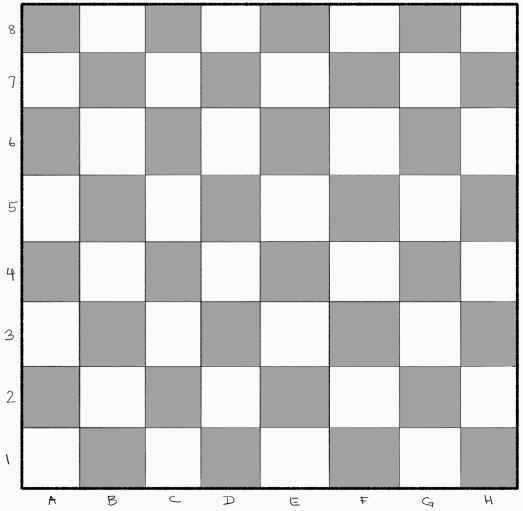
Combinations and permutations

CS 206: Discrete Structures II

Fall 2020

Chess: pawn, knight, and bishop



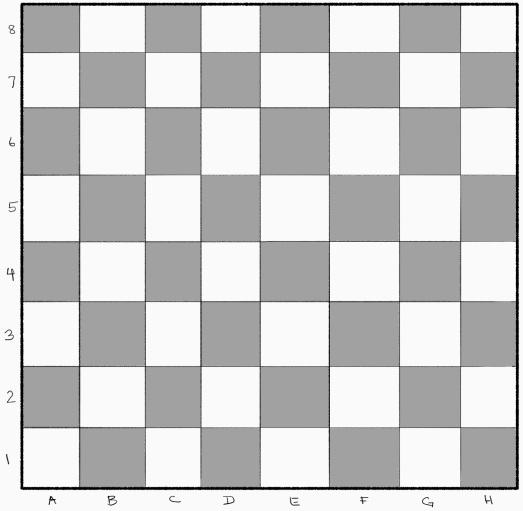
Division rule

$f : A \to B$ is k-to-1

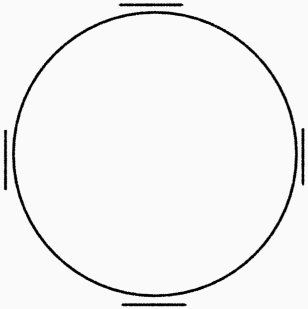
$\Rightarrow |A| = k \cdot |B|$

$\Rightarrow |B| = |A|/k$

Chess: two rooks



Round table example



Permutations

I have three meals prepared:

- tacos, enchiladas, and burritos

How many different ways can I eat breakfast, lunch, and dinner?

Permutations

A reordering of elements is called a permutation.

Given n elements, the number of permutations is

$$n! = n \cdot (n-1) \cdot \cdots \cdot 2 \cdot 1$$

Falling factorials

I have five meals prepared:

- tacos, enchiladas, burritos, pizza, and curry

How many different ways can I eat breakfast, lunch, and dinner?

k-element subsets

How many k-element subsets are there of an n-element set?

- choose 5 books from 100 books
- choose a 13-card hand from 52 cards
- choose 5 pizza toppings from 14 available

k-element subsets

How many k-element subsets are there of an n-element set?

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

k-element subsets

$$\binom{n}{0} =$$

$$\binom{n}{1} =$$

$$\binom{n}{2} =$$

$$\binom{n}{3} =$$

Example

How many 10-bit binary strings have exactly two 1s?

$$\binom{10}{2} = \frac{10 \cdot 9}{2} = 45$$

Example

How many 10-bit binary strings have exactly two 1s?

$$\binom{10}{2} = \frac{10!}{2!8!} = \frac{10 \cdot 9}{2} = 45$$

How many 10-bit binary strings have exactly eight 1s?

$$\binom{10}{8} = \frac{10!}{8!2!} = \frac{10 \cdot 9}{2} = 45$$

k-element subsets

In general,

$$\binom{n}{k} = \binom{n}{n-k}$$

Example

In a graph with 5 vertices, we connect every vertex to every other.

How many edges are there?

Combinations

From 100 apps, choose 4 to live on your phone's dock.

Does order matter?

Donut shop example

A donut shop has 12 varieties of donuts available.

We want to buy 5 donuts (order doesn't matter).

How many different orders can we make?

Donut shop example

Attempt 1:

Sequence of varieties: $(v_1, v_2, v_3, v_4, v_5)$

12 choices for each

$\Rightarrow 12^5$ possible orders

Donut shop example

Let's check with a simpler instance.

Two varieties: chocolate (c) and glazed (g)

Donut shop example

So is this a k-to-1 function?

Consider buying 5 donuts:

Donut shop example

Consider five boxes, one for each variety.

We can put our 12 donuts in any box.

Donut shop example

We can rewrite this as a binary string, using 0 for donut and 1 for divider:

Permutations and combinations

Does order matter?

Can you repeat elements?