

## October 6

### Announcements

- Watch 3rd video
- Section 2.1, 2.2 due next Thursday
- Worksheet. 1 due today, fill out form
- Worksheet 2 will be posted tonight

### Linear Independence

Let  $v, w$  be any vectors in  $\mathbb{R}^n$ . How does the span of  $\{v, w\}$  compare to the span of  $\{v, w, 2v + 3w\}$ ?

Consider the matrix this 3x3 matrix where the last row is the sum of the first two. What's the echelon form?

In these 2 examples, there were some redundant information.

**Definition:** Let  $S = \{u_1, u_2, \dots, u_m\}$  be a set of vectors in  $\mathbb{R}^n$ . We say that  $S$  is *linearly independent* if the only solution to the vector equation

$$x_1 u_1 + x_2 u_2 + \dots + x_m u_m = 0$$

is the trivial solution -  $x_1 = x_2 = \dots = x_m = 0$ . If a set is not linearly independent then it is linearly dependent.

A set is linearly dependent iff some vector is in the span of the others. A set is linearly independent iff no vector is in the span of the others.

Any set containing the zero vector is linearly dependent.

**Example:** Is the set  $\{(16, 2, 8), (22, 4, 4), (18, 0, 4), (18, 2, 6)\}$  linearly independent?

work out example in class using a linear system

Let  $S = \{u_1, \dots, u_m\}$  be a set of vectors in  $\mathbb{R}^n$  and  $A = [u_1 \ u_2 \ \dots \ u_m]$  be the matrix formed by these vectors. Then  $S$  is linearly independent if and only if the only solution is the trivial solution.

**Theorem:** Let  $S = \{u_1, \dots, u_m\}$  be a set of vectors in  $\mathbb{R}^n$ . Suppose

$$A = [u_1 \ u_2 \ \dots \ u_m] \sim B,$$

where  $B$  is in echelon form. Then  $S$  spans  $\mathbb{R}^n$  exactly when  $B$  has a pivot position in every row.  $S$  is linearly independent exactly when  $B$  has a pivot position in every column.

A set with fewer than  $n$  vectors will never span  $\mathbb{R}^n$ . A set with more than  $n$  vectors will never be linearly independent.

## Homogenous Systems

Let  $A$  be a matrix. Then  $A(x + y) = Ax + Ay$  and  $A(x - y) = Ax - Ay$ .

**Example:** Find a general solution for the linear system \*\*

$$2x_1 - 6x_2 - x_3 + 8x_4 = 7 \quad (1)$$

$$x_1 - 3x_2 - x_3 + 6x_4 = 6 \quad (2)$$

$$-x_1 + 3x_2 - x_3 + 2x_4 = 4. \quad (3)$$

\*\* Using row reduction, we see that a general solution is of the form  $x = (1, 0, -5, 0) + s_1(3, 1, 0, 0) + s_2(-2, 0, 4, 1)$ .

The solution to the homogenous system is  $x = s_1(3, 1, 0, 0) + s_2(-2, 0, 4, 1)$ .

Let  $x_p$  be a particular solution  $Ax = b$ . Then solutions have the form  $x_g = x_p + x_h$ , where  $x_p$  is a particular solution and  $x_h$  is the general solution to the homogenous equations.

**Theorem:** Let  $A = [a_i]$  and  $b$  be a vector in  $\mathbb{R}^n$ . Then the following are equivalent (if one is true then they are all true, if one is false then they are all false). \* The set  $\{a_1, \dots, a_m\}$  are linearly independent. \* The vector equation  $x_1a_1 + x_2a_2 + \dots + x_ma_m = b$  has at most one solution. \* The linear system  $[a_1 \ a_2 \ \dots \ a_m | b]$  has at most one solution. \* The equation  $Ax = b$  has at most 1 solution.

**Example:** Consider the vectors  $a_1 = (1, 7, -2)$ ,  $a_2 = (3, 0, 1)$ , and  $a_3 = (5, 2, 6)$ . Set  $A = [a_i]$ . Show that the columns of  $A$  are linearly independent and that  $Ax = b$  has a unique solution for every  $b$  in  $\mathbb{R}^3$ .