Worksheet 7 - 11/17

1. Is the union of two subspaces always a subspace? If not, give a counterexample.
2. Is the intersection of two subspaces always a subspace? If not, give a counterexample.
3. Let's compute a determinant using guassian elimination. Let

$$A = \begin{bmatrix} 2 & -1 & 2 \\ 2 & -1 & 1 \\ 0 & 3 & 1 \end{bmatrix}$$

   We will determine what row operations do to the determinant of a matrix.

   (a) What is $\det(A)$?

   (b) What is the effect of swapping two rows of a matrix on the determinant? Try swapping the first two rows of A and computing the determinant of the resulting matrix.

   (c) What is the effect of scale multiplying a row of a matrix on the determinant? Try scale multiplying the 2nd row of A by 2 and computing the determinant of the resulting matrix.

   (d) What is the effect of adding a multiple a row to another row on the determinant? Try adding twice the first row to the second and computing the determinant of the resulting matrix.

   (e) Use these ideas to compute the determinant of A using guassian elimination. If you are stuck, see the following links:

   - `https://en.wikipedia.org/wiki/Gaussian_elimination#Computing_determinants`
   - `https://math.stackexchange.com/questions/714974/determinant-by-applying-gaussian-elimination`