Worksheet 4 - Due 10/27

1. Find an example of each of the following. If it is not possible, write NOT POSSIBLE.

   (a) Give an example of 2 linear transformations $S, T : \mathbb{R}^3 \to \mathbb{R}^3$ (this means they are both from $\mathbb{R}^3$ to $\mathbb{R}^3$) such that S is onto but $S \circ T$ (this is the function given by $(S \circ T)(x) = S(T(x))$) is not.

   Solution: Let S be the identity and T be the zero transformation.

   (b) Give an example of 2 linear transformations $S, T : \mathbb{R}^3 \to \mathbb{R}^3$ such that T is onto but $S \circ T$ is not.

   Solution: Let T be the identity and S be the zero transformation.

   (c) Give an example of 2 linear transformations $S, T : \mathbb{R}^3 \to \mathbb{R}^3$ such that S is one-to-one but $S \circ T$ is not.

   Solution: Let S be the identity and T be the zero transformation.

   (d) Give an example of 2 linear transformations $S, T : \mathbb{R}^3 \to \mathbb{R}^3$ such that T is one-to-one but $S \circ T$ is not.

   Solution: Let T be the identity and S be the zero transformation.

   (e)

2. Give a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(1, 1) = (2, 3)$ and $T(-1, 2) = (0, 1)$. Do this using matrix inverses.

   Solution: Let $u_1 = (1, 1), u_2 = (-1, 2)$ and $v_1 = (2, 3), v_2 = (0, 1)$. Let $U = [u_1\ u_2]$ be the matrix formed by writing u_1, u_2 as columns and $V = [v_1\ v_2]$ be the matrix formed by writing v_1, v_2 as columns.

   Let $F(x) = Ux$ and $G(x) = Vx$. Then we know that $F(e_1) = u_1, F(e_2) = u_2$ and $G(e_1) = v_1$ and $G(e_2) = v_2$. Since $\{u_1, u_2\}$ is linearly indepedent, we know that U and F are invertible. So $G \circ F^{-1}$ sends u_1 to v_1 and u_2 to v_2, as desired. The associated matrix to $G \circ F^{-1}$ is VU^{-1}.

3. Find an example of each of the following. If it is not possible, write NOT POSSIBLE.

   (a) Give an example of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects every point about the x-axis.

   Solution: $T(x, y) = (x, -y)$

   (b) Give an example of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects every point about the $x = y$ line.

   Solution: We know that $T(1, 1) = (1, 1)$ and $T(-1, 1) = (1, -1)$. So we can figure out what T is by doing the process in question 2.

   (c) Give an example of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that shifts every point up by one unit.

Solution: NOT POSSIBLE. This function does not map zero to zer.