

## Jan 24

- Introduce nullspace and columns space
- Draw picture of domain, range, kernel, codomain

### Theorem related to linear independence

Let  $S = \{a_1, \dots, a_n\} \subseteq \mathbb{R}^m$  be a set of vectors. Let  $A$  be the  $m \times n$  matrix formed by writing the elements of  $S$  as columns. Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  be the linear transformation defined by  $T(x) = Ax$ .

- $S$  is linearly independent
- $Ax = 0$  has only the trivial solution
- For any  $b$ ,  $Ax = b$  has either no solution or exactly one solution.
- $\text{null}(A) = \{0\}$
- $T$  is one-to-one
- $\ker(T) = \{0\}$

### Theorem related to spanning

Let  $S = \{a_1, \dots, a_n\} \subseteq \mathbb{R}^m$  be a set of vectors. Let  $A$  be the  $m \times n$  matrix formed by writing the elements of  $S$  as columns. Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  be the linear transformation defined by  $T(x) = Ax$ .

- $S$  is spanning
- $Ax = b$  has a solution for any  $b$
- $\text{col}(A) = \mathbb{R}^m$
- $T$  is onto
- $\text{range}(T) = \mathbb{R}^m$

### Theorem related to the square case

Let  $S = \{a_1, \dots, a_n\} \subseteq \mathbb{R}^n$  be a set of vectors. Let  $A$  be the  $n \times n$  matrix formed by writing the elements of  $S$  as columns. Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$  be the linear transformation defined by  $T(x) = Ax$ .

- $S$  is a basis
- $S$  is linearly independent
- $S$  is spanning
- $Ax = b$  always has a unique solution
- $\text{col}(A) = \mathbb{R}^n$
- $\text{null}(A) = \{0\}$
- $T$  is invertible
- $A$  is invertible

## 3.2 Matrix Algebra

### Matrix multiplication is weird

- $AB \neq BA$
- Explain what  $AB = 0$  means in terms of columnspace and nullspace

### Transpose of a matrix

- Teach how to transpose
- $(A + B)^t = A^t + B^t$
- $(sA)^t$
- $(AC)^t = C^t A^t$

### Diagonal matrices and upper triangular matrices is a thing

- Give definition
- The product of diagonal is diagonal. Discuss the effects of multiplying a matrix by a diagonal matrix
- The product of upper triangulars is upper triangular

### Powers of matrices is a thing

- Powers of diagonal is easy
- Wouldn't it be great if  $A = UDU^{-1}$

## 3.3 Inverses

- Explain what an inverse is.
- Derive inverse formula.