STAT40720 Intro. to Data Analytics

Assignment 3

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Question 1

(a) Mean Bulb Lifetime longer than 1,000 hours

H_0: $\mu = 1000$ hours
H_A: $\mu > 1000$ hours

This is an upper-tailed test. If the test statistic lies in the rejection region, i.e. $z > z_{CRITICAL}$ value, the null hypothesis should be rejected. There is then sufficient evidence to state the mean lifetime of the bulbs is greater than 1000 hours.

(b) Mean Male/Female Population Proportions of Gym Attendees equal

H_0: $\hat{p}_{MALE} - \hat{p}_{FEMALE} = 0$
H_A: $\hat{p}_{MALE} - \hat{p}_{FEMALE} \neq 0$

…where $\hat{p}_{MALE}$ and $\hat{p}_{FEMALE}$ are the proportions of male and female students respectively who attend the gym 4 or more times per week.

This is a two-tailed test. If the test statistic lies in the rejection regions, i.e. $z > z_{CRITICAL}$ or $z < -z_{CRITICAL}$ values, the null hypothesis should be rejected. There is then sufficient evidence to state the proportion of students that attend the gym 4 or more times per week is different from males to females.

Question 2

(a) Hypothesis Construction

If the restaurant want to test if the entire batch is spoiled, they should conduct a hypothesis test with the following Null and Alternate hypotheses:

H_0: $\hat{p}_{\text{SPOILED}} < 1$
H_A: $\hat{p}_{\text{SPOILED}} = 1$

…where $\hat{p}_{\text{SPOILED}}$ is the proportion of chicken packages that are spoiled.

However it is more likely the chicken is mostly unspoiled, and the restaurant wants to randomly sample for spoiled chicken. In this case, the following hypotheses should be adopted:

H_0: $\hat{p}_{\text{SPOILED}} > 0$
H_A: $\hat{p}_{\text{SPOILED}} = 0$

…where $\hat{p}_{\text{SPOILED}}$ is the proportion of chicken packages that are spoiled.

Here, the Null Hypothesis (H_0) assumes the true proportion of spoiled chicken is greater than zero. After randomly sampling the chicken for spoiled packages, if there is not sufficient evidence that the true spoiled proportion is greater than zero, the Null Hypothesis should be rejected, and accept the Alternate Hypothesis, i.e. the entire batch is safe.

(b) Type I & Type II Error

Proceeding with the more likely scenario above, i.e.:

H_0: $\hat{p}_{\text{SPOILED}} > 0$
H_A: $\hat{p}_{\text{SPOILED}} = 0$

A Type I error refers to the incorrect rejection of the Null Hypothesis, and acceptance of the Alternative Hypothesis, when the Null Hypothesis is, in fact, true. In this scenario, it refers to the assumption that the entire batch of chicken is safe, when in fact, there is a spoiled package.

A Type II error refers to the failure to reject a false Null Hypothesis when the Alternative Hypothesis is true. In the context of this question, it is a failure to reject the assumption that there is a proportion of the population that is spoiled, when in fact, the entire population is not spoiled, and safe to eat.

Question 3

(a) 95% Confidence Interval

The $z_{CRITICAL}$ value for a 95% confidence interval is 1.96.

$$
\begin{aligned}
95\%CI &= \bar{x} \pm 1.96 \times \frac{s}{\sqrt{n}} \\
&= 45 \pm 1.96 \times \frac{5.39}{\sqrt{31}} \\
&= 45 \pm 1.89742 \\
&= (43.1026, 46.8974)
\end{aligned}
$$

(b) Hypothesis Test

The following hypotheses should be used:

H_0: μ = 40 packets
H_A: μ > 40 packets

This is an upper-tailed test.

$$
\begin{aligned}
z &= \frac{\bar{x} - a}{s/\sqrt{n}} \\
&= \frac{45 - 40}{5.39/\sqrt{31}} \\
&= 5.1649
\end{aligned}
$$

The $z_{CRITICAL}$ value for a significance $\alpha = 0.05$ is 1.6449. The z-statistic is greater than the $z_{CRITICAL}$ value, therefore the Null Hypothesis should be rejected. There is sufficient evidence to state the mean quantity of packets sold each week is greater than 40.

(c) Profit Assurance

The P-value (obtained from Table 9, NCST) associated with a $z_{CRITICAL}$ value of 5.1649, with 30 degrees of freedom is greater than 0.9999. The tables stop listing values above $z_{CRITICAL}$ = 4.4. Therefore, the pharmaceutical company can be more than 99.99% certain the mean sales in the Kilkenny pharmacy are greater than 40 per week, and equally that average profits will exceed €400.