October 27

Announcements

- Webassign 3.3, 4.1, 4.2 due this Thursday
- Worksheet 4 solutions posted
- Worksheet 5 posted, due this Friday

4.2 Basis and Dimension

Recall that a subspace S satisfies 2 properties.

- It contains the zero vector.
- It is closed under linear combinations. If $u, v \in S$ then $au + bv \in S$ as well.

It is always the case that S is the span of a set of vectors because $S = span(S)$. What is the minimal number of vectors required to span S? What is the maximal number of vectors that are linearly indepedent in S? It turns out these numbers are equal and that number is called the dimension.

Definition: A set $B = \{u_1, \ldots, u_m\}$ is a *basis* for a subspace S if

- B spans S
- B is linearly indepedent

In this case, the *dimension* of S is m. Dimension is an invariant independent on the choice of basis.

The empty set is the basis for the zero subspace $\{0\}$. It has dimension 0.

Theorem:

- A set B is a basis for S if it is a maximal linearly independent set. This means you can't add any vectors to B and have it still be linearly independent.
- A set B is a basis for S if it is a minimal set that spans S. This means you can't substract any vectors from B and have it still be spanning.

Example:

Let S be the subspace given by $w + x + y + z = 0$. What is the dimension? Give 2 3 different basis.

Definition:

The row space of a matrix A is the span of the rows. It is denote $row(A)$.

Theorem:

Let A and B be equivalent matrices. Then the

Proof: The matrix A is equivalent to B if A can be obtained from B using elementary row opeartions. There are 3 such operations. None of them change the span.

Example:

Let S be the subspace spanned by $u_1 = (-1, 2, 3, 1), u_2 = (-6, 7, 5, 2), u_3 = (4, -3, 1, 0)$. Find a basis for S. What is the dimension of S?

(just find rref, and use the rows)

Notice we just take the nonzero rows.

Theorem:

Suppose $U = [u_1\ u_2\ \ldots\ u_m]$ and $V = [v_1\ \ldots\ v_m]$ be equivalent matrices. Then any relation between the u_i exists between the v_i. For example,

If $2u_1 - u_2 = u_3$ then $2v_1 - v_2 = v_3$.

Proof: Relations correspond to solutions to $Ux = 0$ and $Vx = 0$. Since they are equivalent, the relations must be the same.

Example:

Let S be the subspace spanned by $u_1 = (-1, 2, 3, 1), u_2 = (-6, 7, 5, 2), u_3 = (4, -3, 1, 0)$. Find a basis for S. What is the dimension of S?

(use rref to determine relations between columns)

Notice we only take the columns with leading variables.

4.3 row and column spaces

Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transform given by $T(x) = Ax$. Recall that the the range is the span of the columns of A and the kernel is the space of solutions to $Ax = 0$. The dimension of the range is called the rank of A, the dimension of the kernel is called the nullity.

Do an example of this in class.