Worksheet 3

Due 10/20

1. During the October 13th lecture, I wrote down many statements equivalent to "S is a linearly independent set". Do the same for "S is a spanning set". The answer is in the notes but see what you can do from memory.

ANSWER: See notes

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation. We know that there exists a matrix A such that $T(x) = Ax$.

- Suppose we know that $T(1, 0) = (2, 3, 4)$ and $T(0, 1) = (-1, 2, 1)$. Can we determine A? If so, what is it? If not, why not?
- Suppose instead we know that $T(1, 0) = (2, 3, 4)$ and $T(2, 0) = (4, 6, 8)$. Can we determine A? If so, what is it? If not, why not?
- Suppose instead we know that $T(1, 0) = (2, 3, 4)$ and $T(1, 1) = (-1, 2, 1)$. Can we determine A? If so, what is it? If not, why not?
- Suppose instead we know that $T(x) = u$ and $T(y) = v$. Under what conditions on x and y, can we determine A?

ANSWER:

- Yes. The columns of the matrix are $(2, 3, 4)$ and $(-1, 2, 1)$.
- No. We don't know what $T(0, 1)$ is. There are infinitely many possibilities for A. Just set $T(0, 1)$ to be whatever you like.
- Yes. We need to determine what $T(0, 1)$ is. But $(0, 1) = (1, 1) - (1, 0)$. So by linearity, $T(0, 1) = T(1, 1) - T(1, 0) = (2, 3, 4) - (-1, 2, 1) = (3, 1, 3)$.
- When x, y are spanning (which is equivalent to linearly independent here!). More on this later.

3. Come up with a linear transform that is:

- One-to-one and onto
- One-to-one but not onto
- Onto but not one-to-one
- Not one-to-one nor onto

ANSWER:

- $T(x, y) = (x, y)$.
- $T(x) = (x, 0)$.
- $T(x, y) = x$.
- $T(x, y) = (0, 0)$.

4. Is differentiation a linear transformation? The answer is yes. I just want you to think about why this is true.

5. Do a full exam from the exam archive here under test like conditions.