

## Worksheet 3

Due 10/20

1. During the October 13th lecture, I wrote down many statements equivalent to “ $S$  is a linearly independent set”. Do the same for “ $S$  is a spanning set”. The answer is in the notes but see what you can do from memory.

**ANSWER:** See notes

2. Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation. We know that there exists a matrix  $A$  such that  $T(x) = Ax$ .
  - Suppose we know that  $T(1, 0) = (2, 3, 4)$  and  $T(0, 1) = (-1, 2, 1)$ . Can we determine  $A$ ? If so, what is it? If not, why not?
  - Suppose instead we know that  $T(1, 0) = (2, 3, 4)$  and  $T(2, 0) = (4, 6, 8)$ . Can we determine  $A$ ? If so, what is it? If not, why not?
  - Suppose instead we know that  $T(1, 0) = (2, 3, 4)$  and  $T(1, 1) = (-1, 2, 1)$ . Can we determine  $A$ ? If so, what is it? If not, why not?
  - Suppose instead we know that  $T(x) = u$  and  $T(y) = v$ . Under what conditions on  $x$  and  $y$ , can we determine  $A$ ?

**ANSWER:**

- Yes. The columns of the matrix are  $(2, 3, 4)$  and  $(-1, 2, 1)$ .
  - No. We don't know what  $T(0, 1)$  is. There are infinitely many possibilities for  $A$ . Just set  $T(0, 1)$  to be whatever you like.
  - Yes. We need to determine what  $T(0, 1)$  is. But  $(0, 1) = (1, 1) - (1, 0)$ . So by linearity,  $T(0, 1) = T(1, 1) - T(1, 0) = (2, 3, 4) - (-1, 2, 1) = (3, 1, 3)$ .
  - When  $x, y$  are spanning (which is equivalent to linearly independent here!). More on this later.
3. Come up with a linear transform that is:
    - One-to-one and onto
    - One-to-one but not onto
    - Onto but not one-to-one
    - Not one-to-one nor onto

**ANSWER:**

- $T(x, y) = (x, y)$ .
  - $T(x) = (x, 0)$ .
  - $T(x, y) = x$ .
  - $T(x, y) = (0, 0)$ .
4. Is differentiation a linear transformation? The answer is yes. I just want you to think about why this is true.
  5. Do a full exam from the exam archive here under test like conditions.