October 4

Announcements

- Section 1.1, 1.2 due tomorrow
- Worksheet 1 due Friday - there are html issues, see pdf version
- Section 2.1, 2.2 due next Thursday
- Office hours today after class and tomorrow 12-1

Linear combinations and span

Definition: If $u_1, u_2, \ldots, u_m$ are vectors and $c_1, c_2, \ldots, c_m$ are scalars, then

$$c_1 u_1 + c_2 u_2 + \ldots + c_m u_m$$

is a *linear combination* of $u_1, \ldots, u_m$. Note that the constants can be negative or zero.

Definition: Let $S = \{u_1, u_2, \ldots, u_m\}$ be a set of vectors. Then the span of S, span$S$, is the set of all linear combinations of $u_1, u_2, \ldots, u_m\}$.

What vectors in $\mathbb{R}^2$ are a linear combination of $(1,0)$ and $(0,1)$? In other words, what vectors are in the span of $(1,0)$ and $(0,1)$?

What vectors in $\mathbb{R}^2$ are a linear combination of $(1,2)$ and $(0,1)$? Talk about lines and averages here.

Is $(3,4)$ a linear combination of $(1,2)$ and $(0,1)$? In other words, is $(3,4)$ in the span of $(1,2)$ and $(0,1)$? In other words, does there exists $x_1, x_2 \in \mathbb{R}$ such that $x_1(1,2) + x_2(0,1) = (3,4)$? In other words, system of equations!

Every system of equation can be interpeted in this way.

Theorem: Let $u_1, \ldots, u_m$ and v be vectors in $\mathbb{R}^n$. Then $v \in \text{span}(\{u_1, \ldots, u_m\})$ if and only if the linear system with augmented matrix $[u_1\ u_2\ \ldots\ u_m|v]$ has a solution.

The solution space can be expressed as a linear combination.

Theorem: Let $u_1, u_2, \ldots, u_m$ be vectors in $\mathbb{R}^n$. If $u \in$span$(\{u_1, \ldots, u_m\})$, then span$(\{u_1, \ldots, u_m\})$ =span$(\{u_1, \ldots, u_m, u\})$.

When does a set of vectors span $\mathbb{R}^n$?

Theorem: Let $u_1, u_2, \ldots, u_m$ be vectors in $\mathbb{R}^n$. Let $A = [u_1\ u_2\ \ldots\ u_m]$ and $B \sim A$, where B is in echelon form. Then $\text{span}(\{u_1, \ldots, u_m\}) = \mathbb{R}^n$ if and only if B has a pivot position in every row.

give outline of proof

We can write linear systems as $Ax = b$.