Kev Sharma
Professor Ames
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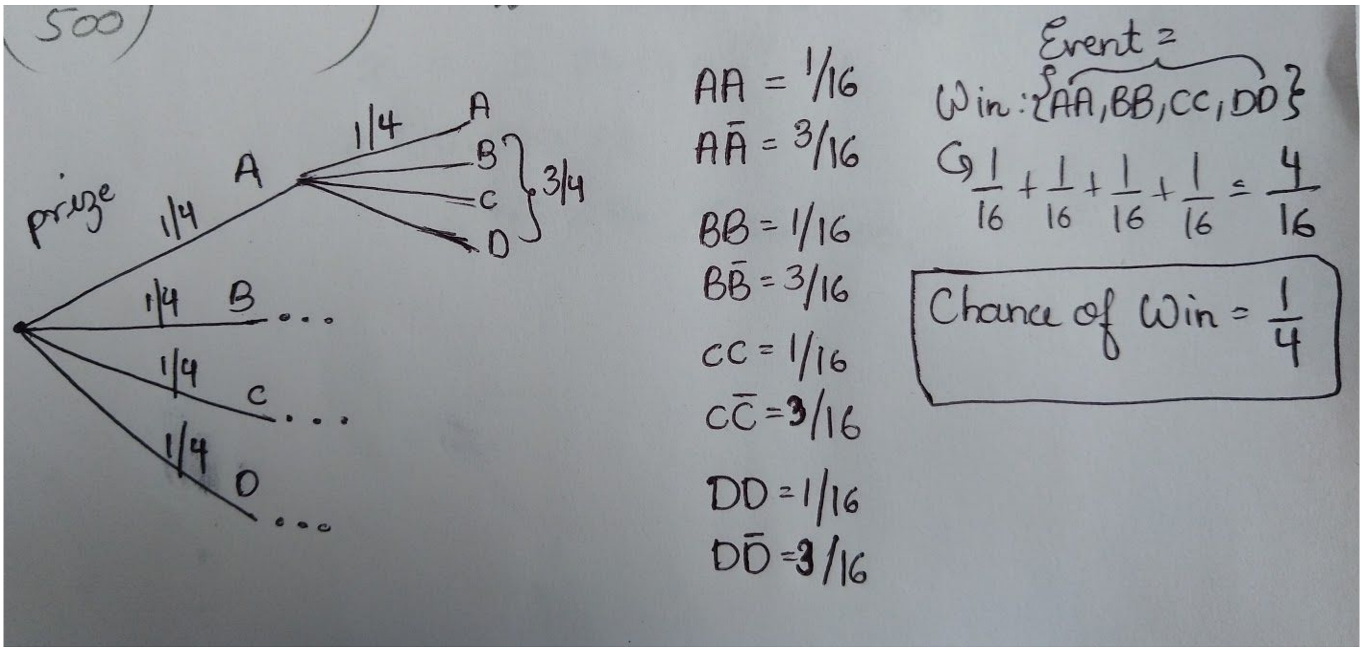
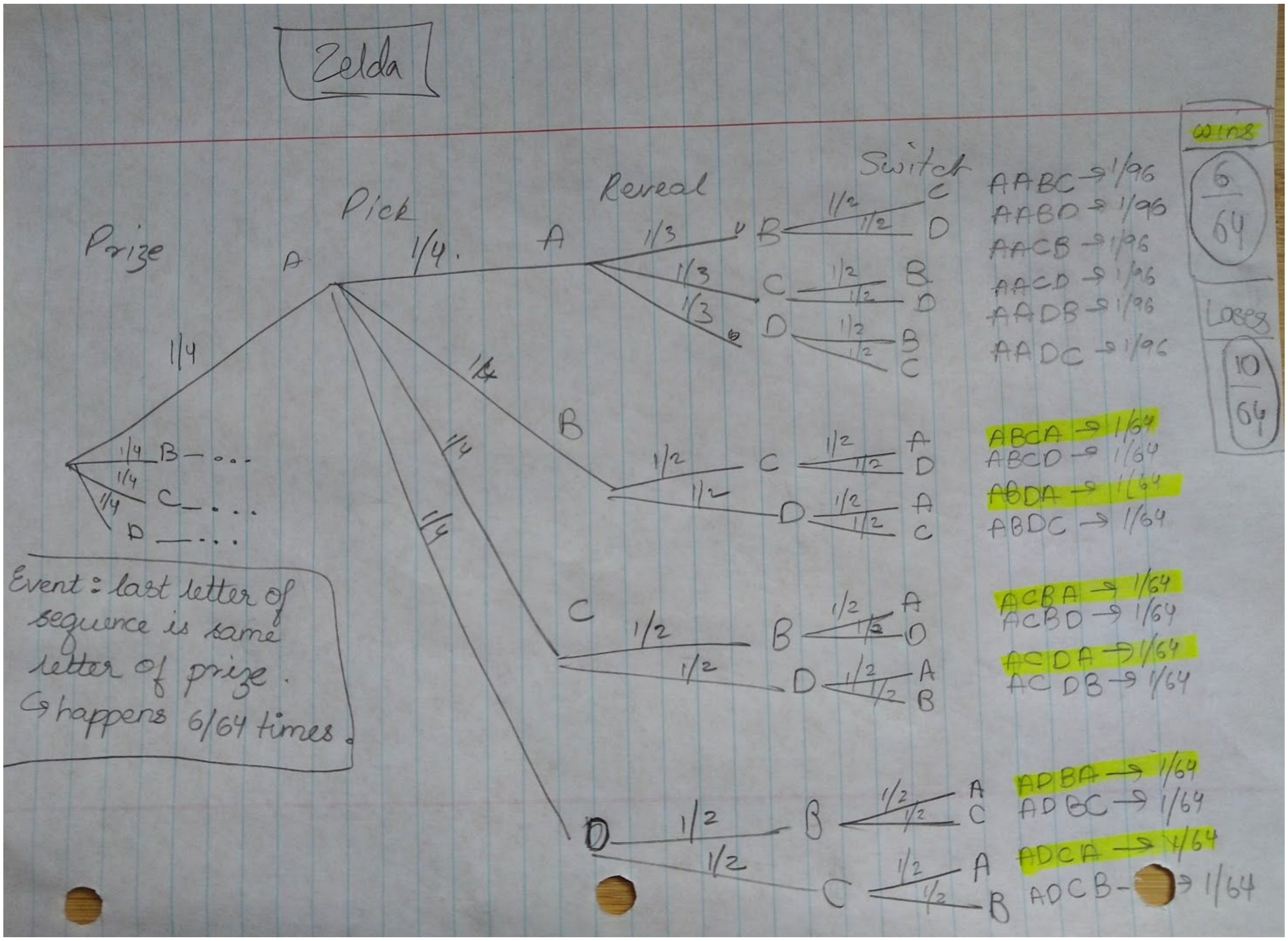
1. We have a uniform sample space (every outcome is equally likely). Let us define outcomes in the form of (<door prize is behind>,<door player picks>,<door host reveals>, <door player picks out of remaining 2>). We have four letters to represent each door : a,b,c,d.

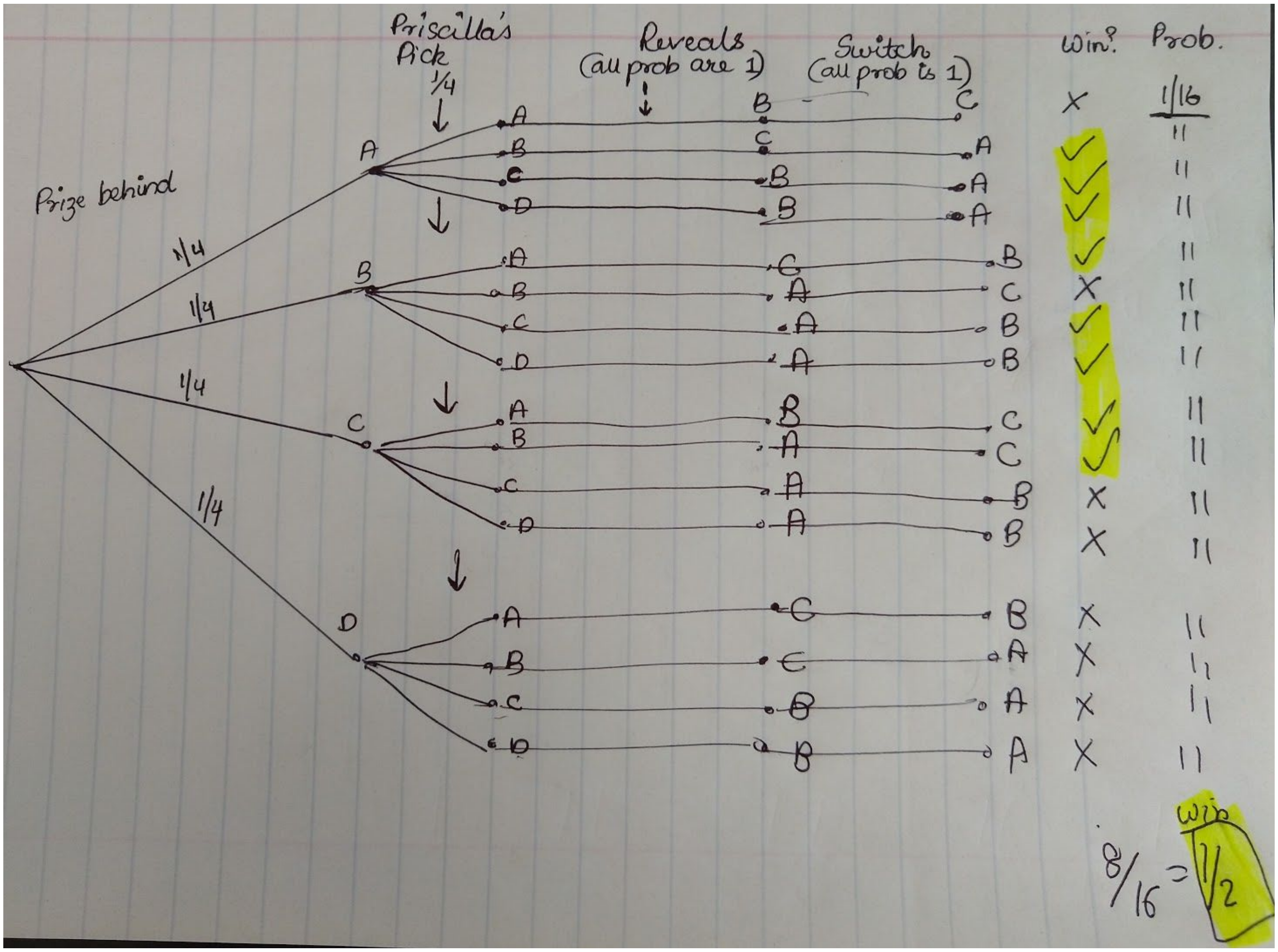
1) The probability that Stu wins is the event that the door he picks first holds the prize behind it. The probability of Stu winning is ¼.





B. Zelda wins: Let the events in consideration be all outcomes where the last letter matches the first letter. By extrapolating the same result for when the prize is behind B,C, and D, we have 6/64 times where this event occurs for each prize. The sum of these events (they are disjoint because different prize) is 6*4 / 64 = 24/64. Similarly the event of Zelda losing when prize behind a door of any given letter is 10/64. For all doors this is 10*4/64 = 40/64. We can confirm that Zelda winning + Zelda losing = 24/64 + 40/64 = 64/64 has a probability of 1. Hence for part 2, Zelda has a ⅜ probability of winning by switching to one of the remaining two doors.

C. Based on the given specifications, we know that if the prize is in the same place as the door she picked, she will lose (she always swaps). Also based on the constraints, the edges now have different probabilities because of the host picking a certain way and then Priscrilla switching to the earliest of remaining choices.

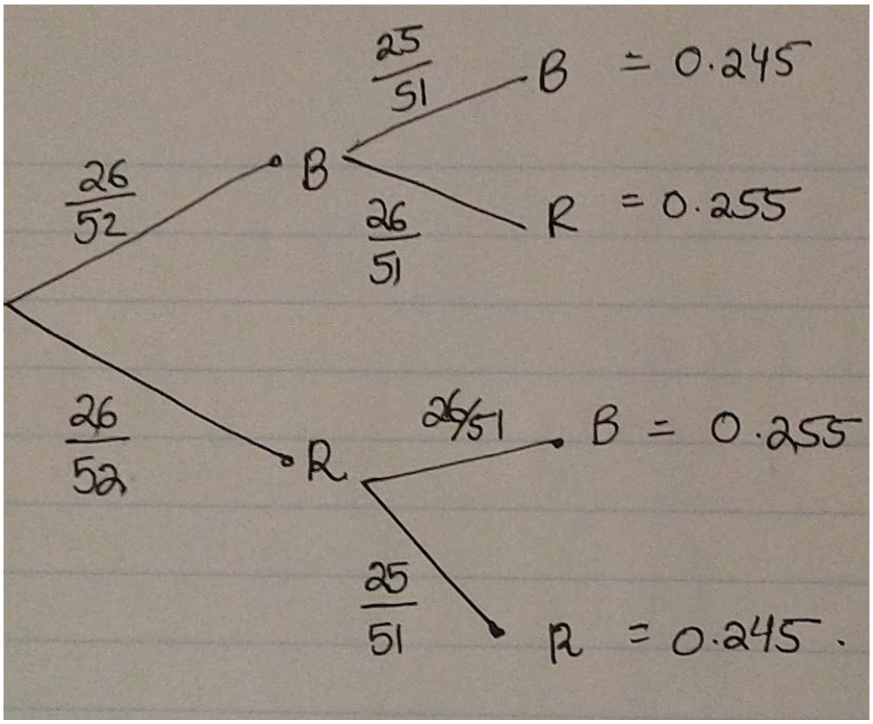


The event is that she picked correctly, there are 8 outcomes where this happens. Uniform sample space with each outcome having a probability of 1/16. She has a 8/16 = ½ probability of winning.

2. We would like to think that waiting until the deck is advantageous for us (i.e has less red cards than black) is when we ought to decide to pick the card.

However the probability that there are more black cards is conditioned on the probability that we have previously passed and the card revealed was red.

The probability of us winning is the probability that the previous card reveals puts us at an advantage AND the probability of the next card being black given the results of the previous card reveals.



The simplest advantage is when the first card is revealed to be red. Then we pick the black card thinking the probability of us winning is greater than half. This is false.

The probability of us winning is actually the probability of the first card being red AND the probability of the second card being black conditioned on the probability that the first card is red. This is really 26/51 * 26/52 = 0.255. In short, a strategy of waiting to get Red first then choosing and hoping its Black is actually 0.255 which is less than 0.5. And waiting for more Red to then eventually select black doesn't improve the odds either.

If A is the event that the first card is Red, and B is the event that the second card is Black and we pick it, then the Probability of Winning is actually P(A and B) which is P(A) * P(B conditioned upon A). Let us base our inductive proof off that:

Base case:

P(A|B) = P(A and B)/P(B).

Hence P(A and B) {winning}= P(B)*P(A|B).

P(A and B) does not exceed 0.255 for any A (black/red first card) or B (black/red second card).

Inductive Step:

P(C | (A and B)) = P(C and (A and B))/P(A and B)

P(C and (A and B)) {winning}= P(C | (A and B)) * P(A and B)

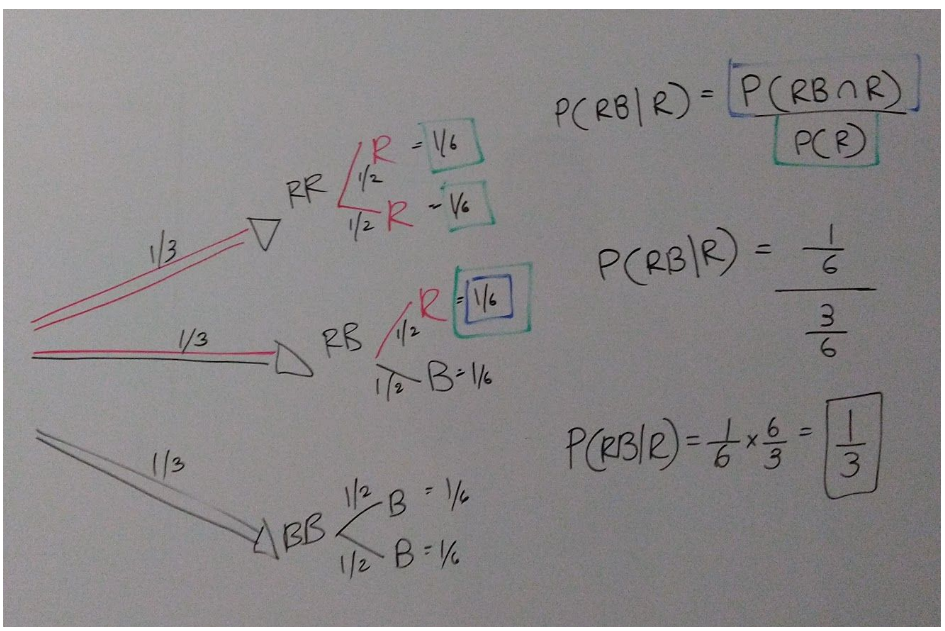
But P(A and B) for our diagram above is always less than or nearly equal to 0.255 for any given A and B. There is no probability P(C | (A and B)), thus, that when multiplied with P(A and B) results in a probability of winning **[P(C and (A and B))]** greater than 0.255 let alone greater than 0.5.

0.255 < P(C | (A and B)) * 0.255.

No value for P(C | (A and B)) satisfies this equation (not even the greatest probability of an event -> i.e 1.).

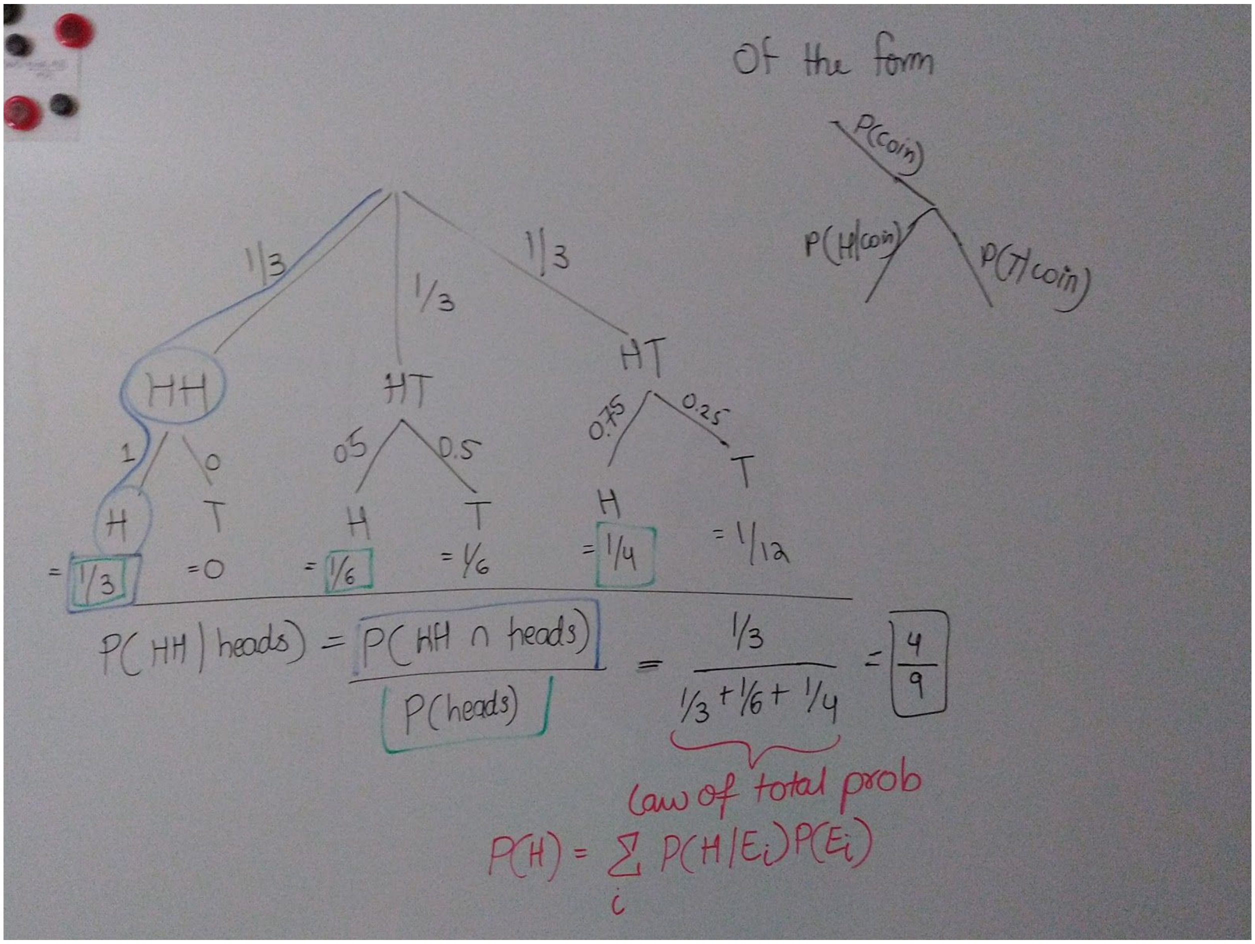
This result continues with the probability of winning decreasing and decreasing (never going above 0.5). It is best instead to simply select the first card.

3. The probability that the card is Red-Black (RB) given the side we have chosen is Red is ⅓.



There is 3 * ⅙ chance of getting Red side. There is only ⅙ chance of getting red when the card is red-black. Hence P(Red-Black | Red) = (⅙)/(3/6) = ⅓.

4.



The probability that the coin we picked had both sides Heads conditioned on the fact that flipping it gave us Heads is 4/9.

5.

<u>4/7 Chance that TA grades it.</u>
<u>2/7 Chance that Instructor grades it.</u>
<u>1/7 Chance that it gets assigned a score of 84.</u>

a) Let us consider the expected score if a TA grades the exam:

Part 1)

For each T/F Question, a student has ¾ chance of getting 2 points, ¼ chance of getting 0 pts. Therefore the expected score for each T/F question is ¾ * 2 + 0 = 1.5. For 10 such questions, the expected score is 10 * 1.5 = 15.

Part 2)

For 4 questions, two fair dice are rolled and their results summed and incremented by 3. Let R be a fair roll + 3. Then R = {1,2,3,4,5,6}. Then our outcomes are:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Adding 3 to all the blue highlights:

5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12
8	9	10	11	12	13
9	10	11	12	13	14
10	11	12	13	14	15

Expected Value of each of those outcomes (each being equally likely) =>
$[5+ 2*6 + 3*7 + 4*8 + 5*9 + 6*10 + 5*11 + 4*12 + 3*13 + 2*14 + 15]/36 = 10$
The expected score for each of the 4 questions is 10. Hence the expected score for this part is $4 * 10 = 40$.

Part 3:

$0.5 * 12 + 0.5 * 18 = 15$. The expected score for part 3 is 15.

The expected score on the exam graded by the TA is thus the sum of the expected scores on each part => $15 + 40 + 15 =$ 70 points.

b) Let us consider the score when the exam is graded by the instructor:

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

$$E(R \times R) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} +$$
$$\frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{8}{36} + \frac{10}{36} + \frac{12}{36} +$$
$$\frac{3}{36} + \frac{6}{36} + \frac{9}{36} + \frac{12}{36} + \frac{15}{36} + \frac{18}{36} +$$
$$\frac{4}{36} + \frac{8}{36} + \frac{12}{36} + \frac{16}{36} + \frac{20}{36} + \frac{24}{36} +$$
$$\frac{5}{36} + \frac{10}{36} + \frac{15}{36} + \frac{20}{36} + \frac{25}{36} + \frac{30}{36} +$$
$$\frac{6}{36} + \frac{12}{36} + \frac{18}{36} + \frac{24}{36} + \frac{30}{36} + \frac{36}{36}$$
$$= 12.25$$

4/10 (40 + 12.25) + 3/10 (50 + 12.25) + 3/10 (60 + 12.25) = 61.25.
The expected score on an exam graded by the instructor is 61.25.

c) The expected score on the final exam is the sum of the expected scores of the exams graded by the TA/instructor/randomly given 84:

E(exam) = 4/7 * E(TA) + 2/7 * E(instructor) + 1/7 * 84
E(exam) = 69.5.

The expected score on the final exam is 69.5