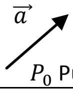
FORMULARIO DE CÁLCULO II

KAIZEN SOFTWARE

GEOMETRÍA ANALÍTICA EN EL ESPACIO

VECTOR $\vec{a} = (x_a, y_a, z_a)$



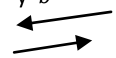
P_1 Punto final

$\vec{a} = P_1 - P_0$

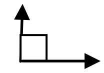
P_0 Punto inicial

Norma (magnitud, módulo) $||\vec{a}|| = \sqrt{{x_a}^2 + {y_a}^2 + {z_a}^2}$

Vectores paralelos $\vec{a}$ y $\vec{b}$: $\vec{a} = k\,\vec{b}$, $k \in R$



Vectores ortogonales $\vec{a}$ y $\vec{c}$: $\vec{a} \circ \vec{c} = 0$



Producto escalar

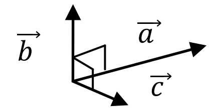
$\vec{a} = (x_a, y_a, z_a)$

$\vec{c} = (x_c, y_c, z_c)$

$\vec{a} \circ \vec{c} = x_a x_c + y_a y_c + z_a z_c$

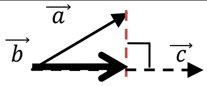
Producto vectorial $\vec{a} = (x_a, y_a, z_a)$ $\quad \vec{c} = (x_c, y_c, z_c)$

$$\vec{b} = \vec{a} \times \vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_a & y_a & z_a \\ x_c & y_c & z_c \end{vmatrix} = \left(\begin{vmatrix} y_a & z_a \\ y_c & z_c \end{vmatrix}, -\begin{vmatrix} x_a & z_a \\ x_c & z_c \end{vmatrix}, \begin{vmatrix} x_a & y_a \\ x_c & y_c \end{vmatrix} \right)$$



Proyección ortogonal

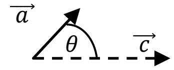
$$\vec{b} = Proy_{\vec{c}}\,\vec{a} = \left(\frac{\vec{a} \circ \vec{c}}{||\vec{c}||^2}\right)\vec{c}$$



Ángulo entre dos vectores

$\vec{a} \circ \vec{c} = \|\vec{a}\|\|\vec{c}\| \cos\theta$

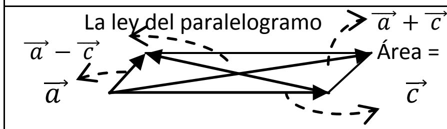
$\|\vec{a} \times \vec{c}\| = \|\vec{a}\|\|\vec{c}\| \sin\theta$



Vector unitario de $\vec{a}$

$$\vec{u} = \frac{1}{\|\vec{a}\|}\vec{a}$$

La ley del paralelogramo



$\vec{a} + \vec{c}$; $\vec{a} - \vec{c}$; Área = $\|\vec{a} \times \vec{c}\|$

Vector bisectriz entre $\vec{a}$ y $\vec{c}$: $\vec{b} = \frac{1}{\|\vec{a}\|}\vec{a} + \frac{1}{\|\vec{c}\|}\vec{c} = \frac{\|\vec{c}\|\,\vec{a} + \|\vec{a}\|\,\vec{c}}{\|\vec{a}\|\|\vec{c}\|}$

Volumen paralelepípedo $V = \left|\vec{a} \circ (\vec{b} \times \vec{c})\right|$; **Volumen tetraedro** $V = \frac{1}{6}\left|\vec{a} \circ (\vec{b} \times \vec{c})\right|$

FAMILIA DE PLANOS (HAZ DE PLANOS) $\alpha(ax + by + cz + d) + \beta(px + qy + rz + s) = 0$

LA RECTA Punto $P_0(x_0, y_0, z_0)$; Vector dirección $\vec{V}\ (v_1, v_2, v_3)$

$$R: P_0 + \vec{V}t \qquad \begin{aligned} x &= x_0 + v_1 t \\ y &= y_0 + v_2 t \\ y &= y_0 + v_3 t \end{aligned} \qquad \frac{x - x_0}{v_1} = \frac{y - y_0}{v_2} = \frac{z - z_0}{v_3}$$

EL PLANO Punto del plano $P_0 = (x_0, y_0, z_0)$ Vector normal $\vec{n} = (a, b, c)$

$$(P - P_0)\circ\vec{n} = 0 \qquad a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$ax + by + cz + d = 0$$

DISTANCIA PUNTO – RECTA

Punto $P_e = (x_e, y_e, z_e)$ Recta $R: P_0 + \vec{V}t$

$$D = \frac{\|\vec{V} \times (P_e - P_0)\|}{\|\vec{V}\|} = \left\| (P_e - P_0) - \left(\frac{(P_e - P_0)\circ\vec{V}}{\|\vec{V}\|^2}\right)\vec{V} \right\|$$

DISTANCIA PUNTO – PLANO

Punto $P_e = (x_e, y_e, z_e)$ Plano $(P - P_0)\circ\vec{n} = 0$

$ax + by + cz + d = 0$

$$D = \frac{|\vec{n} \circ (P_e - P_0)|}{\|\vec{n}\|} = \frac{|ax_e + by_e + cz_e + d|}{\sqrt{a^2 + b^2 + c^2}}$$

DISTANCIA ENTRE DOS RECTAS (Alabeadas)

NO PARALELAS QUE NO SE CORTAN

Rectas $R_1: P_1 + \vec{a}\, t_1 \qquad R_2: P_2 + \vec{b}\, t_2$

$$D = \frac{|(\vec{a} \times \vec{b}) \circ (P_1 - P_2)|}{\|\vec{a} \times \vec{b}\|}$$

SUPERFICIES

ESFERA $(x - h)^2 + (y - k)^2 + (z - m)^2 = R^2$ Centro $C = (h, k, m)$ Radio R

COMPLETAR CUADRADOS $x^2 \pm a\,x = \left(x \pm \frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2$

SUPERFICIES CUADRÁTICAS

Elipsóide $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} + \frac{(z-m)^2}{c^2} = 1$	Hiperboloide de dos hojas $-\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} + \frac{(z-m)^2}{c^2} = 1$
Cono recto $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = \frac{(z-m)^2}{c^2}$	Paraboloide elíptico $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = c(z - m)$
Hiperboloide de una hoja $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} - \frac{(z-m)^2}{c^2} = 1$	Paraboloide hiperbólico $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = c(z - m)$

FUNCIONES CURVILÍNEAS $f(t) = \overrightarrow{r(t)}$ $\quad$ Longitud de curva $L = \int_{t_1}^{t_2} ||f'||dt$ $\quad$ Tangente $\vec{T} = \frac{\vec{r'}}{\|\vec{r'}\|}$ $\quad$ Binormal $\vec{B} = \frac{\vec{r'} \times \vec{r''}}{\|\vec{r'} \times \vec{r''}\|}$

Curvatura $k = \frac{\|\vec{r'} \times \vec{r''}\|}{\|\vec{r'}\|^3}$ $\quad$ Radio de curvatura $\rho = \frac{1}{k}$ $\quad$ Normal $\vec{N} = \frac{(\vec{r'} \times \vec{r''}) \times \vec{r'}}{\|(\vec{r'} \times \vec{r''}) \times \vec{r'}\|}$ $\quad$ Torsión $\tau = \frac{(\vec{r'} \times \vec{r''}) \circ \vec{r'''}}{\|\vec{r'} \times \vec{r''}\|^2}$

FUNCIONES DE VARIAS VARIABLES

LIMITES ITERADOS $\lim_{x \to a}\left(\lim_{y \to b} f(x, y)\right) = \lim_{y \to b}\left(\lim_{x \to a} f(x, y)\right)$

Si los límites iterados son $\neq$ entonces no existe el límite en el punto (a, b)

PRIMERA DERIVADA DE $F\ (x, y, z)$ $\qquad F' = \vec{\nabla F} = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right)$

SEGUNDA DERIVADA (MATRIZ HESSIANA) DE $F\ (x, y)$ **Y DE** $F\ (x, y, z)$

$$F'' = H = \begin{pmatrix} \frac{\partial^2 F}{\partial x^2} & \frac{\partial^2 F}{\partial x \partial y} \\ \frac{\partial^2 F}{\partial y \partial x} & \frac{\partial^2 F}{\partial y^2} \end{pmatrix} \quad ; \quad F'' = H = \begin{pmatrix} \frac{\partial^2 F}{\partial x^2} & \frac{\partial^2 F}{\partial x \partial y} & \frac{\partial^2 F}{\partial x \partial z} \\ \frac{\partial^2 F}{\partial y \partial x} & \frac{\partial^2 F}{\partial y^2} & \frac{\partial^2 F}{\partial y \partial z} \\ \frac{\partial^2 F}{\partial z \partial x} & \frac{\partial^2 F}{\partial z \partial y} & \frac{\partial^2 F}{\partial z^2} \end{pmatrix}$$

DIFERENCIAL DE $F\ (x, y, z)$

$$dF = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right)\circ(dx, dy, dz) = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz$$

2da DIFERENCIAL DE $F\ (x, y, z)$ $\qquad d^2F = (dx, dy, dz) \cdot H \cdot \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}$

DERIVADA IMPLÍCITA

$F\ (x, y) = 0,\ \ x\ \ indep.;\ y\ dependiente \qquad \frac{\partial y}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$

$F\ (x, y, z) = 0,\ \ x, y\ \ indep.; z\ dependiente \qquad \frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} \quad , \quad \frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$

$F\ (x, y, u, v) = 0$

$G\ (x, y, u, v) = 0$ $\quad x, y\ \ independientes;\ u, v\ dependientes$

$$\frac{\partial u}{\partial x} = -\frac{J\left(\frac{F,G}{x,v}\right)}{J\left(\frac{F,G}{u,v}\right)} = -\frac{\begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial v} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial v} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial v} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial v} \end{vmatrix}} \ ;\ \frac{\partial u}{\partial y} = -\frac{J\left(\frac{F,G}{y,v}\right)}{J\left(\frac{F,G}{u,v}\right)} = -\frac{\begin{vmatrix} \frac{\partial F}{\partial y} & \frac{\partial F}{\partial v} \\ \frac{\partial G}{\partial y} & \frac{\partial G}{\partial v} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial v} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial v} \end{vmatrix}} \ ;\ \frac{\partial v}{\partial x} = -\frac{J\left(\frac{F,G}{u,x}\right)}{J\left(\frac{F,G}{u,v}\right)} = -\frac{\begin{vmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial x} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial x} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial v} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial v} \end{vmatrix}} \ ;\ \frac{\partial v}{\partial y} = -\frac{J\left(\frac{F,G}{u,y}\right)}{J\left(\frac{F,G}{u,v}\right)} = -\frac{\begin{vmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial y} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial v} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial v} \end{vmatrix}}$$

REGLA DE LA CADENA $D(F\circ G)=D\,F(G)\cdot DG$

Desarrollando tenemos $F(u,v)$; $u(x,y)$; $v(x,y)$

$\frac{\partial F}{\partial x}=\frac{\partial F}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial F}{\partial v}\frac{\partial v}{\partial x}$; $\frac{\partial F}{\partial y}=\frac{\partial F}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial F}{\partial v}\frac{\partial v}{\partial y}$

DERIVADAS PARCIALES DE $F(x,y)$

$\frac{\partial F}{\partial x}=\lim_{h\to 0}\frac{F(x+h,y)-F(x,y)}{h}$

$\frac{\partial F}{\partial y}=\lim_{h\to 0}\frac{F(x,y+h)-F(x,y)}{h}$

DERIVADA DIRECCIONAL ($\vec{u}$ debe ser vector unitario)

Por definición $D_{\vec{u}}F(x,y)=\lim_{h\to 0}\frac{F((x,y)+h\cdot\vec{u})-F(x,y)}{h}$

Por cálculo directo $D_{\vec{u}}F(x,y,z)=\overrightarrow{\nabla F}\cdot\vec{u}$

SIGNIFICADO DE LA DERIVADA (donde $h\approx 0, k\approx 0$) $F(x+h,y+k)-F(x,y)\cong\left(\frac{\partial F}{\partial x},\frac{\partial F}{\partial y}\right)(h,k)$

$F(x+h,y)-F(x,y)\cong\frac{\partial F}{\partial x}h$ $\quad F(x,y+k)-F(x,y)\cong\frac{\partial F}{\partial y}k$ $\quad F\big((x,y)+h\vec{u}\big)-F(x,y)\cong D_{\vec{u}}F(x,y)\ h$ (usando derivada direccional)

CRITERIO PARA HALLAR MÁXIMOS Y MÍNIMOS PARA FUNCIONES DE 2 VARIABLES

$\Delta_1=\frac{\partial^2F}{\partial x^2}$; $\Delta_2=\det(H)=det\begin{pmatrix}\frac{\partial^2F}{\partial x^2} & \frac{\partial^2F}{\partial x\partial y}\\ \frac{\partial^2F}{\partial y\partial x} & \frac{\partial^2F}{\partial y^2}\end{pmatrix}$

$Si\ \ \Delta_1>0\ \ y\ \Delta_2>0\ , Existe\ mínimo\ local$

$Si\ \ \Delta_1<0\ \ y\ \Delta_2>0\ , Existe\ máximo\ local$

$Si\ \ \Delta_2<0\ , Existe\ un\ punto\ silla$

$Si\ \ \Delta_2=0$, El criterio no da ninguna información

CRITERIO PARA HALLAR MÁXIMOS Y MÍNIMOS PARA FUNCIONES DE 3 VARIABLES

$\Delta_1=\frac{\partial^2F}{\partial x^2}$; $\Delta_2=det\begin{pmatrix}\frac{\partial^2F}{\partial x^2} & \frac{\partial^2F}{\partial x\partial y}\\ \frac{\partial^2F}{\partial y\partial x} & \frac{\partial^2F}{\partial y^2}\end{pmatrix}$; $\Delta_3=\det(H)=det\begin{pmatrix}\frac{\partial^2F}{\partial x^2} & \frac{\partial^2F}{\partial x\partial y} & \frac{\partial^2F}{\partial x\partial z}\\ \frac{\partial^2F}{\partial y\partial x} & \frac{\partial^2F}{\partial y^2} & \frac{\partial^2F}{\partial y\partial z}\\ \frac{\partial^2F}{\partial z\partial x} & \frac{\partial^2F}{\partial z\partial y} & \frac{\partial^2F}{\partial z^2}\end{pmatrix}$

$Si\ \ \Delta_1>0\ \ ,\Delta_2>0\ y\ \Delta_3>0\ , Existe\ mínimo\ local$

$Si\ \ \Delta_1<0\ \ ,\Delta_2>0\ y\ \Delta_3<0\ , Existe\ máximo\ local$

MÁXIMOS Y MÍNIMOS CONDICIONADOS (MULTIPLICADORES DE LAGRANGE) Función: $F(x,y)$ Condición: $g(x,y)=0$; $\overrightarrow{\nabla F}=\boldsymbol{\lambda}\overrightarrow{\nabla g}$

INTEGRALES MÚLTIPLES

ÁREA $A=\iint dydx=\iint dxdy$ $\quad$ MASA: $m=\iint\delta(x,y)dydx$ $\quad$ DENSIDAD MEDIA: $\bar{\delta}=\frac{masa}{area}=\frac{m}{A}$ $\quad$ TEOREMA DE PAPPUS: $V=2\pi\,\bar{y}\,A$

CENTRO DE MASAS (CENTROIDE, CENTRO DE GRAVEDAD)

$\bar{x}=\frac{\iint x\,\delta\,dydx}{\iint\delta\,dydx}$, $\bar{y}=\frac{\iint y\delta\,dydx}{\iint\delta\,dydx}$

INERCIAS

$I_x=\iint y^2\delta\,dydx,$

$I_y=\iint x^2\delta\,dydx,$

$I_o=I_x+I_y$

VOLUMEN DE REVOLUCIÓN

$Alrededor\ del\ eje\ x\quad V=2\pi\iint y\,dydx$

$Alrededor\ del\ eje\ y\quad V=2\pi\iint x\,dydx$

Alrededor de la recta $y=b$ $\quad V=2\pi\iint|y-b|\,dydx$

Alrededor de la recta $x=a$ $\quad V=2\pi\iint|x-a|\,dydx$

VOLUMEN

$V=\iint(z_s-z_i)dydx=\iiint dzdydx$

MASA

$m=\iiint\delta(x,y,z)\,dzdydx$

CENTRO DE MASAS (CENTROIDE, CENTRO DE GRAVEDAD)

$\bar{x}=\frac{\iiint x\,\delta\,dzdydx}{\iiint\delta\,dzdydx},\quad\bar{y}=\frac{\iiint y\,\delta\,dzdydx}{\iiint\delta\,dzdydx},\quad\bar{z}=\frac{\iiint z\,\delta\,dzdydx}{\iiint\delta\,dzdydx}$

INERCIAS CON LOS PLANOS COORDENADOS $I_{YZ}=\iiint x^2\,\delta\,dV,\ I_{XZ}=\iiint y^2\,\delta\,dV,\ I_{XY}=\iiint z^2\,\delta\,dV$

INERCIA POLAR $I_O=\iiint(x^2+y^2+z^2)\,\delta\,dV$

INERCIAS CON LOS EJES COORDENADOS

$I_Z=\iiint(x^2+y^2)\,\delta\,dV\ ,I_Y=\iiint(x^2+z^2)\,\delta\,dV\ ,\ I_X=\iiint(y^2+z^2)\,\delta\,dV\ \ ,I_O=(I_X+I_Y+I_Z)/2$

AREAS DE SUPERFICIES

$A=\iint\sqrt{1+(\frac{\partial z}{\partial x})^2+(\frac{\partial z}{\partial y})^2}dydx$

COORDENADAS POLARES

$x=r\cos\theta$; $x^2+y^2=r^2$

$y=r\sin\theta$; $\theta=\tan^{-1}\left(\frac{y}{x}\right)$;

$J\left(\frac{x,y}{r,\theta}\right)=r$

COORDENADAS CILÍNDRICAS

$x=r\cos\theta$ $\quad x^2+y^2=r^2$

$y=r\sin\theta$; $\theta=\tan^{-1}\left(\frac{y}{x}\right)$; $J\left(\frac{x,y,z}{r,\theta,z}\right)=r$

$z=z$ $\quad z=z$

COORDENADAS ESFÉRICAS

$x=r\sin\emptyset\cos\theta$ $\quad x^2+y^2+z^2=r^2$

$y=r\sin\emptyset\sin\theta$; $\theta=\tan^{-1}(\frac{y}{x})$; $J\left(\frac{x,y,z}{r,\emptyset,\theta}\right)=r^2\sin\emptyset$

$z=r\cos\emptyset$ $\quad\emptyset=\tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

INTEGRALES DE LINEA Y SUPERFICIES $\int F(x,y)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\int F(x,y)\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy=\int F(x,y)\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

$\int P(x,y)dx+Q(x,y)\frac{dy}{dx}dx=\int P(x,y)\frac{dx}{dy}dy+Q(x,y)dy=\int P(x,y)\frac{dx}{dt}dt+Q(x,y)\frac{dy}{dt}dt$

TEOREMA DE GREEN $\oint P(x,y)dx+Q(x,y)dy=\iint\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dydx$

AREAS POR INTEGRALES DE LINEA $A=\frac{1}{2}\oint x\,dy-y\,dx$

TEOREMA DE LA DIVERGENCIA (GAUSS) $\oiint\vec{A}\circ\vec{n}ds=\iiint\vec{\nabla}\circ\vec{A}\,dV$

TEOREMA DE STOKES $\oint\vec{A}.d\vec{r}=\iint(\vec{\nabla}\times\vec{A})\circ\vec{n}\,ds$ $\quad ds=\frac{dy\,dx}{|\vec{n}\circ\vec{k}|}$

SERIE P

$S_a=\sum_{n=1}^{\infty}\frac{1}{n^P}$ Si: $P>1\ \ S_a\ \ es\ Cv$; $P\le 1\ \ S_a\ \ es\ Dv$

SERIE GEOMÉTRICA

$S_a=\sum_{n=1}^{\infty}a\,r^{n-1}$ Si: $r<1\ \ S_a\ \ es\ Cv$; $r\ge 1\ \ S_a\ \ es\ Dv$

Si es Cv la suma se halla con:

$\sum_{n=1}^{\infty}r^{n-1}=\frac{1}{1-r},\quad\sum_{n=1}^{\infty}r^n=\frac{r}{1-r},\quad\sum_{n=0}^{\infty}r^n=\frac{1}{1-r}$

CRITERIOS DE CONVERGENCIA

Criterio de comparación

Si: $a_n\le b_n\ \ si\ \ S_b\ \ es\ Cv, S_a\ \ es\ Cv$

$a_n\ge b_n\ \ si\ \ S_b\ \ es\ Dv, S_a\ \ es\ Dv$

Criterio de límite de comparación

$\lim_{n\to\infty}\frac{a_n}{b_n}=k,$ $\quad k\,\epsilon\,R\ Ambas\ son\ Cv\ o\ Dv$; $k=0\ \ si\ \ S_b\ \ es\ Cv, S_a\ \ es\ Cv$; $k=\infty\ si\ \ S_b\ \ es\ Dv, S_a\ \ es\ Dv$

Criterio del cociente

$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=k,$ $\quad k<1\ \ S_a\ \ es\ Cv$; $k=1\ \ falla$; $k>1\ \ S_a\ \ es\ Dv$

Criterio de la raíz

$\lim_{n\to\infty}\sqrt[n]{a_n}=k,$ $\quad k<1\ \ S_a\ \ es\ Cv$; $k=1\ \ falla$; $k>1\ \ S_a\ \ es\ Dv$

Criterio de Raabe

$\lim_{n\to\infty}n(1-\frac{a_{n+1}}{a_n})=k,$ $\quad k>1\ \ S_a\ \ es\ Cv$; $k=1\ \ falla$; $k<1\ \ S_a\ \ es\ Dv$

Criterio de la integral

$\lim_{m\to\infty}\int_1^m a(x)\,dx=k$ $\quad k\,\epsilon\,R\ \ S_a\ \ es\ Cv$; $k=\infty\ \ S_a\ \ es\ Dv$

SERIE DE TÉRMINOS ALTERNOS

La serie alterna $\sum a_n$ es Cv si:

$|a_{n+1}|<|a_n|\ la\ serie\ es\ decreciente$

$\lim_{n\to\infty}a_n=0\ \ el\ ultimo\ término\ es\ 0$

Si: $\sum a_n$ es Cv y $\sum|a_n|$ es Cv ; $\sum a_n$ $es\ absolutamente\ Cv$

Si: $\sum a_n$ es Cv y $\sum|a_n|$ es Dv ; $\sum a_n\ es\ condicionalmente\ Cv$

SERIE DE POTENCIAS Para determinar el intervalo de convergencia $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|<1$

Serie de Taylor ($x=a$) $F(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\ldots+\frac{f^{(n)}(a)}{n!}(x-a)^n$

Serie de Mc-Laurin $F(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\ldots+\frac{f^{(n)}(0)}{n!}x^n$