DATA304 Project Group 4: A study of the LAB cafe at Victoria University

Kevin Ye, Vivian Dong, Patrick Quito, Tama Hoare

May 2022

1 Introduction

The LAB Food and Beverage Co. is a Cafe located at ground level of the Easterfield Building at Kelburn campus of Victoria University of Wellington 6012, New Zealand. As part of a project for DATA304 course at Victoria University, our group decided to model this cafe system by analyzing customer interarrival time and service times.

The aims of this study were to measure, analyse, and model this real queueing system. We looked to analyse the customer arrival process, the service channel discipline and service times for this chosen system. From here we looked to model the interarrival time and service times of customers by fitting a best fit distribution to these times. We then went on to develop SimPy models in Python to be able to simulate the cafe system and assess how good each model is.

The cafe is open from 7.30am to 3pm every weekday and its main customers are students and staff from the university. The cafe sells a variety of coffee and brunch items such as rolls, sandwiches, salads, noodles etc. The manager of the cafe is Daniel and he can be contacted from email imdan97@outlook.com or by mobile 0226775474.

Note: The structure of the git repository that contains this report and all other relevant files were structured according to here[3].

2 Data Collection Process

2.1 Collecting the Data (Tama)

On March 28th, Kevin gained permission from the manager at LAB Cafe for us to collect the customer arrival and service data.

At the café there are 3 servers by the counter ready to serve. When a customer arrives they choose a server to order. If all the servers are busy the customer will wait in a queue. After ordering the customer waits at the café for their order to be ready. They are notified by a staff member when their order is ready, then they will depart the service unit. In order to observe this system we collected data on the time at which the customer arrived, the time at which their service began and the time at which they had finished their service. We catergorised each of these events as follows:

Customer Arrival (a) - The timing at which a customer arrives at the café with the intention to order.

Service Begin (s) - Moment when the customer begins to place their order with a server.

Service Completion/Departure (d) - Time when the customer receives the order and proceeds to leave the café.

This customer data was collected this data over a period of about 11 hours, which was split across 4 separate days. The times at which the data was recorded was between 9am and 3pm. In order to collect the data we used a Python program called monitor.py which was provided by our DATA304 lecturer[7]. I had to edit this program slightly so it would be effective to be used more the most recent version of python. This program automatically recorded times which made it easier for us to be able to monitor the system effectively. For each customer, we recorded 3 seperate times for their arrival, service beginning and departure. In total we recorded 324 customers at the cafe.

The monitor.py program was very useful for us to be able to record the data. It allowed us to pay more attention to the customer flow. As the cafe is in a public area it sometimes became different to recognise who was at the cafe and who had no intention to be served at the cafe. It also allowed us to be able to monitor the different customers effectively so was had the correct service times for each one.

2.2 Cleaning Data and Reading Data

After collecting the data, it was noticed there were some missing and incorrect values in the file; such as when customers arrive, get served, and depart. There were also some doubled values that were put by mistake which was also easily fixed. This meant we had to clean the raw data to fix these incorrect and missing values. This required some intiative from the group to realise where the mistakes were and what the corrected values were meant to be. In addition to this we require the data on how we tracked and monitored the customer to all be in the same format. This was so we could then produce a read_data file which would read each individual file and separate the interarrival times, and service times. To make it more efficient, we edited the file to have it read multiple files at once. Not only that, we have updated it so it calculates some of the performance measures like W and L for us to compare our models to.

3 Performance Measures of collected data (Tama)

Table 1: Performance Measures of collected data

Performance Measures	**Values calculated from data**
Average time in system (seconds), W	140.07
Average number of customers in the system, L	1.1819
Proportion of time servers are busy, B	0.611 48
Effective arrival rate (per second), λ_{eff}	0.008 438 1

Assessing the performance measures of the collected data is key modelling the performance of the servicing unit. As shown in the tables above there were seven key performance measures assessed from the collected data.

Firstly, we can see that the average time that a customer spent at the café service unit, meaning the total time of queuing plus service, was 140,07 seconds. The average number of customers in the system at any given point was 1.18. We can see from the data that the server at the café was busy serving customers 61.15% of the time. The remaining time is spent with no customers being served at the café.

The performance measure of effective arrival rate corresponds to the rate at which customers enter the service unit. We can see that 0.0084 customers entered the café per second. The correlates to 30.38 customers coming to the café per hour.

Table 2: Other calculated parameters from collected data

Other parameters	**Values calculated from data**
Average Inter-arrival time $\frac{1}{\lambda}$ (seconds)	120.329 450 453 125 02
Average Service time, W_s (seconds)	120.778 903 981 481 69
Average Queue Time, W_q (seconds)	19.295 767 959 876 45

The average inter-arrival time is the average time difference between arrival of one customer and then the next customer. It is a time elapse between the arrival of the person and one following it in the queue. The value calculated from this in the data was 120.33 seconds.

The time it takes for one customer to be served is referred to as the service time. This is the time between a single customer reaching the server (no longer in the queue) and leaving the service unit. Hence it took on average 120.77s for a customer to be served at the café once they had reached the server.

The average queue time is the time between a customer arriving at the service system and getting to the point at which they start to get served. They spend that time in a queue. At the café it took on average 19.295 seconds for a customer to get to the server once they had arrived at the café. Hence they spent of average 19.295 seconds in a queue at the café.

4 Data analysis

4.1 Fitting best fit distributions (Vivian)

We tried to approximate ”inter-arrival time” and ”service time” using the following 12 Distributions: Weibull Minimum Extreme Value distribution, Normal distribution, Weibull Maximum Extreme Value distribution, Beta distribution, Inverse Gaussian distribution, Uniform distribution, Gamma distribution, Exponential distribution, Log-normal distribution, Pearson Type III distribution, Triangular distribution, Erlang distribution. After fitting different distributions, we checked the Goodness of fit based on Chi-square Statistics. The outputs for "inter-arrival time" sorted in order of Goodness of fit looks like this:

Table 3: Distributions listed by Betterment of fit

Distribution	chi square
Pearson Type III distribution	9.155
Weibull Minimum Extreme Value distribution	13.245
Beta distribution	21.708
Log Normal distribution	25.596
Inverse Gaussian distribution	29.390
Exponential distribution	29.515
Gamma distribution	48.359
Triangular distribution	209.930
Normal distribution	332.531
Uniform distribution	510.690
Erlang distribution	672.400
Weibull Maximum Extreme Value distribution	1137.915

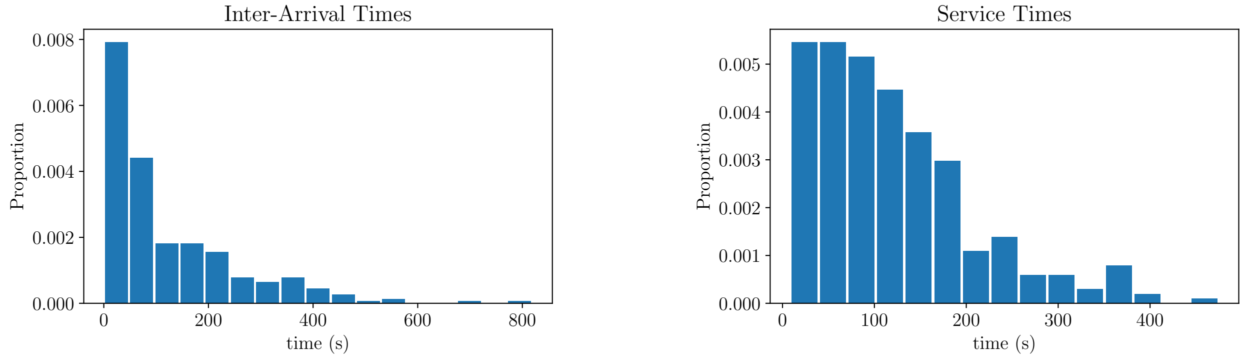
The outputs for "service time" sorted in order of Goodness of fit looks like this:

Table 4: Distributions listed by Betterment of fit

Distribution	chi square
Beta distribution	1.231
Weibull Minimum Extreme Value distribution	2.831
Pearson Type III distribution	4.130
Gamma distribution	4.132
Erlang distribution	4.132
Inverse Gaussian distribution	10.561
Log Normal distribution	11.689
Exponential distribution	29.775
Triangular distribution	39.441
Normal distribution	140.195
Uniform distribution	305.594
Weibull Maximum Extreme Value distribution	1080.829

Since the test measures how a model compares to actual observed data, the Chi-square statistics suggest that the Pearson Type III distribution best approximates inter-arrival time as it has the smallest error score of 9.155. We can see that Beta distribution has the smallest Chi-square statistics value of 1.231, which indicates that Beta distribution is the best fit for service time. The python code using the Scipy Library to fit the distribution is from here[1] Suppose we had more time to do this part. In that case, we will add more distributions to fit our data and find a better fit distribution of the interarrival/service times. And use scipy.stats.chisquare function instead of using the math formula to calculate the Chi-Square test so that I can consider both test output and p-value. Furthermore, we can also use the Anderson-Darling test or other goodness-of-fit tests to compare whether we will get the same results.

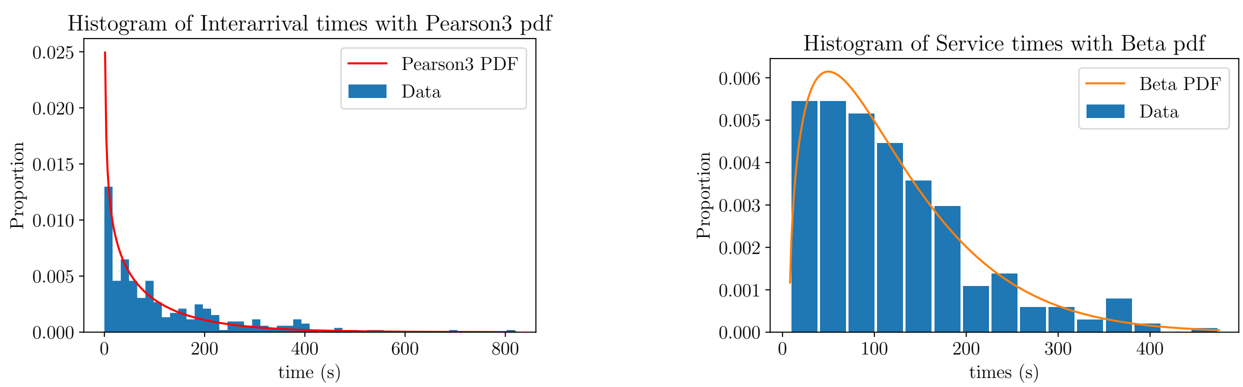
4.2 Histogram plots for visual evaluation (Patrick)



(a) **(b)**

Figure 1: Histograms of inter-arrival times and service times

Here, we have divided the inter-arrival time and the service time data into a number of phases, such that the time occupied by each phase has a negative exponential distribution. Using the Matplotlib.pyplot.hist function for python, I plotted the data and it produced these two histograms. The inter-arrival is the interval of time between each arrival. Assuming that the arrivals are independent, their distribution is exponential. The service times defines as the time required to serve a customer. These assumptions are further confirmed above by the histograms of the observations. We have observed the arrival of customers for a few hours.



(a) **(b)**

Figure 2: Histograms with best fit distribution pdf overlayed

The second pair of histograms above are fitted with the best fit distributions. We have fitted the inter-arrival and service time histograms with a list of distributions listed by Betterment of fit. Based on the Chi-square Statistics, it suggests that Pearson3 is the best in approximating 'inter-arrival time' Data. For our 'service times' data, it suggests that beta distribution is the best fit for the data.

5 Simulation models

5.1 M1 model (Patrick)

The M1 model is a task where we investigate and use either M/M/1 or M/M/C depending on our structure. An M/M/1 queue represents the queue length in a system having a single server, where arrivals are determined by a Poisson process and service times gave an exponential distribution. In the case of our project, we have an M/M/3 queue as there were 3 servers from where we gathered our data form.

$$\pi_0 = \frac{1}{\sum_{k=0}^{s-1} \frac{\rho^k}{k!} + \frac{\rho^s}{s!}\frac{1}{1-\frac{\rho}{s}}}$$

$$\pi_0 = \frac{1}{\frac{\rho^0}{0!} + \frac{\rho^1}{1!} + \frac{\rho^2}{2!} + \frac{\rho^3}{3!}\frac{1}{1-\frac{\rho}{3}}}$$

$$\pi_0 = \frac{1}{1 + \rho + \frac{\rho^2}{2} + \frac{\rho^3}{6}\frac{1}{1-\frac{\rho}{3}}}$$

$$\pi_0 = 0.369$$

$$B = 1 - \pi_0 = 0.631$$

$$L = \pi_0 \frac{\frac{\rho^{s+1}}{s!s}}{(1-\frac{\rho}{s})^2} + \rho$$

$$L = \pi_0 \frac{\frac{\rho^4}{3!3}}{(1-\frac{\rho}{3})^2} + \rho$$

$$L = 1.034$$

$$W = \frac{L}{\lambda} = 124.390$$

Mathematics used were from works published on operations research[4][5]

Upon generating all the values (check the table below), we have used the formula to compare to the original data we've gathered. This is used to confirm that the values from the M1 model are correct after doing the math.

The log relative error is one of the easiest way to compare 2 numbers. See here[2] for more detail on how to use LRE, published by one of our lecturers, Alejandro.

The LRE is defined as

$$LRE(x,c) = \begin{cases} -\log_{10}\frac{|x-c|}{|c|}, & \text{if } c \neq 0. \\ -\log_{10}|x|, & \text{otherwise.} \end{cases}$$

Table 5: Comparing performance measures of Collected data and M1 model

Performance Measures	**Collected Data**	**M1 model**	**LRE**
Average time in system (seconds), W	140.075	124.333	0.949
Average number of customers in the system, L	1.182	1.044	0.932
Proportion of time servers are busy, B	0.611	0.641	1.314
Effective arrival rate (per second), λ_{eff}	0.008	0.008	2.295

In creating the M1 model, we took inspiration from the assignments and labs to be able to produce these outcomes. We have passed our data on arrival and service rate through the model and got the values above. We had a mean arrival rate of 0.008310517468785045 per second, and a mean service rate of 0.008279591609419837 per second. The LAB Cafe is open from 7:30 am to 3:00 pm a total of 7.5 hours per day which is equivalent to 27000 seconds which would be the max time of each simulation. W is denoted as the average waiting of customers, L is the average number of customers in the system, B is the proportion of time that servers are busy, and lambdaEff is the effective arrival rate.

5.2 M2 model (Vivian)

The M2 model is a simulation model developed using SimPy to model the LAB cafe customer waiting and serving system. M2 model uses the interarrival times best fit distribution (Pearson3) and the service times best fit distribution (Beta) to simulate the performance. Below is the comparison between the original data performance measure estimates and the performance measures estimates produced by M2 model.

Table 6: Comparing performance measures of Collected data and M2 model

Performance Measures	Collected Data	M2 model	LRE
Average time in system (seconds), W	140.075	143.288	1.639
Average number of customers in the system, L	1.182	1.628	0.423
Proportion of time servers are busy, B	0.611	0.714	0.776
Effective arrival rate (per second), λ_{eff}	0.008	0.011	0.462

From the table above, we can see that the estimated W from the M2 model has a difference of approximately 3.21 from the estimate provided by the original data collected. The L difference between the two estimations by the collected data and the M2 model is about 0.45. The difference in the proportion of time servers are busy (B) was 0.1 between the two estimates of the collected data and the M2 model. The effective arrival rate λ_{eff} has a difference of 0.0029 approximately between the original data and the M2 estimate. We can see that the M2 model is a decent fit for the actual data as the differences are relatively small. But let's take a look at the LRE values. Two digits are computed correctly for Average time in the system, one digit for proportions of time servers are busy, and approximately 0 digits for the average number of customers in the system and effective arrival rate. So based on the LRE values, we could say that the M2 model isn't the best model to simulate the system among these three models.

5.3 M3 model (Kevin)

Table 7: Comparing performance measures of Collected data and M3 model

Performance Measures	Collected Data	M3 model	LRE
Average time in system (seconds), W	140.075	127.144	1.035
Average number of customers in the system, L	1.182	1.085	1.087
Proportion of time servers are busy, B	0.611	0.625	1.667
Effective arrival rate (per second), λ_{eff}	0.008	0.009	1.970

The M3 model is a simulation model developed using SimPy to model the LAB cafe customer waiting and serving system. The distribution of interarrival and service times are modelled after the empirical distributions of the interarrival times and services times recorded from the original data. A python script called draw_emp.py was provided by our lecturer[6] from DATA304 to assist in building this model. From the M3 model produced some performance measures estimates in the table above which we can compare to the original data performance measure estimates to gauge how well of a fit this M3 model is at simulating the nature of the real life system.

From the table we can see that estimated W from the M3 model has a difference of approximately 13 to the estimate provided by the original data collected. The L difference between the two estimations by the colletced data and the M3 model is about 0.1. The difference in the B, proportion of time servers are busy was 0.01 between the two estimates of the collected data and the M3 model. The effective arrival rate λ_{eff} has a difference of 0.0001 approximately between the original data and the M3 estimate. We can see that The M3 model is a decent fit for the original data as the differences are around about 10% of the original data estimates.

Note: all simulation models were constructed using inspiration from SimPy material from the DATA304 course.[8]

6 Conclusion

To reiterate the objective, our group collected data about customer arrival and service times from the LAB café to determine their best fit distributions for interarrival times and service times. Then we compared the simulated values of 3 different models to the collected data using performance measures to determine each model's fit.

The fitted distributions were Pearson3 for interarrival times and Beta distribution for service times. This contradicts the assumption that interarrival and service times are exponentially distributed due to the Poisson Process.

Problems encountered during the project involved having to clean the raw data for missing values as sometimes there were too many customers and orders to keep track of. Lots of bug fixing in the python code to ensure results were accurate. Sometimes the computer ran out of power during monitoring days.

In the future we could analyze a more complex system that could involve a max number of customers allowed in the system and the issue of customers balking when there are too many customers. We would also develop a singular metric or index that describes how well one simulation model is at simulating the data collected. Then we can compare the models against each other and determine the best model for simulation. When we first collected the data we noticed that there are usually two types of customers those that get coffees with their orders and those that don't. Coffee customers might wait longer so the next time the data is collected we could classify the customers as either coffee orders or not.

This raises many questions after studying this system. What could be done to make the process more efficient for customers to order so their time in the system in minimized? What would happen if we were to hypothetically add more servers? More interesting questions like this can be answered if the project was of wider scope and more time was allowed. The customer arrivals were assumed to be independent of one another, however in our café system, many of the customers often came to order in groups of two or more. Even if they came alone to order they sometimes ran into people they knew and started talking which influenced the time of service and departure. This further reinforces our analysis on the distributions of interarrival times and service times. The distributions of these times are expected to be exponential but they are not due to the arrivals not being independent. So thus can not assume the customers arriva via a poisson process.

References

[1] Mungoli Abhishek. Distribution fitting used car dataset. `https://github.com/mungoliabhishek/Distribution-Fitting-Used_Car_Dataset`, 2020.

[2] Marcelo G Almiron, Bruno Lopes, Alyson LC Oliveira, Antonio C Medeiros, and Alejandro C Frery. On the numerical accuracy of spreadsheets. *Journal of Statistical Software*, 34:1–29, 2010.

[3] Alejandro C Frery, Luis Gomez, and Antonio C Medeiros. A badging system for reproducibility and replicability in remote sensing research. *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, 13:4988–4995, 2020.

[4] Frederick S Hillier and Gerald J Lieberman. Introduction to operations research. 2001.

[5] John DC Little. A proof for the queuing formula: L= λ w. *Operations research*, 9(3):383–387, 1961.

[6] Binh Nguyen. draw_emp.py file. `https://blackboard.vuw.ac.nz/`, 2022.

[7] Binh Nguyen. monitor.py file. `https://blackboard.vuw.ac.nz/`, 2022.

[8] Binh Nguyen. Simpy material. `https://blackboard.vuw.ac.nz/`, 2022.