380 -0.0242015 0.00750052 0.0299108 1.00000e
##  72:     4010.8009: 0.0614036 -0.446372 0.417919 -0.00370750 -0.0212583 0.00701056 0.0290710 1.00000e
##  73:     4010.7442: 0.0617189 -0.448094 0.419828 -0.00318327 -0.0210344 0.00704308 0.0288864 1.00000e
##  74:     4010.6996: 0.0618749 -0.449562 0.423372 -0.00262040 -0.0225813 0.00717705 0.0290457 1.00000e
```

```
##  75:     4010.6780: 0.0621924 -0.451929 0.426155 -0.00224907 -0.0227033 0.00720369 0.0288526 1.00000e
##  76:     4010.6483: 0.0625438 -0.454673 0.428247 -0.00209097 -0.0219649 0.00702564 0.0285238 1.00000e
##  77:     4010.5427: 0.0667274 -0.500006 0.472876 -0.000928701 -0.0203748 0.00652829 0.0273960 1.00000
##  78:     4010.4547: 0.0708405 -0.544821 0.521164 -0.000405678 -0.0228610 0.00657378 0.0275593 1.00000
##  79:     4010.4164: 0.0756968 -0.561299 0.537709 -0.00306507 -0.0225235 0.00658754 0.0273408 1.00000e
##  80:     4010.3894: 0.0783709 -0.628847 0.604918 -0.00429012 -0.0208613 0.00648614 0.0272565 1.00000e
##  81:     4010.3815: 0.0791815 -0.635608 0.613480 -0.00276813 -0.0191614 0.00643207 0.0272847 1.00000e
##  82:     4010.3798: 0.0800103 -0.636912 0.614531 -0.00228583 -0.0188829 0.00645791 0.0273573 1.00000e
##  83:     4010.3796: 0.0806729 -0.645219 0.622930 -0.00270775 -0.0189182 0.00641661 0.0272926 1.00000e
##  84:     4010.3794: 0.0806966 -0.641674 0.619440 -0.00251924 -0.0190116 0.00644132 0.0273206 1.00000e
##  85:     4010.3794: 0.0805144 -0.640627 0.618328 -0.00251647 -0.0189697 0.00643522 0.0273162 1.00000e
##  86:     4010.3794: 0.0805741 -0.641539 0.619262 -0.00252894 -0.0189578 0.00643824 0.0273157 1.00000e
##  87:     4010.3794: 0.0805794 -0.641358 0.619079 -0.00253328 -0.0189762 0.00643561 0.0273156 1.00000e
##  88:     4010.3794: 0.0805711 -0.641331 0.619051 -0.00252902 -0.0189693 0.00643666 0.0273157 1.00000e
##
## Final Estimate of the Negative LLH:
##  LLH:  -7157.571    norm LLH:  -2.371627
##             mu           ar1           ma1           ma2           ma3
##  1.991111e-03 -6.413312e-01  6.190512e-01 -2.529024e-03 -1.896928e-02
##          omega        alpha1        alpha2         beta1         beta2
##  3.930907e-06  2.731573e-02  1.000000e-08  2.259997e-01  7.394421e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                    mu           ar1           ma1           ma2           ma3
## mu     -2.656118e+06 -2.914106e+03     365.17528  4.568877e+02      20.36714
## ar1    -2.914106e+03 -4.391774e+03   -4250.00800  2.844280e+03   -1702.71730
## ma1     3.651753e+02 -4.250008e+03   -4134.16219  2.644020e+03   -1473.99578
## ma2     4.568877e+02  2.844280e+03    2644.02020 -4.372850e+03    2618.63134
## ma3     2.036714e+01 -1.702717e+03   -1473.99578  2.618631e+03   -4167.25150
## omega  -2.741141e+07  2.643665e+06 2697939.15295 -1.765842e+06 1491091.93127
## alpha1 -1.081590e+04  1.457795e+02     152.64185 -2.140655e+02     166.53494
## alpha2  2.113932e+03  9.019478e+02     845.20799 -1.562648e+03     476.46275
## beta1  -1.572107e+04  3.360479e+01      77.65221  7.391271e+00     -10.58190
## beta2  -1.665376e+04  1.103027e+00      49.61753  7.354710e+01     -44.89382
##                 omega        alpha1        alpha2         beta1         beta2
## mu     -2.741141e+07 -1.081590e+04  2.113932e+03 -1.572107e+04 -1.665376e+04
## ar1     2.643665e+06  1.457795e+02  9.019478e+02  3.360479e+01  1.103027e+00
## ma1     2.697939e+06  1.526419e+02  8.452080e+02  7.765221e+01  4.961753e+01
## ma2    -1.765842e+06 -2.140655e+02 -1.562648e+03  7.391271e+00  7.354710e+01
## ma3     1.491092e+06  1.665349e+02  4.764628e+02 -1.058190e+01 -4.489382e+01
## omega  -7.461983e+12 -2.443130e+09 -2.451746e+09 -3.102652e+09 -3.108419e+09
## alpha1 -2.443130e+09 -1.124963e+06 -1.113126e+06 -1.219690e+06 -1.220756e+06
## alpha2 -2.451746e+09 -1.113126e+06 -1.132167e+06 -1.225231e+06 -1.225443e+06
## beta1  -3.102652e+09 -1.219690e+06 -1.225231e+06 -1.458111e+06 -1.460454e+06
## beta2  -3.108419e+09 -1.220756e+06 -1.225443e+06 -1.460454e+06 -1.462866e+06
## attr(,"time")
## Time difference of 0.192426 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.8911309 secs
```

```
summary(gfit1)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = diff(log(amazon_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x7ffdee191270>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1
## 0.00102317  0.00046397  0.25469121
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.023e-03   4.108e-04    2.491   0.0128 *
## omega  4.640e-04   1.513e-05   30.675  < 2e-16 ***
## alpha1 2.547e-01   3.225e-02    7.897 2.89e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7014.796    normalized:  2.324319
##
## Description:
##  Sun Apr 17 22:16:33 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  17306.54  0
##  Shapiro-Wilk Test  R    W      0.88609   0
##  Ljung-Box Test     R    Q(10)  10.02377  0.4384106
##  Ljung-Box Test     R    Q(15)  14.46638  0.4904921
##  Ljung-Box Test     R    Q(20)  17.13716  0.6440499
##  Ljung-Box Test     R^2  Q(10)  28.09652  0.001742111
##  Ljung-Box Test     R^2  Q(15)  44.58694  8.895056e-05
##  Ljung-Box Test     R^2  Q(20)  64.74061  1.283501e-06
##  LM Arch Test       R    TR^2   33.53507  0.0007982695
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.646651 -4.640674 -4.646653 -4.644502
```

```
summary(gfit2)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = diff(log(amazon_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7ffdecae3260>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 1.2579e-03  2.5530e-06  1.6523e-02  9.7887e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.258e-03   3.851e-04    3.266 0.001090 **
## omega  2.553e-06   6.794e-07    3.758 0.000171 ***
## alpha1 1.652e-02   2.013e-03    8.207 2.22e-16 ***
## beta1  9.789e-01   2.682e-03  364.948  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7149.486    normalized:  2.368948
##
## Description:
##  Sun Apr 17 22:16:33 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  21686.27  0
##  Shapiro-Wilk Test  R    W      0.8876144 0
##  Ljung-Box Test     R    Q(10)  7.01244   0.72427
##  Ljung-Box Test     R    Q(15)  7.714981  0.9347004
##  Ljung-Box Test     R    Q(20)  10.34651  0.9614448
##  Ljung-Box Test     R^2  Q(10)  10.08525  0.4330465
##  Ljung-Box Test     R^2  Q(15)  11.08575  0.7464924
##  Ljung-Box Test     R^2  Q(20)  11.37271  0.9359785
##  LM Arch Test       R    TR^2   10.07812  0.609107
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
```

```
## -4.735246 -4.727277 -4.735250 -4.732381
```

```
summary(gfit3)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 0), data = diff(log(amazon_open)))
##
## Mean and Variance Equation:
##  data ~ garch(2, 0)
## <environment: 0x7ffdef1cb438>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2
## 0.00143446  0.00043255  0.17812036  0.11345799
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.434e-03   4.131e-04    3.472 0.000517 ***
## omega  4.326e-04   1.537e-05   28.140  < 2e-16 ***
## alpha1 1.781e-01   2.935e-02    6.069 1.29e-09 ***
## alpha2 1.135e-01   2.448e-02    4.634 3.58e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7040.066    normalized:  2.332693
##
## Description:
##  Sun Apr 17 22:16:33 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  18683.59  0
##  Shapiro-Wilk Test  R    W      0.8855655 0
##  Ljung-Box Test     R    Q(10)  9.212649  0.5120485
##  Ljung-Box Test     R    Q(15)  12.00985  0.6782836
##  Ljung-Box Test     R    Q(20)  14.3352   0.8131022
##  Ljung-Box Test     R^2  Q(10)  14.27447  0.1608363
##  Ljung-Box Test     R^2  Q(15)  18.81062  0.2224444
##  Ljung-Box Test     R^2  Q(20)  32.45948  0.03864172
##  LM Arch Test       R    TR^2   16.78111  0.1580185
##
## Information Criterion Statistics:
```

```
##       AIC       BIC       SIC      HQIC
## -4.662734 -4.654766 -4.662738 -4.659869
```

```
summary(gfit4)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 1), data = diff(log(amazon_open)))
##
## Mean and Variance Equation:
##  data ~ garch(2, 1)
## <environment: 0x7ffdecb87e68>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2       beta1
## 1.2588e-03  2.5672e-06  1.6581e-02  1.0000e-08  9.7878e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.259e-03   3.852e-04    3.268 0.001081 **
## omega  2.567e-06   7.036e-07    3.649 0.000263 ***
## alpha1 1.658e-02   4.974e-03    3.334 0.000856 ***
## alpha2 1.000e-08   5.672e-03    0.000 0.999999
## beta1  9.788e-01   2.981e-03  328.393  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7149.64    normalized:  2.368999
##
## Description:
##  Sun Apr 17 22:16:34 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  21691.33  0
##  Shapiro-Wilk Test  R    W      0.8876379 0
##  Ljung-Box Test     R    Q(10)  7.008764  0.7246173
##  Ljung-Box Test     R    Q(15)  7.716306  0.934652
##  Ljung-Box Test     R    Q(20)  10.35279  0.9613145
##  Ljung-Box Test     R^2  Q(10)  10.05196  0.4359466
##  Ljung-Box Test     R^2  Q(15)  11.05571  0.748635
##  Ljung-Box Test     R^2  Q(20)  11.34225  0.9368595
##  LM Arch Test       R    TR^2   10.04723  0.6118174
```

```
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.734685 -4.724724 -4.734690 -4.731103
```

```
summary(gfit5)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 2), data = diff(log(amazon_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 2)
## <environment: 0x7ffdeca31ad0>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       beta2
## 1.2224e-03  4.1845e-06  2.7810e-02  2.4353e-01  7.2099e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.222e-03   3.850e-04    3.175  0.00150 **
## omega  4.185e-06   1.353e-06    3.092  0.00199 **
## alpha1 2.781e-02   3.965e-03    7.013 2.33e-12 ***
## beta1  2.435e-01   1.345e-01    1.810  0.07022 .
## beta2  7.210e-01   1.329e-01    5.425 5.80e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7154.806    normalized:  2.370711
##
## Description:
##  Sun Apr 17 22:16:34 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  21848.3   0
##  Shapiro-Wilk Test  R    W      0.8879689 0
##  Ljung-Box Test     R    Q(10)  6.938406  0.7312468
##  Ljung-Box Test     R    Q(15)  7.725324  0.934322
##  Ljung-Box Test     R    Q(20)  10.37544  0.9608422
##  Ljung-Box Test     R^2  Q(10)  5.960515  0.8185696
##  Ljung-Box Test     R^2  Q(15)  6.920751  0.9598116
```

```
##  Ljung-Box Test     R^2  Q(20)  7.164063  0.9961113
##  LM Arch Test       R    TR^2   6.007351  0.915711
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.738109 -4.728148 -4.738114 -4.734527
```

```
summary(gfit6)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(2, 2), data = diff(log(amazon_open)))
##
## Mean and Variance Equation:
##  data ~ garch(2, 2)
## <environment: 0x7ffdecc8d8a8>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1      alpha2       beta1       beta2
## 1.2224e-03  4.1845e-06  2.7810e-02  1.0000e-08  2.4353e-01  7.2099e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.222e-03   3.850e-04    3.175  0.00150 **
## omega  4.185e-06   1.512e-06    2.768  0.00563 **
## alpha1 2.781e-02   7.655e-03    3.633  0.00028 ***
## alpha2 1.000e-08   1.096e-02    0.000  1.00000
## beta1  2.435e-01   2.378e-01    1.024  0.30577
## beta2  7.210e-01   2.324e-01    3.103  0.00192 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7154.806    normalized:  2.370711
##
## Description:
##  Sun Apr 17 22:16:35 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  21848.28  0
##  Shapiro-Wilk Test  R    W      0.8879689 0
##  Ljung-Box Test     R    Q(10)  6.938411  0.7312463
##  Ljung-Box Test     R    Q(15)  7.725329  0.9343218
```

```
##  Ljung-Box Test     R    Q(20)  10.37545  0.960842
##  Ljung-Box Test     R^2  Q(10)  5.960547  0.8185669
##  Ljung-Box Test     R^2  Q(15)  6.92078   0.9598108
##  Ljung-Box Test     R^2  Q(20)  7.164093  0.9961112
##  LM Arch Test       R    TR^2   6.007379  0.9157096
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.737446 -4.725493 -4.737454 -4.733148
```

```
summary(gfit7)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 3) + garch(1, 0), data = diff(log(amazon_open)))
##
## Mean and Variance Equation:
##  data ~ arma(1, 3) + garch(1, 0)
## <environment: 0x7ffdf1c82d58>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ma1          ma2          ma3        omega
##  0.00209758  -0.97517747   0.98725847   0.01664843   0.00028586   0.00045748
##      alpha1
##  0.27009911
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      2.098e-03   8.158e-04    2.571   0.0101 *
## ar1    -9.752e-01   9.736e-03 -100.162  < 2e-16 ***
## ma1     9.873e-01   2.506e-02   39.393  < 2e-16 ***
## ma2     1.665e-02   2.928e-02    0.569   0.5696
## ma3     2.859e-04   1.832e-02    0.016   0.9875
## omega   4.575e-04   1.534e-05   29.831  < 2e-16 ***
## alpha1  2.701e-01   3.468e-02    7.787 6.88e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7020.429    normalized:  2.326186
##
## Description:
##  Sun Apr 17 22:16:36 2022 by user:
##
##
```

```
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  17046.93  0
##  Shapiro-Wilk Test  R    W      0.8870857 0
##  Ljung-Box Test     R    Q(10)  9.719911  0.4654006
##  Ljung-Box Test     R    Q(15)  14.09977  0.5179762
##  Ljung-Box Test     R    Q(20)  17.4059   0.6264812
##  Ljung-Box Test     R^2  Q(10)  27.72441  0.001997973
##  Ljung-Box Test     R^2  Q(15)  43.8919   0.0001143316
##  Ljung-Box Test     R^2  Q(20)  62.93215  2.481441e-06
##  LM Arch Test       R    TR^2   33.43068  0.0008289382
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.647733 -4.633788 -4.647744 -4.642719
```

```
summary(gfit8)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 3) + garch(1, 1), data = diff(log(amazon_open)))
##
## Mean and Variance Equation:
##  data ~ arma(1, 3) + garch(1, 1)
## <environment: 0x7ffdeca69ce0>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ma1          ma2          ma3        omega
##  2.0400e-03  -6.3633e-01   6.1358e-01  -7.2505e-03  -2.1001e-02   2.3604e-06
##      alpha1        beta1
##  1.5992e-02   9.7973e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      2.040e-03   7.674e-04    2.658 0.007855 **
## ar1    -6.363e-01   3.690e-01   -1.724 0.084654 .
## ma1     6.136e-01   3.695e-01    1.660 0.096832 .
## ma2    -7.251e-03   2.447e-02   -0.296 0.766986
## ma3    -2.100e-02   2.384e-02   -0.881 0.378348
## omega   2.360e-06   6.184e-07    3.817 0.000135 ***
## alpha1  1.599e-02   1.899e-03    8.420  < 2e-16 ***
## beta1   9.797e-01   2.462e-03  397.920  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Log Likelihood:
##  7152.099    normalized:  2.369814
##
## Description:
##  Sun Apr 17 22:16:36 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  21224.38  0
##  Shapiro-Wilk Test  R    W      0.8884117 0
##  Ljung-Box Test     R    Q(10)  2.14502   0.995103
##  Ljung-Box Test     R    Q(15)  2.85131   0.9997071
##  Ljung-Box Test     R    Q(20)  6.088923  0.998772
##  Ljung-Box Test     R^2  Q(10)  10.72851  0.3790574
##  Ljung-Box Test     R^2  Q(15)  11.72609  0.6996323
##  Ljung-Box Test     R^2  Q(20)  11.95526  0.9176072
##  LM Arch Test       R    TR^2   10.69928  0.5548509
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.734327 -4.718389 -4.734341 -4.728596
```

```
summary(gfit9)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 3) + garch(2, 0), data = diff(log(amazon_open)))
##
## Mean and Variance Equation:
##  data ~ arma(1, 3) + garch(2, 0)
## <environment: 0x7ffdedef2270>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ma1          ma2          ma3        omega
##  0.00235689  -0.63150139   0.61528636   0.00144157  -0.01381240   0.00043196
##      alpha1       alpha2
##  0.17261762   0.11849384
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      2.357e-03   7.294e-04    3.231  0.00123 **
## ar1    -6.315e-01   2.071e-01   -3.049  0.00230 **
## ma1     6.153e-01   2.087e-01    2.949  0.00319 **
## ma2     1.442e-03   2.720e-02    0.053  0.95773
```

```
## ma3    -1.381e-02   2.283e-02   -0.605  0.54518
## omega   4.320e-04   1.552e-05   27.841  < 2e-16 ***
## alpha1  1.726e-01   2.916e-02    5.920 3.21e-09 ***
## alpha2  1.185e-01   2.593e-02    4.570 4.89e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7042.185    normalized:  2.333395
##
## Description:
##  Sun Apr 17 22:16:38 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  18570.35  0
##  Shapiro-Wilk Test  R    W      0.8857043 0
##  Ljung-Box Test     R    Q(10)  5.210325  0.8766924
##  Ljung-Box Test     R    Q(15)  8.077649  0.9206319
##  Ljung-Box Test     R    Q(20)  10.63667  0.9551062
##  Ljung-Box Test     R^2  Q(10)  14.84957  0.1376433
##  Ljung-Box Test     R^2  Q(15)  19.5798   0.1886512
##  Ljung-Box Test     R^2  Q(20)  32.90055  0.03459818
##  LM Arch Test       R    TR^2   17.56832  0.1294403
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.661488 -4.645550 -4.661502 -4.655757
```

```
summary(gfit10)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 3) + garch(2, 1), data = diff(log(amazon_open)))
##
## Mean and Variance Equation:
##  data ~ arma(1, 3) + garch(2, 1)
## <environment: 0x7ffdece1a4c0>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ma1          ma2          ma3        omega
##  2.0407e-03  -6.3560e-01   6.1286e-01  -7.2878e-03  -2.1079e-02   2.3715e-06
##      alpha1       alpha2        beta1
##  1.6044e-02   1.0000e-08   9.7966e-01
##
## Std. Errors:
##  based on Hessian
```

```
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      2.041e-03   7.673e-04    2.659 0.007826 **
## ar1    -6.356e-01   3.688e-01   -1.723 0.084823 .
## ma1     6.129e-01   3.693e-01    1.659 0.097026 .
## ma2    -7.288e-03   2.446e-02   -0.298 0.765769
## ma3    -2.108e-02   2.381e-02   -0.885 0.376082
## omega   2.371e-06   6.349e-07    3.735 0.000188 ***
## alpha1  1.604e-02   4.848e-03    3.310 0.000934 ***
## alpha2  1.000e-08   5.455e-03    0.000 0.999999
## beta1   9.797e-01   2.699e-03  362.910  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7152.251    normalized:  2.369864
##
## Description:
##  Sun Apr 17 22:16:39 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  21223.46  0
##  Shapiro-Wilk Test  R    W      0.8884412 0
##  Ljung-Box Test     R    Q(10)  2.144447  0.9951084
##  Ljung-Box Test     R    Q(15)  2.85678   0.9997035
##  Ljung-Box Test     R    Q(20)  6.09724   0.9987597
##  Ljung-Box Test     R^2  Q(10)  10.70233  0.3811745
##  Ljung-Box Test     R^2  Q(15)  11.70318  0.7013427
##  Ljung-Box Test     R^2  Q(20)  11.93276  0.9183709
##  LM Arch Test       R    TR^2   10.67516  0.5569433
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.733765 -4.715835 -4.733782 -4.727317
```

```
summary(gfit11)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 3) + garch(1, 2), data = diff(log(amazon_open)))
##
## Mean and Variance Equation:
##  data ~ arma(1, 3) + garch(1, 2)
## <environment: 0x7ffdee233dd0>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
```

```
## Coefficient(s):
##          mu          ar1          ma1          ma2          ma3        omega
##  1.9911e-03  -6.4134e-01   6.1906e-01  -2.5284e-03  -1.8969e-02   3.9308e-06
##      alpha1        beta1        beta2
##  2.7315e-02   2.2601e-01   7.3944e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      1.991e-03   7.508e-04    2.652  0.00800 **
## ar1    -6.413e-01   3.501e-01   -1.832  0.06696 .
## ma1     6.191e-01   3.507e-01    1.765  0.07753 .
## ma2    -2.528e-03   2.409e-02   -0.105  0.91642
## ma3    -1.897e-02   2.454e-02   -0.773  0.43952
## omega   3.931e-06   1.258e-06    3.126  0.00177 **
## alpha1  2.732e-02   3.808e-03    7.173 7.33e-13 ***
## beta1   2.260e-01   1.343e-01    1.683  0.09241 .
## beta2   7.394e-01   1.327e-01    5.572 2.52e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7157.571    normalized:  2.371627
##
## Description:
##  Sun Apr 17 22:16:40 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  21431.13  0
##  Shapiro-Wilk Test  R    W      0.8888058 0
##  Ljung-Box Test     R    Q(10)  2.078511  0.9957022
##  Ljung-Box Test     R    Q(15)  2.879982  0.9996881
##  Ljung-Box Test     R    Q(20)  6.150334  0.9986787
##  Ljung-Box Test     R^2  Q(10)  6.206496  0.7976261
##  Ljung-Box Test     R^2  Q(15)  7.168019  0.9528184
##  Ljung-Box Test     R^2  Q(20)  7.356706  0.9953404
##  LM Arch Test       R    TR^2   6.24153   0.9034202
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.737290 -4.719361 -4.737308 -4.730843
```

```
summary(gfit12)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 3) + garch(2, 2), data = diff(log(amazon_open)))
##
```

```
## Mean and Variance Equation:
##  data ~ arma(1, 3) + garch(2, 2)
## <environment: 0x7ffdef3df848>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ar1          ma1          ma2          ma3        omega
##  1.9911e-03  -6.4133e-01   6.1905e-01  -2.5290e-03  -1.8969e-02   3.9309e-06
##      alpha1       alpha2        beta1        beta2
##  2.7316e-02   1.0000e-08   2.2600e-01   7.3944e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      1.991e-03   7.508e-04    2.652 0.008003 **
## ar1    -6.413e-01   3.501e-01   -1.832 0.066972 .
## ma1     6.191e-01   3.507e-01    1.765 0.077540 .
## ma2    -2.529e-03   2.413e-02   -0.105 0.916541
## ma3    -1.897e-02   2.458e-02   -0.772 0.440353
## omega   3.931e-06   1.395e-06    2.819 0.004821 **
## alpha1  2.732e-02   7.495e-03    3.644 0.000268 ***
## alpha2  1.000e-08   1.059e-02    0.000 0.999999
## beta1   2.260e-01   2.335e-01    0.968 0.333207
## beta2   7.394e-01   2.285e-01    3.236 0.001211 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7157.571    normalized:  2.371627
##
## Description:
##  Sun Apr 17 22:16:41 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  21431.24  0
##  Shapiro-Wilk Test  R    W      0.8888057 0
##  Ljung-Box Test     R    Q(10)  2.078513  0.9957022
##  Ljung-Box Test     R    Q(15)  2.879984  0.9996881
##  Ljung-Box Test     R    Q(20)  6.150326  0.9986787
##  Ljung-Box Test     R^2  Q(10)  6.206363  0.7976377
##  Ljung-Box Test     R^2  Q(15)  7.167893  0.9528222
##  Ljung-Box Test     R^2  Q(20)  7.356581  0.9953409
##  LM Arch Test       R    TR^2   6.241406  0.903427
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -4.736627 -4.716706 -4.736649 -4.729464
```

According to the AIC(no big difference) and the significance, we find that generally we should have a ARMA model as mean, here the best model is GARCH(1,2)

```
gfit = garchFit(~garch(1,1),data = diff(log(amazon_open)))
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          3018
##  Recursion Init:            mci
##  Series Scale:              0.02471247
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V     params includes
##     mu     -0.43108945   0.4310894 0.04310894     TRUE
##     omega   0.00000100 100.0000000 0.10000000     TRUE
##     alpha1  0.00000001   1.0000000 0.10000000     TRUE
##     gamma1 -0.99999999   1.0000000 0.10000000    FALSE
##     beta1   0.00000001   1.0000000 0.80000000     TRUE
##     delta   0.00000000   2.0000000 2.00000000    FALSE
##     skew    0.10000000  10.0000000 1.00000000    FALSE
##     shape   1.00000000  10.0000000 4.00000000    FALSE
##  Index List of Parameters to be Optimized:
##     mu  omega alpha1  beta1
##      1      2      3      5
##  Persistence:                 0.9
##
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     4042.9458: 0.0431089 0.100000 0.100000 0.800000
##   1:     4042.4973: 0.0431115 0.0954600 0.0999818 0.801181
##   2:     4041.8609: 0.0431140 0.0947808 0.101781 0.805460
##   3:     4040.7912: 0.0431203 0.0869021 0.101822 0.810553
##   4:     4038.6661: 0.0431352 0.0783656 0.0998654 0.827147
##   5:     4034.2588: 0.0431736 0.0624904 0.0719497 0.861564
##   6:     4034.2388: 0.0431815 0.0593079 0.0744078 0.870187
```

```
##   7:     4032.8380: 0.0431847 0.0547550 0.0731000 0.870628
##   8:     4032.4272: 0.0431905 0.0524926 0.0731374 0.874812
##   9:     4031.9603: 0.0432072 0.0471452 0.0697264 0.881895
##  10:     4031.1275: 0.0432385 0.0465096 0.0637570 0.889243
##  11:     4030.7681: 0.0432850 0.0435581 0.0574993 0.895687
##  12:     4030.0072: 0.0433501 0.0393592 0.0557322 0.903905
##  13:     4029.6089: 0.0434257 0.0340884 0.0530551 0.911153
##  14:     4027.5678: 0.0437053 0.0256380 0.0410100 0.931861
##  15:     4026.9140: 0.0437113 0.0222427 0.0381367 0.939629
##  16:     4024.0604: 0.0438437 0.0158058 0.0299452 0.953862
##  17:     4022.7838: 0.0438856 0.0136363 0.0272785 0.958313
##  18:     4020.7412: 0.0439695 0.00929387 0.0219554 0.967220
##  19:     4020.1500: 0.0439906 0.00857662 0.0209301 0.969811
##  20:     4019.0131: 0.0440855 0.00405146 0.0150598 0.979964
##  21:     4018.8344: 0.0440855 0.00418688 0.0152110 0.980095
##  22:     4018.7596: 0.0440840 0.00415944 0.0153064 0.979877
##  23:     4018.7084: 0.0440810 0.00425126 0.0156942 0.979614
##  24:     4018.6620: 0.0440743 0.00445672 0.0160962 0.978774
##  25:     4018.6320: 0.0440850 0.00414197 0.0157388 0.979577
##  26:     4018.5878: 0.0440851 0.00394387 0.0159577 0.979590
##  27:     4018.5827: 0.0440851 0.00398366 0.0159931 0.979620
##  28:     4018.5756: 0.0440848 0.00397482 0.0160159 0.979564
##  29:     4018.5721: 0.0440847 0.00397086 0.0161267 0.979511
##  30:     4018.5644: 0.0440838 0.00402591 0.0162391 0.979301
##  31:     4018.5613: 0.0440845 0.00411101 0.0163703 0.979111
##  32:     4018.5603: 0.0440943 0.00412180 0.0164059 0.979028
##  33:     4018.5592: 0.0441048 0.00413508 0.0164207 0.979018
##  34:     4018.5361: 0.0455334 0.00441944 0.0168915 0.978235
##  35:     4018.4640: 0.0506278 0.00417157 0.0164932 0.978906
##  36:     4018.4638: 0.0509037 0.00418039 0.0165236 0.978868
##  37:     4018.4638: 0.0509034 0.00418057 0.0165232 0.978869
##  38:     4018.4638: 0.0509026 0.00418039 0.0165229 0.978869
##
## Final Estimate of the Negative LLH:
##  LLH:  -7149.486    norm LLH:  -2.368948
##           mu        omega       alpha1        beta1
## 1.257929e-03 2.552992e-06 1.652295e-02 9.788691e-01
##
## R-optimhess Difference Approximated Hessian Matrix:
##                  mu         omega        alpha1         beta1
## mu      -6745240.65 -9.536841e+07 -3.347588e+04 -6.165248e+04
## omega  -95368410.02 -2.021762e+13 -6.523022e+09 -8.263471e+09
## alpha1    -33475.88 -6.523022e+09 -2.997924e+06 -3.236610e+06
## beta1     -61652.48 -8.263471e+09 -3.236610e+06 -3.880869e+06
## attr(,"time")
## Time difference of 0.02741885 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.1245601 secs

## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
```

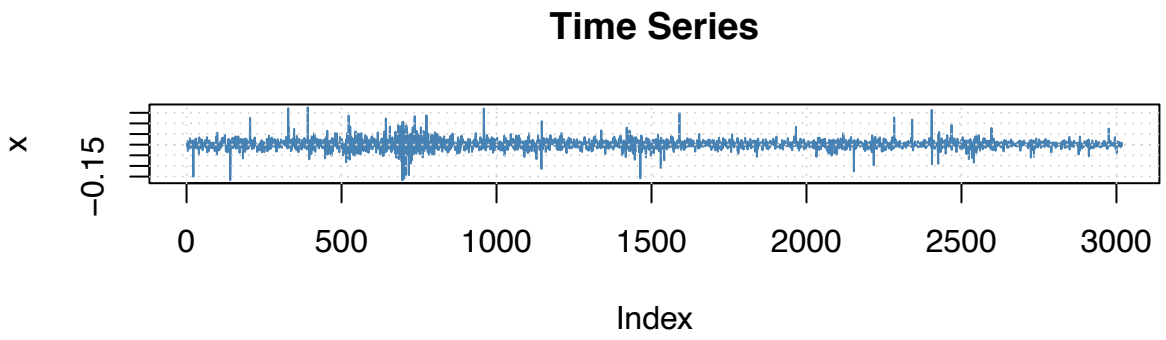
```
##   Consider formula(paste(x, collapse = " ")) instead.
```

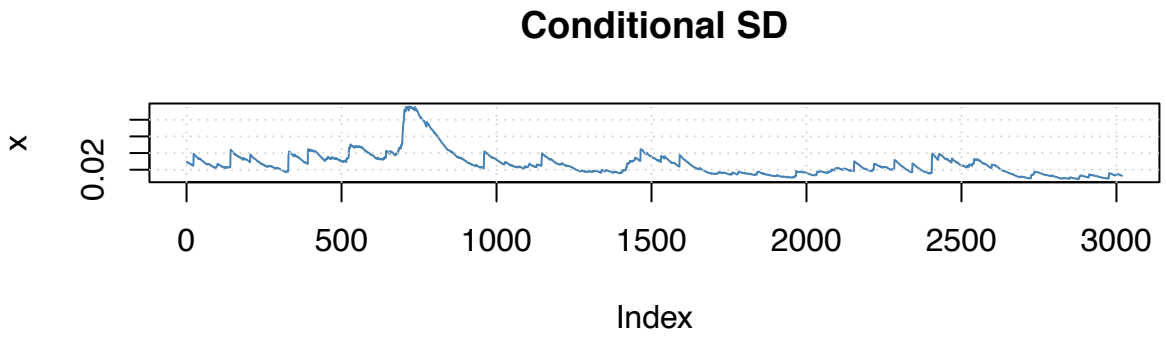
```
summary(gfit)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = diff(log(amazon_open)))
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7ffdeddf92e0>
##  [data = diff(log(amazon_open))]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 1.2579e-03  2.5530e-06  1.6523e-02  9.7887e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.258e-03   3.851e-04    3.266 0.001090 **
## omega  2.553e-06   6.794e-07    3.758 0.000171 ***
## alpha1 1.652e-02   2.013e-03    8.207 2.22e-16 ***
## beta1  9.789e-01   2.682e-03  364.948  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7149.486    normalized:  2.368948
##
## Description:
##  Sun Apr 17 22:16:41 2022 by user:
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  21686.27  0
##  Shapiro-Wilk Test  R    W      0.8876144 0
##  Ljung-Box Test     R    Q(10)  7.01244   0.72427
##  Ljung-Box Test     R    Q(15)  7.714981  0.9347004
##  Ljung-Box Test     R    Q(20)  10.34651  0.9614448
##  Ljung-Box Test     R^2  Q(10)  10.08525  0.4330465
##  Ljung-Box Test     R^2  Q(15)  11.08575  0.7464924
##  Ljung-Box Test     R^2  Q(20)  11.37271  0.9359785
##  LM Arch Test       R    TR^2   10.07812  0.609107
##
## Information Criterion Statistics:
```

```
##       AIC       BIC       SIC      HQIC
## -4.735246 -4.727277 -4.735250 -4.732381
```

```
par(mfrow=c(2,1))
plot(gfit, which = 1)
plot(gfit, which = 2)
```



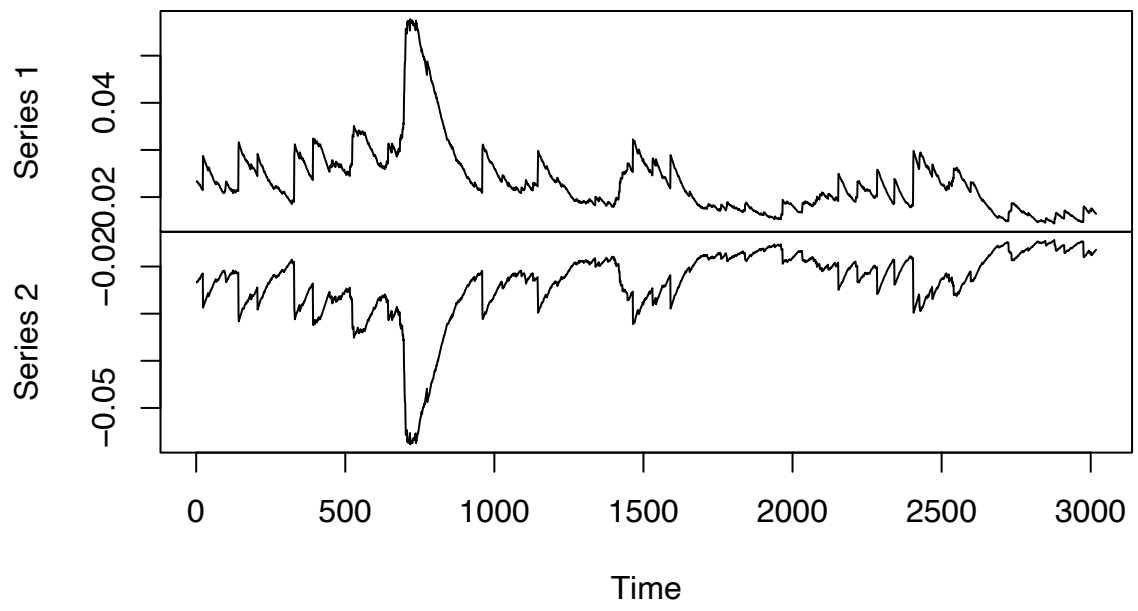



```
gfit = garch(diff(log(amazon_open)),order = c(1,1))
```

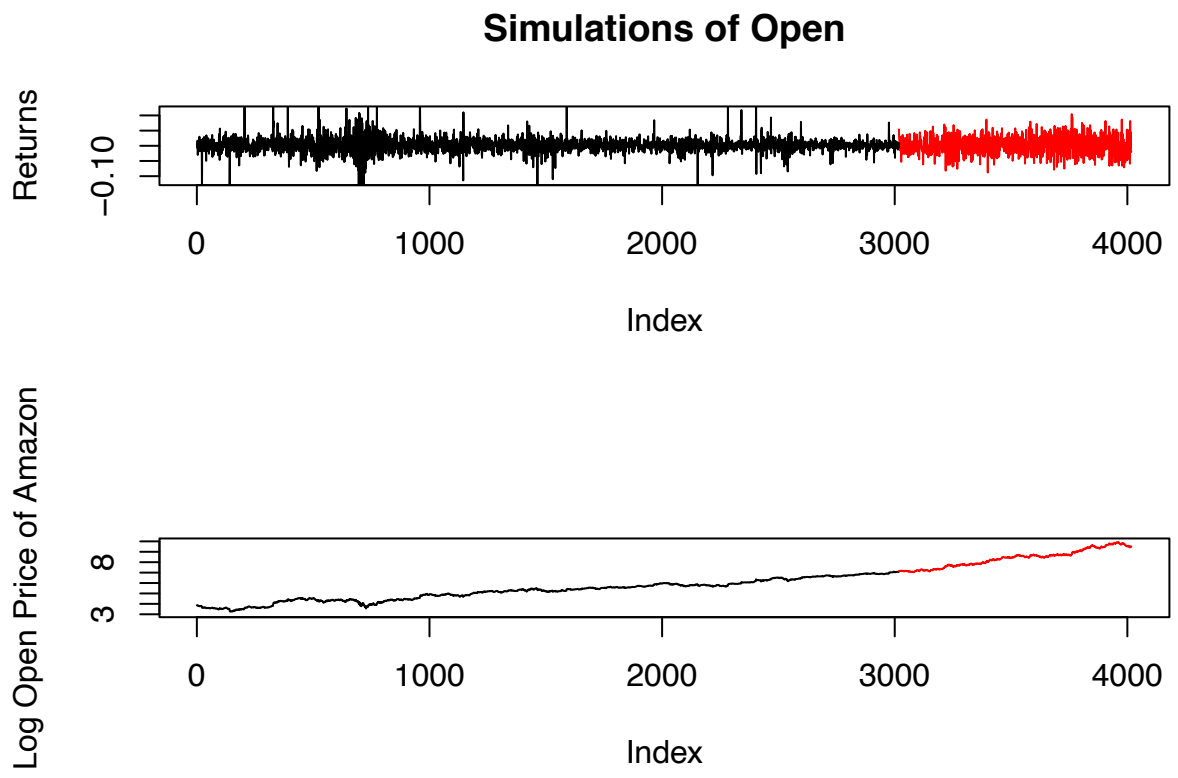
```
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     5.496355e-04     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1 -9.736e+03
##      1    7 -9.740e+03  4.04e-04  6.98e-04  3.7e-04  5.0e+09  3.7e-05  1.75e+06
##      2    8 -9.740e+03  7.78e-06  8.69e-06  3.6e-04  2.0e+00  3.7e-05  3.60e+01
##      3   14 -9.763e+03  2.38e-03  3.23e-03  2.7e-01  2.0e+00  3.8e-02  3.57e+01
##      4   18 -9.838e+03  7.65e-03  8.13e-03  8.5e-01  1.9e+00  6.0e-01  9.12e-01
##      5   28 -9.841e+03  2.87e-04  8.11e-03  4.5e-05  2.4e+00  5.9e-05  5.90e-02
##      6   29 -9.860e+03  1.93e-03  1.31e-03  1.6e-05  2.0e+00  3.0e-05  1.48e-02
##      7   30 -9.862e+03  2.20e-04  4.20e-04  1.9e-05  2.0e+00  3.0e-05  2.40e-01
##      8   31 -9.863e+03  7.63e-05  9.02e-05  2.2e-05  2.0e+00  3.0e-05  1.30e-01
##      9   32 -9.863e+03  2.04e-06  2.00e-06  2.2e-05  2.0e+00  3.0e-05  1.42e-01
##     10   40 -9.884e+03  2.15e-03  2.62e-03  8.4e-02  1.9e+00  1.2e-01  1.43e-01
```

```
##     11   42 -9.887e+03  2.39e-04  3.33e-04  1.3e-02  1.6e+00  2.0e-02  1.20e-03
##     12   43 -9.888e+03  1.82e-04  2.20e-04  1.2e-02  1.4e+00  2.0e-02  7.70e-04
##     13   45 -9.894e+03  5.43e-04  3.93e-04  2.9e-02  0.0e+00  5.8e-02  3.93e-04
##     14   46 -9.900e+03  6.31e-04  7.04e-04  2.4e-02  6.3e-01  5.8e-02  8.41e-04
##     15   57 -9.902e+03  2.09e-04  4.32e-04  1.5e-06  3.3e+00  2.7e-06  9.92e-04
##     16   58 -9.902e+03  8.75e-06  7.64e-06  1.2e-06  2.0e+00  2.7e-06  1.94e-04
##     17   59 -9.902e+03  1.53e-07  1.32e-07  1.3e-06  2.0e+00  2.7e-06  2.44e-04
##     18   68 -9.905e+03  3.13e-04  1.95e-04  2.0e-02  0.0e+00  4.3e-02  2.42e-04
##     19   70 -9.906e+03  7.86e-05  7.71e-05  4.0e-03  2.0e+00  8.7e-03  1.60e-01
##     20   71 -9.906e+03  3.15e-05  5.09e-05  3.9e-03  2.0e+00  8.7e-03  1.15e-02
##     21   77 -9.906e+03  2.74e-06  6.04e-06  6.3e-08  5.2e+01  1.2e-07  4.72e-03
##     22   87 -9.912e+03  5.42e-04  2.43e-04  8.2e-03  0.0e+00  1.7e-02  2.43e-04
##     23   89 -9.913e+03  8.47e-05  1.14e-04  3.1e-03  2.0e+00  6.7e-03  1.30e-02
##     24   96 -9.913e+03  1.39e-07  3.51e-07  6.8e-09  5.0e+01  1.3e-08  1.16e-03
##     25  105 -9.914e+03  9.08e-05  1.32e-04  4.4e-04  1.7e+00  8.7e-04  1.08e-03
##     26  107 -9.914e+03  1.31e-05  2.01e-05  7.9e-04  8.7e-01  1.8e-03  3.89e-05
##     27  108 -9.914e+03  9.74e-06  1.04e-05  2.7e-04  0.0e+00  5.3e-04  1.04e-05
##     28  109 -9.914e+03  1.11e-07  1.77e-07  7.9e-05  0.0e+00  1.6e-04  1.77e-07
##     29  124 -9.914e+03  1.10e-13  2.40e-11  8.5e-12  4.1e+01  1.7e-11  2.10e-09
##     30  131 -9.914e+03  3.67e-16  1.78e-13  6.4e-14  5.8e+03  1.3e-13  2.11e-09
##     31  132 -9.914e+03  1.28e-15  8.88e-14  3.2e-14  1.2e+04  6.3e-14  2.11e-09
##     32  133 -9.914e+03 -2.02e-15  4.44e-14  1.6e-14  2.3e+04  3.1e-14  2.11e-09
##
##  ***** FALSE CONVERGENCE *****
##
##  FUNCTION    -9.913826e+03   RELDX        1.597e-14
##  FUNC. EVALS     133         GRAD. EVALS      32
##  PRELDF       4.442e-14      NPRELDF      2.113e-09
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    2.517335e-06     1.000e+00    -1.409e+04
##      2    1.633744e-02     1.000e+00    -6.617e+00
##      3    9.791323e-01     1.000e+00    -5.782e+00
```

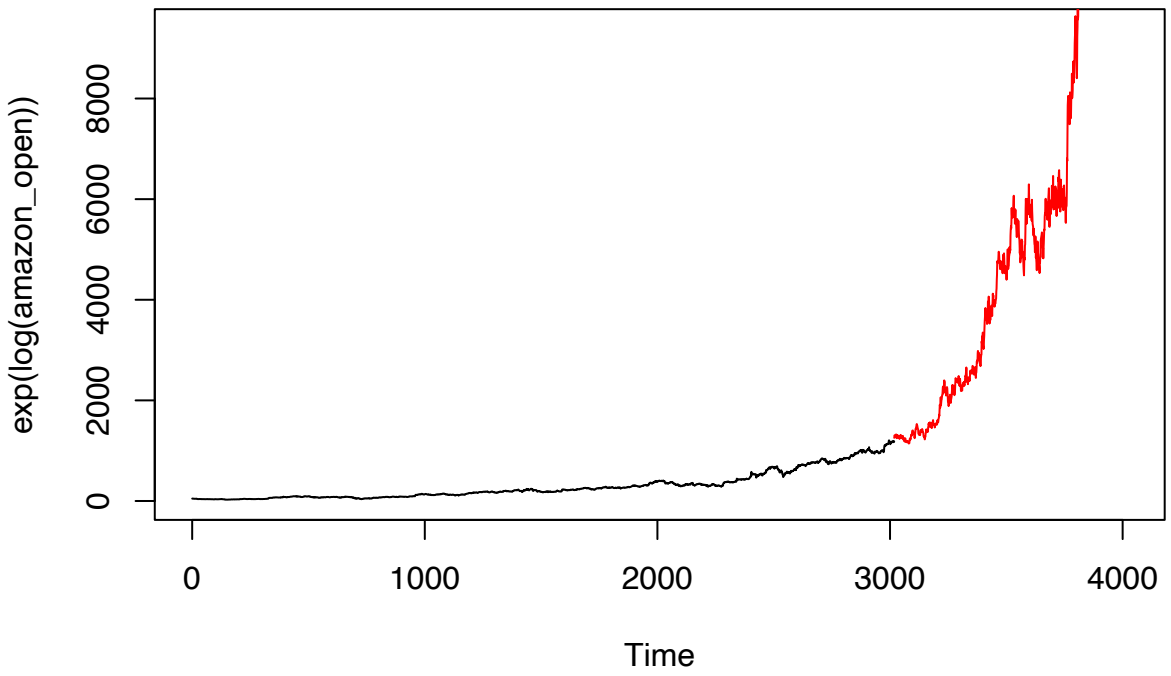
```
plot(predict(gfit))
```



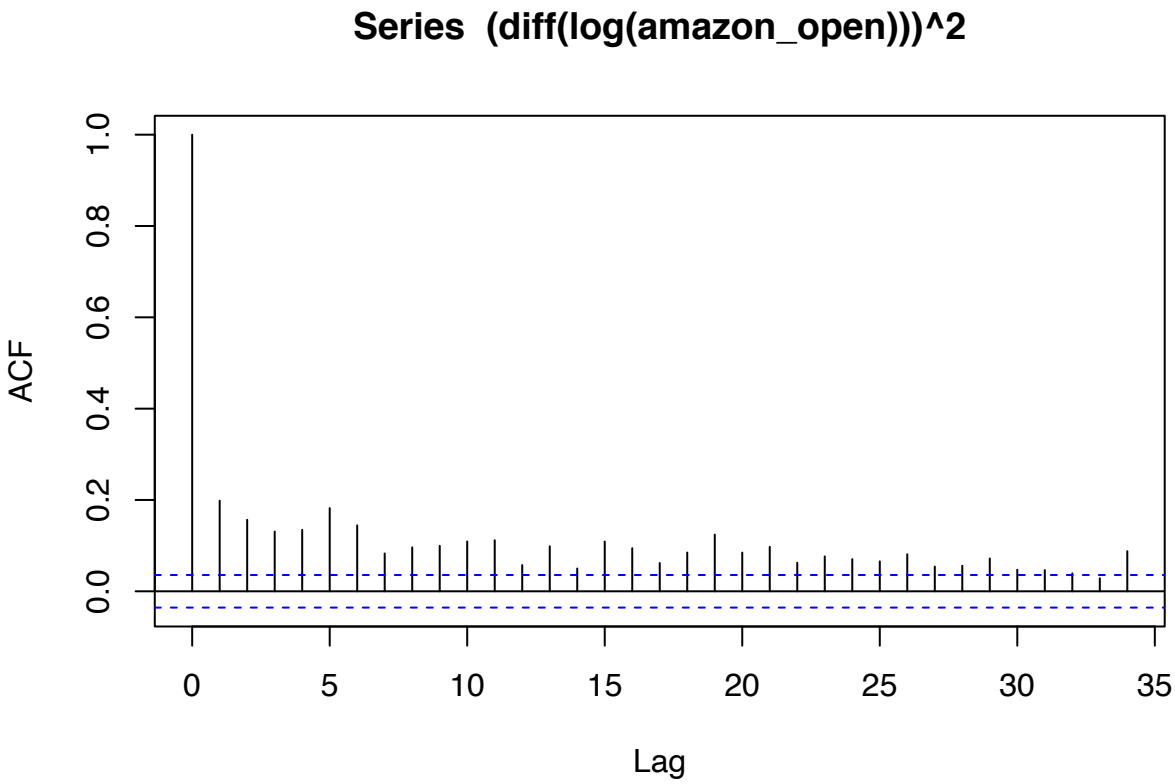

```
library(rugarch)
par(mfrow=c(2,1))
default_spec <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1,1)))
default_garch <- ugarchfit(spec = default_spec, data = diff(log(amazon_open)))
set.seed(22232224)
sim = ugarchsim(default_garch,n.sim=1000)
sim_rst = sim@simulation$seriesSim[,1]
matplot(y = sim@simulation$seriesSim[,1],x=3018:4017, type = "l",
       main = "Simulations of Open",col = "red",xlim=c(0,4020),ylab="Returns",xlab="Index",ylim=c(-0.12
lines(diff(log(amazon_open)))


sim = rep(0,1000)
first = log(amazon_open)[3018]
for(i in 1:1000){
  if(i==1){
    sim[i]=first+sim_rst[1]
  }
  else{
  sim[i]=sim[i-1]+sim_rst[i]}
}

plot(log(amazon_open),xlim = c(0,4020),ylim = c(3,10),ylab="Log Open Price of Amazon",xlab="Index")
lines(x=3018:4017,y=sim,col="red",type="l")
```



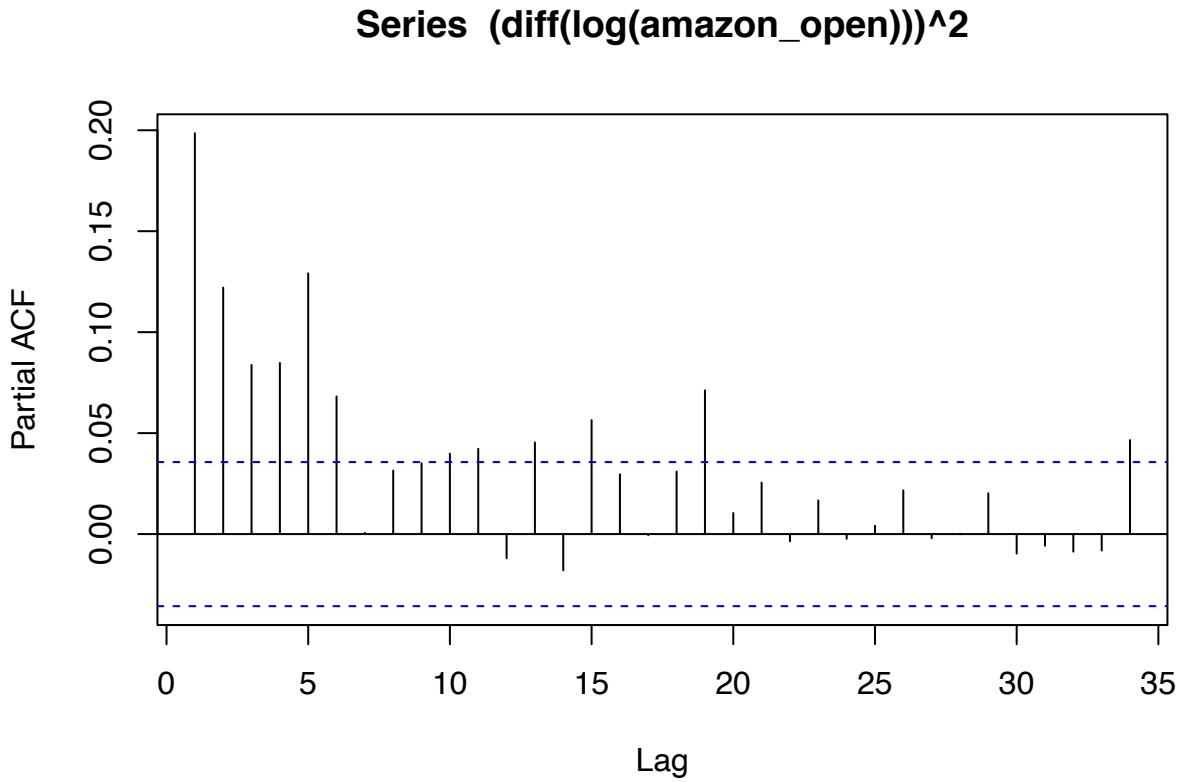

```
plot(exp(log(amazon_open)),xlim = c(0,4020),ylim=c(0,9400))
lines(x=3018:4017,y=exp(sim),col="red",type="l")
```




```
acf((diff(log(amazon_open)))^2)
```




```
pacf((diff(log(amazon_open)))^2)
```




```
model = auto.arima(diff(log(amazon_open))^2, max.p = 3, max.q = 3,max.order = 10, stationary = T, seasor
```

```
##
```

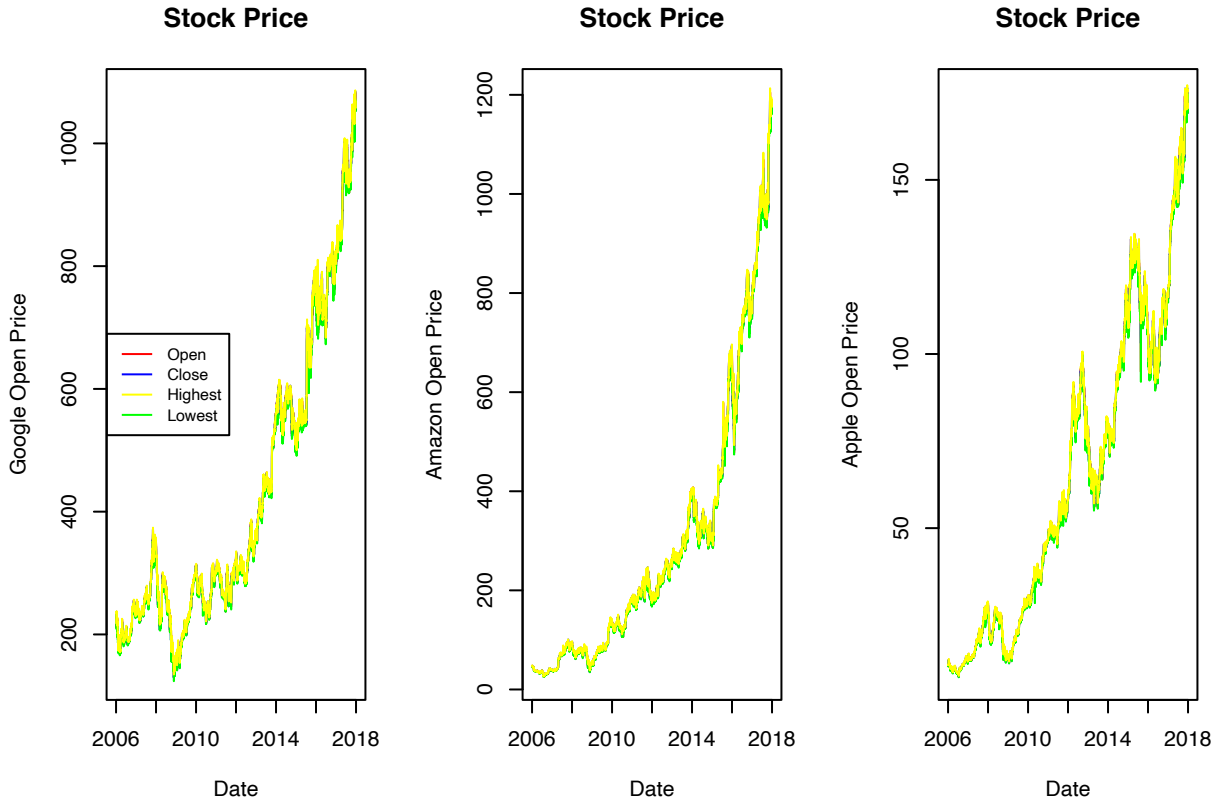
```
##  ARIMA(0,0,0) with zero mean     : -28235.22
##  ARIMA(0,0,0) with non-zero mean : -28465.03
##  ARIMA(0,0,1) with zero mean     : -28388.72
##  ARIMA(0,0,1) with non-zero mean : -28560.08
##  ARIMA(0,0,2) with zero mean     : -28461.79
##  ARIMA(0,0,2) with non-zero mean : -28602.16
##  ARIMA(0,0,3) with zero mean     : -28488.25
##  ARIMA(0,0,3) with non-zero mean : -28614.7
##  ARIMA(1,0,0) with zero mean     : -28440.74
##  ARIMA(1,0,0) with non-zero mean : -28584.37
##  ARIMA(1,0,1) with zero mean     : -28721.91
##  ARIMA(1,0,1) with non-zero mean : -28738.01
##  ARIMA(1,0,2) with zero mean     : -28729.26
##  ARIMA(1,0,2) with non-zero mean : -28741.98
##  ARIMA(1,0,3) with zero mean     : -28729.34
##  ARIMA(1,0,3) with non-zero mean : -28740.8
##  ARIMA(2,0,0) with zero mean     : -28520.41
##  ARIMA(2,0,0) with non-zero mean : -28627.64
##  ARIMA(2,0,1) with zero mean     : -28729.82
##  ARIMA(2,0,1) with non-zero mean : -28742.21
##  ARIMA(2,0,2) with zero mean     : Inf
##  ARIMA(2,0,2) with non-zero mean : Inf
##  ARIMA(2,0,3) with zero mean     : -28727.27
##  ARIMA(2,0,3) with non-zero mean : Inf
##  ARIMA(3,0,0) with zero mean     : -28559.52
##  ARIMA(3,0,0) with non-zero mean : -28646.88
##  ARIMA(3,0,1) with zero mean     : -28729.36
##  ARIMA(3,0,1) with non-zero mean : -28740.69
##  ARIMA(3,0,2) with zero mean     : Inf
##  ARIMA(3,0,2) with non-zero mean : Inf
##  ARIMA(3,0,3) with zero mean     : -28735.83
##  ARIMA(3,0,3) with non-zero mean : -28744.22
##
##
##
##  Best model: ARIMA(3,0,3) with non-zero mean
```

garch(1,1) is better

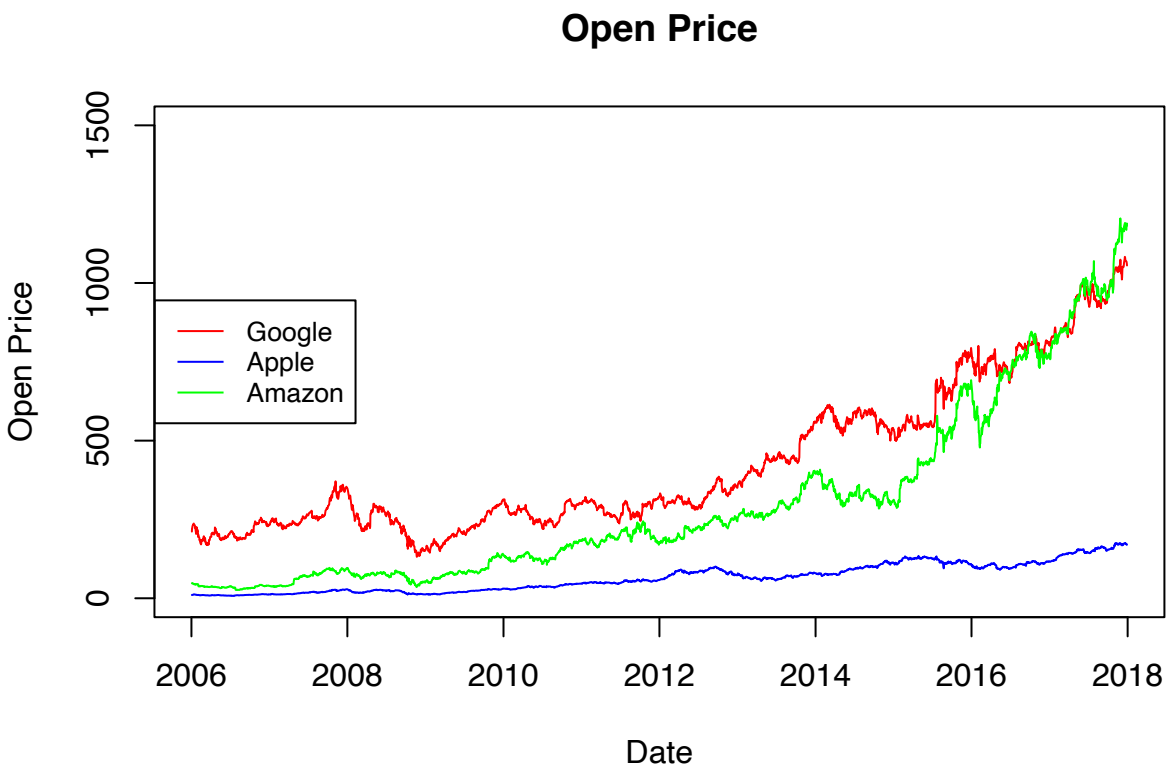
comparing the three

```
par(mfrow = c(1,3))
plot(x=as.Date(google$Date),y=google$Open,col="red",main = "Stock Price",type="l",xlab="Date",ylab = "G
legend("left", legend=c("Open", "Close","Highest","Lowest"),
       col=c("red", "blue","yellow","green"), lty=1, cex=0.8)
lines(x=as.Date(google$Date),y=google$Close,col="blue",main = "Stock Price",type="l")
lines(x=as.Date(google$Date),y=google$Low,col="green",main = "Stock Price",type="l")
lines(x=as.Date(google$Date),y=google$High,col="yellow",main = "Stock Price",type="l")
plot(x=as.Date(amazon$Date),y=amazon$Open,col="red",main = "Stock Price",type="l",xlab="Date",ylab = "A
lines(x=as.Date(amazon$Date),y=amazon$Close,col="blue",main = "Stock Price",type="l")
lines(x=as.Date(amazon$Date),y=amazon$Low,col="green",main = "Stock Price",type="l")
lines(x=as.Date(amazon$Date),y=amazon$High,col="yellow",main = "Stock Price",type="l")
```

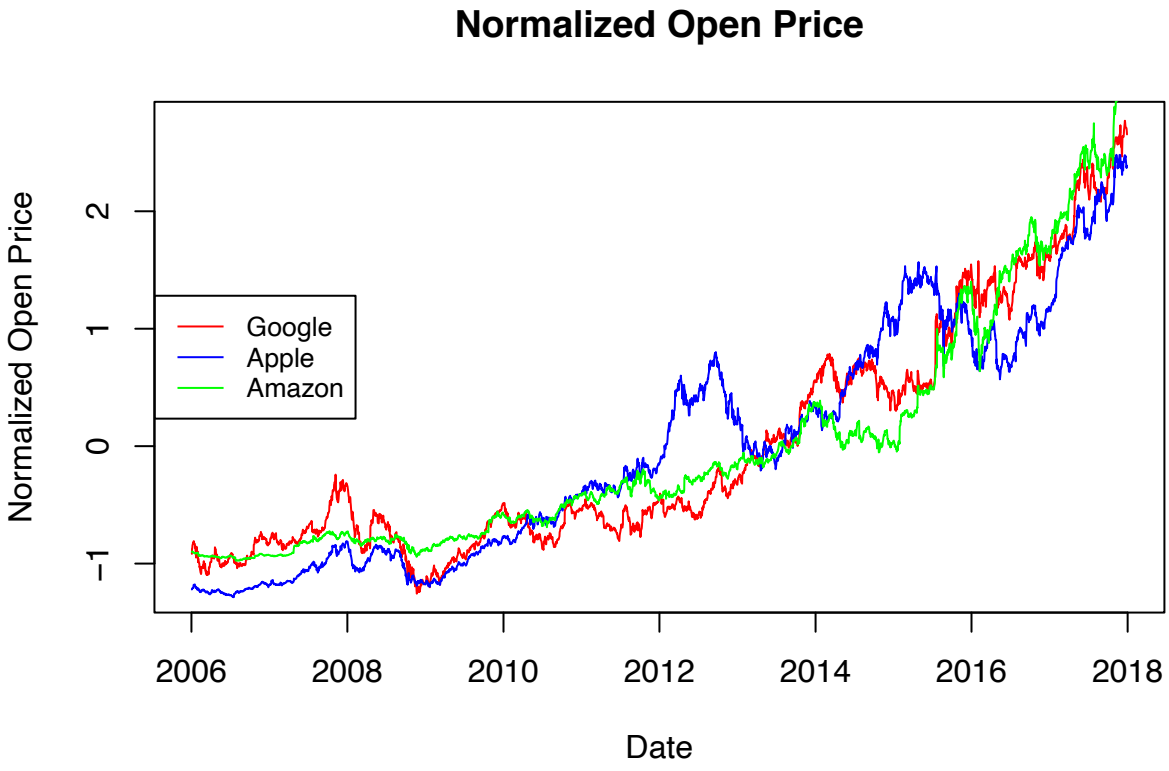
```
plot(x=as.Date(apple$Date),y=apple$Open,col="red",main = "Stock Price",type="l",xlab="Date",ylab = "Appl
lines(x=as.Date(apple$Date),y=apple$Close,col="blue",main = "Stock Price",type="l")
lines(x=as.Date(apple$Date),y=apple$Low,col="green",main = "Stock Price",type="l")
lines(x=as.Date(apple$Date),y=apple$High,col="yellow",main = "Stock Price",type="l")
```




```
plot(y=google_open,x=as.Date(google$Date),col="red",ylab="Open Price", main="Open Price",type="l",xlab=
lines(y=apple_open,x=as.Date(google$Date),col="blue")
lines(y=amazon_open,x=as.Date(google$Date),col="green")
legend("left", legend=c("Google", "Apple","Amazon"),
       col=c("red", "blue","green"), lty=1, cex=0.8)
```




```
plot(y=(google_open-mean(google_open))/sd(google_open),x=as.Date(google$Date),col="red",ylab="Normalize
lines(y=(apple_open-mean(apple_open))/sd(apple_open),x=as.Date(google$Date),col="blue")
lines(y=(amazon_open-mean(amazon_open))/sd(amazon_open),x=as.Date(google$Date),col="green")
legend("left",legend=c("Google", "Apple","Amazon"),
       col=c("red", "blue","green"), lty=1, cex=0.8)
```



```
library(vars)
```

```
## Loading required package: strucchange
```

```
## Loading required package: sandwich
```

```
## Loading required package: urca
```

```
## Loading required package: lmtest
```

```
x = cbind(diff(log(google_open)), diff(log(apple_open)), diff(log(amazon_open)))
VARselect(x, lag.max = 14, type = 'none')
```

```
## $selection
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      6      1      1      6
##
## $criteria
##                   1             2             3             4             5
## AIC(n) -2.376312e+01 -2.376135e+01 -2.375942e+01 -2.375923e+01 -2.376800e+01
## HQ(n)  -2.375664e+01 -2.374841e+01 -2.373999e+01 -2.373333e+01 -2.373563e+01
## SC(n)  -2.374512e+01 -2.372536e+01 -2.370542e+01 -2.368723e+01 -2.367800e+01
## FPE(n)  4.784195e-11  4.792642e-11  4.801945e-11  4.802852e-11  4.760898e-11
##                   6             7             8             9            10
## AIC(n) -2.376871e+01 -2.376578e+01 -2.376864e+01 -2.376787e+01 -2.376307e+01
## HQ(n)  -2.372987e+01 -2.372046e+01 -2.371685e+01 -2.370961e+01 -2.369834e+01
## SC(n)  -2.366072e+01 -2.363978e+01 -2.362465e+01 -2.360588e+01 -2.358308e+01
## FPE(n)  4.757510e-11  4.771489e-11  4.757863e-11  4.761525e-11  4.784417e-11
##                  11            12            13            14
## AIC(n) -2.376004e+01 -2.376402e+01 -2.376266e+01 -2.376041e+01
## HQ(n)  -2.368883e+01 -2.368633e+01 -2.367850e+01 -2.366978e+01
## SC(n)  -2.356205e+01 -2.354803e+01 -2.352867e+01 -2.350843e+01
## FPE(n)  4.798937e-11  4.779911e-11  4.786431e-11  4.797180e-11
```

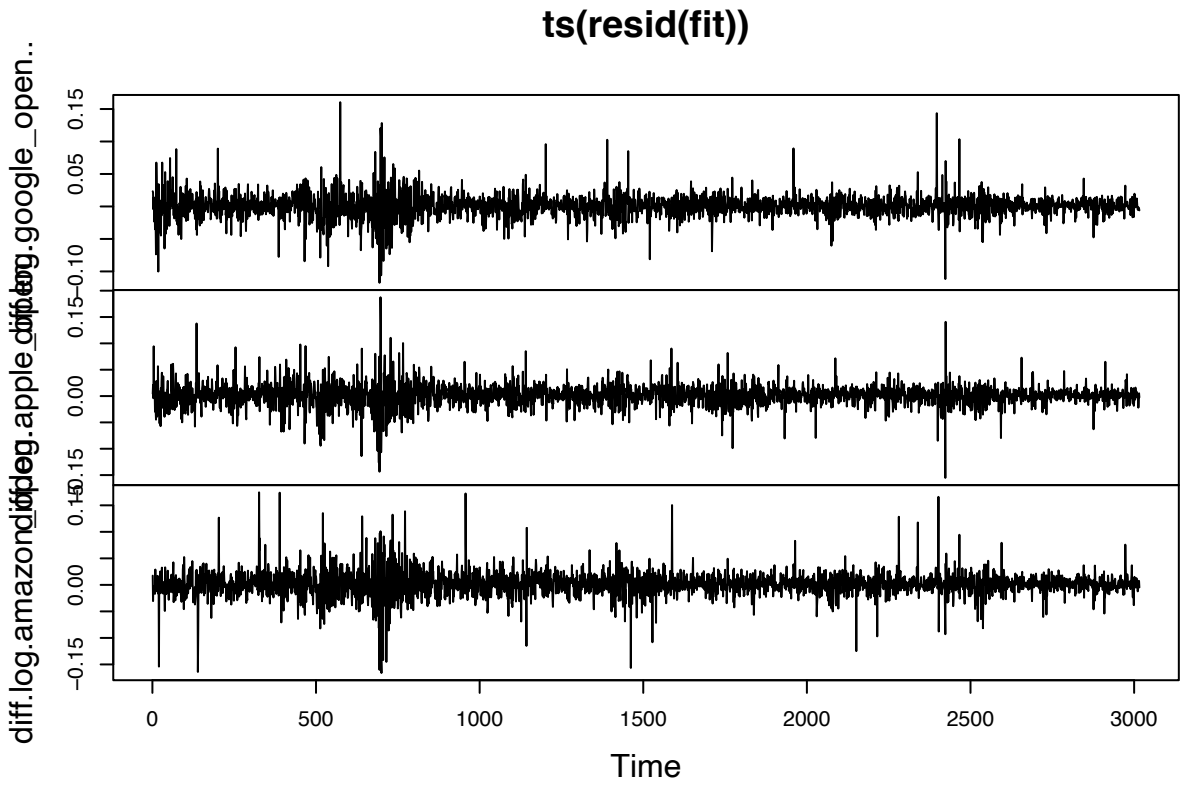
```
fit = VAR(x, p = 2, type = 'none')
summary(fit)
```

```
##
## VAR Estimation Results:
## ========================
## Endogenous variables: diff.log.google_open.., diff.log.apple_open.., diff.log.amazon_open..
## Deterministic variables: none
## Sample size: 3016
## Log Likelihood: 22980.499
## Roots of the characteristic polynomial:
## 0.2263 0.1618 0.1618 0.1553 0.1333 0.1333
## Call:
## VAR(y = x, p = 2, type = "none")
##
##
## Estimation results for equation diff.log.google_open..:
## ======================================================
## diff.log.google_open.. = diff.log.google_open...l1 + diff.log.apple_open...l1 + diff.log.amazon_open
##
##                             Estimate Std. Error t value Pr(>|t|)
## diff.log.google_open...l1 -3.829e-03  2.306e-02  -0.166   0.8681
## diff.log.apple_open...l1  -1.460e-02  1.874e-02  -0.779   0.4358
```

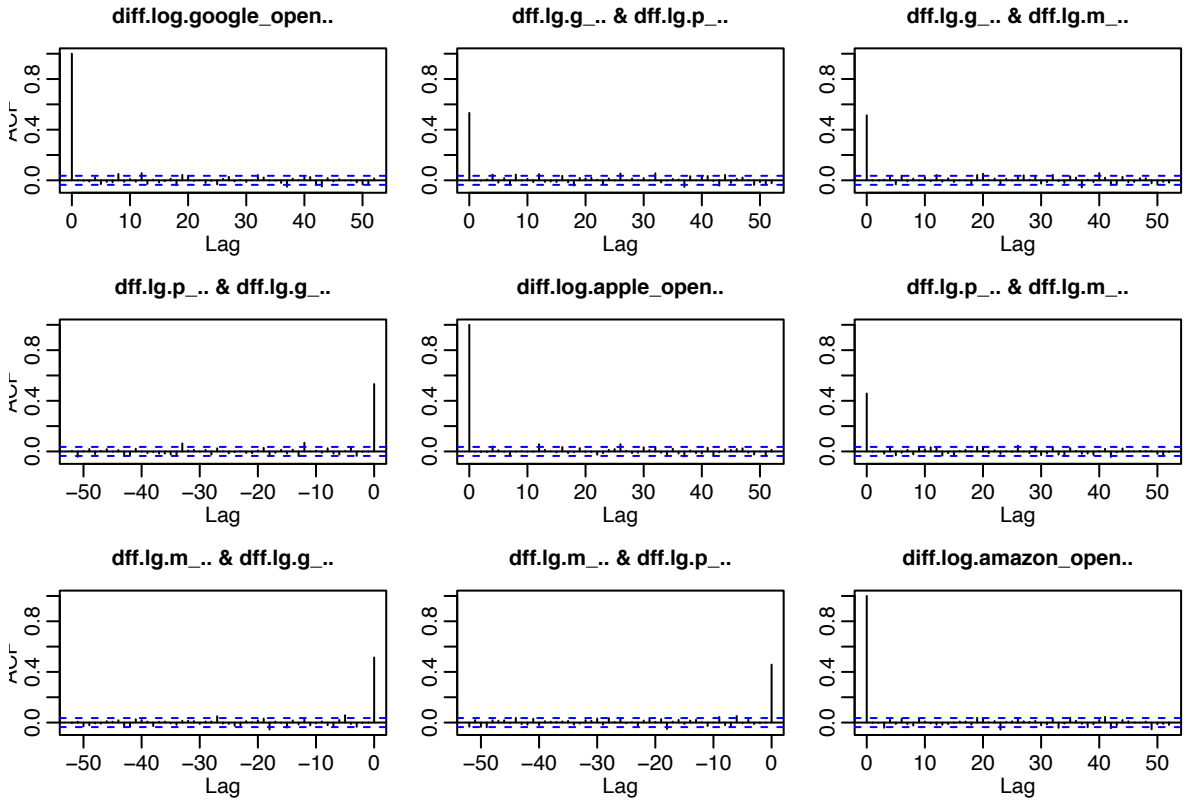
```
## diff.log.amazon_open...l1 -1.947e-02  1.637e-02  -1.189   0.2345
## diff.log.google_open...l2 -1.763e-02  2.304e-02  -0.765   0.4441
## diff.log.apple_open...l2   3.712e-02  1.872e-02   1.983   0.0475 *
## diff.log.amazon_open...l2 -4.311e-05  1.635e-02  -0.003   0.9979
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01841 on 3010 degrees of freedom
## Multiple R-Squared: 0.003334,    Adjusted R-squared: 0.001347
## F-statistic: 1.678 on 6 and 3010 DF,  p-value: 0.1222
##
##
## Estimation results for equation diff.log.apple_open..:
## ======================================================
## diff.log.apple_open.. = diff.log.google_open...l1 + diff.log.apple_open...l1 + diff.log.amazon_open.
##
##                            Estimate Std. Error t value Pr(>|t|)
## diff.log.google_open...l1 -0.033631   0.027390  -1.228    0.220
## diff.log.apple_open...l1  -0.101406   0.022257  -4.556 5.42e-06 ***
## diff.log.amazon_open...l1  0.005075   0.019450   0.261    0.794
## diff.log.google_open...l2  0.005817   0.027367   0.213    0.832
## diff.log.apple_open...l2   0.023666   0.022237   1.064    0.287
## diff.log.amazon_open...l2  0.012975   0.019418   0.668    0.504
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.02187 on 3010 degrees of freedom
## Multiple R-Squared: 0.01564, Adjusted R-squared: 0.01368
## F-statistic:  7.97 on 6 and 3010 DF,  p-value: 1.503e-08
##
##
## Estimation results for equation diff.log.amazon_open..:
## =======================================================
## diff.log.amazon_open.. = diff.log.google_open...l1 + diff.log.apple_open...l1 + diff.log.amazon_open
##
##                           Estimate Std. Error t value Pr(>|t|)
## diff.log.google_open...l1  0.06082    0.03092   1.967 0.049258 *
## diff.log.apple_open...l1  -0.09566    0.02512  -3.807 0.000143 ***
## diff.log.amazon_open...l1 -0.02244    0.02196  -1.022 0.306825
## diff.log.google_open...l2 -0.01108    0.03089  -0.359 0.719981
## diff.log.apple_open...l2  -0.01657    0.02510  -0.660 0.509121
## diff.log.amazon_open...l2  0.01141    0.02192   0.520 0.602807
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.02468 on 3010 degrees of freedom
## Multiple R-Squared: 0.006402,    Adjusted R-squared: 0.004421
## F-statistic: 3.232 on 6 and 3010 DF,  p-value: 0.003621
##
##
##
```

```
## Covariance matrix of residuals:
##                         diff.log.google_open.. diff.log.apple_open..
## diff.log.google_open..               0.0003386             0.0002140
## diff.log.apple_open..                0.0002140             0.0004772
## diff.log.amazon_open..               0.0002331             0.0002464
##                         diff.log.amazon_open..
## diff.log.google_open..               0.0002331
## diff.log.apple_open..                0.0002464
## diff.log.amazon_open..               0.0006080
##
## Correlation matrix of residuals:
##                         diff.log.google_open.. diff.log.apple_open..
## diff.log.google_open..                  1.0000                0.5323
## diff.log.apple_open..                   0.5323                1.0000
## diff.log.amazon_open..                  0.5138                0.4575
##                         diff.log.amazon_open..
## diff.log.google_open..                  0.5138
## diff.log.apple_open..                   0.4575
## diff.log.amazon_open..                  1.0000
```

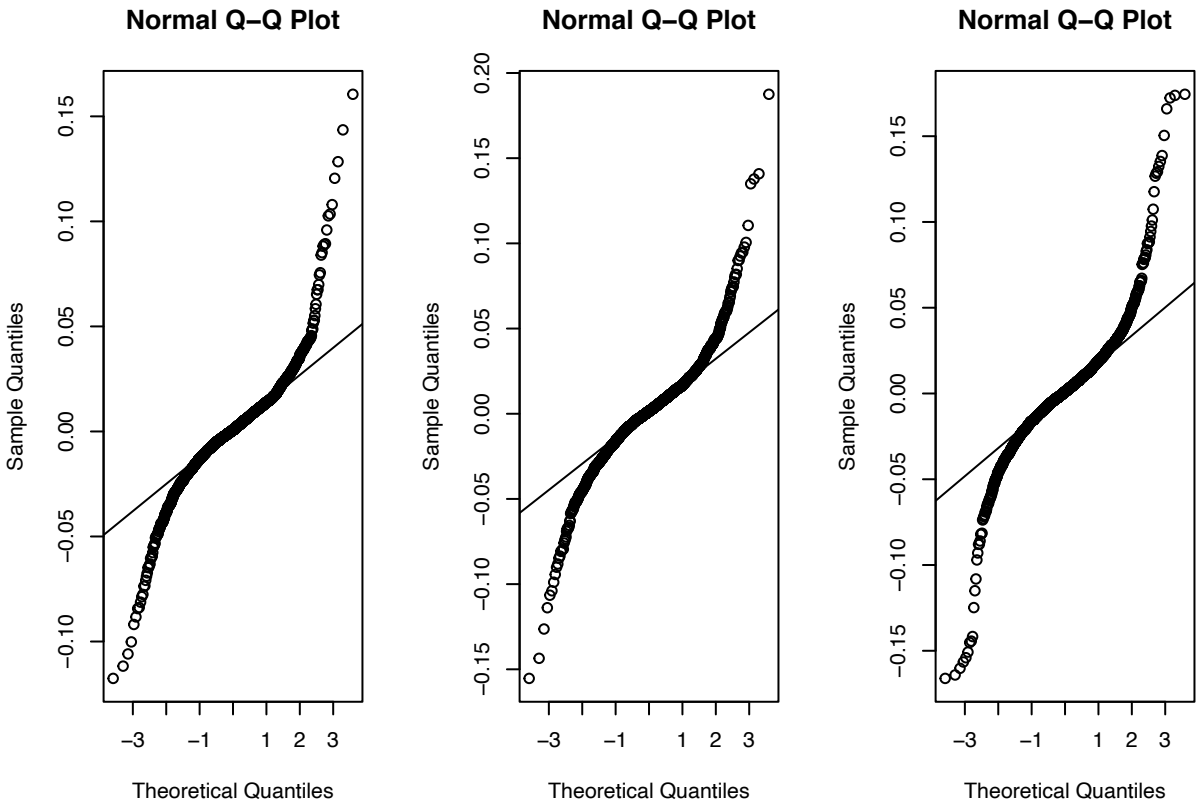
```
plot(ts(resid(fit)))
```




```
acf(resid(fit), 52)
```




```
par(mfrow = c(1,3))
qqnorm(resid(fit)[,1])
qqline(resid(fit)[,1])
qqnorm(resid(fit)[,2])
qqline(resid(fit)[,2])
qqnorm(resid(fit)[,3])
qqline(resid(fit)[,3])
```


Normal Q–Q Plot
Sample Quantiles
Theoretical Quantiles
−0.10
−0.05
0.00
0.05
0.10
0.15
−3
−1
1
2
3
Normal Q–Q Plot
Sample Quantiles
Theoretical Quantiles
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20
−3
−1
1
2
3
Normal Q–Q Plot
Sample Quantiles
Theoretical Quantiles
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
−3
−1
1
2
3