ESP Math 179

Worksheet 15

More Extreme Values

5 March 2020

Spring 2020

For reference, we list the four steps of George Polya's problem solving process.

- Understand the problem
- Devise a plan
- Carry out the plan
- Look back

1. (**Review**) A thin sheet of ice is in the shape of a circle. Suppose it is melting in this warm weather in a way so that the area is decreasing at a rate of 0.5 m^2/sec. What is the rate at which the radius is decreasing when the are of the sheet is 12 m^2?

2. (**Warm-up**) For the function $f(x) = x^2$, what are the intervals of increase/decrease? What are the critical point(s)?

3. Find the intervals of increase/decrease and absolute maximum/minimum values of $f(x) = x^3 + 3x^2 - 24x + 5$ on the interval $[-4, 4]$. Use the first derivative test to find any local maxima and minima.

4. Let a, b, c, d be fixed real numbers. Find the value of x that minimizes

   (a) $f(x) = (x-a)^2 + (x-b)^2$

   (b) $f(x) = (x-a)^2 + (x-b)^2 + (x-c)^2$

   (c) $f(x) = (x-a)^2 + (x-b)^2 + (x-c)^2 + (x-d)^2$

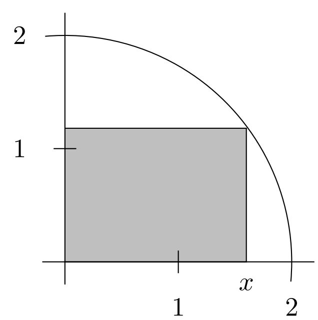
(**!**) (d) Do you see a pattern in the above? If $c_1, c_2, \ldots, c_n$ are fixed real numbers, what do you think is the value of x that minimizes

$$f(x) = (x-c_1)^2 + (x-c_2)^2 + \cdots + (x-c_n)^2$$

5. The vertex of a parabola is the highest or lowest point on the parabola (depending on which way the parabola opens). In pre-calc, you learned that the x-value of the vertex of the parabola $f(x) = ax^2 + bx + c$ is $x = -\dfrac{b}{2a}$. Using what we've learned in calculus, how can we justify this?

6. Suppose you have 90,000 pairs of special limited edition sneakers that you're trying to sell. After doing some market research, you find that if you set the price for each pair at d dollars, the number of sneakers you'll be able to sell is $90,000 - d^2$. What price should you sell the sneakers at in order to maximize your revenue?

(**!**) 7. A rectangle has its lower left corner at $(0, 0)$ and its upper right corner on the circle of radius 2 centered at $(0, 0)$, as in the picture below.



Find the value of x between 0 and 2 that maximizes the area of the rectangle.