

## Worksheet 2

Due 10/13

1. We know how to obtain the general solution from a linear system. Let's try to reverse it. Find a linear system whose general solution is

$$(x_1, x_2, x_3, x_4) = (1, 2, 3, 4) + s_1(5, 6, 7, 8) + s_2(9, 0, 1, 2).$$

2. Suppose  $A$  is a matrix. Let  $v, w$  be distinct (meaning  $x \neq y$ ) vectors that solve  $Ax = 0$  so  $Av = 0$  and  $Aw = 0$  (0 here of course means the zero vector!). Let  $L$  be the line that passes through  $v$  and  $w$ . If  $u$  is on  $L$ , then  $Au = 0$ . Why? This exercise suggests that solution spaces are convex.
3. Let  $z_1, z_2 \in \mathbb{R}$  and let  $S = \{(1, z_1, z_2), (2, 1, 0), (1, 0, -1)\}$ .
  - Find some values for  $z_1$  and  $z_2$  such that  $S$  spans  $\mathbb{R}^3$ .
  - Find some values for  $z_1$  and  $z_2$  such that  $S$  does not span  $\mathbb{R}^3$ .
  - Find all values for  $z_1$  and  $z_2$  such that  $S$  spans  $\mathbb{R}^3$ . (In the process of solving this problem, some of you will be tempted to divide by zero. Resist that temptation.)
4. Consider the following linear system that came from the book and the lecture.

$$2x_1 - 6x_2 - x_3 + 8x_4 = 0 \tag{1}$$

$$x_1 - 3x_2 - x_3 + 6x_4 = 0 \tag{2}$$

$$-x_1 + 3x_2 - x_3 + 2x_4 = 0. \tag{3}$$

Using row reduction, we see that a general solution is of the form  $x = s_1(3, 1, 0, 0) + s_2(-2, 0, 4, 1)$ . Let  $v_1 = (2, 1, -1), v_2 = (-6, -3, 3), v_3 = (-1, -1, 2), v_4 = (8, 6, 2)$ .

- Is  $\{v_1, v_2, v_3, v_4\}$  linearly independent set? The answer should be no.
  - Express  $v_1$  as a linear combination of  $v_2, v_3, v_4$ .
  - Express  $v_2$  as a linear combination of  $v_1, v_3, v_4$ .
  - Express  $v_3$  as a linear combination of  $v_1, v_2, v_4$ .
  - Express  $v_4$  as a linear combination of  $v_1, v_2, v_3$ .
5. Suppose  $\{v_1, v_2, v_3\}$  is a linearly dependent set. Is it always the case that we can write  $v_1$  as a linear combination of  $v_2$  and  $v_3$ ? If not, come up with a counterexample.
  6. Come up with an inconsistent linear system whose associated homogeneous linear system is consistent.