

WEEK 6

CONTRAPOSITIVE & CONTRADICTION

February 17, 2021

1. Use a proof by contrapositive to show that if n^2 is even, then n is even.
2. Let $x, y \in \mathbb{R}$. Use a proof by contrapositive to show that if $y^3 + yx^2 \leq x^3 + xy^2$, then $y \leq x$.
3. Use a proof by contradiction to show that $\sqrt{3}$ is irrational. You may use the fact that for any integer n , if n^2 is divisible by 3, then n is divisible by 3.
4. Let $a, b \in \mathbb{R}$, where a is rational and b is irrational. Use a proof by contradiction to show that ab is irrational.
5. Give an example of a statement (doesn't have to be a mathematical statement) of the form $P \Rightarrow Q$ that is true, but the converse is false.
6. Let $a, b, c \in \mathbb{Z}$. Prove that if $a^2 + b^2 = c^2$, then abc is even. (You should decide whether you want to use a direct proof, contrapositive, or contradiction)
7. Let x and y be positive real numbers. Prove that $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$. (You should decide whether you want to use a direct proof, contrapositive, or contradiction)
8. Suppose that r_1 and r_2 are two different roots of the quadratic function $f(x) = x^2 + bx + c$ (recall that r is a root of a polynomial $p(x)$ if $p(r) = 0$). Prove that $r_1 + r_2 = -b$ and $r_1 r_2 = c$. (You should decide whether you want to use a direct proof, contrapositive, or contradiction)
9. **(Challenge)** You've been selected to take part in a new game show. The first game involves you and two other players. Each player has either a red hat or a blue hat placed on their head. There are no mirrors, so you can't see the color of your own hat, but each player can see the other players' hats. Once the hats are placed, each player is supposed to raise their hand if they see at least one red hat. The first player to correctly guess their color wins. Guessing is not allowed, so players must justify their guess in order to win.

When the hats are placed, you see two red hats, so you raise your hand. The other players also both raise their hands. After a few minutes, no one has made any guesses. What color should you guess for your own hat, and what justification should you give?