Cardinality and counting

CS 206: Discrete Structures II

Fall 2020

Functions

- injective
- surjective
- bijective

Sizes of the domain and range

- injective: $|A| \leq |B|$
- surjective: $|A| \geq |B|$
- bijective: $|A| = |B|$

Relations

$$R \subseteq A \times B$$

Function notation

$f : A \to B$ or B^A

Number of functions

$f : B^A$ is $|B|^{|A|}$

Powerset

$\mathcal{P}(X)$

$f : X \to 2$

Product rule

$$A = P_1 \times P_2 \times ... \times P_n$$

then

$$|A| = |P_1| \cdot |P_2| \cdot ... \cdot |P_n|$$

Product rule example

How many n-bit strings are there?

Product rule example

Let's give out 3 prizes to n people. How many ways can we do so?

What if no one can win more than one prize?

Generalized product rule

$$(x_1, x_2, ..., x_k)$$

- n_1 choices for x_1
- n_2 choices for x_2 (for each n_1)
- n_3 choices for x_3 (for each n_2)
- ...
- n_k choices for x_k (for each n_{k-1})

Then #sequences is $n_1 \cdot n_2 \cdot \cdots \cdot n_k$

Sum rule

$$|A \cup B| = |A| + |B|$$

- sets must be disjoint!

Sum rule example

You have

- 10 sick days
- 15 vacation days

How many days can you take off?

Partition

$$A = A_1 \cup A_2 \cup ...$$

Sum rule example

How many license plates can we make that have between 6 and 8 digits/characters?

Red, blue dice

Suppose we roll a red and blue die: (r, b)

How many possible outcomes are there?

Red, blue dice

Suppose we roll a red and blue die: (r, b)

How many possible outcomes are there where $r \neq b$?

Red, blue dice

Suppose we roll a red and blue die: (r, b)

How many possible outcomes are there where $r < b$?

Subsets of $\{1, ..., 3\}$

How many subsets of $\{1, ..., 99\}$ have an odd number of elements?

Difference method

$$|S| = |A| + |S - A| \Rightarrow |A| = |S| - |S - A|$$

Difference method example

How many ways can our dice have different numbers?

Difference method example

How many 10-bit binary numbers have at least two 1s?

How many 10-bit binary numbers have exactly two 1s?