October 27

Announcements

- Webassign 3.3, 4.1, 4.2 due next Thursday
- Sign sheet if you did worksheet 4, there's a good chance a problem similiar to a problem on worksheet 4 will appear on midterm
- Worksheet 5 posted this weekend
- Watch videos 8,9 of 3blue1brown, determinants are mentioned. Think about it as signed volume.

3.3 Inverses

Theorem: Let A and B be invertible matrices and C and D be matrices. Then

- A^{-1} is also invertible.
- AB is invertible. The inverse is given by $(AB)^{-1} = B^{-1}A^{-1}$.
- If $AC = AD$ then $C = D$
- If $CA = DA$ then $C = D$

Theorem: Let A be a $n \times n$ matrix. Let S be the columns of A. Let $T(x) = Ax$. Then the following are equivalent:

- S spans $\mathbb{R}^n$
- S is linearly indepedent
- $Ax = b$ has a unique solution for all $b \in \mathbb{R}^n$ given by $x = A^{-1}b$.
- T is onto
- T is one-to-one
- T is invertible
- A is invertible

The inverse of $[a, b; c, d]$ is $[d, -b; -c, a]/det$.

Example

Solve the linear system $3x_1 + x_2 = 3$ and $x_1 - x_2 = 4$.

4.1 Subspaces

Defintion: A subset S of $\mathbb{R}^n$ is a *subspace* if S satisfies the following 3 properties

- S contains 0.
- (closed under addition) If u and v are in S then so is $u + v$.
- (closed under multiplciation) If $r \in \mathbb{R}$ and $u \in S$, then $ru \in S$.

Nonexamples:

- If $b \neq 0$, then $Ax = b$ is never a subspace.

- The graph $y = x^2$ is not a subspace.

Example:

- The span of any set of vectors are a subspace.
- The solutions to $Ax = 0$ is a subspace.

Consider the matrix $A = [3, -1, 7, -6; 4, -1, 9, -7; -2, 1, -5, 5]$. The general solution to $Ax = 0$ is $x = s_1(-2, 1, 1, 0) + s_2(1, -3, 0, 1)$ So the set of solutions is the span of $(-2, 1, 1, 0)$ and $(1, -3, 0, 1)$.

Definition: The set of solutions to $Ax = 0$ is called the nullspace of A and is denote null(A).

Definition: Let $T : \mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation. Then the set $\{T(x) : x \in \mathbb{R}^m\}$ is called the *range* of T. This is a subspace of the codomain. If T is associated to a matrix A, then the range is the span of the columns of A.

The set $\{x \in \mathbb{R}^m : T(x) = 0\}$ is called the kernel of T. THis is a subspace of the domain.

- A linear transform is onto if it's range is equal to the codomain.
- A linear transform is one-to-one if it's kernel contains only the zero vector.