

October 9

Announcements

- Section 2.1, 2.2 due this Thursday
- Section 2.3, 3.1 due next Thursday
- Midterm next Wednesday in class
 - 1.1 - 3.1 (maybe 3.2)
- Worksheet 2 has been posted, due this Friday

Homogenous Systems

Let A be a matrix. Then $A(x + y) = Ax + Ay$ and $A(x - y) = Ax - Ay$.

Example: Find a general solution for the linear system

$$2x_1 - 6x_2 - x_3 + 8x_4 = 7 \quad (1)$$

$$x_1 - 3x_2 - x_3 + 6x_4 = 6 \quad (2)$$

$$-x_1 + 3x_2 - x_3 + 2x_4 = 4. \quad (3)$$

Using row reduction, we see that a general solution is of the form $x = (1, 0, -5, 0) + s_1(3, 1, 0, 0) + s_2(-2, 0, 4, 1)$.

The solution to the homogenous system is $x = s_1(3, 1, 0, 0) + s_2(-2, 0, 4, 1)$.

Let x_p be a particular solution $Ax = b$. Then solutions have the form $x_g = x_p + x_h$, where x_p is a particular solution and x_h is the general solution to the homogenous equations.

Linear Independence and Span

Theorem: Let $A = [a_i]$ and b be a vector in \mathbb{R}^n . Then the following are equivalent (if one is true then they are all true, if one is false then they are all false).

- The set $\{a_1, \dots, a_m\}$ are linearly independent.
- The vector equation $x_1a_1 + x_2a_2 + \dots + x_ma_m = b$ has at most one solution.
- The linear system $[a_1 \ a_2 \ \dots \ a_m | b]$ has at most one solution.
- The equation $Ax = b$ has at most 1 solution.

Example: Consider the vectors $a_1 = (1, 7, -2)$, $a_2 = (3, 0, 1)$, and $a_3 = (5, 2, 6)$. Set $A = [a_i]$. Show that the columns of A are linearly independent and that $Ax = b$ has a unique solution for every b in \mathbb{R}^3 .

Example: Let $u_1 = (1, -1, 2)$, $u_2 = (2, -1, 2)$, $u_3 = (-2, 5, -10)$, $u_4 = (3, -4, 8)$. The associated matrix has reduced echelon form:

$$\begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Is $\{u_1, \dots, u_4\}$ linearly independent? Can we write u_1 as a linear combination of u_2, \dots, u_4 ?

If a set of vectors is not linearly independent, can every vector be written as a linear combination of the other vectors? In other words, is every vector in the span of the other vectors?

Section 3.1 Linear Transformations

We can write linear equations as $Ax = b$. We can think of it as A sending x to b .

Definition: A function $T : \mathbb{R}^m \rightarrow \mathbb{R}^n$ is a linear transformation if for all vectors $u, v \in \mathbb{R}^m$ and all scalars r , we have

- $T(u+v) = T(u) + T(v)$
- $T(ru) = rT(u)$.

Examples:

- What are some examples of functions that aren't linear transforms? quadratic, $ax+b$
- Consider the function given by $T(x_1, x_2) = (3x_1 - x_2, 2x_1 + 5x_2)$. What is $T(1, 2)$? Show that this is a linear transformation. Is it associated to a matrix?
- Projections are linear transforms.
- Let A be some matrix. Then $T(x) = Ax$ is a linear transform. Make up some example in class.

A matrix, A , is said to be an $n \times m$ matrix if it has n rows and m columns. If $m = n$, then A is a square matrix.

Theorem: Let A be an $n \times m$ matrix, and define $T(x) = Ax$. Then $T : \mathbb{R}^m \rightarrow \mathbb{R}^n$ is a linear transformation. Moreover, all linear transformations are of this form.

Example: Consider the linear transformation with matrix

$$A = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & 5 \end{bmatrix}.$$

Is $(3, 4)$ in the range of A ?

Theorem: Let $A = [a_1 \ a_2 \ \dots \ a_m]$ be a $n \times m$ matrix, and let $T : \mathbb{R}^m \rightarrow \mathbb{R}^n$ with $T(x) = Ax$ be a linear transformation. Then

- A vector w is in the range of T if and only if $Ax = w$ is a consistent linear system.
- The range of T is the span of the columns (this is also called the column space).

If time, talk about 1-1 and onto