Worksheet 4 - Due 10/27

1. Find an example of each of the following. If it is not possible, write NOT POSSIBLE.
   (a) Give an example of 2 linear transformations $S, T : \mathbb{R}^3 \to \mathbb{R}^3$ (this means they are both from $\mathbb{R}^3$ to $\mathbb{R}^3$) such that S is onto but $S \circ T$ (this is the function given by $(S \circ T)(x) = S(T(x))$) is not.
   (b) Give an example of 2 linear transformations $S, T : \mathbb{R}^3 \to \mathbb{R}^3$ such that T is onto but $S \circ T$ is not.
   (c) Give an example of 2 linear transformations $S, T : \mathbb{R}^3 \to \mathbb{R}^3$ such that S is one-to-one but $S \circ T$ is not.
   (d) Give an example of 2 linear transformations $S, T : \mathbb{R}^3 \to \mathbb{R}^3$ such that T is one-to-one but $S \circ T$ is not.

2. Give a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(1, 1) = (2, 3)$ and $T(-1, 2) = (0, 1)$. Do this using matrix inverses.

3. Find an example of each of the following. If it is not possible, write NOT POSSIBLE.
   (a) Give an example of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects every point about the x-axis.
   (b) Give an example of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects every point about the $x = y$ line.
   (c) Give an example of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that shifts every point up by one unit.