September 26

Announcements

- Read the syllabus
- Watch first 3blue1brown video
- First Webassign is due Tuesday
- Conor's notes
- Discussion problems

1.1 Lines and Linear Equations

Linear equation

Definition: A *linear equation* is an equation of the form

$$c_1x_1 + c_2x_2 + \ldots + c_nx_n = d$$

, where the c_i's are constants and the x_i's are variables. The solutions to a linear equation are the possible x_i's that satisfy the equation.

When we talk about solutions of linear equations, we have an ambient space in mind. In other words, the number of variables should be specified. For example, $6x = 5$ can be considered a linear equation in just x, or in x, y. The space of solutions will depend on this.

Examples:

- $6x = 5$: The solution space in $\mathbb{R}$ is a point. The solution space in $\mathbb{R}^2$ is a linear.
- $3x + 2y = 6$: Think about solution space.
- $4x + 2y + z = 0$: Think about solution space.
- An equation in n variables yields a $n - 1$-dimensional space.

We can see that geometrically, the solution space will a point, a line, a plane, or some other straight object.

Systems of linear equations

Definition: A *system of linear equations* is a list of linear equations. The solutions to a system of linear equation is the possible x_i's that satisfy all linear equations on the list.

The solution space of a system of linear equations is the intersection of the solution space to each linear equation in the system.

Theorem: The number of solutions to a system of linear equations will be either zero, one, or infinity.

The number of soluitions can be determined. In this course, we will learn how.

Examples:

Think of some examples here with class.

Definition: A system of linear equations is said to be *consistent* is there exist at least one solution. It is *inconsistent* if it is not consistent.

Special forms of linear systems

Definition: Given an ordering of the variables, the *leading variable* of a linear equation is the first variable with a nonzero coefficient in that linear equation.

Definition: A linear system of equations is *triangular* if the number of variables is equal to the number of equations the leading variable of the ith equation is x_i.

Examples:

Think of some examples here with class. Remember to solve them with back substitution.

Definition: A linear system is in *echelon form* if the leading variable are strictly increasing from top to bottom. Equations without variables are place at the bottom. In such a linear system, any variable that is not a leading variable is called a *free variable.*

Examples:

Think of some examples here with class. Be sure to explain why a free variable is called a free variable.