CS 206 Homework 4

Fall 2020

1. Let's see what happens when the Monty Hall game show is played with four doors. A prize is hidden behind one of the four doors. Then the contestant picks a door. Next, the host opens an unpicked door that has no prize behind it. The contestant is allowed to stick with their original door or to switch to one of the two unopened, unpicked doors. The contestant wins if their final choice is the door hiding the prize.

   Let's make the same assumptions as in the original problem:

   - The prize is equally likely to be behind each door.
   - The contestant is equally likely to pick each door initially, regardless of the prize's location.
   - The host is equally likely to reveal each door that does not conceal the prize and was not selected by the player.

   Find the following probabilities. If the tree diagram is too large, you can draw just enough of it for the structure to be clear.

   (a) Contestant Stu stays with his original door. What is the probability that Stu wins the prize?

   (b) Contestant Zelda switches to one of the remaining two doors with equal probability. What is the probability that Zelda wins the prize?

   (c) Now let's revise our assumptions about how contestants choose doors. Say the doors are labeled A, B, C, and D. Suppose that the host always opens the earliest door possible (the door whose label is earliest in the alphabet) with the restriction that the host can neither reveal the prize nor open the door that the player picked. This gives contestant Priscilla just a little more information about the location of the prize. Suppose that Priscilla always switches to the earliest door, excluding her initial pick and the one the host opened.

What is the probability that Priscilla wins the prize?

2. We play a game with a deck of 52 regular playing cards, of which 26 are red and 26 are black. They're randomly shuffled and placed face down on a table. You have the option of "taking" or "skipping" the top card. If you skip the top card, then that card is revealed and we continue playing with the remaining deck. If you take the top card, then the game ends; you win if the card you took was revealed to be black, and you lose if it was red. If we get to a point where there is only one card left in the deck, you must take it. Prove that you have no better strategy than to take the top card – which means your probability of winning is 1/2.

   Hint: Prove by induction the more general claim that for a randomly shuffled deck of n cards that are red or black – not necessarily with the same number of red cards and black cards – there is no better strategy than taking the top card.

3. Suppose we have three cards. One is colored red on both sides, one is black on both sides, and one is red on one side and black on the other. We place these three cards in a hat and shuffle them. We draw one card and place in on the table, so only one side is visible. If that side is red, what is the probability that it's the red-black card?

4. There are 3 coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. When one of the 3 coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

5. The final quiz in this course will be graded according to the following procedure:

   With probability 4/7 the exam is graded by a TA, with probability 2/7 it is graded by the instructor, and with probability 1/7, it is arbitrarily given a score of 84.

   TAs score an exam by scoring each problem individually and then taking the sum.

   - There are ten true/false questions worth 2 points each. For each, full credit is given with probability 3/4, and no credit is given with probability 1/4.
   - There are four questions worth 15 points each. For each, the score is determined by rolling two fair dice, summing the results, and adding 3.
   - The single 20 point question is awarded either 12 or 18 points with equal probability.

   The instructor scores an exam by rolling a fair die twice, multiplying the results, and then adding a "general impression" score.

   - With probability 4/10, the general impression score is 40.
   - With probability 3/10, the general impression score is 50.
   - With probability 3/10, the general impression score is 60.

   Assume all random choices during the grading process are independent.

   (a) What is the expected score on an exam graded by a TA?

   (b) What is the expected score on an exam graded by the instructor?

   (c) What is the expected score on a final exam?