Nonlocal Centralized Sparsity and Rank Minimization for Image Super Resolution

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Proyecto Final - Modelado Matematico

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Sparsity Model

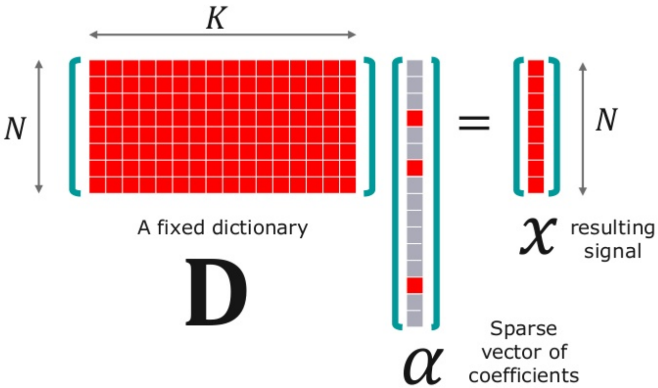


Figure: A signal **X** can be represented sparsely by a vector α on a representation basis **D**

Nonlocal similarities

$$\boldsymbol{\beta}_i = \sum_{p \in \boldsymbol{\Omega}_i} \omega_p \boldsymbol{\alpha}_p, \quad \omega_p = \frac{1}{\varphi} exp\left(\frac{-\|\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_p\|_2^2}{h}\right) \tag{1}$$

Dictionary learning

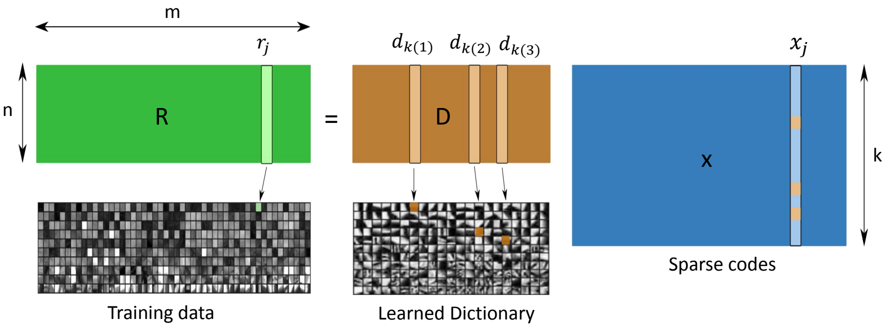


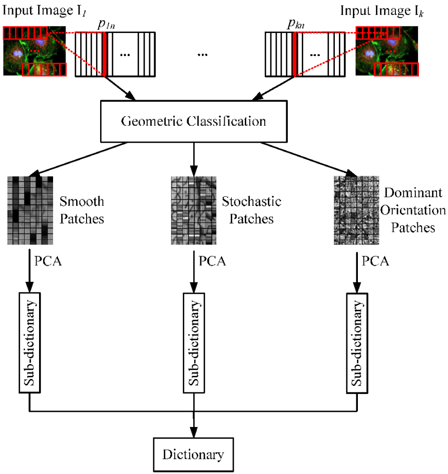
Figure: Dictionary learning by patches

$$\hat{\mathbf{D}}, \hat{\mathbf{X}} \in \underset{\mathbf{D}\in\mathbb{R}^{n\times k},\ \mathbf{X}\in\mathbb{R}^{k\times m}}{\operatorname{argmin}} \frac{1}{2} \| \mathbf{R} - \mathbf{DX} \|_F^2 + \tau \| \mathbf{X} \|_1 \tag{2}$$

where $\| \mathbf{X} \|_1 = \max_{1\leq j\leq m} \sum_{i=1}^{k} | x_{i,j} |$

Dictionary learning by PCA

A simple test on the Columbia multispectral image dataset reveals that 10,000 random 7 7 spatial patches require only 6 principal components to capture 99% of the variance. We also train the spatial bases using PCA from monochrome patches.



Proposed

State-of-art:

$$\boldsymbol{\alpha}_y \in \underset{\boldsymbol{\alpha}}{\operatorname{argmin}} \frac{1}{2} \| \mathbf{y} - \mathbf{H\Phi} \circ \boldsymbol{\alpha} \|_2^2 + \lambda \sum_i \| \boldsymbol{\alpha}_i - \boldsymbol{\beta}_i \|_1 \tag{3}$$

Proposed:

$$\begin{aligned} \boldsymbol{\alpha}_y \in \underset{\boldsymbol{\alpha}}{\operatorname{argmin}} \frac{1}{2} \| \mathbf{y} - \mathbf{H\Phi} \circ \boldsymbol{\alpha} \|_2^2 + \lambda \sum_i \| \boldsymbol{\alpha}_i - \boldsymbol{\beta}_i \|_1 \\ \rho \sum_i \| \mathbf{Z}_i \|_{w,*} \end{aligned} \tag{4}$$

where $\mathbf{x} = \mathbf{\Phi} \circ \boldsymbol{\alpha}$ and $\mathbf{Z}_i$ is the set of similarities patches to $\mathbf{x}_i$, $\mathbf{Z}_i = [\mathbf{z}_{i,1}, \mathbf{z}_{i,2}, ..., \mathbf{z}_{i,1}]$.

State-of-art | Proposed | Original image

PSNR = 29.9
SSIM = 0.941

PSNR = 31.0
SSIM = 0.948

PSNR = 26.8
SSIM = 0.872

PSNR = 27.5
SSIM = 0.886

PSNR = 31.0
SSIM = 0.951

PSNR = 32.2
SSIM = 0.965

The End