Lab4

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11/27/2016

1a.

```
#State 10 after 10 transitions - at state 1
library(expm)
```

```
## Loading required package: Matrix

##
## Attaching package: 'expm'

## The following object is masked from 'package:Matrix':
##
##     expm
```

```
transition <- read.table('markov100.txt',header=F)
P <- as.matrix(transition)
a <- c(1,rep(0,99))
dist_a <- a %*% (P %^% 10)
print(dist_a[,5])
```

```
##       V5
## 0.045091
```

1b.

```
#State 10 after 10 transitions - equal probabilites of 1,2,3
b <- c(rep(1/3,3),rep(0,97))
dist_b <- b %*% (P %^% 10)
print(dist_a[,10])
```

```
##        V10
## 0.08126983
```

1c.

```
#Steady State Probablity of State 1
Q <- t(P) - diag(100)
Q[100,] = c(rep(1,100))
rhs <- c(rep(0,99),1)
steady <- solve(Q) %*% rhs;
print(steady[1])
```

```
## [1] 0.01256589
```

1d.

```
#State 1 to 100 Mean Passage Time
B <- P[1:99,1:99]
Q_m <- diag(99) - B
e <- c(rep(1,99))
m <- solve(Q_m) %*% e
print(m[1])
```

```
## [1] 254.9395
```

2a.

```r
library(Matrix)
web <- read.table('webtraffic.txt',header=T)
traffic_temp <- colSums(web)
traffic <- as.matrix(traffic_temp)
dim(traffic) <- c(9,9)
traffic <- t(traffic)
print(traffic)
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    0  447  553    0    0    0    0    0    0
##  [2,]    0   23  230  321    0    0    0    0   63
##  [3,]    0  167   43  520    0    0    0    0   96
##  [4,]    0    0    0   44  158  312  247    0  124
##  [5,]    0    0    0    0   22   52   90  127  218
##  [6,]    0    0    0    0   67   21    0  294   97
##  [7,]    0    0    0    0    0   94    7  185   58
##  [8,]    0    0    0    0  262    0    0   30  344
##  [9,]    0    0    0    0    0    0    0    0    0
```

2b.

```r
traffic[9,1] <- 1000
P_traffic <- matrix(nrow=9,ncol=9)
for (i in seq(1,9,1)) {
  for (j in seq(1,9,1)){
    P_traffic[i,j] <- (traffic[i,j] / sum(traffic[i,]))
  }
}
print(P_traffic)
```

```
##       [,1]       [,2]       [,3]       [,4]      [,5]       [,6]
##  [1,]    0 0.44700000 0.55300000 0.00000000 0.0000000 0.00000000
##  [2,]    0 0.03610675 0.36106750 0.50392465 0.0000000 0.00000000
##  [3,]    0 0.20217918 0.05205811 0.62953995 0.0000000 0.00000000
##  [4,]    0 0.00000000 0.00000000 0.04971751 0.1785311 0.35254237
##  [5,]    0 0.00000000 0.00000000 0.00000000 0.0432220 0.10216110
##  [6,]    0 0.00000000 0.00000000 0.00000000 0.1398747 0.04384134
##  [7,]    0 0.00000000 0.00000000 0.00000000 0.0000000 0.27325581
##  [8,]    0 0.00000000 0.00000000 0.00000000 0.4119497 0.00000000
##  [9,]    1 0.00000000 0.00000000 0.00000000 0.0000000 0.00000000
##             [,7]       [,8]      [,9]
##  [1,] 0.00000000 0.00000000 0.0000000
##  [2,] 0.00000000 0.00000000 0.0989011
##  [3,] 0.00000000 0.00000000 0.1162228
##  [4,] 0.27909605 0.00000000 0.1401130
##  [5,] 0.17681729 0.24950884 0.4282908
##  [6,] 0.00000000 0.61377871 0.2025052
##  [7,] 0.02034884 0.53779070 0.1686047
##  [8,] 0.00000000 0.04716981 0.5408805
##  [9,] 0.00000000 0.00000000 0.0000000
```

2c.

```r
Q_traffic <- t(P_traffic) - diag(9)
Q_traffic[9,] = c(rep(1,9))
rhs_2 <- c(rep(0,8),1)
steady_traffic <- solve(Q_traffic) %*% rhs_2
print(steady_traffic)
```

```
##             [,1]
##  [1,] 0.15832806
##  [2,] 0.10085497
##  [3,] 0.13077897
##  [4,] 0.14012033
##  [5,] 0.08058898
##  [6,] 0.07583914
##  [7,] 0.05446485
##  [8,] 0.10069664
##  [9,] 0.15832806
```

2d.

```r
averages <- c(0.1,2,3,5,5,3,3,2)
time <- c()
for (i in averages) {
  time <- append(time,averages[i] * steady_traffic[i])
}
print(sum(time))
```

```
## [1] 2.38632
```

2e.

```r
traffic_2 <- traffic
outgoing1 <- traffic[2,3] * 0.30
outgoing2 <- traffic[2,4] * 0.20
traffic_2[2,3] <- traffic[2,3] - outgoing1
traffic_2[2,4] <- traffic[2,4] - outgoing2
traffic_2[2,6] <- traffic[2,6] + outgoing1
traffic_2[2,7] <- traffic[2,7] + outgoing2

P_traffic2 <- matrix(nrow=9,ncol=9)
for (i in seq(1,9,1)) {
  for (j in seq(1,9,1)){
    P_traffic2[i,j] <- (traffic_2[i,j] / sum(traffic_2[i,]))
  }
}

Q_traffic2 <- t(P_traffic2) - diag(9)
Q_traffic2[9,] = c(rep(1,9))
rhs_3 <- c(rep(0,8),1)
steady_traffic2 <- solve(Q_traffic2) %*% rhs_3
print(steady_traffic2)
```

```
##             [,1]
##  [1,] 0.16162840
##  [2,] 0.10034341
##  [3,] 0.12104331
##  [4,] 0.12275720
```

```
##  [5,] 0.08164613
##  [6,] 0.08250884
##  [7,] 0.06003218
##  [8,] 0.10841213
##  [9,] 0.16162840
```

```r
print(var(steady_traffic))
```

```
##             [,1]
## [1,] 0.001410675
```

```r
print(var(steady_traffic2))
```

```
##             [,1]
## [1,] 0.001219604
```

The variance of the steady state for the first distribution is higher, since $0.0014 > 0.0012$. Therefore, the new links helped the website traffic by lowering the variance.